The Hardest Problems on the 2018 AMC 8 are Nearly Identical to Previous Problems on the AMC 8, 10, 12, and MathCounts

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We developed a comprehensive, integrated, non-redundant, well-annotated database "**CMP**" consisting of various competitive math problems, including all previous problems on the AMC 8/10/12, AIME, MATHCOUNTS, Math Kangaroo Contest, **Math Olympiads for Elementary and Middle Schools (MOEMS),** ARML, USAMTS, Mandelbrot, Math League, Harvard–MIT Mathematics Tournament (HMMT), Princeton University Mathematics Competition (PUMaC), Stanford Math Tournament (SMT), Berkeley Math Tournament (BmMT), the Caltech Harvey Mudd Math Competition (CHMMC), the **Rice Math Tournament**, the **Carnegie Mellon** Informatics and **Mathematics Competition** (CMIMC), the **Australian Mathematics Competition, and the United Kingdom Mathematics Trust (UKMT)** . The **CPM** is an invaluable **"big data" system we use for our research and development, and is a golden resource for our students, who are the ultimate beneficiaries.**

Based on artificial intelligence (AI), machine learning, and deep learning, we also created a **data mining and predictive analytics tool for** math problem similarity searching. Using this powerful tool, we can align a set of query math problems against all in the database "**CPM**," and then detect those similar problems in the **CMP** database.

AMC 8
American Mathematics Competitions

Ivy League Education Center

The AMC 8 is a 25-question, 40-minute, multiple choice examination in middle school mathematics designed to promote the development and enhancement of problem solving skills. The problems generally increase in difficulty as the exam progresses. Usually the last 5 problems are the hardest ones.

Among the final 5 problems on the 2018 AMC 8 contest, there are 3 discrete math problems (which contains number theory and counting): **Problems 21, 23, and 25**; and there are 2 geometry problems: **Problems 22 and 24**.

For those hardest problems on the 2018 AMC 8, we found:

- **2018 AMC 8 Problem 21 is very similar to the following 9 problems:**
  - *1985 Australian Mathematics Competition Junior #23*
  - *2004 AMC 8 Problem 19*
  - *2012 AMC 8 Problem 15*
  - *2006 AMC 8 Problem 23*
  - *1951 AHSME #37*
  - *2010 Mathcounts State Sprint #8*
  - *2009 Mathcounts National Countdown #77*
  - *2009 Mathcounts School Sprint #19*
  - *2011-2012 MathCounts School Handbook #266*

- **2018 AMC 8 Problem 22 is very similar to the following 3 problems:**
  - *2016 AMC 10A Problem 19*
  - *1991AHSME Problem 23*
  - *2010MathCounts State Team Problem 10*

- **2018 AMC 8 Problem 23 is exactly the same as 2012 MathCounts State Sprint Problem 3, and very similar to the following 4 problems:**
  - *2017 MathCounts Chapter Countdown #49*
  - *2016 MathCounts National Sprint #11*
  - *2016 - 2017 MathCounts School Handbook Problems #195*
  - *2011 MathCounts State Countdown #22*

- **2018 AMC 8 Problem 24 is completely identical to the following 2 problems:**
  - *2008 AMC 10A Problem 21*
  - *2002 Mathcounts National Team Problem 10*

- **2018 AMC 8 Problem 25 is very similarly to the following 5 problems:**
  - *2013 Michigan Mathematics Prize Competition #1*
  - *2007 MathCounts State print #8*
  - *2007 MathCounts State Countdown #52*
  - *2009 MathCounts State Countdown #3*
  - *2010–2011 MathCounts School Handbook Workout 1 #5*

We can see that every problem has strong similarities to previous problems. Particularly, **2018 AMC 8 Problem 23 is exactly the same as 2012 MathCounts State Sprint Problem 3, and 2018 AMC 8 Problem 24 is completely identical to 2008 AMC 10A Problem 21.**

