THE BENEFITS OF PERFORMING A GEOMETRIC NONLINEAR ANALYSIS FOR CURVED BRIDGES DURING ERECTION

PAUL BIJU-DUVAL

BIOGRAPHY

Paul Biju-Duval is a structural engineer from France with four years of professional experience in Dubai, Paris and Lima. Paul just completed his PhD studies at the University of Texas at Austin under the direction of Dr. Todd Helwig, where he specialized on curved bridges and developed UT Bridge, which is a finite element program to analyze those structures during erection and deck placement.

SUMMARY

Curved bridges during erection are flexible structures that exhibit large pre-buckling deformations, making an eigenvalue buckling analysis irrelevant. This paper shows the benefits of performing a geometric nonlinear analysis to estimate the bridge behavior and stability. Several case studies are presented, and general guidelines are provided on the type of analysis that should ideally be conducted by bridge engineers for the erection phase.

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Introduction

Due to large unbraced lengths and open section geometries, steel bridges are flexible structures during the erection and construction phase. This flexibility can potentially lead to instability issues and failure modes such as lateral-torsional buckling or global lateral buckling. In particular, curved bridges have a natural tendency to rotate in order to reach a state of minimal potential energy. The prevention of such a rigid body movement at pin or roller supports implies that those systems are subjected both to bending and torsion, which complicates their analysis. For straight bridges without skew subjected to gravity loads, an eigenvalue buckling analysis (where the first eigenvalue is a multiplier on the external applied load that would cause buckling of the structure) is appropriate, as they are subject to bending only. As the vertical load applied to straight girders increases, the lateral deflection remains equal to zero, until the structure suddenly buckles. Once the bifurcation point is reached, the structure loses all of its lateral stiffness (Figure 1). An eigenvalue buckling analysis is able to capture that critical load and is therefore quite suitable to evaluate the stability of straight systems without skew. Admittedly, straight bridges are sensitive to initial imperfections, but those imperfections are generally small so that the pre-buckling deformations remain limited.

However, due to torsion, curved bridges, and to some extent, straight bridges with skewed support lines, show significant lateral deflections, even for small vertical load levels. This is particularly true for plate girder systems, as their open section provides them with little torsional stiffness, but is also valid for curved tub girder systems. As the load increases, their stiffness gradually decreases. In other words, they do not show a bifurcation type behavior. The critical load obtained by an eigenvalue buckling analysis will yield an upper bound value that may be quite far off the true critical load (Figure 1). In practice, bridge engineers still conduct an eigenvalue buckling analysis to evaluate the stability of curved bridges. The safety of the erection process, however, would improve if engineers conducted a geometric nonlinear analysis instead.

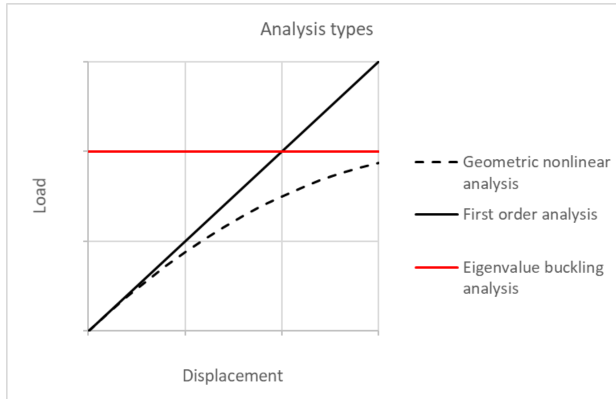

Figure 1: First-order analysis, geometric nonlinear, and eigenvalue buckling analyses

The geometric nonlinear analysis captures geometric nonlinearities, or second-order effects. It differs from the more straightforward first-order linear elastic analysis in that the equilibrium of the structure is calculated in its deformed configuration. As the deflected shape of the structure is itself unknown during the calculation process, it requires the implementation of a Newton-Raphson algorithm of some type. Arc-length methods are another way to implement a geometric nonlinear analysis and are able to capture post-buckling behavior. For curved bridges during erection, however, buckling is the typical failure mode, and traditional Newton-Raphson algorithms are sufficient to capture pre-buckling deformations. It should be recalled that the geometric nonlinear analysis captures one type of nonlinearities only. It does not consider material nonlinearity, but for curved bridges during erection, this is typically a valid assumption as buckling occurs at low-stress levels, well under the steel yield point. It does not consider contact nonlinearities either, but again, this is a valid assumption as uplift issues may arise during concrete placement but are not usually encountered during the erection of the steel superstructure.

UT Bridge

It is widely accepted that the three-dimensional finite element analysis is the most accurate method to analyze curved bridges, particularly during the erection and construction phase (Zureick and Naqib 1999, White et al. 2012). Through an explicit modeling of a complex structure, three-dimensional finite element models are indeed able to capture both global and local behavior, torsion and bending, cross-frame forces, etc. Three-dimensional finite element analysis differs from two-dimensional grid methods, which are approximate analysis methods. Grid methods are accurate enough to analyze curved bridges during the service phase only, once the steel structure acts compositely with the concrete deck, but were shown to be inaccurate for severely curved or skewed systems, although other researchers have tried to mitigate this (Sanchez and White 2017).

Over the past four years, a three-dimensional finite element analysis software targeted at curved plate girder and tub girder bridges during erection and construction has been developed at the University of Texas at Austin. Known as UT Bridge, it has been used by bridge engineers across the United States, and provides a fast yet accurate alternative to expensive, commercial programs. UT Bridge produces three-dimensional models using isoparametric quadratic curved general shell elements for all steel plates, with eight nodes per element and five degrees of freedom per node (the "drilling" degree of freedom is not considered). Curved, complex systems can be modeled within less than an hour. The software capabilities were presented at the previous World Steel Bridge Symposium in Orlando (Biju-Duval and Helwig 2016), as well as the last North American Steel Construction Conference in San Antonio (Biju-Duval and Helwig 2017). However, a major breakthrough from the previous years is that UT Bridge is now available to perform a geometric nonlinear analysis.

