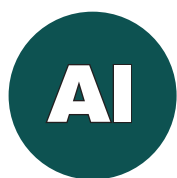


The 2018-2019 Playbook Problems

Start here to choose a math problem to solve! We have reformatted the *2018-2019 MATHCOUNTS School Handbook* exclusively for the Math Video Challenge. In this playbook, all 250 of the handbook problems are organized by math topic.

Because problems are organized by math topic, the problem numbers are not in sequential order. We've written the problem number to the left of each problem. You can find the answer key on [pgs. 36-37](#) and a complete Problem Index, including difficulty level and mapping to the Common Core State Standards, on [pgs. 38-40](#).

If your team would prefer to use the version of the handbook formatted for the Competition Series (divided into Warm-Ups, Workouts and Stretches), you can go to www.mathcounts.org/handbook to download a free copy of that version. **Here is where you can find this year's playbook problems.**



AI Algebraic Expressions
+ Equations
30 problems total
[pgs. 16-17](#)



Nt Number Theory
29 problems total
[pgs. 21-22](#)



Pr Proportional
Reasoning
20 problems total
[pgs. 30-31](#)



Cg Coordinate Geometry
16 problems total
[pgs. 18](#)



Pf Percents + Fractions
12 problems total
[pg. 23](#)



Sp Sequences, Series
+ Patterns
9 problems total
[pgs. 32](#)



Gm General Math
5 problems total
[pgs. 19](#)



Pg Plane Geometry
36 problems total
[pgs. 24-26](#)



Sg Solid Geometry
11 problems total
[pgs. 33](#)



Lo Logic
9 problems total
[pgs. 19-20](#)



Pc Probability, Counting
+ Combinatorics
30 problems total
[pgs. 26-28](#)



Sd Statistics + Data
18 problems total
[pgs. 34-35](#)



Me Measurement
17 problems total
[pgs. 20-21](#)

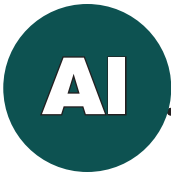


Ps Problem Solving
(Misc.)
8 problems total
[pg. 29](#)



These playbook problems are not in sequential order.

They're based on the *2018-2019 MATHCOUNTS School Handbook*, and we've put the problem number to the left of each math problem. You can find the answers on [pg. 36](#).



Algebraic Expressions + Equations

These playbook problems are not in sequential order.

Get the answers on [pg. 36](#).



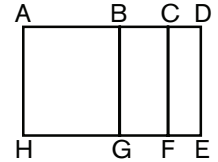
Problem

34. What is the value of $9 + 5 \times 3 - 8 \div 2$?
35. If two more than three times x is equal to five less than ten times x , what is the value of x ?
41. If the value of x is 10, what is the value of $3x + 4x + 5x$?
45. If $ab = 2c$, $bd = c$, $b \neq 0$ and $d = 16$, what is the value of a ?
49. If a and b are real numbers such that $a + b = a - b$ and $a \neq b$, what is the value of $\frac{a^2b + a + b - ab^2}{a - b}$?
55. What is the value of $27^2 - 23^2$?
62. If $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$, what is the value of x when $\begin{vmatrix} x & 4 \\ 3 & 10 \end{vmatrix} = 38$?
67. Photographers often rely on a rule to choose a shutter speed in seconds that is the reciprocal of the effective focal length of the lens in millimeters. If Jackie is shooting at a focal length of 80 mm, Clarise is shooting at 200 mm, Sage is shooting at 400 mm, and they all apply this reciprocal rule, what focal length corresponds to the sum of their shutter speeds?
76. Esther purchased two rock tumblers and one spy pen for \$74. Eli purchased two puzzles and a spy pen for \$50. Sabine purchased a rock tumbler and two puzzles for \$57. Based on this, how much does one puzzle cost?
86. If $x^2 - y^2 = 7$, $x = \frac{12}{y}$ and $y < 0$, what is the value of $x^4 + y^4$?
87. At the county fair, two hot dogs and an ice cream cone cost \$2.50. Two slices of pizza and an ice cream cone cost \$3.50. What is the total cost of one hot dog, one slice of pizza and one ice cream cone at the county fair?
94. The sum of the heights of Alex and his two brothers, Evan and Joel, is 221 inches. Alex is 8 inches taller than Evan and 5 inches taller than Joel. What is Alex's height, in inches?
96. If $f(x) = \frac{4 - x^2}{x + 2}$ and $g(x) = x - 2$, what is the value of $\frac{f(x)}{g(x)}$, for $x \neq \pm 2$?
109. If we define the binary operation ⌘ as $a \text{⌘} b = ab + b$ for all numbers a and b , what positive value of x satisfies the equation $x \text{⌘} (4 \text{⌘} x) = 550$?
111. Thomas is a basketball player who is x inches tall. One season he averaged y points per game. The sum of x and y is 98, and x is 11 more than twice y . Based on these statistics, how many points did Thomas score in the 60 games he played that season?
121. If $\textcircled{x} = 2x + 1$ and $\textcircled{x} = 2x - 1$, what is the value of $\textcircled{\textcircled{\pi}} - \textcircled{\textcircled{\pi}}$?
122. What is the sum of the two values of x for which $|x - 3| + |x - 7| = 6$?

Problem #

- 130.** Two race cars travel in opposite directions in separate lanes of a circular track. Car F takes 120 seconds to complete a lap. Car S takes 240 seconds to complete a lap. Once the two cars pass each other, how long does it take for them to pass each other again?
- 142.** To earn Gold Status on a social media site, Max needs 800 points. He earns a point for every 3 posts he reads, and he gets 15 points for each post he writes. Max can write at most 3 posts a day. He can read up to 120 posts on a weekday and 225 posts on a weekend day. If he starts with 0 points on a Monday morning, what day of the week is the day on which he can first earn Gold Status?
- 147.** If the value of a is 25 and $a - b = \frac{a}{b} - 1$, what is the sum of all possible values of b ?
- 151.** What is the coefficient of abc when the product $(a + 2b)(b + 2c)(c + 2a)$ is expanded and like terms are combined?

- 161.** Rectangles ABGH, BCFG and CDEF are adjacent, as shown, with $AH = n$, $AB = n + 3$, $BC = n - 3$ and $CD = n - 6$. If rectangle BDEG has area 110 units², what is the perimeter of rectangle ACFH?



- 175.** A certain race car consumes 1.3 gallons of fuel during each lap of a race. If each lap is 2.5 miles, and the entire race is 500 miles, how much fuel does the race car consume from start to finish?

186. If $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & k \end{vmatrix} = aek + bfg + cdh - ceg - afh - bdk$, what is the value of $\begin{vmatrix} 2 & -5 & 3 \\ 0 & 4 & -6 \\ -1 & 8 & 7 \end{vmatrix}$?

