THE HISTORY OF THE THEORY OF STRUCTURES.*

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INTRODUCTION.

I propose in this address to invite your consideration of some of the principles upon which our present day structural design is based and to remember some of the scientists of bygone days to whom we are indebted for the discovery of these principles.

In these days of industrial and government subsidised research we do well to remember the army of famous men who laid the foundations of engineering science quite apart from and often oblivious to the possible commercial application of their researches.

We shall find that the principles of design that we use so freely in modern structures are the result of contributions made by men of different countries since the middle of the 17th century ; in Italy, Switzerland, Germany, France, Holland, Great Britain and the United States of America—a league of nations in the truest sense, for science knows no barriers of race or clime.

(1) ELASTICITY.

ELASTICITY

Robert Hooke	(1635-1702)
Hooke's Law	1678
The Bernouillis	18th Century
Dr. Thomas Young	(1773-1829)
Young's Modulus	1807
Resilience	1807

Robert Hooke (1635-1702). Hooke's Law of Elasticity is one of the basic principles of design of structures in which we employ materials which more or less follow this Law.

Robert Hooke was born at Freshwater, Isle of Wight, in 1635 and for the first seven years of his life was so weakly that it was doubtful whether he would grow to manhood. In spite of this early setback he lived to the age of 67 and became one of the most versatile contributors to the proceedings of the Royal Society in London. Educated at Christ Church, Oxford, he became Professor of Geometry in Gresham College at the age of 30.

He was a contemporary of Isaac Newton and Christopher Wren and assisted in the rebuilding of London after the great fire of 1666.

In 1678 Robert Hooke published a treatise "*de Potentia Restitutiva*" and showed experiments to confirm his Theory of Springs and Springy Bodies. One of these experiments was made with a string of brass wire 37 feet long extended by weights hung on the lower end by which he made it evident that the wire extended proportionally to the weight so that if one weight extended it one length, two extended it two lengths and so on.

He showed the same principle to apply to a helical spring and to a watch spring and enunciated his Law of :

"*Ut tensio, sic vis*" ;

i.e., the power of any spring is in the same proportion as the tension thereof.

He is said to have arrived at this Law about the year 1660 when studying the behaviour of springs for watches.

Hooke was also the first man in the world to suggest that fossils might be used to construct a chronology of the earth.

The Bernouillis. James, John and Daniel Bernouilli were members of an illustrious family of which no less than eight members contributed to the advancement of knowledge during the course of the 18th century.

James, as Professor of Mathematics in the University of Basle, studied the problem of the catenary curve of a suspended chain and determined the elastic curve of a cantilever loaded at the free end. The assumption that a cross section of a beam which is plane before bending remains plane after bending is attributed to James Bernouilli.

John was Professor of Mathematics at Basle when Euler was a student at that University.

Daniel Bernouilli in one of his early investigations gave a demonstration of the problem

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* *Chairman's Address to the South Wales and Monmouthshire Branch of The Institution of Structural Engineers. Read at Cardiff, November 3rd,* 1937.

of the parallelogram of forces and his theorem relating to the flow of water was published in 1738.

The Bernouillis like Euler, their contemporary and colleague, studied the elasticity of beams and columns from a mathematical standpoint.

Thomas Young (1773-1829).

Dr. Thomas Young, physician, physicist and student of Egyptian inscriptions wrote a book in 1807 entitled "*A Course of Lectures on Natural Philosophy and the Mechanical Arts.*"

In this book, 130 years after the publication of Hooke's Law, we find the idea of a modulus of elasticity introduced for the first time and also the term resilience.

(2) THEORY OF COLUMNS.

THEORY OF COLUMNS.

Leonardo da Vinci	(1452-1519)
Leonhard Euler	1759
Eaton Hodgkinson	1840
William Fairbairn	1840
W. J. M. Rankine	1858
T. C. Fidler	1886
J. M. Moncrieff	1901
(Straight-Line Formula	1896)

Leonardo da Vinci (1452-1519).

The earliest record of experimental study of the strength of columns is perhaps that contained in the note books of Leonardo da Vinci written about four and a half centuries ago. This remarkable Italian is well known as one of the greatest painters and sculptors in the annals of art, but he was also a scientist and inventor whose theories and discoveries were centuries ahead of his time.

