The 47th Problem of Euclid

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Abstract

The 47th Problem of Euclid is a Masonic emblem, that teaches Masons to be general lovers of the arts and sciences. But there is far more to the 47th Problem of Euclid than a general reminder. The problem solves the Pythagorean Theorem, the greatest scientific discovery of antiquity. The ancients believed that the mental process involved in solving this problem would train the geometer for higher levels of consciousness. This paper will establish its place in Masonic ritual, explore the nature of the problem, its solution, and what consequences its discovery has had on the history of human thought, technology, and Freemasonry.

Contents

Introduction

The purpose of this paper is to establish that the 47th Problem of Euclid, as an emblem of Masonry, is a fit subject for Masonic study, give some understanding of the background of the problem, provide its solution, and show its importance in the history of ideas, more especially in Speculative Masonry. In the first section, its usage in the Preston-Webb version of the ritual will be demonstrated, along with some consideration as to Preston's intention in its inclusion. This is necessary in order to show why the reader should take a deeper look at this problem, and what it represents, not only to the ancient world, but to Operative and Speculative Masons.

Next, after having done this, in the second section some background of Pythagoras and his spiritual philosophy, the relationship that he and his followers had with mathematics in general, and geometry in particular, and the central role that the Pythagorean Theorem had to his belief system. To Pythagoras, this problem was far more than a mathematical discovery, but a profound spiritual truth, and the process of proving the result was in itself a spiritual discipline intended to bring the geometer closer to deeper understanding of the nature of the universe, and man's role in it.

In the third section, a demonstration of the problem and its solution will be provided in some detail. This is necessary in order to guide the reader through the process that the ancient mind would have undergone in order to come to solve the problem, in order to demonstrate firsthand the salutary effect this process has on the mind of the practitioner. Having done this, the paper will show why this knowledge was so highly esteemed in the ancient world, and how such knowledge applies to Speculative Masonry, and why this Emblem is sometimes used as a symbol of the Worshipful Master of a Lodge.

1 The 47th Problem of Euclid in the Preston-Webb Lectures

The 47th Problem of Euclid is an emblem of Freemasonry due to its place in the Preston-Webb lecture. Brothers William Preston and Thomas Smith Webb wrote much of our current ritual. Some of the ritual is secret, and other parts were published openly by them. This paper will only consider those portions which were openly published.

The paragraph about the 47th Problem of Euclid, written by Preston and shaped by Webb, that is part of our ritual, contains many truths, and a few mistakes, but is a masterful piece of prose designed to inspire the candidate to be "general lovers of the arts and sciences." It was Preston's intention that the lectures, and in particular, the inclusion of the 47th Problem of Eucid, would provide intellectual and moral instruction to the Masonic candidate, and by learning these lectures in order to deliver them, this instruction would be reinforced in the Masonic officers. Preston intended to use this method to educate Masons, especially those like himself who were deprived of a formal education.

1.1 William Preston

In his remarkable essay, *The Philosophy of Masonry,* Harvard Law Professor Roscoe Pound wrote about Preston's use of his lectures:

> In Preston's day, there was a general need, from which Preston had suffered, of popular education—of providing the means whereby the common man could acquire knowledge in general. . . Preston's mistakes were the mistakes of his century—the mistake of faith in the finality of what was known to that era, and the mistake of regarding correct formal presentation as the one sound method of instruction.[1]

Preston was orphaned before he could be fully educated, losing his patronage and leaving school at 12. He then became a printer's apprentice, working for the king's printer at 18. He was made a Mason at 23, and at 25 was Worshipful Master of his lodge. At 30, he became the Grand Lecturer of the Grand Lodge of England (Moderns), and in 1774, at the age of 32, his system of lectures and rituals was complete. He strove all of his life for a complete knowledge of Freemasonry, and a very broad general knowledge of the arts and sciences. It was his intention to have his lectures instruct the new Mason in general knowledge. The current Fellowcraft lecture is an abridgement, by Webb, of the lecture Preston wrote. It included, in its original version, a lengthy disquisition on each of the seven Liberal Arts, each of the five orders of Architecture, and much moral instruction.