- 201.** Alice's car travels 29 miles per gallon of gas. Phil's car uses 0.02 gallon of gas per mile traveled. If gas costs \$2.50 per gallon, what is the absolute difference in the amounts Alice and Phil would each spend on the gas needed to drive 113 miles?
- 217.** The two values of x that satisfy the equation $x^2 + bx + c = 0$, for integers b and c , are $x = 6$ and $x = -4$. What is the value of b ?
- 231.** If a number x plus its reciprocal is equal to 4, what is the absolute difference between x and its reciprocal? Express your answer in simplest radical form.
- 237.** Ty flips a fair coin 20 times. What is the probability that it lands heads up and tails up equal numbers of times? Express your answer as a decimal to the nearest thousandth.
- 245.** Suppose (a, b, c) is an ordered triple such that $a^2 + b^2 = c^2$. If $b = x + y\sqrt{2}$ when $a = 12 + 10\sqrt{2}$ and $c = 15 + 8\sqrt{2}$, what is the positive integer value of $x + y$?
- 246.** Camsie has a large number of identical yellow blocks, identical green blocks and identical white blocks. Using a balance scale, she finds that 3 green blocks will balance 1 yellow plus 3 white blocks and that 5 white blocks will balance 1 green plus 5 yellow blocks. How many yellow blocks will balance 30 green plus 60 white blocks?



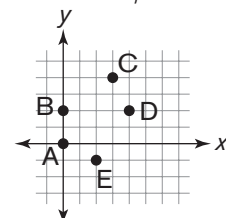
These playbook problems are not in sequential order.

Get the answers on [pg. 36](#).



Problem

- 21. A point $P(-3, 2)$ is translated right 4 units to its image P' . The point P' is then translated up 3 units to its image P'' . What is the distance from P to P'' ?
- 22. A segment has endpoints $A(0, 0)$ and $B(-3, 4)$. Point C is the image of point B translated down 4 units and left 3 units. What is the perimeter of $\triangle ABC$?
- 23. A point $Q(-3, 4)$ is reflected across the x -axis, and then the image Q' is reflected across the line $x = 2$. What are the coordinates of the image Q'' ? Express your answer as an ordered pair.
- 24. A point $S(1, 6)$ is reflected across the line $x - 2y = -6$. What is the sum of the coordinates of the image S' ?
- 25. What are the coordinates of the image of point $D(-5, -3)$ when it is rotated 90 degrees clockwise about the origin? Express your answer as an ordered pair.
- 26. What are the coordinates of the image of the point $E(3, -1)$ when it is rotated 90 degrees counterclockwise about the point $F(5, 4)$? Express your answer as an ordered pair.
- 27. A segment with endpoints $G(-2, 3)$ and $H(4, 7)$ is dilated by a scale factor of $\frac{2}{3}$ with center of dilation $(0, 0)$. What is the sum of all the coordinates of G' and H' ?
- 28. Point $J(4, 8)$ is dilated by a scale factor of $\frac{3}{2}$ with center of dilation $K(2, 2)$. What is the product of the coordinates of J' ?
- 29. A point $L(-2, 4)$ is rotated 90 degrees clockwise about the point $M(3, 2)$. Point N is the image of L' dilated by a scale factor of $\frac{3}{2}$ with center of dilation M . What is the area of $\triangle LMN$? Express your answer as a common fraction.
- 30. A point $R(-5, 3)$ is reflected across the line $y = x - 2$, and then the image R' is rotated 90 degrees clockwise about the origin. What is the distance from R to R'' ? Express your answer in simplest radical form.
- 46. All of the streets in Tom's city are on a regular rectangular grid and run east-west or north-south. Tom starts at the intersection of Poyntz Avenue and Eleventh Street and walks 2 blocks east, then 3 blocks north, then 4 blocks east, then 5 blocks north. Tom then returns to his starting location by walking 6 blocks west, then 8 blocks south. What is the area, in square blocks, enclosed by Tom's path?
- 100. On a coordinate grid, Micah constructs a pentagon with vertices $A(0, 0)$, $B(0, 2)$, $C(3, 4)$, $D(4, 2)$ and $E(2, -1)$. What is the area of pentagon $ABCDE$?
- 123. What is the area of triangle ABC , with vertices $A(6, 8)$, $B(9, 2)$ and $C(17, 6)$?
- 129. If $A(0, 0, 0)$ and $B(2, 2, 2)$ are points in coordinate space, how many paths are there from A to B that move from one lattice point to another in the positive x -, y - or z -direction?
- 148. If square $WXYZ$ has vertices $W(2, 1)$, $X(a, b)$, $Y(4, 7)$ and $Z(c, d)$, what is the value of $|ab - cd|$?
- 158. Rectangle $PQRS$ is drawn on a coordinate plane. The y -coordinate of Q is 24 units greater than the y -coordinate of S , the x -coordinate of R is 25 units greater than the x -coordinate of P , and the y -coordinate of P is equal to the y -coordinate of R . What is the perimeter of rectangle $PQRS$?





These playbook problems are not in sequential order. Get the answers on pg. 36. !

Problem #

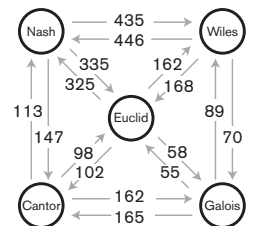
- 52. What is the value of the sum 0.49 + 0.53 + 0.55 + 0.47 + 0.48? Express your answer as a decimal to the nearest hundredth.
63. Every morning, the Sharetrain arrives in Mountain View at 9:19 a.m. It takes Miranda between 17 and 21 minutes to walk to the train station from home. If she wants to guarantee that she will arrive at the station with at least 5 minutes to spare, what is the latest time she can leave home?
69. What is the value of 1 x 12 + 2 x 11 + 3 x 10 + 4 x 9 + 5 x 8 + 6 x 7 + 7 x 6 + 8 x 5 + 9 x 4 + 10 x 3 + 11 x 2 + 12 x 1?
174. Karli paid \$3.00 for lunch every day she attended school. During a six-week period, Karli attended school every Monday through Friday, with the exception of one school holiday. What is the total amount that Karli spent on school lunches during this six-week period?
203. Three consecutive rounds of an X-game last 24 minutes, 48 minutes and 96 minutes. If the game begins at 6:00 a.m., at what time will the third round end?




These playbook problems are not in sequential order. Get the answers on pg. 36. !

Problem #

- 31. Bob has 40 cents in his pocket. If Bob has no pennies, how many different combinations of quarters, dimes and/or nickels could he have?
40. Of 1000 people surveyed, one-third of the 630 people who reported owning a cat also own a dog. If each person surveyed owns a cat, a dog or both, how many own a dog?
72. Alicia needs to make her way from Nash to Wiles. Available flights and their respective costs, in dollars, are indicated in the figure shown. If Alicia can take any sequence of flights, what is the total cost of her least expensive route from Nash to Wiles?
124. A local college has 985 students, of whom 460 play a varsity sport and 571 belong to a club. What is the absolute difference between the least and greatest possible numbers of students who play a varsity sport AND belong to a club?
137. At every lunch, three friends share their cheesy crackers with each other but agree not to eat them until they all have the same number. At the first lunch, Axel gives Ben and Chloe as many crackers as they each already have. At the second lunch, Ben gives Axel and Chloe as many crackers as they each already have. At the third lunch, Chloe gives Axel and Ben as many crackers as they each already have, after which they each have 8 crackers, so they eat them. How many crackers did Axel start with?
153. The vertices of a cube are labeled with the integers 1 through 8, in some order, with each integer used exactly once. Each edge of the cube is labeled with the product of the integers at its two endpoints. Each face of the cube is labeled with the sum of the labels of its four edges. What is the greatest possible absolute difference between the labels of two adjacent faces of the cube?