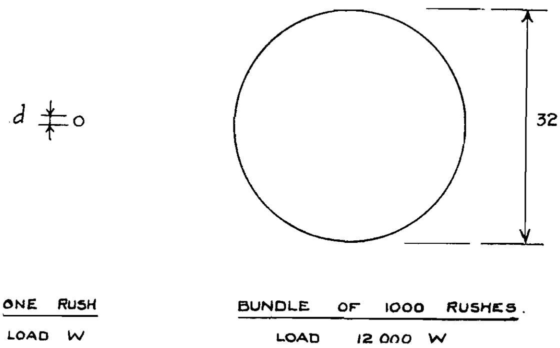
From the age of 30 he practised as a consulting engineer and architect in Milan at which city the manuscripts of his note books have been preserved in a huge miscellany known as Codex Atlanticus. These manuscripts have been deciphered with great difficulty for the four following reasons :—

- (*a*) He wrote from right to left.
- (*b*) His writing was reversed in such a way as would result from looking at a normal script in a mirror.
- (*c*) He used an elaborate system of abbreviations, and
- (*d*) No punctuation.

In his manuscripts he gives the method of finding reactions for a beam with a concentrated load at any point—some of his calculations are incorrect but the errors are due to faulty arithmetic rather than to faulty principles.

His studies of the loading of struts are the first scientific attempt of their kind. He says : "Many little supports held together (in a bundle) are capable of bearing a greater load than if they are separated from each other. Of 1,000 such rushes of the same thickness and length which are separated from one another, each one will bend if you stick it upright and load it with a common weight. And if you bind them together with cords so that they touch each other they will be able to carry a weight such that each single rush is in the position of supporting 12 times more weight than formerly. The increase of carrying capacity is entirely dependent upon the firmness of the binding and if loosely connected the total load becomes merely the sum of the loads each can carry separately."

LEONARDO DA VINCI (1452—1519) TESTS ON COLUMNS OF RUSHES.



Note : The Radius of Gyration of the Bundle is about 32 times that of one Rush, and stress per unit area is increased 12 times.

Figure 1.

This experimental result observed by Leonardo will be found to agree with the results obtained by modern strut formulæ or graphs such as those given by Moncrieff.

Leonhard Euler (1707-1783).

Euler, the son of a Calvinistic Pastor, was a Swiss Mathematician who studied at Basle University under John Bernouilli. In 1733 he succeeded Daniel

Bernouilli in the chair of mathematics at St. Petersburg and in 1741 was invited by Frederick the Great to Berlin as professor of mathematics.

His first important contribution to the theory of columns was a paper in French to the Academy of Berlin in 1759, in which he deals with the least force which will suffice to produce curvature in a column. This was the origin of Euler's well-known formula and it also formed the subject of a further paper in 1780 which he presented in Latin to the Academy of St. Petersburg to which city he had returned in 1766.

It is interesting to note that Euler made the error of assuming that the neutral axis lay at the extreme fibre on the concave side of the deflected strut.

Eaton Hodgkinson (1789-1861).

The experimental researches of Eaton Hodgkinson in the early half of the 19th century were of great importance in determining the formulæ for the practical strength of beams and columns.

Hodgkinson was born at Great Budworth, near Manchester, in 1789, and in spite of adverse circumstances in his early days he rose to be a Fellow of the Royal Society of London, a Professor of Mechanical Engineering at University College London and Consulting Engineer to Robert Stephenson on the design of the Menai and Conway Tubular Bridges.

In 1840 he presented to the Royal Society his paper on "Experimental Researches on the Strength of Pillars of Cast Iron and Other Materials." The investigation which he there described was undertaken in order "to supply the deficiencies of Euler's theory of the strength of pillars if it should appear capable of being rendered practically useful; and if not to endeavour to adapt the experiments so as to lead to useful results."

In this paper Hodgkinson gave the important relationship between the strengths of fixed-ended and round-ended pillars. He said that "a long uniform cast iron pillar with its ends firmly fixed . . . has the same power to resist breaking as a pillar of the same diameter and half the length with the ends rounded or turned so that the force would pass through the axis." He introduced the term "flat-ended" for such fixation as he employed in his tests.

Perhaps the most interesting point in Hodgkinson's paper of 1840 is that he there gives a formula which is of the same type as the well-known Rankine formula and which would appear to be the source from which the latter is derived. Hodgkinson does not usually receive credit for this.

