# THE IMPACT OF ADVERTISING ON MEDIA BIAS 

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#### Abstract

In this study, the authors investigate the role of advertising in affecting the extent of bias in the media. When making advertising choices, advertisers evaluate both the size and the composition of the readership of the different outlets. The profile of the readers matters since advertisers wish to target readers who are likely to be receptive to their advertising messages. It is demonstrated that when advertising supplements subscription fees, it may serve as a polarizing or moderating force, contingent upon the extent of heterogeneity among advertisers in appealing to readers having different political preferences. When heterogeneity is large, each advertiser chooses a single outlet for placing ads (Single-Homing), and greater polarization arises in comparison to the case that media relies on subscription fees only for revenues. In contrast, when heterogeneity is small, each advertiser chooses to place ads in multiple outlets (Multi-Homing), and reduced polarization results.


Keywords: Media Competition; Bias in News; Advertising; Two-Sided Markets

## 1. INTRODUCTION

Bias in news media is well known (e.g., Groseclose and Milyo 2005, and Hamilton 2004) and can be defined as selective omission, choice of words and varying credibility ascribed to the primary source (Gentzkow and Shapiro 2006). In a recent paper by Mullainathan and Shleifer (MS 2005), a link is established between subscription fees and media bias. By assuming that readers prefer news consistent with their political opinions and that newspapers can slant toward these opinions, MS (2005) show that when the papers' sole source of revenue is from subscription fees (i.e., price for news), they slant news toward extreme positions.

For many media outlets, however, $60 \%$ to $80 \%$ of total revenue stems from advertising (Strömberg 2004), as opposed to subscription. Thus, in this study, we aim to complement the work of MS (2005) by recognizing that newspapers rely on revenues that accrue both from subscription fees paid by readers and advertising fees paid by advertisers. We investigate how the existence of these two sources of revenue affect the extent of bias in reporting that is selected by the media.

In order to understand the role of advertising in determining the nature of competition between newspapers, we specify in the model the effectiveness of advertisements to enhance consumers' probability of purchase. We argue that this effectiveness, for some products, may depend upon the political opinions of readers of the ads. It has been long established in the Consumer Behavior literature that products reflect a person's self-concept (Belk 1988). They provide a way for a person to express her self-image, which may be strongly correlated with her political opinions. We introduce, therefore, a product specific variable that measures the extent to which political preferences play a role in enhancing consumers' probability of purchase of the product. While for some products this measure is significant, for others it is trivial. For example,
while "green" products, such as Toyota Prius, or Apple's Mac computer may appeal more to liberals, "American" products, such as the Chevy Truck, may appeal more to conservative consumers. However, there are many products, such as automobile tires or insurance policies, for which political opinions do not affect consumers' choices to a large extent. ${ }^{1}$ When political preferences play an important role in consumers' purchase decisions, advertising the product can be effective if it targets the correct consumers. An advertisement that reminds the consumers that the product is consistent with their political opinions may increase the likelihood that they purchase the product.

Heterogeneity among advertisers with respect to the appeal of their products to consumers having different preferences is distributed in our model over a bounded interval. The length of this interval captures the extent of heterogeneity among advertisers, with longer intervals indicating significant differences in the appeal of products to liberal vs. conservative readers. In our model we show that the degree of heterogeneity among advertisers plays a role in determining whether advertisers choose to place ads with a single newspaper or with both newspapers. The literature on two-sided markets has referred to these two possible outcomes as Single and Double-Homing by advertisers, respectively (See Armstrong (2006), for instance.) While Single-Homing arises as the unique equilibrium when the extent of heterogeneity is large, Double-Homing arises when it is small.

We further investigate the manner in which the advertisers' choice between the newspapers affects the slanting strategies of media outlets. We show that when newspapers rely both on advertising and subscription fees, advertising can serve as a polarizing or moderating force in affecting the reporting of newspapers through two effects. First, adding the advertising market implies that newspapers reduce their reliance on subscribers in favor of advertisers. As a
result, they may choose less slanting in their reporting strategies to improve their appeal to moderate readers, and by doing so, offer a bigger readership to advertisers. This "readership effect" enables the newspapers to charge higher advertising fees.

However, in seeking to lure advertisers a second, counter effect may arise when advertisers choose to Single-Home. Specifically, when downward pressures on subscription fees arise due to reduced slanting of the newspapers, similar downward pressures on advertising fees appear, as well, as each newspaper attempts to defend its market share among advertisers. Hence newspapers may have stronger incentives to polarize in order to alleviate price competition in both markets. This "incremental pricing effect" to polarize is above and beyond the traditional attempt of companies to introduce product differentiation in order to soften price competition in a given market. Due to the two-sided markets we consider, polarization serves to soften price competition in both markets.

We demonstrate that at the Single-Homing equilibrium, the "incremental pricing effect" is stronger than the "readership effect", thus leading to intensified bias in reporting. In contrast, at the equilibrium with Double-Homing the "readership effect" is the only force present, thus giving rise to reduced bias at the equilibrium.

There is a growing body of literature on media bias as implied by the media's attempt to appeal to readers' beliefs. In addition to MS (2005), Gentzkow and Shapiro (2006) and Xiang and Sarvary (2007) also investigate this kind of bias. In Gentzkow and Shapiro (2006) readers who are uncertain about the quality of an information source infer that the source is of higher quality if its reports are consistent with their prior expectations. Xiang and Sarvary assume that there are two types of consumers, those who enjoy reading news consistent with their political opinions and conscientious consumers who care only about the truth. This assumption is
different from MS (2005) or our paper, where each consumer values both some consistency with political opinions and accuracy. The reporting strategy of the newspapers depends then on the relative weights consumers assign to consistency with their political opinions vs. accuracy. In addition, these earlier studies on bias assume that the media's sole source of revenue stems from selling news. In contrast, in the present study we allow the papers to earn revenues from advertising fees as well.

There are two recent papers that consider, like us, a media market with both advertising and subscription fees as sources of revenue. In Gabszewicz, Laussel and Sonnac (2002) and Ellman and Germano (2009), advertisers care only about the size and not the profile of the readership of each newspaper. This assumption is different from our setting, where advertisers wish to target audiences that are receptive to their advertising messages. This targeting objective of advertisers is pursued in Bergemann and Bonatti (2010) in an environment where the sole source of revenues of media outlets is from advertising. In this recent study, the authors investigate how improvements in the targeting technology that is facilitated by online advertising affects the allocation of advertisements across different media and the equilibrium prices of advertising messages. The topic of targeted advertising is also investigated in Iyer, Soberman, and Villas-Boas (2005) in an environment where the firms themselves and not media outlets possess the targeting technology.

Another strand of literature related to our study deals with consumers who may choose one or two of competing products. In Sarvary and Parker (1997) consumers decide whether to rely on a single information source or to diversify their purchases to include competing sources. They show that the segmentation of consumers between those who purchase one or two sources of information depends upon the relative importance consumers assign to obtaining precise
information. In Guo (2006), a similar diversification of the consumption bundle may arise when there is uncertainty about future preferences. Buying competing products simultaneously serves as "insurance" against such uncertainty. The main difference between our study and the previous two is our focus on competition between media outlets in two-sided markets instead of the onesided framework considered in these studies.

## 2. THE MODEL

Consider a market with two newspapers, $i=1,2$, a mass of $A$ advertisers and a mass of $M$ consumers, where $M_{1}$ of these consumers are subscribers to one of these two papers and $M_{2}$ are nonsubscribers. Newspapers provide news and print advertisements. By simultaneously operating in these two markets, newspapers have two potential sources of revenue: subscription fees $\left(P_{i}\right)$ and advertising fees $\left(K_{i}\right)$.

Each of the $M_{l}$ consumers reads either Newspaper 1 or 2 (but not both), and may buy products from the advertisers. We adapt the model developed by MS (2005) to capture the interaction between subscribers and newspapers. Specifically, when reading the newspaper, a subscriber receives information about a certain news item $t$, which is distributed according to $N\left(0, \sigma_{t}^{2}\right)$. Each consumer has some belief about the news item that is affected by her political opinion. We designate this political preference by $b$, and assume that the consumer believes the news item to be distributed according to $N\left(b, \sigma_{t}^{2}\right)$. In comparison to the true distribution of the news item, the consumer's belief is biased. The political opinion parameter $b$ measures the extent and direction of this bias. It is uniformly distributed in the population of readers between $-b_{0}$ and $b_{0}$. For example, readers with beliefs closer to $-b_{0}$ can be considered liberals, and those in the proximity of $b_{0}$ can be considered conservatives.

Newspapers report news about $t$. They receive some data $d=t+\varepsilon$, where the random variable $\varepsilon$ is independently distributed of $t$ according to $\varepsilon \sim N\left(0, \sigma_{\varepsilon}^{2}\right)$. Note that the data received by the newspapers may be different since $t$ and $\varepsilon$ are random variables. Hence, $d \sim N\left(0, \sigma_{d}^{2}\right)$, where $\sigma_{d}^{2}=\sigma_{t}^{2}+\sigma_{\varepsilon}^{2} .{ }^{2}$ Newspapers may choose to report the data with slant $s_{i}$, so the reported news is $n_{i}=d+s_{i}$. Readers incur disutility when reading news inconsistent with their political opinions, as measured by the distance between the reported news and the readers' opinions: ( $n_{i}-$ $b)^{2}$. Holding constant the extent of inconsistency with their opinions, they also prefer less slanting in the news. As in MS (2005), the overall utility of a reader is:

$$
\begin{equation*}
U_{b}{ }^{i}=\bar{u}-\chi s_{i}^{2}-\phi\left(n_{i}-b\right)^{2}-P_{i} \quad \chi, \phi \geq 0, \tag{1}
\end{equation*}
$$

Where $\bar{u}$ is the reservation price of the reader, $\chi$ calibrates her preference for reduced slant, and $\phi$ calibrates the reader's preference for hearing news consistent with her political opinion. Note that the utility of the reader increases the smaller the slant $s_{i}$, and the smaller the discrepancy between the reader's opinion $b$ and the reported news $n_{i}$.

Similar to MS (2005) we also focus on the characterization of the equilibrium with full coverage of the market and linear slanting strategies of the newspapers in the form $s_{i}(d)=$ $\frac{\phi}{\phi+\chi}\left(B_{i}-d\right)$ with $B_{i}$ interpreted as a choice of location of newspaper $i .^{3}$ This location choice of the newspaper can be a point inside or outside of the interval $\left[-b_{0}, b_{0}\right]$ and reflect the newspaper's political preference. Using $s_{i}(d)$, the paper slants data toward its preference $B_{i}$ when reporting news. Notice that the extent of slanting is an increasing function of $\phi$ and a decreasing function of $\chi$. Hence, as readers derive higher utility from hearing news consistent with their political opinions and reduce the importance placed on obtaining accurate information, newspapers choose greater slanting in their reporting. Without loss of generality, we assume that Newspaper

2 is located to the right of Newspaper $1\left(B_{1}<B_{2}\right)$. That is, while Newspaper 1 slants more to the left, Newspaper 2 slants more to the right.

Substituting the linear slanting strategies for $s_{i}$ and $n_{i}$ into Equation 1 and using the distributional properties of the random variable $d$ (specifically, that $E d=0$ and $E d^{2}=\sigma_{d}^{2}$ ), yields the expected payoff of a consumer having opinion $b$ at the time she chooses between the two newspapers. Note that at this time, the realizations of $d$ and $s_{i}(d)$ are yet to be determined due to the fluctuations of the data supporting news stories. At the time of the choice, the reader is aware only of the locations and fees chosen by the newspapers ( $B_{i}$ and $P_{i}$ ) as well as her own political opinion $b$. Since the actual news may fluctuate depending upon the realization of $d$, in evaluating the utility she derives from subscribing to the papers the reader calculates expectation over all possible $d$ realizations in Equation 1. For Newspaper $i$ and reader of type $b$ this yields the following expected utility.

$$
E U_{b}^{i}=\bar{u}-\frac{\phi^{2}}{\phi+\chi}\left(B_{i}-b\right)^{2}-\frac{\chi \phi}{\phi+\chi}\left(b^{2}+\sigma_{d}^{2}\right)-P_{i} .
$$

The consumer who is indifferent between the two newspapers satisfies the equation $E U_{b}{ }^{1}=E U_{b}{ }^{2}$. Solving this equation for $b$ yields:

$$
\begin{equation*}
b_{\text {indif }}=\frac{(\phi+\chi)}{2 \phi^{2}} \frac{\left(P_{2}-P_{1}\right)}{\left(B_{2}-B_{1}\right)}+\frac{B_{1}+B_{2}}{2} . \tag{2}
\end{equation*}
$$

Given the expression derived for $b_{\text {indif }}$, the papers' subscription revenues are:

$$
\begin{equation*}
R_{1, \text { sub }}=M_{1} P_{1} \frac{b_{0}+b_{\text {indif }}}{2 b_{0}} \text { and } R_{2, \text { sub }}=M_{1} P_{2} \frac{b_{0}-b_{\text {indif }}}{2 b_{0}} . \tag{3}
\end{equation*}
$$

The population of advertisers is distributed according to the appeal of their products to consumers having conservative opinions, namely those situated in the positive segment of the distribution of opinions. We designate this appeal parameter by $\alpha$ and assume it is uniformly distributed on the interval $\left[-\alpha_{0}, \alpha_{0}\right], \alpha_{0} \geq 0$. Negative values of $\alpha$ indicate products
unappealing to conservative consumers with opinions in the range $\left[0, b_{0}\right]$, with more negative values indicating increased appeal to liberal consumers with opinions in the range $\left[-b_{0}, 0\right]$. Positive values of $\alpha$ indicate products having the opposite characteristics, with bigger positive values indicating increased appeal to conservatives. Products whose attractiveness to the consumer is unlikely to be determined by political opinions assume an $\alpha$ value in the neighborhood of zero. Given the above specification, the parameter $\alpha_{0}$ can be interpreted as reflecting the extent of heterogeneity of the appeal of different products to consumers with different political opinions.

