The Global Positioning System

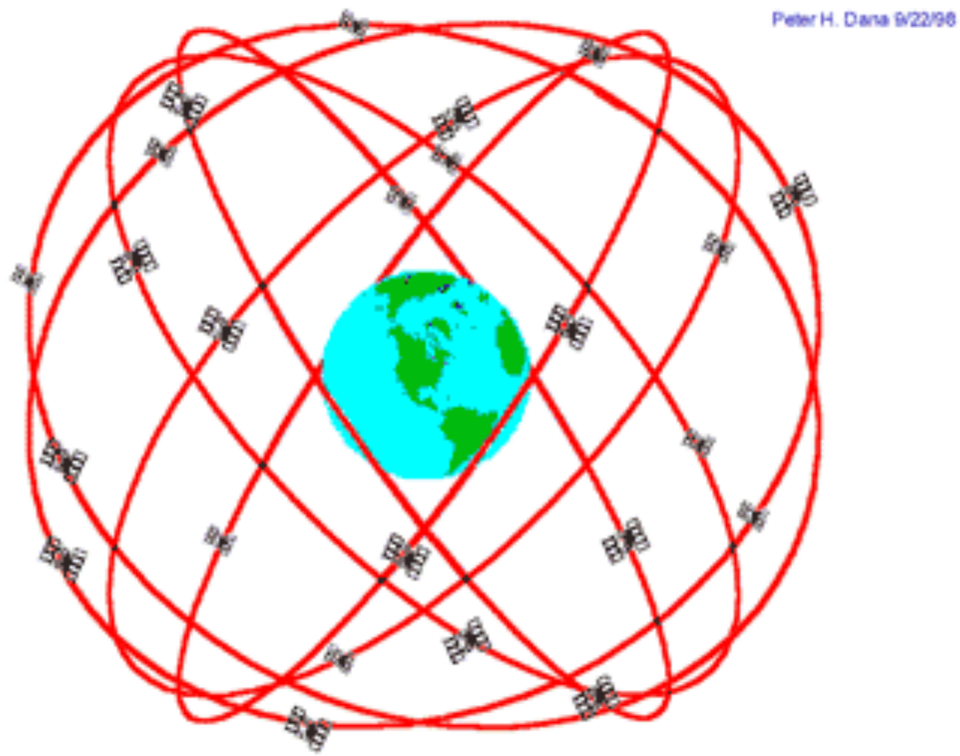
The Global Positioning System

1. System Overview
2. Biases and Errors
3. Signal Structure and Observables
4. Absolute v. Relative Positioning
5. GPS Field Procedures
6. Ellipsoids, Datums and Coordinate Systems
7. Mission Planning

I. System Overview

- GPS is a passive navigation and positioning system available worldwide 24 hours a day in all weather conditions developed and maintained by the Department of Defense
- The Global Positioning System consists of three segments:
  - Space Segment
  - Control Segment
  - User Segment

Space Segment

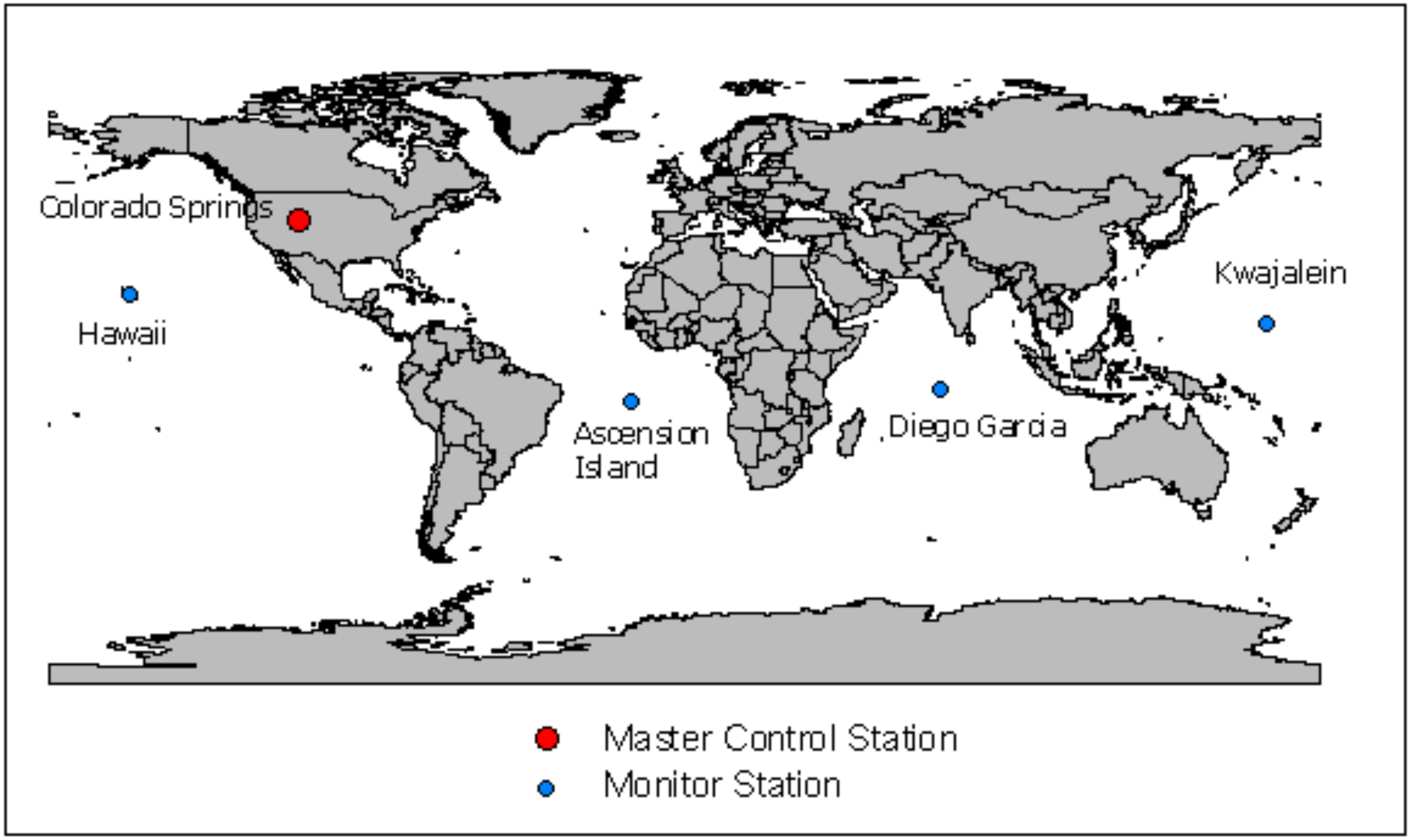


GPS Nominal Constellation
24 Satellites in 6 Orbital Planes
4 Satellites in each Plane
20,200 km Altitudes, 55 Degree Inclination

Space Segment

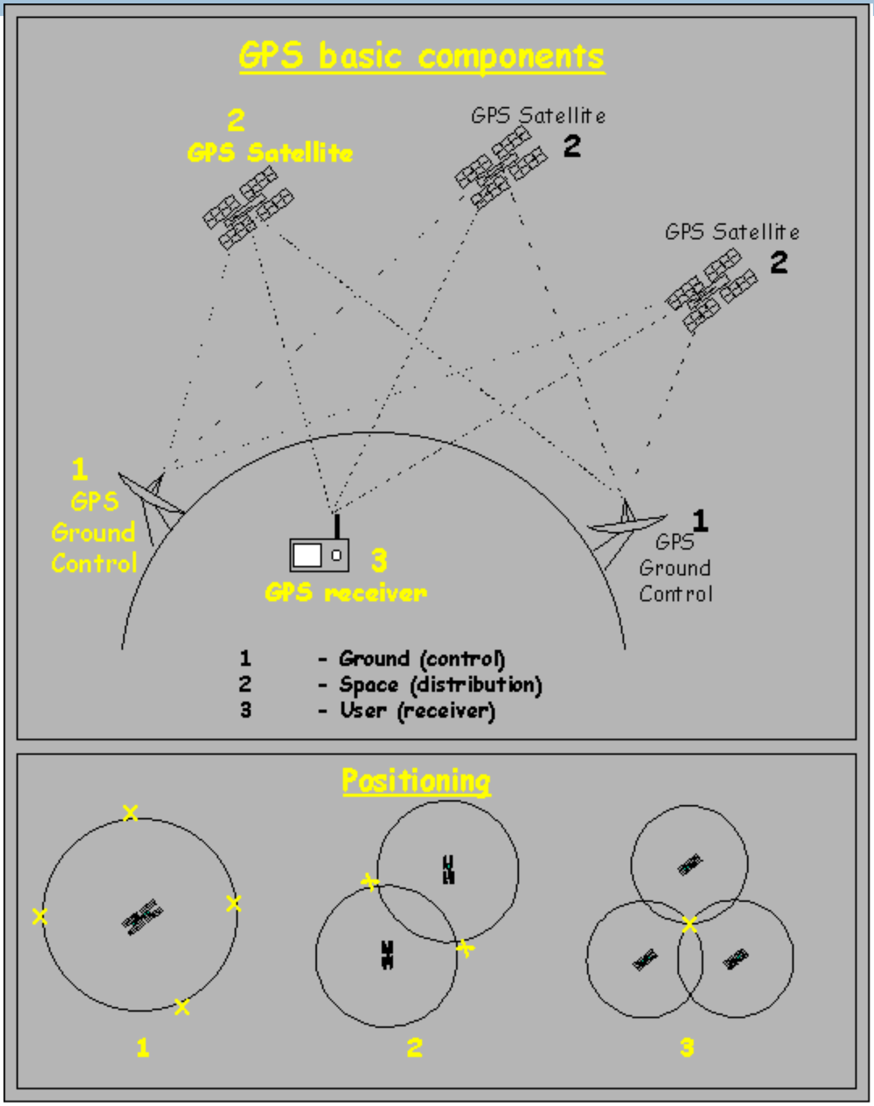
- The current GPS constellation consists of 29 Block II/IIA/IIR/IIR-M satellites. The first Block II satellite was launched in February 1989.

Control Segment



User Segment

How it Works

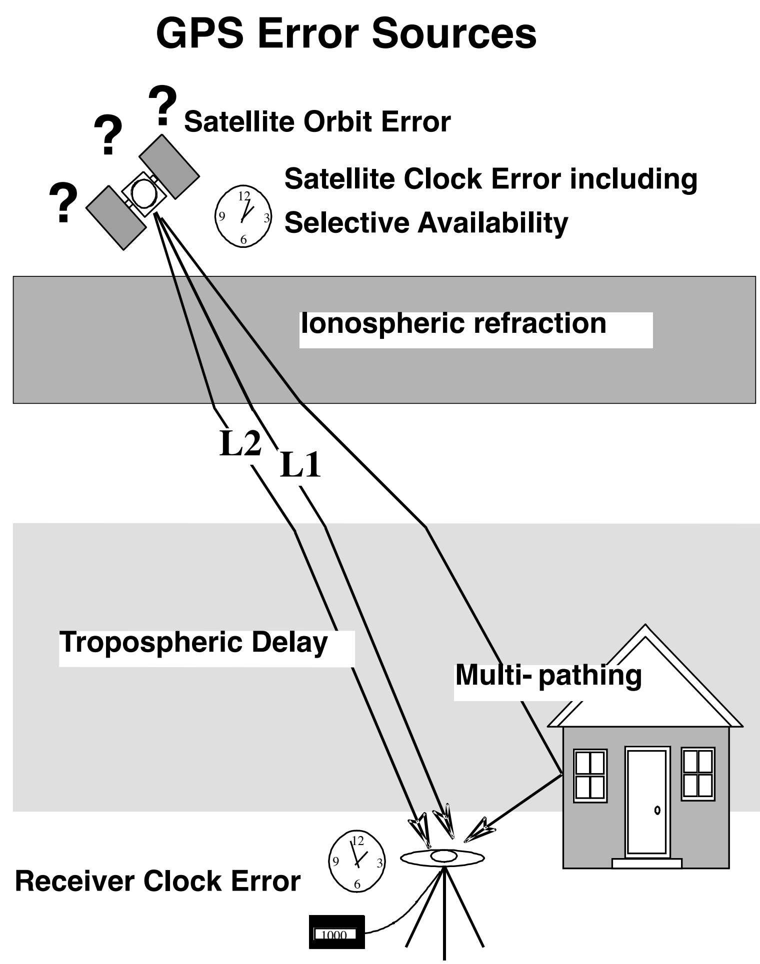


GPS basic components
2
GPS Satellite
GPS Satellite
2
GPS Satellite
2
1
GPS
Ground
Control
3
GPS receiver
1
GPS
Ground
Control
1 - Ground (control)
2 - Space (distribution)
3 - User (receiver)
Positioning
1
2
3

II. Biases and Errors

Biases

- Satellite Dependent
  - Orbit representation biases
  - Satellite clock model biases
- Station Dependent
  - Receiver clock biases
  - Station Coordinates
- Observation Dependent
  - Ionospheric delay
  - Tropospheric delay
  - Carrier phase ambiguity



Satellite Biases

- The satellite is not where the GPS broadcast message says it is.
- The satellite clocks are not perfectly synchronized with GPS time.

