FREGE: EXISTENCE DEFINED AS IDENTIFIABILITY

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The thesis (T) of this article is that in Frege's philosophy existence may be and is best defined as identifiability, where an object is identifiable if every identity statement about it has a determinate truth value. I shall always use the term "identifiable" in this strict sense, since I am concerned with defining existence only within Frege's formal program, or within sufficiently similar formal programs. If we shifted our discussion from Frege's formal notation to ordinary language, then identifiability might be better viewed as admitting of degree. There, a minimal notion of identifiability would be that an object is identifiable if at least one identity statement about it has a determinate truth value. Between this minimal notion and the strict sense I will use, there is plainly much room for degree.

If thesis (T) can be well substantiated, then we will be rewarded not only with a new understanding of Frege on the most fundamental level, but also with a more secure foundation of his place in history as the forerunner of Wittgenstein, Quine, Dummett, Geach, Castañeda, Butchvarov, and others with respect to discussions of connections between identity conditions and existential quantification or reference. But the student of Frege can easily find many difficulties with (T). First, when faced with an insurmountable logical difficulty in *The Foundations of Arithmetic,* Frege seems to abandon the requirement that for numbers to be named, a criterion for their identity must be provided. Second, what may be called Frege's private language argument seems to entail that subjective entities are not identifiable, so that identifiability can hardly be for Frege a sufficient condition of existence. Third, Frege's functions cannot even stand in the identity relation, so how can they be identifiable? Fourth, even if identifiability were a necessary and sufficient condition of existence, why would our definition be preferable to Frege's own suggestion that "Affirming existence is in fact nothing but denial of the number nought"?[1] Fifth, how could our definition be stated generally without violating Frege's type-hierarchy of discourse? In what follows, I shall deal with these five difficulties in order.

I

I shall begin, then, with the first difficulty. In *The Foundations of Arithmetic,* did Frege reject the view stated there that if numbers are to be named, a criterion for their identity must be given? It is the opinion of the leading

Frege scholar that he did. Now there is no doubt that at first Frege tries to provide an identity criterion in the form of a contextual definition of "the Number belonging to the concept *F*."[2] But Michael Dummett holds that when this definition is thwarted by a logical difficulty, Frege abandons contextual definitions in favor of explicit definitions.[3] To be sure, Dummett would admit, as everyone would, that even explicit definitions are contextual in the trivial sense that they tell us how to use the defined term in all sentential contexts.[4] Any criterion of identity provided by such a definition would, of course, be equally trivial. As Arthur Pap, for example, understands contextual definition, an explicit definition fails to be genuinely contextual by definition; an explicit definition does not show us how to get along writing sentences without using a term except in the trivial sense of providing a substitute term.[5] Perhaps it is some such restricted notion of contextual definition that leads Dummett to say that Frege abandoned contextual definitions.

Pap's definition of contextual definition as adefinition that shows us how to write sentences without using the defined term is needlessly syntactical in approach. My definition is somewhat different. I define a contextual definition as a definition whose aim is to fix the sense of, i.e., provide determinate truth values for, a particular, definite, proper subclass of the class of statements in which the defined term occurs, and which may provide determinate truth values for all other statements in which the defined term occurs only in an incidental manner. If that particular, definite, proper subclass is a class of identity statements, then the definition may also be called an identity definition. It is clear that an ordinary explicit definition, while it may be contextual and it may provide a criterion of identity in the trivial sense Dummett admits, is not contextual in the substantive sense I shall use, nor is it an identity definition in my substantive sense. Yet a syntactically explicit definition would be contextual in my sense if its intent were as described.

Why did I define contextual definition in terms of its intent? If there is one thing philosophers such as Wittgenstein and Austin have taught us, it is that our confronting a string of symbols, in the absence of accurate information as to how those symbols are being used, is not enough for our knowing what they mean. When it comes to strings of symbols that are definitions, it is crucial not to be taken in, as Dummett has been, by mere syntactical appearance. One must be wary even of assuming that the function of a given definition is what the function of definitions of its syntactical form usually is. Frege himself points out an instance of this very sort of mistake.[6] To determine the intent of a definition, the thing to look at is the context in which the definition is given. This fact is not based on some philosophical theory of contextual meaning. It is a point of common sense, of responsible scholarship. It shall guide my analysis of the definition of "the Number belonging to the concept *F*" which Frege accepts.

I shall argue that Frege does not give up contextual definition, but rather substitutes one kind of identity definition for another, upholding the requirement of an identity criterion for introducing denoting expressions. The first definition Frege tries may be formulated as follows:

> I. "The Number belonging to the concept *F* is identical with the Number belonging to the concept *G*"
>
> is to mean the same as
>
> "*F* is equal to *G* (there is a one-one correlation between *F*'s and *G*'s)."[7]

It is reasonable to interpret the difficulty Frege finds with this definition as follows. Definition (I) fails to provide a genuinely comprehensive criterion for the identity of Numbers belonging to concepts because it provides a determinate truth value for identity statements about such Numbers only if the statements are of the form "The Number belonging to *F* is identical with the Number belonging to *G*." The general form we should be dealing with is instead "The Number belonging to *F* is identical with *q*," where "*q*" is any subject-term, say, "England." What statement, according to the lights of definition (I), is the statement "The Number belonging to *F* is identical with England" to mean the same as: "*F* is equal to ______"? The sentence cannot be completed. "England" cannot occupy the open argument-place. The first occurrence of "*G*" in definition (I) is only a small part of what "England" would occupy there, namely, the whole of "the Number belonging to the concept *G*." So we can hardly fit "England" into the place of the second occurrence of "*G*" in definition (I).