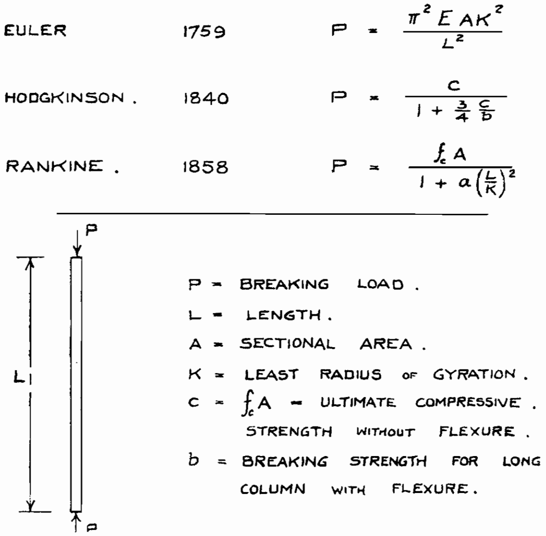


Figure 2. Column Formulæ.

These experimental researches were carried out at the expense of Sir William Fairbairn the famous Scottish engineer who shares with Robert Stephenson the credit of building the tubular bridges at Conway and Menai Straits and who was also one of the founders of the British Association.

The Crumlin Viaduct in South Wales is a remarkable example of wrought iron pin-jointed lattice girder spans supported on towers composed of hollow cast iron columns. The great height of these towers, of which the largest is about 190 feet from base to cap, indicates the confidence of the designer, Mr. T. W. Kennard, in the state of knowledge of the behaviour of columns as long ago as 1853.

This structure was built between the time of Eaton Hodgkinson's first experiments and the time of publication of Rankine's Column formula.

W. J. M. Rankine (1820-1872).

Perhaps no name is better known to engineers than that of Professor Rankine.

We have Rankine's Column Formula, Rankine's Theory of Earth Pressure and Rankine's Formula for combined Bending and Torsion, which figure in all textbooks on structural mechanics.

Important as these are to us it is said that "Rankine's most important contributions to science are in the dynamical theory of heat, in the theory of the steam engine, in that of waves in liquids especially of sea waves and on the resistance and rolling of ships."

In an obituary notice of Rankine in the *Proceedings of the Royal Society* it is stated: "There is probably no scientific writer to be compared with him in the three aspects of the extent covered by his treatises, their scientific accuracy and exhaustiveness, and their immediate adaptation to the use of practical men who may be utterly unable to follow the reasoning by which he arrived at his formulæ."

Rankine was Professor of Civil Engineering and Mechanics in the University of Glasgow from 1855 to 1872.

His Column Formula is given in the first edition of his "*Manual of Applied Mechanics*" in 1858 where the numerical values for the constants are attributed to Gordon, who determined them from Hodgkinson's experiments.

Fidler's Column Formula.

In 1886 Professor Fidler published a paper dealing with the practical strength of columns, in which he derived a formula to fit the results of experiments by Hodgkinson (1840) and Christie of Pencoyd, Pennsylvania (1884).

The basis of his theory was that the physical axis of the column may deviate from the geometrical axis owing to variation of the modulus of elasticity although the column is originally straight and axially loaded.

He says that "experiment has shown that test pieces even when cut from different parts of the same bar will sometimes exhibit considerable differences in the modulus of elasticity; and it may be shown that the deflection of the column will follow a similar law if the modulus is somewhat greater on one side than on the other, as it would if there were an initial curvature of the axis."

He assumed the extreme values of E to apply to the metal on either side of the cross section of the column and derived an expression for the deflection and maximum stress due to direct compression and bending.

J. M. Moncrieff (1865-1931).

One of the most exhaustive studies of column behaviour was that made by Mr. Moncrieff and published in the "*Transactions of the American Society of Civil Engineers*," in 1901. He built up practical formulæ for safe loads on columns of iron, of steel, and of timber based upon the study of the published results of no less than 1,789 tests of columns by various experimenters commencing with Hodgkinson's Tests of 1840.

Like Professor Fidler, Mr. Moncrieff pointed out that the "physical axis" of a practical column—i.e., the axis passing through the centre of resistance of every section of the column—will not be coincident with the geometrical axis. In the case of built-up columns it will be disturbed by cold straightening and riveting whilst in the case of timber and cast metal columns lack of homogeneous structure will have a similar effect. In the Moncrieff Formulæ an equivalent and unintentional eccentricity of loading is assumed to allow for these practical defects, a suitable value having been determined from the tests he examined.

