

# The ABC's and 123's of Laboratory Calculations

Erik Rosenfeldt, P.E., Ph.D.

Associate

Hazen and Sawyer

Richmond, VA

**HAZEN AND SAWYER**  
Environmental Engineers & Scientists

# Workshop Agenda

- **8:45 – 9:00:** Introduction and Welcome
- **9:00 – 10:00:** Part 1 – Review of Chemical Concepts
- **10:00 – 11:00:** Part 2 – Laboratory Practices and Calculations
- **11:00 – 11:45:** Part 3 – Statistics
- Q&A during each section

# Introduction

## Brief Resume

- BS Chemical Engineering
  - Washington University in St. Louis
- Project Engineer
  - ERM, St. Louis MO
- MS, PhD Environmental Engineering
  - Duke University
  - “Post-Doc” at NC State
- Assistant Professor
  - University of Massachusetts, Amherst
- Associate
  - Hazen and Sawyer, PC
- Professional Engineer
  - Massachusetts, Virginia



# Introductions

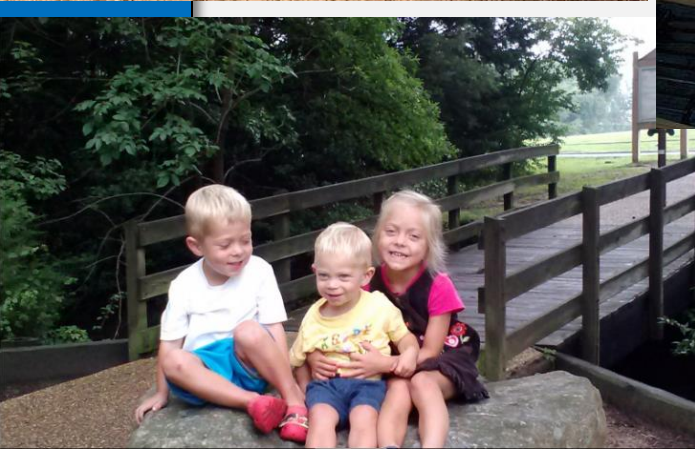
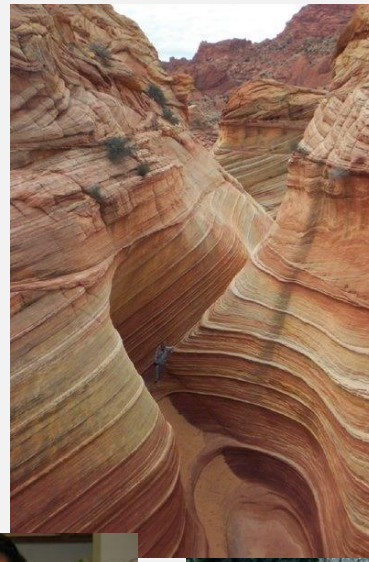
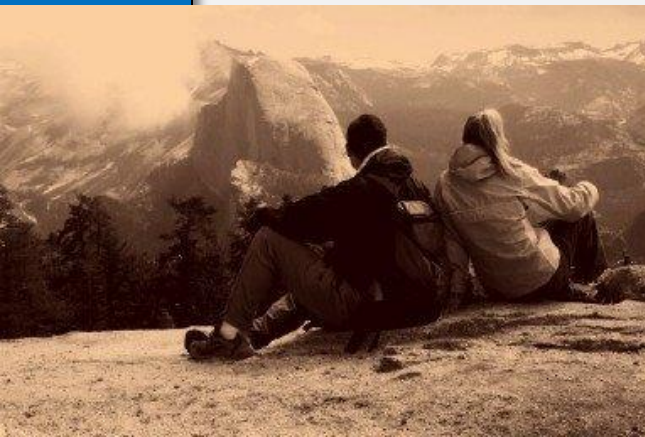
- What I do professionally





# Course Overview - Instructor

- Hobbies include running, hiking, basketball, playing with my wife and kids



# Course Overview - Instructor

- Favorite Teams (in the interest of full disclosure)



Pitchers should HIT!!!



# Course Overview - Instructor

- Least Favorite Teams



- What we will go through today
  - Part 1: Review of Chemical Concepts
  - Part 2: Laboratory Practices and Calculations
  - Part 3: Statistics



- Part 1: Review of Chemical Concepts including:
  - The Periodic Table
  - Units of Measures (moles and equivalents)
  - Concentration (mass, molarity, and normality)
  - Acid/Base
  - Alkalinity and Hardness

- Part 2: Laboratory Practices and Calculations
  - Dimensional Analysis
  - Making Dilutions / Serial Dilutions
  - Making Standards from Stock Solutions
  - Titrations

- Part 3: Statistics
  - Precision and Accuracy
  - Significant Figures
  - Calculating RPD/RSD/% recovery
  - The theory behind correlation coefficients
  - Understanding the 95% Confidence Interval

# Example

- We will create our own data set and work with it:
  - Problem Statement:
    - Height and Wingspan in Human Beings has been observed to be linearly related. Let's see how that relationship holds in this workshop
  - Step 1: Measure every participants height and wingspan in inches (significant figures)?
  - Step 2: Convert height and wingspan to cm.
  - Step 3: Remember age
  - Step 4: Record on Data Sheet
  - Step 5: If you are #s 3, 6, 10, 12, 15 on the data sheet, repeat the measurements.



# PART 1: REVIEW OF CHEMICAL CONCEPTS

The Periodic Table,  
Units of Measures (moles and equivalents)  
Concentration (mass, molarity, and normality)  
Acid/Base  
Alkalinity and Hardness

# The Periodic Table

1 <b>H</b> 1.00	← IA																2 <b>He</b> 4.00											
IIA		Alkali Metals										Alkaline Earth Metals					Transition Metals			Other Metals	Non-Metals	Halogens	Inert Gases	VIIIA →				
3 <b>Li</b> 6.94	4 <b>Be</b> 9.01	Lanthanide										Actinide					5 <b>B</b> 10.8	6 <b>C</b> 12.0	7 <b>N</b> 14.0	8 <b>O</b> 16.0	9 <b>F</b> 19.0	10 <b>Ne</b> 20.2						
Columns = Groups I – VIII = Number of e- in Outer Shell																												
Rows = Periods 1-7 = Outer Shell Number																												
III B		IV B		V B		VI B		VII B		VIII B		I B		II B		13 <b>Al</b> 27.0	14 <b>Si</b> 28.1	15 <b>P</b> 31.0	16 <b>S</b> 32.1	17 <b>Cl</b> 35.4	18 <b>Ar</b> 39.9							
19 <b>K</b> 39.1	20 <b>Ca</b> 40.1	21 <b>Sc</b> 44.9	22 <b>Ti</b> 47.9	23 <b>V</b> 50.9	24 <b>Cr</b> 52.0	25 <b>Mn</b> 54.9	26 <b>Fe</b> 55.8	27 <b>Co</b> 58.9	28 <b>Ni</b> 58.6	29 <b>Cu</b> 63.5	30 <b>Zn</b> 65.3	31 <b>Ga</b> 69.7	32 <b>Ge</b> 72.6	33 <b>As</b> 74.9	34 <b>Se</b> 79.0	35 <b>Br</b> 79.9	36 <b>Kr</b> 83.8											
37 <b>Ru</b> 85.5	38 <b>Sr</b> 87.6	39 <b>Y</b> 88.9	40 <b>Zr</b> 91.2	41 <b>Nb</b> 92.9	42 <b>Mo</b> 96.0	43 <b>Tc</b> 98	44 <b>Ru</b> 101	45 <b>Rh</b> 102	46 <b>Pd</b> 106	47 <b>Ag</b> 108	48 <b>Cd</b> 112	49 <b>In</b> 115	50 <b>Sn</b> 119	51 <b>Sb</b> 122	52 <b>Te</b> 128	53 <b>I</b> 127	54 <b>Xe</b> 131											
55 <b>Cs</b> 132	56 <b>Ba</b> 137	57 <b>La</b> 139	72 <b>Hf</b> 178	73 <b>Ta</b> 181	74 <b>W</b> 184	75 <b>Re</b> 186	76 <b>Os</b> 190	77 <b>Ir</b> 192	78 <b>Pt</b> 195	79 <b>Au</b> 197	80 <b>Hg</b> 200	81 <b>Tl</b> 204	82 <b>Pb</b> 207	83 <b>Bi</b> 210	84 <b>Po</b> 209	85 <b>At</b> 210	86 <b>Rn</b> 222											
87 <b>Fr</b> 223	88 <b>Ra</b> 226	89 <b>Ac</b> 227	104 <b>Rf</b> 267	105 <b>Db</b> 268	106 <b>Sg</b> 271	107 <b>Bh</b> 272	108 <b>Hs</b> 270	109 <b>Mt</b> 276	110 <b>Ds</b> 281	111 <b>Rg</b> 280																		
Lanthanide		58 <b>Ce</b> 140	59 <b>Pr</b> 141	60 <b>Nd</b> 144	61 <b>Pm</b> 145	62 <b>Sm</b> 150	63 <b>Eu</b> 152	64 <b>Gd</b> 157	65 <b>Tb</b> 159	66 <b>Dy</b> 162	67 <b>Ho</b> 165	68 <b>Er</b> 167	69 <b>Tm</b> 169	70 <b>Yb</b> 173	71 <b>Lu</b> 175													
Actinide		90 <b>Th</b> 232	91 <b>Pa</b> 231	92 <b>U</b> 238	93 <b>Np</b> 237	94 <b>Pu</b> 244	95 <b>Am</b> 243	96 <b>Cm</b> 247	97 <b>Bk</b> 247	98 <b>Cf</b> 251	99 <b>Es</b> 252	100 <b>Fm</b> 257	101 <b>Md</b> 258	102 <b>No</b> 259	103 <b>Lr</b> 262													

We can learn a lot from the Periodic Table...

# Other examples of chemistry in action

15

<http://www.youtube.com/watch?v=MTcgo46nxNE>

<http://www.youtube.com/watch?v=7Gp2wx2zIRI>

One more Example:

- Chlorine + Ammonia
  - In drinking water = an effective disinfectant residual
  - In cleaning the bathroom = not good!
  - What's different between the situations?

# More Chemistry



Iron + Sulfur (+ Heat)



REDOX  
(Magnesium + Oxygen)



Combustion



Copper Chloride + Aluminum



Potassium Chlorate + Sugar (+ Heat)

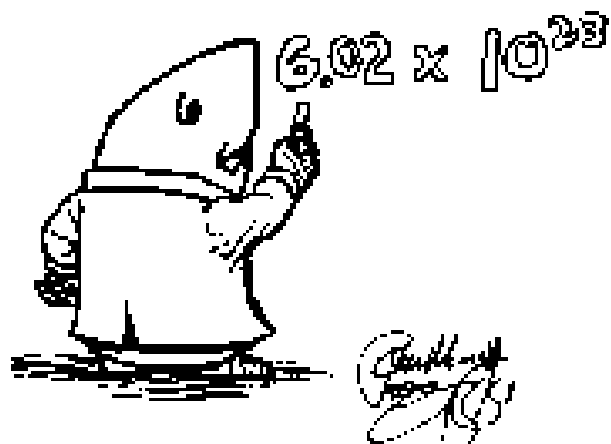
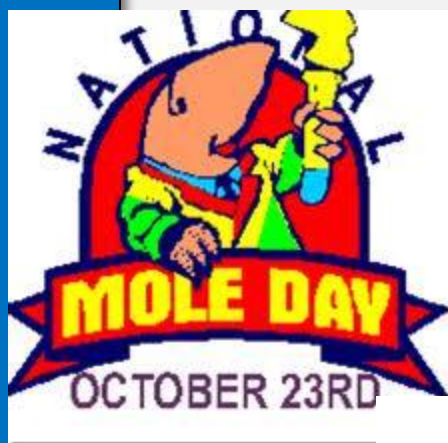


# Guiding principal for today

- To understand laboratory calculations, and avoid “undesirable” outcomes, it is important to understand the terminology, equations, and fundamentals behind important calculations!!!

# Chemical Concepts: The Mole

- “A day without mole calculations is like a day without sunshine!”



# The Mole

- Unit of measure for comparing amounts of atoms and molecules
- 1 mol =  $6.02 \times 10^{23}$  atoms (or molecules)
- Mass of 1 mol = atomic (or molecular) weight
- Examples:
  - 1 mol carbon = 12 g
  - 1 mol  $\text{CO}_3^{2-}$  = 12 g + (16 g x 3) = 60 g

# Equivalents

- The amount of a substance which will either:
  - react with or supply one mole of hydrogen ions ( $\text{H}^+$ ) in an acid–base reaction; or
  - react with or supply one mole of electrons in a redox reaction.
- Examples
  - 1 mol of sulfuric acid ( $\text{H}_2\text{SO}_4$ ) = 2 equivalents
    - 2 x  $\text{H}^+$  supplied
  - 1 mol of Calcium Carbonate ( $\text{CaCO}_3$ ) = 2 equivalents
    - $\text{CO}_3^{-2}$  would consume 2 x  $\text{H}^+$



# Concentrations

- Mass Concentration = mass of a substance in 1L of solution (mg/L)
- Molar Concentration = Number of *moles* of a substance in 1L of solution (M, or moles/L)
- Normal Concentration = Number of *equivalents* of a substance in 1L of solution (N, or Eq/L)
- $c(N) = a(M) \times b(\text{Eq/mol})$

# Concentrations

- Parts per million (ppm)
  - Parts of solute per million parts of total solution (mole per 1,000,000 mole, gram per 1,000,000 gram)
  - ie 30ppm KI = 30 grams of KI in 1,000,000 grams of water
- The water simplification
  - Density of water → 1 kg = 1L
  - To the above example
    - 30 grams KI = 30,000 mg KI
    - 1,000,000 grams water = 1,000 L water
    - 30g KI / 1,000,000g water = 30,000mg KI / 1,000 L water
      - 30 mg/L water
- For WATER only, ppm = mg/L
  - Also, ppb =  $\mu\text{g/L}$ , ppt = ng/L, etc.

# Implications: Water Pollutants

- The water and wastewater industry tends towards mass-based measurements (ppm, ppb, etc.)
- Molecular basis is often a more appropriate measurement
- Disinfection Byproducts (DBPs)
  - Total THMs = 80  $\mu\text{g/L}$
  - HAA5 = 60  $\mu\text{g/L}$
- THMs range from  $\text{CHCl}_3$  to  $\text{CHBr}_3$ 
  - 1  $\mu\text{mol CHCl}_3$  = 120  $\mu\text{g}$ ; 1  $\mu\text{mol CHBr}_3$  = 253  $\mu\text{g}$



# Trichloromethane and Tribromomethane



- For trichloromethane:

$$80 \mu\text{g CHCl}_3 \times \frac{1 \mu\text{mol CHCl}_3}{120 \mu\text{g CHCl}_3} = 0.67 \mu\text{mol CHCl}_3$$

- For tribromomethane:

$$80 \mu\text{g CHBr}_3 \times \frac{1 \mu\text{mol CHBr}_3}{253 \mu\text{g CHBr}_3} = 0.33 \mu\text{mol CHBr}_3$$

- For a chloride-dominated water, potentially 2 x as many DBPs could be present on a molar basis!
- For systems dominated by brominated THMs it is harder to reach mass-based compliance!