Problem #

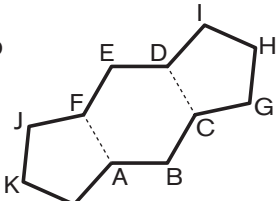
171. A rectangular sheet of paper is cut in half perpendicular to the longer side. One half is discarded, and the other half is cut into thirds as shown. Two of the thirds are discarded, and the remaining third is cut into fourths by vertical lines. Three of the fourths are discarded, and the remaining fourth is cut into fifths by horizontal lines. Four of the fifths are discarded, and the remaining fifth is cut into sixths by vertical lines. Five of the sixths are discarded, leaving a square piece with side length 2 cm. What is the perimeter of the original sheet of paper?
- 
200. Vikram attempted to calculate 67×58 , but he entered exactly one digit incorrectly, resulting in a product of 3596. What is the sum of the two values he actually multiplied?
221. Ronald is attempting to cast a levitation spell. On his first attempt, he has a 20% chance of success. Ronald's likelihood of success improves by 5% after each failed attempt. For example, his second attempt has a 25% chance of success if he fails on the first attempt. What is the minimum number of spells that he must attempt to guarantee a successful casting?



Measurement

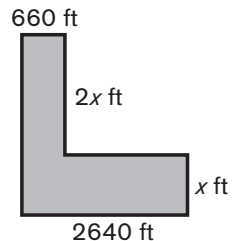
These playbook problems are **not** in sequential order. 
Get the answers on [pg. 36](#).

Problem #

1. Merri places weights of 6 units and 28 units on the right side of a balance and weights of 3 units and 19 units on the left side. If she adds an object to the left side that makes the balance level, how many units does the object weigh?
2. The weight of a small clip is $\frac{2}{3}$ the weight of a large clip. If 2 tacks weigh the same amount as a large clip, how many tacks weigh the same amount as 12 small clips?
3. On the planet Klem, 1 Bem plus 7 Dems equals 4 Pems, and 2 Bems plus 1 Dem equals 1 Pem. How many Dems equal 7 Bems?
4. If a race car is traveling at 99 mi/h, how many meters does it travel in a second, given that 0.305 meter = 1 foot? Express your answer as a decimal to the nearest tenth.
5. If the results when reading a measuring stick can be off by at most 1 cm, what is the maximum percent error when 24 cm is measured? Express your answer to the nearest tenth.
6. Vijay gives Sanjay a set of four weights of 1, 3, 8 and 26 grams. When Sanjay places weights on either side of a balance, what is the smallest positive integer number of grams that he **cannot** measure with this set?
7. If Clem has 2 cups, 7 pints, 8 quarts and 11 half-gallons of lemonade, how many total gallons of lemonade does she have? Express your answer as a mixed number.
8. If 2 Blams equal 15 Droms and 5 Droms equal 28 Klegs, how many Klegs are in a Blam?
9. What is the ratio of 1 ounce to 1000 grams, given that 1 pound equals 454 grams? Express your answer as a decimal to the nearest thousandth.
10. If one order of fries and five burgers cost twice as much as three orders of fries and two burgers, how many times as much does a burger cost compared to one order of fries?
42. In the figure, two regular pentagons have been attached to a regular hexagon to create a 12-gon. If each regular polygon has side length 3 feet, what is the perimeter of the resulting 12-gon?
- 

Problem #

- 61. What is the degree measure of the complement to an angle that is a supplement to an angle of measure 163 degrees?
- 81. When Jim was 10 years old, he was 4 feet 8 inches tall. When he was 20 years old, Jim was 6 feet 2 inches tall. By how many inches did Jim's height increase in those 10 years?
- 97. Freddie Frog traveled 29 yards in 261 jumps, at a constant distance per jump. After hurting a toe, her distance per jump decreased by 1 inch. How many jumps more than 261 will it take for her to travel another 29 yards but with a hurt toe?
- 102. Coordinated Universal Time (UTC) is 5 hours later than Eastern Standard Time (EST). If Nico and Jon start playing a tennis match in London at 8:00 a.m. UTC and finish at 12:15 p.m. EST, for how many minutes did they play?
- 112. Pierre needs 39 minutes to paint a painting. The painting will be dry 55 minutes after Pierre finishes. If Pierre starts painting at 1:00 p.m., what is the earliest time when he can have three finished, dry paintings?
- 248. Farmer John's field has area 60 acres. The field is in the shape of an "L", with right angles at every vertex and dimensions as shown. If one acre is equal in area to a strip of land 40 rods long and 4 rods wide, and one rod is equal in length to 16.5 feet, what is value of x in feet?



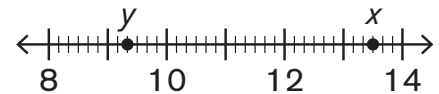
Number Theory

These playbook problems are **not in sequential order.** !

Get the answers on [pg. 36](#).

Problem #

- 32. On the number line shown, what is the value of $x - y$? Express your answer as a mixed number.
- 37. What is the average of the prime numbers between 20 and 30?
- 39. What is the quotient when 1,000,000,000 is divided by $2^8 \times 5^7$?
- 44. How many divisors of 64 are perfect squares?
- 59. When twenty-one-and-a-half trillion is written in scientific notation, what is the exponent needed on the base of 10?
- 64. Ryan picks two different numbers from the set $\{2, 3, 5, 7\}$ and multiplies them. What is the absolute difference between the greatest and the least products that Ryan can get?
- 73. Ancient Egyptians used the symbols shown in Figure 1 to express certain quantities by placing the symbols in any order. What quantity is represented by the symbols in Figure 2?



I	∩	⊙	⋮	∪
Staff	Heel	Rope	Lotus	Finger
1	10	100	1000	10,000



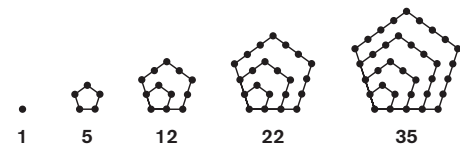
Figure 2

Figure 1

- 74. Yixin thinks of a positive number that is a perfect cube, and Aly thinks of a number that is a perfect square. If the sum of their numbers is 31, what is the product of their numbers?
- 79. If $a = 2 + \frac{3}{4 + \frac{5}{6}}$ and $b = 6 + \frac{5}{4 + \frac{3}{2}}$, what is the value of $\frac{a}{b}$? Express your answer as a common fraction.