UT Bridge has been validated using extensive validation studies using Abaqus, which is a state-of-the-art general-purpose finite element software. UT Bridge can also conduct a first-order linear elastic analysis, as well as an eigenvalue buckling analysis and a modal dynamic analysis. Another significant improvement has also been the ability to model curved tub girder geometries in addition to curved plate girder bridges. A variety of loads are available, from self-weight to point loads, wind loads and thermal loads. Similarly, different brace types can be modeled, such as X-frames, K-frames, lateral trusses and diaphragms (for tub girders).

The approach that was selected to implement the geometric nonlinear analysis in UT Bridge is the modified Newton-Raphson method. At each load increment, the program derives the tangent stiffness matrix based on the bridge deflected shape. Large displacements and rotations are considered, but small strains only, which is a reasonable assumption for buckling problems (Bathe 1982). The program assumes that loads are deformation-independent, meaning that the direction of the loads does not change as the structure gradually deforms. An updated Lagrangian formulation is considered, meaning that all static and kinematic variables are referred to the last computed configuration (Bathe 1982). In particular, the selection of the formulation means that the "true" stresses are calculated at each load increment, unlike other formulations, such as the total Lagrangian method. At each load increment, an algorithm calculates the residual load vector, defined as the difference between the external load vector (R) and the vector of nodal point forces (F) corresponding to the internal stresses at that load increment (Bathe 1982). Convergence of the modified Newton-Raphson solver is considered to be reached when the norm of that residual load vector is less than a tolerance value (ϵ) (Equation 1). For more detailed information on the calculation of the residual load vector and on the algorithmic implementation of the geometric nonlinear analysis into the program, one can refer to the author's dissertation (Biju-Duval 2017).

$$\left\| \frac{R - F}{R} \right\| < \epsilon \qquad \text{(Equation 1)}$$

Case study

A case study is conducted to illustrate the program capabilities and to show how more suitable the geometric nonlinear analysis can be to evaluate the stability of curved bridges compared to the eigenvalue buckling analysis. The structure considered is a five-girder plate girder bridge, with a radius of curvature of 290-ft. for the interior girder, which is a pretty severe curvature. Girder spacing is uniform and equal to 9.67-ft. The cross-frame configuration is radial, with an unbraced length for the interior bay approximately equal to 14-ft. Two different cross-sections are modeled along the bridge, with a uniform web depth of 60-in. In addition, the far-end support line is skewed. Overall, the girder length varies from 152-ft. for the interior girder to 142-ft. for the exterior girder (Figure 2).

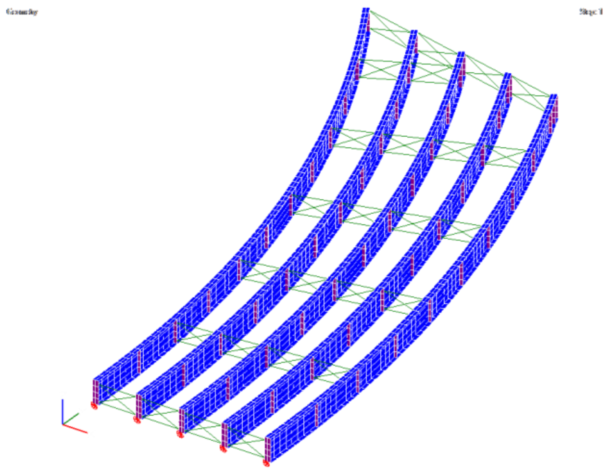

Figure 2: Bridge initial geometry

In a first analysis, however, only half the cross-frames are turned on, which results in an unbraced length approximately equal to 28-ft., which is rather large given the severe curvature (Figure 2). A 2-ft. uniform longitudinal mesh size is specified, resulting in a number of 58,244 unrestrained degrees of freedom. The following figures describe the bridge behavior produced by a first-order linear elastic analysis, with the self-weight of the structure acting as the only external load: deflected shape (Figure 3), longitudinal stress envelope (Figure 4), moment diagram (Figure 5), shear diagram (Figure 6), torsion diagram (Figure 7).

From the figures 3 to 7 it can be observed how the exterior girder displays higher levels of displacements and stresses, and how the longitudinal stress distribution is not uniform across the width of the flanges because of the torsion-induced warping longitudinal stresses. Overall, those figures show how effective UT Bridge can be for modeling complex curved plate girder geometries, although this example remains rather simple, as it is a simply-supported structure only and the cross-frame configuration is radial. Any support or cross-frame configuration can however be modeled with UT Bridge. For the case study presented in this paragraph, modeling of the structure took only about twenty minutes, and finite element calculation about thirty seconds. Important quantities are directly available to the bridge engineer, such as shear, moment and torsion diagrams. Bi-moment diagrams directly related to the warping longitudinal stresses are also available, although they are not presented in this paper. The shear diagram, for example, can be directly used to estimate support reactions. In addition to the longitudinal stresses, UT Bridge can also display the Von Mises stresses as well as the in-plane shearing stress components. UT Bridge can also display any of the three components of the displacement vector, as well as support reactions and cross-frame forces. Post-treatment of the displacements and the stresses is performed automatically, which saves plenty of time for bridge engineers.

