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THE CALCULUS OF THE MIXING ROTATING DEVICES SHAFTS BASED ON THE ENERGONICS METHOD

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Lucrarea prezintă un studiu cu privire la metodologia de calcul de rezistență a arborelui dispozitivelor rotative de amestecare, în condiții de funcționare normale. Scopul acestei lucrări este de a face o comparație între metoda de calcul clasică şi metoda Energonicii (elaborată pe baza principiului energiei critice), evidențiind avantajele celei de a doua metode. Este analizat cazul unui arbore fara discontinuități de structură (defecte de material).

The paper presents a study concerning the methodology of the strength calculation of the mixing rotating devices shaft in normal operating conditions. The aim of this work is to make a comparison between the classical calculation method and Energonics method (developed on the principle of critical energy), showing the advantages of the second method. The case of a shaft without material defects (structural discontinuities) is analyzed.

Keywords: mixing rotating device shaft; strength calculation; principle of critical energy.

1. Introduction

The mechanical design of the mixing rotating devices which operate in liquid medium depends on the exact determination of the mechanical and hydrodynamic loadings of the mixing device (shaft and mixer). These loadings are difficult to evaluate during the mixing process, especially in conditions of unsteady flow. Therefore, in the current design activity one operates with simplified loading models of mixing devices.

Based on the adopted models, different mechanical calculation methods were developed, which, in principle, solve the following problems [1-3]: the driving system design of the mixing device; the strength calculation of the shaft and of the mixers mounted on it; the rigidity calculation of the shaft; the selection and the calculation of the shaft seal; the design of the shaft bearings.

This paper provides a study of the strength calculation methodology of the mixing rotating devices shaft in normal operating conditions. Two different methods can be used to solve these problems: *the classical calculation method*, which is currently more commonly applied, and *the Energonics method* – developed on the principle of critical energy [4-6].

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It is known that the classical method uses the concept of equivalent stress [1-3, 7] for the strength calculation of the shaft.

Unlike the classical calculation method, the Energonics method applies the concept of *the participation of the specific energy due to the shaft loadings* (non-dimensional concept) [4; 5].

In this work, the attention is focused on the use of the Energonics method in the case of a flawless shaft, emphasizing its advantages compared with the classical calculation method.

2. Calculation of the mixing device shaft loadings

One starts from the following hypotheses: the mixing device is multi-staged (n_0 mixers are mounted on the same central shaft, arranged in parallel planes and perpendicular to the longitudinal axis of the shaft); the mixing device shaft is supported on two roller bearings; the mixing device is driven in by a rotating electric motor (with the installed power N_{inst}), via a cylindrical gear drive or belt drive; the drive is provided with a safety overload system and the engine starting is done with the aid of a start-up device; the working medium (liquid) does not have large variations in viscosity and does not adhere to the walls to form crusts; during mixing it does not produce precipitation and there is no danger of accidentally decreasing of the liquid level in the vessel.

For the calculations of the mechanical strength in the two cases analyzed below (shaft *with* and *without* cracks), one chooses *a general model of loading* of the shafts, taking the following loadings into consideration (fig.1):

a) *statically applied loads; b) dynamically applied loads; c) torque* M_t.

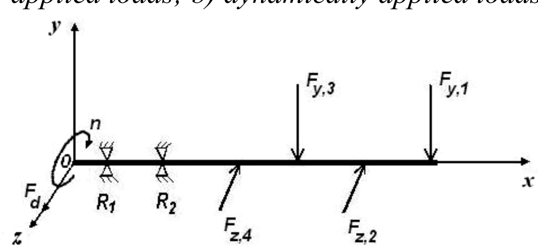

Fig.1. Principle sketch of a statically and dynamically loaded shaft (O – corresponds to the shaft's end, where a driving wheel is placed; R_1, R_2 – bearings; *n* – rotational speed)

a) Statically applied loads

Statically applied loads are the following:

$F_{y,i}$ – in the xOy plan (which produces bending moment $M_{b,y}$);

$F_{z,i}$ – in the *xOz* plan (which produces bending moment $M_{b,z}$).