Mr. Moncrieff was, of course, President of this Institution for the Sessions 1928-9, and 1929-1930.

Straight-Line Formulæ have been extensively used in the design of columns for skyscrapers in America.

Possibly one of the earliest examples was for the Fisher Building of 18 storeys in Chicago in 1896.

(3) THEORY OF BEAMS.

THEORY OF BEAMS.

Galileo Galilei	1633
Mariotte	1680
Leibnitz	1684
James Bernouilli	1705
Coulomb	1776
Peter Barlow	1817
Eaton Hodgkinson	1847
Clapeyron	1857
W. H. Barlow	1859
Otto Mohr	1868

Galileo Galilei (1564-1642).

The earliest writer to interest himself in the theory of beams was Galileo Galilei, the Italian astronomer who invented the thermometer and developed the telescope.

Galileo was lecturer in mathematics in the University of Pisa and later became Professor of Mathematics at Padua.

His attention was drawn to the beam problem during a visit to the arsenal and dockyards at Venice and in a book published in Holland in 1638 (300 years ago) he deals with the fracture of beams and rods. A diagram in this book shows a cantilever loaded at the free end and the problem which he discussed was to determine the breaking load in terms of the dimensions of the beam.

It is remarkable that Galileo supposed the fibres of a strained beam to be inextensible but it must be remembered that Hooke's Law was not discovered until 1678 or 40 years later. His theory was not supported by experiment and was found by later writers to be incorrect.

GALILEO'S BEAM PROBLEM.

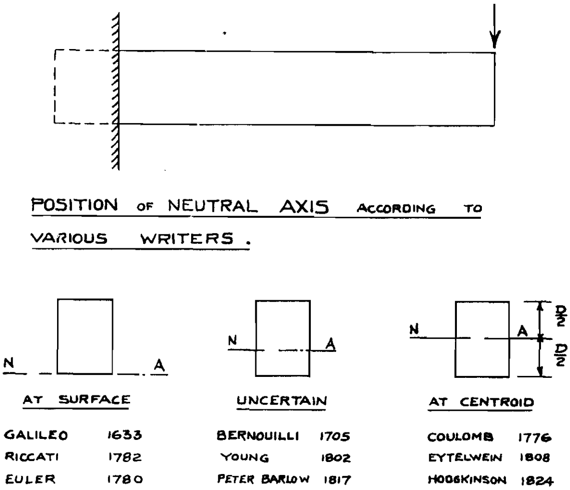


Figure 3. Galileo's Beam Problem.

Galileo's Problem.

The problem which Galileo attempted to solve without having any idea of elasticity was attempted by many subsequent writers until it was finally settled by Coulomb in 1776.

Mariotte, the French physicist, who invented the Rain Gauge in 1677 and formulated the Laws of Gases contemporaneously with Robert Boyle, conducted experiments in 1680 which showed that some of the fibres of a loaded beam are extended whilst others are shortened.

Baron Leibnitz, the German mathematician and philosopher, who shares with Isaac Newton the credit of inventing the Calculus contributed to the solution of Galileo's Beam Problem by applying Hooke's Law to the individual fibres but assumed the material to be strained in tension only and incompressible.

The position of the neutral axis was thus assumed to lie at the concave surface of a deflected beam or cantilever and it is remarkable that Euler made the same assumption in the original publication of his theory of a deflected strut.

In 1705 James Bernouilli stated that the position of the neutral axis was indifferent and unimportant and doubted the application of Hooke's Law.

In 1776 Charles Augustin Coulomb, the French Engineer, gave the three fundamental conditions of the beam theory, namely (*a*) the total tension on the section on one side of the neutral axis = the total compression on the other side, (*b*) the Moment of Resistance of the internal stress = the Bending Moment due to the loads, (*c*) the Shear on a Section = the Load. The Neutral Axis according to Coulomb lies in the middle of the depth for a rectangular section.

It thus took 138 years for mathematicians to solve Galileo's Problem and to arrive at an elementary stage of the beam theory.

Coulomb's theory was not universally accepted and even as late as 1817 or 40 years after its publication there were writers like Peter Barlow, Professor of Mathematics at Royal Military Academy, Woolwich, who neglected Coulomb's method and gave a wrong theory for finding the neutral axis.

