The Acoustics of the Clarinet: An Observation of Harmonics, Frequencies, Phases, Complex Specific Acoustic Impedance, and Resonance

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ABSTRACT

Acoustical measurements were conducted on a wooden Buffet R-13 Bb clarinet to determine its characteristic properties and the effects of its structure. Sound recordings of concert F_3, F_4, F_5, and two separate fingerings of F_6 on the tempered scale were made and analyzed to observe the amplitudes, frequencies, and phases of the harmonics. Complex specific acoustic impedance analyses were conducted over a range of frequencies for the same notes as well as along the bore of the instrument for four harmonics of concert F_4. Data from these experiments as well as longitudinal mechanical resonance measurements identified two formants of the clarinet. Each of these experiments aided in producing a larger picture describing the acoustics of the clarinet.

I. Background and Introduction

A. The Clarinet

The clarinet is a woodwind instrument that uses a reed to induce vibrations in the air column and make sound. The reed is made out of an elastic material called *Arundo Donax*, a variety of cane. This reed is attached over the window of the mouthpiece where the clarinetist blows air into the system. As the clarinet is played, a pressure antinode forms in the mouthpiece, which makes the clarinet similar to a pipe closed at one end and open at the other.

Different notes containing several harmonics are produced by opening and closing various tone holes along the bore of the instrument. The register key, located one third of the way down the clarinet from the mouthpiece end, when opened allows the clarinet to play notes a twelfth (i.e. an octave plus a musical fifth) higher than using the same fingering without the register key. Other effective register keys are created by opening other tone holes near the mouthpiece to raise the pitch.[1]

B. Harmonics

Only certain modes of pressure and longitudinal displacement waves occur in various air column configurations. Since the clarinet is similar to a cylindrical pipe with one end open and one end closed, an open-closed pipe, the focus for this section will be on this air column configuration. Figure 1 depicts the first three modes of displacement and pressure waves within an open-closed pipe. The closed end allows for a displacement node and a pressure antinode while the open end allows the opposite. The distance between a node and antinode is a quarter wavelength, so that

$$\lambda_1 = 4L \tag{1.1}$$

where λ_1 is the wavelength for the first wave mode and L is the length of the pipe.

The fundamental frequency f_1 then becomes,

$$f_1 = \frac{c}{4L} \tag{1.2}$$

where c is the speed of sound.

Using wavelengths and pipe lengths to calculate subsequent mode frequencies in terms of the fundamental frequency, a pattern develops. The wavelengths become

$$\lambda_n = \frac{4L}{n} \tag{1.3}$$

and the corresponding frequencies are

$$f_n = nf_1 \tag{1.4}$$

where n is an odd integer. Therefore, only frequencies that are odd integer multiples of the fundamental are produced in an open-closed pipe. The first mode is referred to as the fundamental or first harmonic, which is followed by the third harmonic, fifth harmonic, and so on.[2]

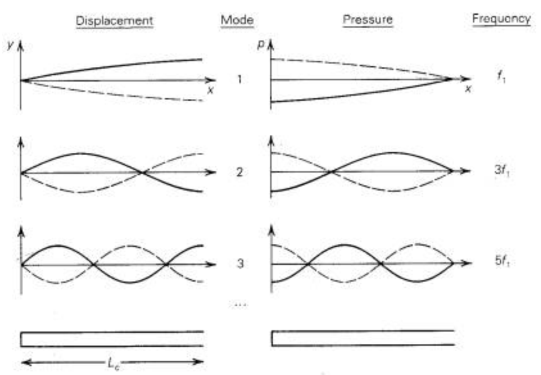

Figure 1: The first three modes of the displacement and pressure waves for an open-closed pipe are pictured including the frequency of each in terms of the fundamental frequency.[3]

C. Inharmonicity

Musical instruments such as the clarinet do not exhibit ideal harmonics because they have a more complex geometry than a cylindrical pipe for example. Inharmonicity is a measure of the deviation of each harmonic from its theoretical frequency. The percent deviation Δ_n is calculated as follows:

$$\Delta_n = 100\left(\frac{n_{exp} - n_{thy}}{n_{thy}}\right) \tag{1.5}$$

where n_{exp} is the experimental ratio between two consecutive harmonic frequencies and n_{thy} is the theoretical ratio between the same consecutive harmonic frequencies.

There are 100 cents between semitones on the tempered musical scale and the inharmonicity can be converted from percent deviation to deviation in cents. If the value of the percent deviation is negative, the conversion uses the equation

$$C_{hi} = 1781.715\left(\frac{n_{exp}}{n_{thy}} - 1\right) \tag{1.6}$$

where C_{hi} is the number of cents high from the theoretical pitch. If the value of the percent deviation is positive, the conversion uses a similar equation

$$C_{lo} = 1681.718\left(\frac{n_{exp}}{n_{thy}} - 1\right) \tag{1.7}$$

where C_{lo} is the number of cents low from the theoretical pitch.[4]

D. Acoustic Impedance

A unique characteristic to every instrument is its specific acoustic impedance. Complex specific acoustic impedance is the ratio of complex pressure to complex particle velocity and is a quality independent of the player of the instrument. Equation 1.8 shows the longitudinal complex specific acoustic impedance $\tilde{Z}(z)$ as a function of complex pressure $\tilde{p}(z)$ and complex particle velocity $\tilde{u}_{\parallel}(z)$ at point z where $0 \leq z \leq L$ and L is a positive distance.

$$\tilde{Z}_{\parallel}(z) = \frac{\tilde{p}(z)}{\tilde{u}_{\parallel}(z)} \tag{1.8}$$

This impedance varies for different frequencies. The clarinet specifically operates at maxima in the impedance, only allowing certain frequencies to be played for each fingering.[5]

II. Method

Four types of experimentation were done on a wooden Buffet R-13 Bb clarinet. These included a harmonic analysis, a frequency spectrum impedance analysis, an analysis of the acoustic impedance along the bore of the clarinet for concert F_4 on the tempered scale, and a spectral analysis of longitudinal mechanical resonance.

A. Harmonic Analysis

Initially, a harmonic analysis was conducted for four octaves of concert F on the tempered scale. These included F_3, F_4, F_5, and two separate fingerings of F_6 as shown in Figure 2.

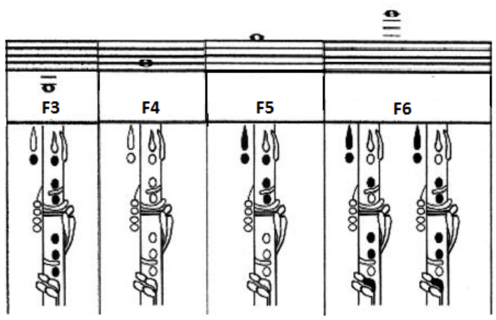

Figure 2: This chart shows the fingerings used for each note used in experimentation where black keys and holes represent finger placement. The key and hole at the top left of each fingering diagram are the thumb hole and register key at the back of the instrument. The notes on the staff are the notes in treble clef that the clarinetist reads for each fingering shown, and the concert pitch is written below each note.