# DBP Example

- For the speciation charts on the next figure, show the differences in THM concentrations (mass basis) for the same molar concentration of  $0.5 \mu\text{M}$  total THMs

# DBP Example, Cont.

System	2004 - 2005	2007-2008	2012
System 1, VA			
System 2, NC		<ul style="list-style-type: none"> <li><span style="color: blue;">■</span> Chloroform</li> <li><span style="color: red;">■</span> Dichlorobromoform</li> <li><span style="color: green;">■</span> Chlorodibromoform</li> <li><span style="color: purple;">■</span> Bromoform</li> </ul>	
System 3, PA			



# DBP Example, Cont.

- DBP Example Solution:
- System 2, NC
  - Step 1: mM of each component:

Component	2004 – 2005 Concentration ( $\mu\text{M}$ )	2012 Concentration ( $\mu\text{M}$ )
$\text{CHCl}_3$	0.395 (79%)	0.035 (7%)
$\text{CHCl}_2\text{Br}$	0.095 (19%)	0.11 (22%)
$\text{CHClBr}_2$	0.01 (2%)	0.24 (48%)
$\text{CHBr}_3$	0 (0%)	0.11 (22%)

- Step 2: Convert to  $\mu\text{g}/\text{L}$  of each and sum

# DBP Example Cont.

Component	Molecular Weight (g/mol)	2004 – 2005 Concentration ( $\mu\text{g/L}$ )	2012 Concentration ( $\mu\text{g/L}$ )
$\text{CHCl}_3$	119.2	$0.395 \times 119.2 = 47.1$	$0.035 \times 119.2 = 4.2$
$\text{CHCl}_2\text{Br}$	163.7	$0.095 \times 163.7 = 15.6$	$0.11 \times 163.7 = 18.0$
$\text{CHClBr}_2$	208.2	$0.01 \times 208.2 = 2.1$	$0.24 \times 208.2 = 50.0$
$\text{CHBr}_3$	252.7	$0 \times 252.7 = \underline{0}$	$0.11 \times 252.7 = \underline{27.8}$
		64.8	100.0

64.8 would be considered “acceptable”,  
100.0 is greater than MCL ( $80 \mu\text{g/L}$ )

# Alkalinity Example

- Sweep Floc Coagulation with Ferric chloride
- $\text{FeCl}_3 + 3\text{HCO}_3^- \rightarrow \text{Fe}(\text{OH})_3 + 3\text{Cl}^- + 3\text{CO}_2$ 
  - 3 moles of  $\text{HCO}_3^-$  are needed per mole of  $\text{FeCl}_3$ 
    - 1 mole  $\text{FeCl}_3 = 161 \text{ mg FeCl}_3$  (neglecting  $\text{H}_2\text{O}$ )
    - 3 mole  $\text{HCO}_3^-$  (alkalinity) = 150 mg Alk as  $\text{CaCO}_3$
- Question 1: If you add 40 mg/L  $\text{FeCl}_3$ , how much alkalinity do you consume (mmoles, mequivalents, mg)
- Question 2: If you have a 1N solution of sodium bicarbonate, how much do you need to add to provide enough alkalinity for a 40mg/L dose of  $\text{FeCl}_3$

# Alkalinity Example

- Question 1 Solution:



$$\frac{40 \text{ mg FeCl}_3}{\text{L}} \times \frac{1 \text{ mmol FeCl}_3}{162.1 \text{ mg FeCl}_3} = 0.246 \text{ mmol FeCl}_3$$

$$\frac{0.246 \text{ mmol FeCl}_3}{1 \text{ mmol FeCl}_3} \times \frac{3 \text{ mmol HCO}_3^-}{1} = 0.74 \text{ mmol HCO}_3^-$$

$$\frac{0.74 \text{ mmol HCO}_3^-}{2 \text{ meq HCO}_3^-} \times \frac{1 \text{ meq Alk as CaCO}_3}{1} = 0.37 \text{ meq Alk as CaCO}_3$$

$$\frac{0.37 \text{ meq Alk as CaCO}_3}{1 \text{ meq Alk as CaCO}_3} \times \frac{100 \text{ mg Alk as CaCO}_3}{1} = 37 \text{ mg Alk as CaCO}_3/\text{L}$$

# Alkalinity Example

- Question 2 Solution:
- Assume 1 L of Solution

$$\frac{40 \text{ mg FeCl}_3}{\text{L}} \times \frac{1 \text{ mmol FeCl}_3}{162.1 \text{ mg FeCl}_3} \times 1 \text{ L} = 0.246 \text{ mmol FeCl}_3$$

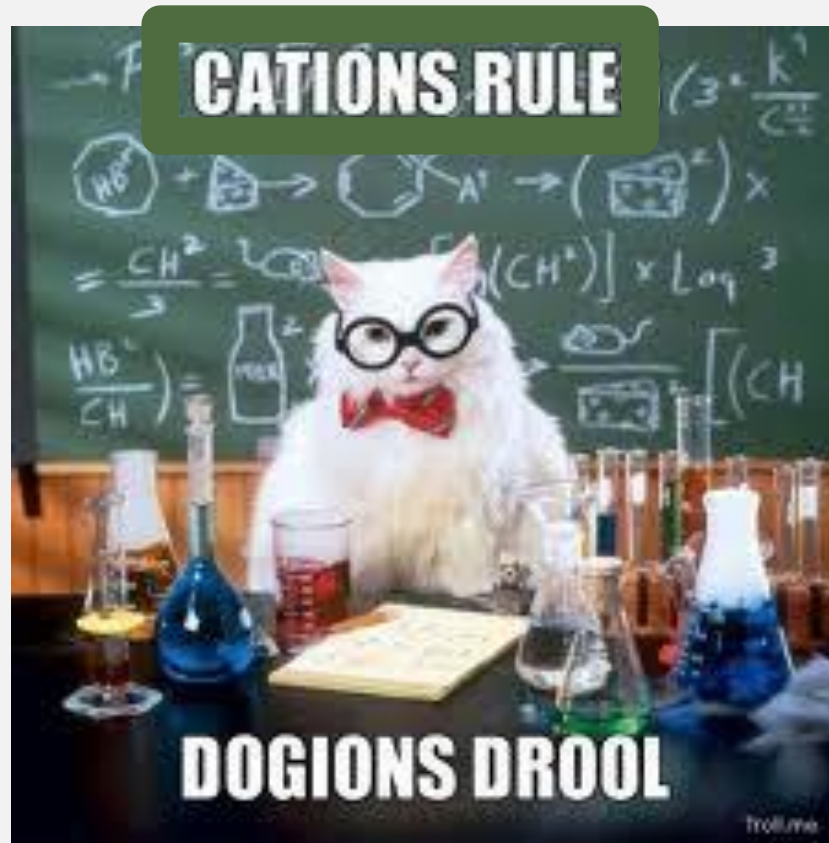
$$\frac{0.246 \text{ mmol FeCl}_3}{1 \text{ mmol FeCl}_3} \times 3 \text{ mmol HCO}_3^- = 0.74 \text{ mmol HCO}_3^-$$



$$\frac{0.74 \text{ mmol HCO}_3^-}{1 \text{ mmol HCO}_3^-} \times \frac{1 \text{ mmol NaHCO}_3}{1 \text{ mmol HCO}_3^-} \times \frac{1 \text{ eq NaHCO}_3}{1000 \text{ mmol NaHCO}_3} = 0.00074 \text{ eq NaHCO}_3$$

$$\frac{0.00074 \text{ eq NaHCO}_3}{1 \text{ eq NaHCO}_3} \times \frac{\text{L}}{1 \text{ L}} \times \frac{1000 \text{ mL}}{1 \text{ L}} = 0.74 \text{ ml NaHCO}_3$$

L solution





# Acids & Bases

Common Household Acids & Bases



Acids

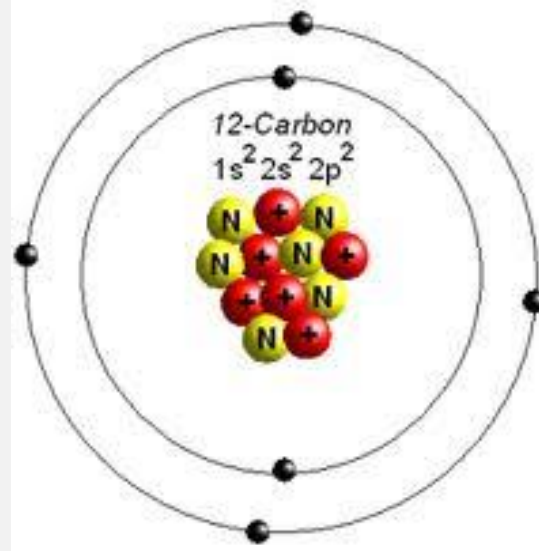


Bases

	[H <sup>+</sup> ]	pH	Common examples
Acids	$1 \times 10^0$	0	Hydrochloric acid
	$1 \times 10^{-1}$	1	Stomach acid
	$1 \times 10^{-2}$	2	Lemon juice
	$1 \times 10^{-3}$	3	Vinegar
	$1 \times 10^{-4}$	4	Soda (carbonic acid)
	$1 \times 10^{-5}$	5	Rainwater
	$1 \times 10^{-6}$	6	Milk
Neutral	$1 \times 10^{-7}$	7	Pure water
Bases	$1 \times 10^{-8}$	8	Egg whites
	$1 \times 10^{-9}$	9	Baking soda
	$1 \times 10^{-10}$	10	Antacid
	$1 \times 10^{-11}$	11	Ammonia
	$1 \times 10^{-12}$	12	Quicklime (calcium hydroxide)
	$1 \times 10^{-13}$	13	Drain cleaner
	$1 \times 10^{-14}$	14	Lye (sodium hydroxide)

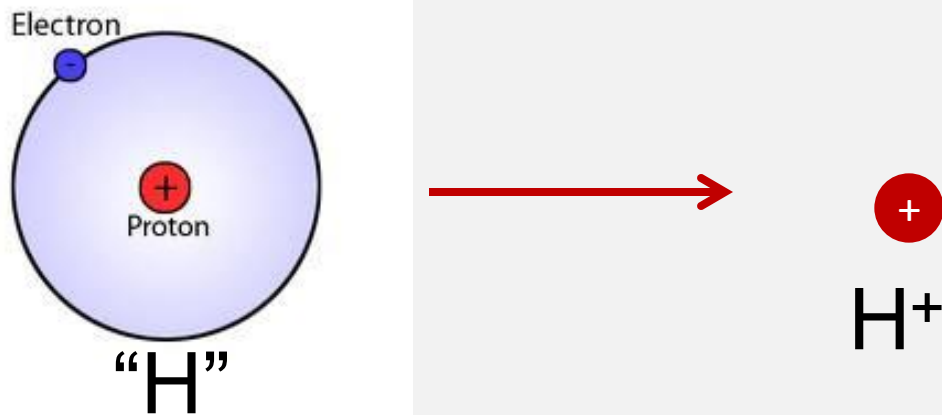
# Acids & Bases

- First, about “protons”
  - Atoms = protons, neutrons, and electrons, right?
  - Carbon: Element 6, Atomic Mass = 12
    - 6 protons + 6 neutrons + 6 electrons



# Acids & Bases

- First, about “protons”
  - Elemental hydrogen: diatomic,  $H_2$
  - Alone, H has 1 proton and 1 electron
  - As a cation,  $H^+$ , the electron is absent, only the proton remains



# Acids & Bases

- Acids and Acidity are generally associated with the availability of free protons ( $\text{H}^+$ ) in solution
- Brønsted-Lowry Acid = Proton Donor
  - e.g.  $\text{HCl} \rightarrow \text{H}^+ + \text{Cl}^-$
  - $\text{HCl} + \text{H}_2\text{O} \rightarrow \text{H}_3\text{O}^+ + \text{Cl}^-$
- Brønsted-Lowry Base = Proton Acceptor
  - e.g.  $\text{OH}^- + \text{H}^+ \rightleftharpoons \text{H}_2\text{O}$
  - $\text{NaOH} + \text{HCl} \rightarrow \text{Na}^+ + \text{Cl}^- + \text{H}_2\text{O}$
- Some species can be both acids and bases:
  - $\text{HCO}_3^- \rightleftharpoons \text{H}^+ + \text{CO}_3^{2-}$
  - $\text{HCO}_3^- + \text{H}^+ \rightleftharpoons \text{H}_2\text{CO}_3$ 
    - The behavior depends on the pH of the solution!

# Strong Acids and Bases

- Strong acids and bases dissociate completely in water
  - $\text{NaOH} \rightarrow \text{Na}^+ + \text{OH}^-$
  - $\text{H}_2\text{SO}_4 \rightarrow 2\text{H}^+ + \text{SO}_4^{2-}$
- Reactions “favor” right hand side
  - “Forward” reactions

# Weak Acids and Bases

- Weak acids and bases dissociate incompletely
  - Some un-ionized fraction remains
  - $\text{H}_3\text{PO}_4 \rightleftharpoons \text{H}^+ + \text{H}_2\text{PO}_4^-$
  - $\text{H}_2\text{PO}_4^- \rightleftharpoons \text{H}^+ + \text{HPO}_4^{2-}$
  - $\text{HPO}_4^{2-} \rightleftharpoons \text{H}^+ + \text{PO}_4^{3-}$
- Ratio of species based on equilibrium constant and pH
  - $K_{a1} = [\text{H}^+][\text{H}_2\text{PO}_4^-] / [\text{H}_3\text{PO}_4]$
  - Generically,  $K_a = [\text{H}^+][\text{A}^-] / [\text{HA}]$



# Concept of pH

- $\text{pH} = -\log[\text{H}^+]$
- Each unit step down in pH is 10 x the hydrogen ion concentration!
- At  $\text{pH} = 7$ ,  $[\text{H}^+] = [\text{OH}^-]$

	$[\text{H}^+]$	pH	Common examples
Acids	$1 \times 10^0$	0	Hydrochloric acid
	$1 \times 10^{-1}$	1	Stomach acid
	$1 \times 10^{-2}$	2	Lemon juice
	$1 \times 10^{-3}$	3	Vinegar
	$1 \times 10^{-4}$	4	Soda (carbonic acid)
	$1 \times 10^{-5}$	5	Rainwater
	$1 \times 10^{-6}$	6	Milk
Neutral	$1 \times 10^{-7}$	7	Pure water
Bases	$1 \times 10^{-8}$	8	Egg whites
	$1 \times 10^{-9}$	9	Baking soda
	$1 \times 10^{-10}$	10	Antacid
	$1 \times 10^{-11}$	11	Ammonia
	$1 \times 10^{-12}$	12	Quicklime (calcium hydroxide)
	$1 \times 10^{-13}$	13	Drain cleaner
	$1 \times 10^{-14}$	14	Lye (sodium hydroxide)

The pH Scale



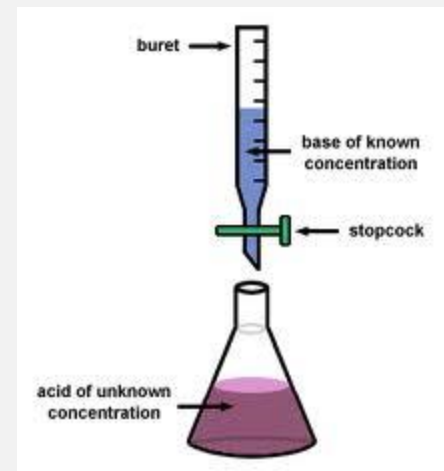
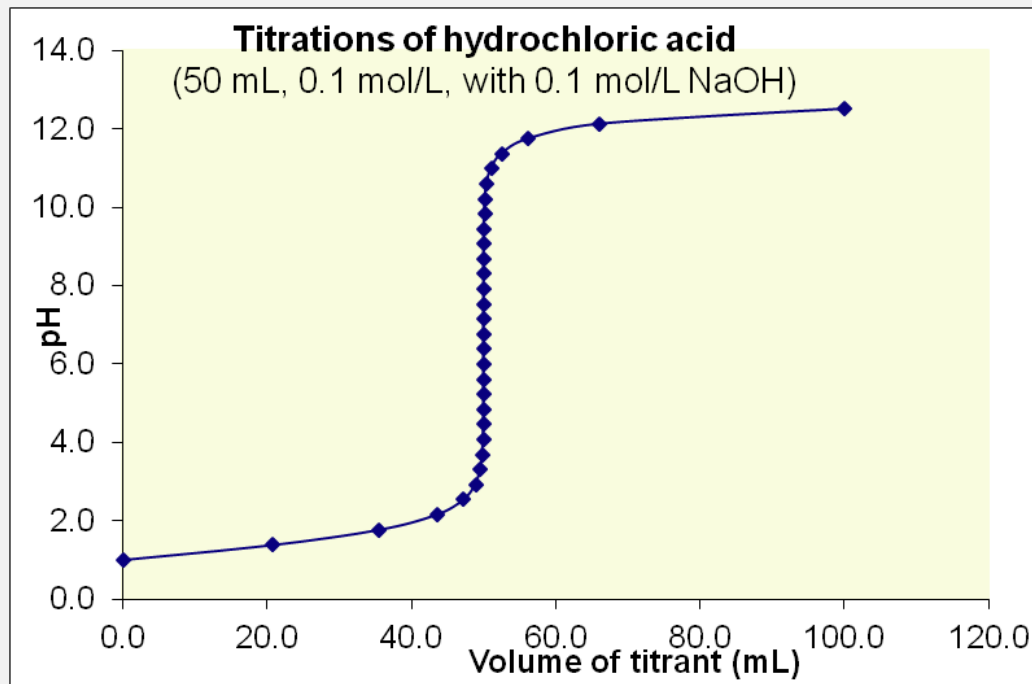
# pH and pKa