Eaton Hodgkinson, whose work on Columns we have previously considered, adopted Coulomb's theory of beams and gave it its true place in English works on practical mechanics.

The experiments carried out by William Fairbairn and Eaton Hodgkinson in connection with the design of Robert Stephenson's tubular railway bridges contributed much information on the behaviour of wrought iron beams.

By the year 1850 we find that the theory of the strength and deflection of beams was sufficiently developed to enable accurate calculations to be made for the Britannia and Conway Tubular Bridges. Thus it was calculated that the central deflection of the 400 feet-span simply supported girder of wrought iron at Conway would be 8.65 inches and the actual deflection was found to be 8.25 inches.

Clapeyron. It is noteworthy that the Britannia Tubular Bridge over the Menai Straits which is 1,511 feet long and continuous over four spans was designed and erected some seven years before Clapeyron published his Theorem of Three Moments for Continuous Beams. This was first given in a memoir published in Paris in 1857 for the case of uniformly loaded spans of uniform cross-section and varying lengths. Its application to other and general cases was developed by later writers.

In 1859 William H. Barlow, a son of Professor Peter Barlow, proposed the construction of continuous girders as a series of beams and cantilevers by introducing joints at the points of contrary flexure to admit of the expansion and contraction of the girder.

This principle was used in the Forth Bridge in 1890 and has been employed in many structures both in steel and in reinforced concrete.

In Rankine's "*Manual of Applied Mechanics*," which was first published in 1858, and in his "*Manual of Civil Engineering*," published in 1862, we find the standard formulæ for the deflection of beams and cantilevers with various types of loading.

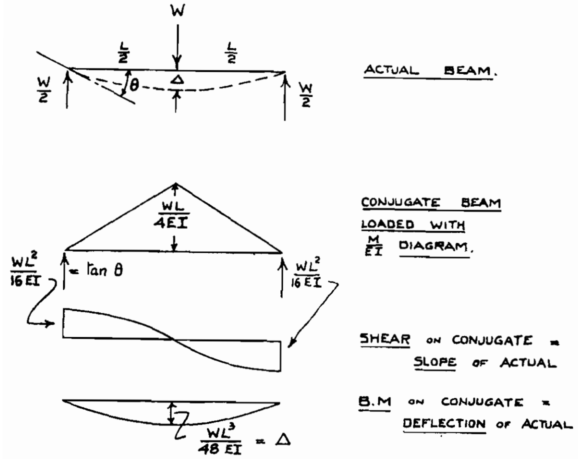


Figure 4. Mohr's Fictitious Loads or Conjugate Beams, 1868.

Otto Mohr (1838-1920). In 1868 Professor Otto Mohr of Dresden, extended the beam theory of deflections by showing that properly chosen fictitious loads on a beam cause bending moments which are numerically equal to the deflections and shear forces numerically equal to the slopes.

This principle of conjugate beams considerably simplifies the study of slopes and deflections by eliminating higher mathematics and is known as the Area-Moment Method. It is the basis of the graphical method of obtaining the elastic line for a loaded beam and also of the so-called Cable Theory of deflection.

(4) THEORY OF FRAMED STRUCTURES.

THEORY OF FRAMED STRUCTURES.

Reciprocal Diagrams :	
W. J. M. Rankine	1856
Clerk Maxwell	1864
Method of Sections :	
Ritter	1863
Bow's Notation :	
Robert Bow	1873
Displacement Diagrams :	
Williot & Mohr	1877

The parallelogram of forces was used by Daniel Bernouilli in the 18th Century and doubtless led to the discovery of the triangle and polygon of forces and the funicular polygon.

It appears to be Rankine to whom we are indebted for the commencement of graphic statics. In 1856 the preparation of his courses of lectures in Civil Engineering and Mechanics at Glasgow University led him to the discovery of reciprocal diagrams of frames and forces. The stress diagram for a simple roof principal is given in Rankine's "*Manual of Applied Mechanics*" in 1858.

In 1863 Professor August Ritter in Germany published his method of sections for finding the stress in a member of a framed structure by taking a section cutting that member and others and applying forces at the cut ends equal to the stresses in the members.