Now this problem of bad format is hardly a reason for rejecting contextual definitions generally or as such. Dummett perceives this fact.[8] He would have done well to have pondered it further.

Now we come to the definition Frege accepts:

> II. The Number which belongs to the concept *F* is the extension of the concept "equal to the concept *F*."[9]

Now definition (II) looks for all the world like a plain, ordinary explicit definition. No wonder Dummett was taken in. In the case of definition (I) at least we could see how it was an attempt to provide determinate truth values for identity statements about Numbers. But (II) does not even appear to indicate in what sense its intent might be to fix the sense of an identity. The thing to look at, however, is not the definition itself, but the context in which it is given.

The best way to find out Frege's aim, I submit, is to look at the reason he gives for accepting definition (II).[10] Frege did not pick the defining extension he did out of a hat. The beauty of definition (II) lies in Frege's having seen that Numbers belonging to concepts, and the extensions in question, are basically identified and differentiated in the same way, in terms of the equality or non-equality of the concepts the Numbers belong to. That is, not only do parallel completions of

A. The Number belonging to *F* is identical with the Number belonging to *G*,
B. *F* and *G* are equal, and
C. The extension of the concept "equal to *F*" is identical with the extension of the concept "equal to *G*"

enjoy logical equivalence and carve up the same thought in different ways, but parallel completions of (A) and (C) each have three components, namely, identity and two subjects, such that identity of content can be very plausibly extended to the components as well. (By component contents I mean, of course, the customary denotations of the subject-names and relation-names.) Once the component contents of (A) and (C) have been identified with each other, the mediation of (B) can be dropped, and the identification of Numbers with certain extensions can take on a life of its own, as it does in definition (II), which does not mention equality.

Frege's reason for giving definition (II) is in effect that parallel completions of (A) and (B) are logically equivalent. Obviously Frege states his reason elliptically, since extensions, which are the basis of the definition, are not even mentioned in completions of (A) or (B). They are mentioned only in completions of (C). That is, the second logical equivalence, i.e., the one between parallel completions of (B) and (C), is unavoidably part of Frege's reason for giving definition (II). If this second equivalence had failed to obtain, definition (II) would have been incorrect. If this second equivalence had not been seen, definition (II) would never have been arrived at.

How, then, is definition (II) an identity definition? Its intent is to provide determinate truth values for the particular, definite class of identity statements about Numbers, by identifying Numbers with just those entities, given as already having a clear criterion of identity,[11] which are demonstrated as having just the identity conditions we want to provide for Numbers. Once Frege is assured that the extensions in question have the right identity conditions for the numerical identities of arithmetic, he identifies Numbers with them without further ado in a way that simply enforces by stipulation the truth values of all other identity statements about Numbers, such as "The Number belonging to *F* is identical with England," which is false because the extension of the concept "equal to *F*" is not England.

Dummett's blithe criticism of definition (II) is that the problem of providing identity conditions for Numbers is merely pushed back to the level of extensions.[12] This completely ignores Frege's intent, and indeed the trouble he must have gone to, to find some entities, given as already having clear identity conditions, such that these conditions were substantively the ones we want for Numbers, and otherwise not inimical to what we want for Numbers. Dummett gives the criticism because he sees definition (II) as merely explicit, while Frege's whole intent in picking the extensions he did was to provide arithmetically adequate and genuinely comprehensive conditions for the identity of Numbers, as opposed to fixing identity conditions for Numbers incidentally in the trivial sense in which a merely explicit definition provides truth values indiscriminately for all the statements in which the defined term occurs. If *that* were all Frege had in mind, we would indeed do well to wonder with Dummett as to what, in turn, were to be the identity conditions for these extensions.

So the immediate context of definition (II), on analysis, shows plainly Frege's intent to provide determinate truth values for a particular, definite, proper subclass of the class of all statements about Numbers, in particular, for identity statements about Numbers. All else is merely incidental, including the final syntactical form of the definition, which is simple and elegant. Frege did not, then, abandon the requirement that for numbers to be named, a criterion for their identity must be provided. Thesis (T), then, does not fall here.