In my AMC 8/MathCounts Prep Class, I ever used Problem 3 on the 2012 MathCounts State Sprint, as a typical example, to elegantly solve discrete probability problems. **In my AMC 10/12 Prep Class** which there were 25 students at grades 4 to 8 to attend, **I ever took** Problem 21 **on**

the 2008 AMC 10A, as a classic example, to show the art of solving 3-D geometry problems. When my students attended the AMC 8 on Nov. 13, 2018, they already knew how to solve Problems 23 and 24 and their answers. So they took two seconds to bubble the correct answers and then got **2 points** easily!

Section 1. 2018 AMC 8 Problem 21

Problem 21

How many positive three-digit integers have a remainder of 2 when divided by 6, a remainder of 5 when divided by 9, and a remainder of 7 when divided by 11?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

This problem is very similar to the following 9 problems:

- **1985 Australian Mathematics Competition Junior #23**
- **2004 AMC 8 Problem 19**
- **2012 AMC 8 Problem 15**
- **2006 AMC 8 Problem 23**
- **1951 AHSME #37**
- **2010 Mathcounts State Sprint #8**
- **2009 Mathcounts National Countdown #77**
- **2009 Mathcounts School Sprint #19**
- **2011-2012 MathCounts School Handbook #266**

1985 Australian Mathematics Competition Junior #23

Find the smallest positive integer which, when divided by 6, gives a remainder of 1 and when divided by 11, gives a remainder of 6.

2004 AMC 8 Problem 19

A whole number larger than 2 leaves a remainder of 2 when divided by each of the numbers $3, 4, 5,$ and 6. The smallest such number lies between which two numbers?

(A) 40 and 49 **(B)** 60 and 79 **(C)** 100 and 129 **(D)** 210 and 249 **(E)** 320 and 369

2012 AMC 8 Problem 15

The smallest number greater than 2 that leaves a remainder of 2 when divided by 3, 4, 5, or 6 lies between what numbers?

(A) 40 and 50 **(B)** 51 and 55 **(C)** 56 and 60 **(D)** 61 and 65 **(E)** 66 and 99

2006 AMC 8 Problem 23

A box contains gold coins. If the coins are equally divided among six people, four coins are left over. If the coins are equally divided among five people, three coins are left over. If the box holds the smallest number of coins that meets these two conditions, how many coins are left when equally divided among seven people?

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 5

1951 AHSME #37

A number which when divided by 10 leaves a remainder of 9, when divided by 9 leaves a remainder of 8, by 8 leaves a remainder of 7, etc., down to where, when divided by 2, it leaves a remainder of 1, is:

(A) 59 (B) 419 (C) 1259 (D) 2519 (E) none of these answers

2010 Mathcounts State Sprint #8

The integer m is between 30 and 80 and is a multiple of 6. When m is divided by 8, the remainder is 2. Similarly, when m is divided by 5, the remainder is 2. What is the value of m?

2009 Mathcounts National Countdown #77

What is the smallest positive integer that has a remainder of 1 when divided by 2, a remainder of 2 when divided by 3 and a remainder of 4 when divided by 5?

2009 Mathcounts School Sprint #19

What is the smallest whole number that has a remainder of 1 when divided by 4, a remainder of 1 when divided by 3, and a remainder of 2 when divided by 5?

2011-2012 MathCounts School Handbook #266

What is the smallest positive integer that is greater than 100 and leaves a remainder of 1 when divided by 3, a remainder of 2 when divided by 5 and a remainder of 3 when divided by 7?

New Problems

Based on **Problem 21**, we raise the following new problems.

New Problem 21. 1.

How many positive 4-digit integers have a remainder of 2 when divided by 6, a remainder of 5 when divided by 9, and a remainder of 7 when divided by 11?