We assume that in the absence of advertising each consumer has a certain probability of purchasing a product. This probability can be modified with advertising. The change in purchase probability for a given reader depends on the extent of compatibility between the political opinion of the reader (her location $b$ ) and the type of the product advertised (its appeal $\alpha$ ). When an ad is successfully targeted to enhance compatibility, the reader's purchase probability of the advertised product increases. However, with lack of compatibility, her purchase probability might actually decrease. We designate by $E(\alpha, b)$ the incremental probability (positive or negative) when a reader of political preference $b$ is exposed to an ad related to product $\alpha$, and specify it as:

$$
\begin{equation*}
E(\alpha, b)=\left(h_{0}+\frac{\alpha b}{b_{0}}\right), \quad \text { where } h_{0}>0 \tag{4}
\end{equation*}
$$

Hence, the effectiveness of advertising is higher when political opinions are more consistent with the appeal parameter of the advertised product, measured by the term $\alpha b$ in Equation 4. Note that the product $\alpha b$ is positive for both liberal consumers of products having a negative measure of appeal $\alpha$ and conservative consumers of products having a positive measure of appeal. The parameter $h_{0}$ is a measure of the basic effectiveness of advertising to increase
consumers' purchase probabilities. The change in the probability of purchase $E(\alpha, b)$ depends also upon the extent of compatibility between the variables $b$ and $\alpha$. For example, when a liberal consumer is exposed to an advertisement of a green product, this will cause an increase in her probability of purchasing this product that is above $h_{0}$, which is the basic increase in purchase probability when the consumer becomes aware of the product due to the advertisement. However, an extremely conservative consumer can respond very negatively to this product in which case the change in her purchase probability due to the advertisement $E(\alpha, b)$ might even become negative. ${ }^{4,5}$

The specification in (4) implies that an advertiser is likely to pursue two objectives in designing its advertising strategy: to obtain a large audience for its ads and to target an audience that is receptive to its advertising message. The first component of the advertising response function motivates the large audience objective and the second motivates the targeting objective. Finally, for simplicity, we assume that advertising has the same effect on a subscriber and nonsubscribers with whom she shares information about advertised products. This assumption is reasonable since subscribers tend to communicate with friends and relatives who normally hold similar political opinions.

The payoff of an advertiser is measured by the average increase in the number of consumers likely to buy its product (average incremental probability times the mass of consumers $M$ ) net of the advertising fees paid to the newspapers. Hence, when an advertiser of appeal parameter $\alpha$ chooses to advertise only in Newspaper 1, its expected payoff as derived from the subscribers of Newspaper 1 is given as:

$$
\begin{equation*}
E_{1}(\alpha)=M \int_{-b_{0}}^{b_{\text {indif }}} \frac{1}{2 b_{0}}\left(h_{0}+\frac{\alpha b}{b_{0}}\right) d b-K_{1}, \tag{5}
\end{equation*}
$$

if it chooses to advertise only in Newspaper 2 its expected payoff is:

$$
\begin{equation*}
E_{2}(\alpha)=M \int_{b_{\text {indif }}}^{b_{0}} \frac{1}{2 b_{0}}\left(h_{0}+\frac{\alpha b}{b_{0}}\right) d b-K_{2}, \tag{6}
\end{equation*}
$$

and if it chooses to advertise in both papers its expected payoff is:

$$
\begin{equation*}
E_{12}(\alpha)=E_{1}(\alpha)+E_{2}(\alpha) \tag{7}
\end{equation*}
$$

By choosing to advertise only in Newspaper 1, an advertiser recognizes that subscribers to this newspaper tend to have left leaning political opinions, lying in the interval $\left[-b_{0}, b_{\text {indif }}\right]$ where $b_{\text {indif }}=0$ at the symmetric equilibrium( when $-B_{1}=B_{2} \geq 0$ ). For instance, if it advertises a green product $(\alpha<0)$ in Newspaper 1, it can expect a positive payoff if the advertising fee paid to the newspaper $\left(K_{1}\right)$ is not too large, given that the average change in these readers' purchase probability due the advertisement is positive (i.e., $\int_{-b_{0}}^{0} \frac{1}{2 b_{0}}\left(h_{0}+\frac{\alpha b}{b_{0}}\right) d b>0$ ). In contrast, by choosing to advertise only in Newspaper 2, the advertiser draws readers who have more right leaning opinions, in the interval $\left[b_{\text {indif }}, b_{0}\right.$ ]. In this case, even though these readers become aware of its product $\left(h_{0}>0\right)$, their political preferences are inconsistent with the product ( $\alpha b \leq 0$ when $\alpha<0$ and $b \in\left[0, b_{0}\right]$ ), thus possibly leading to a negative expected payoff.

When advertising in both newspapers, an advertiser draws the entire population of readers. An advertiser chooses to advertise in a single newspaper $i$ if $E_{i}(\alpha)>E_{12}(\alpha)$ and $E_{i}(\alpha)>0$. From Equations 5-7 it follows that for this advertiser $E_{j}(\alpha)<0$ for $j \neq i$, namely the added benefit from advertising in the second newspaper falls short of the fee newspaper $j$ charges. This may happen if the advertiser's product appeals mostly to readers having extreme political opinions. Advertising in a newspaper whose readership consists mostly of readers with opposing opinions in the political spectrum may not be worthwhile to the advertiser in this case. In contrast, an advertiser whose product's appeal is not highly correlated with political preferences (having an appeal parameter in the neighborhood of zero) may advertise in both
newspapers since the added benefit from advertising in each paper is likely to be positive for this advertiser, implying that $E_{12}(\alpha)>E_{i}(\alpha), i=1,2$. The above discussion indicates that the population of advertisers can be segmented into at most three intervals as described in Figure 1.

## [Insert Figure 1 about here]

Advertised products with appeal parameter less than $\hat{\alpha}_{1}$ are advertised only in Newspaper 1 since the advertisers of these products try to target mostly liberals (from (6) $E_{2}(\alpha)$ is an increasing function of $\alpha$, thus if $E_{2}\left(\hat{\alpha}_{1}\right)=0, E_{2}(\alpha)<0$ for all $\left.\alpha<\hat{\alpha}_{1}\right)$. In contrast, those with appeal parameter bigger than $\hat{\alpha}_{2}$ are advertised only in Newspaper 2, since advertisers wish to reach only conservative readers for such high values of appeal parameter (from (5) $E_{1}(\alpha)$ is a decreasing function of $\alpha$, thus if $E_{1}\left(\hat{\alpha}_{2}\right)=0, E_{1}(\alpha)<0$ for all $\left.\alpha>\hat{\alpha}_{2}\right)$. For intermediate values of $\alpha \in\left[\hat{\alpha}_{1}, \hat{\alpha}_{2}\right]$, advertisers choose to advertise in both newspapers (since both $E_{1}(\alpha)$ and $E_{2}(\alpha)$ are positive in this range). The number of segments in Figure 1 can be smaller than three. If $\hat{\alpha}_{1} \geq \hat{\alpha}_{2}$, no advertiser chooses to advertise in both newspapers (referred to in the literature on two-sided markets as Single-Homing) and if $\hat{\alpha}_{1}=-\hat{\alpha}_{0}$ and $\hat{\alpha}_{2}=\alpha_{0}$ all advertisers choose to advertise in both newspapers, (Double-Homing). Note, in particular that when $\alpha_{0}=0$, the mass of $A$ advertisers is located at $\alpha=0$, and in this case, advertisers do not care about targeting. At the symmetric equilibrium, from Equations 5 and 6 each advertiser derives the net benefit of $\frac{M h_{0}}{2}-K$ when placing an ad with either one of the newspapers. Double-Homing is obviously implied, given that both newspapers offer the same net benefit to each advertiser.

From Equations 5-7 we can derive the expressions for $\hat{\alpha}_{1}$ and $\hat{\alpha}_{2}$ as functions of the locations and advertising fees chosen by the newspapers as follows:

$$
\begin{equation*}
\hat{\alpha}_{1}=\frac{2 b_{0}}{b_{0}+b_{\text {indif }}}\left\{\frac{2 b_{0} K_{2}}{M\left(b_{0}-b_{\text {indif }}\right)}-h_{0}\right\}, \hat{\alpha}_{2}=\frac{2 b_{0}}{b_{0}-b_{\text {indif }}}\left\{h_{0}-\frac{2 b_{0} K_{1}}{M\left(b_{0}+b_{\text {indif }}\right)}\right\} . \tag{8}
\end{equation*}
$$

The appeal parameter $\hat{\alpha}_{1}\left(\hat{\alpha}_{2}\right)$ characterizes an advertiser who is indifferent between advertising in Newspaper 1(2) and advertising in both newspapers (i.e., $E_{2}\left(\hat{\alpha}_{1}\right)=0$ and $E_{1}\left(\hat{\alpha}_{2}\right)=0$ ).

In the Single-Homing equilibrium, the interior segment of Figure 1 disappears and the advertiser who is indifferent between Newspaper 1 and 2 can be derived from Equations 5 and 6 by solving for $\alpha$ in the equation $E_{1}(\alpha)=E_{2}(\alpha)$ :

$$
\begin{equation*}
\alpha_{\text {indif }}=\frac{2 b_{0} b_{\text {indif }}}{\left(b_{0}{ }^{2}-b_{\text {indif }}{ }^{2}\right)} h_{0}-\frac{2 b_{0}{ }^{2}}{\left(b_{0}{ }^{2}-b_{\text {indif }}{ }^{2}\right)} \frac{\left(K_{1}-K_{2}\right)}{M} . \tag{9}
\end{equation*}
$$

From Equation 9 we obtain the advertising revenues that accrue to the newspapers in the equilibrium with Single-Homing as follows:

$$
\begin{equation*}
R_{1, a d v}=A K_{1} \frac{\alpha_{0}+\alpha_{\text {indif }}}{2 \alpha_{0}} \text { and } R_{2, a d v}=A K_{2} \frac{\alpha_{0}-\alpha_{\text {indif }}}{2 \alpha_{0}} \tag{10}
\end{equation*}
$$

When some advertisers Double-Home, the segment of the market covered by Newspaper 1 is $\left(\alpha_{0}+\hat{\alpha}_{2}\right) / 2 \alpha_{0}$ and that covered by Newspaper 2 is $\left(\alpha_{0}-\hat{\alpha}_{1}\right) / 2 \alpha_{0}$. As a result, the advertising revenues of the newspapers are:

$$
\begin{equation*}
R_{1, a d v}=A K_{1} \frac{\alpha_{0}+\widehat{\alpha}_{2}}{2 \alpha_{0}} \text { and } R_{2, a d v}=A K_{2} \frac{\alpha_{0}-\widehat{\alpha}_{1}}{2 \alpha_{0}} . \tag{11}
\end{equation*}
$$

In what follows we will derive symmetric equilibria with the market of advertisers fully covered. At such equilibria, $-\hat{\alpha}_{1}=\hat{\alpha}_{2} \geq 0$, and $-B_{1}=B_{2} \geq 0$. We will focus on two possible cases: equilibrium with Single-Homing, where each advertiser chooses to advertise in a single newspaper ( $\hat{\alpha}_{1}=\hat{\alpha}_{2}=0$ in Figure 1); and Double-Homing, where all advertisers choose to Double-Home $\left(\hat{\alpha}_{1}=-\alpha_{0}, \hat{\alpha}_{2}=\alpha_{0}\right)$.

We formulate the decision process of the newspapers as a two stage game. In the first stage, each newspaper simultaneously announces a strategy $s_{i}(d)$ of how to report the news (its location $B_{i}$ ). In the second stage, the papers choose their prices $P_{i}$ and $K_{i}$ simultaneously. Subsequent to those two stages, advertisers choose where to advertise and readers decide to
which newspaper to subscribe. Next, papers receive data $d$ and report news $d+s_{i}(d)$. Finally, consumers read the news, get exposed to the advertisements, and form new impressions of the advertised products.

Using this framework but with no advertising, MS (2005) show that the equilibrium locations of the newspapers are $B_{1}^{M S}=-3 b_{0} / 2$ and $B_{2}^{M S}=3 b_{0} / 2$. Hence, with subscription fees being the only source of revenues of newspapers, extreme bias in reporting, to the right by Newspaper 2 and to the left by Newspaper 1, are chosen at the equilibrium. Such extreme differentiation in reporting alleviates the extent of competition on subscription fees. In what follows, we investigate how these equilibrium locations change if newspapers earn revenues from advertising as well.

It may be interesting to point out how bias in reporting as a vehicle to introduce differentiation between newspapers is different from other product features aimed at achieving horizontal differentiation. First, the utility of readers depends upon two different attributes of news reports, accuracy and consistency with political opinions, thus introducing potentially opportunities for both vertical and horizontal differentiation. While the location choice of each newspaper $\left(B_{i}\right)$ is the vehicle to introduce horizontal differentiation, the weight assigned to this location in designing the slanting strategy (i.e., $\frac{\phi}{\phi+\chi}$ ) captures the relative importance of the vertical versus the horizontal attributes (i.e., accuracy vs. consistency with political opinions) in the utility function of the consumers. In particular, if the consumers' appreciation for accuracy (the vertical attribute) is infinite, the papers stop slanting the news and don't use reporting bias for horizontal differentiation. Another aspect that distinguishes bias from traditional models of horizontal differentiation is that newspapers attempt to appeal to two different audiences, readers and advertisers. Hence, the positioning of each newspaper has implications for price competition
in both markets. This contrasts with most models of product differentiation, where features are chosen by taking into account competition in a single consumer market.

## 3. ANALYSIS

When both subscription and advertising revenues are available, the objectives of the newspapers are:

Single-Homing $\left(\hat{\alpha}_{1}=\hat{\alpha}_{2}=0\right)$

$$
\begin{equation*}
\pi_{1}=A \frac{\alpha_{0}+\alpha_{\text {indif }}}{2 \alpha_{0}} K_{1}+M_{1} \frac{b_{0}+b_{\text {indif }}}{2 b_{0}} P_{1}, \quad \pi_{2}=A \frac{\alpha_{0}-\alpha_{\text {indif }}}{2 \alpha_{0}} K_{2}+M_{1} \frac{b_{0}-b_{\text {indif }}}{2 b_{0}} P_{2} \tag{12}
\end{equation*}
$$

where $b_{\text {indif }}$ and $\alpha_{\text {indif }}$ are given in Equations 2 and 9 , respectively.
Double-Homing ( $\hat{\alpha}_{1}=-\alpha_{0}, \hat{\alpha}_{2}=\alpha_{0}$ )

$$
\begin{equation*}
\pi_{1}=A K_{1}+M_{1} \frac{b_{0}+b_{\text {indif }}}{2 b_{0}} P_{1}, \quad \pi_{2}=A K_{2}+M_{1} \frac{b_{0}-b_{\text {indif }}}{2 b_{0}} P_{2} \tag{13}
\end{equation*}
$$

where $b_{\text {indif }}$ is given by Equation 2.
The newspapers choose subscription and advertising fees in the second stage to maximize
Objectives 12-13. When the newspapers locate symmetrically so that $-B_{1}=B_{2}=B$, the solution to the maximization is as follows:

Single-Homing $\left(\hat{\alpha}_{1}=\hat{\alpha}_{2}=0\right)$

$$
\begin{equation*}
P_{S}^{* *}=\frac{4 B \phi^{2} b_{0}}{\phi+\chi}-\frac{A \mathrm{~h}_{0}}{\frac{M_{1}}{\mathrm{M}}}, \quad K_{S}^{* *}=\frac{M \alpha_{0}}{2} . \tag{14}
\end{equation*}
$$

Double-Homing ( $\left.\hat{\alpha}_{1}=-\alpha_{0}, \hat{\alpha}_{2}=\alpha_{0}\right)$

$$
\begin{equation*}
P_{D}^{* *}=\frac{4 B \phi^{2} b_{0}}{\phi+\chi}-\frac{A \mathrm{~h}_{0}}{\frac{M_{1}}{\mathrm{M}}}, \quad K_{D}^{* *}=\frac{M}{2}\left[h_{0}-\frac{\alpha_{0}}{2}\right] . \tag{15}
\end{equation*}
$$

Hence, for a fixed symmetric choice of locations, subscription fees are higher if subscribers have greater preference for reports that are consistent with their political opinions
(bigger $\phi$ ), smaller preference for accurate reporting (smaller $\chi$ ), and are more heterogeneous (bigger $b_{0}$ ). Subscription fees are also higher when the advertising market is smaller (smaller $A$ ), the relative size of the population of subscribers is bigger (bigger $M_{1} / \mathrm{M}$ ), and the effectiveness of advertising declines (smaller $h_{0}$ ). In general, the more important advertising revenues in comparison to subscription revenues, the lower the fees newspapers charge to subscribers at the symmetric equilibrium.