Problem #

- 88.** When the positive integer divisors of 385 are arranged from least to greatest, what is the sum of the 4th, 5th and 6th divisors?
- 98.** On the first day of class, Smiley's teacher gave her a piece of paper containing Smiley's seven-digit locker combination. Smiley noticed that the seven-digit number was a multiple of 11. When she went to her locker, she saw that one digit on the paper had been completely smudged, as shown. Smiley had forgotten the smudged digit. What was Smiley's seven-digit locker combination?
- 105.** Liz, Eva and Ace have played trivia as a team 33 times and won 24 times. What is the minimum number of games they must win to have an overall winning percentage of 80%?
- 107.** What is the geometric mean of the median and mode of the set {23, 25, 3, 25, 20, 22, 21, 2, 1, 14, 12}? Express your answer in simplest radical form.
- 113.** Blackbeard and Redbeard take a total of 60 pounds of gold from Treasure Cove. Because Blackbeard outranks Redbeard, Blackbeard gets $\frac{5}{8}$ of the gold and Redbeard gets the remaining $\frac{3}{8}$. How many more pounds of gold does Blackbeard get than Redbeard?
- 114.** A pentagonal number is a number that can be represented as the number of dots on a regular pentagon as in these figures, which show the first five pentagonal numbers. What is the 20th pentagonal number?
- 117.** If the sum of three numbers is equal to their product, and two of the numbers are $\frac{4}{3}$ and 1, what is the third number?
- 127.** How many positive integer divisors of 23,328 are perfect cubes?
- 131.** What is the greatest common divisor of 2563 and 4147?
- 134.** A *prime date* is a date for which the number of the month and the number of the day are both prime numbers. How many prime dates are there in the month of March?
- 145.** Segment AC is a diameter of circle P, which has radius 13 cm. If B is a point on circle P such that AB = 10 cm, what is the length of segment BC?
- 157.** What is the 15th digit to the right of the decimal point in the decimal expansion of $\frac{3}{7}$?
- 173.** Yeong multiplies two-digit positive integers AB and CD. If the digits A, B, C and D are all distinct, what is the greatest possible value of the product?
- 183.** If $x @ y = \text{LCM}(x, y)$ and $x \# y = \text{GCD}(x, y)$, what is the value of $((2 @ 7)^2 \# 42) \# 105$?
- 184.** On a digital clock that shows hours and minutes, for how many minutes, during a single day, between 4:00 a.m. and 6:00 a.m. will the sum of the digits be 10?
- 188.** How many subsets of {1, 2, 3, 4, 6, 8, 10, 15} are there for which the sum of the elements is 15?
- 197.** If $20! = 2^a \times b$ for some integer b , what is the greatest possible value of a ?
- 225.** If A, B and C are the digits of the base-five number ABC, which is equal to the base-ten number 47, what digit does B represent?
- 228.** What is the least positive odd integer with exactly nine natural number divisors?
- 243.** If N is a perfect square and a divisor of $13!$, what is the greatest possible value of N ?





These playbook problems are not in sequential order.

Get the answers on [pg. 36](#).



Problem

47. What is the value of $\frac{\frac{1}{2} + \frac{1}{3}}{\frac{1}{4} - \frac{1}{5}}$? Express your answer as a common fraction.
50. Given that 75% of a certain number is 88, what is $\frac{3}{8}$ of the number?
51. All the jelly beans in Sammi's cup are red, yellow or orange. Five are red and seven are orange. If two-thirds are yellow, how many jelly beans are there in Sammi's cup?
65. Zu's zoo offers a promotional deal: get a free \$3 cotton candy and a free \$2 soda with the purchase of five \$12 admission tickets. A group of 20 students will each purchase an admission ticket. If half of them want cotton candy and one-fourth of them want soda, how many dollars would they save by using the promotional deal?
101. Don's signature coffee blend is 60% dark roast and 40% light roast. He has 10 kg of blend A, which is 80% dark roast and 20% light roast, and 10 kg of blend B, which is 20% dark roast and 80% light roast. How many kilograms of blend A will Don need to use to make 10 kg of his signature blend? Express your answer as a mixed number.
143. If $\frac{a}{3} = \frac{4}{\frac{a}{3}}$, what is the value of a^2 ?
198. At a certain middle school, the 6th-grade class of 390 increased by 10%, the 7th-grade class of 350 increased by 22% and the 8th-grade class of 420 increased by 20%. Overall, what was the percentage increase of students for this middle school? Express your answer to the nearest tenth.
205. How many ordered pairs of positive integers (a, b) satisfy the equation $\frac{2}{a} - \frac{3}{b} = \frac{1}{5}$?
209. On his 15th birthday, Bo invests \$10,000 in a bank that pays 7.5% interest, compounded annually. At this rate and assuming Bo makes no deposits or withdrawals, how old will Bo be on the first birthday when his account balance is at least \$30,000?
219. At Randolph's furniture store, a standard breakfast table has a price of \$1000. Karthik wants to purchase the luxury version of the table, which costs 35% more than the standard one, and he has a coupon that gives him a 15% discount on his purchase. After applying 6% sales tax to the discounted price, how much will Karthik pay for the luxury table?
241. Scott thinks that Miguel is 6 feet 3 inches tall. If Miguel's height is actually 96% of Scott's estimate, how many inches tall is Miguel?
247. A box contains only quarters and dimes. If there were 10% more quarters, there would be 7.5% more money in the box. What is the ratio of the original number of quarters to the original number of dimes in the box? Express your answer as a common fraction.

These playbook problems are **not** in sequential order.

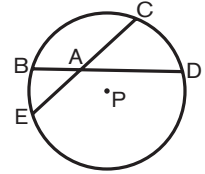
Get the answers on [pg. 36-37](#).



Problem #

38. How many lines of symmetry does an isosceles right triangle have?

56. In circle P, two chords intersect at A, as shown, with $AB = 3$ units, $AC = 8$ units and $AD = 6$ units. What is the value of AE ? Express your answer as a mixed number.



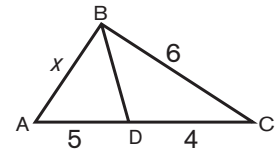
58. The bus from Kevin's home to the middle school travels 8 miles west, then turns and travels 8 miles north, then turns and travels 7 miles west to arrive at school. If the bus were able to travel directly from Kevin's house to the middle school along a straight path, how much shorter would the trip be?

60. If a polygon has 17 sides, how many diagonals does it have?

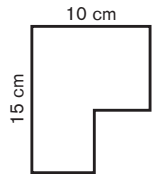
68. Hexagons A and B are geometrically similar. The shortest sides of the two hexagons are 4 inches and 3 inches, respectively. If the area of hexagon A is 48 in^2 , what is the area of hexagon B?

70. What is the measure of an interior angle of a regular polygon with 90 sides?

77. In triangle ABC, shown here, segment BD bisects angle ABC. If $CD = 4$ cm, $AD = 5$ cm and $BC = 6$ cm, what is the value of AB ? Express your answer as a common fraction.

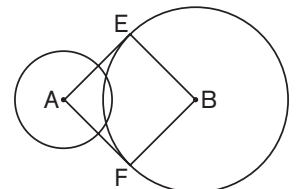


78. In the figure shown, two sides have lengths 10 cm and 15 cm. If all line segments intersect at right angles, what is the perimeter of the figure?



80. If the difference between the measures of an interior angle and an exterior angle of a regular polygon is 100 degrees, how many sides does the polygon have?

82. Circles A and B intersect, and points E and F are on circle B as shown. If quadrilateral AEBF is a square, what is the probability that a randomly chosen line through point A intersects both circles? Express your answer as a common fraction.

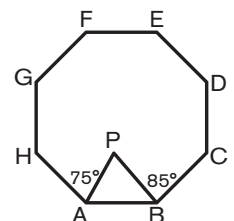


84. Alana draws four squares, five pentagons and six octagons. Marie draws n hexagons and notices that the combined number of diagonals in her hexagons is equal to the combined number of diagonals in Alana's polygons. What is the value of n ?

90. Lines are drawn from the center of a 72-gon to two of its vertices. The smaller central angle formed by these lines includes seven sides of the polygon. What is the degree measure of this central angle?

100. A circle with center P is inscribed in isosceles triangle ABC with apex angle A measuring 34 degrees. What is the degree measure of angle APC? Express your answer as a decimal to the nearest tenth.

