THE
JUST INTONATION
PRIMER

An Introduction to the Theory and Practice of Just Intonation

by David B. Doty

Third Edtion, December 2002

ISBN 0-9726810-0-0

©1993, 1994, 2002 The Just Intonation Network

A Project of Other Music, Inc.
535 Stevenson Street
San Francisco, CA 94103

Phone: (415) 864-8123
Fax: (415) 864-8726

www.justintonation.net
info@justintonation.net

CONTENTS

Preface to the Third Edition (2002) iv
Preface to the Second Edition (1994) iv
INTRODUCTION 1
What is Just Intonation? 1
A Little History 2
Antiquity 2
The Middle Ages and the Renaissance 2
The Common-Practice Period and the Rise of Temperament 3
The End of Common Practice 5
The Twentieth-Century Just Intonation Revival 6
The Purpose of this Publication 7
ACOUSTIC AND PSYCHOACOUSTIC BACKGROUND 9
Introduction 9
Periodic Vibrations 9
What is an Interval? 10
Superposition of Pure Tones 11
Phase Relationships 12
Interference Beats 12
The Harmonic Series 14
Difference Tones 16
Periodicity Pitch 17
Complex Tones with Harmonic Partials 19
Coincident or Beating Harmonics 20
Arthur H. Benade's "Special Relationships" 22
Special Relationships Beyond the Octave .. 23
On the Consonance of Relationships Involving Higher Harmonics 24
BASIC DEFINITIONS, CONVENTIONS, AND PROCEDURES 25
Introduction 25
Rules and Conventions 25
Calculations with Ratios 25
Addition 25
Subtraction 25
Complementation 26
Converting Ratios to Cents 26
Dividing Just Intervals 26
Calculating Absolute Frequencies (Hz) 28
The Harmonic Series and the Subharmonic Series 28
Converting Ratios to a Harmonic or Subharmonic Series Segment 28
Identities 30
Prime Numbers, Primary Intervals, and Prime Limits 30
What is a Chord? 30
Interval Names 32
Notation 32
Anomalies 33
Tetrachords and Tetrachordal Scales 33
THE LADDER OF PRIMES, PART ONE:TWO, THREE, AND FIVE 35
One, the Foundation 36
Two, the Empty Matrix 36
The Three Limit (Pythagorean Tuning) 36
The Pythagorean (Ditone) Diatonic Scale .. 36
Pythagorean Chromatic Scales and the Pythagorean Comma 37
Chords 38
The Five Limit 38
Constructing a Five-Limit Major Scale 39
The Supertonic Problem 40
The Five-Limit Lattice 42
The Great Diesis 44
The Harmonic Duodene 45
Five Limit Chords 46
Condissonant Chords in the Five Limit 47
Deficiencies of the Five Limit 48
THE LADDER OF PRIMES, PART TWO: SEVEN AND BEYOND 51
Seven-Limit Intervals 51
New Consonances 51
Whole Tones 54
Semitones 54
Microtones 54
False Consonances 54
Two Dimensional Planes in the Seven-Limit Fabric 55
Seven Limit Chords 56
Subsets of the Dominant-Seventh and Ninth Chords 56
Diminished Triad 56

Half-Diminished Seventh Chord 57
"4-6-7" Chord 57
Seven-Limit Subharmonic Chords............. 57
Condissonant Chords in the Seven Limit ... 58
Fixed Scales in the Seven Limit........................ 59
The Higher Primes: Eleven, Thirteen, and Beyond.. 59
Eleven and Thirteen 59
Primary Intervals... 61
Chords and Harmony 62
Seventeen and Nineteen............................. 64
Beyond Nineteen... 65
PRACTICAL JUST INTONATION WITH REAL INSTRUMENTS 66
Introduction 66
Fixed-Pitch Instruments 67
Acoustic Keyboards................................... 67
Piano Problems 67
Reed Organs.. 70
Pipe Organs... 70
Electroacoustic Keyboards 70
Electronic Organs 70
Stringed Instruments with One String per Note 70
Fretted Instruments 71
Wind Instruments 73
Continuous-Pitch Instruments 74
"Natural Tendencies"................................. 74
Bowed Strings... 75
Voices.. 76
Trombones 76
MIDI Synthesizers.. 76
Pitch Bend... 76
Instruments with User-Programmable Tuning Tables... 77
Tuning Tables plus Pitch Bend 78
The MIDI Tuning-Dump Specification........ 78
Some Thoughts on Obtaining Satisfactory Performances................................. 79
NOTES 81
INDEX 85