Station Biases

- Receiver clock time differs from satellite clock time.
- Uncertainties in the coordinates of the station.
- Time transfer and orbital tracking.

Observation Dependent Biases

- Those associated with signal propagation

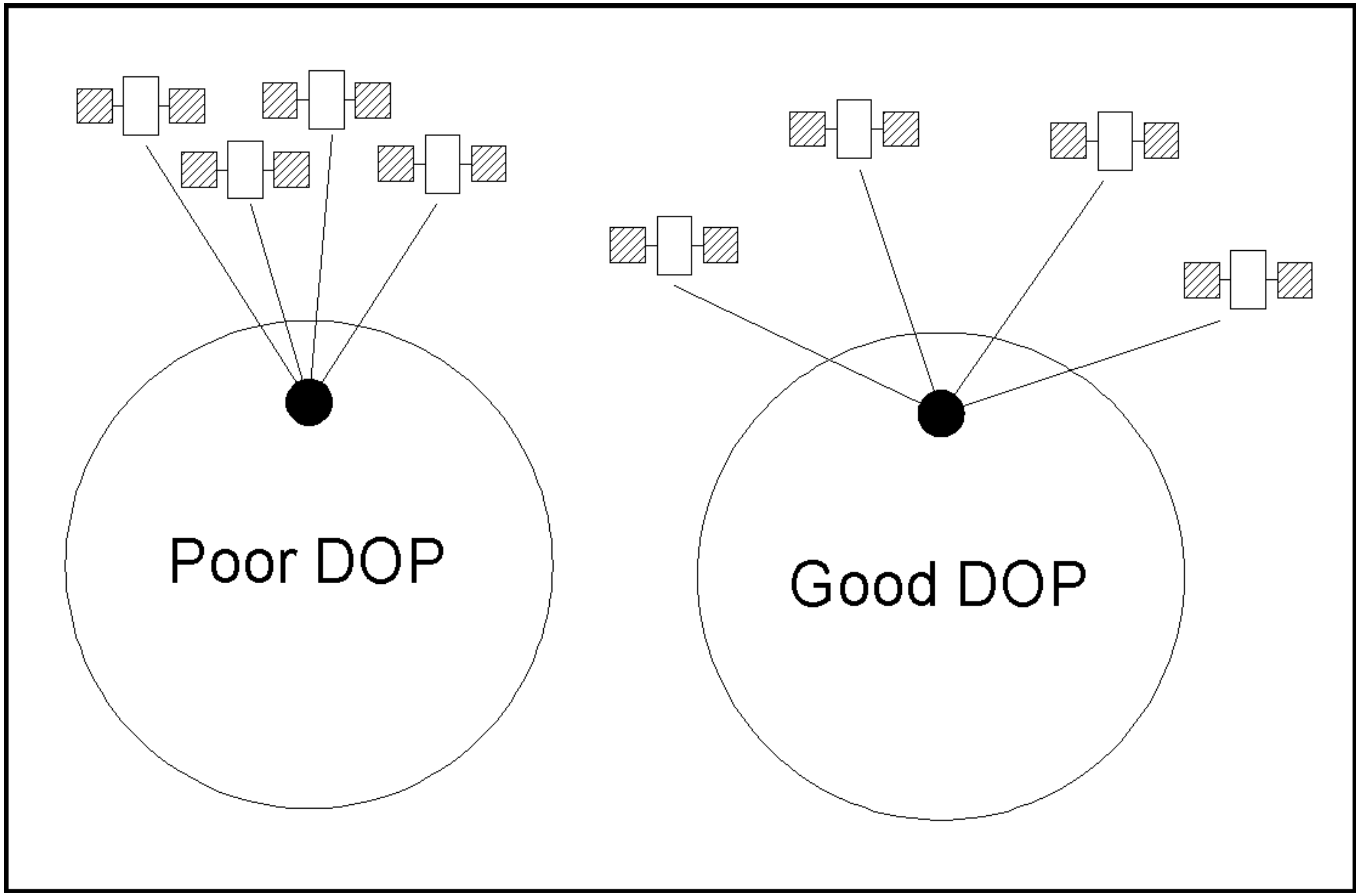
Errors

- Residual Biases
- Cycle Slips
- Multipath
- Antenna Phase Center Movement
- Random Observation Error

Errors

- In addition to biases factors effecting position and/or time determined by GPS is dependant upon:
  - The geometric strength of the satellite configuration being observed (DOP).
  - Remnants from the biases after the major effects have been modeled out.
  - Errors affecting the measurements themselves.

Dilution of Precision
(DOP)



Dilution of Precision (DOP)

- HDOP: for horizontal positioning
- VDOP: for vertical positioning
- PDOP: for 3D positioning
- TDOP: for time determination

III. Signal Structure and Observables

GPS Signal Structure

- Fundamental Frequency
  - f_o = 10.23 MHz
- Carriers
  - L1 = 154 (fo) = 1575.42 MHz $\lambda = 19\text{cm}$
  - L2 = 120 (fo) = 1227.60 MHz $\lambda = 24\text{cm}$
- Both carriers carry the broadcast satellite message, a low frequency stream of data designed to inform the user about the health and position of the satellite.

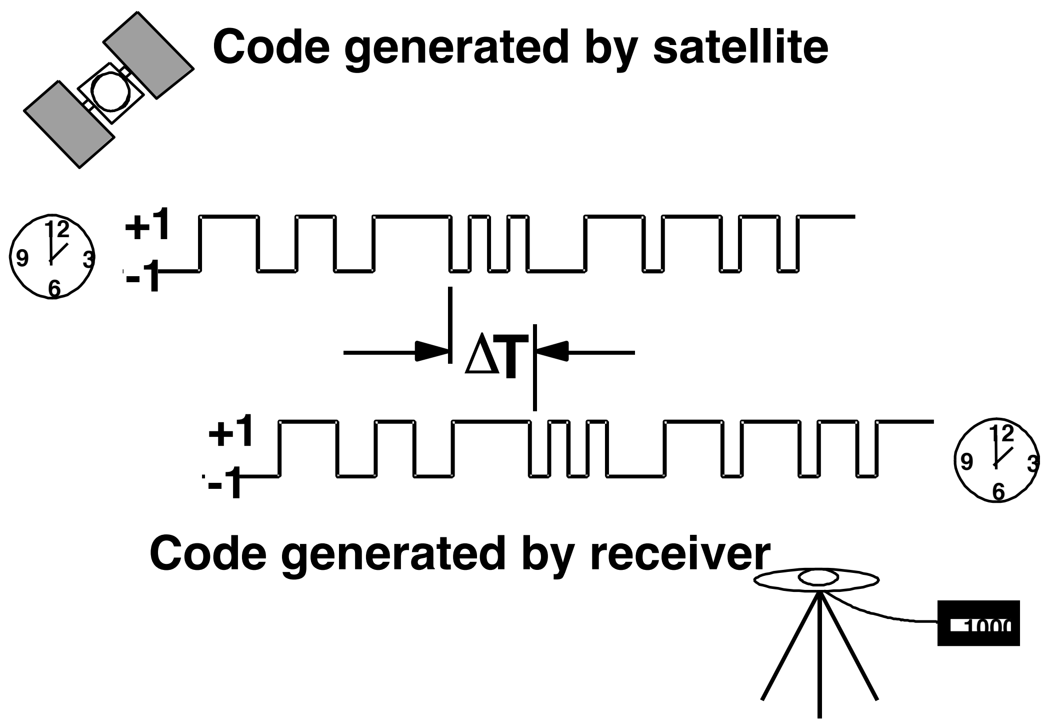
GPS Signal Structure

- Pseudo Random Noise (PRN) Codes
  - C/A (coarse/acquisition) code
    - L1 carrier
    - Repeated every millisecond
    - Emitted at frequency 1.023 MHz
    - Wavelength = 300m
  - P (precise) code
    - L1 and L2 carriers
    - Repeated every 267 days chopped into 38 seven day segments, 32 of which each are assigned to a different satellite
    - Emitted at frequency 10.23 MHz
    - Wavelength = 30m
  - Y code
    - Similar to P code, with secret generating equation
    - L1 and L2 carriers

Observation Types

- Pseudo Range Observable (uses PRN codes)
  - The pseudo range is a measure of the distance between the satellite and the receiver at epochs of transmission and reception of the signals.
- Carrier Phase Observable (uses carrier frequencies)
  - The carrier phase observable is the difference between the phase of the carrier signal of the satellite, measured at the receiver, and the phase of the local oscillator within the receiver at the epoch of measurement.

Pseudo Range Observable



Pseudorange = C*ΔT

Pseudo Range Observable

$$\mathrm{P}_K^P = \rho_K^P + c(dt - dT) + d_{ion} + d_{trop}$$

$$\mathrm{P}_K^P = Pseudo \cdot Range$$

$$\rho_K^P = \sqrt{(X^P - X_K)^2 + (Y^P - Y_K)^2 + (Z^P - Z_K)^2}$$

$$C = Velocity \cdot of \cdot Light$$

$$dt = satellite \cdot clock \cdot error$$

$$dT = receiver \cdot clock \cdot error$$

$$d_{ion} = range \cdot error \cdot due \cdot to \cdot ionospheric \cdot refraction$$

$$d_{trop} = range \cdot error \cdot due \cdot to \cdot tropospheric \cdot refraction$$

The Carrier Phase Observable

Carrier signal generated by Satellite

Phase range = N λ + Φ
N = Phase Ambiguity

Carrier Phase Observable

$$\Phi_K^P = \rho_K^P + c(dt - dT) + \lambda N - d_{ion} + d_{trop}$$

$\Phi_K^P = The \cdot carrier \cdot phase \cdot observable$

$\rho_K^P = The \cdot \text{distance} \cdot between \cdot satellite \cdot and \cdot receiver$