On each mixer i ($i=\overline{1,n_0}$) mounted on the shaft, a bending radial force acts either in the xOy plan (being noted $F_{y,i}$), or in the xOz plan (being noted $F_{z,i}$), depending on the mixer blades orientation as function of the chosen coordinate system. This force is calculated as a resultant of the two following forces: $F_{R,i}$ (the resistance hydrodynamic force of the working fluid, corresponding to the i mixer); $F_{c,i}$ (the centrifugal force caused by the dynamic residual imbalance of the i mixer).

For a i mixer, the two components of the bending radial force have the following mathematical expression [8]:

$$F_{R,i} = c_{R,i} \cdot \rho_l \cdot n^2 \cdot d_{a,i}^4 \quad , \tag{1}$$

$$F_{c,i} = m_{a,i} \cdot r_{d,i} \cdot \omega^2 \quad , \tag{2}$$

where: $d_{a,i}$ - the i mixer diameter (mixer span); $m_{a,i}$ - the i mixer weight; $c_{R,i}$ -the hydrodynamic driving coefficient corresponding to the i mixer (depends on mixer geometry); $r_{d,i}$ - the dynamic residual imbalance of the i mixer; ρ_l - the working fluid density; n – rotational speed of the shaft; ω - the shaft angular speed (velocity of rotation), $\omega = \dfrac{\pi \cdot n}{30}$.

According to [8], one uses the following relationship, in order to calculate $r_{d,i}$:

$$r_{d,i} = \frac{e_i + 0{,}5 \cdot e_b}{1 - \left(\dfrac{\omega}{p_1}\right)^2} \quad , \tag{3}$$

where: e_i - eccentricity of the i mixer mass center; e_b - allowable shaft deflection (usually $e_b \le 1\ mm$); p_1 - basic proper angular frequency of the shaft.

The most disadvantageous variant is that in which $F_{R,i}$ and $F_{c,i}$ act in the same direction and sense. Their resultant module is obtained by algebraic summation: $F_{R,i} + F_{c,i}$.

b) Dynamically applied loads (fatigue loading)

Dynamically applied loads produce bending moment noted $M_b^{(d)}$.

In the category of the dynamically applied loads there are:

- the radial force F_d (fig.1). F_d is either the force resulting from the gears meshing, or the force introduced by the transmission belt, depending on the type of mechanical transmission which is part of the driving system of the shaft);

- the own weight of the shaft and the weights of the elements disposed on it (these are taken into consideration only in case of a horizontal shaft).

c) Torque M_t

Torque M_t is transmitted along the shaft through the mixing device driving system. In design activity, usually one uses calculating torque (noted $M_{t,c}$), considered higher than the torque M_t ($M_{t,c} > M_t$).

$$M_{t,c} = \frac{N_{inst} \cdot \eta_{tr}}{\omega} , \tag{4}$$

where η_{tr} is the overall efficiency of mechanical transmission used.

3. Calculation of the stresses caused by loads acting on the mixing device shaft

Shafts sizing includes identifying dangerous shaft sections and calculating the required minimum diameter in these shaft sections.

The torque transmitted to each j cross-section of the shaft is determined by the relationship:

$$M_{t,j} = \frac{N_j}{\omega} , \tag{5}$$

where N_j is the necessary mixing power in j cross-section of the shaft (fig.2).

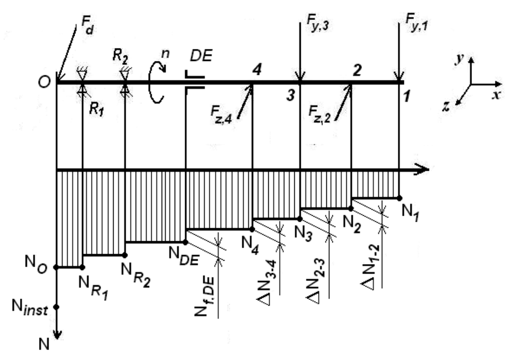

Fig.2. Distribution of the necessary mixing power N along a mixing device shaft provided with sealing device (noted *DE*), and four mixers mounted in the sections 1, 2, 3 and 4.
$N_{f,DE}$ - the power loss in the sealing device as a result of the friction;
ΔN_{1-2}, ΔN_{2-3}, ΔN_{3-4} - the required power increases from a mixer to another.