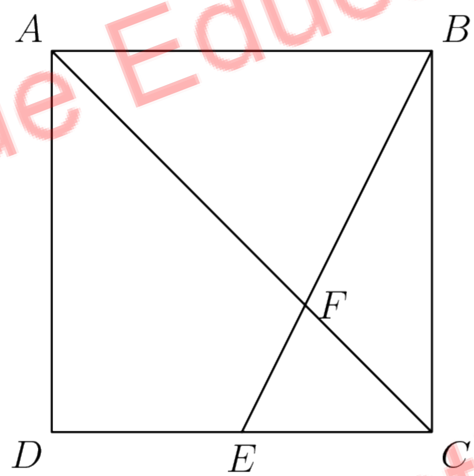
New Problem 21. 2.

How many positive 5-digit integers have a remainder of 2 when divided by 6, a remainder of 5 when divided by 9, a remainder of 7 when divided by 11, and a remainder of 9 when divided by 13?

Section 2. 2018 AMC 8 Problem 22

2018 AMC 8 Problem 22

Point E is the midpoint of side $\overline{CD}$ in square $ABCD$, and $\overline{BE}$ meets diagonal $\overline{AC}$ at F. The area of quadrilateral $AFED$ is 45. What is the area of $ABCD$?



$\textbf{(A)}\ 100 \qquad \textbf{(B)}\ 108 \qquad \textbf{(C)}\ 120 \qquad \textbf{(D)}\ 135 \qquad \textbf{(E)}\ 144$

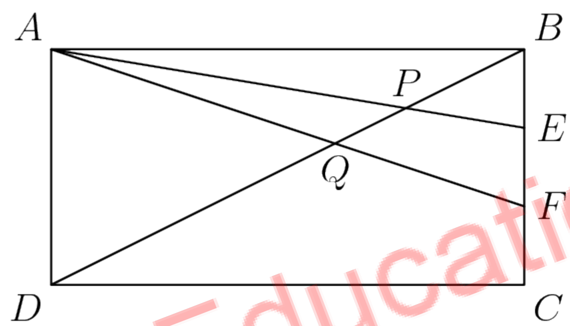
This problem is very similar to the following 3 problems:

- *2016 AMC 10A Problem 19*
- *1991AHSME Problem 23*
- *2010MathCounts State Team Problem 10*

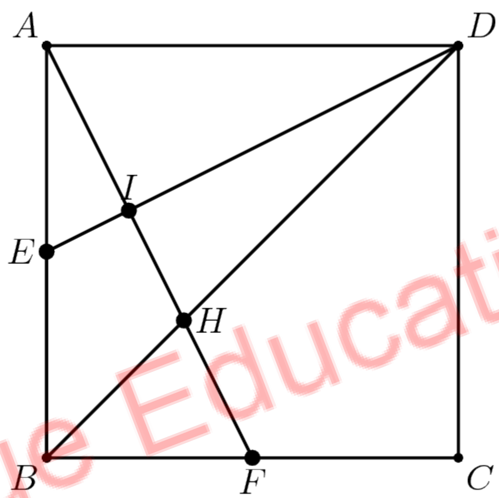
2016 AMC 10A Problem 19

In rectangle $ABCD$, $AB = 6$ and $BC = 3$. Point E between B and C, and point F between E and C are such that $BE = EF = FC$. Segments $\overline{AE}$ and $\overline{AF}$ intersect $\overline{BD}$ at P and Q, respectively. The ratio $BP{:}\,PQ : QD$ can be written as $r : s : t$ where the greatest common factor of r, s, and t is 1. What is $r + s + t$?