$c = The \cdot speed \cdot of \cdot light$

$dt = satellite \cdot clock \cdot error$

$dT = receiver \cdot clock \cdot error$

$\lambda = wavelength$

$N = Integer \cdot ambiguity$

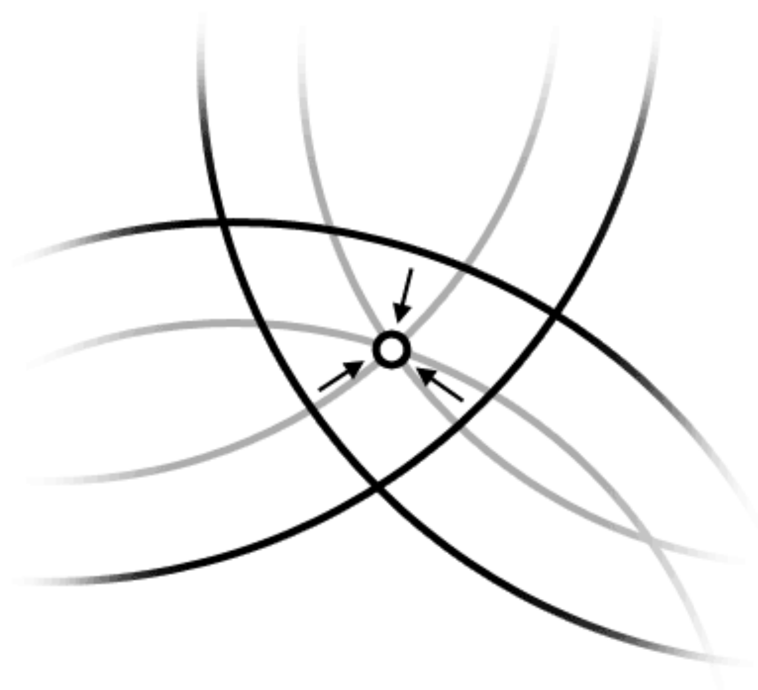
$d_{ion} = error \cdot due \cdot to \cdot ionospheric \cdot refraction$

$d_{trop} = error \cdot due \cdot to \cdot tropospheric \cdot refraction$

V. Absolute v. Relative Positioning

- Absolute Positioning:
  - The position of a single point is directly determined.
- Relative Positioning:
  - Two receivers are used simultaneously to observe satellite signals
  - The vector (dx,dy,dz) joining the two receivers is computed
  - The position of one point is determined relative to the other point.

Absolute Positioning "Ranging"



Relative GPS Positioning

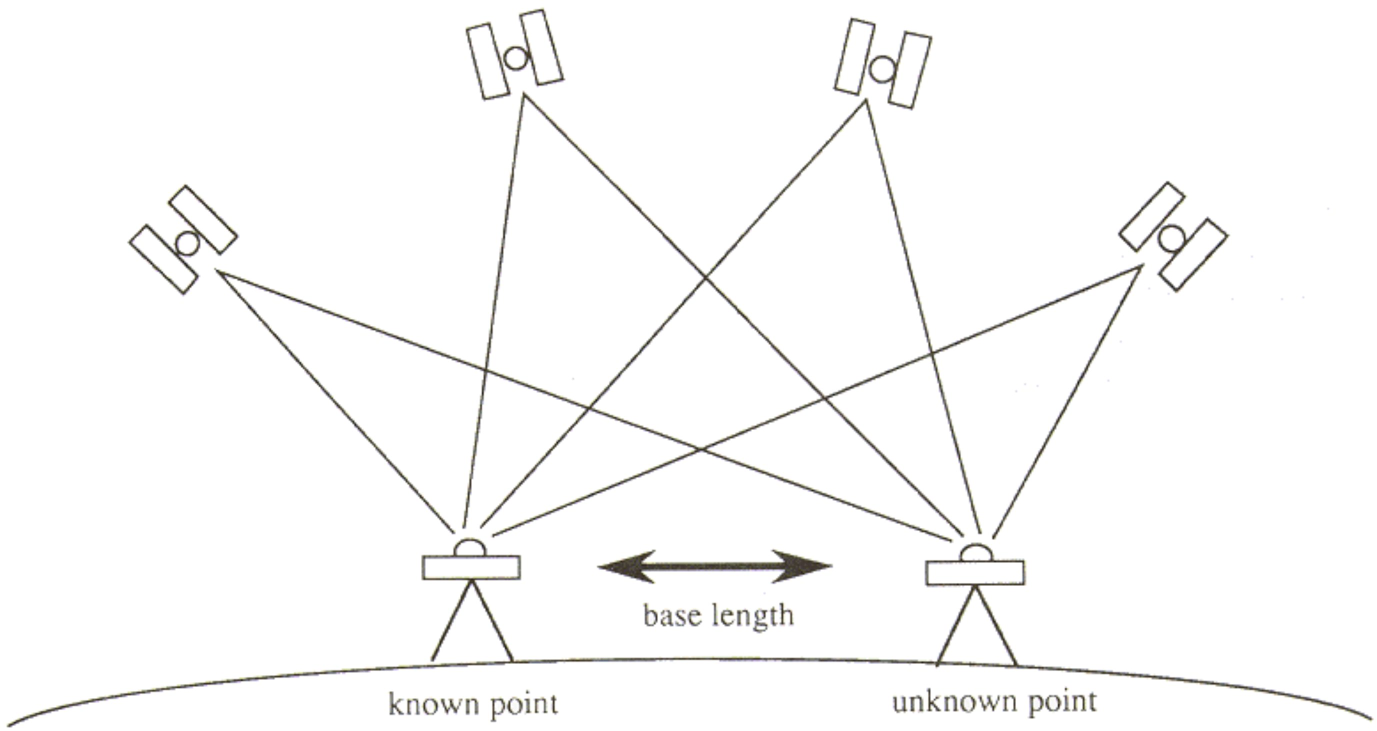


Figure 6.8.2 Relative positioning

GPS Post Processing

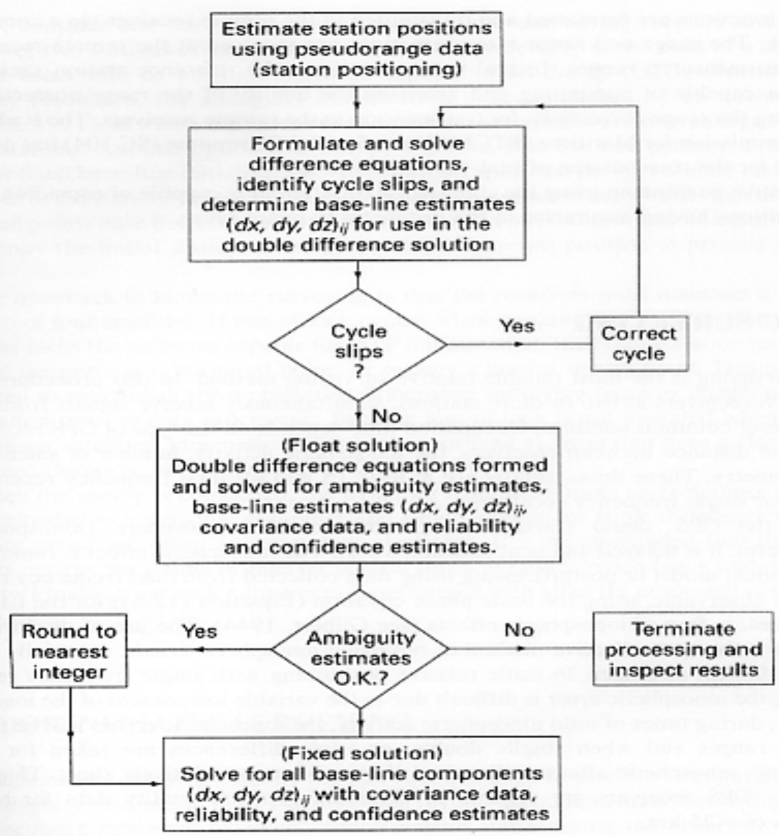
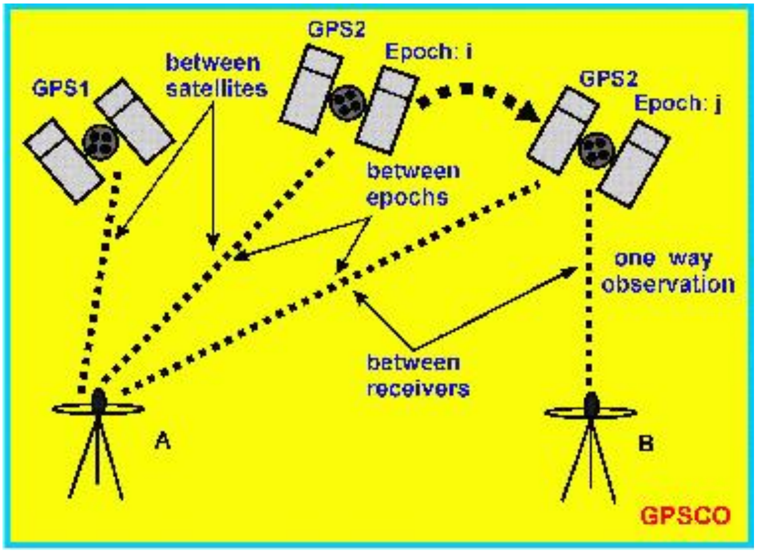


FIGURE 12.12
Data processing flowchart.

ere the conditions governing Equations (12.11) and (12.12) are illustrated in Fig-

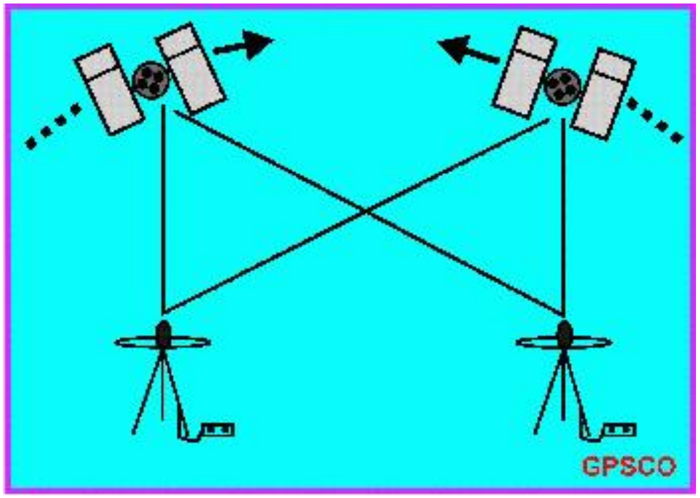
Linear Combinations of Observations

- Single Difference: Takes away some noise and clock bias
  - Between Receiver => Satellite clock bias
  - Between Satellite => Receiver clock bias
  - Between Epochs => Change of phase between two epochs



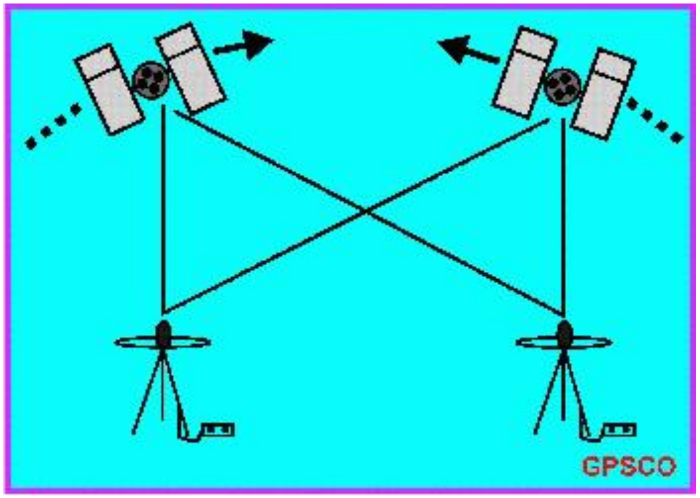
Linear Combinations of Observations

- Double Difference:
  - **Receiver-Time =>** the change from one epoch to the next in the between receiver single difference for the same satellite.
  - Allows for easier editing of cycle slips



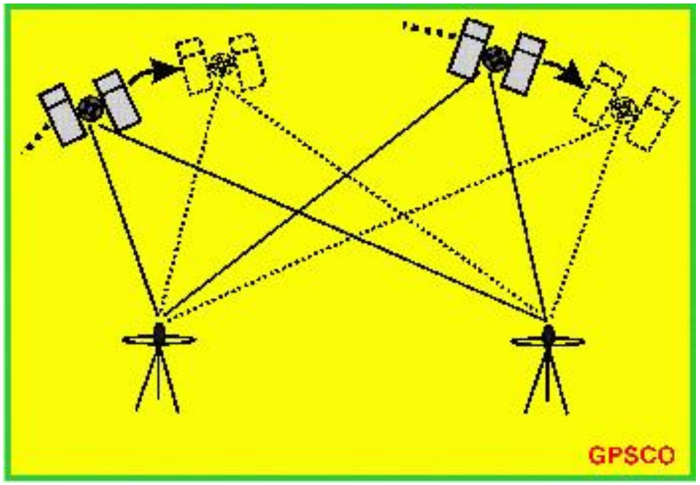
Linear Combinations of Observations

- Double Difference
  - **Receiver-Satellite** => Two receivers and two satellites at the same epoch
  - Removes or greatly reduces the effects of:
    - Receiver clock errors
    - Satellite clock errors



Linear Combinations of Observations

- Triple Difference:
  - **Receiver-Satellite-Time** => change in a receiver-satellite double difference from one epoch to the next.
    - Cycle ambiguity terms cancel allowing easier automatic editing of cycle slips.