What about the larger context of definition (II), the whole of *The Foundations of Arithmetic?* Does it support our claim as to Frege's contextual intent? It does. Frege states in the book as a fundamental principle that words have meaning only in the context of a proposition.[13] This principle is never retracted later in the book. Nor should we expect it to be, since it is stated in the Introduction, presumably written or at least carefully considered as the author surveyed the completed work.[14] With regard to defining Numbers in particular, Frege states as a comment on this fundamental principle, "With numbers of all these types, it is a matter of fixing the sense of an identity."[15] This, too, is in the Introduction. Frege is telling us what to expect him to have done. (Dummett ignores these rather obvious points.) I do not know how Dummett could have missed it, but in the Analysis of Contents, Frege groups the sections concerning the definition of "the Number belonging to the concept *F*" under the heading, "*To obtain the concept of Number, we must fix the sense of a numerical identity.*"[16] Frege even repeats this heading when the sections appear in the text.[17] Right after this repeated heading, Frege gives an argument that since words have meaning only in the context of a proposition, and since number words are to stand for objects, we must give a criterion for the identity of numbers if we are to assign them names.[18] Frege subsequently detects no flaw in this argument, nor does he later reject it. It is still in force when definition (II) is accepted. Nor is this all. At the end of the book, Frege recapitulates what he has done. He tells us three times that the issue is fixing the sense of a numerical identity![19]

My conclusion is that in *The Foundations of Arithmetic,* Frege holds that identifiability is a necessary condition of assigning denotations to expressions. It is also a sufficient condition in that book, since when identifiability is provided, the expressions can be assigned denotations.

II

This brings us to the second difficulty with (T). That identifiability is a necessary and sufficient condition of denotation may be very well in *The Foundations of Arithmetic,* which treats of numbers, shapes, directions, and objects generally. Objects belong to Frege's objective realm, and therefore are already understood to be communicable in a public language, hence interpersonally identifiable, hence identifiable. But what about entities in the subjective realm, such as our ideas? What may be called Frege's private language argument seems to entail that thanks to their essential privacy, ideas are not even minimally identifiable, much less identifiable in the strict, comprehensive sense of the term.[20]

Frege's private language argument can be put as follows. An idea is essentially an item in the consciousness of a single person. Therefore even if an idea vanishes from one person's consciousness at the very moment an idea appears in another person's consciousness, the question whether the ideas are identical is unanswerable. Therefore it is impossible to bring together in one consciousness ideas belonging to different people. Therefore it is impossible to compare such ideas. Therefore a predicate supposed to state a property of an idea of a given person is applicable only within the sphere of that person's consciousness. Therefore, if a predicate is taken to stand for any property of any item that does not belong to a given person's consciousness, all questions as to the applicability of that predicate to any item in that person's consciousness are (likewise) unanswerable and nonsensical. Now this applies to the predicates "is true" and "is false" as well as to any others. Therefore, if thoughts were ideas, then no one could dispute with his friends as to the truth or falsehood of any thought. Since the possibility of such dispute is a necessary condition of there being a science or body of thought common to many, there could then be no such science. But this is absurd. Hence thoughts are not ideas.[21]

How does this argument, which concludes only that thoughts are not ideas, entail in any way that ideas are not even minimally identifiable? Even if the argument states that ideas are not interpersonally identifiable, so that they are not strictly identifiable, surely I can identify at least my own ideas, so that they are minimally identifiable. Well, according to the argument, no predicate can be appointed to characterize an idea, yet express a public sense. Therefore, since "is identifiable" is a predicate of a public language and expresses a public sense, ideas cannot be said to be identifiable. Similarly for the predicate "is minimally identifiable." So it seems there are some entities, ideas, that are not even minimally identifiable, so that (T) cannot be upheld.

The trouble with this objection to (T) is simply that "exists," too, is a predicate of a public language, so that Frege's private language argument entails not only that ideas cannot be said to be identifiable in any sense, but also that they cannot even be said to exist. Frege's argument is therefore actually an excellent argument in favor of (T), since it entails that a thing can be said to exist if and only if it can be said to be identifiable.

If items cannot be said to be the same, then they cannot be said to be anything, or even to be at all. Ontologically, if I cannot identify an item, then I cannot identify what properties it may have either, or even determine that it exists. This ontological principle, I conjecture, is the mainspring of Frege's argument, implied in the very beginning of the argument, where Frege contends that because ideas are not interpersonally identifiable, they cannot be interpersonally compared or described. There is no difference between interpersonal describability and describability, any more than there is any difference between interpersonal identifiability and identifiability, since "is describable," too, is already a public predicate.

Frege himself would have found these implications of his argument unwelcome, and perhaps even bizarre. He seems to show a tendency to avoid them. He seeks to show that perhaps an idea can be taken to be an object.[22] He also argues that different persons' ideas must have at least affinities if art is to be possible.[23] He seems to think that if ideas can be made out to be similar enough to objects, then somehow they can be spoken about after all. But this is nonsense. Not even the public predicates "can be taken as an object" or "has at least affinities with other ideas" can be appointed to characterize an idea. (T) therefore remains secure.

III

The third problem with (T) was this. According to Frege, only objects can stand in the identity relation. Functions cannot stand in the identity relation.[24] Further, identity is given to us so specifically that there cannot be various forms of it.[25] Now if functions cannot even stand in the identity relation, how can they be identifiable?