Clerk Maxwell. James Clerk Maxwell at the age of 25 was Professor of Natural Philosophy at Aberdeen. Four years later he held a similar position at King's College, London, and later was Professor of Experimental Physics at Cambridge.

Most of his work deals with researches in physics on colour and light, electricity and magnetism but between 1864 and 1876 he contributed four papers to various scientific societies dealing with framed structures. In these he simplified and extended Rankine's reciprocal diagrams and gave the Theorem of Reciprocal Deflections which has led to the method of solving problems on statically indeterminate structures by means of measured deformations of elastic models.

Robert Bow, who like Maxwell, was an Edinburgh man, published a treatise in 1850 dealing with stresses in structures and in 1873 gave his method of lettering the spaces instead of the joints or the members—the method known as Bow's Notation.

Williot. The next important development in the study of statically determinate framed structures was the displacement diagram devised by the French Engineer Williot in 1877 and later extended by Professor Mohr so that it is frequently known as the Williot-Mohr Diagram.

The Williot Diagram can be constructed to give the deflection of all panel points of a braced girder under a given fixed system of loads since the vertical displacement of any panel point in relation to a support is the deflection of the point.

With the development of influence lines and the use of Maxwell's Theorem of reciprocal deflections the Williot Diagram is useful to give a construction for influence lines for deflection of braced girders.

(5) THEORY OF ARCHES.

THEORY OF ARCHES.

Robert Hooke	1678
David Gregory	1697
La Hire	1712
Coulomb	1773
Navier	(1785-1836)
Rankine	1862
Castigliano	1875
H. M. Martin	1888

Robert Hooke in 1678 stated that the true form of an arch is an inverted suspension chain.

David Gregory in 1697 published a theory of the arch in which it was assumed that the line of pressure must of necessity coincide with the intrados of the arch.

According to the Wedge Theory of La Hire and Atwood in 1712 it was supposed that the pressure must be perpendicular to the meeting surfaces of the voussoirs or ring stones which were considered as frictionless blocks in equilibrium under the action of the load and normal reactions from adjacent blocks.

Such was the state of Arch Theory when William Edwards built his masonry arch at Pontypridd in 1755 with its span of 140 feet which for three-quarters of a century remained the largest arch in this country.

In 1773 Coulomb corrected the theory of La Hire by allowing for friction between the voussoirs.

The subject was further elucidated by Professor Moseley of King's College, London, in 1843 and a theory based upon the conditions actually existing in a real arch was established.

Navier is credited with the statement that the line of pressure for no tension must lie within the middle third of the ring thickness.

He was also the first to apply the mathematical theory of elasticity to the calculation of Arches.

In Rankine's "*Manual of Civil Engineering*" (1862) equations are developed for an arch rib allowing for elastic distortions. The method of analysis and the assumptions that the span is unchanged and that the supports remain at the same level are the same as in the ordinary elastic theory but Rankine's equations are cumbersome.

Martin. The earliest publication of the three equations of the elastic theory of arch ribs in modern form is probably that in a Student's Paper on "*Arched Ribs and Voussoir Arches*" read at a meeting of the Institution of Civil Engineers by H. M. Martin, Wh.Sc., Stud. Inst. C.E., in 1888.

In this paper the equations are given as :—

$$\Sigma M \frac{\Delta s}{EI} = 0$$

$$\Sigma Mx \frac{\Delta s}{EI} = 0$$

$$\Sigma My \frac{\Delta s}{EI} = 0$$

and proofs are given of their derivation.

The elastic theory of the arch is included in Castigliano's Theorem of Least Work, the equations for which, published in 1875, give the same results as Martin's three equations.

GENERAL FORMULÆ OF ELASTIC THEORY $\left(\frac{d}{EI}\ \text{CONSTANT}\right)$

$$\Sigma_B^A M = 0 \quad (1) \qquad \Sigma_B^A \frac{Myd}{I} - H\,\Sigma_B^A \frac{d}{A} = 0 \quad (2)$$

$$\Sigma_B^A Mx = 0 \quad (3) \qquad \Sigma_B^A My = 0 \ \text{(APPROX:)} \quad (2a)$$