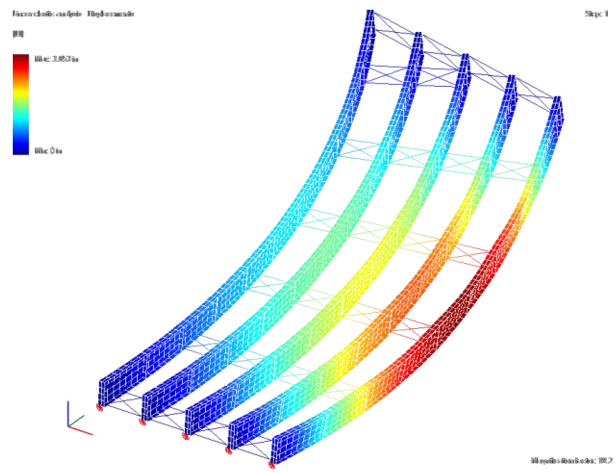

Figure 3: Deflected shape (first-order analysis)

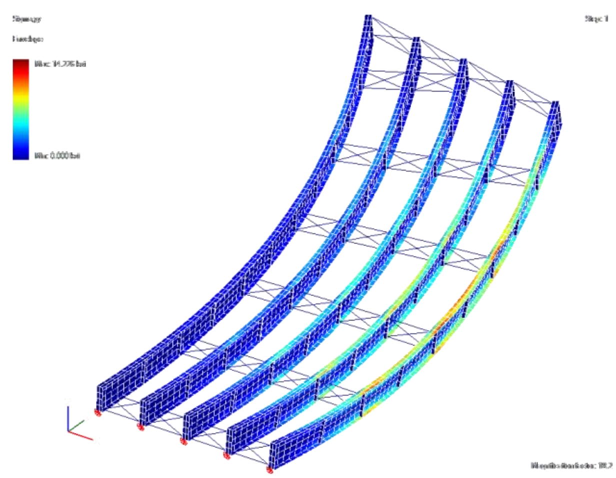

Figure 4: Longitudinal stress envelope

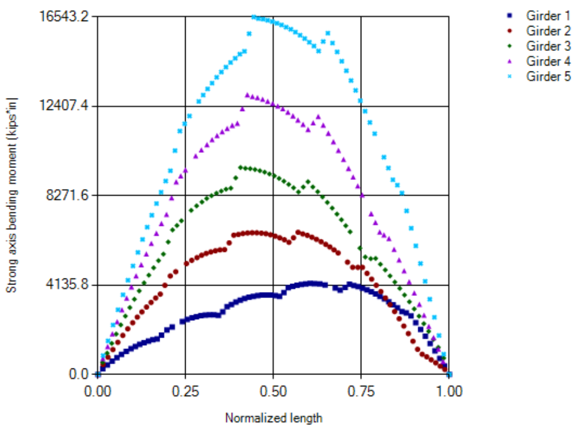

Figure 5: Moment diagram

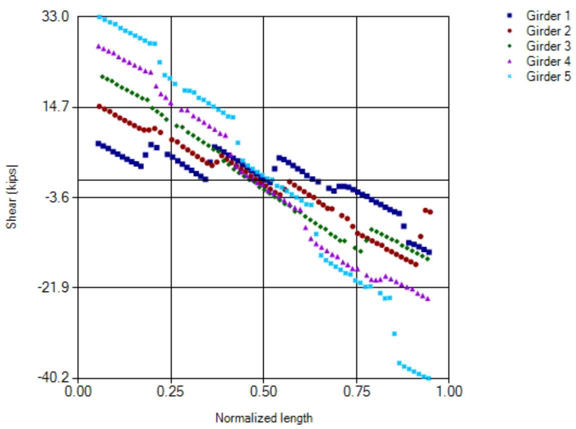

Figure 6: Shear diagram

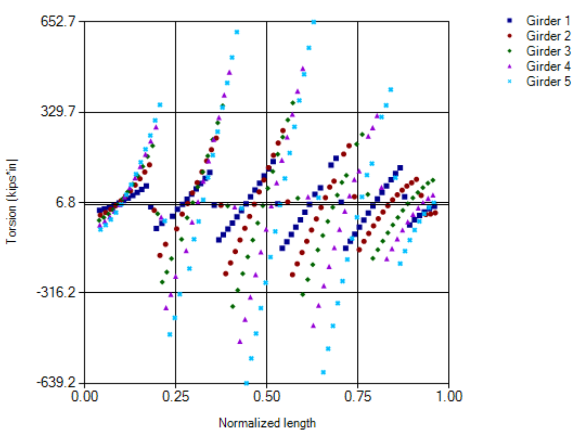

Figure 7: Torsion diagram

Finally, Figures 8 and 9 show some charts that UT Bridge can automatically display to understand the bridge behavior. Figure 8 shows the vertical displacement of the girders, while Figure 9 shows the torsional layovers, meaning the lateral relative displacement between the top and the bottom flanges. The layover diagram gives an indication of the torsional deformation of the structure.