Now we might use Frege's private language argument to dispose of the problem of functions. If functions cannot be said to be the same, then they cannot be said to be anything, or even to be at all. This approach finds support in a recent dispute in the literature as to the ontological status of functions.

A strong doubt that function-expressions are to be understood as denoting arises from Frege's paradoxical discussion of the concept *horse.* Frege says that the expression "the concept *horse*" denotes not a concept but an object, thanks to the role of the definite article "the."[26] William Marshall uses three principles about names to bring out the paradoxical nature of this view. (i) An expression that one can quantify over is a name. (ii) Names of the same entity are intersubstitutable *salva veritate.* (iii) A name's denotation can always be described as the so-and-so. Now by (i), the function-expression "is a man" is a name. If Plato is a man, then there is something that Plato is. But then either (ii) or (iii) must fail to hold. If we try to describe the denotation of "is a man" as the concept *man,* then we have two names of the same entity that are not intersubstitutable *salve veritate.* "Plato is a man" is true, but "Plato the concept *man*" does not even express a thought. Marshall infers that a predicate or function-expression is not a name, and that quantification over a predicate is best seen as implying not denotation but rule-governed use.[27]

Marshall's view makes it easy to defend (T) along the line I indicated, but I think Marshall is wrong about the ontological status of functions. Like Montgomery Furth, I am in favor of ascribing denotation to function-expressions. I am not in favor of Furth's analogical approach, which is based on the fact that proper names denote. The problem is the lack of a clear criterion for the extent or type of formal similarity between proper names and function-expressions for our legitimately being able to say that function-expressions denote in some sense. That is, one can admit all the formal similarities Furth carefully draws and still question his conclusion. As a matter of fact, Furth's final definition of ascribing denotation to function-expressions is unfortunately close to Marshall's explication of quantification over predicates as merely meaning being well governed by rules: both seem to be inspired by the very same passage in Frege.[28]

Michael Dummett has at least the right idea for avoiding the quantificational problem of the concept *horse.* He paraphrases Frege's quantification over predicates into true ordinary sentences such as "There is such a thing as being a philosopher." Unlike "The concept *philosopher* exists," which seems to predicate a first-level concept of an object, Dummett's sentence seems to predicate a second-level concept of a first-level concept.[29] This bypasses Marshall's principle (iii) about names by a direct appeal to ordinary language type-levels.

Nicholas Measor rejects Dummett's paraphrase. For Frege, all first-level predicates in true or false statements denote concepts. This includes predicates which can apply to no object, such as "is round and not round." But there is, precisely because no object can be round and not round, no such thing as being round and not round![30] But while Measor has refuted Dummett's paraphrastic proposal, he has not refuted the idea of a paraphrase. I myself regard the paraphrase "There is such a thing as being a philosopher" as incorrect because it asserts too much. It asserts the existence of the concept but it also asserts that philosophers can exist. My paraphrase of "$(\exists f)[(x)(fx \equiv x \text{ is a philosopher})]$" is instead, "There is something which perhaps nothing can be, namely, being a philosopher." Now in the problem case we can use "There is something which perhaps nothing can be, namely, being round and not round." This, I contend, is a true existential sentence in which the existence of a concept is asserted. All sentences of its form correspond in truth value to the formal statements they are intended to paraphrase. Moreover there is, beyond the mere difference in level, no detectable semantic difference between "There is (something, a thing such as . . .)" when it is completed by a proper name, and the same phrase when it is completed by a function-expression. My conclusion is that function-expressions denote in exactly the way proper names do.

Using Frege's private language argument backwards, as it were, we might now infer that functions must be in some sense identifiable after all, since they exist and we can say truly that they exist. In any case it is certain that we must find some such sense if (T) is to be preserved. Now Frege writes that owing to the predicative nature of concepts, when one needs to assert something about concepts in the sentence form usually used to assert something about something, the reference of the subject-term cannot be the concept itself, but instead is an object that represents the concept.[31] I call this mode of assertion representative assertion. I suggest that identity can be representatively asserted of concepts by means of the sentence "The concept *F* is identical with the concept *G*," where the subject-terms "The concept *F*" and "the concept *G*" denote objects that represent *F* and *G* themselves. (Actually, since I hold that for Frege identity is the relation between names of denoting the same denotation, I should say that the subject-terms customarily denote the representative objects.) This much already gives the main idea of the answer to the question in what sense can functions be said to be identical, and be said to be identifiable. Functions do not stand in the identity relation directly. The identity relation is predicated representatively of them instead.