Not all of the factual content of his lectures was accurate[1]. He often took liberties in order to provide a new teaching point. For example, it was not Pythagoras, but Archimedes, who exclaimed "Eureka!" when he discovered the principle of hydrostatics when testing the purity of the gold in the crown of King Hiero II[2]. He noticed that water rises in the bathtub depending on the density of the item placed in it, and this enabled him to tell if the crown was pure gold, or gold mixed with lead. He was so astonished by his discovery that he leapt out of the bathtub, and ran naked through the streets, shouting "Eureka!"

1.2 Thomas Smith Webb

Thomas Smith Webb was one of the most important early influences on Freemasonry in the USA. A Past Grand Master of Rhode Island, Webb is considered to be the founder of the York Rite. He was the first Eminent Commander of the Knights Templar, and wrote the ritual of their degrees. In 1797, he edited and published the Preston lectures, with quite a bit of his own additions, as *Freemason's Monitor, or Illustrations of Masonry.* Concerning the 47th Problem of Euclid, Webb wrote:

The Forty-Seventh Problem of Euclid was an invention of our ancient friend and Brother, the great Pythagoras, who, in his travels through Asia, Africa and Europe, was initiated into several orders of Priesthood, and raised to the sublime degree of a master mason. This wise philosopher enriched his mind abundantly in a general knowledge of things, and more especially in geometry, or masonry: on this subject he drew out many problems and theorems; and among the most distinguished, he erected this, which, in the joy of his heart, he called *Eureka,* in the Grecian language, signifying, *I have found it*; and upon the discovery of which, he is said to have sacrificed a hecatomb. It teaches masons to be general lovers of the arts and sciences. [3]

> ***The Forty-ſeventh Problem of Euclid.****
>
> **This was an invention of our ancient friend and brother, the great Pythagoras, who, in his travels through Aſia, Africa and Europe, was initiated into ſeveral orders of prieſthood, and raiſed to the ſublime degree of a maſter maſon. This wiſe philoſopher enriched his mind abundantly in a general knowledge of things, and more eſpecially in geometry or maſonry: on this ſubject he drew out many problems and theorems; and among the moſt diſtinguiſhed, he erected this, which, in the joy of his heart, he called *Eureka*, in the Grecian language, ſignifying, *I have found it*; and upon the diſcovery of which, he is ſaid to have ſacrificed a hecatomb. It teaches maſons to be general lovers of the arts and ſciences.**

Webb's *Monitor* has had enormous influence on Freemasonry as practiced in the USA. Indeed, it provides the basis of the majority of the degree work done in almost every Grand Lodge jurisdiction in the USA. While the original words are not the same as the words we currently use, it does provide a strong basis for the current ritual.

2 Who was Pythagoras?

Pythagoras of Samos, born c. 580 BCE, died c. 490 BCE, was a mathematician and religious leader. He founded a religion, afterwards known as Pythagoreanism, in which everything is reduced to number. There are many conflicting legends about him, especially because his religious movement was heavily persecuted, and was dispersed many times without committing its beliefs to writing.

This paper will be using the narrative of Iamblichus[4][1] as a central reference. One of his students, Archytas, made his way to Athens, and was the tutor of Plato, who incorporated the ideas of the Pythagoreans into his philosophy. Pythagoras was the first man to call himself a *philosopher.*

2.1 The life of Pythagoras

While born on the island of Samos, Pythagoras was forced to leave in his youth due to his conflict with the tyrant Polycrates. Under the influence of the great Thales of Miletus, Pythagoras was encouraged to travel to Egypt to study with the priests of Memphis. He also studied in the temples of Tyre[2] and Byblos in Phoenicia. Upon his return, he studied the Orphic Mysteries in Croton, in Sicily before forming his own religious system. He built a cultural center in Croton, and formed a two-level society with secrets, forming both an outer circle and an inner circle, with different rituals and beliefs for each. The colony at Croton was betrayed by a nobleman, Cylon, and destroyed by Dionysius II, Tyrant of Syracuse,[3]and Pythagoras fled to Metapontum, on the Italian mainland, where he later died, around 90 years old, under unknown circumstances.