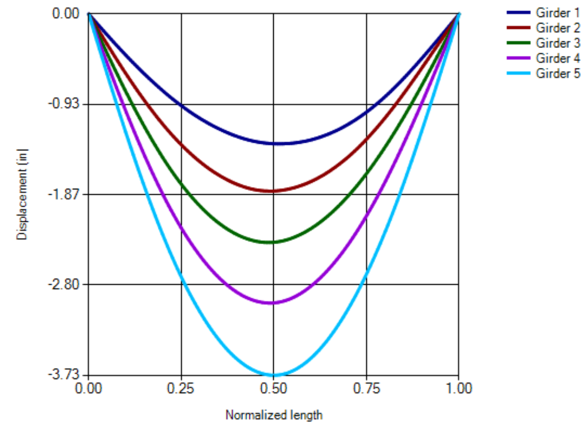

Figure 8: Vertical deflections

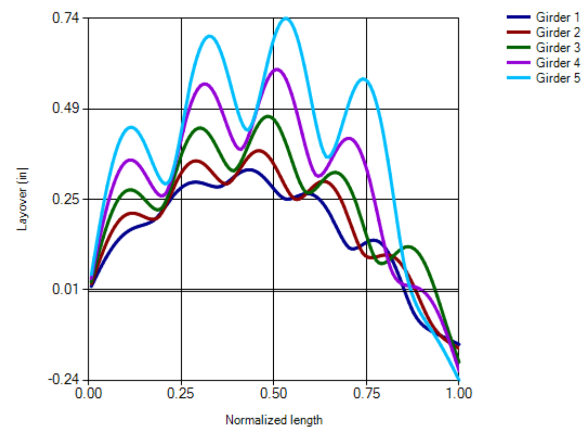

Figure 9: Layovers

In this example, the unbraced length was specified as 28-ft. to have the structure reach a buckling mode for an eigenvalue close to a value of 5, which is a recommended value under which a geometric nonlinear analysis should be conducted (White et al. 2012). For this particular structure, the first buckling mode is a lateral-torsional buckling mode, and the corresponding buckling eigenvalue is calculated equal to 5.5 (Figure 10). It should be recalled here that UT Bridge is selecting FEAST for its eigensolver, which is a powerful eigensolver that was developed at the University of Massachusetts at Amherst (Polizzi 2009) and differs from the traditional Krylov subspace-iteration-based techniques more commonly implemented in structural analysis commercial programs.

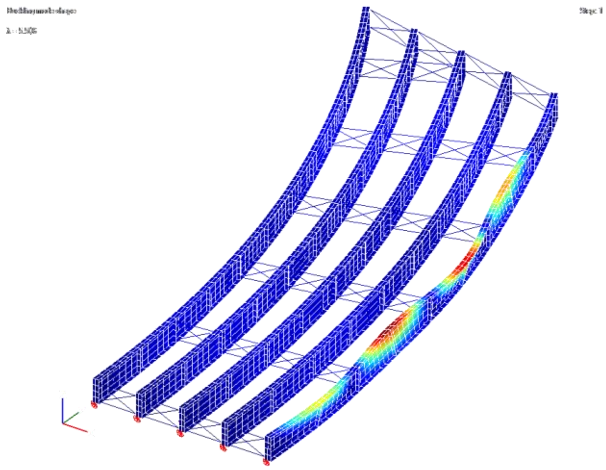

Figure 10: First buckling mode (λ = 5.5)

A geometric nonlinear analysis is then performed, using a number of 25 uniformly spaced load increments, which is the default assumption implemented by the program. This time, the analysis is more computationally expensive, but it is still completed within ten minutes. The deflected shape obtained using a geometric nonlinear analysis is shown in Figure 11. It can be observed that the deformation is slightly amplified from a first-order linear elastic analysis. The maximum deflection is indeed increased from 3.85-in. to 3.93-in. This amplification may seem small, but bridge engineers should not only figure by how much the maximum deflection is amplified. Complex models involve tens of thousands of unrestrained degrees of freedom, and the structural behavior may be severely overlooked if the only result discussed is that maximum displacement.

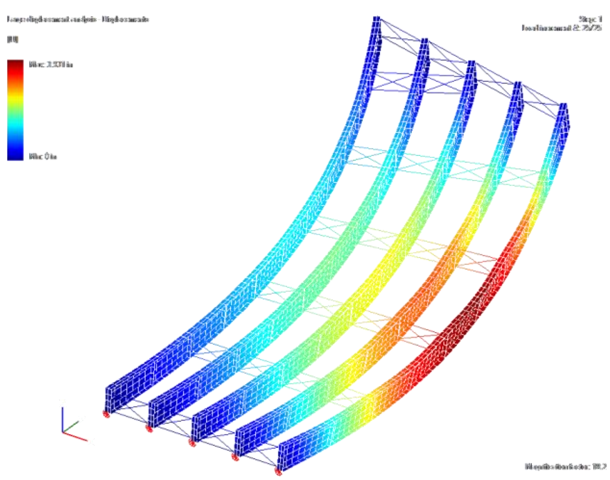

Figure 11: Deflected shape (geometric nonlinear analysis)

After initially moving in a certain direction at low load levels, some points even start moving in the reverse direction at higher load increments. This phenomenon can be observed in Figure 12, which shows the load versus displacement curve of two different displacement degrees of freedom. The blue curve shows the lateral displacement of the top flange node at midspan of the exterior girder. The red curve applies to the same degree of freedom, but this time at the bottom flange node. From the blue curve, it can be observed how the structure gradually loses stiffness. When 100% of the external load is applied, the lateral stiffness at the top flange node has lost about 70% of its original value. As mentioned in the introduction, curved bridges do not exhibit a bifurcation type behavior, and the computed buckling eigenvalue of 5.5 is in this case a very upper bound on the critical load. An eigenvalue buckling analysis is therefore not suitable to evaluate the bridge structural stability. On the other hand, the bottom flange, after initially moving toward the center of curvature of the bridge, then deflects outward (Figure 12), which may seem counter-intuitive at first. At a load ratio close to 0.5, the bottom flange lateral displacement even presents a tangent stiffness close to infinity. This occurs when the bridge lateral movement and the cross-sectional rotation perfectly balance each other.