Now in *The Basic Laws of Arithmetic* a function and its course-of-values represent each other.[32] I shall call any assertion about either a representative assertion about the other. Assertions that represent each other merely carve up the same thought in different ways.[33] So if the assertion that the courses-of-values of concepts *F* and *G* are the same, "$\acute{\varepsilon}F(\varepsilon) = \acute{\varepsilon}G(\varepsilon)$," representatively asserts that *F* and *G* themselves are identical, then there must be a relation *F* and *G* themselves do stand in, "corresponding to identity between objects."[34] This relation is equivalence, or "$(x)[Fx \equiv Gx]$," where "*F*" and "*G*" mark the argument-places. Equivalence obtains between concepts if and only if identity obtains betweentheir courses-of-values.[35]

I shall refer to equivalence as the relation of representative identity. This is not just for the reason that identity and equivalence correspond with each other in mutually representative assertions as described. It is not even for the reason that identity and equivalence can be technically shown to be first- and second-level representatives of each other in Frege's formal notation.[36] The reason I call equivalence representative identity, or if you will, an identity analog, is instead best seen in the fact that for Frege, equivalent functions are interdefinable, that is, they are not, as it were, to be regarded as different functions.[37]

This does not mean that if all and only red objects are round, then the concepts *red* and *round* are the same concept. The concepts themselves would be not identical, but equivalent or representatively identical. Only their courses-of-values would directly stand in the identity relation.[38]

Since mutual representation would be impossible if there were not a one-one correspondence between functions and their courses-of-values, we may say that the representative identity conditions for functions are exactly as sharp as the identity conditions of the objects that represent them. That this one-one correspondence obtains is the famed extensionality thesis, whose name we may honor by saying that on our view representative identity criteria are always extensional.[39]

If, for all that, functions still do not stand in the identity relation, then someone acquainted with our view of Frege's private language argument might object, are not functions still not identifiable, and can the predicate "is representatively identifiable" therefore really be appointed to characterize a function and still express a public sense? The objection is misguided. The representation function, "$\xi \cap \zeta$," is well defined in Frege's formal notation, and its use must be regarded as unexceptionable on the score of public communicability.[40] The said predicate would in any case be appointed to characterize not the function but the object that represents it, since "is representatively identifiable" is a first-level predicate. I therefore defend (T) by saying it is sufficient that functions exist if and only if they are representatively identifiable, where the criterion of sufficiency is that representatively identical entities are incorrectly regarded as different entities.

IV

In the preceding sections of this article it has been made clear that it would have been very natural for Frege to have said that identifiability is a necessary condition of existence. That he would have held that it is also a sufficient condition is clear not only from the last paragraph of Section I, but also from the fact that for Frege, the logical subject-term of every true or false statement must have a denotation, so that if the statement "*a* is identifiable" is true, then "*a*" must denote some object.[41] Similar points hold for functions.[42] Let us concede, then, that identifiability is for Frege a necessary and sufficient condition of existence, so that Frege would find it permissible to define existence as identifiability.

This brings us to the fourth difficulty with (T). Part of (T) was that existence may be defined as identifiability in Frege's philosophy, and this has now been well substantiated. But the rest of (T) was that existence is best so defined in Frege's philosophy, and this has not been shown. Besides a candidate definition Frege himself suggests, denial of the number nought,[43] there is also determinacy (conformance to the law of the excluded middle),[44] and the property of having a property. Horses exist if and only if it is not the case that there are no (0) horses. Horses exist if and only if horses are determinate. Horses exist if and only if horses have properties. In fact there are as many legitimate Fregean definitions of existence as there are concepts equivalent to it. Why then should identifiability be preferred to the others?

The reason lies in what I called the ontological mainspring of Frege's private language argument. Our finding entities identifiable must be prior to our counting them, to our pronouncing them determinate (even if their properties are in some sense the basis for our being able to identify them), and indeed to our being able to say anything at all about them. Identifiability must be prior to counting for fear of counting the same thing twice. And we can hardly determine all or even any of the properties of a thing without being able to identify it in the first place. But this is just to state again the ontological mainspring of Frege's private language argument. On the other hand we can identify, say, people without counting them, without determining whether each person has or does not have each and every property, and even without determining whether any of them has any property in particular.

Now surely it is the case for Frege that an object exists if and only if at least one identity statement about it has a determinate truth value. Why then should we define existence as strict identifiability when we can define it as minimal identifiability? The reason is that it is legitimate to assert that an object exists not because we happen to find some identity statement about it to be true or false, nor even because the object is such that some identity statement about it has a determinate truth value, but because the object is such that no matter what its situation or condition, and no matter which identity statement about it we may be concerned with, that identity statement has a determinate truth value. This is the general ground for the legitimacy of existence assertions, constituting existence's most native mode of presentation, and so telling us uniquely the sense of the word "exists," or what existence is, while the other features equivalent with existence merely constitute conditions that are necessary and sufficient for existence. That is why our definition is to be preferred over any of the others. That is why a genuinely comprehensive criterion of identity, showing strict identifiability, is Frege's correct theoretical requirement in sect. 62 of *The Foundations of Arithmetic* for assigning objects Number-expressions as proper names: "If we are to use the symbol *a* to signify an object, we must have a criterion for deciding in all cases whether *b* is the same as *a,* even if it is not always in our power to apply this criterion."[45]