Each note was tuned and played into a Behringer ECM8000 reference microphone for a duration of about 5 seconds, and five digital recordings were taken. These 24-bit recordings were clipped at the beginning and end to attain a steady sample of each recorded note without the transient effects of articulation or amplitude variations. The harmonics were filtered from each sound clip using a MATLAB-based analysis program created by Joseph Yasi.[6] The data from this analysis included information about frequency, amplitude, and phase for each of the harmonics.

B. Frequency Spectrum Impedance Analysis

The next experiment performed measured the input and output acoustic impedance over a range of frequencies for each concert F. The setup involved simulating playing the clarinet by using a modified mouthpiece and covering the correct tone holes for each fingering. Microphones measuring complex pressure and particle velocity were placed at each end of the instrument to calculate acoustic impedance as a function of frequency.

The mouthpiece used, seen in Figure 3, was a Selmer Bundy student Bb clarinet mouthpiece. A piezoelectric disc transducer was glued to the top of the window on the mouthpiece and a copper plate was glued beneath it to cover the rest of the opening in order to simulate a vibrating reed. The voltage output from a sine wave function generator was applied across the piezoelectric transducer to allow the disc to vibrate at specific frequencies. Additionally, holes were drilled into the reed table and back of the mouthpiece where complex pressure and particle velocity microphones were inserted. These were secured with Apiezon Sealing Compound, which also sealed the remainder of the holes.

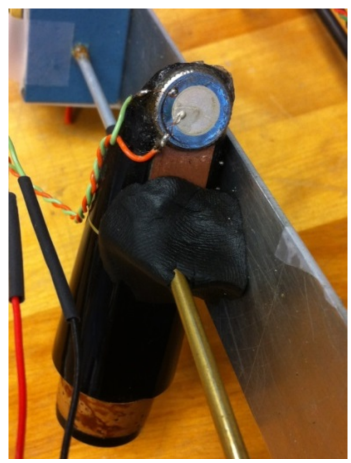

Figure 3: A Selmer Bundy student Bb clarinet mouthpiece was modified to simulate playing the clarinet. A piezoelectric disc transducer was attached to the window of the mouthpiece in addition to a copper plate to simulate a vibrating reed while microphones measuring complex pressure and particle velocity were inserted into the cavity of the mouthpiece.

The tone holes were covered by plugging them with rubber stoppers secured with pieces of PVC-coated wire as depicted in Figure 4 for concert F_3. Any keys that needed to be held down were closed by propping various levers with rubber stoppers or wire. Once the proper holes were covered, the clarinet was played to make sure there were no air leaks in the system.

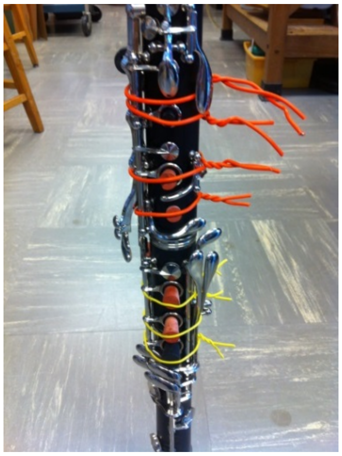

Figure 4: Rubber stoppers and PVC-coated wire was used to close the tone holes necessary for specific notes tested. This picture shows the wooden Buffet R-13 Bb clarinet configured for concert F_3 before doing a frequency spectrum impedance analysis.

The Buffet R-13 Bb clarinet was placed in a wooden box lined with foam in the interior to avoid picking up noise from the room. Two microphones, one measuring complex pressure and one measuring complex particle velocity, were placed at the bell end of the instrument centered at the edge of the bell.

SRS 830 dual-channel lock-in amplifiers were used to measure the pressure and particle velocity at the input and output of the clarinet for specific frequencies of piezoelectric transducer vibration. Measurements were taken from 29.5 Hz to as high as 3030.5 Hz in 1 Hz steps for each of the five fingerings.

C. Acoustic Impedance Along Bore

An input and output acoustic impedance analysis was also done along the bore of the clarinet for concert F_4 on the tempered scale. The modified Selmer Bundy mouthpiece including complex pressure and particle velocity microphones was attached and no tone holes needed to be covered for this note. A pine wood frame was constructed for the clarinet so that the joint below the barrel and the joint above the bell rested on wooden slots coated in foam. This frame was clamped along the y-axis of a near-field acoustic holography apparatus with large C-clamps. A long steel rod held a complex pressure and a particle velocity microphone to be inserted into the bell end of the clarinet along the bore. This rod was clamped onto a ring stand so that it was positioned horizontally. The wooden frame was set to move in 1 mm steps along the y-axis while the steel rod with the microphones remained stationary. This apparatus is shown in Figure 5. Measurements of complex pressure and particle velocity were taken at the mouthpiece and the location of the steel rod microphones for positions of the end of the rod ranging from just between the mouthpiece and barrel to just outside the bell.

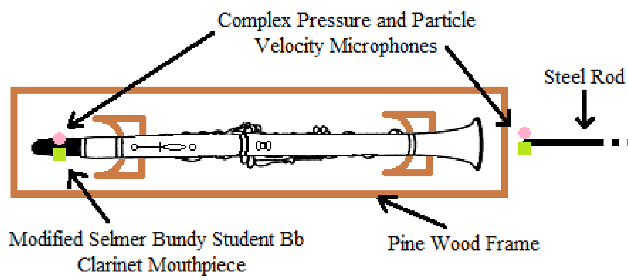

Figure 5: This diagram depicts the apparatus used for experimentation measuring the acoustic impedance along the bore of the clarinet.

D. Spectral Analysis of Longitudinal Mechanical Resonance

The final experiment done on the Buffet R-13 Bb clarinet was a measurement of longitudinal mechanical vibrations due to induced longitudinal vibrations on the bell over a range of frequencies. This experiment used a Pyne Signature Symphonic Bb clarinet mouthpiece, a Rovner Eddie Daniels II Bb clarinet leather ligature, and a size 4 Vandoren V-12 reed. The apparatus, pictured in Figure 6, involved wrapping the clarinet's thumb rest with electrical tape and clamping the tape with a C-clamp to protect the silver from scratches. This clamp was held by a three-pronged clamp attached to a ring stand so that the clarinet was vertical and upside-down. The screw of the ligature rested gently on some foam to keep the clarinet steady, but limit contact to avoid perturbation.

Two piezoelectric transducer discs were attached on the bell of the instrument 180° from each other. A 20 gram brass weight and some electrical tape were placed on top of each transducer to keep them steady. Frequencies ranging from 0 to 10 kHz were applied at constant current amplitude to one piezo disc to induce mechanical vibrations in the instrument. The resulting mechanical vibrations in the clarinet were measured with the other piezoelectric transducer by recording the square of its voltage amplitude for each frequency.

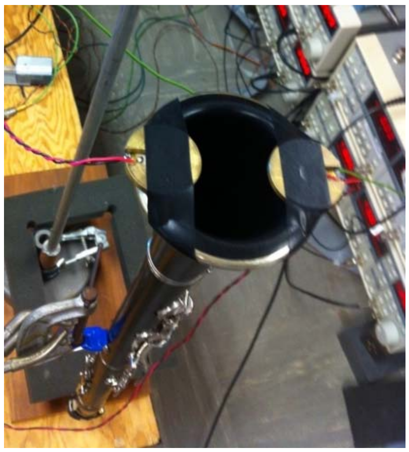

Figure 6: The apparatus for the longitudinal mechanical resonance experiment involved clamping the clarinet upside-down by the thumb rest to a ring stand using electrical tape and a C-clamp. Two piezoelectric transducer discs were placed on the bell of the instrument to induce and measure mechanical vibrations.