$\textbf{(A)}\ 7 \qquad \textbf{(B)}\ 9 \qquad \textbf{(C)}\ 12 \qquad \textbf{(D)}\ 15 \qquad \textbf{(E)}\ 20$



1991 AHSME #23



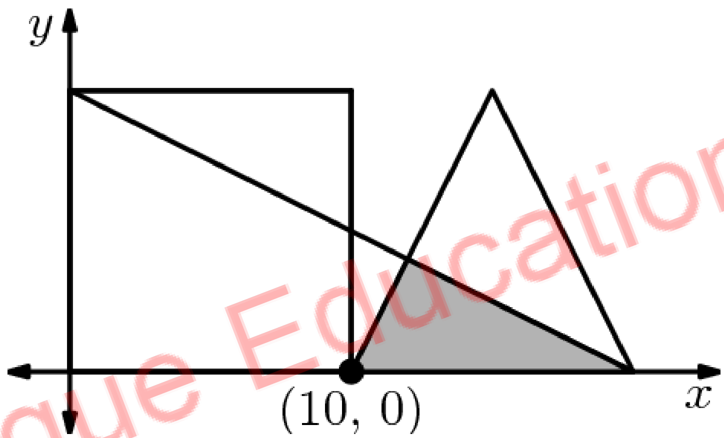
Ivy League Education Center

If $ABCD$ is a 2×2 square, E is the midpoint of $\overline{AB}$, F is the midpoint of $\overline{BC}$, $\overline{AF}$ and $\overline{DE}$ intersect at I, and $\overline{BD}$ and $\overline{AF}$ intersect at H, then the area of quadrilateral $BEIH$ is

(A) $\frac{1}{3}$ (B) $\frac{2}{5}$ (C) $\frac{7}{15}$ (D) $\frac{8}{15}$ (E) $\frac{3}{5}$

2010 MathCounts State Team #10

A square and isosceles triangle of equal height are side-by-side, as shown, with both bases on the x-axis. The lower right vertex of the square and the lower left vertex of the triangle are at $(10, 0)$. The side of the square and the base of the triangle on the x-axis each equal 10 units. A segment is drawn from the top left vertex of the square to the farthest vertex of the triangle, as shown. What is the area of the shaded region?

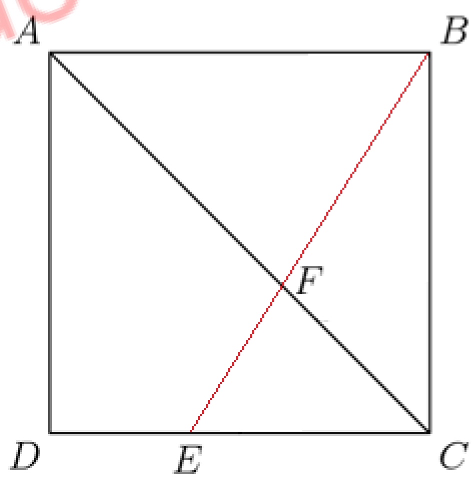


New Problems

Based on **Problem 22**, we propose the following new problems.

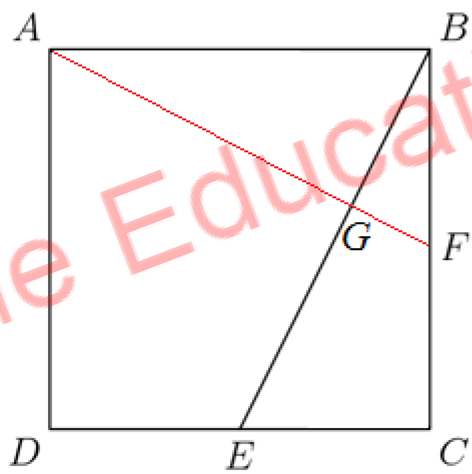
New Problem 22. 1.

Point E is a point of side $\overline{CD}$ in square $ABCD$ such that $\frac{DE}{EC} = \frac{m}{n}$, where m and n are relatively prime positive integers. $\overline{BE}$ meets diagonal $\overline{AC}$ at F. The area of quadrilateral $AFED$ is 45 What is the area of $ABCD$?



