# Chapter 12 Appendix C

# The Algebra of the Aggregate Demand and Supply Model

**Algebraic analysis of the aggregate demand and supply model** allows us to directly obtain all of the results in Chapter 12, as well as a number of additional results.

## Aggregate Demand Curve

In Chapter 10, we derived the aggregate demand curve Equation 4, which we repeat here as Equation 1:[1]

$$Y = [\overline{C} + \overline{I} - d\overline{f} + \overline{G} + \overline{NX} - mpc \times \overline{T}] \times \frac{1}{1 - mpc} - \frac{c + d + x}{1 - mpc} \times (\overline{r} + \lambda\pi) \quad (1)$$

[1]Investment, taxes, and net exports are responsive to income, so more realistic investment, tax, and net export functions are as follows:

$$I = \overline{I} - dr - d\overline{f} + iY$$

because investment should rise with income, with the responsiveness of investment to income represented by the parameter, $i$.

$$T = \overline{T} + tY$$

because taxes rise with income, with the responsiveness of taxes to income represented by the parameter, $t$.

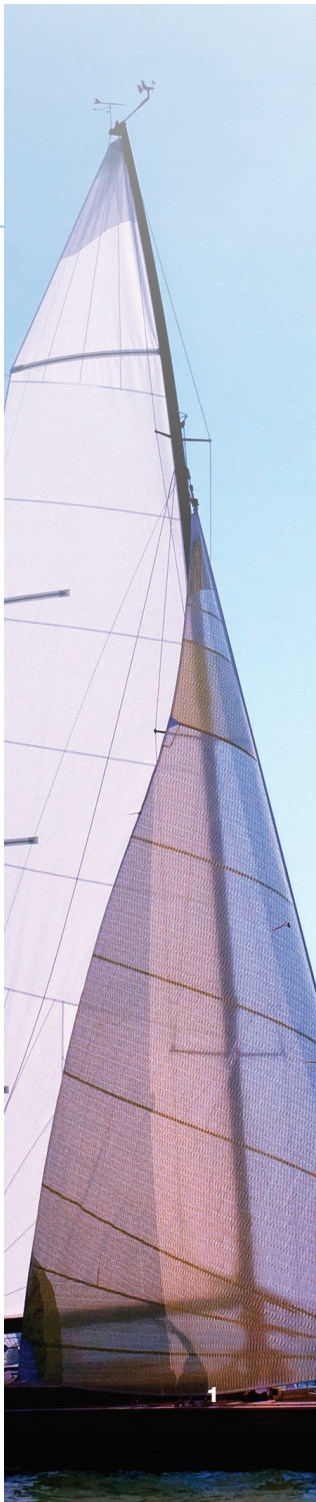
$$NX = \overline{NX} - xr - nY$$

because net exports fall with income since imports rise with income, with the responsiveness of net exports to income represented by the parameter, $n$.

We modify Equation 1 to become

$$Y = [\overline{C} + \overline{I} - d\overline{f} + \overline{G} + \overline{NX} - mpc \times \overline{T}] \times \frac{1}{1 - mpc(1 - t) - i + n} + \frac{c + d + x}{1 - mpc(1 - t) - i + n} \times (\overline{r} + \lambda\pi)$$

Thus the aggregate demand curve is identical to Equation 1 except that we replace $[1 - mpc]$ by $[1 - mpc(1 - t) - i + n]$.



### Implications

There are four immediate implications from the aggregate demand curve Equation 1.

