Student Misconceptions Aligned with the Fractions Learning Pathway

Uncovering Misconceptions Through Student Clinical Interviews

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OTF Grant Learning Goals

1. Review current literature on fraction concepts
2. Make correlations between Fractions Learning Pathway and Ontario Curriculum
3. Create structured interview questions based on Fractions Learning Pathway and Ontario Curriculum to determine student misconceptions
4. Interview students to better understand trends in their misconceptions and areas to focus instruction
5. Create Number Talks and structured lessons to close gaps and clarify misconceptions related to fractions

Context

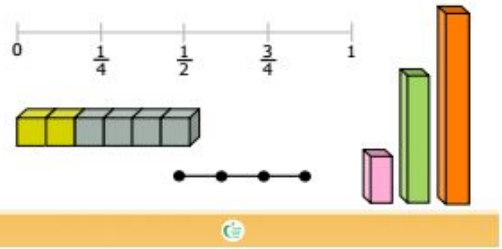
Portage Trail Community School is an inner city school in Toronto. It is currently classified as a “math increased” school receiving targeted coaching support in mathematics. Based on large gaps in conceptual understanding as evidenced in CAT 4 and EQAO results staff wanted to undergo focused research on student misconceptions intending to use this information to carefully focus instruction.

Fractional Models

Paying Attention to Fractions K-12 (2014 Ministry Monograph)– page 4

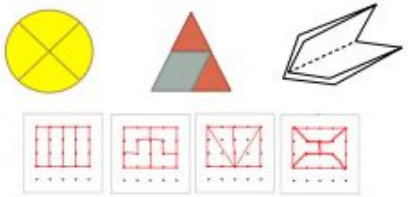
Linear Models/Number line (Continuous)

In a linear model, a length is divided into fractional parts.



Area Model (Continuous)

In an area (region) model, one shape or object represents the whole. The whole is divided into fractional regions.

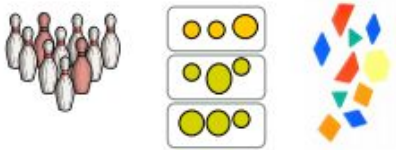


Volume Model

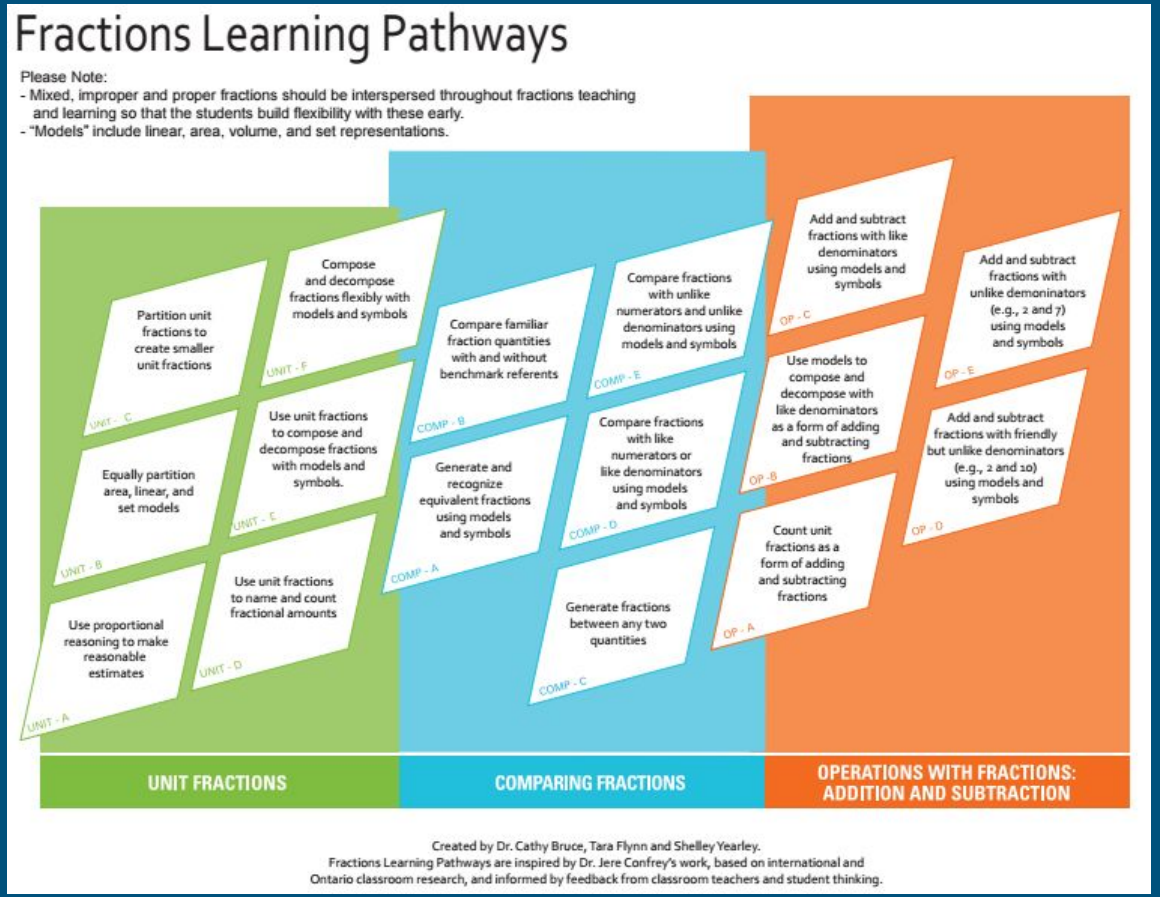
In a volume model, a three-dimensional figure represents the whole. The whole is divided into fractional regions that are occupied by space within the figure.

Set Model (Discrete)

In a set model, a collection of objects represents the whole amount. Subsets of the whole make up the fractional parts.