2.2 The Pythagorean Belief System

The Pythagoreans were divided into an outer circle and an inner circle. The outer circle, the *akousmatikoi,* or listeners, were students from the neighboring area who were allowed to study at the school at Croton, but were not called upon to change their diet or relinquish their private property. The inner circle, the *mathematikoi,* or learners, lived communally, with no private possessions. They were strict vegetarians, and were initiated into the religious and mathematical mysteries of the order. The purpose of this two-tiered system was to protect the secrets of the order while still allowing new people to be recruited. The *akousmatikoi* could study philosophy and mathematics, and if a particular student showed promise, they would be invited to join the *mathematikoi* and learn the secrets of the order.

2.2.1 Pythagorean morality

The Pythagoreans were very concerned with the morality of its members, and of society as a whole. They observed strict moral codes, that often were cryptic

[1] Iamblichus of Chalcis, c. 245–c.325 AD, was a Neoplatonist philosopher, who wrote the most comprehensive treatise on the Pythagorean religion.

[2] Flavius Josephus believed that Pythagoras studied the Jewish religion. In *Against Apion* (c. 97 AD), he notes that some of the ideas in the Pythagorean religion have Jewish origins.

[3] Incidentally, the Tyrant of Syracuse had courtier named Damocles, who flattered him excessively, saying that Dionysius was the most fortunate person in Syracuse, and that anyone would want to trade places with him. The next day, Dionysius offered to switch places with Damocles. Damocles sat on the throne for a sumptuous banquet, at the end of which, he noticed that directly above him, affixed to the ceiling by a single horsehair, was a sharpened sword. Damocles is said to have lost his taste for fine foods and banquets. This is where the expression *a Sword of Damocles over one's head* comes from.

allegories. The *akousmatikoi* were forbidden to look upon Pythagoras, who taught them from behind a veil. This taught them that they had to develop themselves before they could fully behold him, and that what they were learning was veiled allegory of the truth. They were pacifists and vegetarians. Their central rule was of silence, called *echemythia:* if a person was in doubt as to what to say, they should always remain silent. Violations of *echemythia* could be punished with death.

They were encouraged to help their fellow lift a burden, but were forbidden with helping them lay their burden down, as they regarded encouraging indolence as a great sin. Pythagoreans were both men and women, and after Pythagoras died, his wife Theano became the leader of the group.

They were regarded as being excessively superstitious. They would not eat beans, because they believed in the transmigration of souls, and thought that the flatulence that beans cause were trapped souls of the dead.[4] They were forbidden from walking over a cross-bar, poking a fire with a sword, or eating an animal's heart (a strange rule for vegetarians). Aristotle, in his *Metaphysics,* believed that these rules were symbolic: the *akousmatikoi* were ordered to follow these rules, but the *mathematikoi* were given an explanation of their moral symbolism. Stepping over a cross-bar is being covetous; poking a fire with a sword is vexing a man with words who is already swollen with anger, eating a heart is vexing oneself with grief[5].

Incidentally, Iamblichus takes objection to the legend, which existed in his time, that Pythagoras sacrificed a *hecatomb*[5] to celebrate his discovery of the Pythagorean Theorem. Pythagoras was a vegetarian who objected morally to animal sacrifices. "He sacrificed to the gods millet and honeycomb, but not animals. [Again] He forbade his disciples to sacrifice oxen[4]."

2.2.2 Mathematics

The Pythagoreans believed that number was the basis of the universe. The seven liberal arts are divided into the Trivium,[6] consisting of Grammar, Rhetoric and Logic; and the Quadrivium. The Quadrivium consisted of Arithmetic, Geometry, Music and Astronomy. The Pythagorean idea was that arithmetic was number, geometry was number in space, music was number in time, and astronomy was number in space and time. The Pythagoreans made great discoveries in number theory, geometry, and music. Pythagoras discovered that two harp strings, of equal tautness, with one twice the length of the other, will make a musical octave. He invented the Pythagorean tuning, where each string is a perfect fifth relative to the string before it. He coined the phrase "The Music of the Spheres", and believed that the planets had the same relationships with each other and with the sun that musical notes do with each other and with the tonic.