III. Results and Discussion

A. Harmonic Analysis

Concert F_3, F_4, F_5, and two separate fingerings of F_6 were recorded and the sound recordings were clipped at the beginning and end so that a few seconds of steady sound remained. These sound clips were analyzed in a MATLAB-based program that filtered out several harmonics for each note, and determined the mean normalized amplitude and phase of each.

1. F_3 Analysis

The sound clip for the lowest note tested, concert F_3 at a frequency of 174.61 Hz, was analyzed in the MATLAB-based analysis program. The sound clip was very uniform as can be seen in Figure 7 showing the amplitude of the sound wave over time for a total of 0.02 seconds of the sound clip.

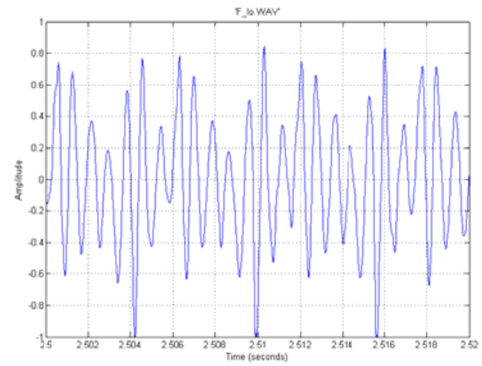

Figure 7: The sound clip for the lowest note tested, concert F_3, was very uniform. The amplitude over time is consistent as can be seen for these 0.02 seconds of the sound clip.

The software filtered the harmonics for F_3 on the wooden Buffet R-13 Bb clarinet and displayed the mean normalized amplitude for each harmonic as seen in Figure 8. The first 9 harmonics clearly follow a trend where the odd harmonics have a larger magnitude than the even harmonics. This can be explained by the basic structure of the clarinet as an essentially open-closed cylindrical pipe. The higher harmonics fall within a formant, which will be discussed in detail in the results section D below.

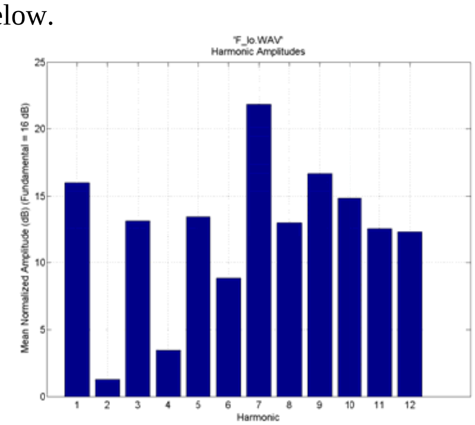

Figure 8: The mean normalized amplitude of the harmonics of concert F_3 played on the Buffet R-13 Bb clarinet were plotted in an analysis program in MATLAB.

Additionally, the software determined the relative phases of each harmonic of concert F_3, setting the fundamental to 0°. The harmonics were plotted as arrows with a magnitude in decibels and an angle corresponding to the phase as seen in Figure 9. The relative phases of the harmonics seem to be clustered roughly 120° from one another with groups of at least two consecutive harmonics in any one cluster excluding the fundamental.

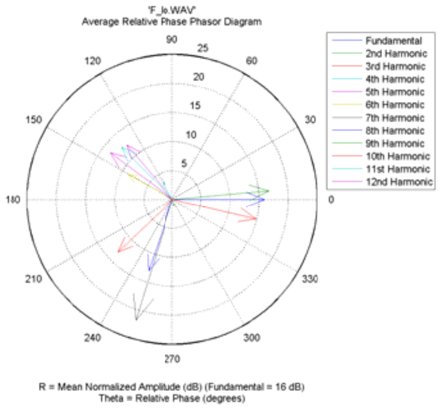

Figure 9: A phasor diagram of concert F_3 was generated from the harmonic analysis done with a MATLAB-based program. Each arrow is a harmonic with the magnitude radially in decibels and the relative phase azimuthally in degrees.

2. F_4 Analysis

Concert F_4 on the tempered scale at 349.23 Hz was played on the clarinet with no keys pressed and recorded. The sound clip used in this analysis was uniform since the amplitude of the sound wave over time was periodic. A plot of the amplitude over a total of 0.02 seconds is pictured in Figure 10.

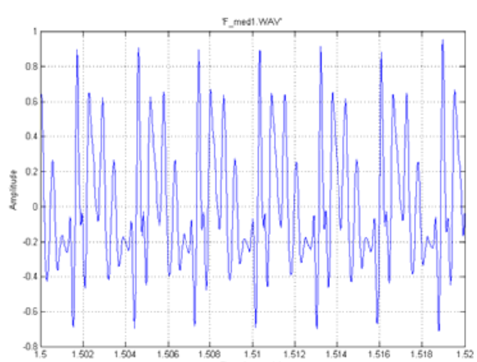

Figure 10: The amplitude of the sound wave over 0.02 seconds is shown in the graph for concert F_4 on the tempered scale played on a wooden Buffet R-13 Bb clarinet. The waveform is periodic and uniform.

The mean normalized amplitudes of each harmonic are plotted in Figure 11 below. The fundamental, 5th, 7th, and 10th harmonics are dominant for this note. This suggests that the odd harmonics are still more prominent because of the clarinet's shape, but at this higher frequency, the pattern is not clearly defined. The second harmonic is much smaller in magnitude than any other harmonic, similar to the data for concert F_3.

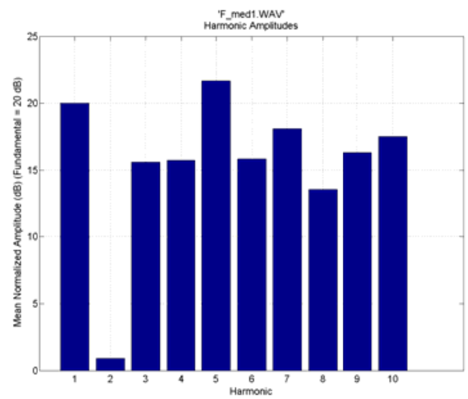

Figure 11: Concert F_4 was played on the clarinet and the harmonics of the recording were filtered. The mean normalized amplitudes, measured in decibels, for the first 10 harmonics are plotted.

The MATLAB harmonic analysis software also measured the relative phases of each harmonic for concert F_4 played on the Buffet R-13 Bb clarinet. Each amplitude is an arrow in the polar plot shown in Figure 12. The mean normalized amplitudes of the harmonics are shown in decibels as magnitudes of each arrow, and the relative phases are shown in degrees. All but the second harmonic are contained within 210° of the fundamental, which is set at 0°. This distribution of relative phases is unique to the specific Buffet R-13 Bb clarinet used, and in part causes it to sound different than other clarinets of the same type.