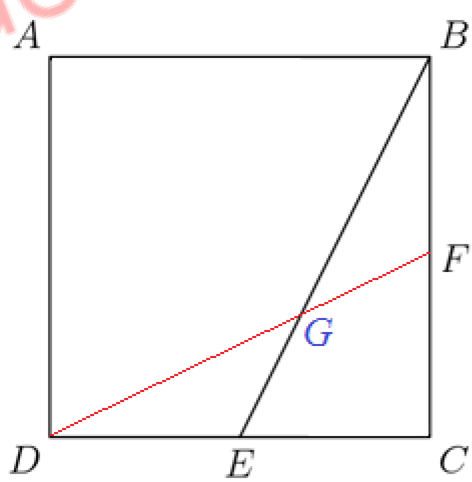
New Problem 22. 2.

Point E is a point of side $\overline{CD}$ and point F is a point of side $\overline{BC}$ in square $ABCD$. $\overline{BE}$ meets $\overline{AF}$ at G. The area of quadrilateral $AGED$ is 45 What is the area of $ABCD$?



New Problem 22. 3.

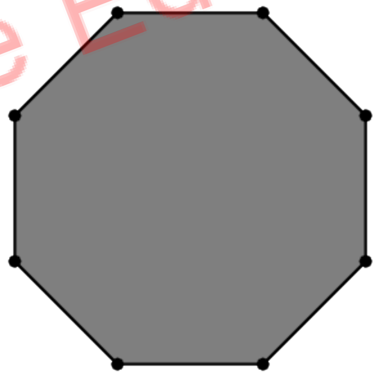
Point E is a point of side $\overline{CD}$ and point F is a point of side $\overline{BC}$ in square $ABCD$. $\overline{BE}$ meets $\overline{DF}$ at G. The area of quadrilateral $ABGD$ is 45 What is the area of $ABCD$?



Section 3. 2018 AMC 8 Problem 23

Problem 23

From a regular octagon, a triangle is formed by connecting three randomly chosen vertices of the octagon. What is the probability that at least one of the sides of the triangle is also a side of the octagon?



(A) $\frac{2}{7}$ **(B)** $\frac{5}{42}$ **(C)** $\frac{11}{14}$ **(D)** $\frac{5}{7}$ **(E)** $\frac{6}{7}$

Problem 23 is exactly the same as 2012 MathCounts State Sprint Problem 3, and very similar to the following 5 problems:

- *2012 MathCounts State Sprint #3*
- *2017 MathCounts Chapter Countdown #49*
- *2016 MathCounts National Sprint #11*
- *2016 - 2017 MathCounts School Handbook Problems #195*
- *2011 MathCounts State Countdown #22*

2012 MathCounts State Sprint #3

Three unique points are chosen at random from the vertices of a convex octagon. What is the probability that they form a triangle, none of whose sides is a side of the octagon? Express your answer as a common fraction.

2017 MathCounts Chapter Countdown #49

Three vertices of a regular octagon are chosen at random to form the vertices of a triangle. What is the probability that such a triangle is isosceles? Express your answer as a common fraction.

2016 MathCounts National Sprint #11

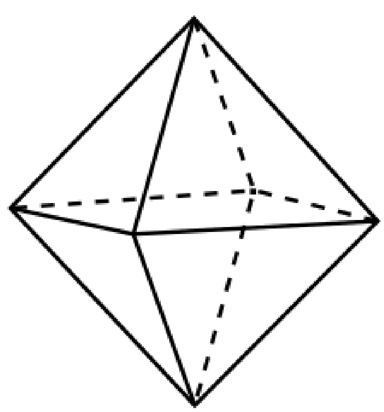
Three distinct vertices of a regular hexagon are chosen at random and a triangle is formed by joining the three vertices. What is the probability that the triangle formed is a right triangle? Express your answer as a common fraction.

2016 - 2017 MathCounts School Handbook Problems #195

Four vertices of a regular octagon are chosen at random. What is the probability that a square can be made by connecting the vertices? Express your answer as a common fraction.