- $\text{pH} = -\log[\text{H}^+]$
- And
- $K_a = [\text{H}^+][\text{A}^-] / [\text{HA}]$
- $K_a = [\text{H}^+] \times [\text{A}^-] / [\text{HA}]$
- $\log (K_a) = \log [\text{H}^+] + \log ([\text{A}^-] / [\text{HA}])$
- $-\text{p}K_a = -\text{pH} + \log ([\text{A}^-] / [\text{HA}])$
- $\text{pH} = \text{p}K_a + \log ([\text{A}^-] / [\text{HA}])$
- Thus, pH controls the amount of  $[\text{A}^-]$  and  $[\text{HA}]$  in solution

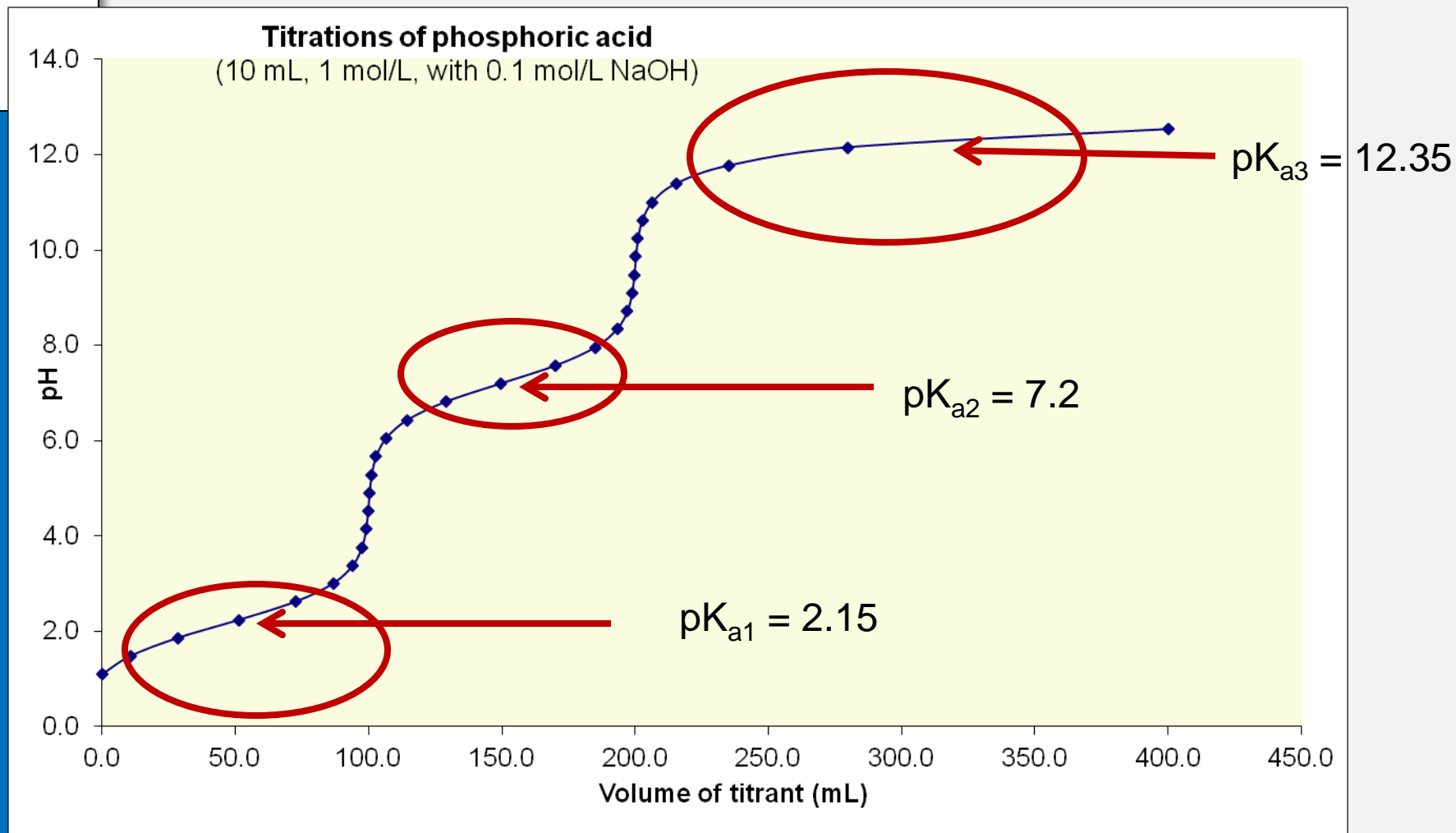


# Titrations (Brief!!!!)

- Acids and bases will neutralize each other
- Titrations can be used to measure acids and bases in solution



# Titrations Show $pK_a$ Values and Buffer Capacity

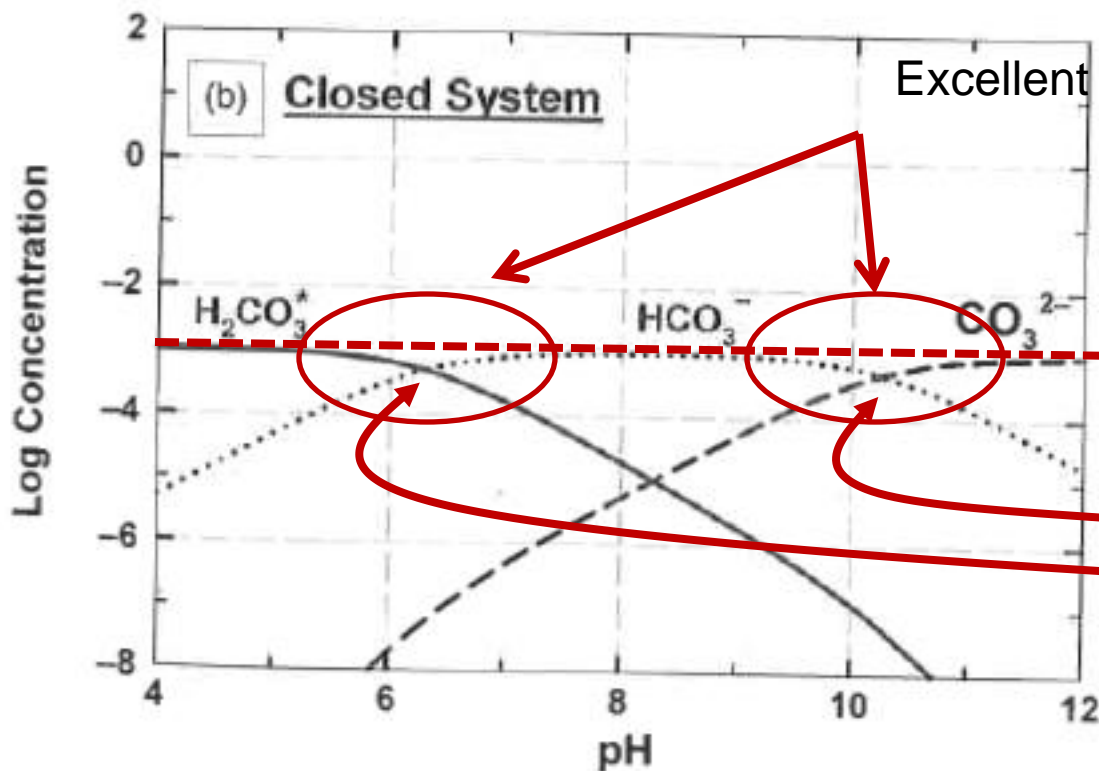


# Buffer Capacity

- Regions that resist change in pH even when acids or bases are added
- Natural waters have different buffer capacities
  - Waters with high concentrations of divalent and trivalent ions tend to resist large shifts in pH
  - Deionized water will rapidly change pH when acids or bases are added

# The Carbonate System, Closed

- One of the largest contributors to buffer capacity in natural waters is carbonate



Excellent Buffer Capacity!!

$$C_{T,C} = [H_2CO_3] + [HCO_3^-] + [CO_3^{2-}] = 10^{-3}M$$

$$pK_{a2} = 10.33$$

$$pK_{a1} = 6.35$$



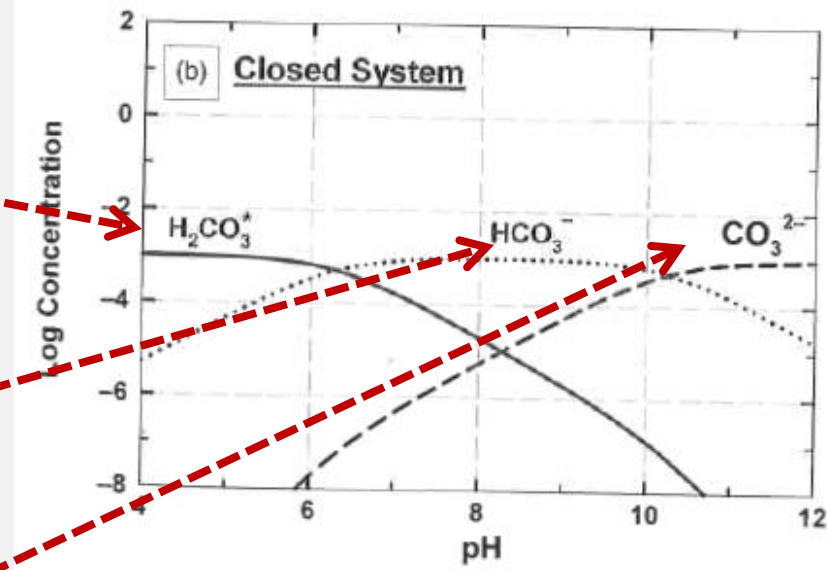
# Calculating Carbonate Species

- Avoiding the details...
  - You can calculate the amount of each carbonate species at a given pH using alpha values which relate to acidity constants ( $K_a$ )
- $\alpha_0 = [\text{H}_2\text{CO}_3^*] / C_{\text{T,C}}$
- $\alpha_1 = [\text{HCO}_3^-] / C_{\text{T,C}}$
- $\alpha_2 = [\text{CO}_3^{2-}] / C_{\text{T,C}}$
- Tables are easier!!

# Carbonate Alpha Values: Water Quality and Treatment Handbook

**TABLE 3-8** Inorganic Acid-Base Alphas for K's at 25°C and Adjusted for 10<sup>-3</sup> M Ionic Strength (pK<sub>1</sub> = 6.30 and pK<sub>2</sub> 10.25)

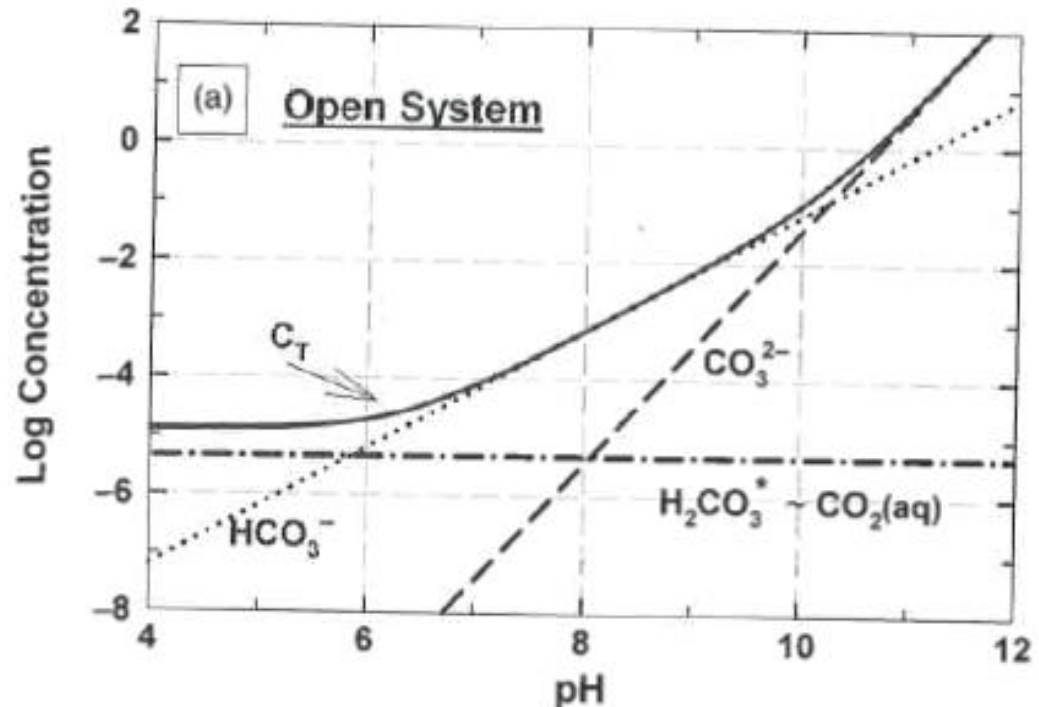
pH	H <sup>+</sup> M	α <sub>0</sub>	α <sub>1</sub>	α <sub>2</sub>
2	1.0 × 10 <sup>-2</sup>	0.9999	5.01 × 10 <sup>-5</sup>	2.82 × 10 <sup>-13</sup>
3	1.0 × 10 <sup>-3</sup>	0.9995	5.01 × 10 <sup>-4</sup>	2.82 × 10 <sup>-11</sup>
4	1.0 × 10 <sup>-4</sup>	0.9950	4.99 × 10 <sup>-3</sup>	2.81 × 10 <sup>-9</sup>
4.5	3.16 × 10 <sup>-5</sup>	0.984	0.0150	2.78 × 10 <sup>-8</sup>
5	1.0 × 10 <sup>-5</sup>	0.952	0.0477	2.69 × 10 <sup>-7</sup>
5.5	3.16 × 10 <sup>-6</sup>	0.863	0.137	2.43 × 10 <sup>-6</sup>
6	1.0 × 10 <sup>-6</sup>	0.666	0.334	1.88 × 10 <sup>-5</sup>
6.3	5.01 × 10 <sup>-7</sup>	0.500	0.500	5.61 × 10 <sup>-5</sup>
7	1.0 × 10 <sup>-7</sup>	0.166	0.833	4.68 × 10 <sup>-4</sup>
7.5	3.16 × 10 <sup>-8</sup>	0.059	0.939	1.67 × 10 <sup>-3</sup>
8	1.0 × 10 <sup>-8</sup>	0.019	0.975	5.48 × 10 <sup>-3</sup>
8.3	5.01 × 10 <sup>-9</sup>	0.010	0.979	0.01
9	1.0 × 10 <sup>-9</sup>	1.88 × 10 <sup>-3</sup>	0.945	0.053
9.5	3.16 × 10 <sup>-10</sup>	5.30 × 10 <sup>-4</sup>	0.849	0.151
10	1.0 × 10 <sup>-10</sup>	1.30 × 10 <sup>-4</sup>	0.640	0.360
10.25	5.62 × 10 <sup>-11</sup>	5.50 × 10 <sup>-5</sup>	0.500	0.500
11	1.0 × 10 <sup>-11</sup>	3.0 × 10 <sup>-6</sup>	0.151	0.849
12	1.0 × 10 <sup>-12</sup>	3.4 × 10 <sup>-8</sup>	0.0175	0.983