Linear Combinations of Observations

- Most common approach is to difference:
  1. Between two receivers
  2. Between two satellites
  3. Between two epochs

Ephemerides

- Ephemeris data is a set of parameters that can be used to accurately calculate the location of a GPS satellite at a particular point in time. It describes the path that the satellite is following as it orbits Earth.
  - Broadcast: Included with the satellite message, based upon a least squared forecast, subject to degradation.
  - Precise: Based upon actual observation, available after observations made.

V. GPS Field Procedures

- Static
  - Traditional Static
  - Rapid Static
  - Re-occupation (pseudo-kinematic)
- Kinematic
  - Stop and Go
  - Traditional Kinematic
  - Real Time Differential (RTK)

Traditional Static Survey

- Method:
  - Base receiver (single or dual frequency) set over point of known (X,Y,Z) coordinates.
  - Additional receiver(s) placed over permanent station(s) to be positioned.
  - Observation times vary from 1 hour to several days depending upon accuracy required, length of vectors, satellite geometry, atmospheric conditions.
  - Post processing required.

Traditional Static Survey

- Uses
  - Long lines for geodetic control
  - Control densification
  - Precise engineering surveys

Rapid Static Survey

- Method
  - Uses dual frequency receivers over short (<15 km) lines.
  - Base station set over known point (X,Y,Z)
  - Rover moves from station to station with pole mounted antenna (no need to maintain lock).
  - Observation times of 5 to 10 minutes are typical.
  - Accuracies of a few millimeters achievable.
  - Requires post processing.

Re-occupation Survey (pseudo-kinematic)

- Method
  - Base station established on known (X,Y,Z) point.
  - Rovers (pole mounted) occupy remote stations for relatively short periods of time (10-15 minutes)
  - Remote stations are re-occupied when satellite geometry has changed (> 1 hour).
  - Solution can be strengthened by setting base station over second know point during re-occupation.
  - Useful when less than ideal number of satellites are available or when GDOP is weak.
  - Single or dual frequency receivers.
  - Post processing required.

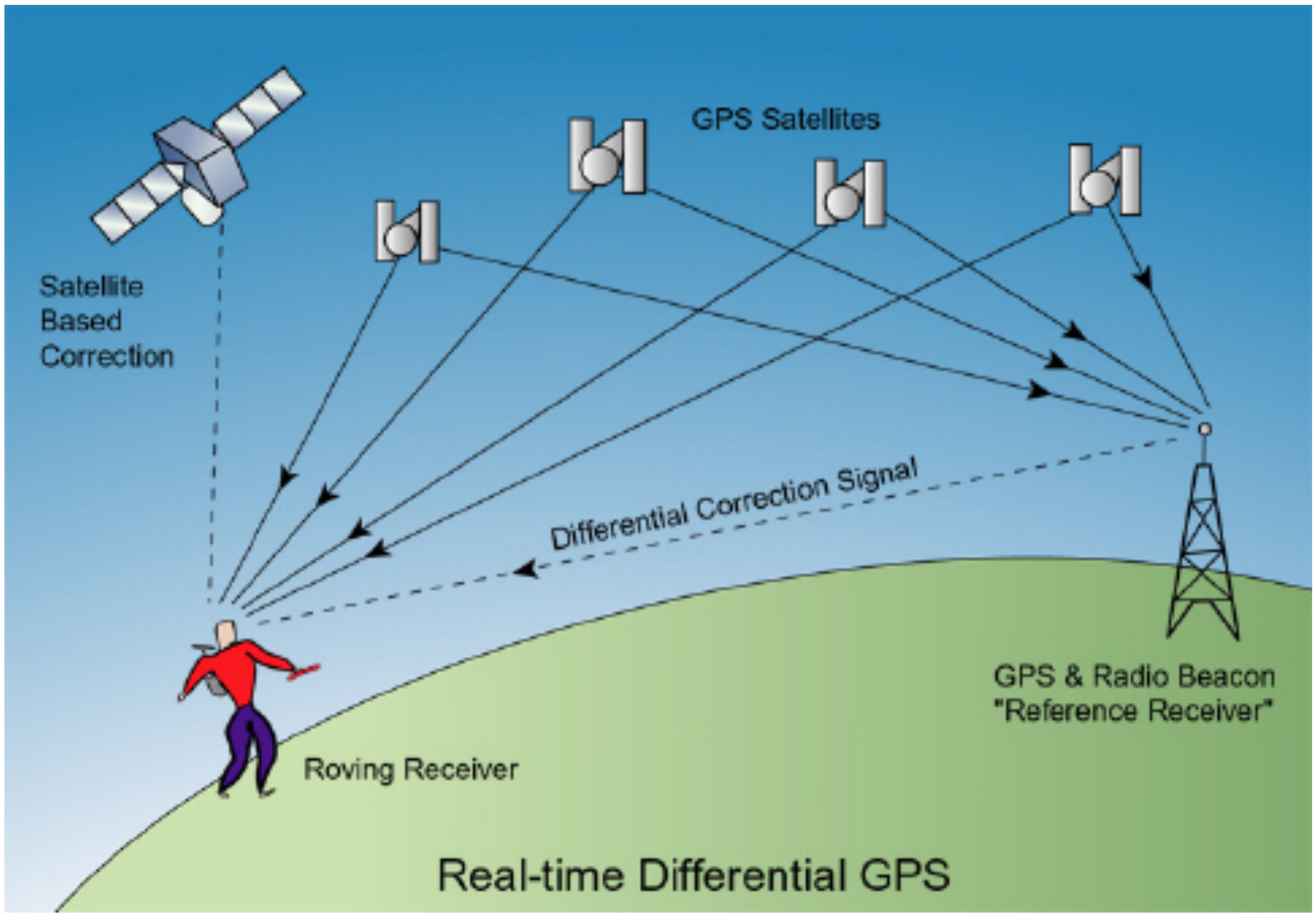
Stop and Go Survey

- Both base and rover receivers occupy known points less than 10km distant until ambiguities are resolved (5-10 minutes)
- Alternatively a short baseline (antenna cable length) with one known position can be used and the technique of “antenna swapping” employed to resolve ambiguities.
  - With the base station in static mode the rover moves to position all detail points.
  - Satellite lock must be maintained
  - Post processing required

Traditional Kinematic Survey

- Computes the relative differential position at preset time intervals instead of at operator selected points.
- Satellite lock must be maintained or must be reestablished when lost.
- Used for road profiling, ship and aircraft positioning.

Real Time Differential GPS (RTK)



Real Time Differential (RTK)

- Combines GPS receivers, mobile data communications, onboard data processing, onboard applications software.
- Base station receiver processes baseline corrections and broadcasts the corrections to any number of rovers via radio transmissions.
- Solution instantaneous with no post processing required.

GPS Observation Methods

Table 5-1
Carrier Phase Tracking Techniques

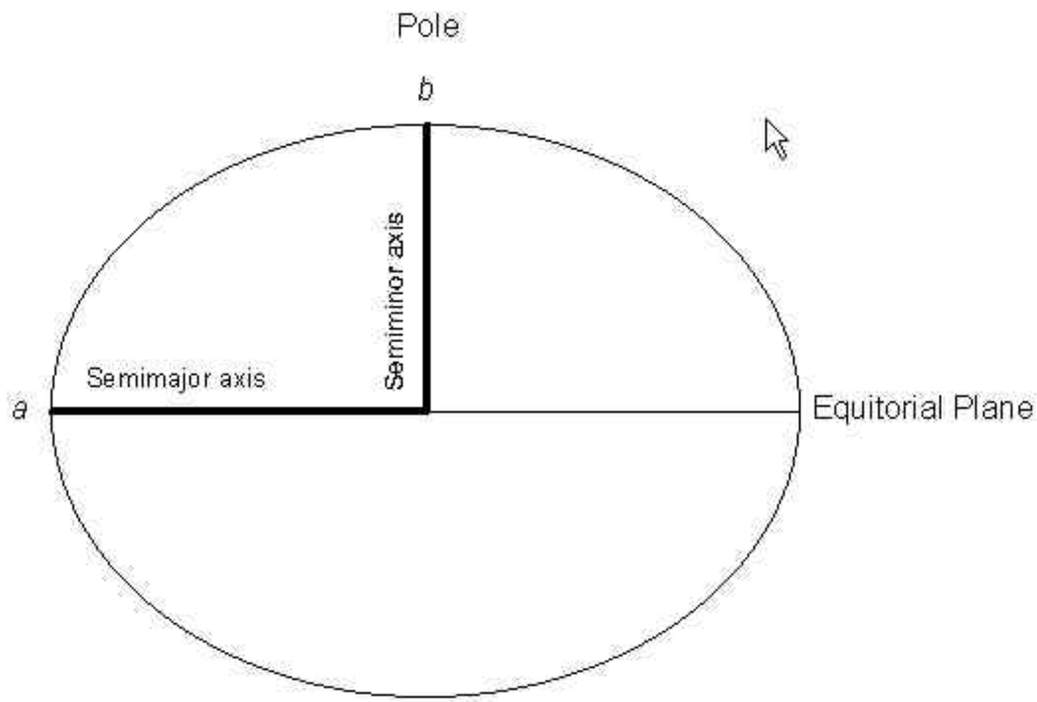
Method	Minimum Requirements	Applications	Accuracy
Classic Static (Post Processing)	• L1 or L1/L2 Receiver • 45 min to 1 hr minimum processing time	• Primary Control (L1/L2) • Secondary Control (L1 or L1/L2)	Sub-centimeter
Rapid Static (Post Processing)	• L1/L2 Receiver • 5 to 20 min observing time (1)	• Secondary Control • Photo Control	Sub-centimeter
Kinematic (2) (Post Processing)	• L1 Receiver with kinematic survey option	• Continuous Topo • Location Surveys	Centimeter
Stop and Go (Post Processing)	• L1 Receiver	• Secondary Control • Photo Control	Centimeter
Pseudo-Kinematic (3) (Post Processing)	• L1 Receiver • 2 5-10 minute periods separated by 1 hour	• Secondary Control • Photo Control	Centimeter
Real Time Kinematic (4) (Real Time or Post Processing)	For post processing • L1/L2 Receiver For real time • L1/L2 Receiver • Internal or External Processor • Min 4800 baud radio/modem data link	• Location Surveys • Continuous Topo	Sub-decimeter