Fractions Learning Pathway



Interview Questions: Unit Fractions

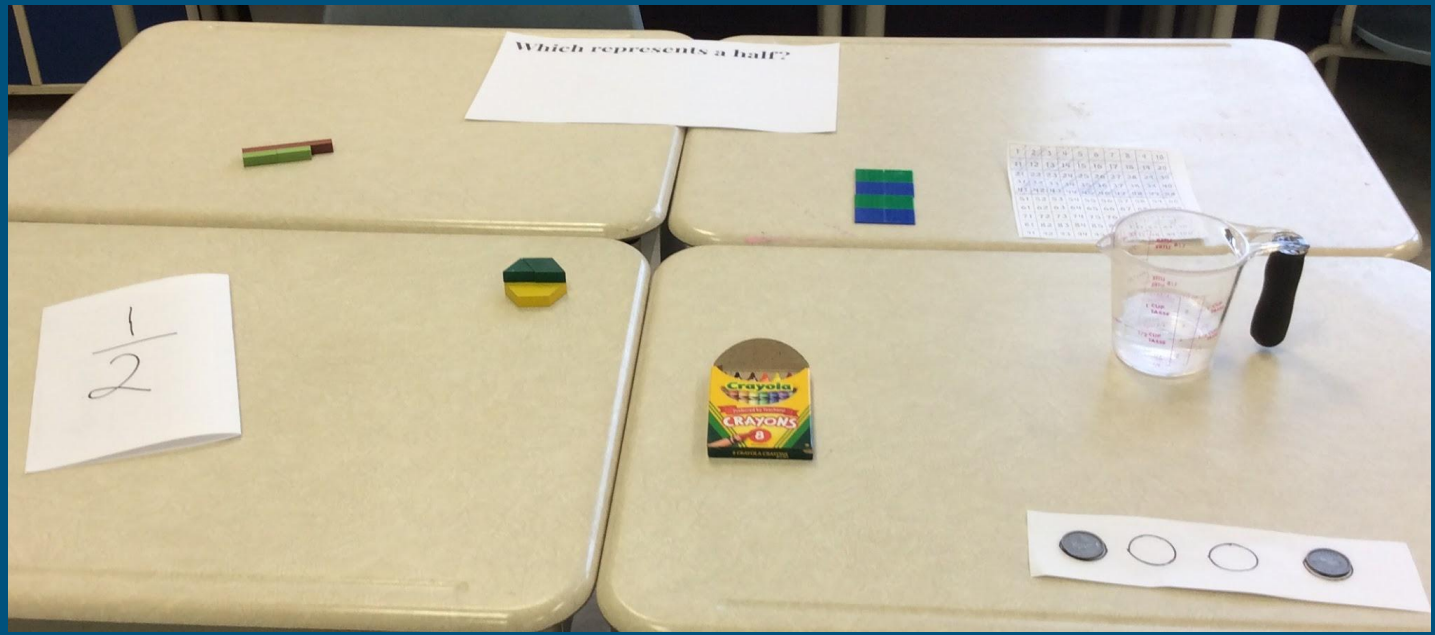
Unit Fractions Unit A Use proportional reasoning to make reasonable estimates	**Grade 2** – determine, through investigation using concrete materials, the relationship between the number of fractional parts of a whole and the size of the fractional parts **Grade 3** – divide whole objects and sets of objects into equal parts, and identify the parts using fractional names (e.g., one half; three thirds; two fourths or two quarters), without using numbers in standard fractional notation	**Which represents a half?** *Materials to be provided:* - *½ (standard notation)* - *Coloured tiles blue & green separated ½ (area)* - *Measuring cup ½ (volume)* - *2 quarters w 2 blank spaces (set)* - *Hundreds chart 50/100 (area)* - *Box of crayons/markers with one removed (set)* - *Cuisenaire rod (brown & two green) (linear)*

Unit Fractions Misconception Trends

Identify the fractions that show ½.

Unit Fractions

Unit A
Use proportional reasoning to make reasonable estimates



Unit Fractions Misconception Trends

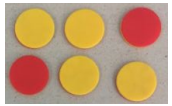
Identify the fractions that show ½.

Unit Fractions

Unit A
Use proportional reasoning to make reasonable estimates

Misconception Trends Unit A

- Does not recognize a fraction when fractional parts are separated by space
  "This is not 2/6"



- Does not recognize a fraction when fractional parts are not all the same shape
  "This is not ¼"

- Misidentifies fractions of a set due to confusion with area model
  "This is not ¼. The star is bigger than the circles"

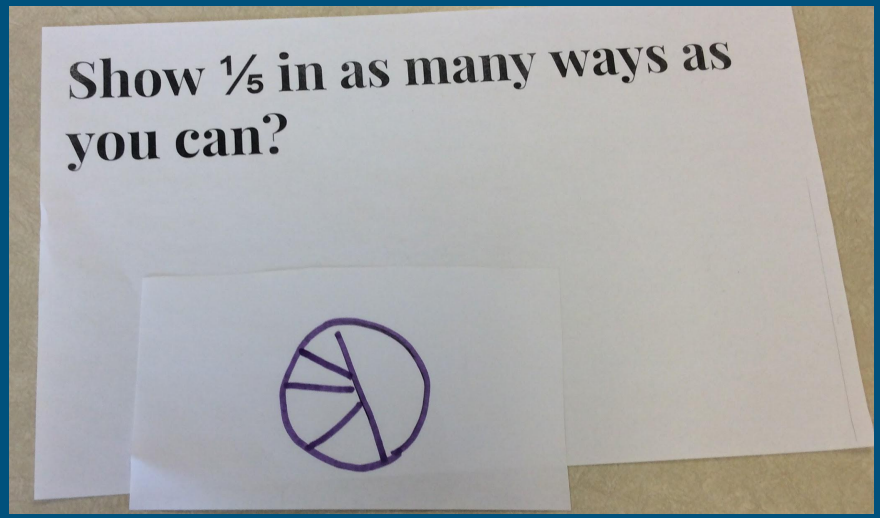
Interview Questions: Unit Fractions

Unit Fractions Unit E Use unit fractions to compose and decompose fractions with models and symbols.	**Grade 4** - represent fractions using concrete materials, words, and standard fractional notation, and explain the meaning of the denominator as the number of the fractional parts of a whole or a set, and the numerator as the number of fractional parts being considered	**Show ⅛ in as many ways as you can?** *Materials to be provided:* - *Drawing paper w pencil (area)* - *Pattern blocks (area)* - *Connecting cubes (volume)* - *Counters (set)* - *Geoboard (area)* - *Coloured tiles (area)* - *Hundreds chart (area)* - *Cuisenaire rods (linear)*

Unit Fractions Supporting Photo/Videos

Unit Fractions

Unit E
Use unit fractions to compose and decompose fractions with models and symbols.