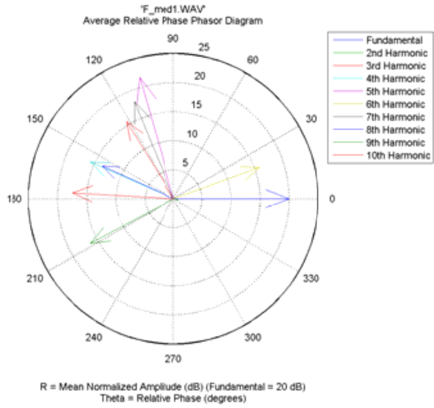

Figure 12: The relative phases for the first 10 harmonics of concert F_4 on the tempered scale are plotted. Each harmonic is displayed as an arrow with a magnitude of mean normalized amplitude and an azimuthal angle of relative phase measured in degrees.

3. F_5 Analysis

Concert F_5 on the tempered scale at 698.46 Hz was played on the clarinet and recorded. This is the first note tested that used the register key. A 0.02 second plot of the sound wave amplitude made with the MATLAB-based harmonic analysis program seen in Figure 13 shows consistency in the sound clip used.

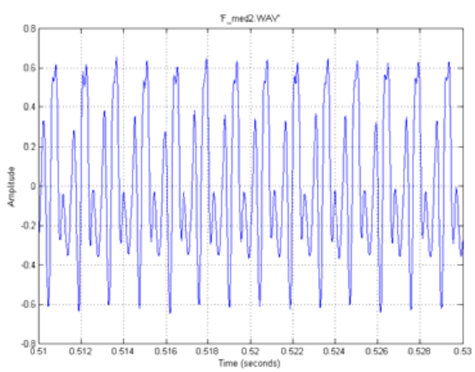

Figure 13: A graph of the amplitude of the waveform for concert F_5 played on the clarinet over a time span of 0.02 seconds demonstrates the uniformity of the analyzed sample.

The harmonics from this sound sample were filtered with the software and the mean normalized amplitudes of the first 7 harmonics were extracted. Figure 14 pictures the plot from this analysis. The amplitude of the second harmonic for this note is much larger than that of the first two notes analyzed. The other even harmonics are minima, however, corresponding with the fact that the clarinet is similar to an open-closed cylindrical pipe.

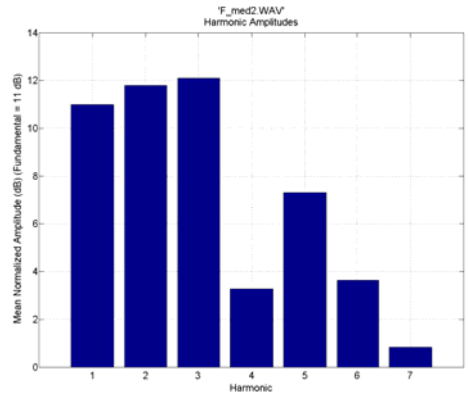

Figure 14: Concert F_5 was played on the clarinet, and the harmonics of the recording were analyzed in MATLAB. The mean normalized amplitudes for the first 7 harmonics are plotted.

An additional analysis done on the recording of concert F_5 was a phase analysis of the filtered harmonics displayed in Figure 15. Each harmonic was represented as an arrow with magnitude equal to the mean normalized amplitude in decibels and angle equal to the phase in degrees relative to the fundamental. There is no recognizable pattern in phases for this note. However, the relative phases of the harmonics quantify a unique characteristic of the clarinet tested, setting its sound apart from that of other Buffet R-13 Bb clarinets.

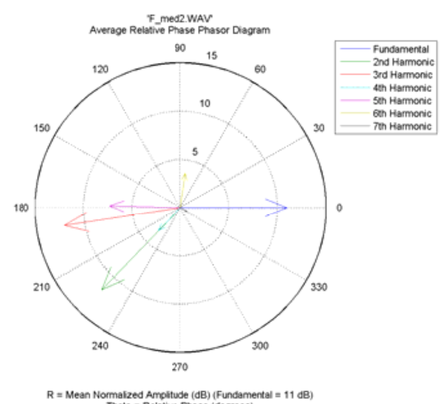

Figure 15: A phasor diagram showing the mean normalized amplitude and relative phase of the first 7 harmonics for concert F_5 is shown. Each harmonic is plotted as an arrow with radial magnitude in decibels and azimuthal relative phase in degrees.

4. F_6 Analysis: First Fingering

A fourth octave of concert F, F_6 at 1396.9 Hz, was played on the Buffet R-13 clarinet. This note used the fingering also used for concert A_5 which incorporates the register key similar to concert F_5 noted previously. The waveform of this recorded note was uniform as seen in Figure 16, a plot of the amplitude of the sound wave over a period of 0.02 seconds. The waveform for this note is much simpler than that of the first three notes analyzed.

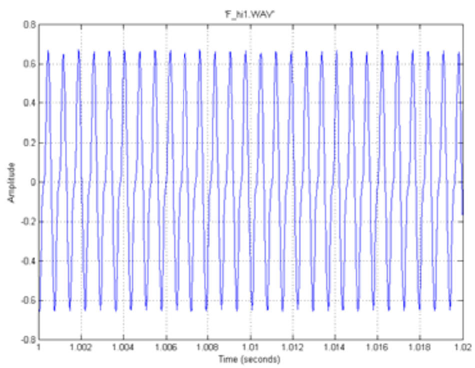

Figure 16: The waveform for 0.02 seconds of the recording of the first fingering of concert F_6 on the tempered scale is graphed. The sound sample was periodic and uniform.

The first three harmonics of the sound clip for concert F_6 using the first fingering were filtered using the MATLAB-based harmonic analysis program. Although there is not a large sample of harmonics to analyze, it is clear from Figure 17 that the fundamental is dominant and the following harmonics have decreasing mean normalized amplitudes.

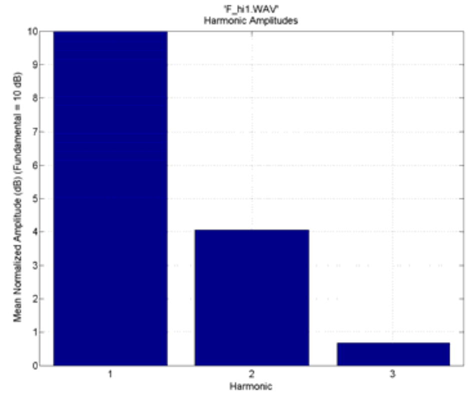

Figure 17: The mean normalized amplitudes of the first three harmonics of concert F_6 on the tempered scale played on the clarinet with the first fingering tested is shown in the plot.

A polar plot of the relative phases of these three harmonics was produced as seen in Figure 18. The second and third harmonics are close to 180° out of phase from the fundamental.

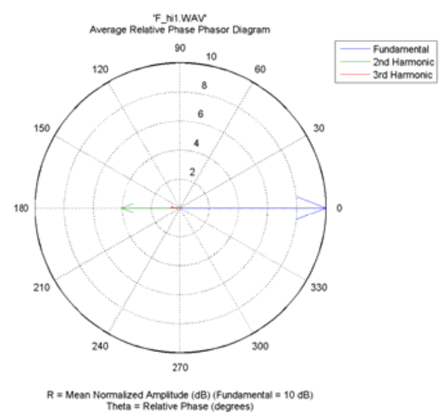

Figure 18: This plot depicts the relative phases of first three harmonics of concert F_6 using the fingering also used for concert A_5. Each harmonic is an arrow with a magnitude equal to the mean normalized amplitude measured in decibels and an angle of phase relative to the fundamental measured in degrees.