[4] In Greek, πνευμα means wind, spirit, and can refer to *flatus.*

[5] One hundred head of oxen.

[6] Where we get the word *trivial.*

Pythagoras discovered a proof that the square root of two is not a rational number.[7] As it turned out, this violated his religious belief system, since here was a value without a number associated with it. As a result, he taught the proof to the *mathematikoi,* but made them swear on pain of death not to reveal the proof to outsiders. A legend tells that one of his followers was on a ship during a bad storm, and he believed they would drown, so he showed the proof to the others on the ship. Another passenger was a Pythagorean as well, and when the ship passed safely through the storm, he threw the first man overboard, drowning him for betraying the secrets of the order.

3 The Pythagorean Theorem

The proof that Pythagoras used to prove his theorem is not known to us. The proof that Euclid gives as the 47th Proposition of Book I of his *Elements* is almost certainly not the same proof. Euclid makes a number of pedagogical choices in his presentation that would not be necessary if the only purpose were to prove the theorem. This paper will explore the proof that Euclid gives. The result was known to the Babylonians, Egyptians, Chinese, and many others, but the Greeks were the first to prove the result. Whereas mathematics everywhere else was an empirical science, the Greeks were concerned with proof. Geometry was a means at arriving at truth, using logic.

This is an important distinction. To the Greek mathematician, the application of their work was secondary to the mental discipline of proof. Starting with a few unproven basic assumptions and common notions, collectively known as *axioms,* the Greek mathematician would define and construct a myriad of mathematical objects, using pr oven theorems, corollaries and lemmas to achieve their result. The whole construction was valued, from the selection of the proper axioms to choice of which theorems to prove in what order. While other mathematical cultures were more interested in the application,[8] the Greeks believed that the method was even more important, because it trained the mind, and allowed mortals to approach absolute truth.

Pythagoras felt that proof had a spiritual purpose, and that the mental rigors of axiomatic thought were morally edifying. Plato, echoing that sentiment, put a sign outside his Academy, reading "Geometers only." He felt that a thorough study of geometry was a prerequisite for any serious philosophical student.

3.1 Euclid's treatment of the problem

Euclid of Alexandria taught at the *Musaeum* at Alexandria, an institute of science and philosophy that included the Great Library. In his youth, he had

[7] That is, there is no fraction $\frac{a}{b}$ such that $\sqrt{2} = \frac{a}{b}$.

[8] The Arabic mathematicians who invented algebra, from *al-jabr,* or bone-setting, played fast and loose with logical rigor. They were able to achieve a lot of results very rapidly, but by doing so, introduced structural flaws into the study of algebra that were not repaired until the early 20th century.

been a student at Plato's Academy. He wrote the *Elements* around 300 BCE. It was designed to be an elementary[9] treatment of the entire field of mathematics known in his day. He wrote many other books, including works on astronomy, optics, catoptrics, logic and fallacies, several works on mechanics, and an advanced treatise on conic sections. Most of his writings do not survive, falling victim to the fire at the Library at Alexandria. Some fragments of his books survive today only in Arabic. The Caliph Haroun Al-Rashid, 763–809 AD, declared Euclid a Muslim saint for his contributions to human thought and culture, and worked to preserve as much of Euclid's writings as he could.

3.1.1 The *Elements*

The *Elements* are a series of thirteen books designed to teach from scratch the basics of every mathematical discipline then known, using an axiomatic approach. The *Elements* begins with five common notions and five postulates. Using only these ten unproven statements or axioms, he builds, using definitions and theorems, a vast edifice. The first book was designed to prove the Pythagorean Theorem and its converse as quickly and succinctly as possible. The second book expounds upon the corollaries of that theorem. The third and forth books cover circles and their properties, inscribed triangles, and regular polygons. The next five books explore ratio and proportion, creating a geometrical algebra, leading up to the treatment of irrational numbers, and a primitive form of integral calculus, called "the method of exhaustion." The last three books are about spatial geometry. They analyze cones, cylinders, spheres, and finally, give a proof that there are five and only five Platonic solids.