In any j cross-section of the shaft, the resulting bending moment $M_{b,j}$ due to the statically applied loads $F_{y,i}$, $F_{z,i}$ has the expression:

$$M_{b,j} = (M_{b,y,j}^2 + M_{b,z,j}^2)^{\frac{1}{2}}, \tag{6}$$

where each of the components $M_{b,y,j}$ and $M_{b,z,j}$ is the algebraic sum of the bending moments determined in *j* section by all the statically applied forces acting in the plane *xOy*, *xOz,* respectively, on one side of the current *j* section.

The diagram of the bending moments due to the dynamically applied loads differs depending on the operating position of the mixing device shaft (vertical or horizontal). For a vertical shaft, for example, in accordance with the loading model presented before, only the force F_d has dynamic effect. In this case, the bending moment $M_{b,j}^{(d)}$ produced in any *j* cross-section is calculated with the relationship:

$$M_{b,j}^{(d)} = F_d \cdot l_d, \tag{7}$$

where l_d is the moment arm. The maximum value $M_{b,\max}^{(d)}$ is obtained in the cross-section corresponding to the support R_1.

For a *shaft without material defects*, in any of its *j* cross-section, the normal stresses $\sigma_{b,j}$ (produced by static bending) and $\sigma_{b,j}^{(d)}$ (produced by dynamic bending), and also the tangential stress $\tau_{t,j}$ (produced by the torsion) are calculated by the relations:

$$\sigma_{b,j} = \frac{M_{b,j}}{W}\ ;\ \sigma_{b,j}^{(d)} = \frac{M_{b,j}^{(d)}}{W};\ \tau_{t,j} = \frac{M_{t,j}}{W_p}, \tag{8}$$

where W - the modulus of resistance of the shaft cross-section; W_p - the polar modulus of resistance of the shaft cross-section.

4. Strength calculation of the mixing device shaft

The strength calculation of the shaft is made for the torsion and bending using both classical and Energonics method.

The classical calculating method comprises two successive stages [1-3]:

- sizing of the shaft subjected to torsion;
- verification of the shaft strength under combined bending and torsion.

The working principle of this method is based on the concept of equivalent stress, whose expression depends on the applied strength theory.

The Energonics method is based on the principle of critical energy and the equivalence of processes and phenomena [5, 9]. It has a high level of generality, offering the following advantages: enables to solve the cases in which loads may be applied both statically and dynamically; enables to take into account the non-linear elastic behaviour of the shaft material in the conditions of the considered loading.

The non-linear elastic behaviour of the shaft material is described by the power-law function [5]:

$$\sigma = M_{\sigma} \cdot \varepsilon^{k}; \quad \tau = M_{\tau} \cdot \gamma^{k}, \tag{9}$$

where $M_{\sigma}; M_{\tau}$ and k are constants of material; $\sigma; \tau$ – normal stress and shear stress, respectively; ε – strain; γ – shear strain.

The application of the critical energy principle consists in the calculation of the specific energy participation in the evolution of a process, with respect to the critical state defined for this analyzed process. One considers the energy accumulated in the material involved in that process, due to the external loadings/ actions.

In the case of the mixing rotating device shaft, reaching the critical state corresponds to its fracture. One determines the total participation P_T with respect to the critical state, calculated as the sum of partial participations caused by each loading exerted on the shaft (i.e. the static and dynamic bending, and torque). The limit is reached when $P_T = P_{cr}$ [5], where $P_{cr} \le 1$ (P_{cr} is the critical value of the participation corresponding to fracture).