(1) Dependant upon satellite constellation and number of satellites in view

VI. Ellipsoids, Datums, Coordinate Systems

Ellipsoid

flattening = f = (a-b)/a



Common Reference Ellipsoids

Selected Reference Ellipsoids

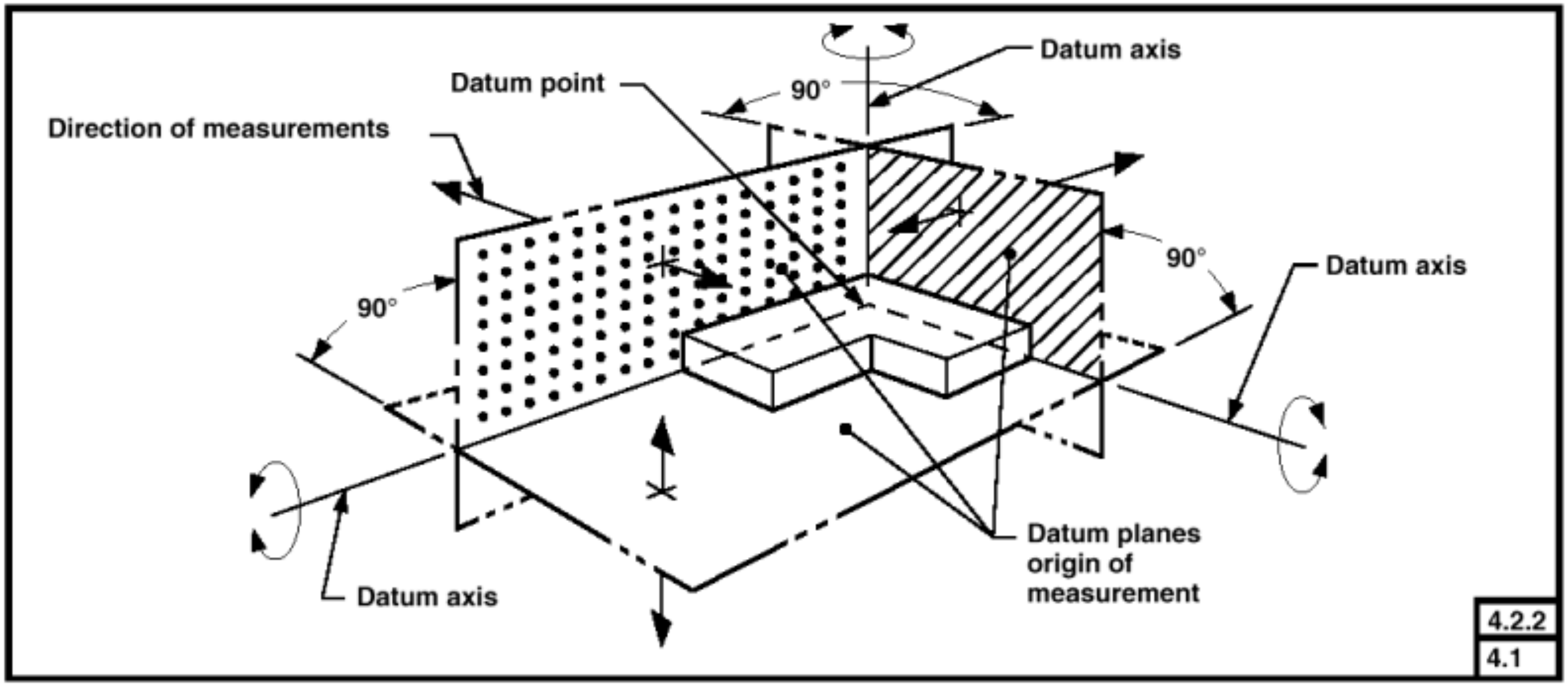
Ellipse	Semi-Major Axis (meters)	1/Flattening
Airy 1830	6377563.396	299.3249646
Bessel 1841	6377397.155	299.1528128
Clarke 1866	6378206.4	294.9786982
Clarke 1880	6378249.145	293.465
Everest 1830	6377276.345	300.8017
Fischer 1960 (Mercury)	6378166.0	298.3
Fischer 1968	6378150.0	298.3
G R S 1967	6378160.0	298.247167427
G R S 1975	6378140.0	298.257
G R S 1980	6378137.0	298.257222101
Hough 1956	6378270.0	297.0
International	6378388.0	297.0
Krassovsky 1940	6378245.0	298.3
South American 1969	6378160.0	298.25
WGS 60	6378165.0	298.3
WGS 66	6378145.0	298.25
WGS 72	6378135.0	298.26
WGS 84	6378137.0	298.257223563

Peter H. Dana 9/1/94

Datums

- A datum is a framework that enables us to define coordinate systems.
- The framework includes the ellipsoid and other parameters.
- The way a datum is defined has changed with the improvement of measurement techniques through technology.

Datums



Datum axis
Datum point
90°
Direction of measurements
90°
Datum axis
90°
Datum axis
Datum planes
origin of
measurement
4.2.2
4.1

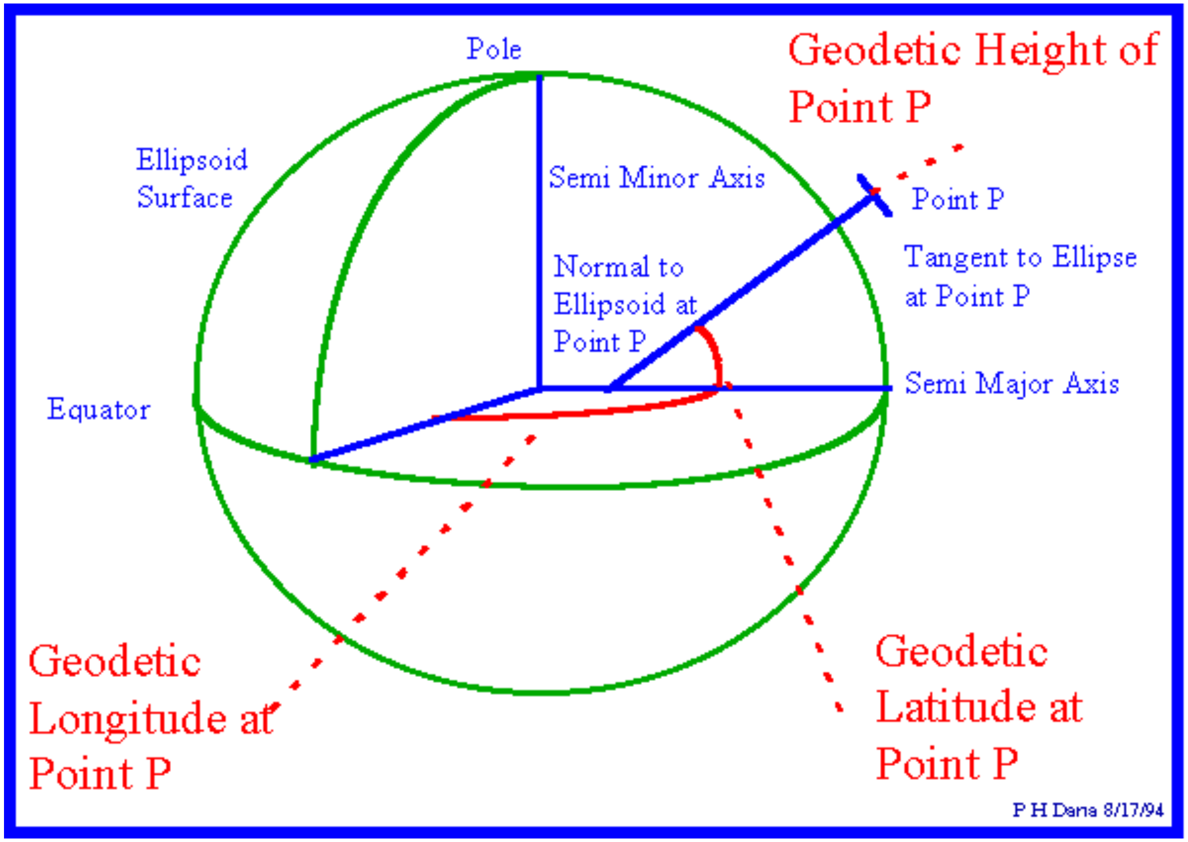
Datums

Table 3-1. Reference Ellipsoids and Related Coordinate Systems

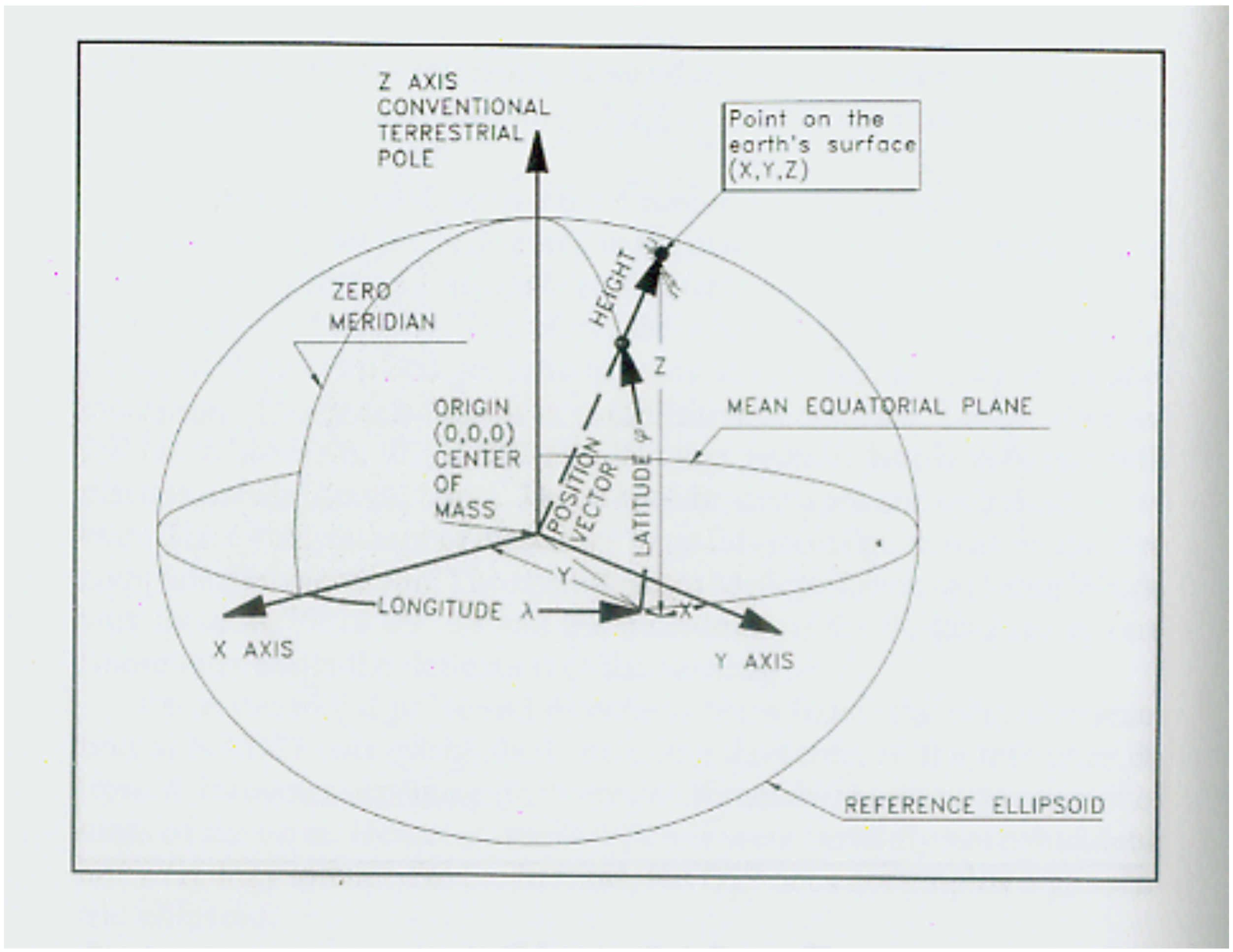
Reference Ellipsoid	Coordinate System (Datum/Frame)	Semimajor axis (meters)	Shape (1/flattening)
Clarke 1866	NAD 27	6378206.4	1/294.9786982
WGS 72	WGS 72	6378135	1/298.26
GRS 80	NAD 83 (XX)	6378137	1/298.257222101
WGS 84	WGS 84 (GXXX)	6378137	1/298.257223563
ITRS	ITRF (XX)	6378136.49	1/298.25645