The *Elements* was designed to be a textbook, and the method he uses is intended to be pedagogical. It was his intention that the thirteen books would prepare the reader for his more advanced works, an intention that later mathematical writers shared. For millenia afterwards, the *Elements* were the prerequisite for any advanced mathematical study. The works of Apollonius of Perga, Archimedes, Pappus, even Kepler, Descartes and Newton, assumed that the reader had studied the *Elements* very thoroughly, and had mastered its propositions.

3.1.2 Euclid's Proof of the Pythagorean Theorem

Euclid tries to prove the Pythagorean Theorem using as few axioms, definitions and theorems as possible. A series of propositions in Book I builds his argument, culminating in the 47th Proposition of Book I, which states and proves the Pythagorean Theorem. Because of this, he will call a rectangle a parallelogram, because he doesn't want to waste time proving that it is a rectangle. Every line of Book I is designed to move the argument forward. The 48th Proposition proves the converse, that if the measure of the area of the square of the longest

[9] Note that elementary does not mean easy. The later books of the *Elements* contain some truly challenging problems, and involve solid geometry, number theory, and a deep exploration of the Platonic solids.

side of a triangle equals the sum of the measures of the areas of the other two sides, then the triangle is a right triangle.

3.2 A statement of the theorem

> In right-angled triangles the square on the side opposite the right angle equals the sum of the squares on the sides containing the right angle[6].

In more modern language, we might say “In a right triangle, the measure of the area of the square of the hypotenuse equals the sum of the measures of the areas of the squares on the other two sides.”

3.3 Towards a proof

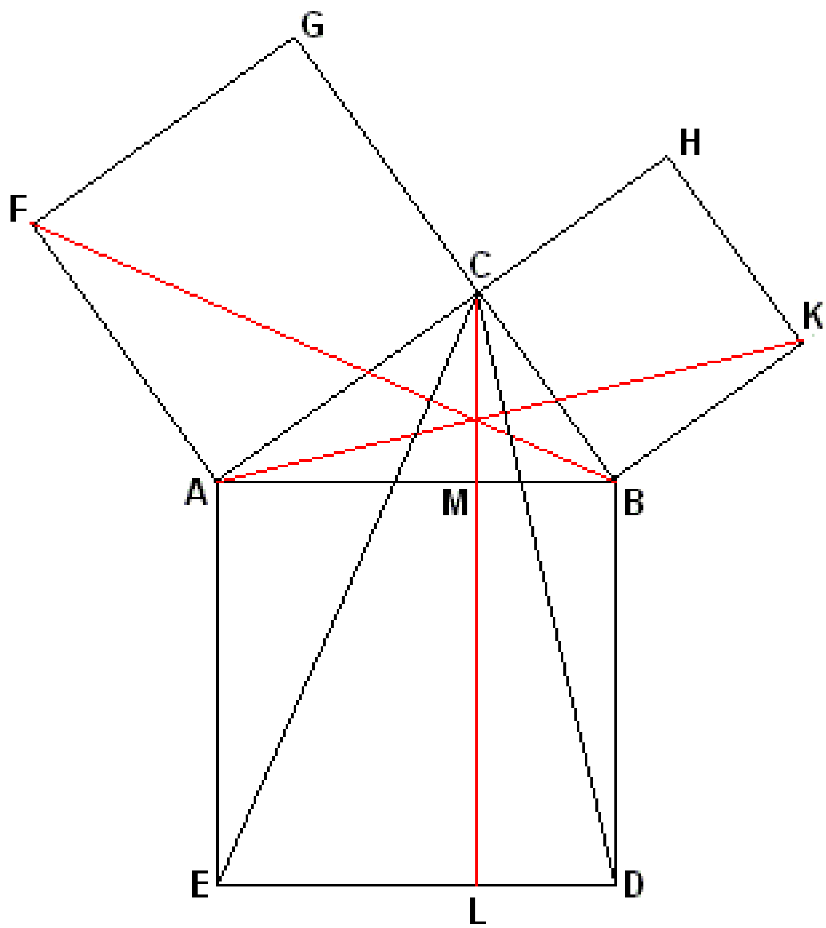
Euclid is going to break the figure up into triangles, and show that different triangles are equal to each other. He’s going to use a rule, known today as “Side-Angle-Side” to do this. Earlier in the *Elements,* he has shown that when two triangles that have two sides that match, and the angles between those sides also match, then they are equal to each other.