One uses the expression of the total participation P_T^* with respect to the allowable state for sizing the mixing device shaft [5]:

$$P_T^* = \left(\frac{\sigma_{b,j}}{\sigma_{b,ad}}\right)^{\alpha+1} + \left(\frac{\sigma_{b,j}^{(d)}}{\sigma_{b,ad}^{(d)}}\right)^{\alpha+1} + \left(\frac{\tau_{t,j}}{\tau_{t,ad}}\right)^{\alpha+1}, \tag{10}$$

where α exponent depends on the rate of load applying. $\alpha = \frac{1}{k}$, for static loading; $\alpha = 0$, for shock loading (the time interval required to reach maximum load is less than half of the structure fundamental period); $\alpha = \frac{1}{2k}$, for rapid loading (the applied load is increased rapidly at a finite rate and the time interval required to reach maximum load is greater than the time required in the case of shock loading. The allowable stresses are:

$$\sigma_{b,ad} = \frac{\sigma_{b,cr}}{c_b}; \qquad \sigma_{b,ad}^{(d)} = \frac{\sigma_{b,cr}^{(d)}}{c_b^{(d)}}; \qquad \tau_{t,ad} = \frac{\tau_{t,cr}}{c_t}, \tag{11}$$

where $\sigma_{b,cr}$, $\sigma_{b,cr}^{(d)}$ are the critical normal stresses under static and dynamic bending, respectively; $\tau_{t,cr}$ - critical tangential stress under the torsion; c_b, $c_b^{(d)}$, c_t - safety coefficients of static bending, dynamic bending and torsion, respectively.

From the relations (8) and (10) it yields:

$$P_T^* = \left(\frac{M_{b,j}}{W \cdot \sigma_{b,ad}}\right)^{\alpha+1} + \left(\frac{M_{b,j}^{(d)}}{W \cdot \sigma_{b,ad}^{(d)}}\right)^{\alpha+1} + \left(\frac{M_{t,j}}{W_p \cdot \tau_{t,ad}}\right)^{\alpha+1} . \quad (12)$$

In general, $W_p = \beta \cdot W$, where β depends on the cross-sectional shape of the shaft (for example, for a shaft with a circle cross-section, $\beta = 2$ [8]).

In Energonics method, the condition for the shaft loading to be allowable is expressed by the relationship:

$$P_T^* \le P_{ad} , \quad (13)$$

where

$$P_T^* = 1 - D^*(t) . \quad (14)$$

$D^*(t)$ is the deterioration (function of time t) with respect to the allowable state produced in the shaft during loading application (for example, in the case of cracks propagation).

If there are not cracks, then

$$P_T^* = 1 . \quad (15)$$

From the relations (12) and (15), one obtains the expression for the necessary modulus of resistance, $W_{nec,j}$, corresponding to any section j of the shaft:

$$W_{nec,j} = \left[\left(\frac{M_{b,j}}{\sigma_{b,ad}}\right)^{\alpha+1} + \left(\frac{M_{b,j}^{(d)}}{\sigma_{b,ad}^{(d)}}\right)^{\alpha+1} + \left(\frac{M_{t,j}}{\beta \cdot \tau_{t,ad}}\right)^{\alpha+1}\right]^{\frac{1}{\alpha+1}} . \quad (16)$$

Expression (16) takes into account: the shaft material behavior (non-linear elastic or linear elastic) under loading conditions; the influence of the type loading (static loading; fatigue loading under pulsating or symmetric alternating stress cycle etc.); the rate of loads application (static, rapid or shock loads).

The minimum required shaft diameter corresponding to the analysed j section (noted $d_{nec,j}$) is calculated depending on the value $W_{nec,j}$ (16), taking into account the geometry of the shaft cross-section. For shafts with full circular section is obtained:

$$d_{nec,j} = \sqrt[3]{\left(\frac{32 \cdot W_{nec,j}}{\pi}\right)} \quad . \tag{17}$$

For linear elastic behavior of the shaft material corresponding to Hooke's law, $\sigma = E \cdot \varepsilon$ and $\tau = G \cdot \gamma$, constant $k = 1$, so $\alpha = 1$ [8]. Equation (16) becomes:

$$W_{nec,j} = \left[\left(\frac{M_{b,j}}{\sigma_{b,ad}}\right)^2 + \left(\frac{M_{b,j}^{(d)}}{\sigma_{b,ad}^{(d)}}\right)^2 + \left(\frac{M_{t,j}}{\beta \cdot \tau_{t,ad}}\right)^2\right]^{\frac{1}{2}} . \tag{18}$$

Practically, the strength calculation of the mixing rotating device shaft in the conditions of the above mentioned hypotheses and of the previously determined loadings is done by applying the algorithms included in Table 1.