2011 MathCounts State Countdown #22

Two of the vertices of a regular octahedron are to be chosen at random. What is the probability that they will be the endpoints of an edge of the octahedron?

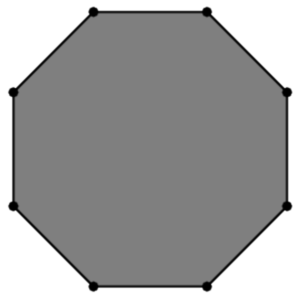


New Problems

Based on **Problem 23**, we propose the following new problems.

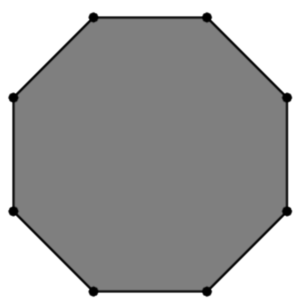
New Problem 23. 1.

From a regular octagon, a quadrilateral is formed by connecting three randomly chosen vertices of the octagon. What is the probability that at least one of the sides of the quadrilateral is also a side of the octagon?



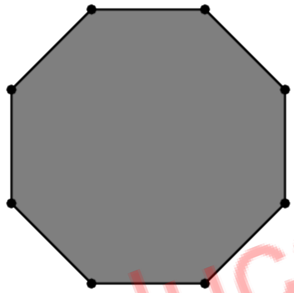
New Problem 23. 2.

From a regular octagon, a convex quadrilateral is formed by connecting three randomly chosen vertices of the octagon. What is the probability that at least one of the sides of the convex quadrilateral is also a side of the octagon?



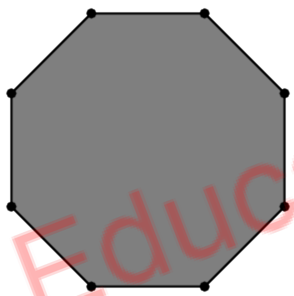
New Problem 23. 3.

From a regular octagon, a pentagon is formed by connecting three randomly chosen vertices of the octagon. What is the probability that at least one of the sides of the pentagon is also a side of the octagon?



New Problem 23. 4.

From a regular octagon, a convex pentagon is formed by connecting three randomly chosen vertices of the octagon. What is the probability that at least one of the sides of the convex pentagon is also a side of the octagon?



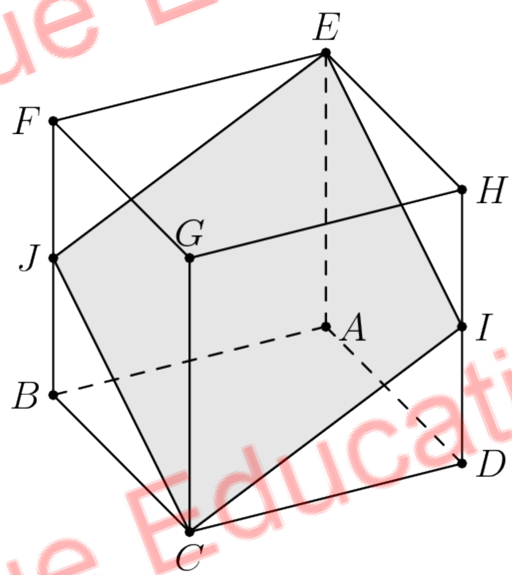
New Problem 23. 5.

From a convex regular n-gon, a triangle is formed by connecting three randomly chosen vertices of the n-gon. What is the probability that at least one of the sides of the triangle is also a side of the n-gon?

Section 4. 2018 AMC 8 Problem 24

2018 AMC 8 Problem 24

In the cube $ABCDEFGH$ with opposite vertices C and E, J and I are the midpoints of edges $\overline{FB}$ and $\overline{HD}$, respectively. Let R be the ratio of the area of the cross-section $EJCI$ to the area of one of the faces of the cube. What is R^2?