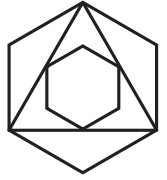
120. Triangle ABP is drawn inside a regular octagon as shown. What is the degree measure of acute angle P if $m\angle HAP = 75$ degrees and $m\angle CBP = 85$ degrees?



125. What is the smallest possible area of an isosceles triangle with side lengths 5 inches and 6 inches? Express your answer as a decimal to the nearest tenth.

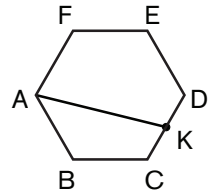
Problem #

126. An equilateral triangle is inscribed in a regular hexagon, and a smaller regular hexagon is inscribed inside the triangle so that three of its vertices are each the midpoint of a side of the triangle, as shown. What is the ratio of the area of the smaller hexagon to that of the larger hexagon? Express your answer as a common fraction.



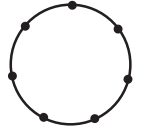
133. Regular pentagon ABCDE has diagonal BE. If P is the midpoint of side ED, what is the degree measure of angle EBP?

136. Regular hexagon ABCDEF has side length 4 cm. Point K is the midpoint of side CD. What is the length of AK? Express your answer in simplest radical form.



144. Segment AC is a diameter of circle P, which has radius 13 cm. If B is a point on circle P such that AB = 10 cm, what is the length of segment BC?

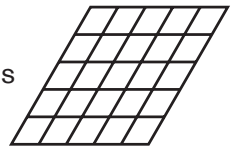
146. Seven points are equally spaced around a unit circle. How many non-congruent convex pentagons can be drawn by using a subset of the points as vertices of the pentagons?



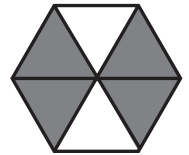
150. If a rhombus with area 26 cm² has one diagonal of length 4 cm, what is the length of its other diagonal?

154. What is the length of the hypotenuse of a right triangle with legs of length $12\sqrt{3}$ units and $23\sqrt{3}$ units? Express your answer in simplest radical form.

166. How many rhombi of any size are in this figure, composed of equally spaced horizontal lines and parallel diagonal lines that are also equally spaced?



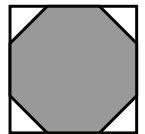
180. Jessie makes butterfly wings by shading four sections of a regular hexagon as shown. If the hexagon has side length $4\sqrt{3}$ units, what is the area of the shaded region? Express your answer in simplest radical form.



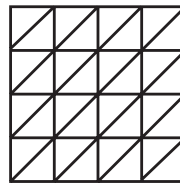
181. What is the area of a triangle with sides of length 16, 30 and 34?

187. A certain trapezoid has these properties: its diagonals are congruent and perpendicular to each other, and its longer base length is equal to the length of a diagonal. What is the sum of the degree measures of the two acute angles of this trapezoid?

189. The figure shows a shaded regular octagon inscribed in a square. What percent of the figure is shaded? Express your answer to the nearest whole number.

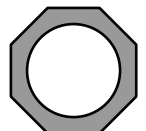


193. How many triangles of any size are in the figure shown here?



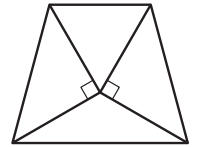
196. What is the ratio of the area of the largest square that can fit inside a circle of diameter 2 cm compared to the area of the largest square that can fit inside a semicircle of diameter 2 cm? Express your answer as a common fraction.

211. A regular octagon with side length 4 cm is concentric with a circle, as shown. If the area of the circle is equal to the area of the shaded region between the shapes, what is the radius of the circle? Express your answer as a decimal to the nearest tenth.



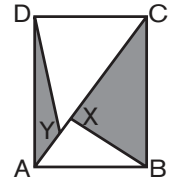
Problem #

220. Tarin and Sonja construct the trapezoid shown, which contains an equilateral triangle and three isosceles triangles, two of which are right triangles. What is the ratio of the length of the shorter base to the length of the longer base of the trapezoid? Express your answer as a common fraction in simplest radical form.

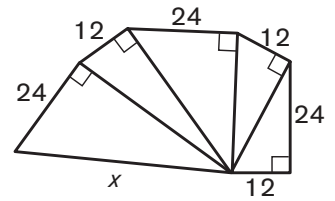


230. What is the area of a triangle with side lengths of 4, 6 and 8 inches? Express your answer in simplest radical form.

233. Rectangle ABCD is shown here with $AB = 6$ and $BC = 8$. If X and Y lie on diagonal AC , and $XY = 1$, what is the total area of shaded triangles BCX and DAY ? Express your answer as a decimal to the nearest tenth.

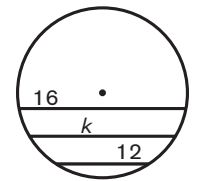


236. Five right triangles are constructed adjacent to each other as shown. All five triangles share a vertex. The longer leg of each triangle, except the rightmost triangle, is the hypotenuse of an adjacent triangle. What is the length of the longest hypotenuse, labeled x ? Express your answer in simplest radical form.




240. Two right triangles have areas in the ratio 1:2. If the smaller triangle has legs of length 3 units and 4 units, and the larger triangle has a side of length 8 units, what is the sum of the possible lengths of the hypotenuse of the larger triangle? Express your answer in simplest radical form.

250. Three parallel chords of lengths 12, k and 16 units are in a circle, as shown. If the chord of length k is equally spaced 2 units away from each of the other chords, what is the value of k ? Express your answer in simplest radical form.



Probability, Counting + Combinatorics

These playbook problems are **not** in sequential order.

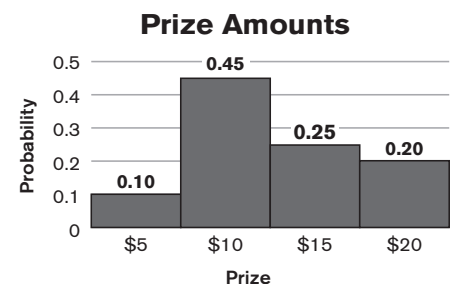
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Problem #

11. An unfair six-sided die with faces labeled 1, 2, 3, 5, 8 and 13 is rolled. The table lists the probability of the die landing with each number showing on the top face. The expected value of the roll is the sum of the products of each face value and its corresponding probability of being rolled. What is the expected value when the die is rolled? Express your answer as a mixed number.

Top Face Value	Probability
1	$\frac{1}{3}$
2	$\frac{1}{15}$
3	$\frac{1}{6}$
5	$\frac{1}{5}$
8	$\frac{2}{15}$
13	$\frac{1}{10}$

12. Terry plays a game with prizes of 5, 10, 15 and 20 dollars. The graph shows each possible prize amount and its corresponding probability. The expected value of her prize is the sum of the products of each prize and the probability of winning that prize. What is the expected value of Terry's prize?



13. A fair 10-sided die with one face labeled 1, two faces labeled 2, three faces labeled 3 and four faces labeled 4 is rolled. What is the expected value when this die is rolled?