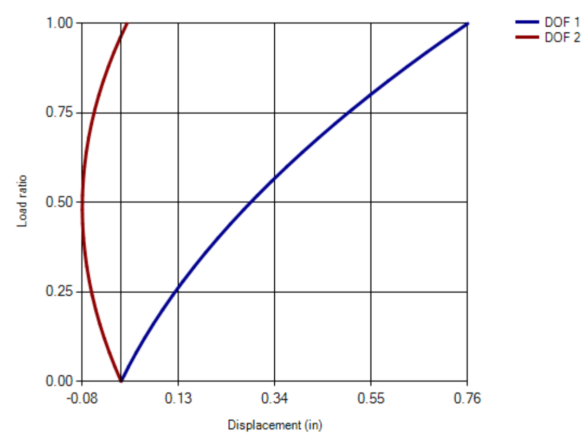

Figure 12: Load vs. deflection curves (geometric nonlinear analysis)

The combination of an unbraced length of 28-ft. together with a radius of curvature of 290-ft. not surprisingly resulted in a flexible structure prone to instability issues, although the buckling eigenvalue was calculated equal to 5.5. In order to assess the performance of a geometric nonlinear analysis on stiffer structures, the unbraced length is now reduced by 50% and is therefore equal to roughly 14-ft. An eigenvalue buckling analysis yields a first global buckling model equal to 17.8 (Figure 13).

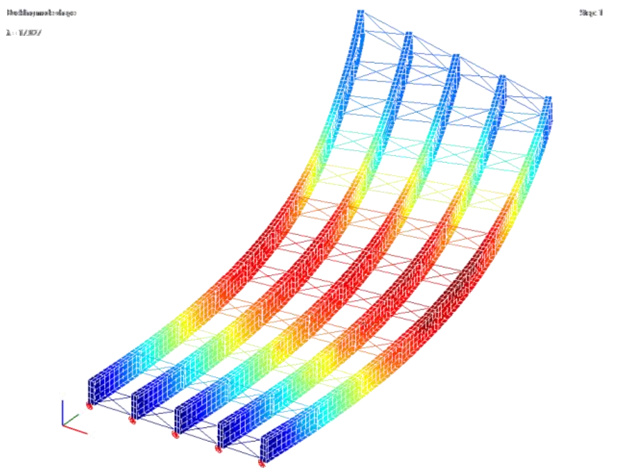

Figure 13: First buckling mode (λ = 17.8)

As expected, reducing the unbraced length significantly increased the structural stability. Although second-order effects are expected to be small, a geometric nonlinear analysis is conducted to assess them. The deflected shape obtained using a geometric nonlinear analysis is barely amplified and is therefore not shown in this paper. However, there are some differences in the value of the cross-frame forces, whether they are evaluated using a first-order linear elastic analysis or a geometric nonlinear analysis. For example, the axial forces in the sixth cross-frame from the left support in the exterior bay are shown in Figures 14 and 15 (axial forces for all members are shown in Figures 16 and 17). This cross-frame is selected because it is close to midspan of the exterior girder, where displacements and stresses are maximal. Although the geometric nonlinear analysis does not show any substantial deflection amplification, it does have an impact on the magnitude of those forces. Cross-frame forces evaluated using a geometric nonlinear analysis are symmetric: top and bottom chords, as well as left and right diagonals, have opposite values. A first-order linear elastic analysis does not yield such a symmetry: the tension forces seem to be slight overestimated, and the compression forces, slightly underestimated. Although cross-frames are usually sized for higher loads such as truck loads, a parametric study is conducted to estimate the influence of the curvature on those axial forces.

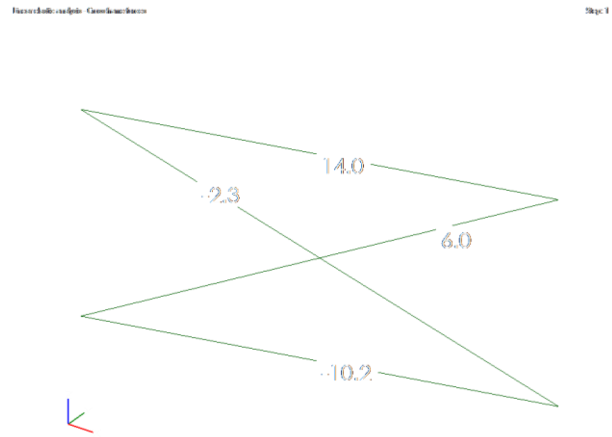

Figure 14: Cross-frame forces (exterior bay, sixth cross-frame from the left support, first-order linear elastic analysis)

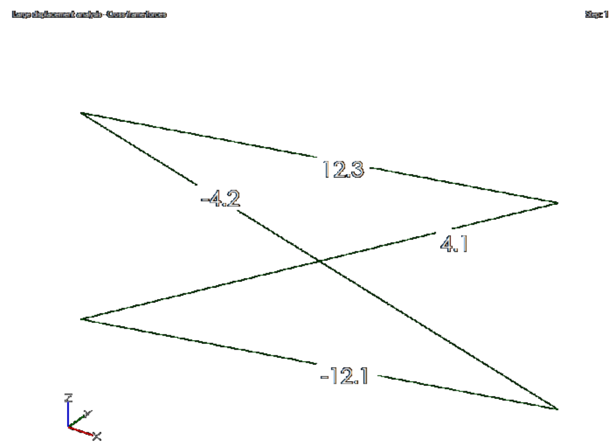

Figure 15: Cross-frame forces (exterior bay, sixth cross-frame from the left support, geometric nonlinear analysis)

Parametric study

The case study described in the previous paragraph has shown how the type of analysis, whether first-order linear elastic or geometric nonlinear, affects the magnitude of the cross-frame forces. Even for large buckling eigenvalues, there may be sizeable difference between results obtained from both analyses. In this paragraph, a parametric study describing the influence of the curvature on the cross-frame forces is presented. The structure considered is a curved, simply-supported three-girder plate girder bridge. It is adapted from a study by Davidson and Yoo (1996). Girder spacing is equal to 9-ft. The exterior girder has an overall length of 100-ft (Figure 18).