PREFACE TO THE THIRD EDITION (2002)

The third edition of *The Just Intonation Primer* contains no significant changes in content from the previous editions, though a few minor errors have been corrected. The type, however, has been completely reset and all of the art has been recreated from scratch. Why? The first and second editions of *The Primer* were produced with software that has long since become obsolete, necessitating the resetting of the type. As for the art, both my skills and the available tools have greatly improved since the production of the previous editions, and my standards, as well, have risen considerably.

This is not to say that I regard *The Primer*, as it now stands, as needing no improvement—quite the contrary. The problem is that once I began making improvements, there is no telling where the process would end. This would be contrary to my purpose for creating this third edition: to keep the publication available until I am able to complete the work that will supersede it. This is intended to be the last "print-only" edition of *The Just Intonation Primer*; any future editions will be in the form of multimedia products. There seems to be little purpose in merely *writing* about unfamiliar intervals, scales, and chords when it is now possible to present students with the actual sounds.

Thanks to Lucy A. Hudson for creating software that simplified the conversion of the text files from the previous edition.

PREFACE TO THE SECOND EDITION (1994)

The writing and publication of *The Just Intonation Primer* were made possible by a gift from a friend of the Just Intonation Network.

The material in this publication was excerpted from a larger work in progress, tentatively entitled *A Composer's Guide to Just Intonation*.

Thanks to Carola B. Anderson and Dudley Duncan for proofreading and commenting on *The Just Intonation Primer*. Thanks to Jim Horton for the use of his collection of obscure publications.

INTRODUCTION

What is Just Intonation?

What is Just Intonation? Although, like most composers working with this unfamiliar tuning system, I am frequently asked this troublesome question, I have yet to devise an answer that is suitable for casual conversation. Technically, Just Intonation is any system of tuning in which all of the intervals can be represented by whole-number frequency ratios, *with a strongly implied preference for the simplest ratios compatible with a given musical purpose.* Unfortunately this definition, while concise and accurate, is more likely to result in a glazed expression indicative of confusion than in the gleam of understanding. It is, in short, a definition that is perfectly clear to the comparative few who have the background to understand it and who could, therefore, formulate it for themselves, and perfectly opaque to everyone else, including, unfortunately, most trained musicians. (It is my experience that most musicians are as ignorant of the details of twelve-tone equal temperament, the predominant tuning system in Western cultures for the past two hundred years, as they are of Just Intonation. If you doubt this, ask the next dozen musicians you meet to explain why there are twelve semitones in a chromatic scale and how to accurately tune those twelve equal semitones.) A detailed answer that incorporates all the necessary background on the physics of sound, the physiology and psychology of human hearing, the history of music, and the mathematics of tuning systems, far exceeds the limits of casual conversation. It could, in fact, fill a book.

A formal definition of Just Intonation may be difficult for the novice to grasp, but the aesthetic experience of just intervals is unmistakable. Although it is difficult to describe the special qualities of just intervals to those who have never heard them, words such as clarity, purity, smoothness, and stability come readily to mind. The supposedly consonant intervals and chords of equal temperament, which deviate from simple ratios to varying degrees, sound rough, restless, or muddy in comparison.

The simple-ratio intervals upon which Just Intonation is based are "special relationships" that the human auditory system is able to detect and distinguish from one another and from a host of more complex stimuli. They are what the human auditory system recognizes as consonance, if it ever has the opportunity to hear them in a musical context. Although the importance of these whole-number ratios is recognized both by musical tradition and by modern acoustic and psychoacoustic research, for the last two hundred years Western music has been burdened with a tuning system in which all of the supposed consonances, with the exception of the octave, deviate significantly from their optimal, integer-ratio forms. Indeed, some consonant intervals are so compromised in twelve-tone equal temperament that they are hardly represented at all.