5. F_6 Analysis: Second Fingering

Concert F_6 on the tempered scale at a frequency of 1396.9 Hz was tested again except this time with a different fingering using the register key again, but also lifting the left index finger to uncover, in essence, a second register hole. The sound clip for the recorded note was very uniform as seen in Figure 19 plotting the amplitude of the sound wave over a period of 0.02 seconds. Although this was F_6 again, the waveform is much more complex as compared to that for the first fingering of F_6.

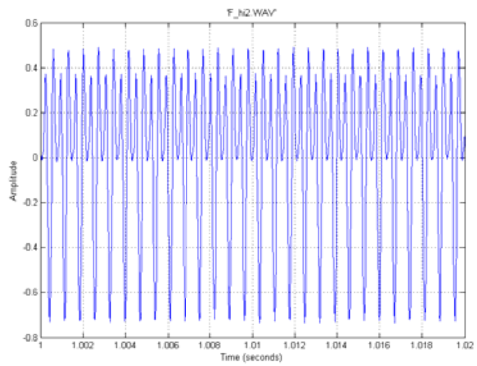

Figure 19: This plot shows the amplitude over a total of 0.02 seconds of the waveform for concert F_6 played on the clarinet with a second fingering using the register key and opening a second essential register hole.

The mean normalized amplitudes of each filtered harmonic were plotted in Figure 20. The first three harmonics for this fingering vary from the first fingering in that the second harmonic has the largest amplitude while the amplitude of the fundamental is still similar in magnitude. Listening to the recordings reveals that this fingering of F_6 has a slightly brighter tone than the first fingering analyzed previously, due primarily to the larger amplitude of the second harmonic.

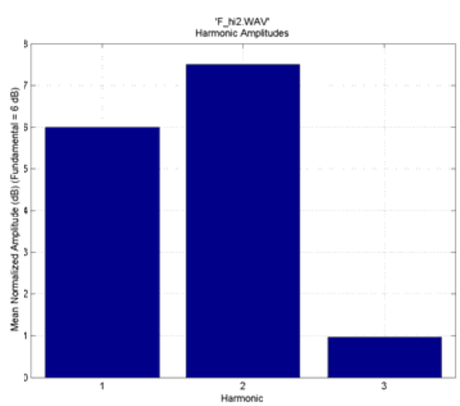

Figure 20: The mean normalized amplitudes of the first three harmonics for the second fingering of concert F_6 played on the clarinet were plotted using MATLAB.

A phasor plot was also generated for the second fingering of concert F_6 and is pictured in Figure 21. The phases of the three harmonics are plotted in degrees relative to the fundamental and the magnitudes represented for each harmonic are the mean normalized amplitudes in decibels. The relative phases are contained within 150° of the fundamental, looking very different from the phases of the same harmonics when the note was played with the first fingering.

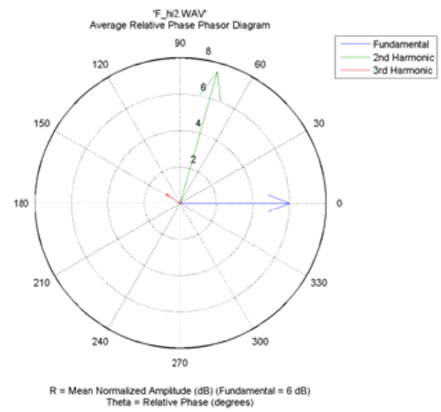

Figure 21: The relative phases of the first three harmonics of concert F_6 on the tempered scale are shown when played on the clarinet with the second fingering tested. The magnitudes of each arrow are the mean normalized amplitudes in decibels and the angles are the phases relative to the fundamental measured in degrees.

6. Harmonic Inharmonicity Analysis

The frequencies of the harmonics for each note were analyzed manually to determine inharmonicities. The reference frequency was set to the measured frequency of the fundamental, and the deviation from the theoretical frequencies was measured in cents and calculated using equations 1.5-1.7. Figure 22 shows the inharmonicities for the five recorded notes measured in cents. This data corresponds very closely with the theoretical harmonicities, varying at most by half a cent for concert F_3.

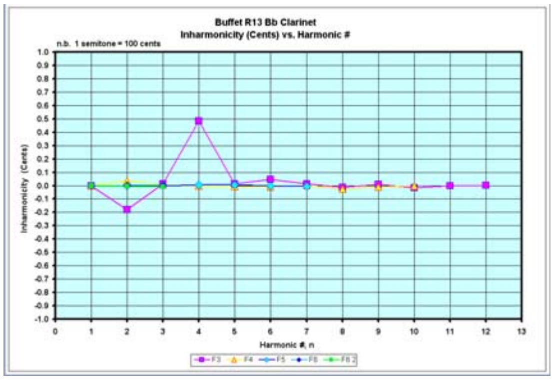

Figure 22: The inharmonicities for the measured harmonic frequencies of concert F_3, F_4, F_5, and two fingerings of F_6 played on the clarinet were determined using equations 1.5-1.7.

B. Frequency Spectrum Acoustic Impedance Analysis

A frequency spectrum analysis was done across a range of frequencies using the apparatus described in section II. B. Complex pressure and particle velocity microphones took measurements at the mouthpiece and bell of the clarinet and the data was used to calculate the input and output impedance using Equation 1.8 for concert F_3, F_4, F_5, and two fingerings of F_6 on the tempered scale. The complex input impedance graphs are documented in Figures 23-27 as they represent the best quality and most information of the data gathered. The impedance maxima represent places of maximum pressure and minimum particle velocity while the impedance minima represent maximum particle velocity and minimum pressure at the bell end of the instrument. The impedance maxima are located at frequencies that produce playable sounds.

1. F_3 Analysis

Figure 23 shows the complex input impedance measured in acoustic Ohms from 29.5 Hz to 2613.5 Hz induced on the piezoelectric transducer. The plot shows a rather ideal pattern of minima and maxima except around 1500 Hz because of mechanical resonances in the clarinet. The first maximum is the fundamental at 185.50 Hz, which is 11.08 Hz higher than concert F_3 on the tempered scale at 174.42 Hz. This could be due to 80°F high and 41% humidity in Urbana which could cause the wood to expand and therefore reduce the diameter of the bore cavity. However, the acoustic physics lab was kept at a much cooler temperature. The higher frequency could also be due to the nature of the apparatus using the modified mouthpiece and rubber stoppers to close necessary tone holes.

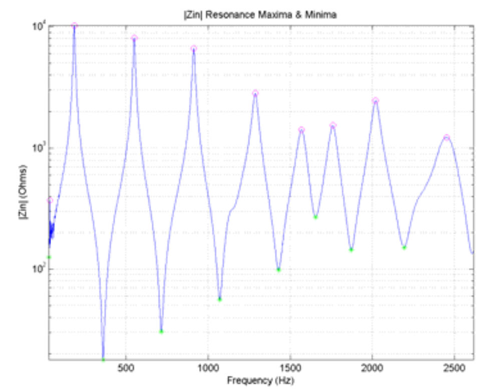

Figure 23: This data was obtained from a frequency spectrum acoustic impedance analysis for concert F_3 on the clarinet. The magnitude input complex acoustic impedance measured in acoustic Ohms and logarithmically scaled is on the y-axis while the frequency of the piezoelectric transducer attached to the modified clarinet mouthpiece is located on the x-axis.

