## The Ballistic Pendulum (approx. 90min.) (12/16/15)

### *Introduction*

In this lab we will use conservation of energy and momentum to determine the velocity of a projectile fired into a pendulum and compare it to the velocity determined by looking at the trajectory of the projectile when it is launched across the room. In the ballistic pendulum (Figure 1) a spring-loaded gun fires a projectile horizontally into the bottom of a pendulum, where it becomes lodged. The pendulum swings to some maximum height where it is stopped. By measuring the maximum height of the pendulum and projectile and using conservation of mechanical energy, we can determine the kinetic energy of the pendulum plus projectile immediately after they collide. Knowing the velocity of the pendulum and projectile immediately after they collide, we can use conservation of momentum to calculate the velocity of the projectile alone before the collision. This velocity will then be compared to that determined by firing the projectile across the room and measuring how far it goes.

### *Equipment*

- ballistic pendulum
- balance
- eye protection
- ruler
- scrap paper
- plumb bob
- sz. 16 rubber band
- ball stop (box)
- masking tape
- level
- 2-meter stick
- meter stick

Note: Carbon paper is not needed for this lab, but is available at the request of the instructor.

**<u>Warning</u>: Projectiles can cause serious injury. Do not load gun until it's safe to fire.**

### *Before the lab*

Review two dimensional projectile motion, conservation of energy and conservation of momentum. Pay particular attention to *when* energy and momentum are conserved. Mechanical energy (kinetic plus potential energy) is conserved when no work is done by forces that generate heat, resulting in a loss of mechanical energy. Momentum is conserved when no *external* forces are acting. In a collision total momentum is conserved because the internal forces cause equal and opposite changes in the momentum of the colliding objects. Is mechanical energy conserved in a collision?

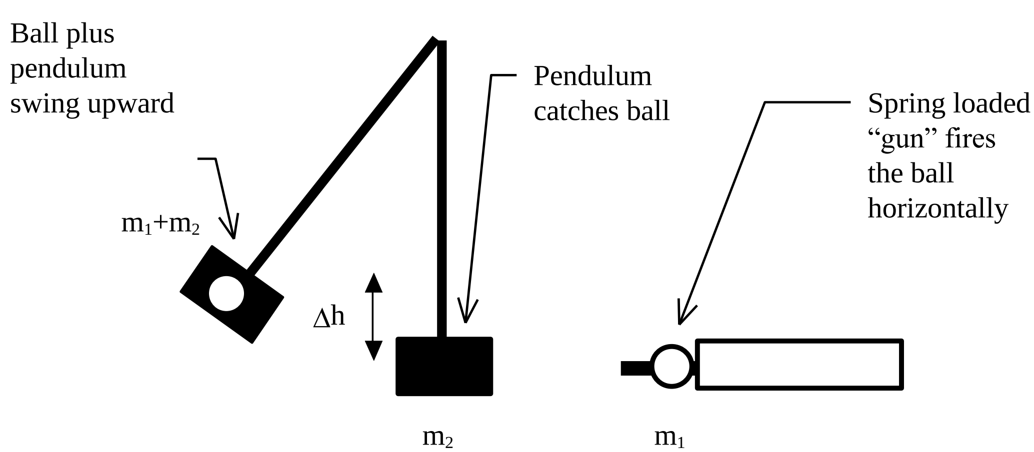


**Figure 1: The Ballistic Pendulum. Momentum in conserved in the collision of projectile and pendulum. Mechanical energy is conserved as the pendulum (+ projectile) rises to its final height.**

## *Theory*

Conservation of Momentum:

One way to state Newton's Second Law: **F**=m**a** is in terms of the momentum, defined by:

$$\mathbf{p} = m\mathbf{v}$$

Since acceleration is the rate of change of velocity, it follows that if the mass is constant force must be the rate of change of momentum. When two objects collide they exert equal and opposite forces on each other. This means that their changes in momentum must be equal and opposite to each other. As long as there is no net externally applied force the momentum of the system must be a constant. This can be illustrated for an inelastic collision (one in which the two objects stick together after the collision as shown in Figure 2.). Suppose that before the collision one object, with mass $m_1$ is moving with velocity $v_0$. It strikes a second, initially stationary, mass, $m_2$, and after the collision they stick together and move with the same speed, $v_{(1+2)}$. . The momentum before the collision is given by $p_{initial}=m_1v_1$ and after the collision by $p_{final}=(m_1+m_2)v_{(1+2)}$ Since there are no external forces acting during the instant of the collision, conservation of momentum requires that