Just Intonation provides a greater variety and superior quality of consonances and concords than equal temperament, but its resources are by no means limited to unrelieved consonance. Just Intonation also has the potential to provide more varied and powerful dissonances than the current system. This is the case in part because the simple, consonant intervals can be compounded in a great many ways to yield more complex dissonant intervals and, in part, because, the consonant intervals being truly consonant, the dissonances are rendered that much sharper in contrast. Further, because dissonances in Just Intonation are the products of concatenations of simpler intervals, consonance and dissonance coexist in a rational framework and their mutual relations are readily comprehensible.

The virtues of Just Intonation and the shortcomings of equal temperament are not limited to the affective properties of their respective intervals and chords. An equally serious problem with twelve-tone equal temperament is that it supplies composers with an artificially simplified, one-dimensional model of musical relationships. By substituting twelve equally spaced fixed tones for a potentially unlimited number of tones, interconnected by a web of subtle and complex musical relationships, equal temperament not only impoverished the sonic palette of Western music, but also deprived composers and theorists of the means for thinking clearly about tonal relationships, causing them to confuse close relationships with remote ones and consonances with dissonances. Not only does Just

Intonation provide a vast array of superior new musical resources, but, when properly understood, provides the tools necessary for organizing and manipulating these greatly expanded resources.

Just Intonation is not a particular scale, nor is it tied to any particular musical style. It is, rather, a set of principles which can be applied to a limited number of musically significant intervals to generate an enormous variety of scales and chords, or to organize music without reference to any fixed scale. The principles of Just Intonation are applicable to any style of tonal or modal music (or even, if you wish, to atonal styles). Just Intonation is not primarily a tool for improving the consonance of existing musics, although it can, in some cases, be used this way. Just Intonation can give rise to new styles and forms of music which, although truly innovative, are, unlike those created by the proponents of the various "avant-garde-isms" of the twentieth century, comprehensible to the ear of the listener as well as to the intellects of the composer and analyst. Ultimately, Just Intonation is a method for understanding and navigating through the boundless reaches of the pitch continuum — a method that transcends the musical practices of any particular culture.

Just Intonation has depth and breadth. Its fundamental principles are relatively simple but its ramifications are vast. At present, the musical realm that comprises Just Intonation remains largely unexplored. A few pioneering composers and theorists have sketched some of its most striking features, but the map still contains many blank spaces where the adventurous composer may search for new musical treasures.

A LITTLE HISTORY

In light of its numerous virtues, why isn't Just Intonation currently in general use? Like so many of our peculiar customs, this is largely an accident of history. A detailed history of tuning in the West would require a book of considerable length in its own right, and is thus far beyond the scope of the current work. No one has, as yet, written a comprehensive study of this subject. Until such becomes available, the reader is advised to consult Harry Partch's *Genesis of a Music*, especially Chapter Fifteen, "A Thumbnail Sketch of the History of Intonation,"[1] and J. Murray Barbour's *Tuning and Temperament.*[2] The following short sketch is intended only to describe, in general terms, how musical intonation in the West achieved its current, peculiar state.