Misconceptions Trends Unit E

Draws an area model of a fraction, where the size of each piece is unequal

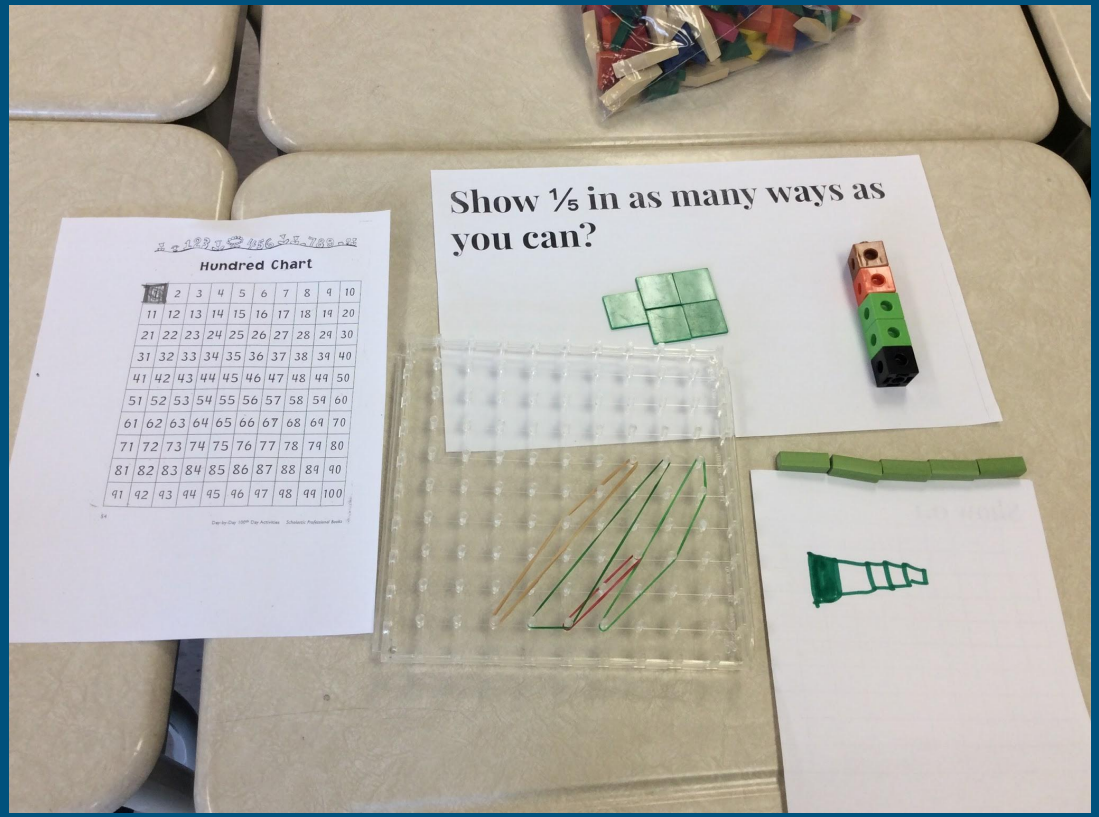
Unit Fractions Supporting Photo/Videos

Unit Fractions

Unit E
Use unit fractions to compose and decompose fractions with models and symbols.

Misconceptions Trends Unit E

Confuses fractions and whole numbers by representing the denominator as a whole number



Unit Fractions Supporting Photo/Videos

Unit Fractions

Unit E
Use unit fractions to compose and decompose fractions with models and symbols.

Misconceptions Trends Unit E

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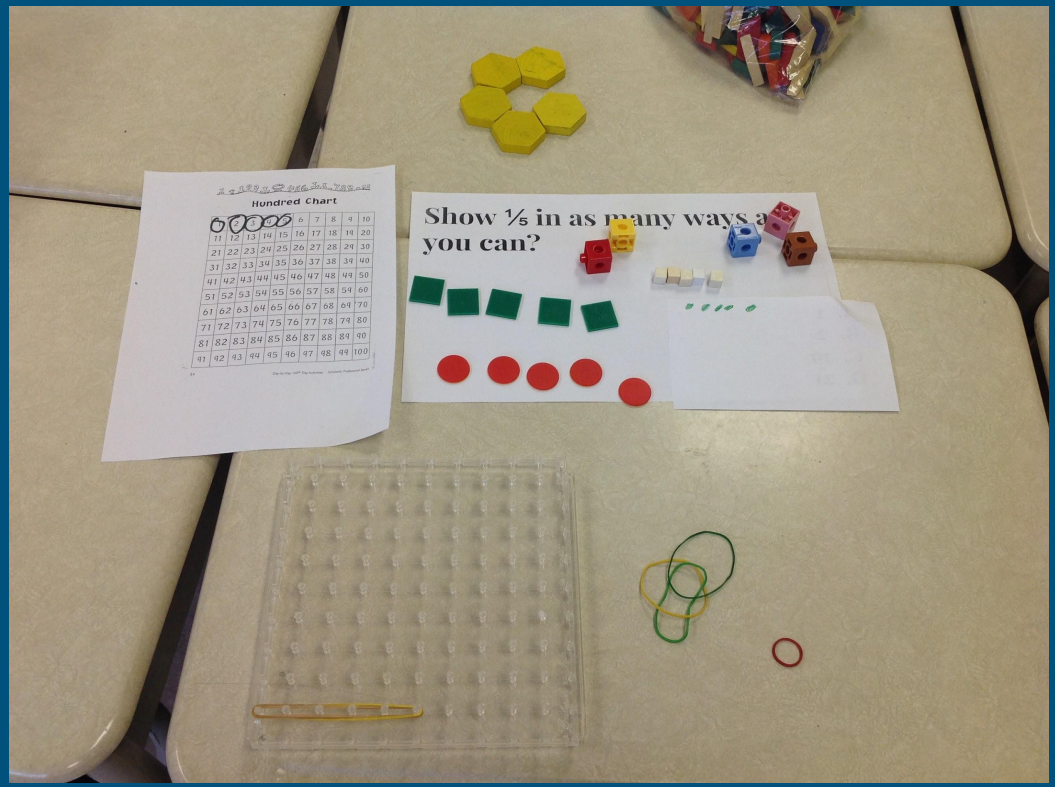


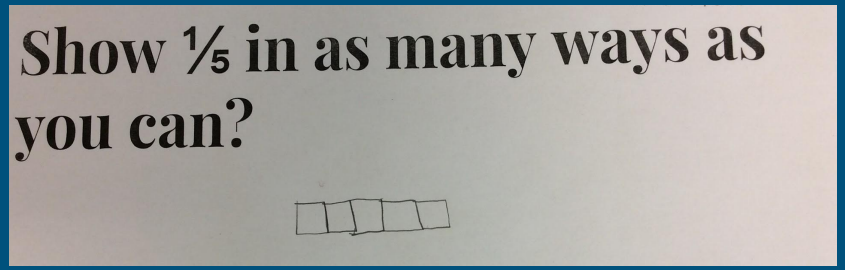
Unit Fractions Supporting Photo/Videos

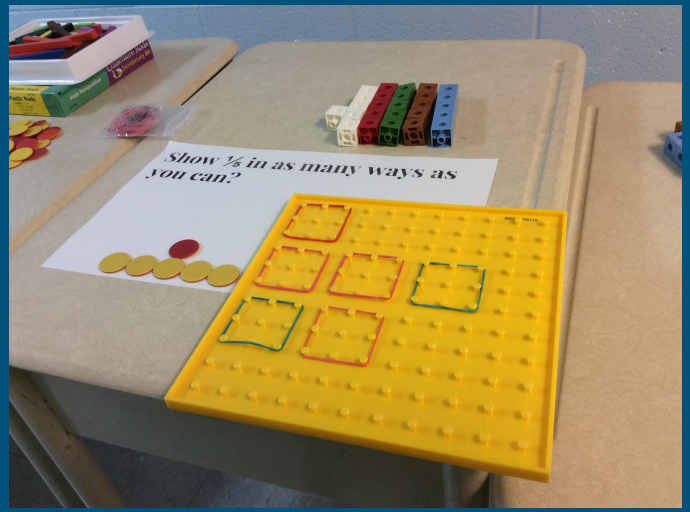
Unit Fractions

Unit E
Use unit fractions to compose and decompose fractions with models and symbols.

Misconceptions Trends Unit E

Creates a fraction where the numerator and denominator are representing two separate whole numbers





Unit Fractions Supporting Photo/Videos

Unit Fractions

Unit E
Use unit fractions to compose and decompose fractions with models and symbols.