Table 1

Relations used for the strength calculation of the mixing devices rotating shaft

Classical Method (equivalent stresses method)	**Energonics Method**
Stage / Computation relations	**Stage / Computation relations**
1. ***Sizing the shaft under torque load*** / $M_{t,c} = \frac{N_{inst} \cdot \eta_{tr}}{\omega}$; $W_{p,nec} = \frac{M_{t,c}}{\tau_{t,ad}}$; $d_{nec} = \sqrt[3]{\frac{16 \cdot W_{p,nec}}{\pi}}$; (for full circular cross - section shaft) $d = d_{nec} + 2c$. It is adopted $d^{*} \geq d$.	***Sizing the shaft*** / For each section j, possibly dangerous, it is calculated: $W_{nec,j} = \left[\left(\frac{M_{b,j}}{\sigma_{b,ad}}\right)^2 + \left(\frac{M_{b,j}^{(d)}}{\sigma_{b,ad}^{(d)}}\right)^2 + \left(\frac{M_{t,j}}{\beta \cdot \tau_{t,ad}}\right)^2\right]^{\frac{1}{2}}$; $d_{nec,j} = \sqrt[3]{\left(\frac{32 \cdot W_{nec,j}}{\pi}\right)}$ (for full circular cross - section shaft). The dangerous section is established: where the calculated value of $d_{nec,j}$
2. ***Sizing the shaft under combined bending and torsion /*** For each section j, possibly dangerous, there are calculated: $M_{b,tot,j} = M_{b,j} + \alpha_d \cdot M_{b,j}^{(d)}$; $(M_{b,ech})_j = \left(M_{b,tot,j}^2 + 0{,}75 \cdot M_{t,j}^2\right)^{\frac{1}{2}}$ The dangerous section is established: where the	

calculated value of $(M_{b,ech})_j$ is maximum. In this section there will be determined: $$W_{nec} = \frac{\left((M_{b,ech})_j\right)_{\max}}{(\sigma_{b,ad})_I};$$ $$d_{nec} = \sqrt[3]{\frac{32 \cdot W_{nec}}{\pi}};\ d' = d_{nec} + 2c.$$ If $d' \le d^*$, a shaft with diameter d^* is chosen. Otherwise, it is adopted $d'' \ge d'$.	is maximum. $$d_{arb} = \left(d_{nec,j}\right)_{\max} + 2c.$$ It is adopted $$d_{final} \ge d_{arb}.$$

Note: The computation relations corresponding to the classical method included in the left column of the Table 1 have been developed according to references [1-3; 7].

List of symbols used in Table 1 (excepting the symbols whose signification has already been mentioned in the paper***):***

● ***corresponding to the classical method:***

$W_{p,nec}$ - the necessary polar modulus of resistance corresponding to the shaft cross-section;

$\tau_{t,ad}$ - the allowable shear stress; in the case of shaft sizing under torque load, it is necessary to adopt a minimum covering value $\tau_{t,ad} = 40\ N/mm^2$ [3], taking into account the fact that a supplementary bending moment is acting (but it is not included in the calculation);

d_{nec} - the required diameter of the shaft; *c* - the corrosion and/or erosion allowance in radial direction; *d* – the shaft diameter resulted from the calculus effected in the first stage; d^* - the standard value adopted for the shaft diameter, in the first stage;

$M_{b,tot,j}$ - the total bending moment in *j* cross-section;

$(M_{b,ech})_j$ - the equivalent bending moment in *j* cross-section;

α_d - the equivalence coefficient of a fatigue loading with a static loading; in case of the symmetric alternating fatigue loading, we have:

$$\alpha_d = \frac{(\sigma_{b,ad})_I}{(\sigma_{b,ad})_{III}};\quad (\sigma_{b,ad})_I = \frac{\sigma_c}{c_b};\quad (\sigma_{b,ad})_{III} = \frac{\sigma_{-1}}{c_{b,III}},$$

where $(\sigma_{b,ad})_I$ - the allowable stress under static bending;

$(\sigma_{b,ad})_{III}$ - the allowable stress under symmetric alternating bending;