Geodetic Coordinate Systems

Longitude, Latitude, Height

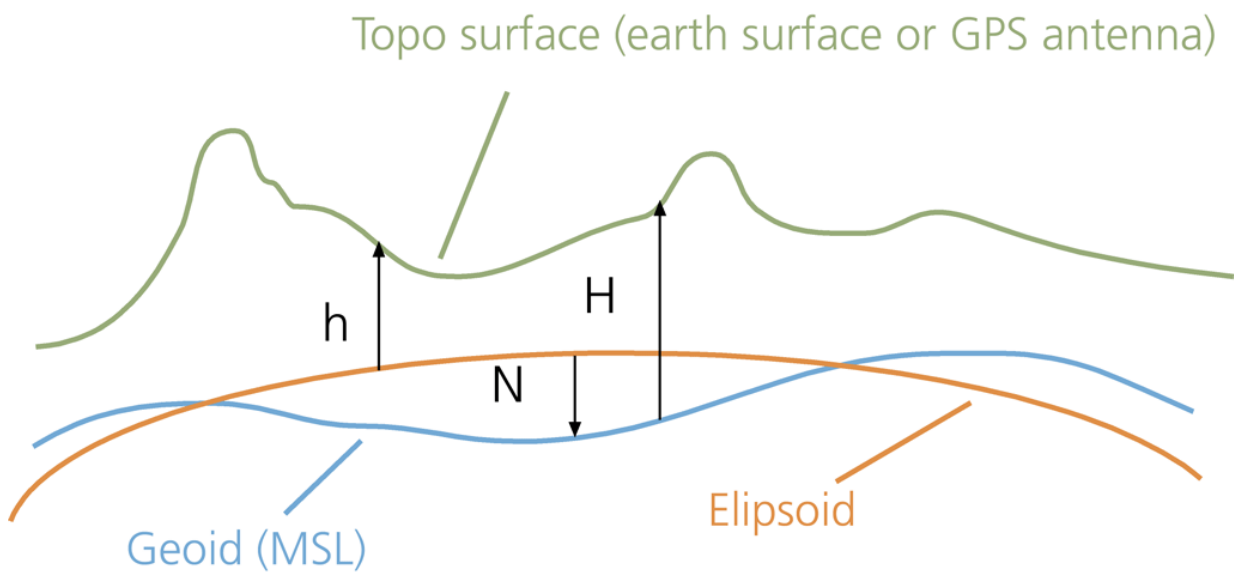


Geodetic Coordinate Systems
(Earth Centered Earth Fixed X,Y,Z)



GPS Heights

h=H+N



h=elipsoid height
H=orthometric height
N=geoid height

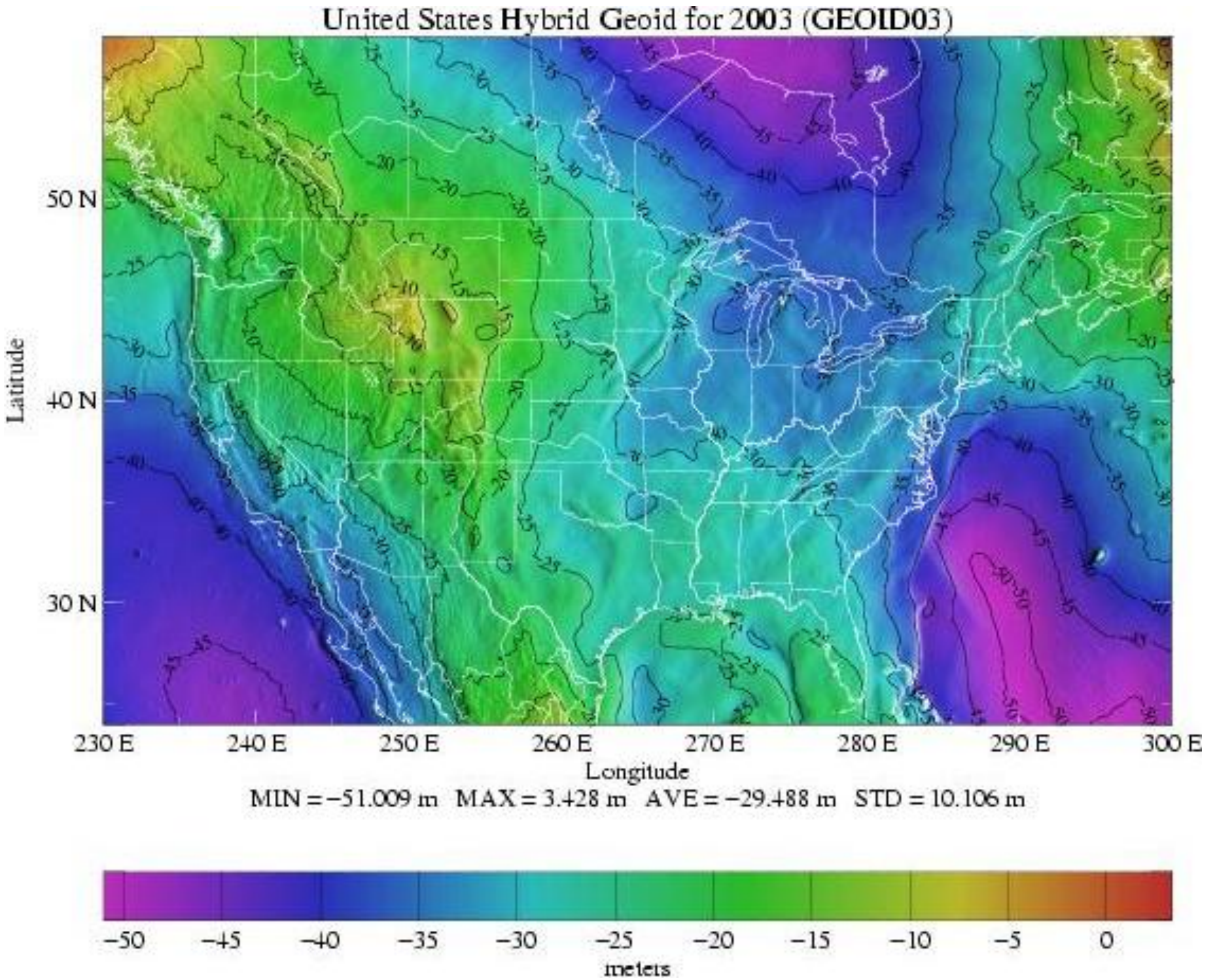
GPS Heights

- The geoid approximates mean sea level. The shape of the ellipsoid was calculated based on the hypothetical equipotential gravitational surface. A significant difference exists between this mathematical model and the real object. However, even the most mathematically sophisticated geoid can only approximate the real shape of the earth.

GPS Heights

- The accuracy of GPS height measurements depends on several factors but the most crucial one is the "imperfection" of the earth's shape.
- Height can be measured in two ways. The GPS uses height (h) above the reference ellipsoid that approximates the earth's surface.
- The traditional, orthometric height (H) is the height above an imaginary surface called the geoid, which is determined by the earth's gravity and approximated by MSL.
- The signed difference between the two heights—the difference between the ellipsoid and geoid—is the geoid height (N).

Geoid Models



Vertical Datums

- Until 1991 the official datum in the United States was the National Geodetic Vertical Datum of 1929 (NGVD29).
- The current datum is the North American Vertical Datum of 1988 (NAVD88).

NAVD 88

- Primary tidal benchmark is Father Point, Rimouski, Quebec.
- Contains leveling from the United States, Canada, Mexico, and includes the International Great Lakes Datum of 1985 (IGLD85)
- Differences between NGVD29 and NAVD88 in the continental United States ranges from -0.040m to 1.50m.

Coordinate Conversions

Coordinate Conversion
Geodetic Latitude, Longitude, and Height to ECEF, X, Y, Z

$$X = (N + h)\cos\phi\cos\lambda$$

$$Y = (N + h)\cos\phi\sin\lambda$$

$$Z = [N(1 - e^2) + h]\sin\phi$$

where:

ϕ, λ, h = geodetic latitude, longitude, and height above ellipsoid

X, Y, Z = Earth Centered Earth Fixed Cartesian Coordinates

and:

$N(\phi) = a / \sqrt{1 - e^2 \sin^2 \phi}$ = radius of curvature in prime vertical

a = semi-major earth axis (ellipsoid equatorial radius)

b = semi-minor earth axis (ellipsoid polar radius)