# The Carbonate System, Open

- Carbon dioxide is a weak acid
- Dissolved readily in high pH solutions
- When open to the air, basic solutions will absorb  $\text{CO}_2$

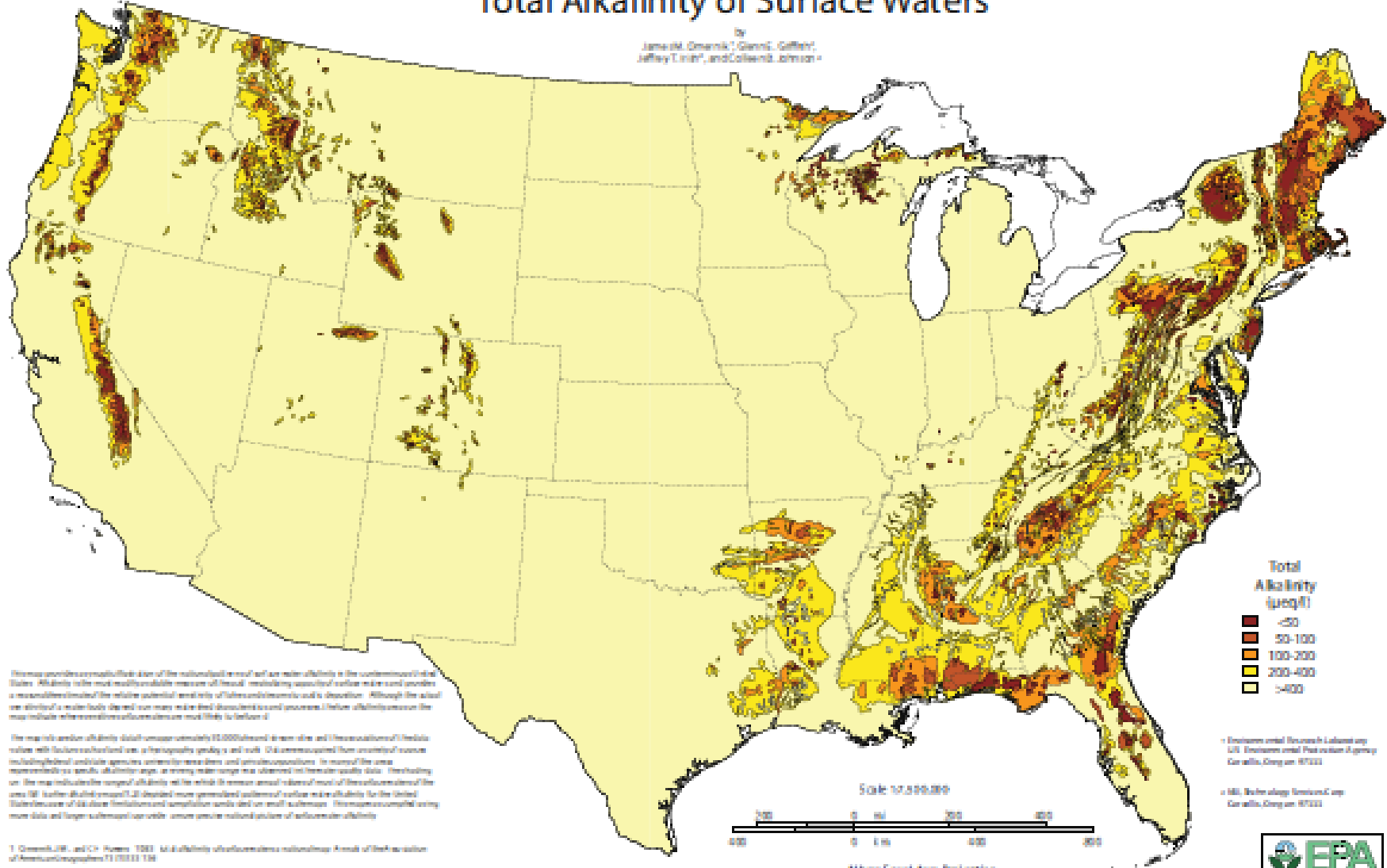
$C_{T,C}$  increases  
as pH  
increases to  
satisfy  
equilibrium



# Alkalinity and Acidity

## Total Alkalinity of Surface Waters

by  
James M. Owens<sup>1</sup>, Glenn C. Giffen<sup>2</sup>,  
Jeffrey T. Miller<sup>2</sup>, and Colleen B. Johnson<sup>2</sup>



**Total Alkalinity (µeq/l)**

- <50
- 50-100
- 100-200
- 200-400
- >400

<sup>1</sup> Environmental Research Laboratory  
U.S. Environmental Protection Agency  
Corvallis, Oregon 97331

<sup>2</sup> EPA Waterways Research Unit  
Corvallis, Oregon 97331

Scale 1:1,000,000



Miles Equal Area Projection

Stream specific conductivity is a function of the total dissolved solids (TDS) in the water. Alkalinity is the most readily available measure of TDS, resulting in a positive correlation between alkalinity and conductivity. The relative potential availability of TDS and conductivity is dependent on the mineral composition of the water body. The map may not be directly comparable to other alkalinity maps that do not include stream specific conductivity data.

The map is based on alkalinity data from approximately 10,000 stream sites and 100,000 stream kilometers. The data were collected from a variety of sources including federal, state, and local agencies, as well as private contractors. In general, the data represent a wide range of alkalinity values, from very low to very high. The map is based on the highest alkalinity value for each stream segment. The map is based on the highest alkalinity value for each stream segment. The map is based on the highest alkalinity value for each stream segment.

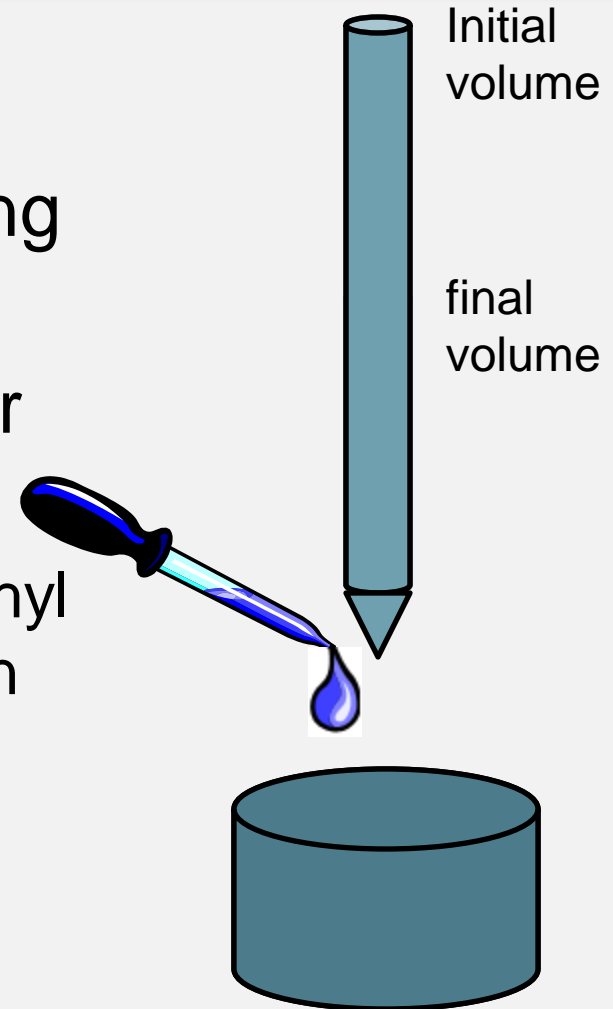
1. Owens, J.M., and G.C. Giffen. 1985. Alkalinity of surface waters in the United States. *Journal of Environmental Quality* 14:113-124.

2. Owens, J.M., G.C. Giffen, and J.T. Miller. 1985. Alkalinity of surface waters. Corvallis: Environmental Research Laboratory, U.S. Environmental Protection Agency, Corvallis, Oregon.



- Alkalinity is the acid neutralization capacity of a water
  - Anything that can consume  $H^+$  (acid) when added to water
  - Common Examples include:
    - Inorganic Carbon ( $HCO_3^-$ ,  $CO_3^{2-}$ )
    - Orthophosphate ( $H_2PO_4^-$ ,  $HPO_4^{2-}$ ,  $PO_4^{3-}$ )
    - Also  $OH^-$  at basic pH
  - The *amount of strong acid* required to reduce pH to 4.5

- Measuring Alkalinity
  - Titrate sample with a strong acid to pH 4.5
  - Measure with pH meter, or with indicator solution
    - Add bromcresol green-methyl red indicator solution to turn sample “pink”
    - Add strong acid (ie  $\text{H}_2\text{SO}_4$ ) until color goes away
    - $\text{vol}_i - \text{vol}_f = \text{meq Alkalinity}$



- Usually see alkalinity in mg/L as  $\text{CaCO}_3$ 
  - $(\text{vol}_{\text{initial}} - \text{vol}_{\text{final}}) = \text{meq Acid}$
  - 1 meq/L Acid = 50 mg/L as  $\text{CaCO}_3$
- For example
  - Initial volume 0.02N  $\text{H}_2\text{SO}_4$  = 52 mls
  - Final volume 0.02N  $\text{H}_2\text{SO}_4$  = 47 mls

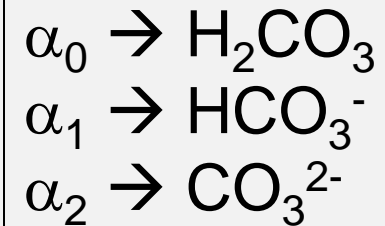
$$\text{Alk} = \frac{(52 - 45)\text{mls} \times 0.02\text{N} \times 50,000 \frac{\text{mgCaCO}_3}{\text{eq}}}{100 \text{ mls}} = 70 \frac{\text{mg}}{\text{L}} \text{ as CaCO}_3$$

- Inorganic Carbon and Alkalinity

$$\text{Alk} = [\text{HCO}_3^-] + 2[\text{CO}_3^{2-}] + [\text{OH}^-] - [\text{H}^+]$$

- Relating to  $C_T$ ,

$$\text{Alk} = (\alpha_1 + 2\alpha_2)C_{T,C} + [\text{OH}^-] - [\text{H}^+]$$



- At near neutral pH (6 – 9)

$$\text{Alk} = [\text{HCO}_3^-] = \alpha_1 C_{T,C}$$



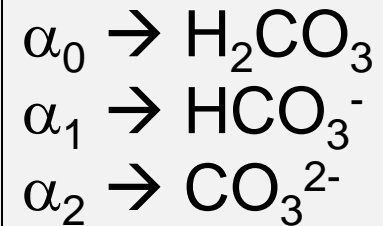
- Acidity is the base neutralization capacity of a water
  - Anything that can donate  $H^+$  (acid) or neutralize  $OH^-$  that is added to water
  - Common Examples include:
    - Inorganic Carbon ( $H_2CO_3$ ,  $HCO_3^-$ )
    - Orthophosphate ( $H_3PO_4$ ,  $H_2PO_4^-$ ,  $HPO_4^{2-}$ )
    - Ammonium Ion ( $NH_4^+$ )
    - Also  $H^+$  at pH
  - The *amount of strong base* required to increase pH to 10.6

- Inorganic Carbon and Acidity

$$\text{Acy} = [\text{H}_2\text{CO}_3] + 2[\text{HCO}_3^-] + [\text{H}^+] - [\text{OH}^-]$$

- Relating to  $C_T$ ,

$$\text{Acy} = (2\alpha_0 + \alpha_1)C_{T,C} + [\text{H}^+] - [\text{OH}^-]$$



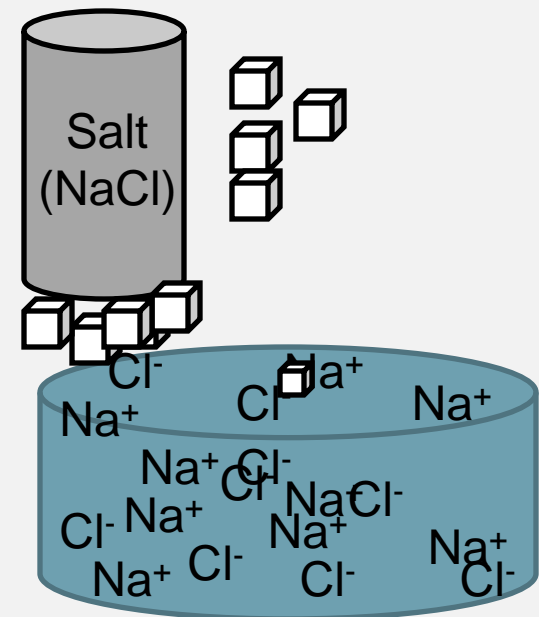
- Alkalinity and Acidity are related

$$\text{Alk} + \text{Acy} = 2(C_{T,C})$$

# Solubility & Precipitation

- Dissolving and forming of solids
- Example: table salt in water
  - Ability to dissolve is a function of:
    - Concentration
    - Temperature
    - pH (not here but sometimes)
    - Ionic strength
- Other examples
  - Sweep Floc formation
  - Sweet Tea
  - Lime Softening

Disolution  
 Undersaturation  
 Supersaturation  
 Precipitation



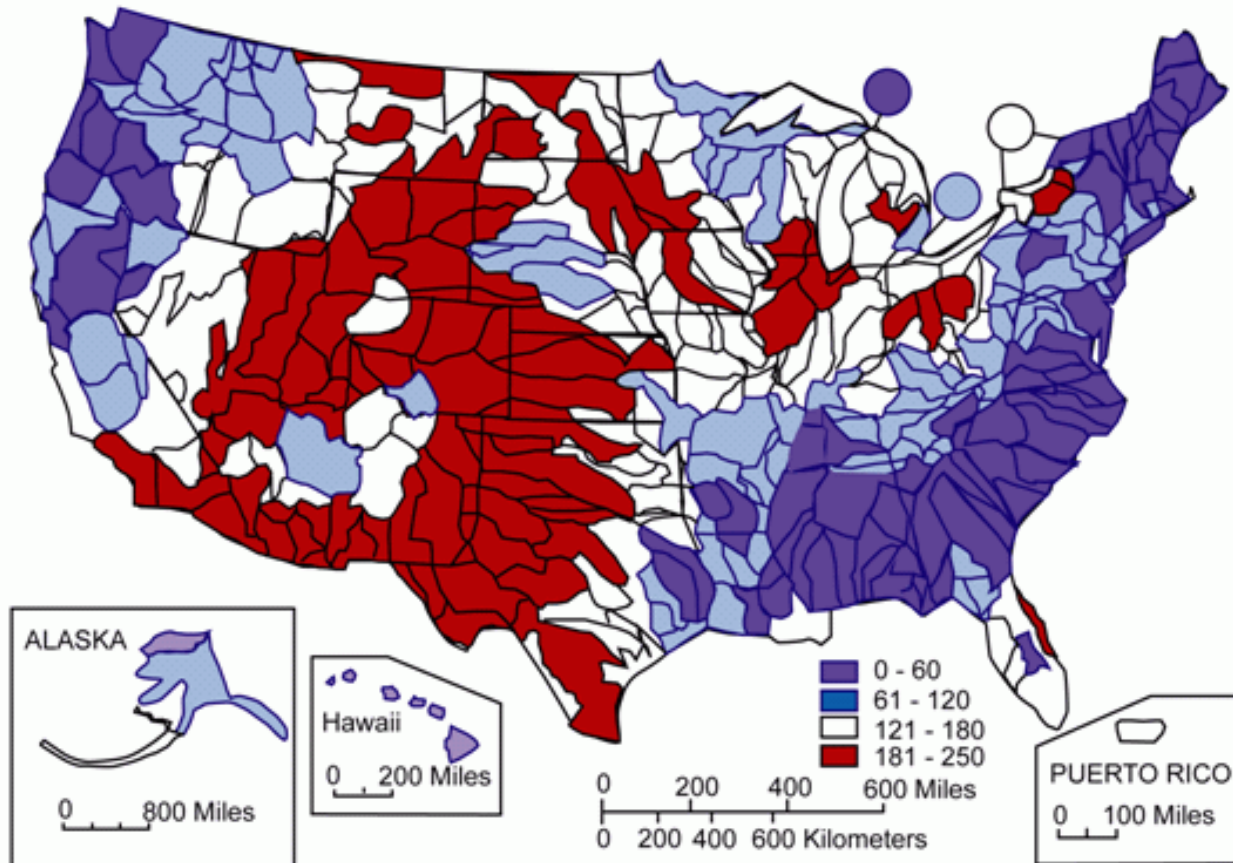
# Solubility & Precipitation

- Solubility Equilibria
  - $A_xB_y (s) \leftrightarrow xA^{y+} + yB^{x-}$
  - $K_{sp} = [A^{y+}]^x [B^{x-}]^y$
- If  $[A^{y+}]^x [B^{x-}]^y > K_{sp}$ , precipitation can occur
- Some  $K_{sp}$  values:

Solid	Formula	$K_{sp}$
Aluminum hydroxide	$Al(OH)_3$	$2 \times 10^{-32}$
Calcium carbonate (calcite)	$Ca(CO_3)_2$	$3.31 \times 10^{-9}$
Calcium hydroxide	$Ca(OH)_2$	$5.02 \times 10^{-6}$
Ferrous carbonate	$FeCO_3$	$3.13 \times 10^{-11}$
Ferrous hydroxide	$Fe(OH)_2$	$4.87 \times 10^{-17}$
Ferric hydroxide	$Fe(OH)_3$	$2.79 \times 10^{-39}$
Lead(II) hydroxide	$Pb(OH)_2$	$1.43 \times 10^{-20}$
Lead (VI) hydroxide	$Pb(OH)_4$	$3.2 \times 10^{-66}$
Magnesium hydroxide (amorphous)	$Mg(OH)_2$	$3.91 \times 10^{-11}$

# Water Hardness

CONCENTRATION OF HARDNESS AS CALCIUM CARBONATE,  
IN MILLIGRAMS PER LITER



# Water Hardness

- Presence of multivalent cations in water (expressed as mg/L as  $\text{CaCO}_3$ )
  - $\text{Ca}^{2+}$
  - $\text{Mg}^{2+}$
- Aesthetic water issues with hardness
  - Scaling of boilers, water heaters, washing machines, coffee pots, and other hot water appliances
  - Conservation of soaps and detergents
  - Taste
    - (Durand and Dietrich, 2009)

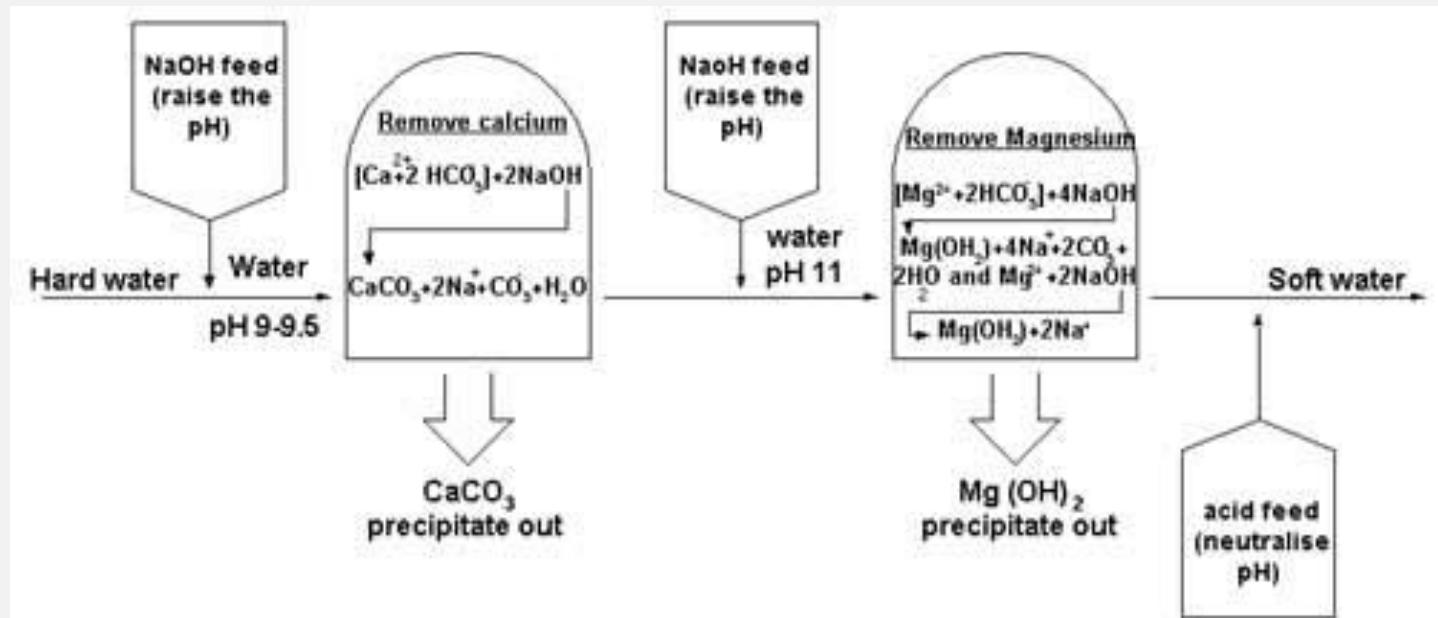
Hardness (mg/L as $\text{CaCO}_3$ )	Degree of Hardness
0 – 75	Soft
75 – 150	Moderately hard
150 – 300	Hard
> 300	Very hard

# Water Hardness

- Carbonate vs. non-carbonate hardness
  - Operational definition tied to alkalinity
  - Addition of lime will change bicarbonate to carbonate, which will precipitate  $\text{Ca}^{2+}$  or  $\text{Mg}^{2+}$ 
    - Carbonate Hardness
  - When alkalinity exhausted,  $\text{Ca}^{2+}$  and  $\text{Mg}^{2+}$  cannot precipitate with lime addition.
    - Noncarbonate Hardness
    - The addition of sodium carbonate or carbon dioxide can remove NCH

# Water Hardness

- Treatment involves precipitation, ion exchange, or membrane softening/desalting
- Precipitative Softening



Water softening by the chemical precipitation process



# Water Hardness Example

- Question 1: Estimate residual  $\text{Mg}^{2+}$  in lime-treated water assuming it is limited by  $\text{Mg}(\text{OH})_2(\text{s})$  solubility, and the pH is 11.0
- Question 2: How much hardness remains in solution (as  $\text{CaCO}_3$ )?

# Water Hardness Example

- Question 1 Solution:
- $K_{sp} = [\text{Mg}^{2+}][\text{OH}^-]^2 = 3.91 \times 10^{-11}$
- $\text{pH} = 11 \rightarrow \text{pOH} = 14 - 11 = 3$
- $-\log[\text{OH}^-] = 3 \rightarrow [\text{OH}^-] = 10^{-3}$
- $[\text{Mg}^{2+}][1 \times 10^{-3}]^2 = 3.91 \times 10^{-11}$
- $[\text{Mg}^{2+}] = 3.91 \times 10^{-11} / 1 \times 10^{-6} = 3.91 \times 10^{-5} \text{ M}$
- $3.91 \times 10^{-5} \text{ M} = 0.95 \text{ mg/L Mg}^{2+}$

# Water Hardness Example

- Question 2 solution:
- Only hardness that remains is 0.95 mg/L  $\text{Mg}^{2+}$

$$\frac{0.95 \text{ mg Mg}^{2+}}{\text{L}} \times \frac{1 \text{ mmol Mg}^{2+}}{24.3 \text{ mmol Mg}^{2+}} \times \frac{2 \text{ meq Hardness as CaCO}_3}{2 \text{ meq Mg}^{2+}} \times \frac{100 \text{ mg Hard}}{\text{meq Hard}}$$

- 3.91 mg/L Hardness as  $\text{CaCO}_3$

# SECTION 2: LABORATORY PRACTICES AND CALCULATIONS

Dimensional Analysis

Making Dilutions / Serial Dilutions

Making Standards from Stock Solutions

Titration

# Dimensional Analysis

- The technique of converting between units
  - Conversion factor – an equation (or fraction) to relate two units
  - Easy Examples:
    - How many milligrams are in a gram?
    - How many centimeters are in a meter?

# Dimensional Analysis

- Step-by-step instructions for converting units
  1. On the left, write the unit you are looking for (box it off)
  2. Next to that, write the value and unit you are given
  3. Choose a conversion factor that relates those 2 units
  4. Place the value with the unit you *have* on the bottom and the value with the unit you are *looking for* on top
  5. Cancel units
  6. Multiply by the numerator, divide by the denominator
  7. Write your answer on the right side, circle it

# Dimensional Analysis

- A more complicated example
  - How many milliliters are in a 17 gallons?

milliliters    17 gallons

- Conversion Factor: 1 gallon = 3785 milliliters

17 gallons	3785 milliliters	64,345 milliliters
	1 gallon	

# A fun (and expensive) example

- The grain is a strange British unit that is still in use today.
  - It derives from the average mass of a barley seed, with 1 gram consisting of 15.43 grains.
  - The carat (as in diamonds!) is the average mass of a carob seed and equal to 3 grain.
- Question: What is the mass in grams of a 1.0 carat diamond?
  - How many Carbon atoms are in a 1.0 carat diamond ring?





# Solution

- How many grams in a 1 carat diamond

grams

1 carat

- Conversion factor: carat  $\rightarrow$  grain  $\rightarrow$  grams
  - 1 carat = 3 grains
  - 1 gram = 15.43 grains

1 carat	3 grains	1 gram	0.19 grams
	1 carat	15.43 grain	

- 1 mol = 12 grams C =  $6.023 \times 10^{23}$  C atoms
  - 1 carat =  $9.5 \times 10^{21}$  C atoms

# Making Dilutions / Serial Dilutions

- A few terms and conversions of note
  - milligram = mg = 1/1000 of a g or  $10^{-3}$  g
  - gram = g
  - kilogram = kg = 1000 g or  $10^3$  g
  - mole =  $6.023 \times 10^{23}$  molecules
  - molarity = moles per liter
    - molar = M = term used to discuss molarity of solutions
  - millimole = 1/1000 of a mole
    - millimolar = mM = term used to discuss molarity in thousandths of a mole
  - w/v = weight (of a solute) per final solution volume
  - v/v = volume (of reagent) per final solution volume

# Simple Dilution

- Aka – dilution factor method
  - A unit volume of a liquid material of interest is combined with an appropriate volume of solvent liquid to achieve the desired concentration
  - Example
    - 1:5 dilution = 1 unit of diluent + 4 units of solvent

# Simple dilution examples

- How do you make a 1:300 dilution of a bacillus spore sample?
  - 1 ml of sample + 299 mls solvent
- How do you dilute a 20X solution to the appropriate concentration before use?
  - Needs to be diluted 1:20
  - 1ml of 20x solution + 19ml of solvent

# Mixing parts or volumes

- Different from simple dilution
- Example
  - Instructions state to mix 1 part acid with 3 parts water. What do you do and how is this different from simple dilution?
  - Solution
    - Literally mix 1 unit acid (eg 1 L) with 3 units water (eg 3 L)
    - Creates 1:4 dilution
- **Remember: Always add acid to water!!!**

# Why not to add water to acid

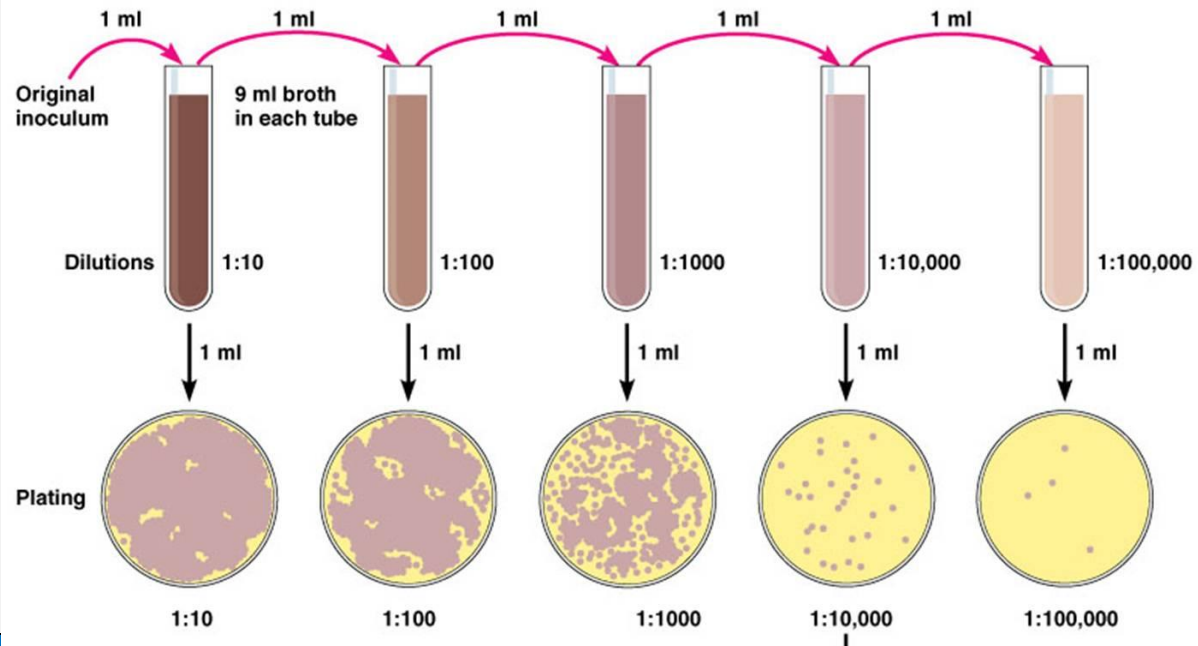
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“Safety Third, is what I always say”

# Serial Dilution

- A series of simple dilutions with amplifies the dilution factor quickly beginning with a small initial quantity of material
  - Good for DNA, RNA, enzyme work
- Final dilution factor (DF) =  $DF1 \times DF2 \times DF3, \text{ etc.}$



# Making Standards from Stock Solutions

- Often need to make fixed volumes of solutions of known concentrations from stock solutions
  - Expensive, limited materials
  - Waste concerns
  - Good lab practice
  - Convenience
- Key equation:
  - $V_1C_1 = V_2C_2$



# Percent Solutions

- Aka, parts per hundred
  - With a dry chemical, it is mixed as *dry mass per volume*, where  $\text{g}/100\text{ml} = \% \text{ concentration}$
  - With liquid reagents,  $\% \text{ concentration}$  based upon *volume per volume* =  $\text{ml}/100\text{ml}$
- Dry chemical example:
  - 3% w/v NaCl
  - Dissolve 3.0 g NaCl in 100ml water
- Liquid chemical example:
  - 70% v/v ethanol
  - Mix 70ml of 100% ethanol with 30ml water
- **Need to keep in mind specific gravity for “non-water like” liquids**

# Example

- Dilute on-site alum for jar tests



# Step 1: Calculating Chemical Concentrations

On-site Alum:

48.5% strength, specific gravity (S.G.) = 1.33

**Stock Concentration (mg/mL) =**  
**Percent Strength x S.G. x 10**

Alum stock =  $48.5 \times 1.33 \times 10$   
= 645 mg/mL  
= 645,000 mg/L



# Making Secondary Stocks

Alum Primary Stock = 645 mg/mL

Need a secondary stock of 20 mg/mL (500 mL)

$$X * 645 = 20 * 500$$

$$X = 15.5 \text{ mL}$$

Add 15.5 mL of primary stock to a 500 mL volumetric flask

Secondary alum stocks are usually good for 2 hours  
(will hydrolyze after that)

# Using Secondary Stock for Jar Tests

Alum Secondary Stock = 20 mg/mL

Each Square Beaker = 2 Liters

1 mL of secondary stock to a beaker:

$$= 20 \text{ mg/mL} * 1 \text{ mL} / 2 \text{ L}$$

$$= \text{Alum Dose of } 10 \text{ mg/L}$$



# Side Note: Calculating Chemical Feed Rates

Alum Primary Stock = 645 mg/mL = 645,000 mg/L

Pilot Plant Flow Rate = 15 gpm

Required Alum Dose = 40 mg/L

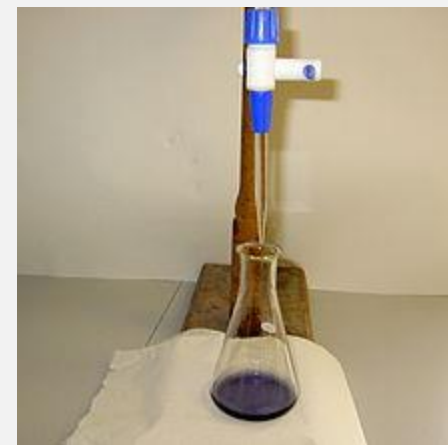
$$\begin{aligned}\text{Alum Pump Setting} &= \frac{15 \text{ gal/min} \times 3.785 \text{ L/gal} \times 40 \text{ mg/L}}{645,000 \text{ mg/L}} \\ &= 0.0035 \text{ L/min} \\ &= 3.5 \text{ mL/min}\end{aligned}$$

# Polymer Stocks

- Polymers are usually very viscous (liquids)
- Easier to weigh them rather than dispense a volume
- Say, need to make a polymer stock of 0.2 mg/mL (500 mL volume):
  - Amount of polymer required =  $0.2 \times 500 = 100$  mg
  - Polymer doses are usually as product
  - Weigh 100 mg of polymer, transfer to a 500 mL volumetric flask, and make the volume to 500 mL

# Titrations

- Common laboratory method of quantitative chemical analysis that is used to determine the unknown concentration of an identified analyte.
- Types of Titrations:
  - Acid/Base
  - Redox
  - Complexation
  - Zeta Potential
  - Assay





- Several Examples

- Alkalinity

- Acid Value

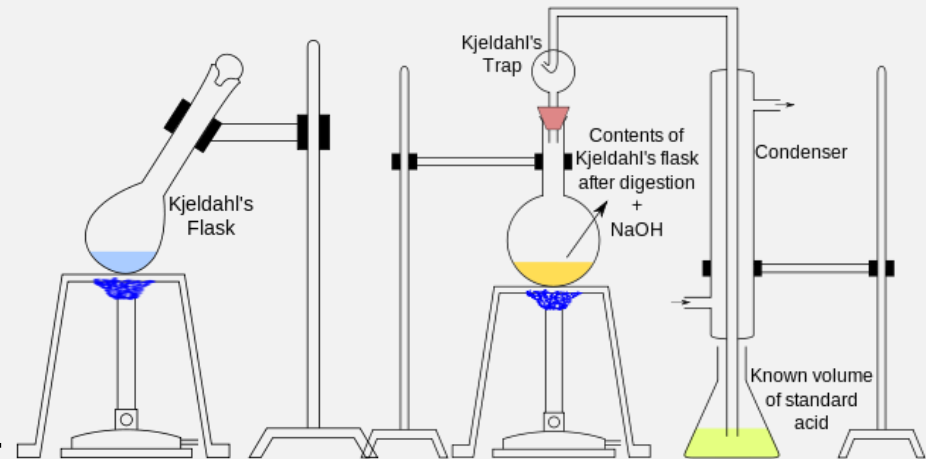
- Kjeldahl Nitrogen

- Dissolved Oxygen

- Benedict's reagent

- Bromine/Iodine Number

- See Alkalinity discussion in Section 1 for example calculations



# SECTION 3: STATISTICS

Precision and Accuracy

Significant Figures

Basic Statistics

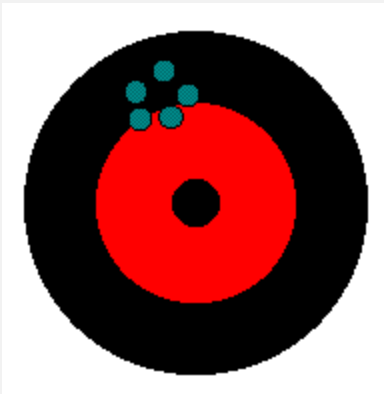
Calculating RPD/RSD/% recovery

Understanding the 95% Confidence Interval

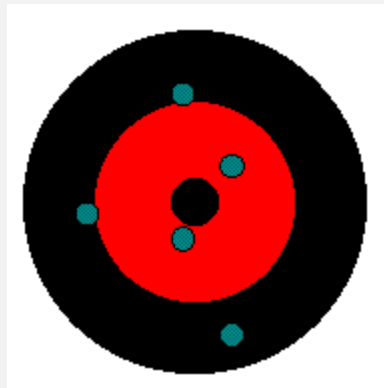
The theory behind correlation coefficients

# Precision and Accuracy

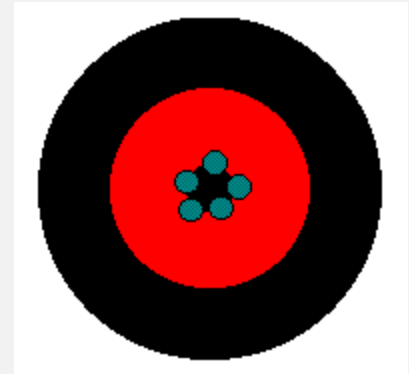
- Precise: After taking a lot of measurements, you notice that they are all very close to each other.
- Accurate: After taking a lot of measurements, you find they agree with the true value
- Dart Board Example:



Precise, not accurate



Neither Precise nor accurate



Precise and accurate

# Significant Figures

- Only report numbers with the correct degree of precision
- Significant Figures are the digits in a number which are known precisely, plus one estimated digit

What is the temperature, to the correct significant digit?

18.5°C

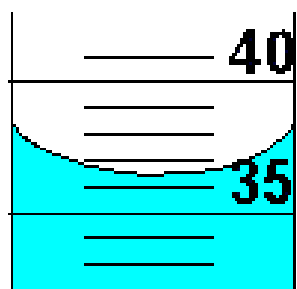


# Counting Significant Figures

- Only a few simple rules
  - All non-zero numbers *are* significant
    - Eg (sig figs in parenthesis): 251 (3), 13.49 (4), 8765.1 (5)
  - Zeros *between* significant digits are significant
    - Eg: 305 (3), 42003 (5), 70201 (5)
  - If there is no decimal point, than *trailing* zeros are *not* significant
    - Eg: 470 (2), 10 (1), 6,000 (1)
  - If a number is less than one, then the first significant figure is the first *non-zero* digit after the decimal point
    - Eg: 0.009 (1), 0.156 (3), 0.01060 (4)

# Measuring with Significant Figures

- Using the correct number of significant digits shows how precise your measurement was
- Result should have all of the digits which you're sure of and then one estimated digit
- What is the volume in the graduated cylinder below:



36.5°C

# Calculating with Significant Figures

- When performing calculations involving lab results, must be aware of significant figures:
- Follow these rules:
  - When adding or subtracting, the number of digits to the right of the decimal point in the answer is equal to the number of digits to the right of the decimal point in the number with the least such digits
  - When multiplying or dividing, the answer should have the same number of sig figs, as the number with the fewest sig figs.

# Calculating with Significant Figures

- Give the answers to the following, with the correct number of significant figures:

$$\begin{array}{r} 36.45 \\ + 1.467 \\ \hline 37.91 \end{array}$$

$$45 \times 21.31111 = 960$$

$$9,081.3 \div 3 = 3,000$$

$$\begin{array}{r} 1.7 \\ - 0.1357 \\ \hline 1.6 \end{array}$$



# Basic Statistics

- Populations, Parameters, Samples, Variables and Statistics

## Population

the aggregate of all arbitrarily defined sample units

- Parameters

constants that describe the population as a whole

## Samples

an aggregation of sample units

- Variables

a characteristic that may vary from one sample to the next

- Statistic

parameter of a sample (sample distribution)

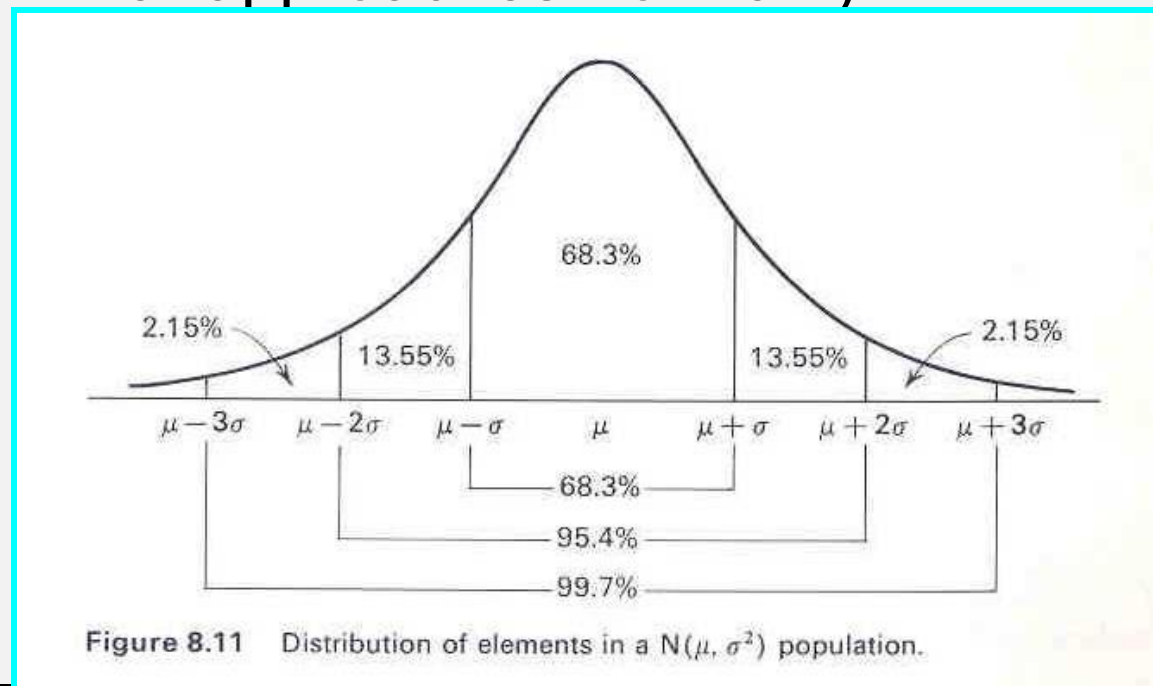
# Basic Stats

- What's a sample?
  - Any subset (or collection) of units from a population of units
    - May be the units themselves (e.g., A handful of Reeses pieces from a large jar of pieces)
  - or more often
    - A measure of the units (e.g., a list of heights from 10 trees occurring in a stand of 1200 trees).
- Samples that are selected at random, or similar to random, may lend themselves to statistical analysis

- **Why sample?**
  - Measuring all units (trees, recreationists, birds, etc.) is impractical, if not impossible.
  - Sampling just a few units saves money.
  - Sampling just a few units saves time.
  - Some measurements are destructive:
    - cutting down trees to inspect ring patterns or stem analysis
    - capturing wildlife to examine their morphology, etc.
- **Sampling makes statistical methods attractive and powerful.**

# Basic Stats – Frequency Distribution

- Normal Distribution
  - Most commonly used by scientists and lab practitioners
  - Means of large samples are expected to have a distribution that approaches normality



# Basic Stats – Frequency Distribution

## Useful Statistics:

Mean,  
Standard Error  
Coefficient of Variance  
Standard Deviation  
Confidence

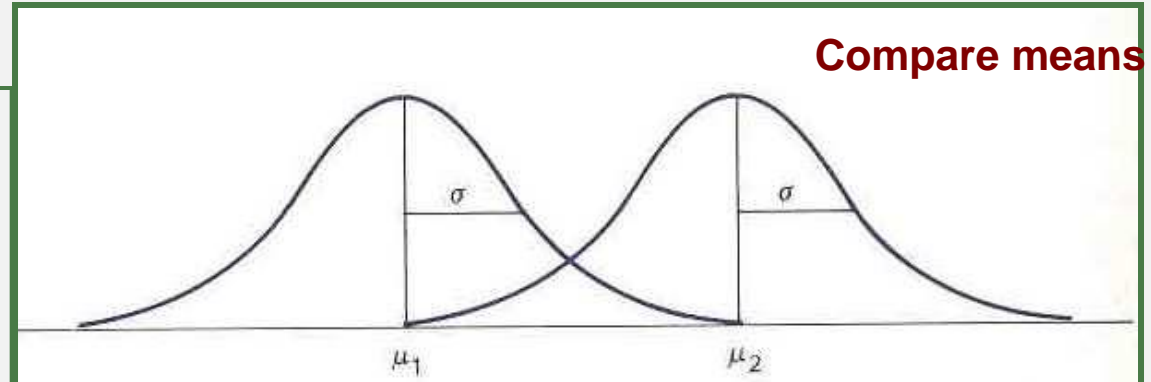


Figure 8.9 Normals with different means, same variance.

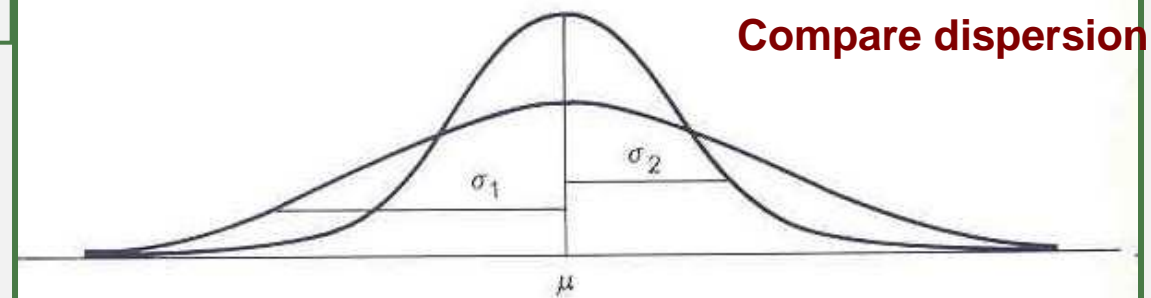


Figure 8.10 Normals with different variances, same mean.