Substituting the equilibrium advertising fees derived in Equations 14 and 15 back into Equation 8 implies different types of homing depending on the extent of heterogeneity among the advertisers (value of $\alpha_{0}$ ). While for large values ( $\alpha_{0}>2 h_{0}$ ), Single-Homing is the unique equilibrium, for small values ( $\alpha_{0} \leq 2 h_{0} / 3$ ), Double-Homing is the unique equilibrium. ${ }^{6}$ As explained earlier, advertisers in our environment care both about the number and profile of readers who are exposed to their ads. When heterogeneity among advertisers is significant, targeting readers who are compatible with advertised products is very important to the advertisers. Single-Homing is more successful than Double-Homing in achieving such targeting. In the absence of targeting, ads might reach consumers with extreme political opinions incompatible with the products advertised. When heterogeneity is large, such lack of targeting is especially costly for advertisers since the product $\alpha b$ might assume very large negative values in Equation 4.

To obtain the equilibrium locations chosen by the newspapers in the first stage, one has to solve first for the second stage fees, $P_{i}\left(B_{i}, B_{j}\right)$ and $K_{i}\left(B_{i}, B_{j}\right)$, as functions of arbitrary location choices selected in the first stage (not necessarily symmetric locations only). The second stage equilibrium strategies have to be substituted back into Equations 12-13 to obtain the first stage payoff functions of the newspapers.

Assuming the existence of an interior equilibrium, next we compare the locations selected at the symmetric equilibrium (designated by $B^{* *}$ ) to those derived when newspapers obtain revenues from subscribers only (denoted as $-B_{1}^{*}=B_{2}^{*}=B^{M S}$ ). When there is no heterogeneity among advertisers, namely when $\alpha_{0}=0$, advertisers Double-Home and $B^{* *}=$ $B^{M S}=3 b_{0} / 2$, meaning that bias remains unaffected when advertising is added as a source of revenue. However, when $\alpha_{0}>0$, adding advertising to supplement subscription fees may moderate or intensify bias. In Lemma 1, we first derive restrictions on the parameters of the model to guarantee that those regimes can be supported with positive streams of revenues from subscribers (namely that $B^{* *}>0$ and $P^{* *}>0$ ). For ease of presentation, we introduce a measure for the importance of advertising relative to subscription as a source of revenue for the papers, $T \stackrel{\text { def }}{=}\left(A M / M_{1}\right)\left((\phi+\chi) /\left(8 \phi^{2}\right)\right)$, where $\left(A M / M_{1}\right)$ represents the size of the advertising market relative to the subscription market and $(\phi+\chi) /\left(8 \phi^{2}\right)$ is a measure of the importance consumers attach to accuracy relative to consistency with their political opinions. If consumers attach great importance to accurate reporting (i.e., $(\phi+\chi) / \phi^{2}$ is large), the papers cannot charge high subscription fees. Hence, if either one of the two components of $T$ increases, the subscription market loses its importance as a source of revenues relative to the advertising market.

LEMMA 1. To ensure positive subscription prices and strict differentiation between newspapers (i.e., $P^{* *}>0$ and $B^{* *}>0$ ):
(i) At the Single-Homing equilibrium: $T<T_{\text {max }}^{S} \xlongequal{\text { def }} \frac{3 b_{0}{ }^{2}\left(9 \alpha_{0}-4 h_{0}\right)}{2 h_{0}\left(9 \alpha_{0}-2 h_{0}\right)}$, and $\alpha_{0}>2 h_{0}$.
(ii) At the Double-Homing equilibrium: $T<T_{\max }^{D} \stackrel{\text { def }}{=} \frac{b_{0}{ }^{2}\left(3 h_{0}-2 \alpha_{0}\right)}{2 h_{0}\left(2 h_{0}-\alpha_{0}\right)}$, and $\alpha_{0}<2 h_{0} / 3$.

Restricting attention to the regions specified in Lemma 1, we derive the optimal locations chosen by the newspapers at the symmetric equilibrium in Equations 16 and 17.

## Single-Homing

$$
(16)-B_{1}^{* *}=B_{2}^{* *}=B_{S}^{* *}=\frac{3 b_{0}}{4}+\frac{T \mathrm{~h}_{0}\left(\frac{1}{2}+\frac{\mathrm{h}_{0}}{3 \alpha_{0}}\right)}{b_{0}}+\sqrt{\left(\frac{3 b_{0}}{4}+\frac{T \mathrm{~h}_{0}}{b_{0}}\left(\frac{1}{2}+\frac{\mathrm{h}_{0}}{3 \alpha_{0}}\right)\right)^{2}-\frac{4 T \mathrm{~h}_{0}{ }^{2}\left(1+\frac{\left.2 \mathrm{Th}_{0}\right)}{3 \mathrm{~b}_{0}{ }^{2}}\right)}{3 \alpha_{0}}}
$$

## Double-Homing

$$
\begin{equation*}
-B_{1}^{* *}=B_{2}^{* *}=B_{D}^{* *}=\frac{3 b_{0}}{4}+T \frac{\alpha_{0}}{2 b_{0}}+\sqrt{\left(\frac{3 b_{0}}{4}+T \frac{\alpha_{0}}{2 b_{0}}\right)^{2}-2 T \alpha_{0}} \tag{17}
\end{equation*}
$$

Proposition 1 follows from the expressions derived in Equations 16 and 17.

PROPOSITION 1. With both advertising and subscription fees contributing to the newspapers' revenues,
(i) When heterogeneity among advertisers is sufficiently large $\left(\alpha_{0}>2 h_{0}\right)$ :

Each advertiser chooses a single newspaper for placing its ads (Single-Homing), and newspapers introduce more bias in their reporting ( $B_{S}^{* *}>B^{M S}$ ). This bias increases as the importance of advertising as a source of revenue increases $\left(\frac{\partial B_{S}^{* *}}{\partial T}>0\right)$.
(ii) When heterogeneity among advertisers is sufficiently small $\left(\alpha_{0}<2 h_{0} / 3\right)$ :

Each advertiser chooses both newspapers for placing its ads (Double-Homing), and newspapers introduce less bias in their reporting $\left(B_{D}^{* *}<B^{M S}\right)$. This bias decreases as the importance of advertising as a source of revenue increases $\left(\frac{\partial B_{D}^{* *}}{\partial T}<0\right)$.

To understand the results reported in Proposition 1, it is important to highlight the new effects influencing the location choice of the newspapers that arise when advertising is added as a source of revenues to supplement subscription fees. The first "readership effect" relates to the intensified incentives of each newspaper to increase its readership (for Newspaper 1 this means increasing $b_{\text {indif }}$, and for Newspaper 2 decreasing $i t$ ). Note that at the symmetric equilibrium
$\left(\right.$ when $\left.b_{\text {indif }}=0\right) \frac{\partial K_{1}^{S}}{\partial b_{\text {indif }}}=\frac{M h_{0}}{3 b_{0}}>0$ and $\frac{\partial K_{1}^{D}}{\partial b_{\text {indif }}}=\frac{M h_{0}}{2 b_{0}}>0 .{ }^{7}$ Hence, irrespective of the type of homing, a newspaper that delivers a bigger readership can command a higher advertising fee from advertisers. This implies that each newspaper has extra incentives to move closer to its competitor's location in order to increase its market share among readers (e.g., $\frac{\partial b_{\text {indif }}}{\partial B_{1}}=\frac{1}{2}>0$ at symmetry, when $P_{1}=P_{2}$ ).

Adding advertising as a source of revenue introduces, though, a second counter force when advertisers Single-Home. We refer to this force as the "incremental pricing effect" to capture the idea that a change in a newspaper's location does not only have a direct effect on the intensity of price competition in the subscription market but may also have an indirect, incremental effect on the intensity of price competition in the advertising market. ${ }^{8}$ When a newspaper modifies its location and advertisers Single-Home, the competing newspaper may have to adjust its advertising fee in order to defend its market share among advertisers. For instance, when Newspaper 1 increases $B_{1}$, it moves closer to the location of Newspaper 2, and due to reduced differentiation, Newspaper 2 is forced to cut subscription fees . In addition, since the new, moderated location of Newspaper 1 offers a larger readership to advertisers, Newspaper 2 has to cut its advertising fee as well in order to defend its market share in the advertising market. ${ }^{9}$ The existence of this "incremental pricing effect" introduces, therefore, incentives for Newspaper 1 to polarize in order to discourage aggressive pricing by Newspaper 2. These incentives are stronger than in an environment where newspapers compete in a single, subscriber market because Newspaper 2 is forced to cut both its advertising and subscription fees.

According to part (i) of Proposition 1, the "incremental pricing effect" present at the Single-Homing equilibrium more than outweighs the objective of increasing readership, thus
leading to intensified bias at the equilibrium when advertising is added as a source of revenues to augment subscription fees. Moreover, this bias increases as the importance of advertising as a source of revenue ( $T$ ) increases. In contrast, according to part (ii) of the Proposition, at the equilibrium with Double-Homing, bias in reporting the news is reduced when advertising supplements subscription fees. At this type of equilibrium, the only additional effect that advertising introduces is the added objective of newspapers to offer bigger readerships to advertisers. Since the market share of each newspaper in the advertising market is fixed at $100 \%$ and the newspapers don't need to defend their market shares among advertisers, the "incremental pricing effect" is non-existent in the Double-Homing environment. Note that the "readership effect" intensifies, in this case, when advertising is a more important source of revenue (large $T$ ). Figure 2 depicts the relationship between the equilibrium locations of the newspapers and the importance of advertising as a source of revenue to the newspapers, as reported in Proposition $1 .{ }^{10}$

## [Insert Figure 2 about Here]

We can use the results reported in Proposition 1 to conjecture how the equilibrium is likely to change in case of less than full coverage of readers. At the Single-Homing equilibrium (when $\alpha_{0}$ is big) bias in reporting is significant. Hence, it is sensible that when the market is less than fully covered, it is consumers with moderate opinions in the neighborhood of $b=0$ who choose to drop out of the market $\left(E U_{b}^{i}<0\right.$ for such consumers). As a result, the subscribers of each newspaper are fewer in number and have more extreme beliefs in comparison to a fully covered market. This new composition of subscribers reduces even further the benefit from Double-Homing. In the Web Appendix, we demonstrate that newspapers may have reduced incentives to polarize as a result of incomplete coverage of the subscriber market. In fact, when
the reservation price of readers is relatively low and their valuation of accurate reporting is high, bias is more moderate than that derived in MS (i.e., smaller than $\frac{3}{2} b_{0}$ ) even though advertisers Single-Home. At the Double-Homing equilibrium (when $\alpha_{0}$ is small) bias is moderate. It is now consumers with very extreme opinions who are likely to drop out of the market. The population of subscribers becomes less heterogeneous, as a result, thus enhancing the benefit from DoubleHoming. In the Web Appendix, we demonstrate, that in this case as well, incomplete coverage may moderate the extent of bias selected by the newspapers if the reservation price of readers (and their valuation of accuracy) is low (high), respectively.

## 4. CONCLUSION

In this paper we extend the work of MS (2005) by investigating media bias when advertising is added as a source of revenue to supplement subscription fees. We show that the additional advertising market introduces two counteracting effects on the behavior of newspapers. First, as newspapers attempt to increase their readership in order to attract advertisers, they moderate slanting in order to appeal to readers having moderate opinions. Second, when advertisers choose to Single-Home a second effect arises that may lead to greater polarization in news reporting. If newspapers moderate bias in this case they are forced to compete more aggressively not only for subscribers, but for advertisers as well. Downward pressure on subscription as well as advertising fees follows. To avoid such intensified price competition, newspapers may choose to increase polarization. We demonstrate that when the heterogeneity among advertisers in appealing to consumers with different political preferences is significant, the attempt to alleviate price competition dominates, thus leading to greater polarization. When this heterogeneity is negligible, reduced polarization is predicted.