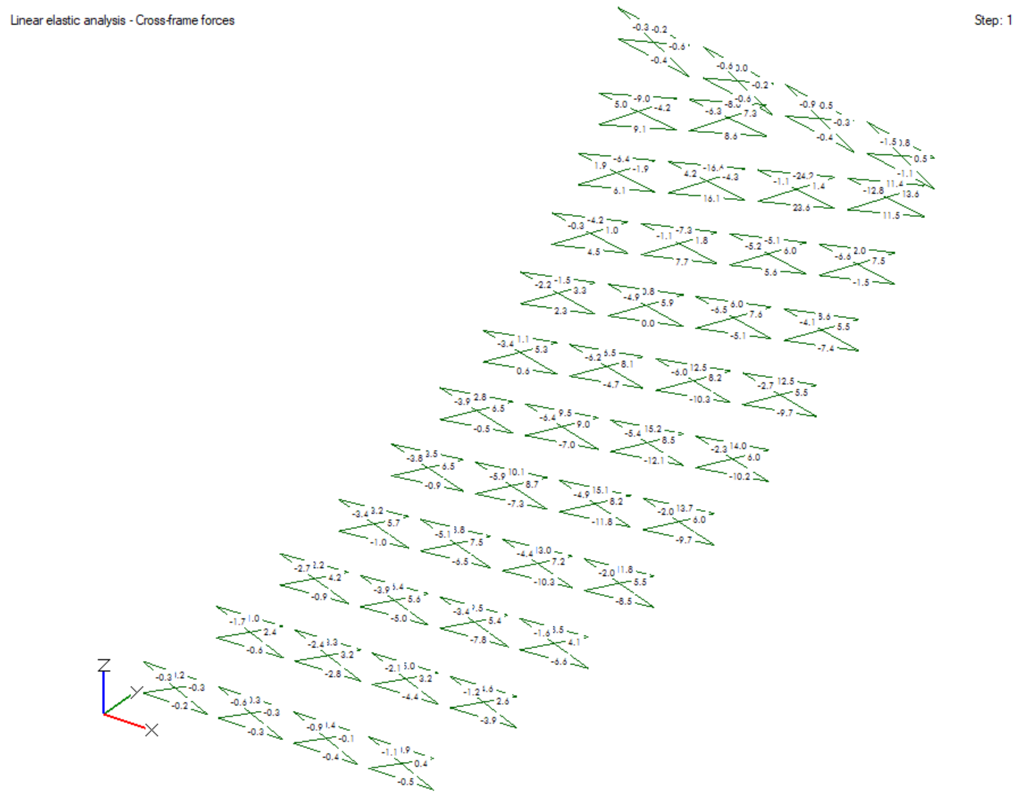

Figure 16: Cross-frame forces (first-order linear elastic analysis)

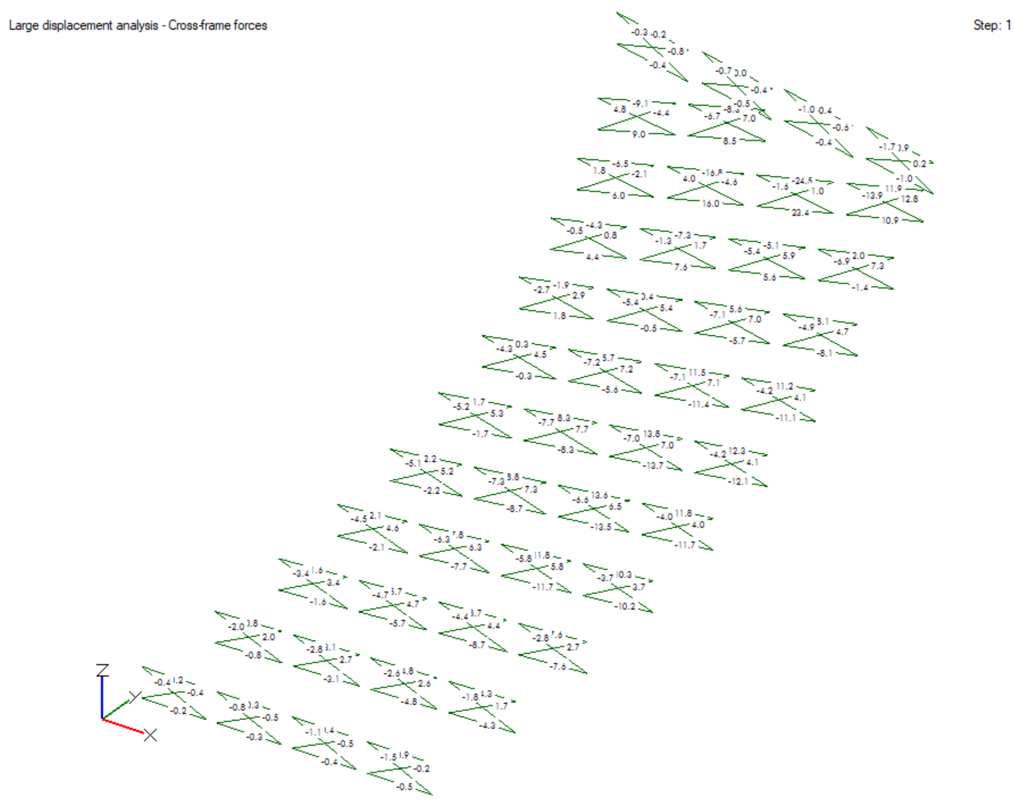

Figure 17: Cross-frame forces (geometric nonlinear analysis)