# Basic Stats – Frequency Distribution

- There are other distributions useful in statistics, including:
  - Chi Square
  - F
  - Student's t-distribution
  - Binomial
  - Negative Binomial
  - Gamma
- Sometimes you may see the Normal referred to as Gaussian.
- We will not consider other distributions in this workshop

- Terms

**$n$  = total number of samples**

**$x$  = sample**

---

**$\bar{x}$  (or  $\mu$ ) = sample average**

---

**$s^2$  = variance**

**$s$  (or  $\sigma$ ) = standard deviation**

---

**$CV$  = coefficient of variation**

**$s_{\bar{x}}$  or  $SE$  = standard error of the mean**

# Basic Stats - Computations

- Mean (the average)
- Median (the middle value)
- Mode (the most frequently appearing value)

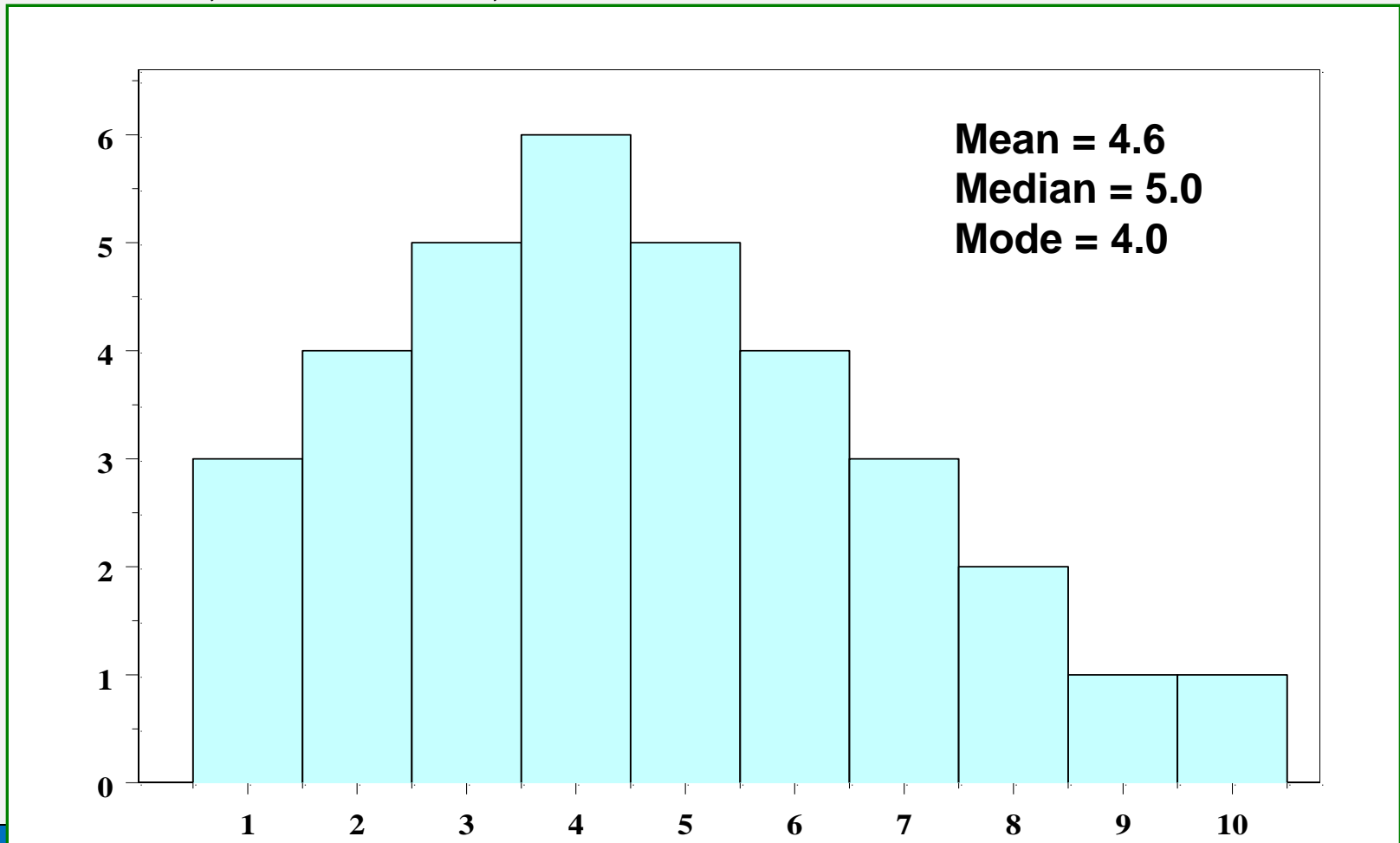
$$\bar{x} = \left( \frac{\sum x}{n} \right)$$

Mean



# Basic Stats – Computations

- Mean, Median, and Mode



# Basic Stats – Computations

- Range
  - easy to compute
  - fails to take into account how the data are distributed

$$\mathbf{Range} = \mathbf{x}_{(max)} - \mathbf{x}_{(min)}$$

# Basic Stats – Computations

- **Measures of Dispersion**

## **Variance**

- A measure of dispersion among individual observations about their average value
- Computed before the standard deviation

$$s^2 = \left( \frac{\sum x^2 - ((\sum x)^2 / n)}{n-1} \right)$$

## **Standard deviation**

- Another measure of dispersion
- 68% of observations should be within  $\pm 1$  standard deviation of the mean
- 95% will be within 1.96 standard deviations

$$s = \sqrt{\frac{\sum x^2 - ((\sum x)^2 / n)}{n-1}}$$

# Basic Stats – Computations

- Alternate formulas for variance and standard deviation

**Variance**

$$s^2 = \left( \frac{\sum (\bar{x} - x)^2}{n - 1} \right)$$

**Standard deviation**

$$s = \sqrt{\left( \frac{\sum (\bar{x} - x)^2}{n - 1} \right)}$$

# Basic Stats – Computations

- Coefficient of variation
  - Permits a comparison of relative variability about means of different sizes, and data collected from different populations
  - A relative measure, expressed as a percentage the ratio of the standard deviation to the mean of a sample

$$CV = \left( \frac{s}{\bar{x}} \right) 100\%$$

# Basic Stats – Computations

- Standard error of the mean
  - A measure of variation of among sample means (place different means on a common reference) calculated from the same population
  - Used to help produce confidence limits, or to determine required sample sizes for a sampling effort
  - Assumes random sampling from an infinite population

$$SE = \left( \sqrt{\frac{s^2}{n}} \right)$$

$$SE = \left( \frac{s}{\sqrt{n}} \right)$$

# Basic Stats – Computations

- Standard error of the mean in percent
  - A measure of variation of among sample means (place different means on a common reference)

$$SE\% = \left( \frac{CV}{\sqrt{n}} \right)$$

# Basic Stats - Example

Sample of student ages:  
19, 21, 21, 24, 30, 20

Sample size:  
 $n = 6$

$n-1 = 5$   
 $\sqrt{n} = 2.45$

1

Mean age:

$$\text{mean} = 19 + 21 + 21 + 24 + 30 + 20 / 6$$

$$\text{mean} = 22.5$$

2

Variance of the mean:

$$s^2 = 81.5 / 5$$

$$s^2 = 16.3$$

3

Standard Deviation of the mean:

$$\text{SD} = \sqrt{16.3}$$

$$\text{SD} = 4.04$$

4

Standard Error of the mean:

$$\text{SE} = 4.04 / 2.45$$

$$\text{SE} = 1.65$$

5

Coefficient of Variation:

$$\text{CV} = 4.04 / 22.5$$

$$\text{SE} = 0.18 \text{ or } 18\%$$

x	n
19	6
21	6
21	6
24	6
30	6
20	6



# Calculating RPD/RSD/% recovery

- RPD = Relative Percent Difference
- RSD = Relative Standard Deviation
- % Recovery = Percent of known *spiked* analyte concentration measured upon analysis

- The RPD is used with two measurements exist.
  - Expresses the precision of duplicates

$$\text{RPD} = \frac{|X_1 - X_2|}{\bar{X}} * 100$$

- $x_1$  = measurement #1
- $x_2$  = measurement #2
- $\bar{x}$  = mean of measurements

- The RSD is used when there are at least three measurements
  - Expresses the precision of measurements

$$\text{RSD} = \frac{S}{\bar{X}} * 100$$

- $S$  = standard deviation
- $\bar{X}$  = mean of measurements

# % Recovery

- Measures the accuracy of a technique

$$\%R = \frac{(SSR - SR)}{SA} * 100$$

- SSR = measured value of spiked sample
  - SR = measured value of sample
  - SA = known value of spike added
- If there is no sample contribution to the measured values,  $SR = 0$

$$\%R = \frac{SSR}{SA} * 100$$

# Understanding 95% Confidence Intervals 113

- If the population is normally distributed, the Central Limit Theorem indicates that 95% of all *sample means* are within 2 Standard Errors (SE) of the *population mean*
- Assuming a Normal Distribution, 95% confidence intervals can be calculated using the sample mean and sample standard deviation

# Calculating 95% Confidence Intervals

- Step 1: Calculate Standard Error (SE)
  - S = sample standard deviation
  - N = sample size

$$SE = \frac{s}{\sqrt{N - 1}}$$

- Step 2:

$$CI\ 95\% = \bar{X} \pm t * (SE)$$

- X = sample mean
- t = 1.96 for normal distribution, 95% CI

# 95% CI Example

Sample mean = 60.5

Sample standard deviation = 24

Sample size N = 101

Calculate the 95% CI

$$SE = \frac{s}{\sqrt{N-1}} = \frac{24}{\sqrt{101-1}} = \frac{24}{\sqrt{100}} = \frac{24}{10} = 2.4$$

95% Confidence Interval:

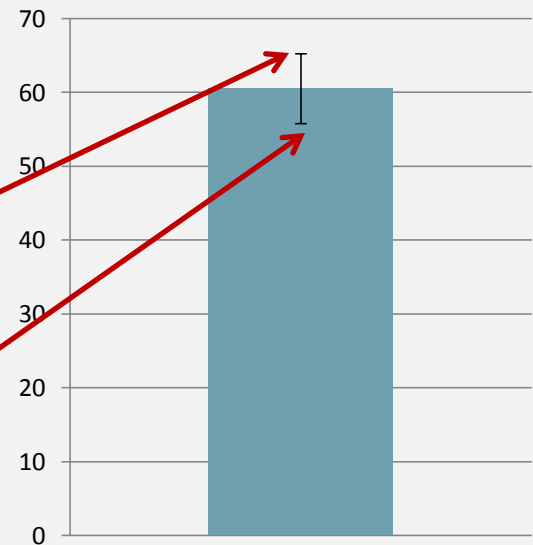
$$60.5 \pm (1.96 * 2.4) = 60.5 \pm 4.7$$

The upper bound of the interval is:

$$60.5 + (1.96 * 2.4) = 65.2$$

The lower bound of the interval is:

$$60.5 - (1.96 * 2.4) = 55.8$$



# Scatter Plots and Correlation

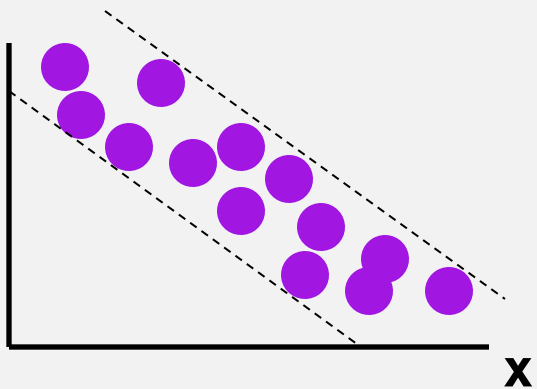
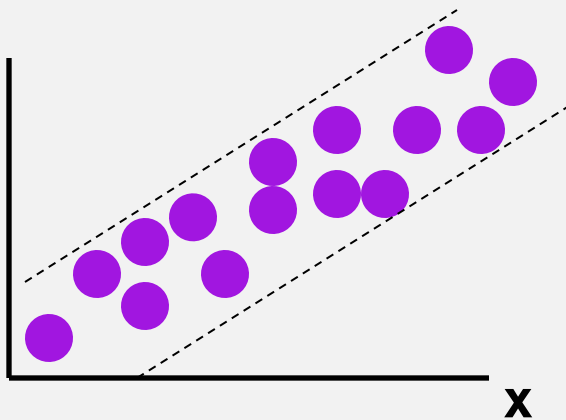
- A **scatter plot** (or scatter diagram) is used to show the relationship between two variables
- **Correlation** analysis is used to measure strength of the association (linear relationship) between two variables
  - Only concerned with strength of the relationship
  - No causal effect is implied



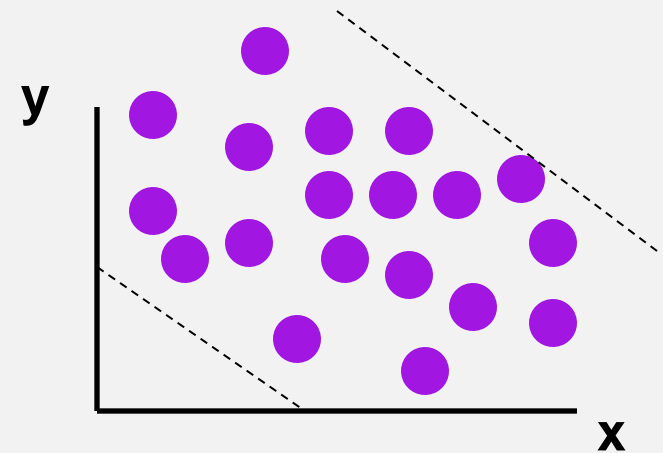
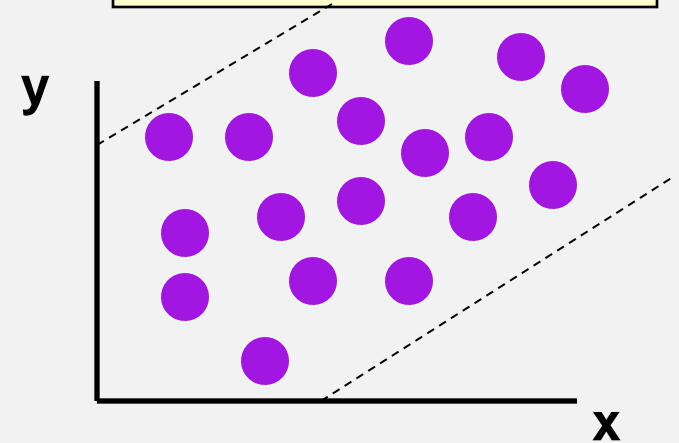
# Scatter Plot Examples

*(continued)*

**Strong relationships**



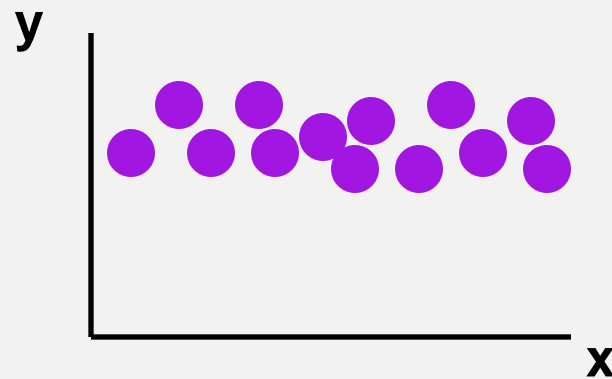
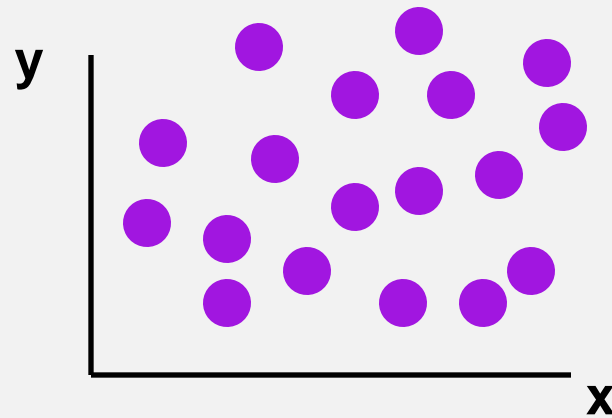
**Weak relationships**