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## FOOTNOTES

${ }^{1}$ The appeal parameter of the advertised product that we introduce in the model assumes a value in the vicinity of zero if political preferences do not play an important role in consumers' purchase decisions.
${ }^{2}$ Notice that there is no vertical differentiation between the newspapers in this setting (i.e., the accuracy of the data received by both newspapers is identical: ${\sigma_{d_{1}}}^{2}={\sigma_{d_{2}}}^{2}={\sigma_{d}}^{2}$ ). In the Web Appendix, we demonstrate that our utility specification may also give rise to a tradeoff between vertical and horizontal differentiation. Specifically, when ${\sigma_{d_{1}}}^{2}<{\sigma_{d_{2}}}^{2},\left|B_{1}\right|<\left|B_{2}\right|$.
${ }^{3}$ In the Web Appendix we show the optimality of linear slanting strategies when the newspapers' sole source of revenue is from subscription fees. However, in our analysis, in which both advertising and subscription fees are sources of revenue, we implicitly assume that the linearity of slanting strategies is still valid.
${ }^{4}$ According to Equation 4, the change in the purchase probability for extreme products and consumers is larger than that for moderate products and consumers. As we mention later in footnote 10 , when this feature of our model is not valid, some of our results may change, even though the strategic effects we identify will continue to operate.
${ }^{5}$ Let $F_{b}$ denote the initial probability of purchase in the absence of advertising by an individual with opinion $b$ and $P_{b}$ denote the probability of purchase after advertising such that $P_{b}=F_{b}+E(\alpha, b)$. In order to guarantee that $0 \leq P_{b} \leq 1$ we assume that $1-F_{b}-\alpha_{0} \geq h_{0} \geq \alpha_{0}-F_{b}$ and $F_{b}+\alpha_{0}<1$. Note that these parameter restrictions do not conflict with those given in Lemma 1.
${ }^{6}$ Note that between $2 h_{0} / 3$ and $2 h_{0}$ there is an equilibrium in which while some advertisers Single-Home (place their ads in a single newspaper), others Double-Home (place ads in both outlets). As well, multiple equilibria may arise in this range. (See Web Appendix for derivations.)
${ }^{7}$ The solution for the advertising fees as functions of the locations are: $K_{1}^{S}=M\left(\frac{\alpha_{0}\left(b_{0}^{2}-b_{\text {indif }}{ }^{2}\right)}{2 b_{0}^{2}}+\frac{b_{\text {indif }} h_{0}}{3 b_{0}}\right), K_{2}^{S}=$ $M\left(\frac{\alpha_{0}\left(b_{0}^{2}-b_{\text {indif }}{ }^{2}\right)}{2 b_{0}^{2}}-\frac{b_{\text {indif }} h_{0}}{3 b_{0}}\right) ;$ and $K_{1}^{D}=\frac{M\left(b_{0}+b_{\text {indif }}\right)}{2 b_{0}}\left(h_{0}-\frac{\alpha_{0}\left(b_{0}-b_{\text {indif }}\right)}{2 b_{0}}\right), K_{2}^{D}=\frac{M\left(b_{0}-b_{\text {indif }}\right)}{2 b_{0}}\left(h_{0}-\frac{\alpha_{0}\left(b_{0}+b_{\text {indif }}\right)}{2 b_{0}}\right)$.
${ }^{8}$ Note that this effect does not exist in standard models of horizontal differentiation in which a change in location has implications on price competition in only one market.
${ }^{9}$ As Newspaper 1 increases its readership by increasing $B_{1}$, Newspaper 2 loses market share among advertisers since at the symmetric equilibrium from Equation 2 and from Equation $9 \frac{\partial \alpha_{\text {indif }}}{\partial b_{\text {indif }}}=\frac{2 h_{0}}{b_{0}}>0$. Thus, Newspaper 2 has an incentive to cut its advertising fee since $\frac{\partial K_{2}^{S}}{\partial B_{1}}=-\frac{M h_{0}}{6 b_{0}}<0$ at symmetry.
${ }^{10}$ Note that with a different advertising response function, which implies that the change in purchase probability for moderate products and consumers is larger than that for extreme products and consumers, the readership effect will be stronger, since in this case, the moderate readers will be more valuable for the advertisers, and therefore the newspapers. We predict that while the results for Double-Homing reported in Proposition 1 will continue to hold in such an environment, the results for Single-Homing may change as the readership effect may outweigh the incremental pricing effect.

## FIGURES



Figure 1: Segmentation of the Advertising Market


Figure 2: Equilibrium Locations as a Function of T

## WEB APPENDIX

## Derivations of Equations 16-17 and Proof of Lemma 1

(i) Single-Homing: Second stage prices are obtained by optimizing (12) with respect to $P_{i}$ and $K_{i}$ as follows:

$$
\begin{align*}
& \frac{\partial \pi_{1}}{\partial P_{1}}=\frac{A}{2 \alpha_{0}}\left[\frac{\partial \alpha_{\text {indif }}}{\partial b_{\text {indif }}} \frac{\partial b_{\text {indif }}}{\partial P_{1}} K_{1}\right]+\frac{M_{1}}{2 b_{0}}\left[\left(b_{0}+b_{\text {indif }}\right)+\frac{\partial b_{\text {indif }}}{\partial P_{1}} P_{1}\right]=0  \tag{A1}\\
& \frac{\partial \pi_{2}}{\partial P_{2}}=\frac{A}{2 \alpha_{0}}\left[-\frac{\partial \alpha_{\text {indif }}}{\partial b_{\text {indif }}} \frac{\partial b_{\text {indif }}}{\partial P_{2}} K_{2}\right]+\frac{M_{1}}{2 b_{0}}\left[\left(b_{0}-b_{\text {indif }}\right)-\frac{\partial b_{\text {indif }}}{\partial P_{2}} P_{2}\right]=0  \tag{A2}\\
& \frac{\partial \pi_{1}}{\partial K_{1}}=\frac{A}{2 \alpha_{0}}\left[\frac{\partial \alpha_{\text {indif }}}{\partial K_{1}} K_{1}+\left(\alpha_{0}+\alpha_{\text {indif }}\right)\right]=0 \text { and }  \tag{A3}\\
& \frac{\partial \pi_{2}}{\partial K_{2}}=\frac{A}{2 \alpha_{0}}\left[-\frac{\partial \alpha_{\text {indif }}}{\partial K_{2}} K_{2}+\left(\alpha_{0}-\alpha_{\text {indif }}\right)\right]=0 \tag{A4}
\end{align*}
$$

From (9):

$$
\begin{align*}
& \frac{\partial \alpha_{\text {indif }}}{\partial b_{\text {indif }}}=\frac{2 b_{\text {indif }}}{\left(\mathrm{b}_{0}{ }^{2}-b_{\text {indif }}{ }^{2}\right)} \alpha_{\text {indif }}+\frac{2 \mathrm{~b}_{0} \mathrm{~h}_{0}}{\left(\mathrm{~b}_{0}{ }^{2}-b_{\text {indif }}{ }^{2}\right)} .  \tag{A5}\\
& \frac{\partial \alpha_{\text {indif }}}{\partial K_{1}}=-\frac{2 \mathrm{~b}_{0}{ }^{2}}{\left(\mathrm{M}_{1}+\mathrm{M}_{2}\right)\left(\mathrm{b}_{0}{ }^{2}-b_{\text {indif }}{ }^{2}\right)}, \text { and }  \tag{A6}\\
& \frac{\partial \alpha_{\text {indif }}}{\partial K_{2}}=\frac{2 \mathrm{~b}_{0}{ }^{2}}{\left(\mathrm{M}_{1}+\mathrm{M}_{2}\right)\left(\mathrm{b}_{0}{ }^{2}-b_{\text {indif }}{ }^{2}\right)} . \tag{A7}
\end{align*}
$$

From (2):

$$
\begin{equation*}
\frac{\partial b_{\text {indif }}}{\partial P_{1}}=\frac{\phi+\chi}{2 \phi^{2}\left(\mathrm{~B}_{1}-\mathrm{B}_{2}\right)} \text { and } \frac{\partial b_{\text {indif }}}{\partial P_{2}}=\frac{\phi+\chi}{2 \phi^{2}\left(\mathrm{~B}_{2}-\mathrm{B}_{1}\right)} . \tag{A8}
\end{equation*}
$$

Substituting (2), (9), (A5), (A6) and (A7), into the first order conditions (A1)-(A4), evaluating them at symmetry ( $-B_{1}=B_{2}=B$ ), and solving for $K_{i}$ and $P_{i}$, we get $P_{S}^{* *}$ and $K_{S}^{* *}$ as given in (14). And substituting (A6), (A7) and (9) into (A3) and (A4) and solving for $K_{1}$ and $K_{2}$, one can get equilibrium advertising fees as a function of the locations:

$$
\begin{equation*}
K_{1}^{S}=M\left(\frac{\alpha_{0}\left(b_{0}^{2}-b_{\text {indif }}{ }^{2}\right)}{2 b_{0}^{2}}+\frac{b_{\text {indif }} h_{0}}{3 b_{0}}\right), K_{2}^{S}=M\left(\frac{\alpha_{0}\left(b_{0}^{2}-b_{\text {indif }}{ }^{2}\right)}{2 b_{0}^{2}}-\frac{b_{\text {indif }} h_{0}}{3 b_{0}}\right) \tag{A9}
\end{equation*}
$$

To obtain the equilibrium locations chosen by the newspapers in the first stage, one has to solve first for the second stage fees, $P_{i}\left(B_{i}, B_{j}\right)$ and $K_{i}\left(B_{i}, B_{j}\right)$ as functions of arbitrary location choices (not only symmetric). Substituting the equilibrium strategies back into (12), we obtain the first stage payoff functions designated as $V_{i}\left(B_{i}, B_{j}\right)$. Differentiating with respect to the locations yields from the Envelope Theorem that:

$$
\begin{equation*}
\frac{\partial V_{i}}{\partial B_{i}}=\frac{\partial \pi_{i}}{\partial B_{i}}+\frac{\partial \pi_{i}}{\partial P_{j}} \frac{\partial P_{j}}{\partial B_{i}}+\frac{\partial \pi_{i}}{\partial K_{j}} \frac{\partial K_{j}}{\partial B_{i}}=0 \quad i, j=1,2, i \neq j . \tag{A10}
\end{equation*}
$$

To illustrate the derivation of the first stage equilibrium, we focus on the optimization of Newspaper 1. For this newspaper, the terms of (A10) can be derived as follows:

$$
\begin{align*}
& \frac{\partial \pi_{1}}{\partial B_{1}}=\frac{M_{1}}{2 b_{0}}\left(\frac{\partial b_{\text {indif }}}{\partial B_{1}}\right) P_{1}+\frac{A}{2 \alpha_{0}}\left(\frac{\partial \alpha_{\text {indif }}}{\partial B_{1}}\right) K_{1},  \tag{A11}\\
& \frac{\partial \pi_{1}}{\partial P_{2}} \frac{\partial P_{2}}{\partial B_{1}}=\frac{M_{1}}{2 b_{0}}\left(\frac{\partial b_{\text {indif }}}{\partial P_{2}} \frac{\partial P_{2}}{\partial B_{1}}\right) P_{1}+\frac{A}{2 \alpha_{0}}\left(\frac{\partial \alpha_{\text {indif }}}{\partial b_{\text {indif }}} \frac{\partial b_{\text {indif }}}{\partial P_{2}} \frac{\partial P_{2}}{\partial B_{1}}\right) K_{1} \text { and } \\
& \frac{\partial \pi_{1}}{\partial K_{2}} \frac{\partial K_{2}}{\partial B_{1}}=\frac{A}{2 \alpha_{0}}\left(\frac{\partial \alpha_{\text {indif }}}{\partial K_{2}} \frac{\partial K_{2}}{\partial B_{1}}\right) K_{1} .
\end{align*}
$$

While the expression for $\frac{\partial K_{2}}{\partial B_{1}}$ in (A13) can be directly derived from (A9), to obtain the expression from $\frac{\partial P_{2}}{\partial B_{1}}$ in (A12), we need to utilize the Implicit Function Approach by totally differentiating the first order conditions (A1) and (A2) that determine subscription fees $\left(\frac{\partial \pi_{1}}{\partial P_{1}}=\right.$ 0 and $\frac{\partial \pi_{2}}{\partial P_{2}}=0$ ). We obtain:

$$
\begin{align*}
& d P_{1}\left(\frac{\partial^{2} \pi_{1}}{\partial \mathrm{P}_{1}^{2}}\right)+d P_{2}\left(\frac{\partial^{2} \pi_{1}}{\partial \mathrm{P}_{1} \partial \mathrm{P}_{2}}\right)+d B_{1}\left(\frac{\partial^{2} \pi_{1}}{\partial \mathrm{P}_{1} \partial \mathrm{~B}_{1}}\right)+d B_{2}\left(\frac{\partial^{2} \pi_{1}}{\partial \mathrm{P}_{1} \partial \mathrm{~B}_{2}}\right)=0 \text { and }  \tag{A14}\\
& d P_{1}\left(\frac{\partial^{2} \pi_{2}}{\partial \mathrm{P}_{1} \partial \mathrm{P}_{2}}\right)+d P_{2}\left(\frac{\partial^{2} \pi_{2}}{\partial \mathrm{P}_{2}^{2}}\right)+d B_{1}\left(\frac{\partial^{2} \pi_{2}}{\partial \mathrm{P}_{2} \partial \mathrm{~B}_{1}}\right)+d B_{2}\left(\frac{\partial^{2} \pi_{2}}{\partial \mathrm{P}_{2} \partial \mathrm{~B}_{2}}\right)=0 . \tag{A15}
\end{align*}
$$

From (A14) and (A15):

$$
\left[\begin{array}{ll}
\frac{\partial \mathrm{P}_{1}}{\partial B_{1}} & \frac{\partial \mathrm{P}_{1}}{\partial B_{2}}  \tag{A16}\\
\frac{\partial \mathrm{P}_{2}}{\partial B_{1}} & \frac{\partial \mathrm{P}_{2}}{\partial B_{2}}
\end{array}\right]=-\left[\begin{array}{cc}
\frac{\partial^{2} \pi_{1}}{\partial \mathrm{P}_{1}{ }^{2}} & \frac{\partial^{2} \pi_{1}}{\partial \mathrm{P}_{1} \partial \mathrm{P}_{2}} \\
\frac{\partial^{2} \pi_{2}}{\partial \mathrm{P}_{1} \partial \mathrm{P}_{2}} & \frac{\partial^{2} \pi_{2}}{\partial \mathrm{P}_{2}{ }^{2}}
\end{array}\right]^{-1}\left[\begin{array}{cc}
\frac{\partial^{2} \pi_{1}}{\partial \mathrm{P}_{1} \partial \mathrm{~B}_{1}} & \frac{\partial^{2} \pi_{1}}{\partial \mathrm{P}_{1} \partial \mathrm{~B}_{2}} \\
\frac{\partial^{2} \pi_{2}}{\partial \mathrm{P}_{2} \partial \mathrm{~B}_{1}} & \frac{\partial^{2} \pi_{2}}{\partial \mathrm{P}_{2} \partial \mathrm{~B}_{2}}
\end{array}\right] .
$$

Using (A1) and (A2) in evaluating (A16) at the symmetric equilibrium yields:

$$
\left[\begin{array}{ll}
\frac{\partial \mathrm{P}_{1}}{\partial B_{1}} & \frac{\partial \mathrm{P}_{1}}{\partial B_{2}}  \tag{A17}\\
\frac{\partial \mathrm{P}_{2}}{\partial B_{1}} & \frac{\partial \mathrm{P}_{2}}{\partial B_{2}}
\end{array}\right]=\left[\begin{array}{cc}
-\frac{\phi+\chi}{2 \phi^{2} \mathrm{~B}}(2-Z) & \frac{\phi+\chi}{2 \phi^{2} \mathrm{~B}}(1-Z) \\
\frac{\phi+\chi}{2 \phi^{2} \mathrm{~B}}(1-Z) & -\frac{\phi+\chi}{2 \phi^{2} \mathrm{~B}}(2-Z)
\end{array}\right]^{-1}\left[\begin{array}{rr}
-Y-W & -Y+W \\
Y-W & Y+W
\end{array}\right] ;
$$

where $\mathrm{Z} \stackrel{\text { def }}{=} \frac{A(\phi+\chi) \mathrm{Mh}_{0}{ }^{2}}{6 M_{1} \phi^{2} b_{0} \alpha_{0} B}, Y \stackrel{\text { def }}{=} 1-\frac{A \mathrm{M}(\phi+\chi) \mathrm{h}_{0}{ }^{2}}{6 M_{1} \alpha_{0} b_{0} \phi^{2} \mathrm{~B}}$ and $W \stackrel{\text { def }}{=}-\frac{\phi+\chi}{4 \phi^{2} \mathrm{~B}^{2}} P^{* *}-\frac{A \mathrm{M}}{M_{1}} \frac{(\phi+\chi) \mathrm{h}_{0}}{4 \phi^{2} B^{2}}$.