σ_c - the shaft material yield limit; σ_{-1} - the fatigue limit under symmetric alternating bending;

c_b - the safety coefficient in static bending;

$c_{b,III}$ - the safety coefficient under symmetric alternating bending;

W_{nec} - the required modulus of resistance corresponding to the dangerous shaft cross-section;

d_{nec} - the minimum required diameter of the dangerous shaft section;

d' - the shaft diameter in the dangerous section resulted from the sizing, in the second stage;

d'' - the standard value of the shaft diameter, adopted in the second stage in case $d' > d^*$;

● **corresponding to the Energonics *method:***

d_{arb} - the shaft diameter in the dangerous section; d_{final} - the standard value adopted for the shaft diameter.

Comparing the algorithms presented parallelly in Table 1, one observes that: in the case of the Energonics method, shaft sizing is made directly, using relation (18). This was obtained by customizing expression (16) which is derived on the basis of critical energy principle. In order to establish relation (16), both dynamic loading and allowable resistance to dynamic loading have been considered. That is why there is no need to effected an additional fatigue verification of the shaft.

5. Numerical Example

One considers a mixing rotating device as the one in fig.3. Its vertical shaft is provided with sealing device (noted *DE*), and three identical mixers mounted in the sections E, F and G, in perpendicular planes, successively.

The following data are known (fig. 3 and 4):

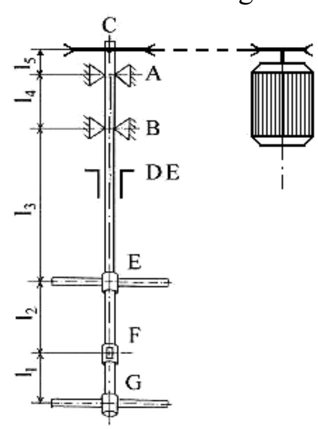

Fig. 3. The mixing rotating device with vertical shaft, provided with sealing device (noted *DE*), and three identical mixers mounted in the sections E, F, G. (C – corresponds to the shaft's end, where a belt pulley is placed; A, B – bearings)

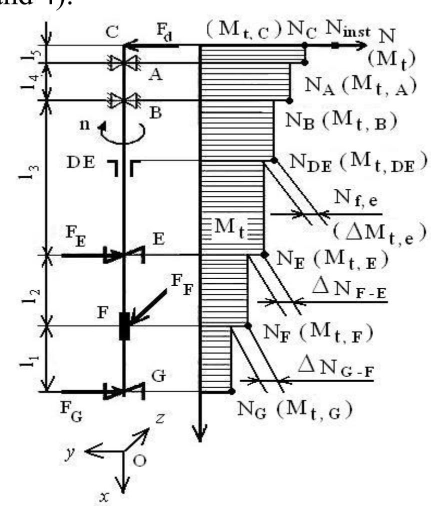

Fig. 4. Distribution of the necessary mixing power N, and of the torque M_t, respectively, along the shaft

$l_1 = 800\ mm$; $l_2 = 1000\ mm$; $l_3 = 1660\ mm$; $l_4 = 500\ mm$; $l_5 = 140\ mm$;

the installed power of the driving electromotor, $N_{inst} = 4\,kW$; the overall efficiency of mechanical transmission, $\eta_{tr} = 0{,}75$; the bearings friction couple efficiency, $\eta_A = \eta_B = 0{,}99$; the rotational speed of the shaft, $n = 60\,rot/\min$; the radial force in the belts, $F_d = 4540\,N$ (*the dynamically applied load*); the bending radial forces, corresponding to each mixer, $F_E \approx F_F \approx F_G \approx 24\,N$ (*the statically applied load*); the power loss in the sealing device as a result of the friction, $N_{f,e} = 0{,}27\ kW$; N_E, N_F, N_G - the necessary mixing power corresponding to each mixer, respectively; $N_F = 1{,}75 \cdot N_G$; $N_E = 2{,}25 \cdot N_G$; the working medium: liquid. The shaft is made from a single piece and it has the constant full circular cross-section. The shaft material is a steel with the following mechanical characteristics: $\sigma_r = 540\,N/mm^2$; $\sigma_c = 320\,N/mm^2$; $\sigma_{-1} = 240\,N/mm^2$; $\tau_0 = 264\,N/mm^2$. In the conditions of the given loading, the shaft material has a linear-elastic behaviour.