ANTIQUITY

Just Intonation is not a new phenomenon. The basic discovery that the most powerful musical intervals are associated with ratios of whole numbers is lost in antiquity.[3, 4] Perhaps it was first discovered by the priestly musicians of Egypt or Mesopotamia in the second or third millennium B.C.E. Some scholars, most notably Ernest G. McClain, regard this discovery as of vital importance to the development of mathematics and religion in these ancient societies. The semimythical Greek philosopher Pythagoras of Samos (c. 560–480 B.C.E.) is generally credited with introducing whole-number-ratio tunings for the octave, perfect fourth, and perfect fifth into Greek music theory in the sixth century B.C.E. In the generations following Pythagoras, many Greek thinkers devoted a portion of their energies to musical studies and especially to scale construction and tuning. These musical philosophers, known collectively as the *harmonists*, created a host of different tunings of the various Greek scales, which they expressed in the form of whole-number ratios. The discoveries of the Greek harmonists constitute one of the richest sources of tuning lore in the world and continue to this day to exercise a significant influence on Western musical thought. Although most of the original writings of the harmonists have been lost, much of their work was summarized by the second century C.E. Alexandrian, Claudius Ptolemy, in his *Harmonics.* Ptolemy made significant contributions in his own right to the field of music theory, as well as to astronomy and geography.

THE MIDDLE AGES AND THE RENAISSANCE

Since the time of the Greek harmonists, the idea of simple ratios as the determinants of musical consonance has never been wholly absent from Western musical thought. Although much Greek music theory was lost to the West with the fall of the Roman Empire, some was retained and passed on to medieval Europe, primarily through the musical writings of the late Roman philosopher Anicius Manlius Severinus Boethius (c. 480–525/6 C.E.). (Greek music theory was also preserved and further developed in the Islamic sphere, but this does not appear to have had much influence on musical developments in the West.) Throughout the Middle Ages, Western music was theoretically based on what is called Pythagorean intonation, a subset of Just Intonation based on ratios composed only of multiples of 2 and 3, which will be described in detail in Chapter Three. Pythagorean tuning is characterized by consonant octaves, perfect fourths, and perfect fifths, b,ased on ratios of the numbers 1, 2, 3, and 4. All other intervals in Pythagorean tuning are dissonant. This property is consistent with the musical practice of the middle ages, in which polyphony was based on fourths, fifths, and octaves, with all other intervals, including thirds and sixths, being treated as dissonances.

In the later Middle Ages and early Renaissance, thirds and sixths were increasingly admitted into polyphonic music as consonances, and music theory was gradually modified to account for the existence of these consonant intervals, although it appears to have lagged considerably behind musical practice. Eventually, theorists were forced to partially abandon the Pythagorean framework of the middle ages in order to explain the existence of consonant thirds and sixths, because the most consonant possible thirds and sixths are based on ratios involving 5. The association of consonant thirds and sixths with ratios involving 5 was first mentioned by the English monk Walter Odington (c. 1300), but it took a long time for this idea to penetrate the mainstream of musical thought and displace the Pythagorean intonational doctrines—indeed, it can be argued that it never wholly succeeded in doing so. In the sixteenth century, the rediscovery of Greek writings on music, especially the writings of Ptolemy, gave considerable added ammunition to the advocates of consonant thirds and sixths based on ratios involving 5. In general, music theorists of the Italian Renaissance came to agree with the proposition of the Venetian Gioseffe Zarlino (1517 – 1590) that consonance was the product of ratios of the integers 1 – 6 (the so-called *senario*). The ratios that define the major and minor triads were discovered in the senario and were acclaimed as the most perfect concords, thereby setting the stage for the development of chordal, harmonic music in the subsequent "common practice" period.

The Common-Practice Period and the Rise of Temperament

Alas, while Renaissance theorists considered just intervals the foundation of melody and harmony, there was also a fly in the proverbial ointment, in the form of the growth of independent instrumental music featuring fixed-pitch fretted and keyboard instruments. The polyphonic music of the Middle Ages and the Renaissance was predominantly vocal music and the human voice, when properly trained and coupled to a sensitive ear, is readily capable of the subtle intonational adjustments required to perform sophisticated music in Just Intonation. The same can hardly be said for fretted strings or keyboard instruments. A player of a lute, guitar, or viol can make some expressive adjustment of pitch, it is true, but certainly has not the same degree of flexibility as a singer.[5] An organ or harpsichord can produce only those tones that its pipes or strings have been tuned to. For reasons that will not be explained here, but which will be made plain in subsequent chapters, a fixed-pitch instrument intended to play in perfect Just Intonation in more than a few closely related keys requires far more than twelve tones per octave, an arrangement that had already become standard by the fifteenth century. In fact, some experimental keyboard instruments with far more than twelve keys per octave were built during the sixteenth and seventeenth centuries but, presumably because of their added cost and complexity, these instruments did not become popular and the mainstream of musical thought and activity adopted a different solution to the problem of intonation on fretted strings and keyboard instruments: that of *temperament.*