$f = \dfrac{a - b}{a}$ = flattening

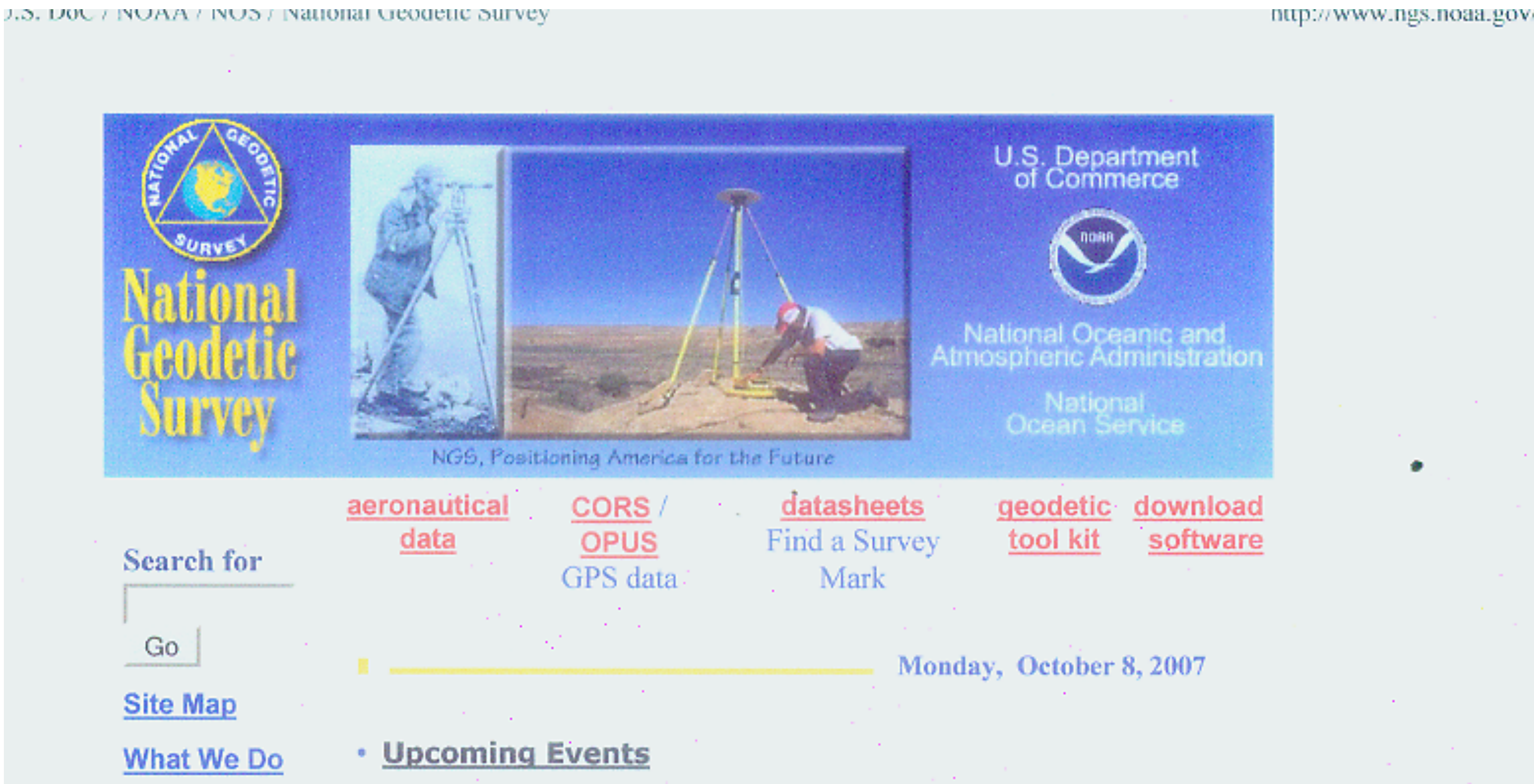
$e^2 = 2f - f^2$ = eccentricity squared

Peter H. Dana 8/3/96

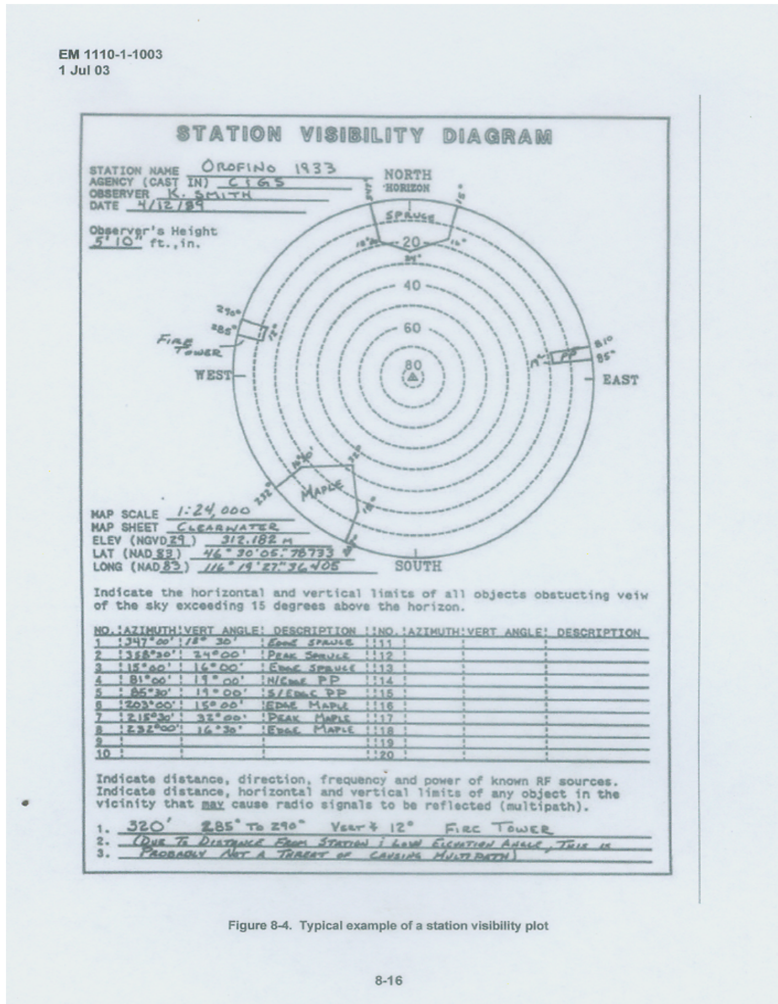
VI. GPS Mission Planning

- Purpose of Survey
- Horizontal and Vertical Accuracy Requirements
- Equipment Selection
- Point Selection
- Satellite Availability and Geometry
- Site Reconnaissance
- Surveying Scheme

Point Selection



Site Reconnaissance



EM 1110-1-1003
1 Jul 03

STATION VISIBILITY DIAGRAM

STATION NAME Orofino 1933
AGENCY (CAST IN) C&GS
OBSERVER K. Smith
DATE 4/12/89

Observer's Height
5'10" ft.,in.

MAP SCALE 1:24,000
MAP SHEET Clearwater
ELEV (NGVD 29) 312.182 m
LAT (NAD 83) 46° 30' 05." 78733
LONG (NAD 83) 116° 19' 27." 36405

Indicate the horizontal and vertical limits of all objects obstucting veiw of the sky exceeding 15 degrees above the horizon.

NO.	AZIMUTH	VERT ANGLE	DESCRIPTION	NO.	AZIMUTH	VERT ANGLE	DESCRIPTION
1	347°00'	18° 30'	Edge Spruce	11			
2	358°30'	24°00'	Peak Spruce	12			
3	15°00'	16°00'	Edge Spruce	13			
4	81°00'	19°00'	N/Edge PP	14			
5	85°30'	19°00'	S/Edge PP	15			
6	203°00'	15°00'	Edge Maple	16			
7	215°30'	32°00'	Peak Maple	17			
8	232°00'	16°30'	Edge Maple	18			
9				19			
10				20			

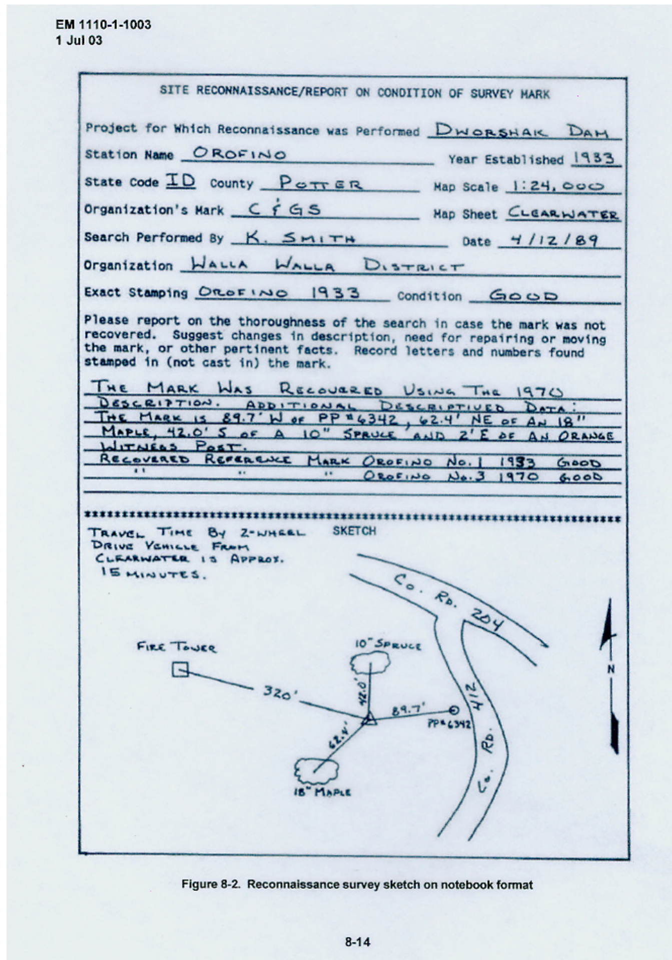
Indicate distance, direction, frequency and power of known RF sources. Indicate distance, horizontal and vertical limits of any object in the vicinity that may cause radio signals to be reflected (multipath).

1. 320' 285° to 290° Vert ∢ 12° Fire Tower
2. (Due to distance from station & low elevation angle, this is
3. probably not a threat of causing multipath)

Figure 8-4. Typical example of a station visibility plot

8-16

Site Reconnaissance



EM 1110-1-1003
1 Jul 03

SITE RECONNAISSANCE/REPORT ON CONDITION OF SURVEY MARK

Project for Which Reconnaissance was Performed DWORSHAK DAM

Station Name OROFINO Year Established 1933

State Code ID County POTTER Map Scale 1:24,000

Organization's Mark C & GS Map Sheet CLEARWATER

Search Performed By K. SMITH Date 4/12/89

Organization WALLA WALLA DISTRICT

Exact Stamping OROFINO 1933 Condition GOOD

Please report on the thoroughness of the search in case the mark was not recovered. Suggest changes in description, need for repairing or moving the mark, or other pertinent facts. Record letters and numbers found stamped in (not cast in) the mark.

THE MARK WAS RECOVERED USING THE 1970 DESCRIPTION. ADDITIONAL DESCRIPTIVED DATA: THE MARK IS 89.7' W OF PP #6342, 62.4' NE OF AN 18" MAPLE, 42.0' S OF A 10" SPRUCE AND 2' E OF AN ORANGE WITNESS POST.
RECOVERED REFERENCE MARK OROFINO No. 1 1933 GOOD
" " " OROFINO No. 3 1970 GOOD

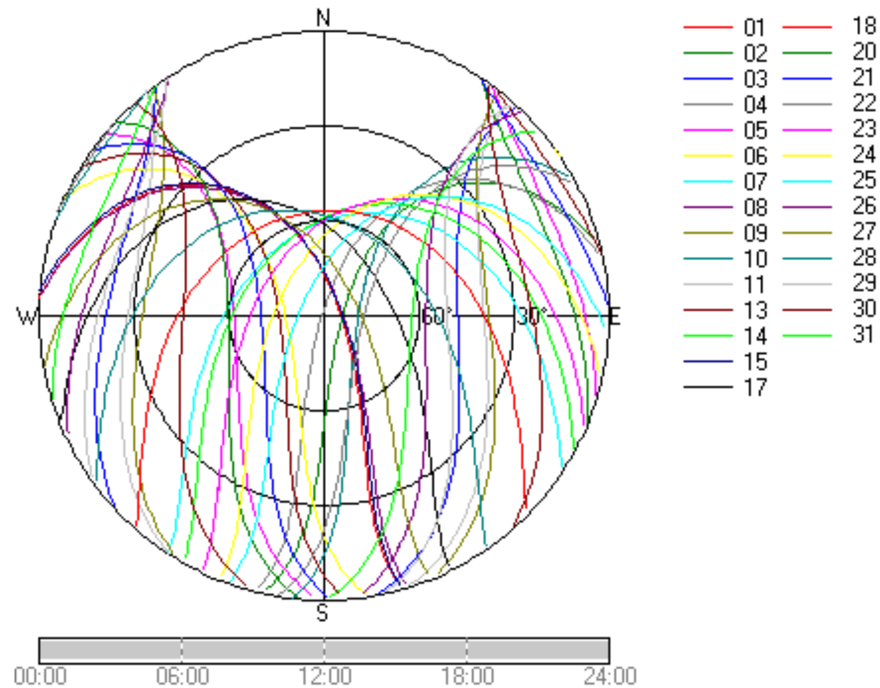
TRAVEL TIME BY 2-WHEEL DRIVE VEHICLE FROM CLEARWATER IS APPROX. 15 MINUTES.