# Scatter Plot Examples

*(continued)*

No relationship



# Correlation Coefficient

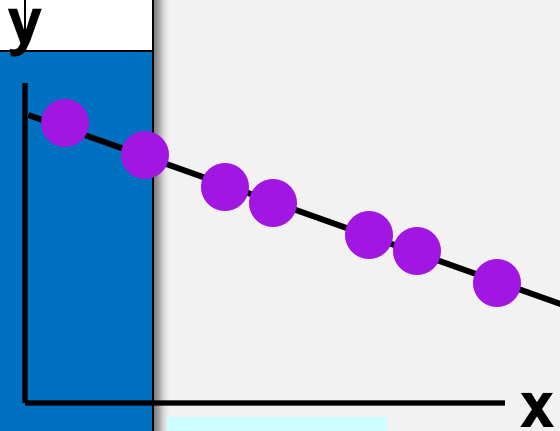
*(continued)*

- The **population correlation coefficient  $\rho$**  (rho) measures the strength of the association between the variables
- The **sample correlation coefficient  $r$**  is an estimate of  $\rho$  and is used to measure the strength of the linear relationship in the sample observations

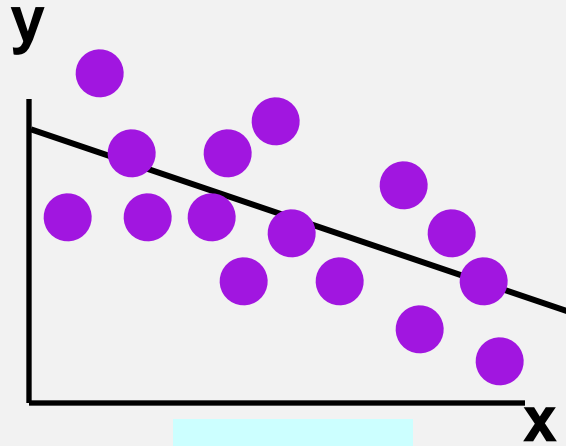
# Features of $\rho$ and $r$

- Unit free
- Range between -1 and 1
- The closer to -1, the stronger the negative linear relationship
- The closer to 1, the stronger the positive linear relationship
- The closer to 0, the weaker the linear relationship

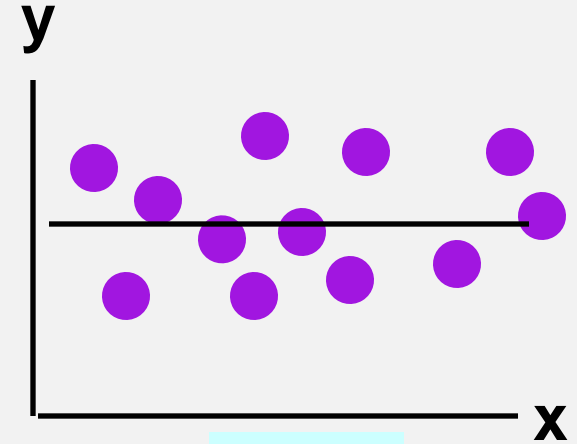
# Examples of Approximate $r$ Values



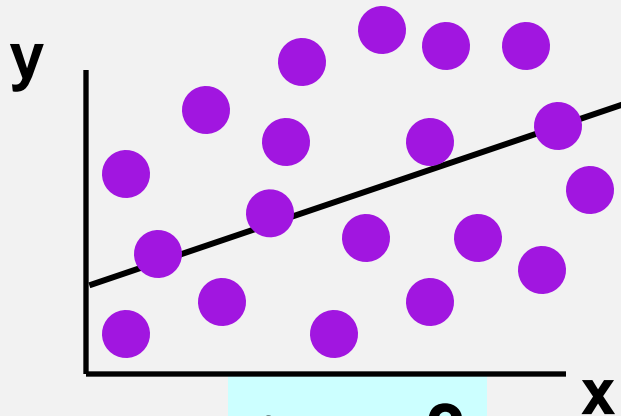
$r = -1$



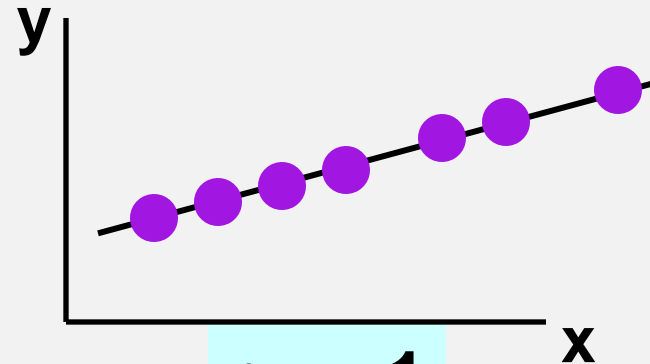
$r = -.6$



$r = 0$



$r = +.3$



$r = +1$

# Calculating the Correlation Coefficient

Sample correlation coefficient:

$$r = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sqrt{[\sum (x - \bar{x})^2][\sum (y - \bar{y})^2]}}$$

or the algebraic equivalent:

$$r = \frac{n \sum xy - \sum x \sum y}{\sqrt{[n(\sum x^2) - (\sum x)^2][n(\sum y^2) - (\sum y)^2]}}$$

where:

$r$  = Sample correlation coefficient

$n$  = Sample size

$x$  = Value of the independent variable

$y$  = Value of the dependent variable

# Coefficient of Determination, $R^2$

- The **coefficient of determination** is the portion of the total variation in the dependent variable that is explained by variation in the independent variable
- The coefficient of determination is also called **R-squared** and is denoted as  $R^2$

$$R^2 = \frac{SSR}{SST}$$

where  $0 \leq R^2 \leq 1$

# Coefficient of Determination, $R^2$

*(continued)*

## Coefficient of determination

$$R^2 = \frac{SSR}{SST} = \frac{\text{sum of squares explained by regression}}{\text{total sum of squares}}$$

**Note:** In the single independent variable case, the coefficient of determination is

$$R^2 = r^2$$

where:

$R^2$  = Coefficient of determination

$r$  = Simple correlation coefficient



- Total variation is made up of two parts:

$$\text{SST} = \text{SSE} + \text{SSR}$$

Total sum of Squares

Sum of Squares Error

Sum of Squares Regression

$$\text{SST} = \sum (y - \bar{y})^2$$

$$\text{SSE} = \sum (y - \hat{y})^2$$

$$\text{SSR} = \sum (\hat{y} - \bar{y})^2$$

where:

$\bar{y}$  = Average value of the dependent variable

$y$  = Observed values of the dependent variable

$\hat{y}$  = Estimated value of  $y$  for the given  $x$  value

*(continued)*

- SST = total sum of squares

- Measures the variation of the  $y_i$  values around their mean  $y$

- SSE = error sum of squares

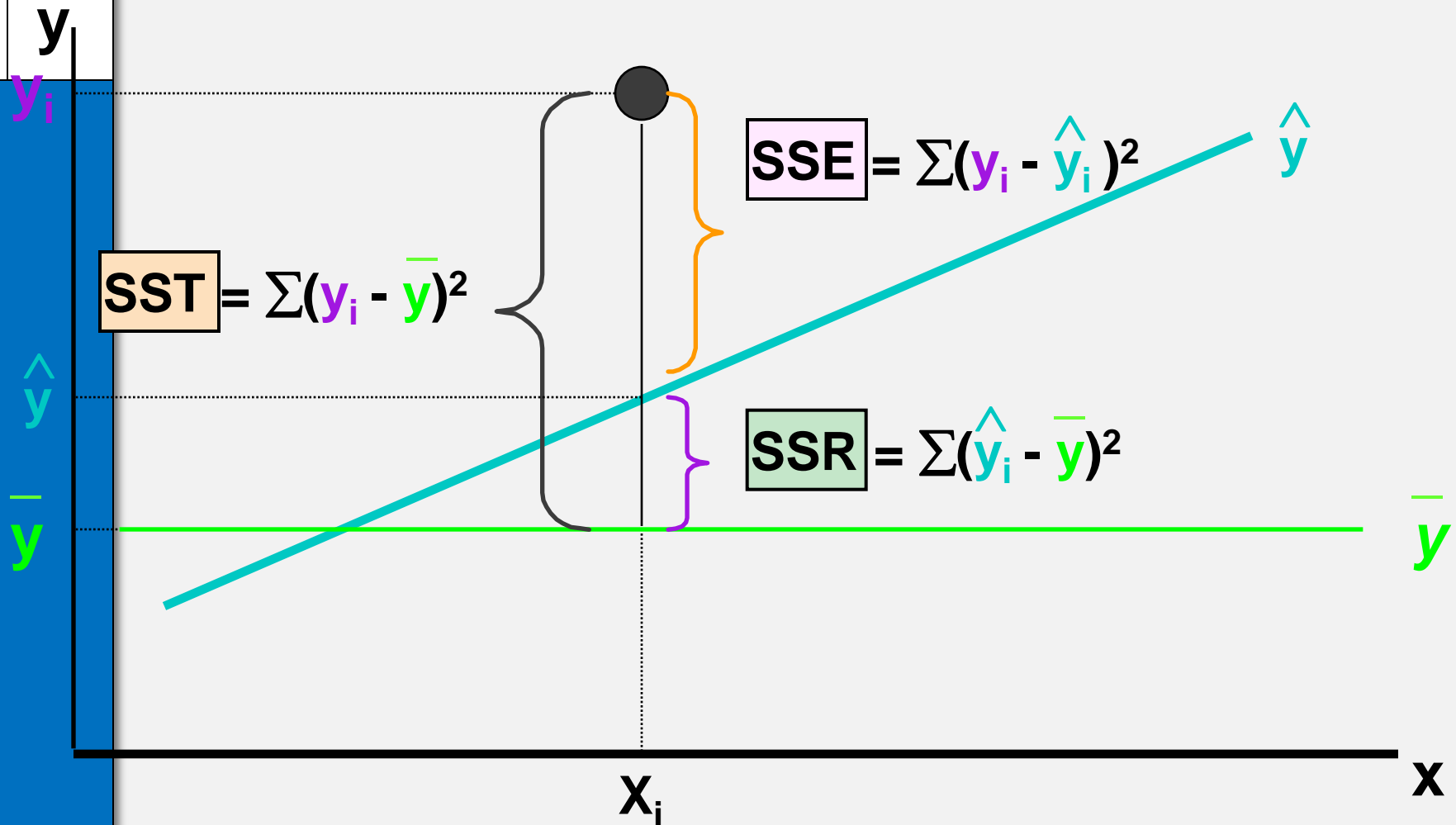
- Variation attributable to factors other than the relationship between  $x$  and  $y$

- SSR = regression sum of squares

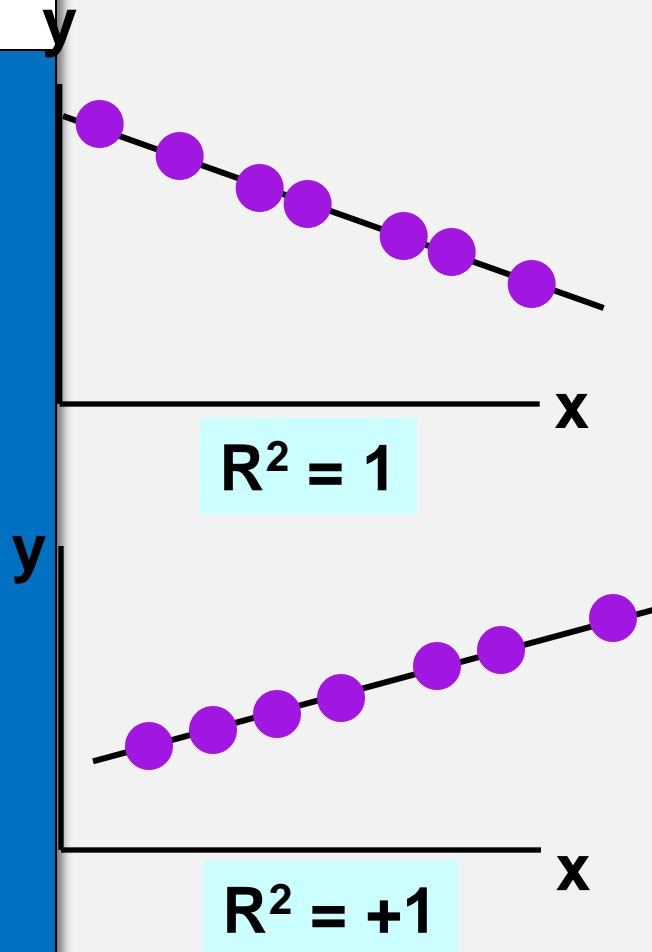
- Explained variation attributable to the relationship between  $x$  and  $y$

# Explained and Unexplained Variation

*(continued)*



# Examples of Approximate $R^2$ Values

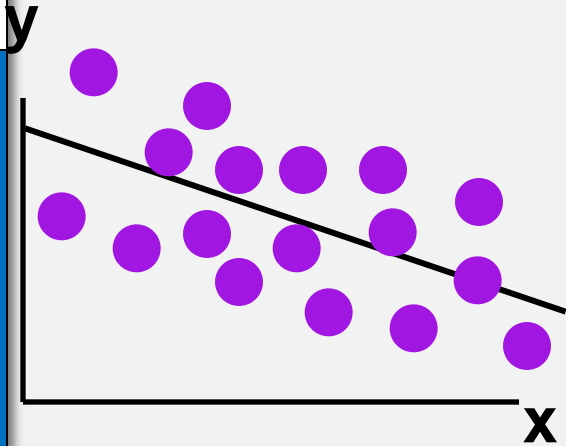


$$R^2 = 1$$

**Perfect linear relationship  
between x and y:**

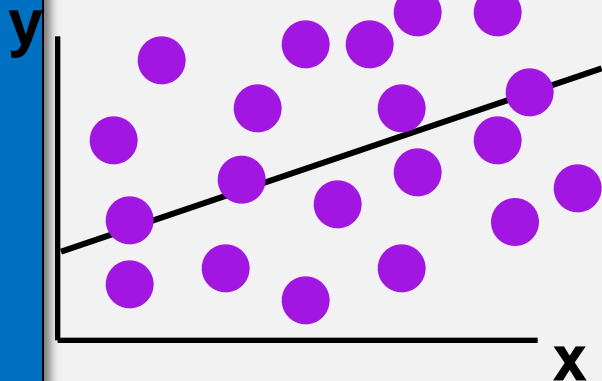
**100% of the variation in y is  
explained by variation in x**

# Examples of Approximate $R^2$ Values



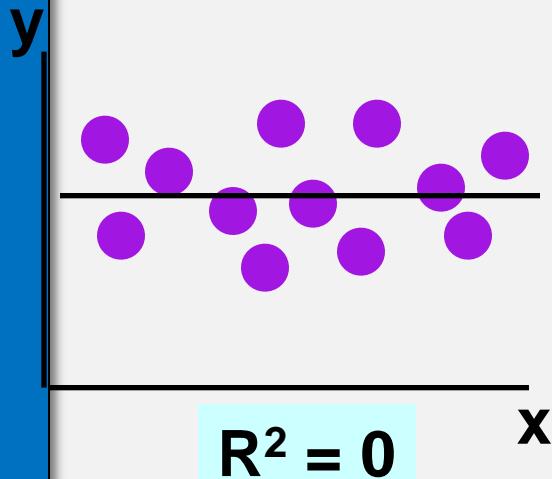
$$0 < R^2 < 1$$

**Weaker linear relationship  
between x and y:**



**Some but not all of the  
variation in y is explained  
by variation in x**

# Examples of Approximate $R^2$ Values



$$R^2 = 0$$

**No linear relationship between  $x$  and  $y$ :**

**The value of  $Y$  does not depend on  $x$ . (None of the variation in  $y$  is explained by variation in  $x$ )**

# Example

- What we will do with the data
  - Plot as a scatter plot
  - Assess mean for replicate data points

# Reference Materials and Acknowledgements

- Aquatic Chemistry by Werner Stumm and James J. Morgan. John Wiley & Sons Publishers, 1996. ISBN 0-471-51184-6
- Water Chemistry by Mark Benjamin. McGraw Hill Publishers, 2002. ISBN 0-072-38390-9
- [www.studyphysics.ca](http://www.studyphysics.ca)
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- Mississippi Genome Exploration laboratory
- UCLA Statistics