Let us solve the sizing problem of this mixing rotating device shaft!

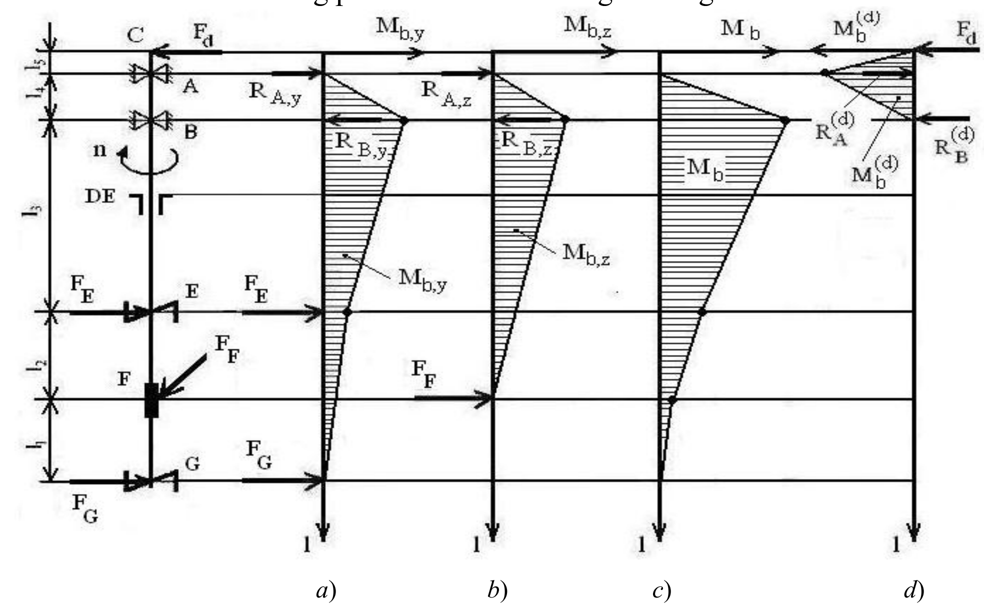

Fig. 5. *a*) The diagram of the bending moments, $M_{b,y}$,due to the statically applied loads acting in the plane *xOy*; *b*) The diagram of the bending moments, $M_{b,z}$, due to the statically applied loads acting in the plane *xOz*; *c*) The diagram of the resulting bending moments, M_b ,due to the static loading; *d*) The diagram of the bending moments, $M_b^{(d)}$,due to the dynamic loading. ($R_{A,y}, R_{B,y}, R_{A,z}, R_{B,z}, R_A^{(d)}, R_B^{(d)}$ - the bearings reactions)

• The distribution of the necessary mixing power N, and of the torque M_t, respectively, along the shaft, is shown in fig.4.
• The diagram of the bending moments M_b due to the static loading, and the diagram of the bending moments $M_b^{(d)}$ due to the dynamic loading, along the shaft, are presented in fig.5.
● The necessary mixing power, torque and bending moments calculated in the main cross-sections of the shaft are registered in table 2.
● The possible dangerous cross-sections of the shaft correspond to the two supports A and B. These are established based on the moments diagrams represented in fig. 4 and 5.
● Shaft sizing is based on the algorithms previously described in Table 1. The results are included in Table 3.

Table 2

The necessary mixing power, torque and bending moments calculated in the main cross-sections of the shaft

The *j* cross-section of the shaft	**C**	**A**	**B**	**DE**	**E**	**F**	**G**
Power, N_j $[kW]$	3	3	2,97	2,94	2,67	2,07	1,18
The torque, $M_{t,j} = \frac{N_j}{\omega}$ $[N \cdot m]$	477,46	477,46	472,69	467,96	425,33	330,81	189,03
The bending moment $M_{b,y,j}$ $[N \cdot m]$	----	0	122,88	----	43,2	19,2	0
The bending moment $M_{b,z,j}$ $[N \cdot m]$	----	0	63,84	----	24	0	----
The resulting bending moment $M_{b,j} = (M_{b,y,j}^2 + M_{b,z,j}^2)^{\frac{1}{2}}$ $[N \cdot m]$	----	0	138,47	----	49,41	19,2	0
The bending moment $M_{b,j}^{(d)}$ $[N \cdot m]$	0	635,6	0	----	----	----	----