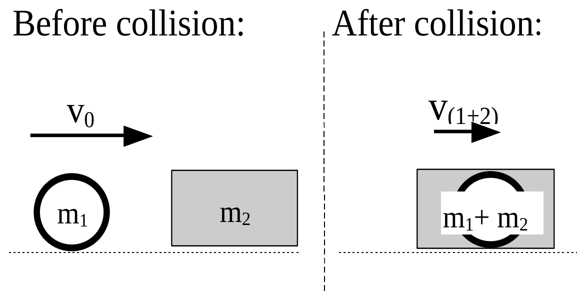


**Figure 2: A totally inelastic collision**

$$m_1v_0 = (m_1+m_2)v_{(1+2)} \quad . \tag{1}$$

**During this collision some of the energy may be dissipated as heat or sound, therefore mechanical kinetic energy is not conserved during the collision.**

Conservation of Energy:

After the collision the pendulum/projectile combination (mass $m=m_1+m_2$) has initial speed $v=v_{(1+2)}$. This means it has kinetic energy $K= ½\ mv^2$. We can determine how much energy this is by seeing how high the pendulum swings: it stops when all the kinetic energy is converted to gravitational potential energy $U=mg\Delta h$. We assume friction is negligible and note that since the arm of the pendulum acts perpendicularly to its motion the work by the tension in the arm is zero. By measuring how high the pendulum/projectile swings we can determine what its velocity was in the instant *after* the collision. From this we can work backwards and use conservation of momentum (Equation 1) to determine the projectile's velocity *before* the collision.

Projectile Motion:

If we move the pendulum away from the projectile we can see how far it travels when we fire it across the room. The motion of the projectile is determined by the initial velocity (in the x-direction) and the acceleration of gravity (in the y-direction). The equations for constant downward acceleration are then:

$\Delta x = x-x_0 = v_{0x}t + ½\ a_x\ t^2 = v_0t$

$\Delta y = y-y_0 = v_{0y}t + ½\ a_y\ t^2 = -\ ½\ gt^2$

From these equations and knowing the height $\Delta y$ we can use a measurement of $\Delta x$ to determine $v_0$ or *visa versa*.

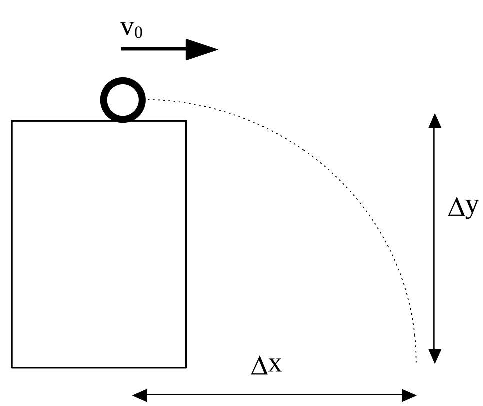


**Figure 3: Projectile motion.**

## *Procedure*

**Ballistic Pendulum**

Determine the mass of the projectile, $m_1$ and the mass of the ballistic pendulum, $m_2$. Record these.

The mass of the ballistic pendulum is not all concentrated on the end. Find the *Center of Mass* of the Pendulum by finding the position where you can hold it horizontally and balance it. Record the distance from the bottom of the bob (catcher) to the center of mass and mark it with a pencil.

Replace the pendulum. Place the projectile ball on the gun.

**CAUTION: EYE PROTECTION RECOMMENDED BEFORE FIRING SPRING GUN!**

Hold the ball release and push the ball back until the gun is cocked. (If this is too difficult, you may need to use the rectangular metal plate to pull the spring all the way back. If so, be sure to remove it before firing.)

Fire the ball into the stationary pendulum bob and record the number of notches up from the bottom at which the pendulum pawl catches.

Repeat for a total of five trials. Calculate the average notch number. This will give you the final position of the pendulum.

Place the pendulum and ball so that they are hanging vertically (i.e. the position they would be in immediately after the collision). Carefully measure the height of the center of the ball and the height of the Center of Mass of the pendulum and record these in the data table. (Heights may be measured relative to either the base or the table.)

Place the pendulum (with ball inside) at the notch most closely corresponding to your average value for the position after the collision. Now carefully measure the height of the ball and the height of the center of mass of the pendulum. These are their final heights. Record them in your table.

**Determine the pre-collision velocity**

You will work backwards from the measured changes in height to find the pre-collision velocity, $v_0$.

Use Conservation of Energy to find the kinetic energy, and then the velocity, $v_{(1+2)}$ of the pendulum/ball combination right after collision (but before the initial height has changed.):

Calculate the change in potential energy of the ball due to its change in height.

Calculate the change in potential energy of the pendulum due to its change in height by treating all the mass, $m_2$, as if it's concentrated at the position of the center of mass of the pendulum. Be sure to include the masses of all moving parts, including the screw adjustment at the top.

Calculate the total change in potential energy and from this find the initial kinetic energy of the pendulum/ball combination. (Here initial means the instant after the collision but before the objects have begun to rise in height.)

Find the corresponding velocity $v_{(1+2)}$

Describe your calculations and record the results.

Now use Conservation of Momentum for the collision between the ball and the pendulum to find the pre-collision velocity of the Ball, $v_0$. (This assumes that, although external forces like gravity and the force on the attachment point of the pendulum due to the apparatus are present, they have no significant effect). You have the common velocity, $v_{(1+2)}$ *after* the collision and know the masses. Describe your calculations and record results.