Problem #

- 14.** Ana has a bowl containing two square tiles, one with side length 2 cm and the other with side length 3 cm. She randomly chooses a tile from the bowl. The expected value of the area of the chosen tile is the sum of the products of each tile's area and its corresponding probability of being chosen. If the probability of choosing a particular tile is proportional to its area, what is the expected value of the area of the tile Ana chooses? Express your answer as a common fraction.
- 15.** For the dartboard shown, the number of points scored when a dart lands in each region is indicated. The innermost circle of the board has radius 1 inch, and each subsequent circle has a radius 2 inches greater than the previous circle. Kane throws a dart that lands randomly somewhere on the board. What is the expected value of the number of points he scores? Express your answer as a decimal to the nearest tenth.
- 
- 16.** Gwen randomly draws a card from a deck of 40 cards numbered 1 through 40. What is the expected value of the number on the card she draws? Express your answer as a decimal to the nearest tenth.
- 17.** Luke paints each face of a $5 \times 5 \times 5$ cube red. He then cuts the cube into 125 unit cubes and randomly chooses a single unit cube. What is the expected value of the number of painted faces on this unit cube? Express your answer as a decimal to the nearest tenth.
- 18.** In each round of a particular game, Dinara can win at most one point. If she has a 70% chance of winning a point in each round, what is the expected value of Dinara's total score after three rounds? Express your answer as a decimal to the nearest tenth.
- 19.** Jo and her four friends each secretly pick a random integer from -5 to 5 , inclusive. What is the expected value of the sum of the five chosen numbers?
- 20.** Allen randomly distributes 1000 jelly beans into 10 jars lined up in a row from left to right. What is the expected value of the number of jelly beans in the leftmost jar?
- 38.** Ted flips a coin that is equally likely to land heads up or tails up. Ted flips the coin 10 times, and each time it lands heads up. What is the probability that the next flip will also land heads up? Express your answer as a common fraction.
- 53.** If an outfit consists of jeans, a shirt, a sweater and a scarf, how many different outfits can Allie make from one pair of jeans, four shirts, three sweaters and two scarves?
- 83.** Lauren randomly picks two cards, without replacement, from a group of six cards numbered 2, 3, 5, 6, 7 and 10. What is the probability that the product of the two numbers on the selected cards is a multiple of 10? Express your answer as a common fraction.
- 99.** How many ways are there to put twelve identical donuts into three differently colored boxes if there must be at least two donuts in each box?
- 103.** While working on a history project, Erika writes down a list of years starting with 1809 and ending with 2019. How many times did Erika write the digit 1?
- 104.** What is the probability that a sequence of five flips of a fair coin will not land heads up twice in a row? Express your answer as a common fraction.
- 106.** A certain vending machine accepts nickels, dimes, quarters and dollar bills, and it provides change using nickels, dimes and quarters. If Sarah selects a healthful snack priced at \$1.30 after inserting \$2.00, in how many ways can the vending machine provide Sarah's change?
- 119.** Cayley has a fair six-sided die whose faces are numbered -2 , -1 , 0 , 0 , 1 and 2 . She rolls the die three times. What is the probability that the sum of the three numbers she rolls is 0? Express your answer as a common fraction.

Problem #

- 138.** Paul and his friends play a version of tag football in which a team earns 5 points for every touchdown and 3 points for every field goal. If Paul's team has a total of 25 points, how many different sequences of field goals and touchdowns could the team have scored?
- 141.** Exactly 512 small cubes perfectly fill a lidless cubical box. All of the small cubes are removed except those touching the bottom of the box and those touching the sides of the box. How many of the small cubes were removed?
- 152.** Jenny has a bag that contains equal numbers of red, green, yellow, orange and purple candies. She likes only the red candies. If she randomly selects a candy from her bag and discards any non-red candy she selects, then she is guaranteed to select a red candy within a minimum of 45 random selections. What is the total number of candies in her bag?
- 157.** Ilana rolls a fair 8-sided die with faces labeled 1 through 8, and Yolanda rolls a fair 12-sided die with faces labeled 1 through 12. What is the probability that Ilana rolls a smaller number than Yolanda? Express your answer as a common fraction.
- 164.** Burt has a set of tiles spelling BANANAS. Ernie randomly chooses a set of five tiles from Burt's set. What is the probability that Ernie can create a palindrome using all five tiles? Express your answer as a common fraction.
- 169.** Two real numbers from 0 to 10, inclusive, are to be chosen at random. What is the probability that the absolute difference between the two numbers will be less than 3? Express your answer as a common fraction.
- 170.** How many permutations of the digits 1, 2, 3 and 4 can be arranged in a line, so that no adjacent digits differ by more than two?
- 210.** How many different two-letter permutations can be made from two distinct letters in the word MATHCOUNTS?
- 213.** Blake has a repertoire of 15 songs, of which 5 are acoustic. In his concert, he plans to perform 10 distinct songs, of which 4 will be acoustic. He wants to perform the 4 acoustic songs consecutively in the second half of the concert but doesn't want to end the concert with an acoustic song. How many different sequences of songs can Blake perform?
- 214.** Each time Chris bats, he has a 37% probability of getting a hit, independent of his previous performance at bat. What is his percent probability of getting at least two hits in four times at bat? Express your answer to the nearest whole number.
- 223.** A cheerleading squad consists of ten cheerleaders of ten different heights. How many ways are there for the cheerleaders to line up for a photo in two rows of five people each so that each cheerleader in the back row is taller than the one immediately in front?
- 238.** Ty flips a fair coin 20 times. What is the probability that it lands heads up and tails up equal numbers of times? Express your answer as a decimal to the nearest thousandth.



These playbook problems are not in sequential order.

Get the answers on [pg. 37](#).



Problem

- 57.** Six people are invited to attend a five-day conference scheduled for Monday through Friday. Grace will attend only on Monday and Thursday. Becca will attend every day except Monday. Carmen will attend every day except Thursday. Davis will attend only on Thursday. Ernie will only attend either Thursday or Friday. Frank will attend every day except Monday and Wednesday. On how many days during the conference will at least three of these six people be in attendance?
- 91.** Two days ago, Neil was assigned a set of problems to solve. Today he solved the final three. Yesterday he solved half of those then remaining plus half a problem. The first day he solved half of those assigned plus half a problem. How many problems were in the assigned set?
- 92.** A deck of 14 cards numbered 1 through 14 is dealt to Ken and Gunther so that each gets 7 cards. Each player's score is the sum of his card values, and the player with the lower score wins. How many different winning scores are possible?
- 152.** The larger of two right triangles has area numerically equal to 3 times the sum of its integer leg lengths, a and b . The other right triangle has area numerically equal to 2 times the sum of its integer leg lengths, c and d . If the triangles have areas in the ratio 3:2, what is the value of $a + b + c + d$?
- 216.** An eight-digit number N has one 2, three 3s, two 4s and two 5s as digits. If N can be expressed as a power of 2, what is the value of N ?
- 232.** For making house numbers, 5-inch digits can be purchased online for \$8.79 each for the digits 1, 3, 5 and 6 and \$5.98 each for the digits 0, 2, 4, 7, 8 and 9. Ashera wishes to create consecutive house numbers beginning with 5100 and ending with 5200. What is the total cost to purchase the necessary digits online, excluding tax and shipping?
- 234.** Fran has four dogs whose ages are 3, 4, 5 and 6 years. She feeds them one at a time according to the rule that a younger dog can never eat right before a dog that is only one year older. In how many different orders can Fran feed her four dogs?
- 244.** When 64 unit cubes are glued together seamlessly to form a $4 \times 4 \times 4$ cube, as shown, there are 76 distinct lines that pass through the centers of exactly four unit cubes. If 1000 unit cubes are glued together seamlessly to form a $10 \times 10 \times 10$ cube, how many distinct lines pass through the centers of ten unit cubes?