Table 1 lists the frequencies at several of the complex input impedance extrema, the theoretical harmonic number for each, and the type of extrema for the concert F_3 acoustic impedance frequency spectrum data. All the odd harmonics with the exception of the 9^{th} are at impedance maxima, meaning that the playable sounds for this fingering only occur at odd harmonics in accordance with the geometry of the clarinet similar to an open-closed cylindrical pipe. The fundamental is the only frequency intentionally played with this fingering, but when the reed vibrates at other input impedance maxima frequencies a squeak is produced at that frequency.

Concert F_3 Impedance Extrema		
Measured Frequency (Hz)	**Theoretical Harmonic Number**	**Type of Local Extrema**
185.50	1	Maximum
361.50	2	Minimum
549.5	3	Maximum
715.5	4	Minimum
912.5	5	Maximum
1070.5	6	Minimum
1286.5	7	Maximum
1428.5	8	Minimum
1654.5	9	Minimum
1871.5	10	Minimum
2019.5	11	Maximum
2192.5	12	Minimum
2452.5	13	Maximum

Table 1: This table represents several data points from Figure 23 describing the complex input impedance extrema of the clarinet for concert F_3.

2. F_4 Analysis

A frequency spectrum impedance analysis was done for concert F_4 from 29.5 Hz to 3030.5 Hz. A plot of the complex input impedance data can be seen in Figure 24. Similar to the data taken for concert F_3, the impedance minima and maxima look uniform until about 1500 Hz due to a formant of the clarinet. The first maximum is at the frequency for the note tested at 382.5 Hz, 33.27 Hz above the theoretical frequency for concert F_4 at 349.23 Hz. This could be due expansion in the clarinet's wood from the 63% humidity in Urbana on the day of experimentation as well as imperfections in the apparatus.

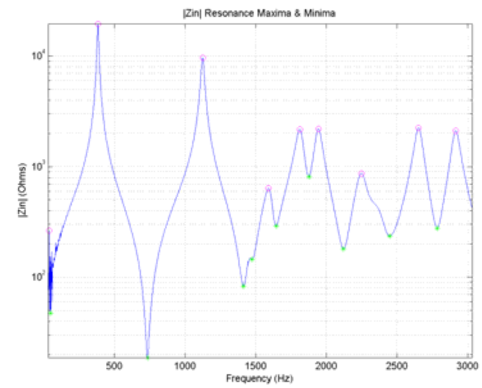

Figure 24: This semi-log plot shows the magnitude of the complex input acoustic impedance data for concert F_4 on the clarinet across a range of frequencies.

The values of the frequencies at several of the complex input impedance extrema are listed in Table 2 along with the theoretical harmonic number for each. The 5^{th} and 6^{th} harmonics are at the opposite extrema than expected, but the frequencies of these harmonics fall within a formant of the clarinet.

Concert F_4 Impedance Extrema		
Measured Frequency (Hz)	**Theoretical Harmonic Number**	**Type of Local Extrema**
382.5	1	Maximum
733.5	2	Minimum
1123.5	3	Maximum
1474.5	4	Minimum
1875.5	5	Minimum
2247.5	6	Maximum
2652.5	7	Maximum

Table 2: The concert F_4 complex input acoustic impedance extrema, measured frequency, and theoretical harmonic number are listed from the data gathered in Figure 24.

3. F_5 Analysis

A frequency spectrum acoustic impedance analysis was also run for concert F_5 on the tempered scale for the wooden Buffet R-13 Bb clarinet from 29.5 Hz to 3030.5 Hz. The complex input impedance data as a function of frequency is displayed in Figure 25, and the second maximum is at the frequency of the note being tested. This maximum occurs at 745.5 Hz, a whole 47.04 Hz higher than the frequency specified for concert F_5 on the tempered scale at 698.46 Hz. The Urbana temperature of 78°F and humidity level of 56% played a large role in this discrepancy along with the apparatus' errors in perfectly simulating a person playing the clarinet. Apart from the frequency values, the maxima and minima look regular until about 1500 Hz where the clarinet experiences mechanical resonance.

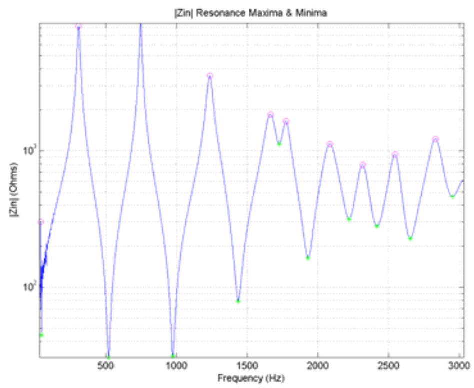

Figure 25: An acoustic impedance frequency spectrum analysis was conducted for concert F_5 on the clarinet where the magnitude of the complex input acoustic impedance was plotted as a function of frequency with a semi-log scale.

Table 3 contains measured frequency values and theoretical harmonic numbers for several complex input acoustic impedance extrema. It is easy to see the regular pattern of maxima and minima before reaching the 9^{th} harmonic. The frequency at each impedance maximum is a playable pitch and the third harmonic frequency is the intended playable note for this fingering.

Concert F_5 Impedance Extrema		
Measured Frequency (Hz)	**Theoretical Harmonic Number**	**Type of Local Extrema**
306.5	1	Maximum
518.5	2	Minimum
745.5	3	Maximum
972.5	4	Minimum
1233.5	5	Maximum
1434.5	6	Minimum
1663.5	7	Maximum
1929.5	8	Minimum
2220.5	9	Minimum
2416.5	10	Minimum
2651.5	11	Minimum

Table 3: This table lists the measured frequencies, theoretical harmonic numbers, and type of extrema for several data points in Figure 25 involving a frequency spectrum acoustic impedance analysis done for concert F_5.

4. F_6 Analysis: First Fingering

The same impedance analysis was performed on the clarinet with the first fingering of concert F_6 shown in Figure 2. This fingering is also intended for concert A_5 on the tempered scale. The complex input acoustic impedance was calculated from the measured complex pressure and particle velocity and plotted across a range of frequencies from 29.5 Hz to 3030.5 Hz as seen in Figure 26. The second impedance maximum is for concert A_5 with a theoretical frequency of 880.00 Hz, and the third impedance maximum is for concert F_6 with a theoretical frequency of 1396.90 Hz. The measured frequency values for these notes were 946.5 Hz and 1523.5 Hz respectively, each higher than the theoretical values by 66.5 Hz and 126.6 Hz. This can be explained by a high humidity of 82% in Urbana on the day of experimentation, non-ideal apparatus, and the register of the clarinet as these notes tend to be sharp on the instrument.