1. The inflation rate $\pi$ is multiplied by $\lambda(c + d + x)/(1 - mpc)$, and so equilibrium output is negatively related to the inflation rate. That is, the aggregate demand curve slopes downward.
2. The more monetary policy makers raise real interest rates as inflation goes up (i.e., the larger $\lambda$ is), the more responsive equilibrium output is to the inflation rate, as is indicated by the larger coefficient on inflation in Equation 1. In other words, the more willing monetary policy makers are to raise interest rates when faced by inflation, the steeper the *AD* curve is.
3. Since $\overline{C}$, $-mpc \times \overline{T}, \overline{I}, \overline{df}, \overline{G}$, and $\overline{NX}$ all are multiplied by the same term, they all shift the aggregate demand curve exactly the same amount and in the same direction as they shift the *IS* curve.
4. Since $\overline{r}$ is multiplied by $-(c + d + x)/(1 - mpc)$, an autonomous tightening of monetary policy (i.e., a rise in $\overline{r}$) leads to a lower level of equilibrium output, shifting the aggregate demand curve to the left. An autonomous easing of monetary policy (a fall in $\overline{r}$) results in a higher level of equilibrium output, shifting the aggregate demand curve to the right.

## Aggregate Supply Curves

As we saw in Chapter 11, there are two types of aggregate supply curves, the short run and the long run.

### Short-Run Aggregate Supply Curve

The short-run aggregate supply curve shows that inflation responds to expected inflation, the output gap, and price shocks:

$$\pi = \pi_{-1} + \gamma(Y - Y^P) + \rho \tag{2}$$

### Long-Run Aggregate Supply Curve

The long-run aggregate supply curve is a vertical line at potential output:

$$Y = Y^P \tag{3}$$

where $Y^P$ = potential output.

## Aggregate Demand and Supply Model

The aggregate demand and supply model has two equilibria, the short run and the long run.

## Short-Run Equilibrium

Solving for the short-run equilibrium involves substituting the short-run aggregate supply curve Equation 2 into the aggregate demand curve Equation 1:

$$Y = [\overline{C} + \overline{I} - d\overline{f} + \overline{G} + \overline{NX} - mpc \times \overline{T}] \times \frac{1}{1 - mpc} - \frac{c + d + x}{1 - mpc} \times (\overline{r} + \lambda[\pi_{-1} + \gamma(Y - Y^P) + \rho]) \tag{4}$$

Multiplying both sides of Equation 4 by $(1 - mpc)$ and collecting the $Y$ terms on the left-hand side results in the following:

$$Y[1 - mpc + \lambda\gamma(c + d + x)] = \overline{C} + \overline{I} - d\overline{f} + \overline{G} + \overline{NX} - mpc \times \overline{T} - (c + d + x) \times (\overline{r} + \lambda[\pi_{-1} - \gamma Y^P + \rho]) \tag{5}$$

Dividing both sides by $[1 - mpc + \lambda\gamma(c + d + x)]$ solves for aggregate output in the short run:

$$Y = [\overline{C} + \overline{I} - d\overline{f} + \overline{G} + \overline{NX} - mpc \times \overline{T} - (c + d + x) \times (\overline{r} + \lambda[\pi_{-1} - \gamma Y^P + \rho])] \times \frac{1}{1 - mpc + \lambda\gamma(c + d + x)} \tag{6}$$

We solve for the inflation rate in the short run by substituting Equation 6 into the short-run aggregate supply curve Equation 2:

$$\pi = \pi_{-1} - \gamma Y^P + \rho + [\overline{C} + \overline{I} - d\overline{f} + \overline{G} + \overline{NX} - mpc \times \overline{T} - (c + d + x) \times (\overline{r} + \lambda[\pi_{-1} - \gamma Y^P + \rho])] \times \frac{\gamma}{1 - mpc + \lambda\gamma(c + d + x)} \tag{7}$$

Collecting terms in $\pi_{-1}$, $Y^P$, and $\rho$:

$$\pi = [\overline{C} + \overline{I} - d\overline{f} + \overline{G} + \overline{NX} - mpc \times \overline{T} - (c + d + x) \times \overline{r}] \times \frac{\gamma}{1 - mpc + \lambda\gamma(c + d + x)} + (\pi_{-1} - \gamma Y^P + \rho) \times \left[1 - \frac{\lambda\gamma(c + d + x)}{1 - mpc + \lambda\gamma(c + d + x)}\right] \tag{8}$$

which we can rewrite as follows:

$$\pi = [\overline{C} + \overline{I} - d\overline{f} + \overline{G} + \overline{NX} - mpc \times \overline{T} - (c + d + x) \times \overline{r}] \times \frac{\gamma}{1 - mpc + \lambda\gamma(c + d + x)} + (\pi_{-1} - \gamma Y^P + \rho) \times \left[\frac{1 - mpc}{1 - mpc + \lambda\gamma(c + d + x)}\right] \tag{9}$$