For second order condition, the determinant of the inverted matrix on the RHS of (A17) should be positive implying that $\mathrm{Z}<1.5$. From (A17), therefore:

$$
\begin{equation*}
\frac{\partial \mathrm{P}_{2}}{\partial B_{1}}=\frac{\partial \mathrm{P}_{1}}{\partial B_{2}}=\frac{2 \phi^{2} \mathrm{~B}}{\phi+\chi}\left(W-\frac{Y}{3-2 Z}\right) . \tag{A18}
\end{equation*}
$$

We can now complete the characterization of the optimal location choice of Newspaper 1. Using (A11) - (A13), as well as the derivation for $\frac{\partial \mathrm{K}_{2}}{\partial B_{1}}$ from (A9) and $\frac{\partial \mathrm{P}_{2}}{\partial B_{1}}$ from (A18) in (A10), we obtain at the symmetric equilibrium:

$$
\begin{equation*}
\frac{\partial V_{1}}{\partial B_{1}}=\frac{M_{1}}{4 b_{0}} P_{S}^{* *}+\frac{M_{1}}{2} \frac{\partial P_{2}}{\partial B_{1}}+\frac{A}{3 \alpha_{0}} K_{S}^{* *} \frac{h_{0}}{\mathrm{~b}_{0}}=0, \text { where } \frac{\partial P_{2}}{\partial B_{1}} \text { is given by (A18). } \tag{A19}
\end{equation*}
$$

At the symmetric equilibrium when $-B_{1}=B_{2}=B$, we obtain from (A19) a quadratic equation as follows:

$$
\begin{equation*}
B^{2}-B\left(\frac{3 b_{0}}{2}+2 T\left(\frac{h_{0}}{3 \alpha_{0}}+\frac{1}{2}\right) \frac{h_{0}}{\mathrm{~b}_{0}}\right)+4 T \frac{\mathrm{~h}_{0}{ }^{2}}{3 \alpha_{0}}\left(1+\frac{2 T h_{0}}{3 \mathrm{~b}_{0}{ }^{2}}\right)=0 . \tag{A20}
\end{equation*}
$$

The two roots of this quadratic equation are:
$B_{S}^{* *}=\frac{3 b_{0}}{4}+T \frac{\mathrm{~h}_{0}}{b_{0}}\left(\frac{1}{2}+\frac{\mathrm{h}_{0}}{3 \alpha_{0}}\right) \mp \sqrt{\Delta} ;$ where $\Delta \stackrel{\text { def }}{=}\left(\frac{3 b_{0}}{4}+T \frac{\mathrm{~h}_{0}}{b_{0}}\left(\frac{1}{2}+\frac{\mathrm{h}_{0}}{3 \alpha_{0}}\right)\right)^{2}-\frac{4 T \mathrm{~h}_{0}{ }^{2}}{3 \alpha_{0}}\left(1+\frac{2 T h_{0}}{3 \mathrm{~b}_{0}{ }^{2}}\right)$.
Only the bigger root guarantees stability of reaction functions (i.e. $\frac{\partial^{2} V_{1}}{\partial B_{1}{ }^{2}}<0$.) As a result, the optimal location at the Single-Homing equilibrium is given in (16). Note that if $\Delta<0$ the quadratic expression (A20) is positive for all values of $B$. Hence, $\frac{\partial V_{1}}{\partial B_{1}}>0$ for all $B$ and the optimal location is the corner solution $B^{* *}=0$. Hence, $B^{* *} \neq 0$ if:

$$
\begin{equation*}
\Delta=\left(\frac{3 b_{0}}{4}+T \frac{\mathrm{~h}_{0}}{b_{0}}\left(\frac{1}{2}+\frac{\mathrm{h}_{0}}{3 \alpha_{0}}\right)\right)^{2}-\frac{4 T \mathrm{~h}_{0}{ }^{2}}{3 \alpha_{0}}\left(1+\frac{2 T h_{0}}{3 \mathrm{~b}_{0}{ }^{2}}\right)>0 . \tag{A21}
\end{equation*}
$$

Inequality (A21) holds if:

$$
\begin{equation*}
T<\frac{3 b_{0}{ }^{2} \alpha_{0}}{2 \mathrm{~h}_{0}\left(2 \mathrm{~h}_{0}-\alpha_{0}\right)} . \tag{A22}
\end{equation*}
$$

We next investigate the conditions under which $P^{* *}$ is positive. From (14):

$$
\begin{equation*}
P^{* *}=\frac{4 B \phi^{2} b_{0}}{\phi+\chi}-\frac{A \mathrm{Mh}_{0}}{M_{1}}=\frac{4 \phi^{2}}{\phi+\chi}\left(B b_{0}-2 T \mathrm{~h}_{0}\right) . \tag{A23}
\end{equation*}
$$

$P^{* *}>0$ implies $B>2 \frac{T \mathrm{~h}_{0}}{b_{0}}$ or equivalently from (16):

$$
\begin{equation*}
\underbrace{\sqrt{\left(\frac{3 b_{0}}{4}+T\left(\frac{h_{0}}{3 \alpha_{0}}+\frac{1}{2}\right)\right)^{2}-\frac{4 T \mathrm{~h}_{0}{ }^{2}}{3 \alpha_{0}}\left(1+\frac{2 T h_{0}}{3 \mathrm{~b}_{0}{ }^{2}}\right)}}_{L H S}>\underbrace{2 \frac{T \mathrm{~h}_{0}}{b_{0}}-\frac{3 b_{0}}{4}-T \frac{\mathrm{~h}_{0}}{b_{0}}\left(\frac{1}{2}+\frac{\mathrm{h}_{0}}{3 \alpha_{0}}\right)}_{R H S} . \tag{A24}
\end{equation*}
$$

Given that the LHS is positive, there are two cases where this inequality can hold: when RHS is negative (Case 1) and when both sides are positive but the LHS is bigger (Case 2). Case

1 implies that $T<\frac{9 b_{0}{ }^{2} \alpha_{0}}{2 \mathrm{~h}_{0}\left(9 \alpha_{0}-2 \mathrm{~h}_{0}\right)}$. For Case 2, squaring both sides of (A24) and solving for $T$ yields $T<\frac{3 b_{0}{ }^{2}\left(9 \alpha_{0}-4 \mathrm{~h}_{0}\right)}{2 \mathrm{~h}_{0}\left(9 \alpha_{0}-2 \mathrm{~h}_{0}\right)}$. Combining the two cases, yields that $P^{* *}>0$ if:
(A25) $\quad T<\frac{3 b_{0}{ }^{2}\left(9 \alpha_{0}-4 \mathrm{~h}_{0}\right)}{2 \mathrm{~h}_{0}\left(9 \alpha_{0}-2 \mathrm{~h}_{0}\right)}$.
Combining (A25) and (A22) yields the condition of part (i) of Lemma 1.
(ii) Double-Homing: Using a very similar approach to that developed when advertisers SingleHome, we obtain the following first order condition for the choice of location in the first stage.

$$
\begin{equation*}
\frac{d \pi_{1}}{d B_{1}}=A\left[\frac{\partial K_{1}}{\partial b_{\text {indif }}} \frac{\partial b_{\text {indif }}}{\partial B_{1}}+\frac{\partial K_{1}}{\partial B_{1}}\right]+\frac{M_{1} P_{1}}{2 b_{0}} \frac{\partial b_{\text {indif }}}{\partial B_{1}}+\left[\frac{A \partial K_{1}}{\partial b_{\text {indif }}} \frac{\partial b_{\text {indif }}}{\partial P_{2}}+\frac{M_{1} P_{1}}{2 b_{0}} \frac{\partial b_{\text {indif }}}{\partial P_{2}}\right] \frac{\partial P_{2}}{\partial B_{1}}=0, \tag{A26}
\end{equation*}
$$

where the expression for $K_{1}$, which follows from the maximization of (11) is:

$$
\begin{equation*}
K_{1}^{D}=\frac{M\left(b_{0}+b_{\text {indif }}\right)}{2 b_{0}}\left(h_{0}-\frac{\alpha_{0}\left(b_{0}-b_{\text {indif }}\right)}{2 b_{0}}\right), \quad K_{2}^{D}=\frac{M\left(b_{0}-b_{\text {indif }}\right)}{2 b_{0}}\left(h_{0}-\frac{\alpha_{0}\left(b_{0}+b_{\text {indif }}\right)}{2 b_{0}}\right) . \tag{A27}
\end{equation*}
$$

At the symmetric equilibrium (A26) reduces to:

$$
\begin{equation*}
\frac{d \pi_{1}}{d B_{1}}=\frac{A M \mathrm{~h}_{0}}{4 b_{0}}+\frac{M_{1}}{2}\left[\frac{P}{2 b_{0}}+\frac{\partial P_{2}}{\partial B_{1}}\right]=0 . \tag{A28}
\end{equation*}
$$

To derive the expression for $\frac{\partial P_{2}}{\partial B_{1}}$, we have to use, once again, the Implicit Function Approach, by totally differentiating the first order condition for the subscription fees $P_{i}$. Those conditions are:

$$
\begin{align*}
& \frac{\partial \pi_{1}}{\partial P_{1}}=M_{1}\left(b_{0}+b_{\text {indif }}\right)-\frac{(\phi+\chi)}{2 \phi^{2}\left(B_{2}-B_{1}\right)}\left\{M A\left(h_{0}+\frac{\alpha_{0}}{h_{0}} b_{\text {indif }}\right)+M_{1} P_{1}\right\}=0,  \tag{A29}\\
& \frac{\partial \pi_{2}}{\partial P_{2}}=M_{1}\left(b_{0}-b_{\text {indif }}\right)-\frac{(\phi+\chi)}{2 \phi^{2}\left(B_{2}-B_{1}\right)}\left\{\operatorname{MA}\left(\mathrm{h}_{0}-\frac{\alpha_{0}}{h_{0}} b_{\text {indif }}\right)+M_{1} P_{2}\right\}=0 .
\end{align*}
$$

Total differentiation of the first order conditions yields the following system of equations for $\frac{d \mathrm{P}_{1}}{d B_{1}}$
 order conditions $R<\frac{3}{4} M_{1}$ or $T<\frac{3}{4} \frac{b_{0} B}{\alpha_{0}}$. Solving for $\frac{d \mathrm{P}_{2}}{d B_{1}}$, we obtain: $\frac{d \mathrm{P}_{2}}{d B_{1}}=\frac{2 \phi^{2}}{(\phi+\chi)}\left[-b_{0}+\right.$ $\left.\frac{\frac{B}{2}\left(\frac{T \alpha_{0}}{b_{0}}-\frac{B}{2}\right)}{\frac{3}{4} B-\frac{T \alpha_{0}}{b_{0}}}\right]$. Substituting back into (A28), yields a quadratic equation in $B$ as follows:

$$
\begin{equation*}
B^{2}-B\left(\frac{T \alpha_{0}}{b_{0}}+\frac{3}{2} b_{0}\right)+2 T \alpha_{0}=0 \tag{A30}
\end{equation*}
$$

There are two roots to this equation. However, only one satisfies also the condition for stability of reaction function. It is given in equation (17). The discriminant of the solution in (17) is
positive if $T<\frac{b_{0}{ }^{2}}{2 \alpha_{0}}$. As well, to guarantee that $P^{* *}>0$, it follows from (15) that $T<\frac{B b_{0}}{2 h_{0}}$. Using the expression for $B$ from (17) in the last inequality, yields $T<\frac{\left(3 h_{0}-2 \alpha_{0}\right) b_{0}{ }^{2}}{2 h_{0}\left(2 h_{0}-\alpha_{0}\right)}$. This is a more demanding constraint than the one necessary to insure that the discriminant is positive, thus yielding part (ii) of Lemma 1.

## Proof of Proposition 1

First note from (16) and (17) that $B_{S}{ }^{* *}=B_{D}{ }^{* *}=\frac{3}{2} b_{0}$ when $T=0$. Differentiating the expressions of $B_{S}{ }^{* *}$ and $B_{D}{ }^{* *}$ with respect to $T$ for the range of parameters that support each type of equilibrium yields $\frac{\partial B_{S}^{* *}}{\partial T}>0$ and $\frac{\partial B_{D}^{* *}}{\partial T}<0$. Hence, $B_{S}^{* *}>B^{M S}$ and $B_{D}^{* *}<B^{M S}$.