Cross-sectional dimensions are 12-in.x1-in. for the flanges and 58-in.x0.5-in. for the web. A radial cross-frame configuration is applied. A uniform surface load equal to 1.0k/ft^2 for the interior and exterior girders, and 1.2k/ft^2 for the intermediate girder is applied (Figure 18). This load represents the self-weight of the steel superstructure, as well as an 8-in. thick wet concrete deck. A fine mesh is specified, with eight shell elements through the web depth, and a longitudinal mesh size of 9-in, resulting in an overall number of 80,976 unrestrained degrees of freedom. Several values for the radius of curvature of the exterior girder are considered: 200-ft., 500-ft., 700-ft., 1,000-ft. and 2,000-ft. The length of the first two interior girders is modified accordingly to keep a uniform girder spacing. A straight bridge is also modeled in order to model a radius of curvature close to infinity. The axial forces in the third cross-frame from the left support on the right bay are investigated. As shown in the previous paragraph, deflections and stresses are indeed maximal at midspan of the exterior girder.

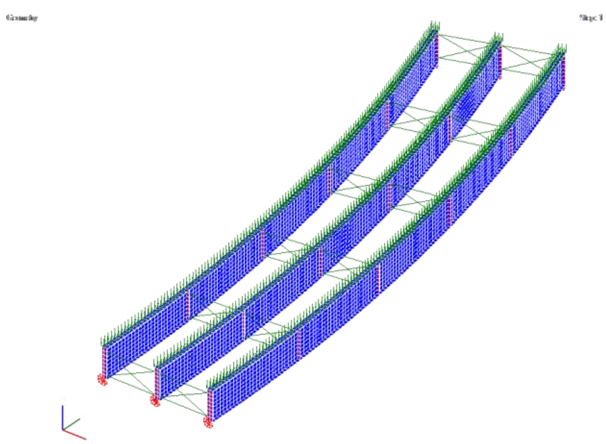

Figure 18: Bridge initial geometry and loading (R = 100-ft.)

The influence of the subtended angle L/R, where L is equal to the length of the exterior girder (100-ft.), and R is equal to the radius of curvature of the exterior girder, is shown in Figure 19. Several observations can be made. First, as expected, cross-frame forces are small for straight and mildly curved bridges. But as the subtended angles gradually increases, cross-frame forces increase dramatically, which justifies why cross-frames are considered as primary structural components for severely curved bridges. Second, the discrepancy between the first-order and the geometric nonlinear analysis increases with the subtended angle. As expected, second-order effects are negligible for straight systems under gravity loads. On the other hand, second-order effects become significant for more severe curvatures. Interestingly, although the discrepancy steadily increases, it does not affect tension and compression components in the same manner. At least for this particular cross-frame, tension forces are overestimated using a first-order linear elastic analysis, while compression forces are underestimated using a first-order linear elastic analysis. This statement shows how unconservative a first-order linear elastic analysis may be for severely curved bridges. Compression forces may be significantly underestimated, resulting in undersized braces potentially leading to buckling failures. Third, Figure 19 shows that diagonal forces increase linearly with curvature, while top and bottom chord forces increase more rapidly. Last but not least, and similarly to the case study presented in the previous paragraph, a geometric nonlinear analysis yields a symmetric distribution of the cross-frame forces: top and bottom chords show opposite forces of equal magnitude, and the same applies to the left and right diagonals. A first-order linear elastic analysis does not produce such a symmetric distribution.

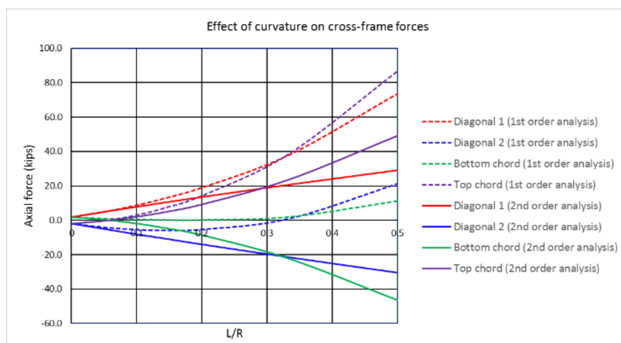

Figure 19: Effect of curvature on cross-frame forces (third cross-frame from the left support, exterior bay)

Another way to evaluate of the benefit of performing a geometric nonlinear analysis can be obtained by comparing the maximal deflection produced by a first-order linear elastic analysis versus a geometric nonlinear analysis. The effect of curvature on both deflections is shown in Figure 20. For straight bridges under gravity loads, both analyses yield equivalent displacements (Figure 20). But as far curvature gradually increases, second-order magnify the displacements, and the structural response is amplified (Figure 20). As shown in the case study though, second-order effects do not only mean a structural amplification.