## Long-Run Equilibrium

We have already seen from the long-run aggregate supply curve that in long-run equilibrium,

$$Y = Y^P$$

We then solve for the long-run inflation rate by substituting $Y^P$ for $Y$ in the aggregate demand curve Equation 1:

$$Y^P = [\,\overline{C} + \overline{I} - d\overline{f} + \overline{G} + \overline{NX} - mpc \times \overline{T} - (c + d + x)(\overline{r} + \lambda\pi)\,] \times \frac{1}{1 - mpc} \quad (10)$$

Multiplying by $(1 - mpc)$ and collecting terms in $\pi$,

$$(c + d + x)\lambda\pi = \overline{C} + \overline{I} - d\overline{f} + \overline{G} + \overline{NX} - mpc \times \overline{T} - (c + d + x)\,\overline{r} - (1 - mpc)Y^P \quad (11)$$

Dividing by $(c + d + e)\lambda$,

$$\pi = [\,\overline{C} + \overline{I} - d\overline{f} + \overline{G} + \overline{NX} - mpc \times \overline{T} - (c + d + x)\,\overline{r}\,] - (1 - mpc)\,Y^P\,] \times \frac{1}{(c + d + x)\lambda} \quad (12)$$

## Implications

There are five implications for aggregate output from the short- and long-run equilibrium output Equations 3 and 6:

1. Increases in $\overline{C}$, $\overline{I}$, $\overline{G}$, and $\overline{NX}$, or decreases in $\overline{T}$, $\overline{f}$ and $\overline{r}$, lead to higher aggregate output in the short run, but not in the long run.
2. Because $\lambda$ enters positively in the denominator in Equation 6, the more aggressively monetary policy makers respond to inflation (a higher $\lambda$), the less impact demand shocks ($\overline{C}$, $\overline{I}$, $\overline{G}$, $\overline{NX}$, $\overline{T}$, $\overline{f}$, and $\overline{r}$) have on aggregate output in the short run.
3. Positive price shocks, $\rho$, and increases in expected inflation, $\pi_{-1}$, lead to a decline in aggregate output in the short run, but not in the long run.
4. A rise in potential output leads to a rise in actual aggregate output in both the short and long run.
5. The more aggressively monetary policy makers respond to inflation (a higher $\lambda$), the larger are the effects of price shocks, $\rho$, and changes in expected inflation, $\pi_{-1}$, on aggregate output in the short run.

There are five implications for inflation from the short- and long-run inflation Equations 9 and 12:

1. Increases in $\overline{C}$, $\overline{I}$, $\overline{G}$, and $\overline{NX}$, or decreases in $\overline{T}$, $\overline{f}$, and $\overline{r}$, lead to higher inflation in both the short and the long run.

2. Positive price shocks, $\rho$, and increases in expected inflation, $\pi_{-1}$, lead to higher inflation in the short run, but not in the long run.
3. A rise in potential output leads to a decline in inflation in both the short and long run.
4. The more aggressively monetary policy makers respond to inflation (a higher $\lambda$), the less impact demand shocks ($\overline{C}$, $\overline{I}$, $\overline{G}$, $\overline{NX}$, $\overline{T}$, $\overline{f}$, and $\overline{r}$) have on inflation in the short run.
5. The more aggressively monetary policy makers respond to inflation (a higher $\lambda$), the smaller are the effects of price shocks and changes in expected inflation, $\pi_{-1}$, on inflation in the short run.