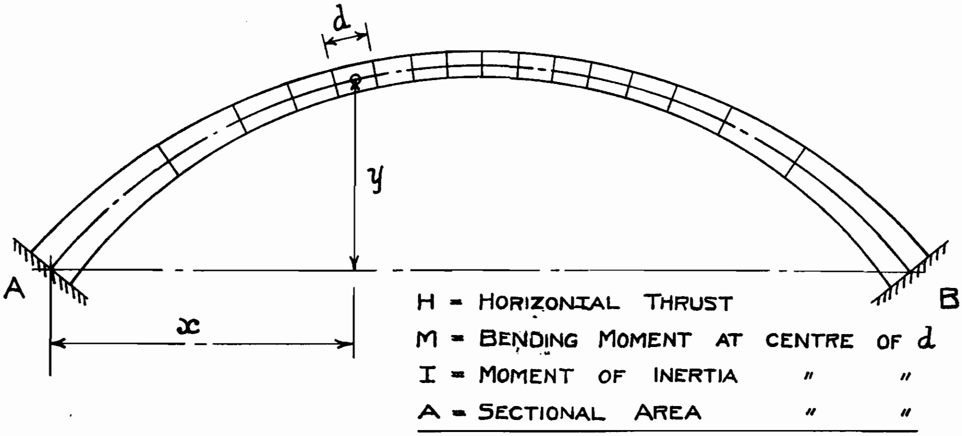


Figure 5.

(6) MODERN STRUCTURES AND METHODS.

THEORY OF MODERN STRUCTURES.

(Statically Indeterminate)

Principle of Work	1833	Clapeyron
,, Virtual Work	1874	Mohr
,, Least Work	1875	Castigliano
Reciprocal Deflections	1864	Maxwell
Slope-Deflection Equations	1868	Mohr
Influence Lines	1867	Winkler
Mechanical Analysis :		
(A) Slope-Deflection	1915	Wilson & Maney
(B) Reciprocal Deflection	1922	Beggs
Photo Elasticity	1911	Coker

Statically Indeterminate Structures. The principles employed in the design of modern structures were discovered during the second half of the nineteenth century many years before their practical applications were needed.

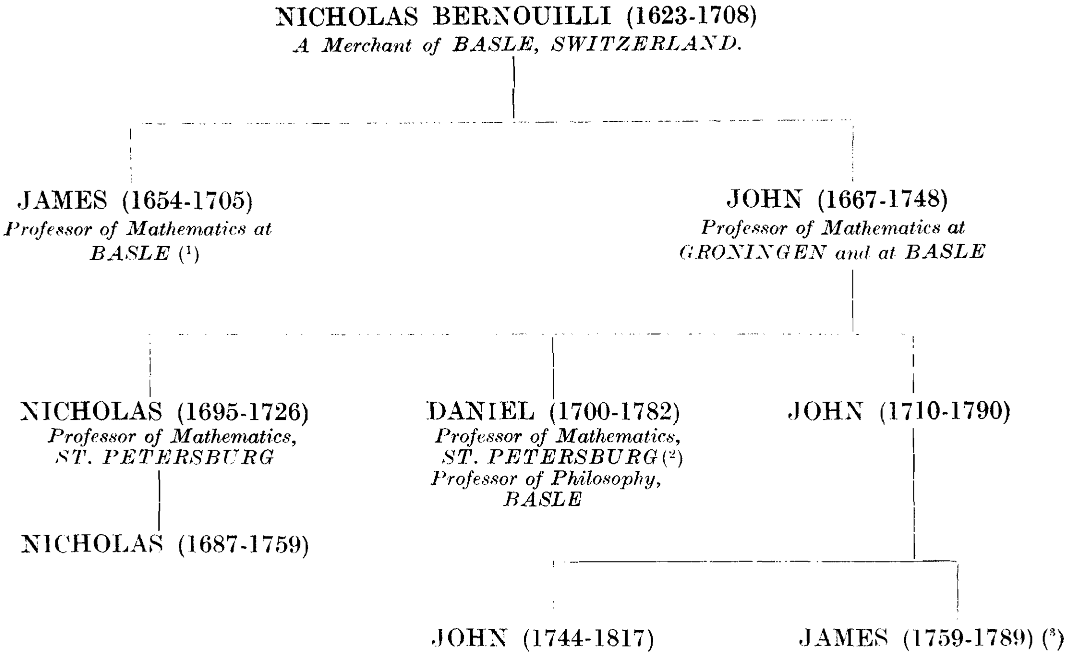
Clapeyron's Theorem of the equality of internal and external work was published in France in 1833. This Theorem states that the internal work done by the stresses in straining the members of a structure is equal to the external work done by the loads which produce the stresses. It is a fundamental principle in the calculation of deflections.