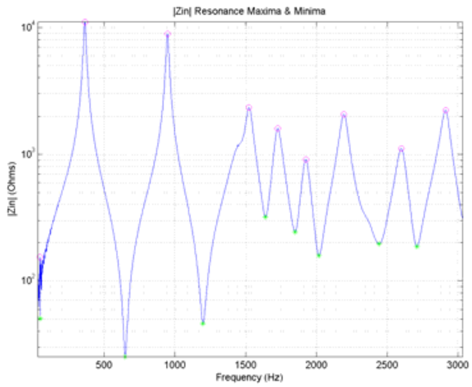

Figure 26: This is a semi-log plot of the magnitude of the complex input acoustic impedance data across a range of vibration frequencies of the piezoelectric transducer on the modified clarinet mouthpiece for testing done with the first fingering of concert F_6 which can also be used for playing concert A_5.

Values for the measured frequencies at various impedance extrema are listed in Table 4 along with the theoretical harmonic number and type of local extrema for each one. The 8^{th} harmonic shows strange behavior as it falls at a local maximum instead of a local minimum.

Concert F_6: First Fingering Impedance Extrema		
Measured Frequency (Hz)	**Theoretical Harmonic Number**	**Type of Local Extrema**
362.5	1	Maximum
647.5	2	Minimum
946.5	3	Maximum
1195.5	4	Minimum
1523.5	5	Maximum
1846.5	6	Minimum
2192.5	7	Maximum
2598.5	8	Maximum
2911.5	9	Maximum

Table 4: The frequency spectrum acoustic impedance analysis done for the first fingering of concert F_6 produced the complex input impedance data found in Figure 26. The measured frequencies, theoretical harmonic numbers, and type of local extrema are listed in the table for several data points in this plot.

5. F_6 Analysis: Second Fingering

The second fingering for concert F_6 was used to conduct a final frequency spectrum acoustic impedance analysis. The complex input acoustic impedance data from this experiment are shown in Figure 27 over a frequency range from 29.5 Hz to 3030.5 Hz. The 4^{th} local impedance maximum is at the 7^{th} harmonic, the playable note for this fingering. Instead of the ideal frequency of 1396.90 Hz, the measured frequency was 85.6 Hz higher at 1482.5 Hz. This was due in part to the 73°F temperature and 52% humidity in Urbana causing the wood of the clarinet to expand as well as the non-ideal apparatus used and the register containing concert F_6.

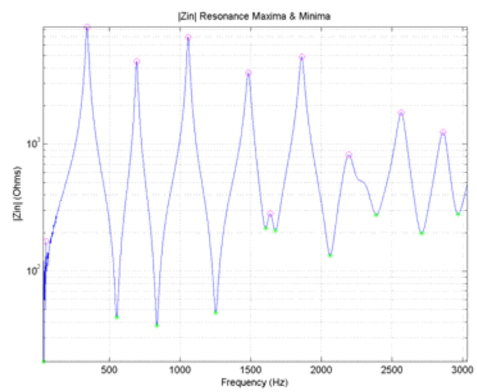

Figure 27: This semi-log plot displays the magnitude of the complex input acoustic impedance measured in acoustic Ohms as a function of frequency in Hertz for the second fingering for concert F_6.

The measured frequencies, theoretical harmonic numbers, and type of local extrema for several extrema in Figure 27 are listed in Table 5. The maxima and minima follow a regular pattern besides the 8^{th} harmonic which falls within a range of mechanical resonance for the clarinet.

Concert F_6: Second Fingering Impedance Extrema		
Measured Frequency (Hz)	**Theoretical Harmonic Number**	**Type of Local Extrema**
340.5	1	Maximum
550.5	2	Minimum
691.5	3	Maximum
835.5	4	Minimum
1056.5	5	Maximum
1251.5	6	Minimum
1482.5	7	Maximum
1636.5	8	Maximum
1859.5	9	Maximum
2061.5	10	Minimum
2194.5	11	Maximum
2387.5	12	Minimum
2564.5	13	Maximum
2709.5	14	Minimum

Table 5: This table displays the measured frequencies, theoretical harmonic numbers, and type of local extrema for several complex input impedance extrema from the frequency spectrum acoustic impedance analysis with the second fingering of concert F_6 on the clarinet.

6. Acoustic Impedance Inharmonicity Analysis

An inharmonicity analysis was repeated except instead of using the filtered harmonic frequencies from the harmonic analysis, certain frequencies from the complex input acoustic impedance extrema were used. The reference frequencies were set to the measured frequency of the first playable note, and the subsequent theoretical frequencies were determined based on these reference values. The experimental harmonic numbers were then calculated with the reference frequency as an integer harmonic, and equations 1.5-1.7 were used to determine the inharmonicities.

Figure 28 shows the deviation in cents from the ideal harmonics for each harmonic number of each note tested. The data for the first fingering of F_6 is labeled as A_5 on the graph, and the measured frequency for A_5 was also used as the reference frequency for this note's inharmonicity data. Clearly this plot varies from Figure 22 based on the harmonic analysis in that the harmonics deviate even beyond 500 cents. The deviations are less dramatic close to the reference harmonic, and the worst inharmonicities occur at the lowest harmonics for the highest notes. This is relatively insignificant in the playing of the clarinet, however, because the notes at those harmonics are not intended to be played.

This input impedance-based inharmonicity plot differs significantly from the harmonic analysis-based inharmonicity plot, which is disconcerting. One cause for this discrepancy is the inability to simulate a person's lips in interaction with the mouthpiece and reed as well as the effects of blowing into the instrument. The data from the three highest notes that use the register key have the greatest inharmonicities at the lowest harmonics which could be due to the open hole one third of the way down the clarinet.

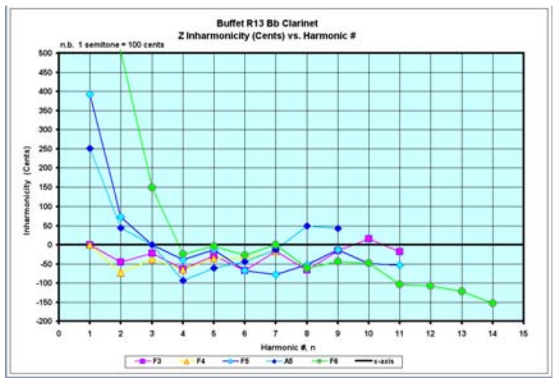

Figure 28: The frequency values at complex input acoustic impedance extrema were used to determine inharmonicities in the concert pitches of F_3, F_4, F_5, A_5, and F_6.

C. Acoustic Impedance Along Bore for Concert F_4

The complex specific acoustic impedance was calculated from measurements of complex pressure and particle velocity in the mouthpiece and along the bore of the clarinet for concert F_4 using the apparatus pictured in Figure 5. Figures 29-32 show the complex specific acoustic impedance as a function of distance from the mouthpiece. Just past 10 cm, the vent hole that is opened by the register key has a short metal tube protruding into the bore to avoid clogging the hole with condensed moisture from the player's breath.[7]

1. 382.5 Hz Analysis

The piezoelectric transducer on the modified mouthpiece was set to vibrate at 382.5 Hz. This frequency was chosen by manually scanning frequencies near the fundamental for concert F_4 until a complex specific acoustic impedance maximum was reached in the mouthpiece. Figure 29 shows the acoustic impedance data as a function of distance from the mouthpiece. The spike near 10 cm is the impedance near the protrusion in the vent hole showing a pressure maximum and particle velocity minimum for this frequency. The data becomes rather noisy starting around 45 cm from the mouthpiece of the clarinet as the microphones become more exposed to room and ventilation noise.