Misconceptions Trends Unit E

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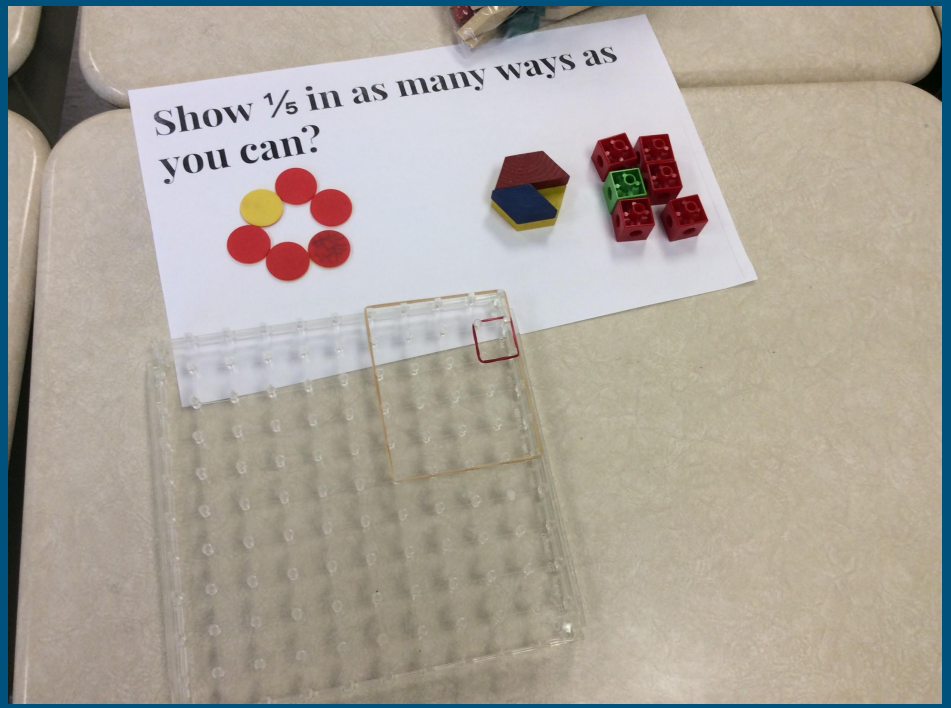


Unit Fractions Misconception Trends

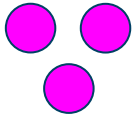
Unit Fractions

Unit E
Use unit fractions to compose and decompose fractions with models and symbols.

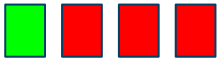
Represent ⅛ in as many ways as you can.

Misconception Trends Unit E

- Confuses fractions and whole numbers by representing the denominator as a whole number
  "This is a third"



- Creates a fraction where the numerator and denominator are representing two separate whole numbers
  "This is ⅓"



- Draws an area model of a fraction, where the size of each piece is unequal
  "This is a third"

Interview Questions: Unit Fractions

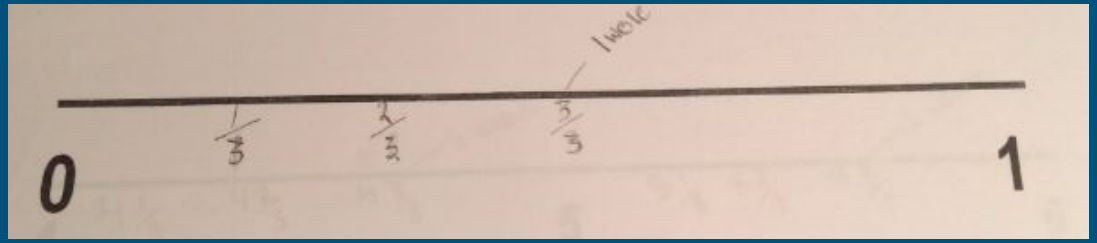
Unit Fractions Unit B Equally partition area, linear and set models	**Grade 4** – represent fractions using concrete materials, words, and standard fractional notation, and explain the meaning of the denominator as the number of the fractional parts of a whole or a set, and the numerator as the number of fractional parts being considered – count forward by halves, thirds, fourths, and tenths to beyond one whole, using concrete materials and number lines	**How could you use this number line to count up by thirds?** **Mark the fractions as you count on the line.** *(Have 0 to 1 marked on a number line.)*
Unit Fraction Unit B Equally partition area, linear and set models	**Grade 5** – represent, compare, and order fractional amounts with like denominators, including proper and improper fractions and mixed numbers, using a variety of tools	**How could you use this number line to count up by thirds?** **Mark the fractions as you count on the line.** *(Have numbers 4 to 6 marked on a number line)*

Unit Fractions Supporting Photos

Unit Fractions

Unit B
Equally partition area, linear and set models

Prompt: “Count up by ⅓ using this number line”



Misconceptions Trends Comp B

Does not correctly partition the linear space relative to the fraction being shown.

Does not relate fractional amount to a benchmark whole number.

Unit Fractions Supporting Photos

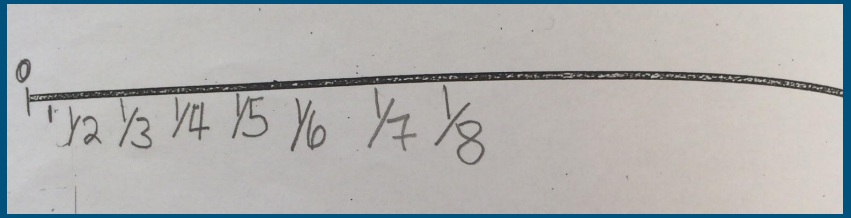
Unit Fractions

Unit B
Equally partition area, linear and set models

Misconceptions Trends Comp B

Misunderstands “counting up” as involving changing the denominator rather than the numerator.

Prompt: “Count up by ⅛ using this number line”



Unit Fractions Supporting Photos

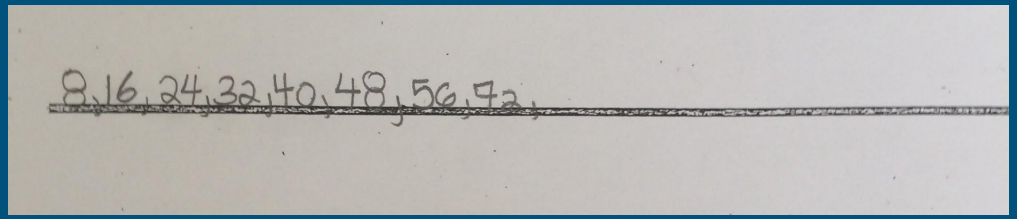
Unit Fractions

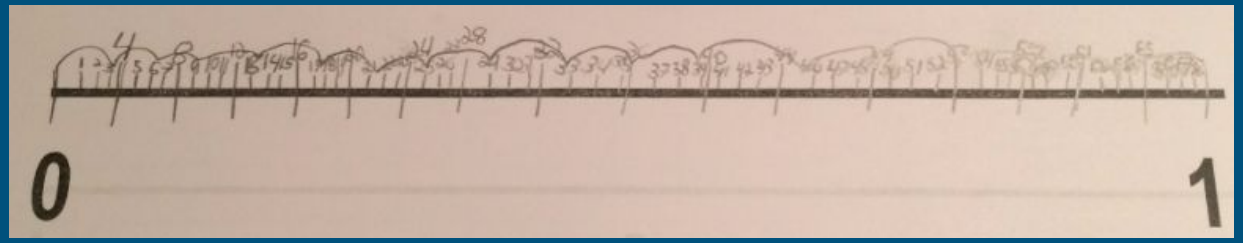
Unit B
Equally partition area, linear and set models

Misconceptions Trends Comp B

Confuses the unit fraction with a whole number when counting.