**Shooting the Ball across the room.**

You now have a good estimate of the pre-collision velocity, $v_0$, of the ball when launched by the gun. Measure the height of the gun off the floor and use $v_0$ as the initial velocity to predict where the ball will land on the floor when the pendulum isn't in the way. Describe your calculations and record your result.

**SAFETY PRECAUTION: Each team should designate a "watcher" and a "shooter". The watcher must make sure the range is clear. The shooter should load the gun only <u>after</u> being given the "all clear" from the watcher, and should announce "firing" beforehand.**

Move the pendulum up the notched rack until it is out of the way. Making sure the space in front of your projectile is clear, measure out the distance to where you think the projectile will land. (NOTE: You should use the plumb bob to make an accurate measurement here. How? )
Tape a piece of paper to the floor and mark the spot with an X.

Place a "Ball Stop" (i.e. a box) just downrange of where you expect the ball to land.

Fire your projectile and see if your prediction is accurate. Check your calculation and re-measure distances if you are off.

If you hit the paper, fire the projectile five more times to make marks where the ball lands. (Optional: You may place carbon paper over your paper, carbon face down, with another piece of protective paper taped on top, to get clearer, darker markings. Do not tape the carbon paper. Carefully remove and return the carbon paper when all finished.)

Carefully record the positions, $\Delta x$, where the ball landed. ($\Delta x$ is the horizontal distance from the initial ball position to where it lands.) Compare this to your predicted value.

(Note: Be sure your gun is <u>not</u> cocked before you return it to the cart.)

### *Lab Questions and Conclusions*

Write a description of how you used the equation for Conservation of Energy, Conservation of Momentum and Projectile Motion in this lab (next page). What conditions made it valid to use these equations in these parts of the experiment?

Compare your prediction for the projectile range and the measured results. Discuss any differences and reasons for the differences. Are there systematic reasons you might expect the results to be larger or smaller than the prediction?

Is momentum conserved (constant) after the collision (as the pendulum swings upwards)? Recall that a) momentum is a vector $\mathbf{p}=m\mathbf{v}$ and has both *magnitude* and *direction* and b) total momentum is a constant if there are no external forces acting on the object.

In the collision itself kinetic energy is not conserved. Calculate the change in kinetic energy in the collision (from just before the ball hits to just after, but before there is any change in height). Where might this energy have gone?
What fraction of the initial kinetic energy is the final kinetic energy?

BONUS PROBLEM: Consider a perfectly <u>in</u>elastic collision between a mass, m, moving with velocity u, and a stationary mass M. Derive an expression for the fraction of the initial kinetic energy of m that is transferred into the combined kinetic energy of (m+M) moving with velocity v. Use this expression to calculate the theoretical value for the collision of the ball and the pendulum in this lab. How does this value compare to the value calculated previously (above)?

### *1. Conservation of Energy*

Describe (in words) how you used conservation of energy in this lab. You must also explain *why* you were able to use conservation of energy for this part of the problem.

### *2. Conservation of Momentum*

Describe (in words) how you used conservation of momentum in this lab. You must also explain *why* you were able to use conservation of momentum for this part of the problem.

### *3. Projectile Motion (Constant Acceleration)*

Describe (in words) how you used the equations for constant acceleration in this lab. Show how you derived the result, which you included in your Calculation Table.

**Ballistic Pendulum Data**

| | | | |
|---|---|---|---|
| Mass of Ball | | Notch Numbers for Five Trials | 1) 2)<br>3) 4)<br>5) |
| Mass of Pendulum | | | |
| Location of Center of Mass of Pendulum | | Average Notch Number | |
| Initial Height of Ball | | Final Height of Ball | |
| Initial Height of Center of Mass of Pendulum | | Final Height of Center of Mass of Pendulum | |

**Ballistic Pendulum Calculations:**

| | Formula or Equation used: | Result of Calculation: |
|---|---|---|
| Change in Potential Energy of Pendulum | | |
| Change in Potential Energy of Ball | | |
| Initial Kinetic Energy of Pendulum Plus Ball (right after collision) | | |
| Velocity of Ball and Pendulum Right After Collision | | |
| Momentum After Collision | | |
| Momentum Before Collision | | |
| Velocity of Ball Before Collision | | |

**Predictions and Results for Projectile Motion (Ball Launched across room)**

| | | |
|---|---|---|
| Measured Height from floor, $\Delta y$ | | |
| Initial Velocity, $v_0$ (determined above) | | |
| Calculation of $\Delta x$ (This is your prediction using $v_0$.) | Formula or Equation used: | Result: |
| Actual $\Delta x$ measured | Five Trials:<br>1) 2)<br>3) 4)<br>5) | Average $\Delta x$ Measured |
| Percent Difference: $\% = [(x_1 - x_2)/x_2] \times 100\%$ | | |