Castigliano's Principle of Least Work was published in Italy in 1875 in the Transactions of the Academy of Science of Turin. It is explained in detail with applications to actual indeterminate structures in Castigliano's book published about 1879 and translated into English by Mr. Ewart S. Andrews in 1918 under the title "*Elastic Stresses in Structures.*"

Maxwell's Principle of Reciprocal Deflections was published in 1864 in a paper "*On the Calculation of the Equilibrium and Stiffness of Frames,*" but its practical value in structural design was not appreciated for many years, until the development of influence lines and mechanical methods of stress analysis.

The Principle of Virtual Work was developed by Professor Mohr, of Dresden, in dealing with redundant frames and the scope of this method has been extended by Professor Müller-Breslau, of Berlin.

This Principle is useful in the determination of the horizontal reactions for a two-hinged braced arch and for other like problems.



(1) *Theory of Elastic Curve for Beams and Catenary Curve for Chains.*

(2) *Theory of Vibration of Beams (collaborated with Euler) and Hydrodynamics.*

(3) *Vibration of Plates.*

Influence Lines. The method of dealing with the effects of rolling loads by influence lines originated with Professor Winkler, of Berlin, in 1867 and was followed up by Weyrauch in Germany in 1873. It was introduced in America in 1887. The earliest published reference to the subject in this country was a paper by Professor F. C. Lea contributed to the Institution of Civil Engineers in 1905. In this paper Professor Lea shows examples of the construction of influence lines for stress in members of lattice girder bridge trusses. Since that date the subject has developed considerably and is dealt with in all standard works on advanced structural design.

Mechanical Stress Analysis. Two methods are in use :—

(*a*) Direct loading of an elastic model and measurement of changes of slope and deflection at each point of a framed structure.

The analysis depends upon the fact that bending moments can be obtained mathematically when these slopes and deflections are known. The theory is called the "slope-deflection" method and was used in America by Wilson and Maney in 1915.

(*b*) The indirect method based upon Clerk Maxwell's Theorem of Reciprocal Deflections also uses an elastic model but no loads are applied.

Instead of loading the model, accurately known deformations are applied to various points and the resulting distortions at other points are measured. This is the principle of the Beggs Deformeter, by means of which influence lines can be drawn for the bending moment thrust and shear at the point where the deformations are applied.

Photo Elasticity. The discovery that glass and other transparent bodies can acquire the property of double-refraction by being placed under stress was announced by Sir David Brewster to the Royal Society in 1816.

The development of the subject by Professor Coker and others has led to interesting and useful results with all manner of structures. In a test carried out by Professor Coker with a transparent model of a beam examined by polarised light it has been shown that the accepted theory that stress is proportional to distance from the neutral axis is proved by the colourbands. At the commencement of loading the neutral axis is at the centre of gravity and as the loading is increased the neutral axis moves slightly away from the compression side and towards the tension side, but not sufficiently to affect practical design calculations. Such is the application of the most modern method of investigation to Galileo's problem.

(7) APPLIANCES.

It is perhaps not inappropriate to close this paper with a brief reference to some of the aids to structural design calculations without which our work would be much more difficult if not indeed impossible.

Logarithms were discovered by John Napier, a Scottish landowner, who devoted his leisure to mathematics. The idea originated in 1594 and his Napierian system was published in 1614.

Henry Briggs, Professor of Geometry at Gresham College, followed up Napier's discovery by developing the system of logs. to the base 10 which we commonly employ.

The Calculus was discovered at the close of the 17th century by Newton and Leibnitz.

Slide Rules date from 1815 and the Amsler Planimeter was invented by Professor Jacob Amsler in 1856.

Conclusion. In St. Paul's Cathedral there is an inscription in honour of Sir Christopher Wren ending with the words :—

"*si monumentum requiris circumspice.*"

In a similar way many of the great engineers of the past like Telford, Rennie, Stephenson and Brunel, have left lasting memorials of their skill in the bridges they built up and down our land.

But the men of whom we have been thinking to-night are no less famous although they have left no material monuments to remind us of the benefits they have conferred upon us. They are mostly men of the Universities of the world whose monument we are, whose minds have been trained to make use of the principles they discovered.

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