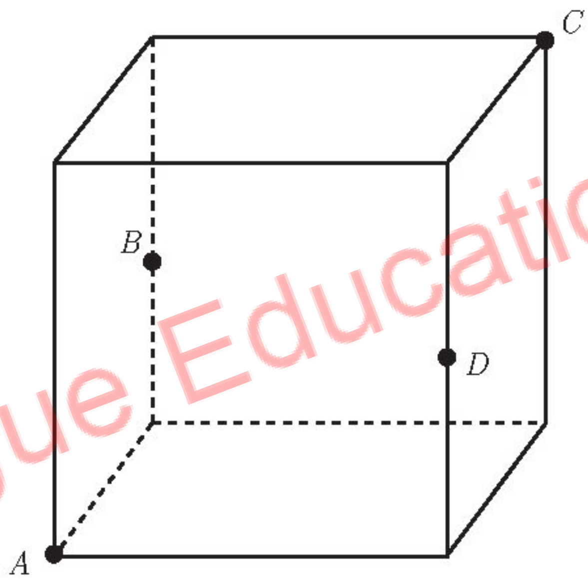
(A) $\frac{5}{4}$ **(B)** $\frac{4}{3}$ **(C)** $\frac{3}{2}$ **(D)** $\frac{25}{16}$ **(E)** $\frac{9}{4}$

This problem **is nearly identical to the following 2 problems:**

- *2008 AMC 10A Problem 21*
- *2002 Mathcounts National Team Problem 10*

2008 AMC 10A #21

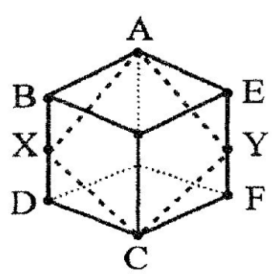
A cube with side length 1 is sliced by a plane that passes through two diagonally opposite vertices A and C and the midpoints B and D of two opposite edges not containing A or C, as shown. What is the area of quadrilateral $ABCD$?



(A) $\frac{\sqrt{6}}{2}$ (B) $\frac{5}{4}$ (C) $\sqrt{2}$ (D) $\frac{5}{8}$ (E) $\frac{3}{4}$

2002 Mathcounts National Team #10

A cube with edge length 6 inches is sliced by a plane to create quadrilateral $AXCY$ where X and Y are the midpoints of segments BD and BF, respectively. What is the number of square inches in the area of $AXCY$? Express your answer as a decimal to the nearest tenth.

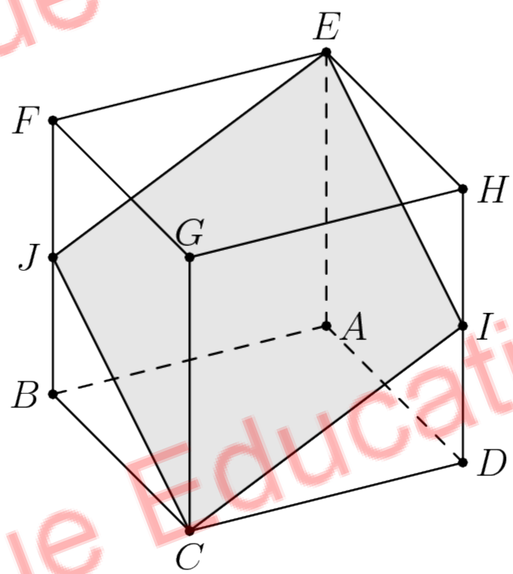


New Problems

Based on **Problem 24**, we propose the following new problems.