Prompt: “Count up by ⅛ using this number line”





Unit Fractions Misconception Trends

Prompt: “Count up by ⅛ using this number line”

Unit Fractions

Unit B
Equally partition area, linear and set models

Misconceptions Trends Unit B

- Confuses the unit fraction with a whole number when counting.
  “1, 2, 3, 4, 5, etc.”
- Misunderstands “counting up” as involving changing the denominator rather than the numerator.
  “ 1/1, ½ , ⅓ , ¼ , ⅕ , ⅙ , etc.”
- Does not correctly partition the linear space relative to the fraction being shown.
- Does not relate fractional amount to a benchmark whole number.
  “⅓, ⅔, 3/3, 4/3, 5/3, 1”

Interview Questions: Comparing Fractions

Comparing Fractions Comp B Compare familiar fraction quantities with and without benchmark referents	Grade 4 – compare fractions to the benchmarks of 0, ½ and 1	**Is this closer to 0, ½, or 1?** ¼ ⅜ **Explain your thinking.** *(Give number line with benchmarks listed. Give fractions written on post-its to be placed. Prompt to use manipulatives if needed)*

Comparing Fractions Supporting Photo/Videos

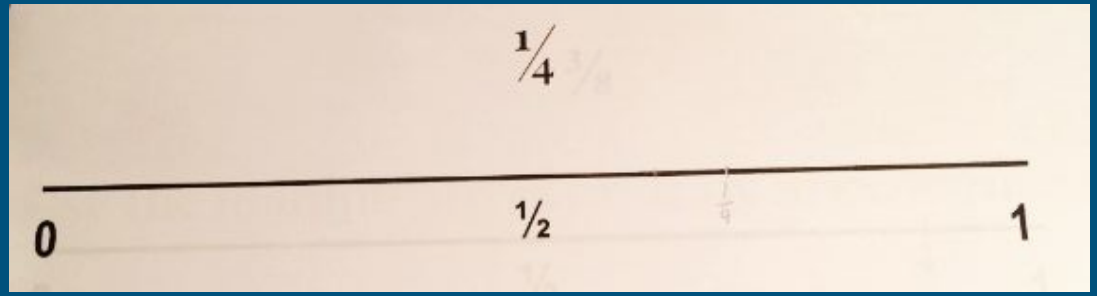
Comparing Fractions

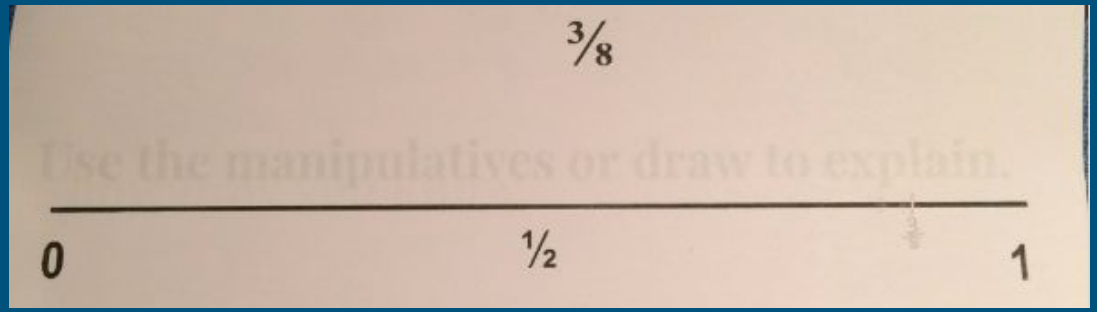
Comp B
Compare familiar fraction quantities with and without benchmark referents

Misconceptions Trends Comp B

Does not look at the relationship between the size of the numerator and denominator when comparing fractions. Misidentifies the fraction as being larger than it is, because the “bottom number is big”.

Is this closer to 0, ½, or 1?





Comparing Fractions Supporting Photo/Videos

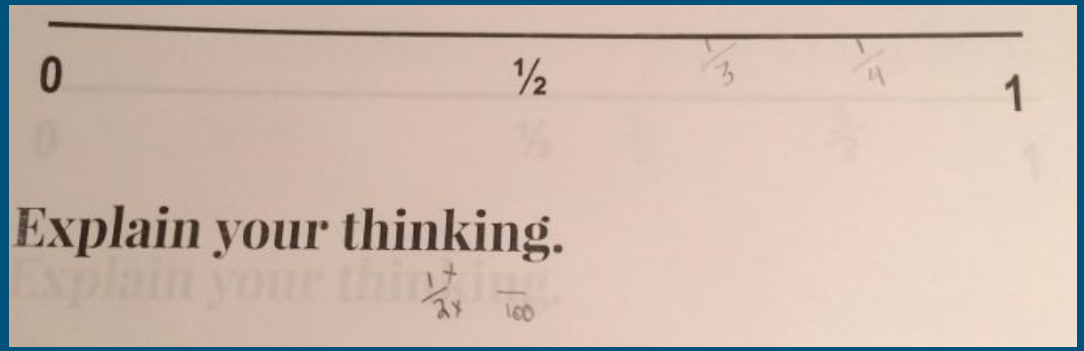
Comparing Fractions

Comp B
Compare familiar fraction quantities with and without benchmark referents

Misconceptions Trends Comp B
Does not look at the relationship between the size of the numerator and denominator when comparing fractions. Misidentifies the fraction as being larger than it is, because the “bottom number is big”.

Is this closer to 0, ½, or 1?

¼



Comparing Fractions Misconception Trends

Is ¼ closer to 0, ½, or 1?

Comparing Fractions

Comp B
Compare familiar fraction quantities with and without benchmark referents

Misconceptions Trends Comp B

- Does not look at the relationship between the size of the numerator and denominator when comparing fractions. Misidentifies the fraction as being larger than it is, because the denominator or "bottom number" is bigger.

  "¼ is closer to 1, because 4 is a big number"

Interview Questions: Comparing Fractions

Comparing Fractions Comp A Generate and recognize equivalent fractions using models and symbols	Grade 4 – demonstrate and explain the relationship between equivalent fractions, using concrete materials	**Are 2/6 equivalent to ⅓?** **Use the manipulatives or draw to explain.**

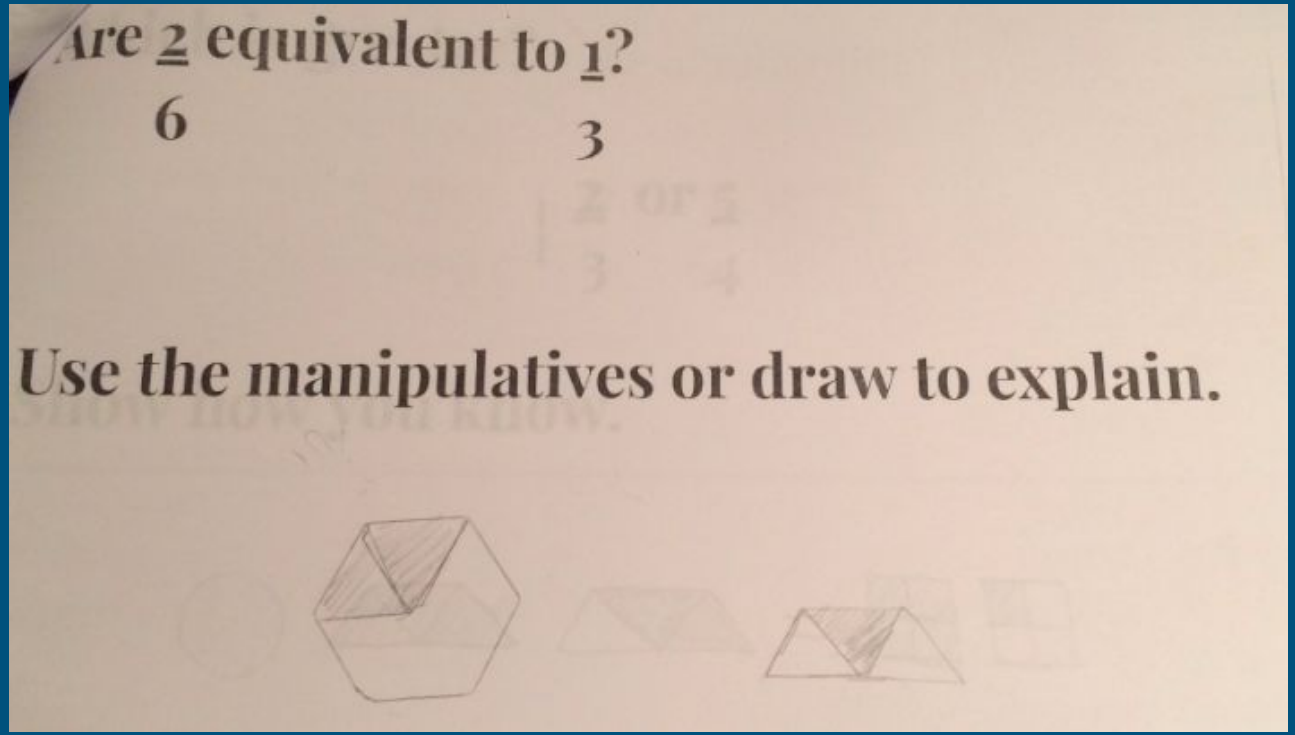
Comparing Fractions Supporting Photo/Videos