These playbook problems are not in sequential order.

Get the answers on [pg. 37](#).

**Problem #**

- 43.** A speed of 60 miles per hour is equal to 88 feet per second. If the speed limit in a school zone is 15 miles per hour, what is the speed limit in this zone in feet per second?
- 54.** Destiny can run one-eighth of a mile in three-quarters of a minute, and she can walk one-quarter of a mile in 4 minutes. How many minutes total will it take for Destiny to run one mile and then walk one mile?
- 71.** For each lawn Kayla mows, she charges a flat fee of \$15, plus an amount proportional to the area of the lawn. If Kayla charges the owner of a 20-foot by 40-foot lawn \$24, how much would she charge the owner of a 60-foot by 80-foot lawn?
- 89.** Lines are drawn from the center of a 72-gon to two of its vertices. The smaller central angle formed by these lines includes seven sides of the polygon. What is the degree measure of this central angle?
- 135.** Bennie and Flossie are traveling on a straight road, going in the same direction and starting at the same place. Bennie is traveling at an average speed of 62 mi/h, and Flossie is traveling at an average speed of 75 mi/h. After 2 hours, Flossie stops for a 20-minute break before continuing. After a total of 5 hours, what is the distance between them?
- 155.** It takes a crew of eight people three days to paint an aircraft. At this rate, how many people are needed to paint five aircraft in six days?
- 165.** Screen sizes of rectangular television sets are measured on the diagonal. If Joe owns a television with a 62-inch screen, and Daniel owns a geometrically similar television with a 31-inch screen, what is the ratio of the area of Daniel's screen to the area of Joe's screen? Express your answer as a common fraction.
- 172.** In 1969, the Apollo 10 mission set the record for the fastest crewed space travel, at 39,897 km/h. At that speed, how many days would it take to travel 54.6 million km, the minimum distance from Earth to Mars? Express your answer to the nearest whole number.
- 177.** One television screen measures 56 inches long and 33 inches wide. A smaller, geometrically similar television screen measures 48 inches long. What is the width of the smaller screen? Express your answer as a decimal to the nearest tenth.
- 178.** Lindsay starts at the peak of a mountain, and it takes her 50 minutes to hike 15,000 feet. What was her average walking speed, in miles per hour, given 1 mile = 5280 feet? Express your answer as a decimal to the nearest tenth.
- 185.** The density of an object is the mass of the object per unit volume. The density of water is 1 gram/cm³, and the density of ice is 91.67% that of water. If 1 kilogram of water completely freezes, what is the volume, in cubic centimeters, of the resulting ice block? Express your answer to the nearest whole number.

Problem #

- 190.** Spike can dig 8 holes in 3 hours. Butch can dig 7 holes in 4 hours. Lucky can dig 6 holes in 5 hours. How many minutes will it take them to dig 3 holes if all three work together? Express your answer to the nearest whole number.
- 194.** Rex reads a novel at a constant rate while Wren writes a report at a constant rate. Rex finishes page 313 of his novel at the same time Wren finishes page 5 of her report. Rex later finishes page 409 of his novel when Wren finishes page 9 of her report. What page of his novel will Rex finish when Wren finishes page 12 of her report?
- 205.** R.J.'s pedometer indicates that he has walked 10,002 steps and equates that to traveling 4.11 miles. Based on this, how many additional steps must he walk to travel the equivalent of 5 miles total? Express your answer to the nearest whole number.
- 207.** A liter of fuel cost £1.33. At that rate, what would be the cost, in U.S. currency, of 1 gallon of fuel, given that 1 gallon \approx 3.78541 liters and US\$1.30 \approx £1.00?
- 215.** The Bobcats had a win-loss record of 41-23 before their star player, Melissa, got injured. Melissa didn't play in any more games, and the Bobcats finished the season with an overall win-loss record of 54-34. What is the absolute difference in the Bobcats' win percentages with and without Melissa? Express your answer to the nearest whole number.
- 224.** The math club purchased a number of gadgets to sell. Each box of gadgets cost the club \$75 and contained 30 gadgets. In the first 15 days, the club members sold 22 gadgets each day at the regular price of \$6.80 each. In the next 10 days, they sold 17 gadgets per day at a discount of 25% off. They then sold the remaining 220 gadgets at 30% off the discounted price. After all the gadgets were sold, what was the math club's total profit?
- 227.** In his first 319 career games, goaltender Braden Holtby had 200 wins and 119 losses. If he were to maintain this rate of winning games, what is the least number of games he would need to play in his career to pass Martin Brodeur's career record of 691 wins?
- 229.** Noa completes $5\frac{1}{3}$ laps around a track in the same amount of time it takes Lev to complete 6 laps. Lev is running at a pace of 7.5 minutes per mile. How many seconds longer will it take Noa to run 1 mile, compared to Lev? Express your answer to the nearest whole number.
- 239.** One hundred years ago, a newly minted Walking Liberty Half Dollar contained 0.36 troy ounce of silver, a Standing Liberty Quarter contained 90% silver (by mass) and a Mercury Dime was made from the same alloy as the other two coins. If the mass of each coin was proportional to its face value, what was the total mass of a Mercury Dime? Express your answer as a decimal to the nearest hundredth.



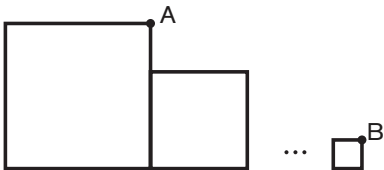
Sequences, Series + Patterns

These playbook problems are not in sequential order.

Get the answers on [pg. 37](#).



Problem

- 93.** What is the units digit of the sum $1! + 2! + 3! + \dots + n!$ when $n = 2019$?
- 132.** Hongyi is making a triangular display of widgets. The bottom layer will contain n widgets. The layer immediately above it will contain $n - 1$ widgets. Each subsequent layer will contain one fewer widget than the previous layer, and the top layer will contain one widget. If widgets are packed 25 per case, and Hongyi must use all of the widgets in every case he opens, what is the smallest positive number of widgets Hongyi can use to build the display?
- 139.** Five numbers form an arithmetic sequence with a mean of 18. If the mean of the squares of the five numbers is 374, what is the greatest of the five original numbers?
- 155.** In a particular sequence, the ratio between consecutive terms remains constant. If the 2nd term is 2, and the 5th term is 5, what is the value of the 11th term of the sequence? Express your answer as a common fraction.
- 160.** If the coefficients c and d of $x^3 - 19x^2 + cx + d = 0$ are chosen so that the roots of the equation are positive integers that form a geometric sequence, what is the value of $c + d$?
- 195.** The Fibonacci sequence is defined by the function $F(n) = F(n - 2) + F(n - 1)$, for $n > 2$ and $F(1) = F(2) = 1$. What is the value of $F(11)$?
- 222.** In a sequence of adjacent squares, a portion of which is shown, the side length of each successive square is two-thirds that of the preceding square. In the completed sequence, the areas of the largest and smallest squares are 6561 units² and 256 units², respectively. What is the length of the segment with endpoints A and B? Express your answer to the nearest whole number.
- 
- 235.** What is the arithmetic mean of the terms of a five-term geometric sequence of positive numbers whose second term is 15 and whose fourth term is 135?
- 249.** Notice that 2019 can be written as the sum of consecutive integers in increasing order in numerous ways, for example $1009 + 1010 = 2019$ and $(-2018) + (-2017) + \dots + 2018 + 2019 = 2019$. Including these two examples, in how many ways can 2019 be written as the sum of at least two consecutive integers in increasing order?