Euclid is going to find a triangle half the area of one of the little squares, and then show that he can split the big square into two rectangles. The triangle he finds will also be half the area of one of the rectangles. This means that the square is equal in area to that rectangle. He will then show that he can do the same thing for the other square, showing that it is the same area of the second rectangle he made out of the big square. That means that the big square is made of two pieces. One piece is the same area as one square, and the other piece is the same area as the other square. This proves the proposition, since the the measure of the area of the big square is equal to the sum of the measures of the areas of the two smaller squares.

3.4 The proof

From Euclid:

> I say that the square on AB equals the sum of the squares on AC and CB. Describe the square AEDB on AB, and the squares GA and HB on AC and CB. Draw CL through C parallel to either AE or BD, and join CE and FB.[6]



> Since each of the angles ACB and ACG are right, it follows that with a straight line AC, and at the point A on it, the two straight lines CB and CG not lying on the same side make the adjacent angles equal to two right angles, therefore BC is in a straight line with CG.[6]

This is a bit fancy, but really what Euclid is saying is that G, C and B all lie on the same line, because the angle between segment CB and segment CG is 180 degrees. Euclid tells us: “For the same reason AC is also in a straight line with CH.”[6]

> Since the angle EAB equals the angle FAC, for each is right, add the angle CAB to each, therefore the whole angle EAC equals the whole angle FAB.[6]

Those angles are going to be the angles he will use to get “Side-Angle-Side”. Because the sides EA and AB come from the same square, the sides are going to be equal. Also, for the same reason, Side AC is going to be equal to Side FA. Watch how he does it.

> Since EA equals AB, and FA equals AC, the two sides CA and AE equal the two sides FA and AB respectively, and the angle CAE equals the angle FAB, therefore the base CE equals the base FB, and the triangle CAE equals the triangle FAB.[6]

That’s the “Side-Angle-Side” you were warned about. The next argument is a bit tricky. Remember from geometry class that a parallelogram[10] has an area equal to the product of the length of its base and the length of its altitude. And also recall that a triangle has an area equal to the product of one-half of the length of its base and the length of its altitude. In short, the area formula for a parallelogram is $A = bh$ and for a triangle is $A = \frac{1}{2}bh$. They’re nearly the same formula, except the triangle is one-half the area of the parallelogram. One-half. That’s the trick that Euclid is going to use in this next bit. He’s going to show that one of those triangles we were talking about has the same base and height as the little square. Then he’s going to show that the other triangle, which is equal in measure to its partner, has the same base and height as the rectangle[11] made from splitting the big square. That will mean that the little square has the same area as the rectangle. Once he shows that for both sides, he’ll have proved his theorem.

> Now the parallelogram AL is double the triangle CAE, for they have the same base AE and are in the same parallels AE and CL. And the square GA is double the triangle FAB, for they again have the same base FA and are in the same parallels FA and GB.
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> Therefore the parallelogram AL also equals the square GA.[6]

He’s done all the work for one side. The arguments on the other side are exactly the same, so he illustrates how to do the same thing on the other side without being as detailed about it.

[10] A four-sided plane figure with opposites sides parallel to each other.

[11] He calls it a parallelogram because he doesn’t want to prove that it is a rectangle, and it doesn’t matter for the purposes of the proof. Remember, all rectangles are parallelograms, but the reverse is not true.