Note: ω - the shaft angular speed , $\omega = \frac{\pi \cdot n}{30} = \frac{\pi \cdot 60}{30} = 2\pi, \ s^{-1}$

Table 3

The results of the shaft strength calculation using both classical method and Energonics method

Classical Method (equivalent stresses method)	Energonics Method
Stage	**Stage**
1. ***Sizing the shaft under torque load*** $M_{t,c} = 477{,}46\ N\cdot m$; $d_{nec} = 39{,}32\ mm$; $d = 43{,}32\ mm$. It is adopted $d^* = 45\ mm$. **2.** ***Sizing the shaft under combined bending and torsion*** For the sections A and B, possibly dangerous, there are calculated: $M_{b,tot,A} = 1302{,}98\ N\cdot m$; $M_{b,tot,B} = 138{,}47\ N\cdot m$; $(M_{b,ech})_A = 1367{,}01\ N\cdot m$; $(M_{b,ech})_B = 432{,}14\ N\cdot m$. The calculation demonstrates that A is the dangerous section. $W_{nec,A} = 5{,}55\cdot 10^{-6}\ m^3$; $d_{nec,A} = 38{,}37\ mm$ $d'_A = 42{,}37\ mm < 45\ mm = d^*$. Therefore, a shaft with the diameter $d^* = 45\ mm$ is chosen.	***Sizing the shaft*** / For the sections A and B, possibly dangerous, there are calculated: $W_{nec,A} = 5{,}42\cdot 10^{-6}\ m^3$; $d_{nec,A} = 38{,}07\ mm$; $W_{nec,B} = 1{,}29\cdot 10^{-6}\ m^3$; $d_{nec,B} = 23{,}59\ mm$. The calculation demonstrates that A is the dangerous section. $d_{arb} = 42{,}07\ mm$. It is adopted $d_{final} = 45\ mm$. Therefore, a shaft with the diameter $d_{final} = 45\ mm$ is chosen.

Note : The results of the shaft strength calculation have been obtained taking into consideration the following supplementary data: $c = 2\ mm$ (the corrosion allowance in radial direction); $\beta = 2$ (for full circular cross – section of the shaft); $\sigma_{b,ad} = 246{,}15\ N/mm^2$; $\sigma_{b,ad}^{(d)} = 120\ N/mm^2$; $\tau_{t,ad} = 203{,}07\ N/mm^2$ ($\sigma_{b,ad}$, $\sigma_{b,ad}^{(d)}$ - the allowable normal stresses under static and dynamic bending, respectively; $\tau_{t,ad}$ - the allowable tangential stress under the torsion); $\alpha_d = \dfrac{\sigma_{b,ad}}{\sigma_{b,ad}^{(d)}} = 2{,}05$ (the equivalence coefficient of a fatigue loading with a static loading).

6. Conclusions

The analysis realised in this paper shows that the classical method of strength calculation of the shaft is more laborious compared to the Energonics method.

Thus, as it was specified in this study, the classical calculation method is based on the concept of equivalent stress (but the calculation result depends on the choice of strength theory used).

Unlike the classical procedure, the Energonics method, using the concept of *the participation of the specific energy due to the shaft loadings*, proves to be very simple to use. Sizing shafts is made by applying relationship (16), taking into account the dependance $d(W)$, where d – the shaft diameter; W - the modulus of resistance corresponding to the shaft cross-section.

The shaft sizing effected by the two methods (the classical one and the one based on the principle of critical energy) lead to almost identical final results. Moreover, the algorithm of the second method is much simpler, and consequently, the calculation time is much shorter.

It is also important to note that, the Energonics method enables to solve the cases in which the classical method cannot be applied (non-linear behaviour of the shaft material; rapid or shock loading).

Therefore, it is obvious that the calculation method based on the principle of critical energy is superior to the calculation classical method of the mixing rotating device shafts.

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