V

The fifth difficulty was how to formulate a definition of existence as identifiability. Now one can define the existential quantifier, "(∃x)[*F*x]," "*F*" marking the argument-place, by saying that it is to mean the same as "(*I*x)[*F*x]," meaning "All F's are identifiable." Now someone might object to this definition (D) that while, say, unicorns do not exist, surely all unicorns are identifiable if horses are, since there is not much difference between a unicorn and a horse. But this objection to (D) is ill founded. I might stipulate concerning identity statements ostensibly about unicorns that each one of the form "$a=b$" is to be false while each one of the form "$a=a$" is to be true, so that every identity statement ostensibly about unicorns would have a truth-value and unicorns would be identifiable in my strict sense. But the stipulations would not amount to provisions of determinate truth-values in any meaningful sense. If, in "$a=b$," "a" has no customary denotation to be identical or not with the customary denotation of "b," it is hard to see how "$a=b$" could be true or false. An object that is not actual is not an object at all for Frege, and cannot be compared with anything to see if it is the same. There is not true or false identity statement that is concerned at all with unicorns, then, since there are no unicorns to be concerned with. Unicorns-expressions are part not of science, but of fiction.[46]

Our definition (D) resembles Frege's definition (II) of the Number belonging to the concept *F*. In both cases the syntactical form is that of an explicit definition. But also in both cases the entity defined is defined by being identified with an entity already given to us as having clear identity conditions, the defining entity being chosen for the sole intent of providing identity conditions for the defined entity that are substantially the correct ones and otherwise inconsequential.[47] (I take the second-level concept *identifiable* as already having clear identity conditions, since its strict sense is quite clear.) Beyond the mere difference in level, then, (D) and (II) are identity definitions of exactly the same sort, and satisfy (T) by securing the denotations of their defined terms in exactly the same way.

One may object to (D) that it defines only the second-level concept of existence, while (T) is concerned with existence in the broadest possible way. What about the existence of functions of all levels, not to mention the existence of senses and forces? I believe that like functions, senses and forces can be talked about by means of the definite article only representationally, owing to the very same necessity of form of language that is the reason why "the concept *horse*" must denote an object.[48] Further, all of Frege's own categorial pronouncements, such as that functions are incomplete or that senses (complete and incomplete) are objective, are also best viewed as representations of, as it were, general features of the world that cannot be directly described in Frege's notation, thanks to the subject-predicate hierarchy. Similarly on the transcategorial level: no broad definition of existence can be directly stated in the formal notation. Even the English formula, (F), "To exist is to be identifiable," must be regarded as a representation. Representation (F) is definitional in character since identifiability really is a necessary and sufficient condition of existence in Frege's philosophy, even if this cannot be directly expressed in his notation or in ordinary language any more than his own express articulations of general metaphysical views can. But indirectly, one may still advance (F) or even (D) as representations of something beyond the ability of language to denote.

Somebody may object that the formula (F) does not represent a general feature of the world, insofar as Frege expressly derided views such as that existence is absolute being,[49] or that existence is the concept superordinate to all other concepts.[50] However, referring the problem of seeming transcategoriality to a distinction between object languages and ordinary or meta-languages, or else to the idea of a representational definition as a mere shorthand summary of a series of types and levels of definitions, may not be ultimately adequate. It is usually not best to view a meta-language as an arena in which somehow one can say things that cannot be said in one's object language. Indeed, for Frege it is ordinary language that gives rise to pitfalls or illusions such as the views about existence he derided.[51] Nor does it seem best to think of formula (F) or of Frege's own general ontological statements as mere shorthand summaries.

Other objections remain. What about the type-hierarchy with its series of different existential quantifiers? What about Frege's view that a concept under which all objects fall would have no content? And beyond such objections, what positive ground can be given for saying that (F) really represents anything? Is (F) even intelligible?

The question of how to say what it seems cannot be said remains, of course, one of the most fruitful issues of comparison of Frege to Wittgenstein. As the famous criticism of the *Tractatus Logico-Philosophicus* goes, if you cannot say it, you cannot whistle it either. But unlike Wittgenstein's obscure ladder metaphor, perhaps a responsible account of Frege's representation would prove to be an adequate whistle. My view is that R1, given as denotable, may represent R2 if R2 exists and can be put in one-one correspondence with R1, and if our representing R2 by denoting R1 can be distinguished from our merely denoting R1. This is why a concept, which due to its predicative nature cannot be denoted by means of the definite article, may be represented by an object which can be so denoted. Our account also applies to forces and senses. Frege has good reason for asserting their existence. Forces ground the distinction between assertion, question, command, and supposition. Senses make informative identities, fiction, and false belief possible. Yet there are good reasons for saying that forces and senses, due to their respective natures, cannot be denoted at all.[52] But they can be identified and distinguished between, so they can be put in one-one correspondence with functions or objects, and can be represented in language. It seems evident that they must be represented for us to be able to speak about them at all. Here representation may be indicated by underlining the names in question, since Frege's representation function, $\xi \cap \zeta$, relates only objects and can be used only to represent functions or objects.