These playbook problems are **not in sequential order**.

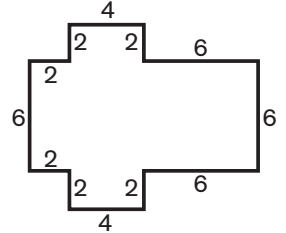
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Problem #

36. What is the volume of a rectangular prism of height 5 cm, width 7 cm and depth 3 cm?

116. The figure shown can be folded into a rectangular prism. What is the absolute difference between the numerical values of the surface area and volume of the prism?



153. The dimensions of a packing box are 15 inches by 13 inches by 10 inches. Every edge of the box is to be secured with packing tape. If the length of tape along each edge is equal to the edge length, how many feet of tape are needed? Express your answer as a mixed number.

182. How many gallons are in the volume of a cylinder with height 12 inches and diameter 14 inches if 1 gallon = 231 in³? Express your answer to the nearest whole number.

191. Let q be the sum of the lengths of all 12 edges of a cube, and let A be the total surface area of the cube. If $q \times A$ is k times the volume of the cube, what is the value of k ?

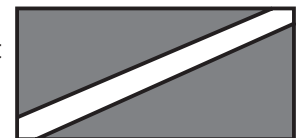
192. Alta's favorite sphere has six times as much surface area compared to her favorite cylinder, which is right circular. If the height of her favorite cylinder is equal to its diameter, what is the ratio of the volume of Alta's favorite cylinder to the volume of her favorite sphere? Express your answer as a common fraction.

199. A 6-8-10 right triangle is rotated about the side of length 10 units. What is the volume of the resulting solid? Express your answer as a decimal to the nearest tenth.

202. A cheese cube is sliced diagonally from one edge to another, as shown. One piece of the cheese is melted and then reformed into a perfect sphere. What is the ratio of the side length of the original cheese cube to the radius of the cheese sphere? Express your answer as a decimal to the nearest hundredth.

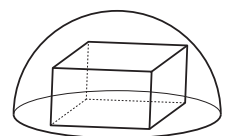


208. A 10-meter by 20-meter garden is going to have a concrete walkway constructed diagonally through the middle as shown. The longest sides of the walkway each start 1 meter from the closest corner of the garden. If the walkway will have a uniform depth of 10 cm and the concrete costs \$70 per cubic meter, what will be the cost of the concrete used for the walkway?



212. Jonas has a $1\frac{1}{2}$ -inch by 4-inch by 3-inch rectangular block of tofu weighing 16 ounces. He cuts the block into smaller pieces, each of which is a triangular prism having a right triangle base with legs of length $1\frac{1}{3}$ inch and $\frac{3}{4}$ inch. If each tofu prism has height $\frac{1}{2}$ inch, what is the weight of one tofu prism? Express your answer as a common fraction.

242. A hemisphere of radius 6 inches lies flat on its circular base, and a rectangular prism with two square bases lies inside the hemisphere, with one square base on the base of the hemisphere and all four vertices of the other square base touching the upper boundary of the hemisphere. The height of the prism is one-third of the length of the square base. What is the volume of the prism? Express your answer as a decimal to the nearest tenth.





These playbook problems are **not** in sequential order. 
Get the answers on [pg. 37](#).

Problem #

- 48. What is the arithmetic mean of $\frac{2}{3}$, $\frac{7}{9}$, $\frac{1}{4}$ and $\frac{5}{16}$? Express your answer as a common fraction.

- 65. Preston has four potatoes, each of which weighs a whole number of ounces. The median weight of his potatoes is 11 ounces, and the mean weight of his potatoes is 12 ounces. What is the greatest possible difference between the weight of the heaviest and lightest of his potatoes?

- 73. Jalacia erases some integers from the following list: 5, 9, 2, 3, 7, 8, 6, 5, 6, 4, 1, 7, 9, 8. When she is done, the range of the remaining list is 2. What is the least possible number of integers that Jalacia could have erased?

- 85. Consider the set of all four-digit positive integers less than 2000 whose digits have a sum of 24. What is the median of this set of integers?

- 95. If all test scores are integers from 0 to 100, inclusive, what is the least possible median of five test scores that add to 204?

- 108. What is the mean of all three-digit positive integers whose digits are in the set {2, 0, 1, 9}?

- 115. What is the arithmetic mean of all 3-digit numbers whose digits are distinct and nonzero?

- 118. Alvin plays mini-golf with four friends. The median and unique mode of all their scores are both 10, and the range of their scores is 16. If the mean of all their scores is 15, what is the sum of all possible values of the lowest score?

- 128. Saila took nine exams, each scored out of 100 points. She received a passing score of 60 or above on each of them. If the mean of her nine scores was 80 points, what is the greatest possible value of the median of her nine scores?

- 140. An oceanographer tags every dolphin that she sees over a one-week period. At the end of the week, she has tagged 1000 dolphins. One month later, when she returns and examines 2000 dolphins, she finds that 400 of them are ones that she tagged. Assuming that each time, she saw a random subset of the local dolphin population, what is the expected number of dolphins in the local population?

Problem #

- 149.** The ten guppies in Vida's tank are 5, 5, 8, 8, 8, 9, 10, 11, 11 and 33 weeks old. A guppy of age n weeks is added to her tank. The sum of the mean, median and unique mode of the eleven guppies' ages equals the range. What is the sum of all possible integer values of n ?
- 159.** Hannah ran five races, each of which was a different positive integer number of miles. The mean of her distances was 4 miles. What is the maximum possible distance of her longest race?
- 168.** If 6, a , b , c , d , e , 34 is a list of seven different positive integers written in increasing numerical order such that c is both the mean and the median of the list, what is the least possible value for c ?
- 175.** What is the sum of the mean, median, mode and range of the numbers 15, 33, 24, 10, 20 and 24?
- 179.** Let a be the arithmetic mean of 3.27 and 17.95. Let b be the product of 32.7 and 0.4382. Let c be the quotient of 2.637 and 0.316. Let d be the absolute difference between 793.241 and 804.3692. What is the numerical value of the median of a , b , c and d ? Express your answer as a decimal to the nearest hundredth.

- 204.** Ms. Santa asks the 25 students in her math class to measure their heights in meters. The students' heights are recorded in the stem-and-leaf plot shown here, where 1.2 | 1 represents 1.21 meters. What is the median height of the students in Ms. Santa's class? Express your answer as a decimal to the nearest hundredth.

1.2	1 3 6 6
1.3	2 4 5 5 7 9
1.4	0 1 4 8
1.5	0 3 4 6 8 8 9
1.6	0 5 7
1.7	1

- 216.** On a test, Walter scored 86, Bev scored 72, Claire scored 61 and Rachel scored 93. Tyler didn't remember his score, but he knew that his score was both the mean and the median of all five students' scores. What was Tyler's score?

- 226.** The figure, which shows half of the cross section of a pool, can be rotated 360 degrees around the dotted line to create a three-dimensional pool with depth 10 feet in the center and 5 feet at the outer edge. What is the average depth of the pool? Express your answer as a decimal to the nearest tenth.