Comparing Fractions

Comp A
Generate and recognize equivalent fractions using models and symbols

Misconceptions Trends Comp A

Mistakes the fraction with the larger numerator & denominator as the larger fraction. Does not pay attention to the relationship between numerator & denominator when estimating



Comparing Fractions Misconception Trends

Comparing Fractions

Comp A
Generate and recognize equivalent fractions using models and symbols

Are 2/6 equivalent to ⅓?

Use the manipulatives or draw to explain.

Misconceptions Trends Comp A

- Mistakes the fraction with the larger numerator & denominator as the larger fraction. Does not pay attention to the relationship between numerator & denominator when estimating

Interview Questions: Comparing Fractions

Comparing Fractions Comp E Compare fractions with unlike numerators and unlike denominators using models and symbols	Grade 5 – represent, compare, and order fractional amounts with like denominators, including proper and improper fractions and mixed numbers, using a variety of tools	**Which is greater?** **1 ⅔ or 5/4** **Show how you know.**
Comparing Fractions & DECIMALS Comp E Compare fractions with unlike numerators and unlike denominators using models and symbols	Grade 5 – represent, compare, and order fractional amounts with like denominators, including proper and improper fractions and mixed numbers, using a variety of tools (e.g., number lines) and using standard fractional notation – represent, compare, and order whole numbers and decimal numbers from 0.01 to 100 000, using a variety of tools s (e.g., number lines with appropriate increments)	**Where would these go on the number line?** **90/100, 0.3, ⅝, 0.75** **Explain.** *(Provide a number line with benchmarks: 0, ½ , 1.* *Have the fractions and decimals on post-its for students to place)*

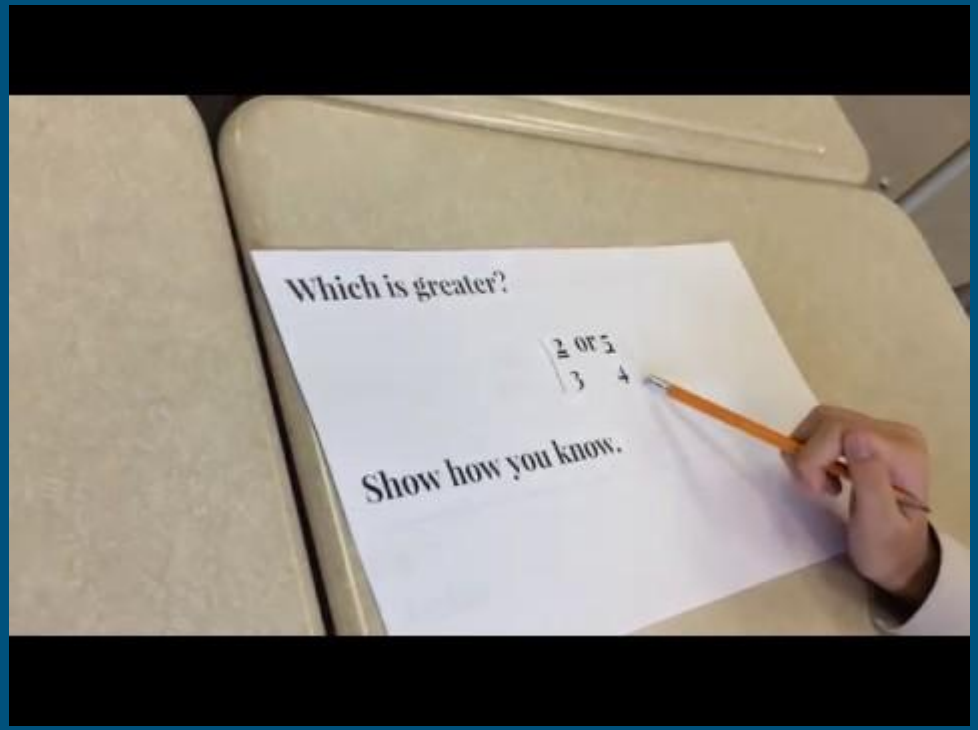
Comparing Fractions Supporting Photo/Videos

Comparing Fractions

Comp E
Compare fractions with unlike
numerators and unlike denominators using models and symbols

Misconceptions Trends Comp E

Incorrectly judges that a mixed number is always greater than an improper number because of the whole number in front.



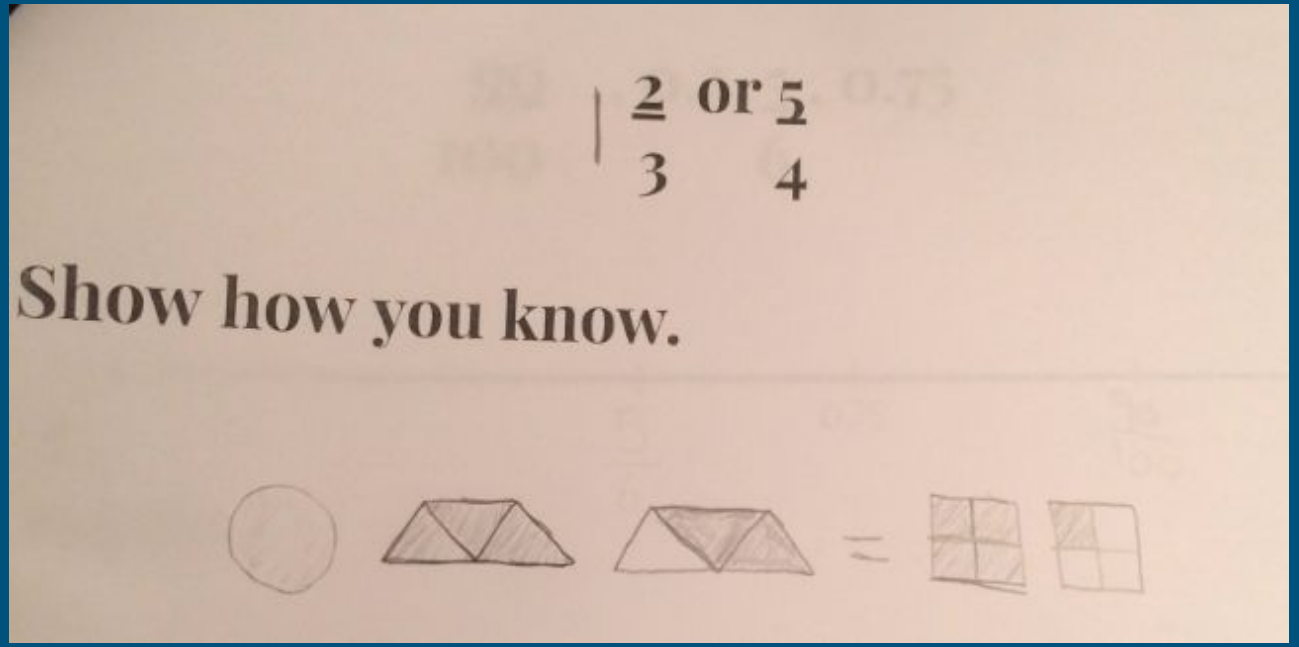
Comparing Fractions Supporting Photo/Videos