Let us now apply our account of representation to transcategorial entities. There is a tremendous intrinsic plausibility to saying that everything exists, is identifiable, and has at least one property. Even Frege, despite his having a type-hierarchy with a series of existential quantifiers, and his disbarring concepts under which all objects fall from having any content, seems to think at bottom of existence, logic, and determinateness as being the same for all entities. Perhaps, then, Frege is best seen not as successfully denying that existence is a transcategorial, but as successfully denying only that existence in its transcategorial comprehensiveness can be denoted by a predicate. Let the underlined names " '*a*' " and "*F* ( )" respectively denote the name "*a*" and the concept *F* ( ), where both denotations represent the object *a*. Let "denotes" and "(∃x) x"respectively denote denoting and instantiation, where both denotations represent existence. Then the English sentence, "*a* exists," may be rewritten as either " '*a*' denotes" or "(∃x)*F*(x)". On either analysis the subject-term of "*a* exists" will have a denotation even if the object *a* does not exist. The denotation of that term will simply not represent anything in that case; the sentence, of course, will be false. Also on either analysis, existence need not be absolute being or the most superordinate concept, but may instead be just being the case, which seems best understood as identifiability. Frege's type-hierarchy and requirements for predicates are fully observed.

Now let "(*I*x) x" denote direct identifiability, which will represent identifiability in general. (Only objects are directly identifiable.) Let "*F*( )" denote *F*( ), which will now represent any entity: object, function, sense, or force. Understanding "(∃x) x" as before, we may now replace definition (D) with (D1): "(∃x)*F*(x)" is to mean the same as "(*I*x)*F*(x)." (D1) will be our analysis of formula (F). My conclusion is that thesis (T) is both understandable and correct.

Value, beauty, causation, God, and the thinking subject are not even minimally identifiable as denotable entities. I may identify a thing and fully describe it, and still ask is it good, beautiful, or causally efficient. I cannot identify myself as an object of introspection, and who has seen God? The hints, metaphors, and suggestions Frege allows as explications of logically simple denotations do not help, as we still cannot identify these things.[53] Wittgenstein was right that they are transcendental, at least so far as the denotable world is concerned.[54] But we have seen that even the daily workings of language cannot be ontologically understood by means of denotation alone. If these things can in any sense still be distinguished from each other and recognized again, and if they can be put in one-one correspondence *salva veritate* with what can be denoted, then our account of representation applies to them.[55] Perhaps those who found "that the sense of life became clear to them" did so by ceasing their search for denotable entities, and by letting things present themselves in their own way.[56]

1 Gottlob Frege, *The Foundations of Arithmetic*, J. L. Austin, trans., Evanston, Northwestern University Press, 1974, p. 65.

2 *Ibid.*, pp. 73-74.

3 Michael Dummett, *Frege: Philosophy of Language*, New York, Harper and Row, 1973, pp. 495-96, and "Frege," in *The Encyclopedia of Philosophy*, Vol. 3, New York, MacMillan, 1967, p. 235.

4 See *Frege: Philosophy of Language*, pp. 6, 194, 501.

5 Arthur Pap, *Semantics and Necessary Truth*, New Haven, Yale University Press, 1966, p. 425.

6 Frege, *The Foundations of Arithmetic*, p. 76.

7 Frege's actual discussion is in terms of the definition of direction referred to in my last Note.

8 Dummett, *Frege: Philosophy of Language*, p. 496.

9 Frege, *The Foundations of Arithmetic*, pp. 79-80.

10 *Ibid.*, p. 79 ("If line *a* . . ."). Frege's discussion remains in terms of how to define direction.

11 *Ibid.*, p. 80, footnote 1.

12 Dummett, *Frege: Philosophy of Language*, p. 501.

13 Frege, *The Foundations of Arithmetic*, p. X.

14 In fact, Frege reiterates it. *Ibid.*, p. 73.

15 *Ibid.*, p. X.

16 *Ibid.*, sects. 62-69.

17 *Ibid.*, p. 73.

18 *Ibid.*, p. 73.

19 *Ibid.*, pp. 115, 117, 119. Frege says on p. 117, "I attach no decisive importance even to bringing in the extensions of concepts at all."

20 Dummett, *Frege: Philosophy of Language*, p. 496.

21 Frege, "The Thought," A. M. Quinton, trans., in *Mind*, LXV (1956), pp. 299-300, 301-02.

22 Frege, "On Sense and Reference," in Peter Geach and Max Black, trans. and eds., *Translations from the Philosophical Writings of Gottlob Frege*, Oxford, Basil Blackwell, 1970, p. 60.

23 *Ibid.*, p. 61.

24 Frege, *The Foundations of Arithmetic,* pp. 89-90; Illustrative extracts from Frege's review of Husserl's *Philosophie der Arithmetik,* in *Translations from the Philosophical Writings of Gottlob Frege,* p. 80, footnote *.

25 Frege, *The Basic Laws of Arithmetic,* Montgomery Furth, trans. and ed., Berkeley, University of California Press, 1967, p. 129.