## Regions of the Parameter $\alpha_{0}$ that Support Single and Double-Homing

(i) Single-Homing: To ensure that Single-Homing is an equilibrium, we use $K_{S}^{* *}$ from (14) in (8) and evaluate (8) at the symmetric equilibrium (i.e., $\left.b_{\text {indif }}=0\right)$ to obtain $\hat{\alpha}_{1}=2\left(\alpha_{0}-h_{0}\right)$ and $\hat{\alpha}_{2}=2\left(h_{0}-\alpha_{0}\right)$. To guarantee that the interior interval in Figure 1 disappears, we impose the restriction that $\hat{\alpha}_{1} \geq \hat{\alpha}_{2}$, which happens when $\alpha_{0} \geq h_{0}$.
(ii) Partial Double-Homing: In Partial Double-Homing equilibrium, in which some advertisers Double-Home and some Single-Home (i.e., $\alpha_{0}>\hat{\alpha}_{2}>\hat{\alpha}_{1}>-\alpha_{0}$ ), the newspapers choose subscription and advertising fees in the second stage to maximize the objectives:

$$
\pi_{1}=A \frac{\alpha_{0}+\widehat{\alpha}_{2}}{2 \alpha_{0}} K_{1}+M_{1} \frac{b_{0}+b_{\text {indif }}}{2 b_{0}} P_{1}, \quad \pi_{2}=A \frac{\alpha_{0}-\widehat{\alpha}_{1}}{2 \alpha_{0}} K_{2}+M_{1} \frac{b_{0}-b_{\text {indif }}}{2 b_{0}} P_{2} .
$$

When the newspapers locate symmetrically so that $-B_{1}=B_{2}=B$, the solution to this maximization can be obtained as: $P_{D}^{* *}=\frac{4 B \phi^{2} b_{0}}{\phi+\chi}-\frac{A \mathrm{~h}_{0}}{\frac{M_{1}}{\mathrm{M}}} \frac{1}{2 \alpha_{0}}\left[h_{0}+\frac{\alpha_{0}}{2}\right]$ and $K_{D}^{* *}=\frac{M}{4}\left[h_{0}+\frac{\alpha_{0}}{2}\right]$. Using $K_{D}^{* *}$ in (8) and evaluating it at the symmetry (i.e., $b_{\text {indif }}=0$ ) we obtain $\hat{\alpha}_{1}=\frac{\alpha_{0}}{2}-h_{0}$ and $\hat{\alpha}_{2}=h_{0}-\frac{\alpha_{0}}{2}$. Hence, $\hat{\alpha}_{2} \geq \hat{\alpha}_{1}$ if $\alpha_{0} \leq 2 h_{0}$, and $\hat{\alpha}_{2}<\alpha_{0}$ if $\alpha_{0}>\frac{2}{3} h_{0}$.
(iii) Double-Homing: All advertisers will Double-Home when $\hat{\alpha}_{2} \geq \alpha_{0}$. It follows from part (ii) that this happens if $\alpha_{0} \leq \frac{2}{3} h_{0}$.

From (i), (ii) and (iii) we conclude that while for $\alpha_{0}>2 h_{0}$ Single-Homing is the unique equilibrium, for $\alpha_{0} \leq \frac{2}{3} h_{0}$ Double-Homing is the unique equilibrium. It also follows from the
above that between $h_{0}$ and $2 h_{0}$ Single-Homing and Partial Double-Homing equilibria may coexist.

## Optimality of Linear Decision Rules

We show the optimality of linear slanting strategies when the newspapers' sole source of revenue is subscription fees. To this end we first derive the first order conditions that follow from the newspapers' first stage location choices without restricting the functional form of the slanting strategies $s_{i}\left(B_{i}, d\right), i=1,2$. Then we show that a linear slanting rule satisfies these first order conditions.

The consumer who is indifferent between the two newspapers satisfies the equation $E U_{b}{ }^{1}=E U_{b}{ }^{2}$ where $U_{b}{ }^{i}$ is given by (1). Solving this equation and using the distributional properties of the random variable $d\left(d \sim N\left(0, \sigma_{d}^{2}\right)\right)$ and the uniform distribution of the parameter $b$, yields:

$$
\begin{equation*}
b_{\text {indif }}=\frac{\left(P_{2}-P_{1}\right)}{2 \phi\left(E s_{2}-E s_{1}\right)}+\frac{(\phi+\chi)}{2 \phi} \frac{\left(E s_{2}^{2}-E s_{1}^{2}\right)}{\left(E s_{2}-E s_{1}\right)}+\frac{E\left(d\left(s_{2}-s_{1}\right)\right)}{\left(E s_{2}-E s_{1}\right)} . \tag{A31}
\end{equation*}
$$

In the second stage, newspapers set their prices $P_{1}$ and $P_{2}$ to maximize (3). First order conditions for this maximization are:

$$
\begin{align*}
& \frac{M_{1}}{2 b_{0}}\left(\left(b_{\text {indif }}+b_{0}\right)-\frac{P_{1}}{2 \phi\left(E s_{2}-E s_{1}\right)}\right)=0  \tag{A32}\\
& \frac{M_{1}}{2 b_{0}}\left(\left(b_{0}-b_{\text {indif }}\right)-\frac{P_{2}}{2 \phi\left(E s_{2}-E s_{1}\right)}\right)=0 . \tag{A33}
\end{align*}
$$

At the symmetric equilibrium (i.e., $B_{1}=B_{2}=B, b_{\text {indif }}=0$ ) the solution to (A32) and (A33) is:

$$
\begin{equation*}
P_{1}=P_{2}=P^{*}=2 \phi\left(E s_{2}-E s_{1}\right) b_{0} \tag{A34}
\end{equation*}
$$

To obtain the equilibrium locations chosen by the newspapers in the first stage, one has to solve first for the second stage fees, $P_{i}\left(B_{i}, B_{j}\right)$ as functions of arbitrary location choices (not only symmetric). Substituting the equilibrium strategies back into (3), we obtain the first stage payoff functions $V_{i}\left(B_{i}, B_{j}\right)$. Differentiating with respect to the locations yields from the Envelope Theorem that:

$$
\begin{equation*}
\frac{\partial V_{i}}{\partial B_{i}}=\frac{\partial \pi_{i}}{\partial B_{i}}+\frac{\partial \pi_{i}}{\partial P_{j}} \frac{\partial P_{j}}{\partial B_{i}}=0 \quad i, j=1,2, i \neq j \tag{A35}
\end{equation*}
$$

It follows from (A35) and (3) that,

$$
\begin{align*}
& \frac{\partial b_{\text {indif }}}{\partial B_{1}}+\frac{\partial b_{\text {indif }}}{\partial P_{2}} \frac{\partial P_{2}}{\partial B_{1}}=0,  \tag{A36}\\
& -\frac{\partial b_{\text {indif }}}{\partial B_{2}}-\frac{\partial b_{\text {indif }}}{\partial P_{1}} \frac{\partial P_{1}}{\partial B_{2}}=0 . \tag{A37}
\end{align*}
$$

From (A31):

$$
\begin{align*}
\frac{\partial b_{\text {indif }}}{\partial P_{1}}= & \frac{-1}{2 \phi\left(E s_{2}-E s_{1}\right)},  \tag{A38}\\
\frac{\partial b_{\text {indif }}}{\partial B_{2}}= & -\frac{\left(P_{2}-P_{1}\right) E s_{2}{ }^{\prime}\left(B_{2}\right)}{2 \phi\left(E s_{2}-E s_{1}\right)^{2}}+\frac{(\phi+\chi)}{2 \phi} \frac{2 E s_{2} s_{2}{ }^{\prime}\left(B_{2}\right)\left(E s_{2}-E s_{1}\right)-E s_{2}{ }^{\prime}\left(B_{2}\right)\left(E s_{2}^{2}-E s_{1}^{2}\right)}{\left(E s_{2}-E s_{1}\right)^{2}}+ \\
& \frac{E\left\{d\left[s_{2}{ }^{\prime}\left(B_{2}\right)\left(E s_{2}-E s_{1}\right)-E s_{2}{ }^{\prime}\left(B_{2}\right)\left(s_{2}-s_{1}\right)\right]\right\}}{\left(E s_{2}-E s_{1}\right)^{2}}, \text { where } s_{2}^{\prime}\left(B_{2}\right) \stackrel{\text { def }}{=} \frac{\partial s_{2}\left(B_{2}, d\right)}{\partial B_{2}} .
\end{align*}
$$

To obtain $\frac{\partial P_{1}}{\partial B_{2}}$, we utilize, once again, the Implicit Function approach by totally differentiating the first order conditions (A32) and (A33), and solving the resulting system of equations using $P^{*}$ from (A34). This procedure yields

$$
\begin{equation*}
\frac{\partial P_{1}}{\partial B_{2}}=\frac{2 \phi\left(E s_{2}-E s_{1}\right)}{3} \frac{\partial b_{\text {indif }}}{\partial B_{2}}+2 \phi b_{0} E s_{2}{ }^{\prime}\left(B_{2}\right) . \tag{A40}
\end{equation*}
$$

Suppose the firms use a linear decision rule:

$$
\begin{align*}
& s_{i}\left(B_{i}, d\right)=A_{1} B_{i}+A_{2} d, \text { then: }  \tag{A41}\\
& s_{i}{ }^{\prime}\left(B_{i}\right)=A_{1}  \tag{A42}\\
& E s_{2}-E s_{1}=A_{1}\left(B_{2}-B_{1}\right)  \tag{A43}\\
& E s_{2}^{2}-E s_{1}^{2}=A_{1}^{2}\left(B_{2}^{2}-B_{1}^{2}\right) \tag{A44}
\end{align*}
$$

Substituting (A42)-(A44) into (A38)-(A40), and using these in (A37) at the symmetric equilibrium yields:

$$
\begin{equation*}
-\frac{(\phi+\chi)}{3 \phi} A_{1}+\frac{b_{0}}{2 B}=0 . \tag{A45}
\end{equation*}
$$

From (A45):

$$
\begin{equation*}
B=\frac{3}{2} b_{0} \frac{\phi}{(\phi+\chi) A_{1}} . \tag{A46}
\end{equation*}
$$

Thus, the linear slanting rule (A41) allows us to solve the first order condition in (A37) and find an optimal location as given in (A46). Since the newspapers are symmetric, the same rule also satisfies the first order condition for Newspaper 1(i.e., (A36)) as well.

We will now show that this rule can be expressed as $s_{i}(d)=\frac{\phi}{(\phi+\chi)}\left(B_{i}-d\right)$ for newspaper $i$. We will assume that Newspaper 1 follows such a decision rule and demonstrate that the best response of Newspaper 2 is to follow such a rule, as well. Specifically, assuming that
$s_{1}(d)=\frac{\phi}{(\phi+\chi)}\left(B_{1}-d\right)$ we designate the linear rule followed by Newspaper 2 as $s_{2}(d)=m+$ $A_{2} d$, where $m \stackrel{\text { def }}{=} A_{1} B_{2}$.

Given the assumed behavior of Newspaper 1, a consumer who chooses to subscribe to it derives the expected payoff: $E U_{b}{ }^{1}=\bar{u}-\frac{\phi^{2}}{\phi+\chi}\left(B_{1}-b\right)^{2}-\frac{\chi \phi}{\phi+\chi}\left(b^{2}+\sigma_{d}^{2}\right)-P_{1}$. Her payoff when choosing Newspaper 2 is:

$$
E U_{b}^{2}=\bar{u}-(\phi+\chi)\left(m^{2}+A_{2}^{2} \sigma_{d}^{2}\right)-\phi\left(b^{2}+\sigma_{d}^{2}\left(2 A_{2}+1\right)-2 b m\right)-P_{2}
$$

To find the consumer who is indifferent between the two newspapers we solve the equation $E U_{b}{ }^{1}=E U_{b}{ }^{2}$ for $b$ as follows:

$$
\begin{equation*}
b_{\text {indif }}=\frac{(\phi+\chi) m^{2}-\frac{\phi^{2}}{\phi+\chi} B_{1}{ }^{2}+(\phi+\chi) \sigma_{d}^{2}\left(A_{2}+\frac{\phi}{\phi+\chi}\right)^{2}+P_{2}-P_{1}}{2 \phi\left(m-\frac{\phi}{\phi+\chi} B_{1}\right)} \tag{A47}
\end{equation*}
$$

Assuming without loss of generality that $m>\frac{\chi}{\phi+\chi} B_{1}$, in the second stage the newspapers choose their subscription fees to maximize: $\pi_{1}=\frac{\left(b_{0}+b_{\text {indif }}\right)}{2 b_{0}} P_{1}$ and $\pi_{2}=\frac{\left(b_{0}-b_{\text {indif }}\right)}{2 b_{0}} P_{2}$. Note that $m>\frac{\chi}{\phi+\chi} B_{1}$ simply guarantees that Newspaper 2 serves the upper end of subscribers above $b_{\text {indif }}$ and Newspaper 1 serves the lower end. Once the coefficients are derived this assumption is indeed satisfied as $B_{2}>B_{1}$. Optimizing with respect to $P_{1}$ and $P_{2}$, yields the second stage prices as a function of $B_{1}, m$ and $A_{2}$ as follows:

$$
\begin{align*}
& P_{1}=2 \phi\left(m-\frac{\phi}{\phi+\chi} B_{1}\right) b_{0}+\frac{\left[(\phi+\chi) m^{2}-\frac{\phi^{2}}{\phi+\chi} B_{1}^{2}+(\phi+\chi) \sigma_{d}^{2}\left(A_{2}+\frac{\phi}{\phi+\chi}\right)^{2}\right]}{3},  \tag{A48}\\
& P_{2}=2 \phi\left(m-\frac{\phi}{\phi+\chi} B_{1}\right) b_{0}-\frac{\left[(\phi+\chi) m^{2}-\frac{\phi^{2}}{\phi+\chi} B_{1}^{2}+(\phi+\chi) \sigma_{d}^{2}\left(A_{2}+\frac{\phi}{\phi+\chi}\right)^{2}\right]}{3} .
\end{align*}
$$

It follows, therefore, that:

$$
\frac{\partial P_{1}}{\partial m}=2 \phi b_{0}+\frac{2(\phi+\chi) m}{3}, \quad \frac{\partial P_{1}}{\partial A_{2}}=\frac{2(\phi+\chi) \sigma_{d}^{2}\left(A_{2}+\frac{\phi}{\phi+\chi}\right)}{3} .
$$

When Newspaper 2 chooses its slanting strategy rule in stage 1 , namely $m$ and $A_{2}$, it optimizes its payoff function $\pi_{2}$, given the prices established subsequently in the second stage. Substituting for $P_{1}\left(m, A_{2}, B_{1}\right)$ and $P_{2}\left(m, A_{2}, B_{1}\right)$ back into the payoff functions, yields the first stage payoff function for Newspaper 2, $V_{2}\left(m, A_{2}, B_{1}\right)$. Using the Envelope Theorem:

$$
\begin{align*}
& \frac{\partial V_{2}}{\partial m}=-\frac{P_{2}}{2 b_{0}}\left[\frac{\partial b_{\text {indif }}}{\partial m}+\frac{\partial b_{\text {indif }}}{\partial P_{1}} \frac{\partial P_{1}}{\partial m}\right]=-\frac{P_{2}}{2 b_{0}} \frac{\left[-b_{0}+\frac{2(\phi+\chi)}{3} m-b_{\text {indif }}\right]}{\left[m-\frac{\phi}{\phi+\chi_{1}}\right]},  \tag{A49}\\
& \frac{\partial V_{2}}{\partial A_{2}}=-\frac{P_{2}}{2 b_{0}}\left[\frac{\partial b_{\text {indif }}}{\partial A_{2}}+\frac{\partial b_{\text {indif }}}{\partial P_{1}} \frac{\partial P_{1}}{\partial A_{2}}\right]=-\frac{P_{2}}{3 b_{0}} \frac{\left[(\phi+\chi) \sigma_{d}^{2}\left(A_{2}+\frac{\phi}{\phi+\chi}\right)\right]}{\phi\left[m-\frac{\phi}{\phi+\chi} B_{1}\right]} .
\end{align*}
$$

From the second equation of (A49), it follows that $\frac{\partial V_{2}}{\partial A_{2}}=0$ when $A_{2}=-\frac{\phi}{\phi+\chi}$. Hence, the decision rule of Newspaper $2 s_{2}\left(B_{2}, d\right)=A_{1} B_{2}+A_{2} d$ can be written as $s_{2}(d)=\frac{\phi}{(\phi+\chi)}\left(B_{2}-d\right)$ where $A_{1}$ has been normalized to $\frac{\phi}{(\phi+\chi)}$. To find the value of $B_{2}$, we further restrict our attention to symmetric Bayesian equilibria, which implies that $b_{\text {indif }}=0$. Substituting into the first equation of (A49), implies that $m=\frac{3 b_{0}}{2} \frac{\phi}{\phi+\chi}$, thus, $B_{2}=\frac{3 b_{0}}{2}$, and by the symmetry assumption, $B_{1}=-\frac{3 b_{0}}{2}$.