SKETCH

Figure 8-2. Reconnaissance survey sketch on notebook format

8-14

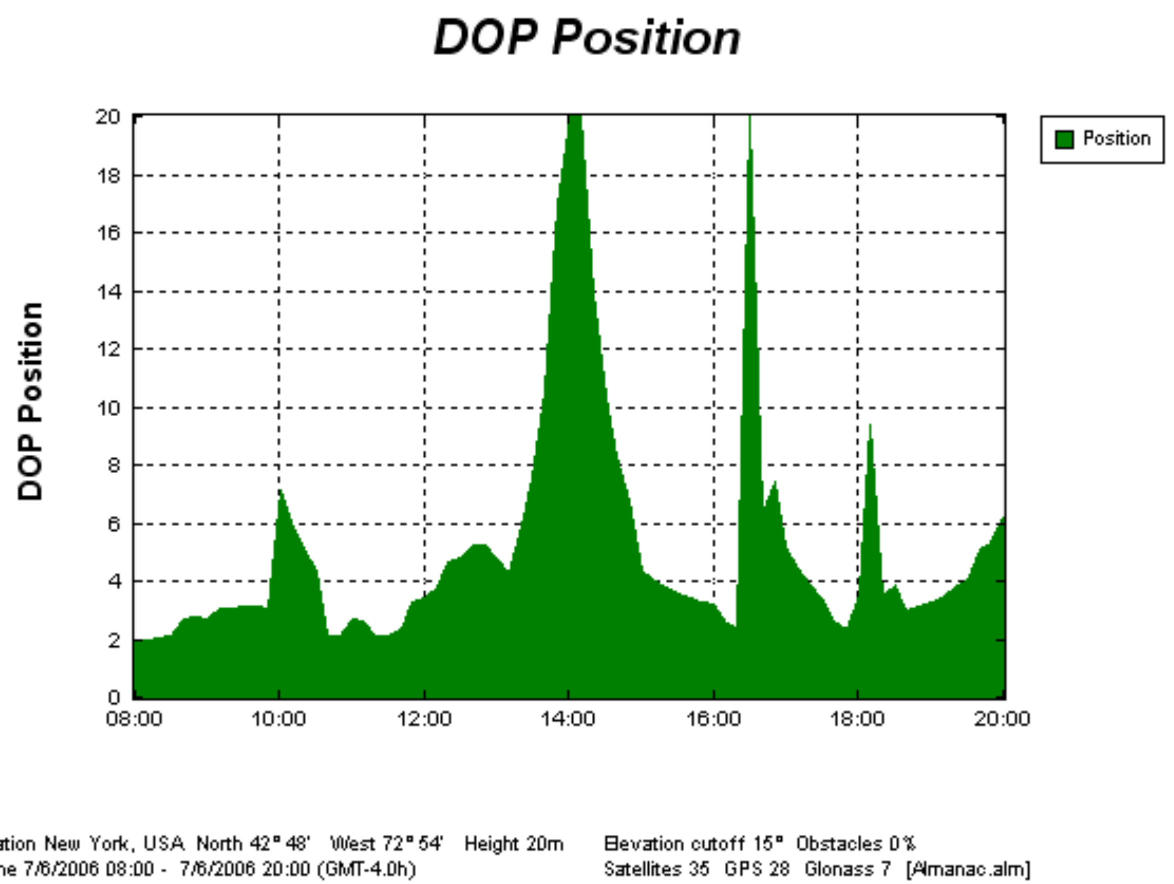
Satellite Availability



User Location:
Latitude: 29°48'10.257"N
Longitude: 097°54'04.49"W
Date: 27th January 2003

THALES ®

Dilution of Precision
(DOP)



Network Design

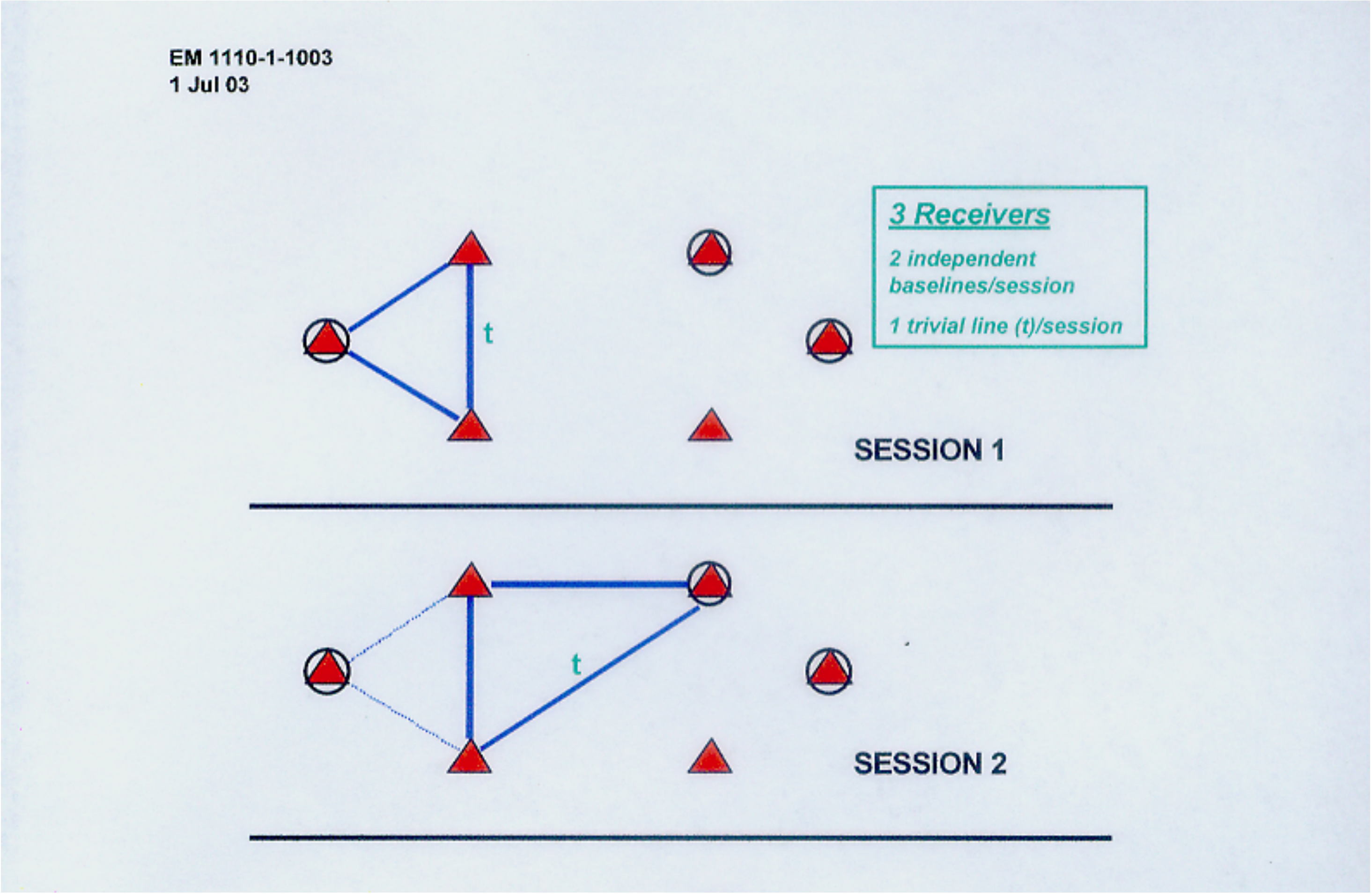
- Loops (i.e. traverses) made from GPS baseline observations provide the mechanism for performing field data validation as well as final adjustment accuracy analysis.
- Loops must not include trivial baselines.
- Design loops to meet closure requirements.
- Keep within the limits of your control.
- Avoid radial spurs at all costs. These contribute nothing and are impossible to assess qualitatively.

Surveying Scheme

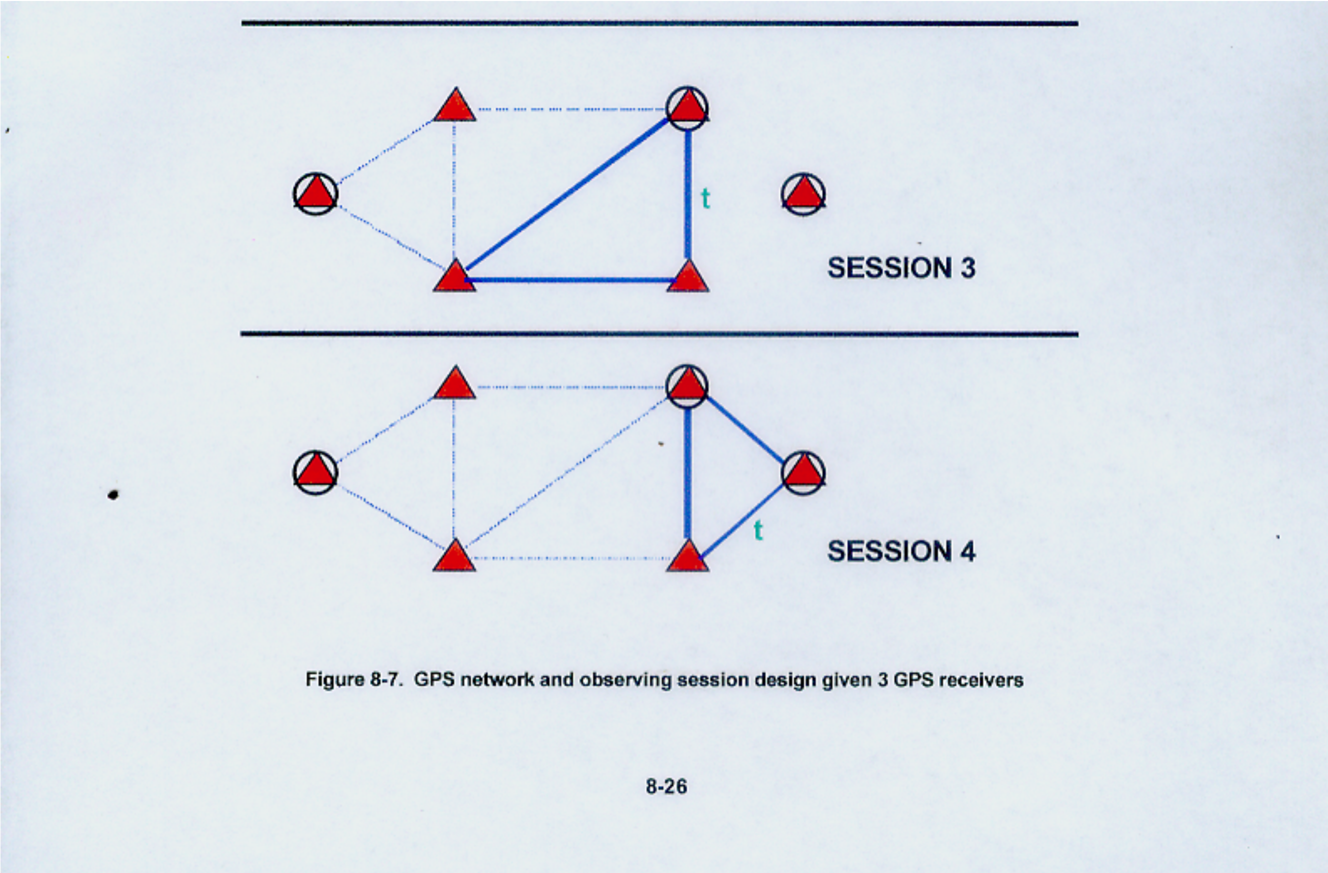
- Independent Baselines
  - Number of Baselines per session = n(n-1)/2
  - Independent Baselines per session = n-1

    where n = number of receivers deployed
- Example: 4 receivers deployed simultaneously
  - Number of Baselines = 4(4-1)/2 = 6 Baselines
  - Independent Baselines = 4-1 = 3 independent (non-trivial) baselines

Surveying Scheme



Surveying Scheme



Surveying Scheme

- Finding the Number of Sessions
- $S = (m*n)/r + [(m*n)(p-1)]/r + (k*m)$

S = the number of observing sessions

R = the number of receivers

m = the total number of stations involved

n = planned redundancy

p = production factor (varies by firm. Typical is 1.1)

k = safety factor (0.1 w/in 100 km of home base, 0.2 otherwise)

Network Adjustment

- After all the baselines have been processed and their reliabilities verified, they are adjusted within the framework of a network using least squares.
- Minimally constrained adjustment: One point held fixed with all other stations adjusted relative to it.
- Determines the quality of the observations.
- Constrained: Determines the quality of control.