26 Frege, "On Concept and Object," in Translations from the Philosophical Writings of Gottlob Frege, pp. 45-46.

27 William Marshall, "Sense and Reference: A Reply," in *The Philosophical Review,* LXV (1956), pp. 359-60.

28 Furth, "Two Types of Denotation," in Nicholas Rescher, ed., *Studies in Philosophical Theory, American Philosophical Quarterly* Monograph Series, Monograph No. 2, Oxford, Basil Blackwell, 1968, pp. 41-43; compare Marshall, "Sense and Reference: A Reply," p. 357, and Frege, *The Basic Laws of Arithmetic,* sect. 29, par. 2, p. 84.

29 Dummett, *Frege: Philosophy of Language,* pp. 216-18.

30 Nicholas Measor, "Frege, Dummett, and the Philistines," in *Analysis,* Vol. 38, No. 1 (January 1978), p. 15.

31 "On Concept and Object," p. 46.

32 Frege, *The Basic Laws of Arithmetic,* pp. 92-94.

33 For carving and related metaphors, see *The Foundations of Arithmetic,* pp. 34, 43, 73-74, 100-01. See "On Concept and Object," p. 49, on the analysis of thoughts. Compare *The Basic Laws of Arithmetic,* pp. 92-94.

34 Illustrative extracts from Frege's review of Husserl's *Philosophie der Arithmetik,* p. 80.

35 Frege, *The Basic Laws of Arithmetic,* pp. 43-44, 45. I use material equivalence where Frege uses identity.

36 The second-level relation of equivalence, (A) $(x)[\Phi x \equiv \psi x]$, would be mutually represented by the first-level relation (B) $(x)[(x \cap \xi) \cap ((x \cap \zeta) \cap \dot{\alpha}\dot{\varepsilon}(\varepsilon = \alpha))]$. But (B) can be rewritten as (C) $\xi \cap (\zeta \cap \dot{\alpha}\dot{\varepsilon}(\varepsilon = \alpha))]$, and (C) can be rewritten as (D) $\xi = \zeta$. I use "Φ," "Ψ," "ξ," and "ζ" to mark the argument-places. See *The Basic Laws of Arithmetic,* pp. 94-95.

37 Illustrative extracts from Frege's review of Husserl's *Philosophie der Arithmetik,* p. 80.

38 *Ibid.,* p. 80, and p. 80, footnote *.

39 Furth, Editor's Introduction, *The Basic Laws of Arithmetic,* pp. xl-xliv.

40 Frege, *The Basic Laws of Arithmetic,* p. 92.

41 Frege, "On Sense and Reference," p. 62.

42 On the extensionality thesis, Frege's definitional program in *The Basic Laws of Arithmetic,* sect. 29, par. 2, p. 84, states what is in effect a sufficient condition of representative identifiability for simple predicates as a sufficient condition of their denoting. Also the truth of "The concept *F* is representatively identifiable" is a sufficient condition of the existence of *F* itself, since it must exist to be represented by the object denoted by "The concept *F*."

43 Frege, *The Foundations of Arithmetic,* p. 65. Definition of the existential quantifier in terms of negation and the universal quantifier is a variant. See Frege, "Begriffsschrift, a formula language, modeled upon that of arithmetic, for pure thought," in Jean van Heijenoort, ed., *From Frege to Gödel,* Cambridge, Harvard University Press, 1967, p. 27, footnote 15, p. 28, footnote 16.

44 See Frege, "Function and Concept," in *Translations from the Philosophical Writings of Gottlob Frege,* p. 33, on the determinacy of objects. See *The Basic Laws of Arithmetic,* p. 84, on the determinacy of functions.

45 Frege, *The Foundations of Arithmetic,* p. 73.

46 Compare Frege, "On Sense and Reference," pp. 62-63.

47 See Note 19.

48 Frege, "On Concept and Object," p. 46.

49 Frege, "Dialogue with Pünjer on Existence," in *Gottlob Frege: Posthumous Writings,* Peter Long and Roger White, trans., Hans Hermes, Friedrich Kambartel, and Friedrich Kaulbach, eds., Chicago, The University of Chicago Press, 1979, p. 64.

50 *Ibid.,* p. 63.

51 *Ibid.,* p. 67.

52 On forces, see *The Basic Laws of Arithmetic,* pp. 37-38, and "The Thought," pp. 293-95. On senses, see Jan Dejnozka, "Frege on Identity," *International Studies in Philosophy* (Spring 1981), pp. 36-37.

53 Frege, "On the Foundations of Geometry" (1906), in Eike-Henner W. Kluge, ed., *Frege: On the Foundations of Geometry and Formal Theories of Arithmetic,* New Haven, Yale University Press, 1971, pp. 59-60.

54 Compare Ludwig Wittgenstein, *Tractatus Logico-Philosophicus,* D. F. Pears and B. F. McGuinness, trans., London, Routledge & Kegan Paul, 1961, pp. 117, 141, 145, 147, 149.

55 The correspondences might include: ethical or aesthetic act—ethical or aesthetic pleasure or displeasure, *Ibid.,* p. 147; God—the world, pp. 143, 149; causation—constant conjunction, p. 143; the thinking subject—the human being, p. 119.

56 *Ibid.,* pp. 149-51.