Comparing Fractions

Comp E
Compare fractions with unlike
numerators and unlike denominators using models and symbols

Misconceptions Trends Comp E

Creates an incorrect proof by creating two fractions that show different models.



Comparing Fractions Misconception Trends

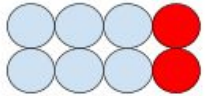
Comparing Fractions

Comp E
Compare fractions with unlike numerators and unlike denominators using models and symbols

Misconceptions Trends Comp E

- Incorrectly judges that a mixed number is always greater than an improper number because of the whole number in front.
  “1 ⅔ is bigger than 5/4, because it has 1 whole in front”

- Creates an incorrect proof by creating two fractions that show different models.
  “2/8 is bigger, because the circles take up more space in the picture”



Interview Questions: Fraction Operations

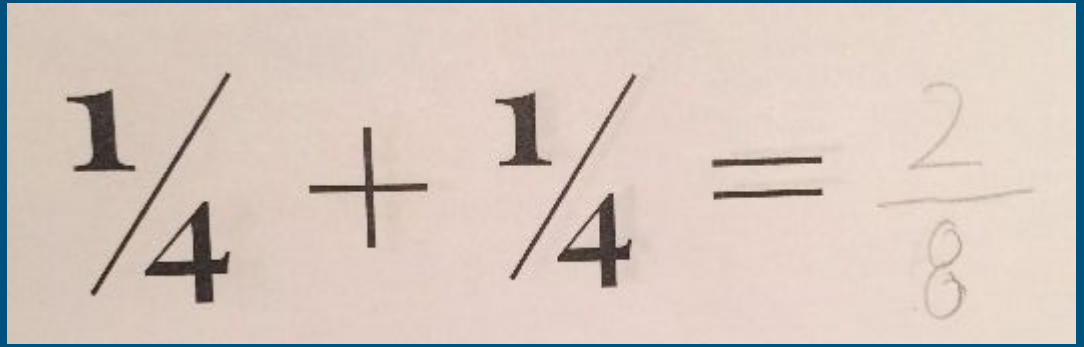
Operation with Fractions: Addition & Subtraction OP C Add and subtract fractions with like denominators using models and symbols	Grade 7 – add and subtract fractions with simple like and unlike denominators, using a variety of tools and algorithms	**Give the answer:** ¼ + ¼ =
Operation with Fractions: Addition & Subtraction OP D Add and subtract fractions with friendly but unlike denominators (e.g., 2 and 10) using models and symbols	Grade 7 – add and subtract fractions with simple like and unlike denominators, using a variety of tools and algorithms	**Give the answer:** ½ + ¼ =

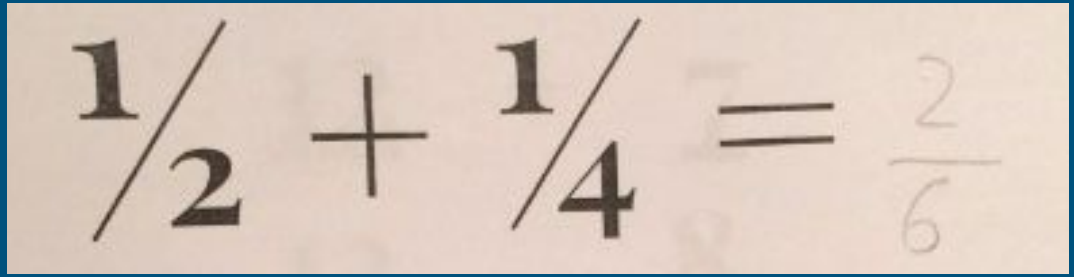
Fraction Operations Supporting Photo/Videos

Operation with Fractions: Addition & Subtraction

OP C
Add and subtract fractions with like denominators using models and symbols

OP D
Add and subtract fractions with friendly but unlike denominators (e.g., 2 and 10) using models and symbols





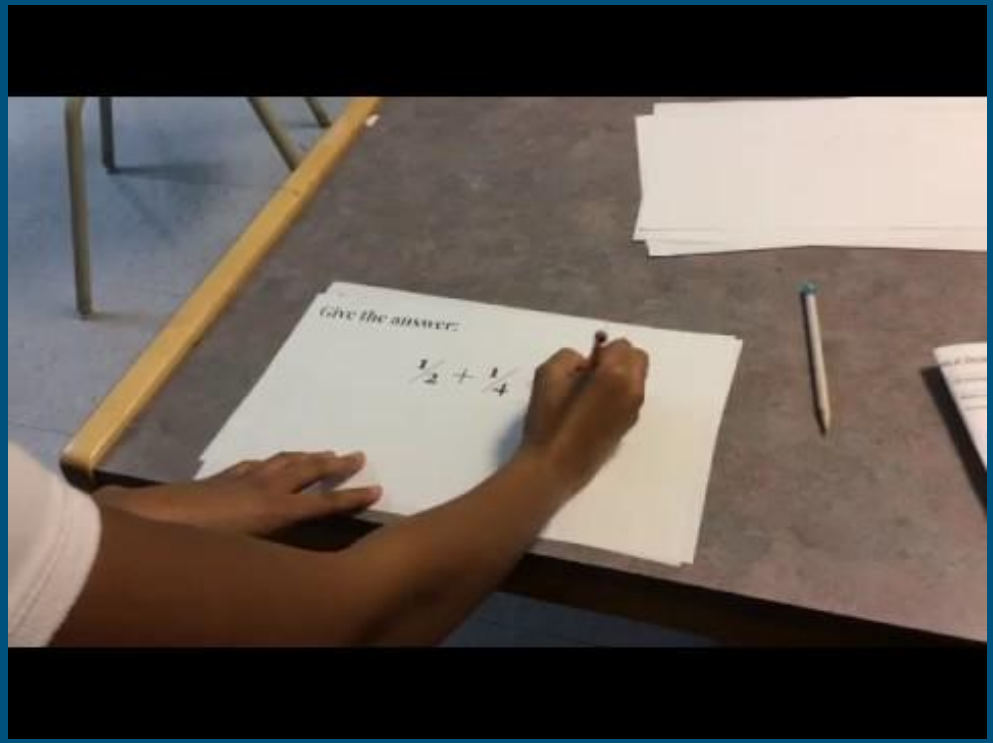
Fraction Operations Supporting Photo/Videos