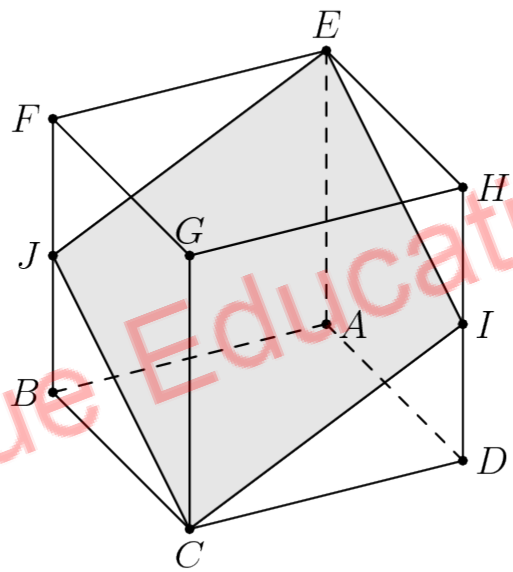
New Problem 24. 1.

In the cube $ABCDEFGH$ with opposite vertices C and E, J and I are the midpoints of edges $\overline{FB}$ and $\overline{HD}$, respectively. Let R be the ratio of the area of the cross-section $EJCI$ to the area of a circle inscribed in $EJCI$. What is R^2?



New Problem 24. 2.

In the rectangular prism $ABCDEFGH$ with opposite vertices C and E, J and I are the midpoints of edges $\overline{FB}$ and $\overline{HD}$, respectively. Let $AB = x, BC = y,$ and $z = AE$. What is the area of the cross-section $EJCI$?



New Problem 24. 3.

In the cube $ABCDEFGH$ with opposite vertices C and E, J and I are the midpoints of edges $\overline{FB}$ and $\overline{HD}$, respectively. Find a plane through the center of the cube that yields a cross-section area larger than the cross-section $EJCI$. Which plane through the center of the cube yields the cross-section of largest area of all? (And what is that largest area?) Which plane through the center yields a cross-section of smallest area?

Section 5. 2018 AMC 8 Problem 25

2018 AMC 8 Problem 25

How many perfect cubes lie between $2^8 + 1$ and $2^{18} + 1$, inclusive?

(A) 4 **(B)** 9 **(C)** 10 **(D)** 57 **(E)** 58

2018 AMC 8 Problem 25 is very similar to the following 5 problems:

- *2013 Michigan Mathematics Prize Competition #1*
- *2007 MathCounts State print #8*
- *2007 MathCounts State Countdown #52*
- *2009 MathCounts State Countdown #3*
- *2010–2011 MathCounts School Handbook Workout 1 #5*

2013 Michigan Mathematics Prize Competition #1

How many three-digit numbers are perfect cubes?

A) 5 B) 6 C) 7 D) 8 E) 9

2009 MathCounts State Countdown #3

How many perfect squares are there between 20 and 150?

2007 MathCounts State print #8

How many perfect squares less than1000 have a ones digit of 2, 3, or 4?

2007 MathCounts State Countdown #52

How many perfect squares have a value between 10 and 1000?

2010–2011 MathCounts School Handbook Workout 1 #5

How many positive three-digit perfect cubes are even?

New Problems

Based on **Problem 25**, we propose the following new problems.

New Problem 25. 1.

How many perfect squares lie between $2^8 + 1$ and $2^{18} + 1$, inclusive?

New Problem 25. 2.

How many perfect cubes lie between $2^8 + 1$ and $2^{2018} + 1$, inclusive?

New Problem 25. 3.

How many perfect cubes lie between $3^8 + 1$ and $3^{2018} + 1$, inclusive?

New Problem 25. 4.

How many perfect cubes lie between $5^8 + 1$ and $5^{2018} + 1$, inclusive?

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Section 6. Conclusions

This year's AMC 8 was much more difficult than the last year's AMC 8. Some hard problems were even at the AMC 10 level. For example, Problems 21, 22, and 24 on the 2018 AMC 8 are three typical hard level AMC 10 problems.

Because the AMC 8 problems are getting harder, we must practice not only previous AMC 8 problems but also easy, medium, and even high difficulty level problems from previous AMC 10/12 to do well on the AMC 8.