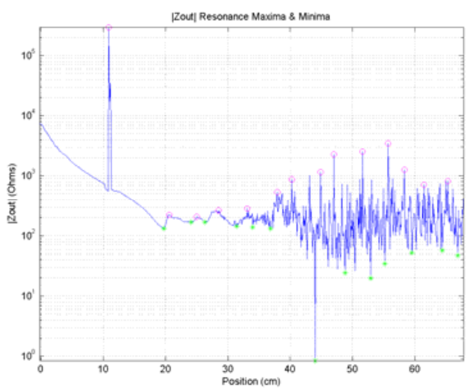

Figure 29: This semi-log plot displays the magnitude of the complex specific acoustic impedance in acoustic Ohms as a function of distance in centimeters from the mouthpiece of the clarinet for the fundamental frequency of concert F_4.

2. 738.5 Hz Analysis

A second data set, seen in Figure 30, was obtained for the second harmonic frequency of concert F_4 located at a maximum in input acoustic impedance. This data is similar to that for 382.5 Hz in that a maximum in the impedance occurs at the vent hole and the data becomes noisy further along the bore, in this case starting around 30 cm. This would indicate that there is a pressure antinode and particle velocity node at the vent hole.

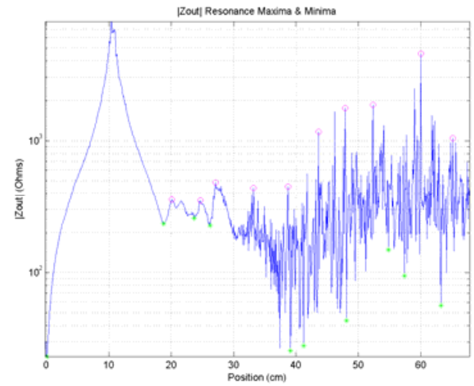

Figure 30: The magnitude of the complex specific acoustic impedance measured along the bore of the clarinet in 1 mm steps is plotted for 738.5 Hz, the second harmonic of concert F_4.

3. 1123.5 Hz Analysis

The third harmonic frequency was found manually to be at 1123.5 Hz, again based on a maximum in the input impedance. The complex specific acoustic impedance was determined along the bore and displayed in the semi-log plot in Figure 31. This time the vent hole seems to be the location of an inflection point in the acoustic impedance.

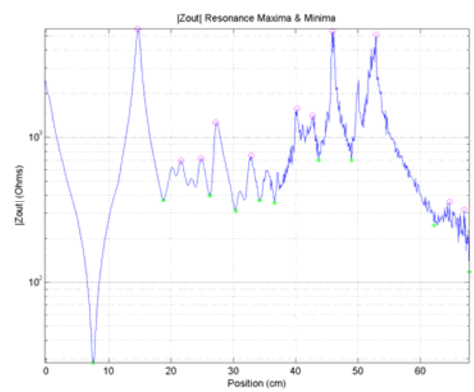

Figure 31: This semi-log plot shows the magnitude of the complex specific acoustic impedance in as a function of distance from the mouthpiece of the clarinet for the third harmonic frequency of concert F_4.

4. 1413.5 Hz Analysis

The final impedance analysis done for F_4 was at the fourth harmonic frequency found manually to be 1413.5 Hz. Figure 32 shows the plot for the magnitude of the complex specific acoustic impedance along the bore of the clarinet for this frequency. There are two clearly defined maxima near the mouthpiece end of the clarinet and one minimum in between them at the vent hole. From about 19 to 30 cm, the impedance goes through several extrema before the signal becomes noisy. This behavior could be due to the fact that this frequency is near a mechanical resonance, causing the pattern of complex pressure and particle velocity extrema to be excited.

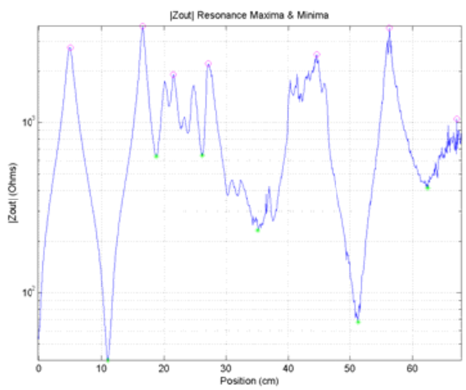

Figure 32: The magnitude of the complex specific acoustic impedance was calculated as a function of position along the bore of the clarinet for the fourth harmonic frequency of concert F_4.

D. Spectral Analysis of Longitudinal Mechanical Resonance

The longitudinal mechanical resonance analysis produced results that explain behavior found in other analyses. Figure 33 shows the amplitude squared of the voltage signal for the vibration-measuring piezoelectric transducer across a range of frequencies from 0 to 5 kHz. The blue curve is data collected without inducing longitudinal vibrations, and the pink curve is the data observed when inducing constant voltage longitudinal vibrations using a piezoelectric transducer across the range of frequencies specified.

Two distinct formants appear, one ranging from about 1500 Hz to 2700 Hz and the other ranging from about 3700 Hz to 4500 Hz. The effects of these resonant frequencies are evident in data taken in the harmonic analysis and frequency spectrum acoustic impedance experiment.

Two formants are even present in the noise signal without induced vibrations since the clarinet resonates simply from vibrations in the room such as those caused from the air ventilation system.

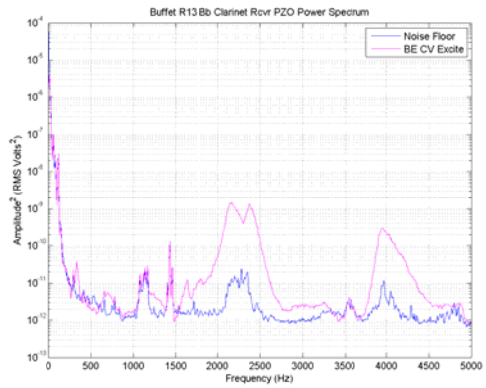

Figure 33: The amplitude squared of the voltage signal on a piezoelectric transducer measuring longitudinal mechanical vibrations as a function of the frequency of induced vibrations was plotted.

IV. Conclusions

All the analyses conducted on the wooden Buffet R-13 Bb clarinet give a broader picture of the acoustics of the instrument. These experiments have allowed for a deeper understanding of characteristic qualities of the clarinet including the amplitudes, frequencies, and phases of different harmonics, inharmonicities, complex specific acoustic impedance, and mechanical resonance.

Analysis on the complex specific acoustic impedance along the bore of the clarinet could be continued for several harmonics of other concert pitches. This would allow for a more complete observation of the complex pressure and particle velocity extrema within the instrument for different fingerings.

Further research into the effects of various effective register keys and cross fingerings would bring even more insight into the physical characteristics of the clarinet based on which tone holes are opened and closed. This study could be extended by conducting the same analyses for different clarinet keywork systems such as the Albert systems, Oehler system, and Boehm systems.[8]

V. Acknowledgments

I would like to thank Dr. Steven Errede for his advisement in this research and the opportunity to utilize his sound analysis equipment and computer programs.

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VI. Footnotes, Endnotes and References

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