Operation with Fractions: Addition & Subtraction

OP D
Add and subtract fractions with friendly but unlike denominators (e.g., 2 and 10) using models and symbols

Misconceptions Trends OP D

Adds numerator and denominator (without creating common denominator)



Fraction Operations Misconception Trends

Operation with Fractions: Addition & Subtraction

OP C
Add and subtract fractions with like denominators using models and symbols

OP D
Add and subtract fractions with friendly but unlike denominators (e.g., 2 and 10) using models and symbols

$\frac{1}{4} + \frac{1}{4} =$

Misconceptions Trends OP C

- Adds numerator and denominator (without recognizing like denominators)
  e.g. ¼ + ¼ = 2/8

$\frac{1}{2} + \frac{1}{4} =$

Misconceptions Trends OP D

- Adds numerator and denominator (without recognizing unlike denominators & creating common denominator)
  e.g. ½ + ¼ = 2/6

Interview Questions: Fraction Operations

Operation with Fractions: Addition & Subtraction OP E Add and subtract fractions with unlike demoninators (e.g., 2 and 7) using models and symbols	Grade 7 – add and subtract fractions with simple like and unlike denominators, using a variety of tools and algorithms	**Estimate the answer:** **12/13 + ⅞** **A. 1** **B. 2** **C. 19** **D. 21**

Fraction Operations Supporting Photo/Videos

Operation with Fractions: Addition & Subtraction

OP E
Add and subtract fractions with unlike denominators (e.g., 2 and 7) using models and symbols

Misconceptions Trends OP E

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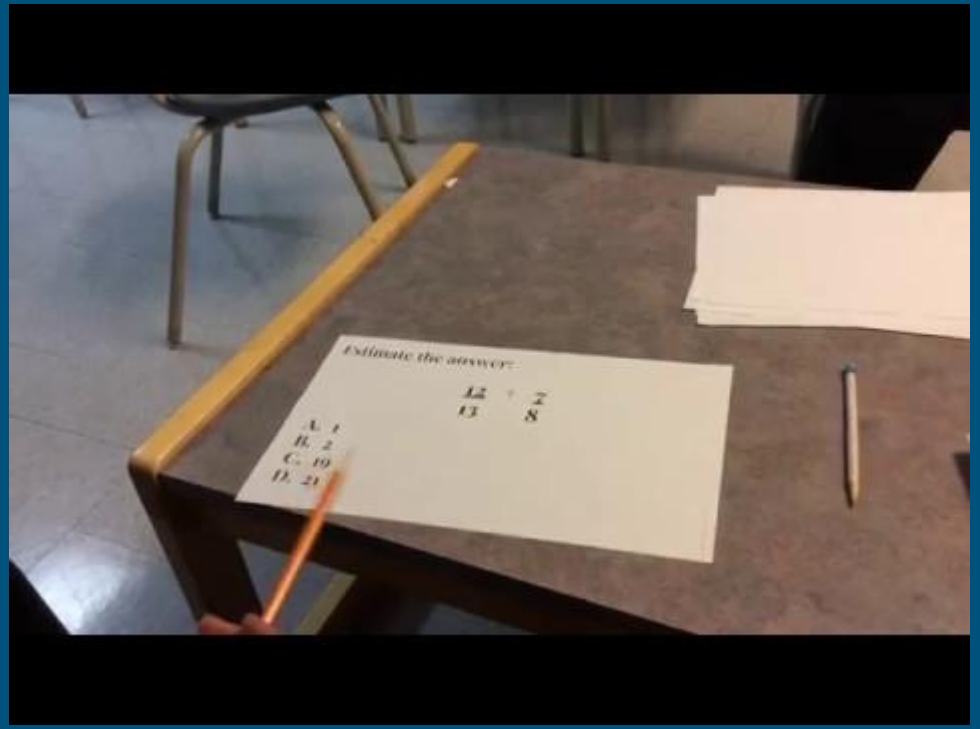

Fraction Operations Supporting Photo/Videos

Operation with Fractions: Addition & Subtraction

OP E
Add and subtract fractions with unlike denominators (e.g., 2 and 7) using models and symbols

Misconceptions Trends OP E

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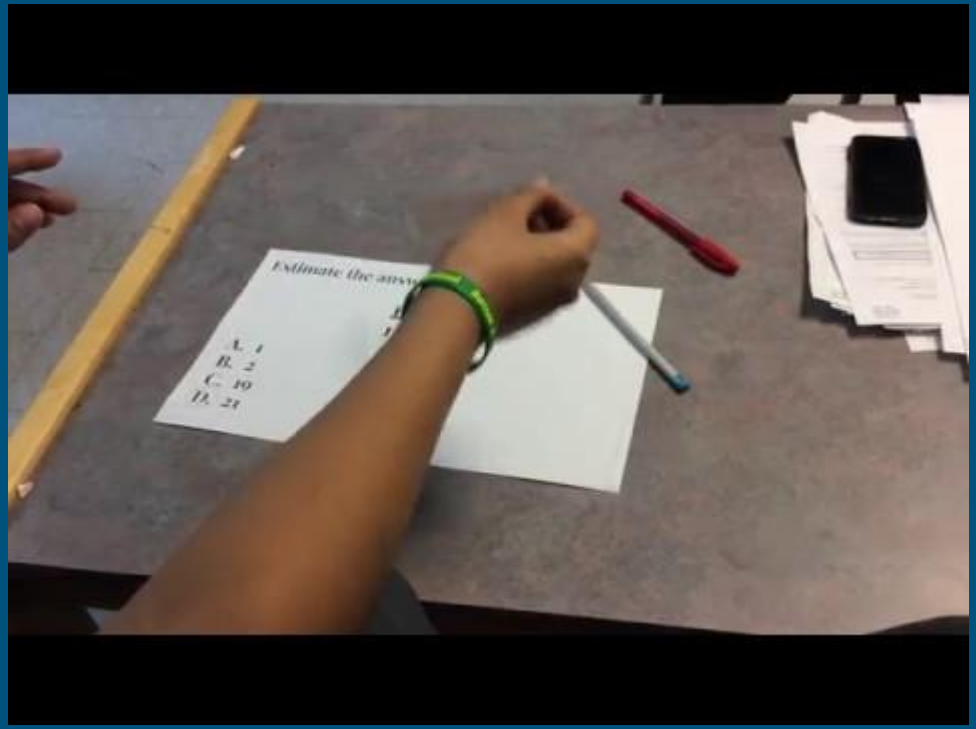

Fraction Operations Misconception Trends

Operation with Fractions: Addition & Subtraction

OP E
Add and subtract fractions with unlike denominators (e.g., 2 and 7) using models and symbols

Estimate the answer: $\frac{12}{13} + \frac{7}{8}$

A. 1 **B. 2** **C. 19** **D. 21**

Misconceptions Trends OP E

- Adds numerators without estimating or creating common denominator
  e.g. 12/13 + ⅞ = 19 “because 12 + 7 = 19”

- Adds denominators without estimating or creating common denominator
  e.g. 12/13 + ⅞ = 21 “because 13 + 8 = 21”