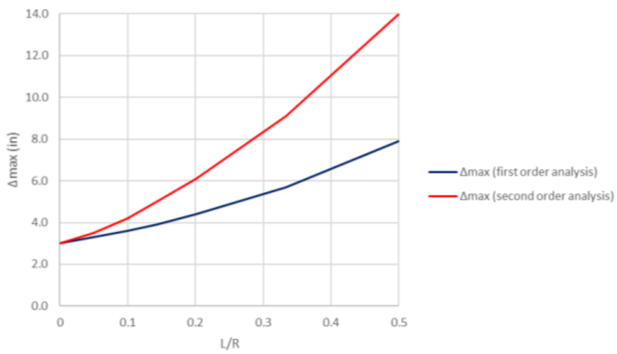

Figure 20: Effect of curvature on the maximal deflection

Case of straight tub girder bridge

The previous paragraphs have shown how conducting a geometric nonlinear analysis can lead to a better estimation of displacements and cross-frame forces. It was also shown that an eigenvalue buckling analysis is not truly meaningful for curved structures, as they exhibit large pre-buckling deformations. The eigenvalue buckling analysis, however, is quite useful for straight systems during erection and deck placement to evaluate structural stability. For example, the Marcy Bridge collapse in the state of New York in 2002 could have been prevented if such an analysis had been conducted (Yura and Widianto 2005). In this paragraph, a similar failure is analyzed. The Y1504 Bridge in Sweden collapsed the same year as the Marcy Bridge, also during deck placement. It was a straight, 213-ft. long tub girder with three different profiles along its length, with eleven uniformly spaced diaphragms to prevent cross-sectional distortion (Ohlin 2016). The bridge is modeled in UT Bridge and loaded with its self-weight as well as a uniform pressure of 0.67k/ft^2 on the first two interior panels to model the weight of the wet concrete (Ohlin 2016). A view of the model and the loading is given in Figure 21. An eigenvalue buckling analysis is conducted on the structure, leading to a first buckling eigenvalue equal to 1.0, which shows how sensitive to buckling the structure was (Figure 22). When a couple of lateral trusses are added at each end, the buckling eigenvalue increases to 2.7 (Figure 23). As expected, lateral trusses provide a warping restraint to the bridge, dramatically increasing its stability.

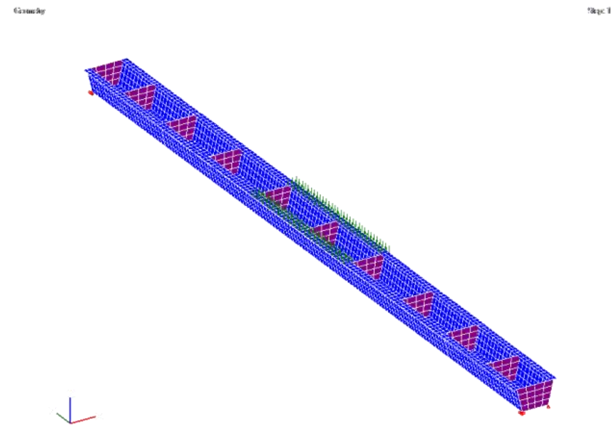

Figure 21: Y1504 Bridge model and loading

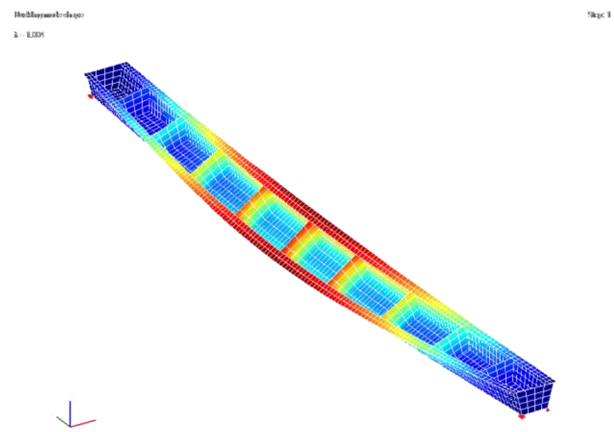

Figure 22: Y1504 Bridge buckled shape (λ = 1.0)

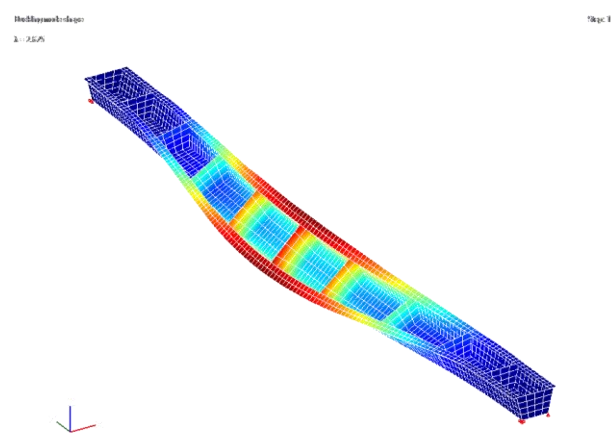

Figure 23: Y1504 Bridge (with two lateral trusses added at each end) buckled shape (λ = 2.7)

Conclusions

Although eigenvalue buckling analyses are conducted on a daily basis for curved bridges during erection, they are not truly suitable to evaluate the structural stability during erection, as curved systems gradually lose stiffness and do not exhibit a bifurcation type buckling behavior like straight bridges. It is highly recommended that bridge engineers perform a geometric nonlinear analysis when the computed buckling eigenvalue is less than 5. Second-order effects result in an amplification of the deformations, but also in different cross-sectional rotations, leading to different cross-frame forces. For buckling eigenvalues between 5 and 15, second-order effects are still sizeable but will not affect the bridge stability, therefore a geometric nonlinear analysis is not necessary, although it may be conducted to estimate the bridge displacements and cross-frame forces with increased accuracy. For buckling eigenvalues larger than 15, a geometric nonlinear analysis is not necessary: second-order effects are small and will only affect slightly the magnitude of the cross-frame forces. For non-skewed straight bridges however, an eigenvalue buckling analysis is suitable to address any instability issues and should be conducted whenever necessary.

As far as UT Bridge, next developments include the ability to model initial imperfections in order to conduct meaningful geometric nonlinear analyses even on straight systems.

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