## Incomplete Coverage of the Subscribers Market

## Single-Homing:

In this section, we demonstrate that when advertisers Single-Home and when the subscriber market is not covered as in Figure (A1), there exist conditions under which $B_{2}<\frac{3}{2} b_{0}$


## Figure A1: Segmentation of the Subscriber Market

The reader who is indifferent between buying Newspaper 1 and not buying at all satisfies the equation:

$$
\begin{equation*}
E U_{b}{ }^{1}=0 \text { where } \tag{A50}
\end{equation*}
$$

$E U_{b}{ }^{1}=-\phi b^{2}+\frac{2 b \phi^{2} B_{1}}{\phi+\chi}+\bar{u}-P_{1}-\frac{B_{1}^{2} \phi^{2}}{\phi+\chi}-\frac{\sigma_{d}^{2} \chi \phi}{\phi+\chi}$ from (1).
The above equation has two roots:

$$
\begin{equation*}
b_{1}=\frac{\phi B_{1}}{\phi+\chi} \mp \sqrt{\frac{\phi^{2} B_{1}^{2}}{(\phi+\chi)^{2}}+\frac{1}{\phi}\left(\bar{u}-P_{1}-\frac{B_{1}^{2} \phi^{2}}{\phi+\chi}-\frac{\sigma_{d}^{2} \chi \phi}{\phi+\chi}\right)} \tag{A51}
\end{equation*}
$$

Notice that for the segmentation given in Figure A1 to hold we need $\frac{\partial E U_{b}{ }^{1}}{\partial b}=-2 \phi b+\frac{2 b \phi^{2} B_{1}}{\phi+\chi}<$ 0 at $b=b_{1}$ which implies:
(A52) $\quad b_{1}>\frac{\phi B_{1}}{(\phi+\chi)}$.
Further at $b=0$, we should have $E U_{b}{ }^{1}<0$ and at $b=-b_{0}, E U_{b}{ }^{1}>0$, thus :

$$
\begin{equation*}
\bar{u}-P_{1}-\frac{B_{1}^{2} \phi^{2}}{\phi+\chi}-\frac{\sigma_{d}^{2} \chi \phi}{\phi+\chi}<0, \text { and } \bar{u}-P_{1}-\frac{B_{1}^{2} \phi^{2}}{\phi+\chi}-\frac{\sigma_{d}^{2} \chi \phi}{\phi+\chi}-\phi b_{0}^{2}-\frac{2 b_{0} \phi^{2} B_{1}}{\phi+\chi}>0 . \tag{A53}
\end{equation*}
$$

The root in (A51) that satisfies (A52) is:

$$
\begin{equation*}
b_{1}=\frac{\phi B_{1}}{\phi+\chi}+\sqrt{\frac{\phi^{2} B_{1}^{2}}{(\phi+\chi)^{2}}+\frac{1}{\phi}\left(\bar{u}-P_{1}-\frac{B_{1}^{2} \phi^{2}}{\phi+\chi}-\frac{\sigma_{d}^{2} \chi \phi}{\phi+\chi}\right)}=\frac{\phi B_{1}}{\phi+\chi}+\sqrt{\frac{\bar{u}-P_{1}}{\phi}-\frac{\chi}{\phi+\chi}\left(\sigma_{d}^{2}+\frac{B_{1}^{2} \phi}{\phi+\chi}\right)} . \tag{A54}
\end{equation*}
$$

Similarly, the location of the reader who is indifferent between buying Newspaper 2 and not buying at all can be calculated as:

$$
\begin{equation*}
b_{2}=\frac{\phi B_{2}}{\phi+\chi}-\sqrt{\frac{\bar{u}-P_{2}}{\phi}-\frac{\chi}{\phi+\chi}\left(\sigma_{d}^{2}+\frac{B_{2}^{2} \phi}{\phi+\chi}\right)}, \tag{A55}
\end{equation*}
$$

and $0<b_{2}<b_{0}$ if

$$
\begin{equation*}
\bar{u}-P_{2}-\frac{B_{2}^{2} \phi^{2}}{\phi+\chi}-\frac{\sigma_{d}^{2} \chi \phi}{\phi+\chi}<0, \text { and } \bar{u}-P_{2}-\frac{B_{2}^{2} \phi^{2}}{\phi+\chi}-\frac{\sigma_{d}^{2} \chi \phi}{\phi+\chi}-\phi b_{0}^{2}+\frac{2 b_{0} \phi^{2} B_{2}}{\phi+\chi}>0 . \tag{A56}
\end{equation*}
$$

When an advertiser of appeal parameter $\alpha$ chooses to advertise in Newspaper 1, its expected payoff is given as:

$$
\begin{equation*}
E_{1}(\alpha)=M \int_{-b_{0}}^{b_{1}} \frac{1}{2 b_{0}}\left(h_{0}+\frac{\alpha b}{b_{0}}\right) d b-K_{1} . \tag{A57}
\end{equation*}
$$

If it chooses to advertise in Newspaper 2 its expected payoff is:

$$
\begin{equation*}
E_{2}(\alpha)=M \int_{b_{2}}^{b_{0}} \frac{1}{2 b_{0}}\left(h_{0}+\frac{\alpha b}{b_{0}}\right) d b-K_{2} . \tag{A58}
\end{equation*}
$$

The advertiser who is indifferent between Newspaper 1 and 2 can be derived from (A57) and (A58) by solving for $\alpha$ in $E_{1}(\alpha)=E_{2}(\alpha)$ :

$$
\begin{equation*}
\alpha_{\text {indif }}=\frac{2 b_{0} h_{0}\left(b_{1}+b_{2}\right)}{\left(2 b_{0}^{2}-b_{1}^{2}-b_{2}^{2}\right)}+\frac{\left(K_{2}-K_{1}\right) 4 b_{0}^{2}}{M\left(2 b_{0}^{2}-b_{1}^{2}-b_{2}^{2}\right)} . \tag{A59}
\end{equation*}
$$

In the last stage the newspapers set their subscription and advertising fees to maximize their profits:

$$
\begin{equation*}
\pi_{1}=A \frac{\alpha_{0}+\alpha_{\text {indif }}}{2 \alpha_{0}} K_{1}+M_{1} \frac{b_{0}+b_{1}}{2 b_{0}} P_{1}, \quad \pi_{2}=A \frac{\alpha_{0}-\alpha_{\text {indif }}}{2 \alpha_{0}} K_{2}+M_{1} \frac{b_{0}-b_{2}}{2 b_{0}} P_{2}, \tag{A60}
\end{equation*}
$$

which yields the following first order conditions:

$$
\begin{align*}
& \frac{A}{2 \alpha_{0}}\left[\left(\alpha_{0}+\alpha_{\text {indif }}\right)-\frac{4 b_{0}^{2} K_{1}}{M\left(2 b_{0}^{2}-b_{1}^{2}-b_{2}^{2}\right)}\right]=0,  \tag{A61}\\
& \frac{A}{2 \alpha_{0}}\left[\left(\alpha_{0}-\alpha_{\text {indif }}\right)-\frac{4 b_{0}^{2} K_{2}}{M\left(2 b_{0}^{2}-b_{1}^{2}-b_{2}^{2}\right)}\right]=0,  \tag{A62}\\
& M_{1}\left(\frac{b_{1}+b_{0}}{2 b_{0}}-\frac{P_{1}}{4 b_{0} \phi\left(b_{1}-\frac{\phi B_{1}}{(\phi+\chi)}\right)}\right)-\frac{2 A K_{1} b_{0} h_{0}\left(2 b_{0}^{2}+b_{1}^{2}-b_{2}^{2}+2 b_{1} b_{2}\right)}{4 \alpha_{0} \phi\left(2 b_{0}^{2}-b_{1}^{2}-b_{2}^{2}\right)^{2}\left(b_{1}-\frac{\phi B_{1}}{(\phi+\chi)}\right)}=0,  \tag{A63}\\
& M_{1}\left(\frac{b_{0}-b_{2}}{2 b_{0}}-\frac{P_{2}}{4 b_{0} \phi\left(\frac{\phi B_{2}}{(\phi+\chi)}-b_{2}\right)}\right)-\frac{2 A K_{2} b_{0} h_{0}\left(2 b_{0}^{2}+b_{2}^{2}-b_{1}^{2}+2 b_{1} b_{2}\right)}{4 \alpha_{0} \phi\left(2 b_{0}^{2}-b_{1}^{2}-b_{2}^{2}\right)^{2}\left(\frac{\phi B_{2}}{(\phi+\chi)}-b_{2}\right)}=0 . \tag{A64}
\end{align*}
$$

Simultaneously solving (A61) and (A62) we get:

$$
\begin{align*}
& K_{1}=M\left(\frac{\alpha_{0}\left(2 b_{0}^{2}-b_{1}^{2}-b_{2}^{2}\right)}{4 b_{0}^{2}}+\frac{1}{6} \frac{h_{0}}{b_{0}}\left(b_{1}+b_{2}\right)\right),  \tag{A65}\\
& K_{2}=M\left(\frac{\alpha_{0}\left(2 b_{0}^{2}-b_{1}^{2}-b_{2}^{2}\right)}{4 b_{0}^{2}}-\frac{1}{6} \frac{h_{0}}{b_{0}}\left(b_{1}+b_{2}\right)\right) . \tag{A66}
\end{align*}
$$

Thus, at the symmetric equilibrium (i.e., $b_{1}=-b_{2}$ ):

$$
\begin{equation*}
K^{*}=\frac{M\left(b_{0}^{2}-b_{2}^{2}\right) \alpha_{0}}{2 b_{0}^{2}} \tag{A67}
\end{equation*}
$$

Using (A67) in (A64) at the symmetric equilibrium yields:

$$
\begin{equation*}
P^{*}=2 \phi\left(\frac{\phi B_{2}}{(\phi+\chi)}-b_{2}\right)\left(b_{0}-b_{2}\right)-\frac{A M h_{0}}{2 M_{1}} . \tag{A68}
\end{equation*}
$$

To obtain the equilibrium locations with incomplete coverage we differentiate the first stage payoff functions $V_{i}\left(B_{i}, B_{j}\right)$ with respect to the locations and use the Envelope Theorem:

$$
\begin{equation*}
\frac{\partial V_{i}}{\partial B_{i}}=\frac{\partial \pi_{i}}{\partial B_{i}}+\frac{\partial \pi_{i}}{\partial P_{j}} \frac{\partial P_{j}}{\partial B_{i}}+\frac{\partial \pi_{i}}{\partial K_{j}} \frac{\partial K_{j}}{\partial B_{i}}=0 . \quad i, j=1,2, i \neq j . \tag{A69}
\end{equation*}
$$

For Newspaper 2, it follows from (A69) that,

$$
\begin{equation*}
\frac{\partial V_{2}}{\partial B_{2}}=\frac{\partial \pi_{2}}{\partial B_{2}}+\frac{\partial \pi_{2}}{\partial P_{1}} \frac{\partial P_{1}}{\partial B_{2}}+\frac{\partial \pi_{2}}{\partial K_{1}} \frac{\partial K_{1}}{\partial B_{2}}=0 \tag{A70}
\end{equation*}
$$

Using the Implicit Function approach by totally differentiating the first order conditions (A63) and (A64) and solving the resulting system of equations using $K^{*}$ from (A67) and $P^{*}$ from (A68) at the symmetric equilibrium we obtain:

$$
\begin{align*}
& \frac{\partial P_{1}}{\partial B_{2}}=-R \frac{\partial K_{1}}{\partial B_{2}} \text { where }  \tag{A71}\\
& R=\frac{A b_{0} h_{0}}{4 \alpha_{0} \phi\left(b_{0}^{2}-b_{2}^{2}\right)\left(\frac{B_{2}^{2} \phi}{\phi+\chi}-b_{2}\right)} /\left[\frac{M_{1}\left(\frac{2 B_{2}^{2} \phi}{\phi+\chi}-2 b_{2}+b_{0}-b_{2}\right)}{4 b_{0} \phi\left(\frac{B_{2}^{2} \phi}{\phi+\chi}-b_{2}\right)^{2}}+\frac{A h_{0} M b_{2}}{8 b_{0}\left(b_{0}^{2}-b_{2}^{2}\right) \phi^{2}\left(\frac{B_{2}^{2} \phi}{\phi+\chi}-b_{2}\right)^{2}}\right] .
\end{align*}
$$

From (12), (A59), and (A71), at the symmetric equilibrium (A70) becomes:

$$
\begin{equation*}
\frac{\partial V_{2}}{\partial B_{2}}=-\frac{\partial b_{2}}{\partial B_{2}}\left[\left(\frac{P^{*} M_{1}}{2 b_{0}}+\frac{M A h_{0}}{4 b_{0}}\right)-\frac{\partial K_{1}}{\partial b_{2}} \frac{M A}{4}\left(\frac{2}{M}-\frac{h_{0} R}{2 b_{0} \phi\left(\frac{B_{2}^{2} \phi}{\phi+\chi}-b_{2}\right)}\right)\right] \tag{A72}
\end{equation*}
$$

From (A55), $\frac{\partial b_{2}}{\partial B_{2}}>0$. Notice that the first term inside the brackets in (A72) is always positive and the second term is positive if $\frac{\partial K_{1}}{\partial b_{2}}<0$ and if $b_{2}>\frac{b_{0} h_{0}}{2 \alpha_{0}}$. From (A65) $\frac{\partial K_{1}}{\partial b_{2}}<0$ if $b_{2}>\frac{b_{0} h_{0}}{3 \alpha_{0}}$. Therefore, if $b_{2}>\frac{b_{0} h_{0}}{2 \alpha_{0}}, \frac{\partial V_{2}}{\partial B_{2}}<0$, and the newspaper will continue to reduce bias. Since $\frac{\partial b_{2}}{\partial B_{2}}>0$ it follows that bias will be reduced until $b_{2} \leq \frac{b_{0} h_{0}}{2 \alpha_{0}}$. Because at the Single-Homing equilibrium with complete coverage $\alpha_{0}>h_{0}$, it follows that $b_{2}<\frac{b_{0}}{2}$. Hence, incomplete coverage of the type depicted in Figure A1, will never lead to an equilibrium where each newspaper covers less than half of the segment of readers who prefer its location best (for 1 this segment is $b<0$, and for 2 this segment is $b>0$ ).