Similarly, if CD and AK are joined, the parallelogram BL can also be proved equal to the square HB. Therefore the whole square AEDB equals the sum of the two squares GA and HB. And the square AEDB is described on AB, and the squares GA and HB on AC and CB.

Therefore the square on AB equals the sum of the squares on AC and CB. Therefore *in right-angled triangles the square on the side opposite the right angle equals the sum of the squares on the sides containing the right angle.*

Q.E.D.[12][6]

Conclusion

The Ancient Greeks rightly regarded the knowledge of the Pythagorean Theorem to be one of the hallmarks of the civilized mind, and the measure of a civilized man. For eighteen centuries after Euclid wrote the *Elements,* it was taught to every math student, and was prized as one of the most important ideas of humanity. It provides the basis for the entire concept of measure, because, by making a right triangle, one can measure the lengths of sides that will not submit to tape measure. The converse is true as well, which was known to Egyptian builders in ancient times. They would use a knotted rope to measure right angles. The knots would be spaced equally, and so a length of three units, four units, and five units could be shaped into a triangle of sides 3, 4, and 5. Because $3^2 + 4^2 = 5^2$, the angle between the side of length 3 and the side of length 4 would be a right triangle. Thus a knotted rope could become a builder's square.

The science of trigonometry is based on a repeated application of the Pythagorean Theorem, over and over. From there, we get surveying, architecture and engineering. Astronomy is also based on trigonometry. Signal processing, as used in radio, television, cellular telephones and the Internet, comes from Fourier Analysis, which uses trigonometry, which is just the Pythagorean Theorem applied repeatedly. Electricity, weather, cartography, acoustics, all are studied using trigonometric principles. The Theory of Relativity, of Albert Einstein, was the first improvement on the subject since Euclid. In Einstein's theory, the Pythagorean Theorem fails when a subject is moving at a velocity very close to the speed of light. But at normal speeds, the Pythagorean Theorem still governs our understanding of nature, engineering, harmonics; pretty much every branch of science uses the Pythagorean Theorem in some way.

It is not an accident that in Freemasonry, the builder's square represents the Worshipful Master. The knowledge of how to make right angles was known only to a select few, and among Operative Masons, this knowledge was one of the secrets of Masonry. A building's structural integrity depended upon the cross beams being truly perpendicular to the walls. On a work site, it was the job of

[12] *Quod erat demonstratum.* That which was to be proved. Proofs conclude with this statement to let you know the proof is completed.

the master to determine if a right angle were truly square, and the Pythagorean Theorem ensured that if a builder's square were ever broken, a master could make a new one that would be true. That is why, in Pennsylvania, and in the Ancient York Lodges, a representation of the diagram used for the 47th Problem of Euclid is used as a Past Master's Jewel.

As this is a Masonic paper, it must be noted that President James Garfield, himself a Freemason, discovered a new proof of the Pythagorean Theorem. He drew a trapezoid[13] consisting of two copies of the right triangle separated by a half-square, split along the diagonal. The proof uses the area of the trapezoid, and uses algebra to manipulate that area to show that the area of the square of the hypotenuse is equal to the sum of the areas of the squares on the other two sides.

References

[1] Pound, Roscoe. *The Philosophy of Masonry.* Published serially in *The Builder, Vol. 1, Nos. 1–5,* 1915. The section on Preston is the first, published in Vol. 1, No. 1. A copy exists online at The Builder, January 1915. `http://www.phoenixmasonry.org/the_builder_1915_january.htm`

[2] Marcus Vitruvius Pollio. *On Architecture, Book IX, Section 9.* c. 15 BCE.

[3] Webb, Thomas Smith. *Freemason's Monitor, or Illustrations of Masonry.* 1797, 1818.

[4] Iamblichus of Chalcis. *Collection of Pythagorean Doctrines.* Especially, *Life of Pythagora*s *XVIII*, pp. 108–9. c.300 AD.

[5] Aristotle. *Metaphy*sics. *Book I, Section 5.* c. 350 BCE.

[6] Euclid. *Elements. Book I, Proposition 47.* c. 300 BCE.

[13] A four-sided figure with one pair of sides parallel to each other.