We now derive the conditions on the parameters of the model to guarantee that less than full coverage moderates the extent of bias at the equilibrium in comparison to $B^{M S}=\frac{3}{2} b_{0}$. To obtain the conditions we substitute the equilibrium price from (A68) back into (A55) and solve for $B_{2}$ in terms of $b_{2}$ as follows:

$$
\begin{equation*}
B_{2}=\sqrt{\left(b_{0}-2 b_{2}\right)^{2}+\frac{\phi+\chi}{\phi}\left(\frac{\bar{u}}{\phi}-\frac{\chi \sigma_{d}^{2}}{\phi+\chi}+\frac{A M h_{0}}{2 M_{1} \phi}+2 b_{2} b_{0}-3 b_{2}^{2}\right)}-\left(b_{0}-2 b_{2}\right) \tag{A73}
\end{equation*}
$$

It is very easy to show that $\frac{\partial B_{2}}{\partial b_{2}}$ in the above expression is positive when $b_{2}<\frac{b_{0}}{2}$. Evaluating the right-hand side of (A73) at $b_{2}=\frac{b_{0}}{2}$ yields therefore, that:

$$
\begin{equation*}
B_{2}<\sqrt{\frac{\phi+\chi}{\phi}\left(\frac{\bar{u}}{\phi}-\frac{\chi \sigma_{d}^{2}}{\phi+\chi}+\frac{A M h_{0}}{2 M_{1} \phi}+\frac{b_{0}^{2}}{4}\right)} . \tag{A74}
\end{equation*}
$$

The right-hand side of (A74) is smaller than $\frac{3}{2} b_{0}$ if:

$$
\begin{equation*}
R \xlongequal{\text { def }} \frac{\bar{u}}{\phi}-\frac{\chi \sigma_{d}^{2}}{\phi+\chi}+\frac{A M h_{0}}{2 M_{1} \phi}<\frac{b_{0}^{2}}{4} \frac{8 \phi+\chi}{\phi+\chi} . \tag{A75}
\end{equation*}
$$

Hence, as long as $R$ is sufficiently small (e.g., when the reservation price of readers $\bar{u}$ is low and their valuation of accurate reporting $\chi$ is high), incomplete coverage may yield moderation of bias below $B^{M S}$. Note that condition (A75) does not necessarily contradict (A53) and (A56), conditions necessary to support the type of incomplete coverage we consider.

## Double-Homing:

Suppose that the newspapers, this time, cover only the middle of the market (i.e, between $b_{1}$ and $b_{2}$ in Figure A2) and advertisers Double-Home.


## Figure A2: Segmentation of the Subscriber Market

We demonstrate that at the limit as $b_{1} \rightarrow-b_{0}$ and $b_{2} \rightarrow b_{0}$, the newspapers may have incentives to moderate their bias in reporting in comparison to the full market coverage case that is analyzed in Section 4.

Again, the reader who is indifferent between buying Newspaper 1 and not buying at all satisfies equation (A50). For the segmentation given in Figure A2 to hold we need: $\frac{\partial E U_{b}{ }^{1}}{\partial b}=-2 \phi b+\frac{2 b \phi^{2} B_{1}}{\phi+\chi}>0$ at $b=b_{1}$ which implies:

$$
\begin{equation*}
b_{1}<\frac{\phi B_{1}}{(\phi+\chi)} . \tag{A76}
\end{equation*}
$$

Further at $b=0$, we should have $E U_{b}{ }^{1}>0$ and at $b=-b_{0}, E U_{b}{ }^{1}<0$. The root in (A51) that satisfies (A76) is:
(A77) $b_{1}=\frac{\phi B_{1}}{\phi+\chi}-\sqrt{\frac{\phi^{2} B_{1}^{2}}{(\phi+\chi)^{2}}+\frac{1}{\phi}\left(\bar{u}-P_{1}-\frac{B_{1}^{2} \phi^{2}}{\phi+\chi}-\frac{\sigma_{d}^{2} \chi \phi}{\phi+\chi}\right)}=\frac{\phi B_{1}}{\phi+\chi}-\sqrt{\frac{\bar{u}-P_{1}}{\phi}-\frac{\chi}{\phi+\chi}\left(\sigma_{d}^{2}+\frac{B_{1}^{2} \phi}{\phi+\chi}\right)}$.
Similarly, the location of the reader who is indifferent between buying Newspaper 2 and not buying at all can be calculated as:

$$
\begin{equation*}
b_{2}=\frac{\phi B_{2}}{\phi+\chi}+\sqrt{\frac{\bar{u}-P_{2}}{\phi}-\frac{\chi}{\phi+\chi}\left(\sigma_{d}^{2}+\frac{B_{2}^{2} \phi}{\phi+\chi}\right)} . \tag{A78}
\end{equation*}
$$

One can show that in this case as well, the newspapers will choose to cover at least one half of the readers who prefer their locations best, namely $b_{2}>\frac{b_{0}}{2}$ and $b_{1}<-\frac{b_{0}}{2}$. Note from (A78) that:

$$
\frac{\partial b_{2}}{\partial B_{2}}=\frac{\left(b_{2}-B_{2}\right) \frac{\chi}{\phi+\chi}}{b_{2}-\frac{\chi B_{2}}{\phi+\chi}}\left\{\begin{array}{l}
>0 \text { if } b_{2}>B_{2}  \tag{A79}\\
<0 \text { if } b_{2}<B_{2} .
\end{array}\right.
$$

Equation (A79) implies that newspapers may have incentives to moderate their bias, once again. The benefit to advertisers in this case can be derived as: $E_{1}(\alpha)=M \int_{b_{1}}^{b_{\text {indif }}} \frac{1}{2 b_{0}}\left(h_{0}+\frac{\alpha b}{b_{0}}\right) d b-$ $K_{1}$, and $E_{2}(\alpha)=M \int_{b_{\text {indif }}}^{b_{2}} \frac{1}{2 b_{0}}\left(h_{0}+\frac{\alpha b}{b_{0}}\right) d b-K_{2}$.
When advertisers Double-Home, advertising fees are determined by the requirement that $E_{1}\left(\alpha_{0}\right)=0$ and $E_{2}\left(-\alpha_{0}\right)=0$, thus yielding:

$$
\begin{align*}
& K_{1}=M\left(\frac{h_{0}\left(b_{\text {indif }}-b_{1}\right)}{2 b_{0}}+\frac{\alpha_{0}\left(b_{\text {indif }}^{2}-b_{1}^{2}\right.}{4 b_{0}^{2}}\right),  \tag{A80}\\
& K_{2}=M\left(\frac{h_{0}\left(b_{2}-b_{\text {indif }}\right)}{2 b_{0}}-\frac{\alpha_{0}\left(b_{2}^{2}-b_{\text {indif }}^{2}\right.}{4 b_{0}^{2}}\right) . \tag{A81}
\end{align*}
$$

Newspapers' profits when they cover only the middle of the market (and when advertisers Double-Home) are:

$$
\begin{equation*}
\pi_{1}=A K_{1}+M_{1} \frac{b_{\text {indif }}-b_{1}}{2 b_{0}} P_{1}, \quad \pi_{2}=A K_{2}+M_{1} \frac{b_{2}-b_{\text {indif }}}{2 b_{0}} P_{2} . \tag{A82}
\end{equation*}
$$

where $K_{1}$, and $K_{2}$ are as given in (A80) and (A81). Let $\pi_{2}^{c o v}$ and $\pi_{2}^{\text {non-cov }}$ denote Newspaper 2's profits when it covers and does not cover the market as given in (13) and (A82), respectively. Then, when $b_{1} \rightarrow-b_{0}$ and $b_{2} \rightarrow b_{0}$ :

$$
\begin{equation*}
\frac{\partial \pi_{2}^{n o n-c o v}}{\partial \mathrm{~B}_{2}}-\frac{\partial \pi_{2}^{c o v}}{\partial \mathrm{~B}_{2}}=\frac{1}{2 b_{0}}\left(M_{1} P_{2}+A M\left(h_{0}-\alpha_{0}\right)\right) \frac{\partial b_{2}}{\partial B_{2}} . \tag{A83}
\end{equation*}
$$

Recall that at the Double-Homing equilibrium $h_{0}>\alpha_{0}$. Hence, the sign of the above difference depends only on the sign of $\frac{\partial b_{2}}{\partial B_{2}}$. From (A79), $\frac{\partial b_{2}}{\partial B_{2}}<0$ if $b_{2}<B_{2}$. This inequality is more likely when the variable $R$, defined in (A75), is relatively small (e.g., when the reservation price of readers $\bar{u}$ is low and their valuation of accurate reporting $\chi$ is high). If the sign of (A83) is negative, newspapers have incentives to moderate when $b_{1} \rightarrow-b_{0}$ and $b_{2} \rightarrow b_{0}$. Note that even when $\frac{\partial b_{2}}{\partial B_{2}}>0$, in which case the newspapers have incentives to polarize, bias will at most be equal to $b_{0}$, because, $\frac{\partial b_{2}}{\partial B_{2}}>0$ when $B_{2}<b_{2}$ and $b_{2} \leq b_{0}$.

## Asymmetric Accuracy

We investigate the impact of asymmetry in newspapers' data accuracy on reporting bias when the papers' sole source of revenue is subscription fees. Specifically, we assume that Newspaper 1
has access to more accurate data than Newspaper 2: ${\sigma_{d_{1}}}^{2}<{\sigma_{d_{2}}}{ }^{2}$. In this case location of reader who is indifferent between the two newspapers is:

$$
\begin{equation*}
b_{\text {indif }}=\frac{(\phi+\chi)}{2 \phi^{2}} \frac{\left(P_{2}-P_{1}\right)}{\left(B_{2}-B_{1}\right)}+\frac{B_{1}+B_{2}}{2}+\frac{\chi}{2 \phi} \frac{\left(\sigma_{d_{2}}{ }^{2}-\sigma_{d_{1}}{ }^{2}\right)}{\left(B_{2}-B_{1}\right)} . \tag{A84}
\end{equation*}
$$

For the newspapers' second stage pricing decisions, we optimize (3) with respect to $P_{1}$ and $P_{2}$ which yields the following first order conditions:

$$
\begin{align*}
& \frac{M_{1}}{2 b_{0}}\left(\left(b_{\text {indif }}+b_{0}\right)-\frac{(\phi+\chi)}{2 \phi^{2}} \frac{P_{1}}{\left(B_{2}-B_{1}\right)}\right)=0,  \tag{A85}\\
& \frac{M_{1}}{2 b_{0}}\left(\left(b_{0}-b_{\text {indif }}\right)-\frac{(\phi+\chi)}{2 \phi^{2}} \frac{P_{2}}{\left(B_{2}-B_{1}\right)}\right)=0 . \tag{A86}
\end{align*}
$$

Substituting (A84) in (A85) and (A86) and simultaneously solving for $P_{1}$ and $P_{2}$ we get equilibrium subscription fees as functions of the newspapers locations:

$$
\begin{align*}
& P_{1}=\frac{(\phi \chi)\left(\sigma_{d_{2}}^{2}-\sigma_{d_{1}}^{2}\right)}{3(\phi+\chi)}+\frac{\phi^{2}\left(B_{2}-B_{1}\right)}{(\phi+\chi)}\left(2 b_{0}+\frac{B_{1}+B_{2}}{3}\right),  \tag{A87}\\
& P_{2}=\frac{-(\phi \chi)\left(\sigma_{d_{2}}^{2}-\sigma_{d_{1}}^{2}\right)}{3(\phi+\chi)}+\frac{\phi^{2}\left(B_{2}-B_{1}\right)}{(\phi+\chi)}\left(2 b_{0}-\frac{B_{1}+B_{2}}{3}\right) . \tag{A88}
\end{align*}
$$

Substituting these second stage equilibrium strategies $P_{i}\left(B_{i}, B_{j}\right)$ into (3) we obtain the first stage payoff functions $V_{i}\left(B_{i}, B_{j}\right)$. Differentiating with respect to the locations yields from the Envelope Theorem:
(A89) $\frac{\partial V_{i}}{\partial B_{i}}=\frac{\partial \pi_{i}}{\partial B_{i}}+\frac{\partial \pi_{i}}{\partial P_{j}} \frac{\partial P_{j}}{\partial B_{i}}=0 \quad i, j=1,2, i \neq j$.
It follows from (A89) that,

Using (A87), (A88) and (A84) to find the terms of (A90) and (A91) and solving them

$$
\begin{array}{ll}
\text { (A90) } & \frac{\partial b_{\text {indif }}}{\partial B_{1}}+\frac{\partial b_{\text {indif }}}{\partial P_{2}} \frac{\partial P_{2}}{\partial B_{1}}=0, \\
\text { (A91) } & -\frac{\partial b_{\text {indif }}}{\partial B_{2}}-\frac{\partial b_{\text {indif }}}{\partial P_{1}} \frac{\partial P_{1}}{\partial B_{2}}=0 . \tag{A90}
\end{array}
$$


#### Abstract

simultaneously for the newspapers' first stage location choices yields:


$$
\begin{align*}
& B_{1}=-\frac{3}{2} b_{0}+\frac{\chi\left(\sigma_{d_{2}}{ }^{2}-\sigma_{d_{1}}{ }^{2}\right)}{6 \phi b_{0}},  \tag{A92}\\
& B_{2}=\frac{3}{2} b_{0}+\frac{\chi\left(\sigma_{d_{2}}{ }^{2}-\sigma_{d_{1}}{ }^{2}\right)}{6 \phi b_{0}} . \tag{A93}
\end{align*}
$$

The equilibrium locations derived illustrate that Newspaper 1, which has access to more accurate data, introduces less bias in reporting and the opposite is true about Newspaper 2, which has less precise data available. Hence, the utility formulation introduces some tradeoff between
vertical differentiation (in our case precision of data) and horizontal differentiation (in our case bias in reporting). There may be different reasons why a newspaper has access to more accurate data, including lower cost of conducting investigations due to greater experience in investigative reporting. Hence, if ${\sigma_{d_{i}}}^{2}$ can be chosen endogenously, the newspaper facing lower cost will likely choose greater accuracy in gathering information.


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