The basic premise of temperament is that the number of pitches required to play in different keys can be reduced by compromising the tuning of certain tones so that they can perform different functions in different keys, whereas in Just Intonation a slightly different pitch would be required to perform each function. In other words, temperament compromises the quality of intervals and chords in the interest of simplifying instrument design and construction and playing technique. Many different schemes of temperament were proposed in the Renaissance and baroque eras, but, at least where keyboard instruments were concerned, they eventually coalesced into a type of tuning known as *meantone temperament.* (According to many writers, equal temperament was always the preferred system for lutes and viols, because it greatly simplified the placement and spacing of the frets.) Meantone temperament aims to achieve perfect major thirds and acceptable major and minor triads in a group of central keys, at the expense of slightly flatted fifths in those same central keys and some bad thirds and triads and one very bad fifth in more remote keys. The exemplary variety of meantone temperament, called quarter-comma meantone, produced, in a twelve-tone realization, eight good major triads and eight good minor triads, with the remaining four triads of each type being badly mistuned.[6] Meantone tunings satisfied the needs of composers for a time, but as instrumental music became more complex and the desire to modulate to more remote keys increased, the bad triads became a barrier to progress. As a result, musicians gradually adopted another system, twelve-tone equal temperament.

There is some uncertainty as to who deserves the credit or blame for the invention of equal temperament. It seems to have been the product of many minds working along convergent lines over a number of decades, if not centuries.[7] The French monk and mathematician Marin Mersenne (1588 – 1648) gave an accurate description of equal temperament and instructions for tuning it on a variety of instruments in his most important work, the *Harmonie Universelle* (1639), thereby contributing substantially to its popularization, but the practical adoption of equal temperament, like its invention, was a gradual process, occurring at different rates in different countries. Equal temperament seems to have first found favor for keyboard instruments in Germany, where some organs were so tuned as early as the last quarter of the seventeenth century, although it was still a subject of debate there seventy-five years later. Meantone seems still to have been the predominant system in France in the mid-eighteenth century, and in England meantone continued to be the predominant tuning, at least for organs, until the middle of the nineteenth century. The commonly held assumption that J.S. Bach was an advocate of equal temperament and that he wrote the twenty-four preludes and fugues of *The Well-Tempered Clavier* to demonstrate its virtues is at least debatable. The term “well temperament” was used in the eighteenth century to describe a species of temperament in which all keys were usable and in which the principal consonances of the most central keys often retained their just forms. In well temperaments, different keys had different characters, depending on their closeness to or remoteness from the key on which the tuning was centered. This latter characteristic was considered desirable by many baroque composers and theorists, who believed that different keys had characteristic colors and emotional effects.

Twelve-tone equal temperament, unlike meantone, mistunes all consonant intervals except the octave. Also unlike meantone, twelve-tone equal temperament favors perfect fifths over thirds. The equally tempered perfect fifth is approximately two cents narrower than the just perfect fifth (one cent = $^{1}/_{100}$ tempered semitone or $^{1}/_{1200}$ octave), whereas the equally tempered major third is approximately fourteen cents wider than the just major third, and the equally tempered minor third is approximately sixteen cents narrower than the just minor third. In a sense, the rise of equal temperament can be seen as a partial resurgence of the old Pythagorean doctrine, since the Pythagorean tuning also produced good perfect fifths (and fourths), wide major thirds, and narrow minor thirds. The major advantage of equal temperament over meantone is that every key in equal temperament is equally good (or equally bad). There is no contrast in consonance between keys, so all twelve tones can serve equally well as keynotes of major or minor scales or as the roots of major or minor triads.

Equal temperament was not adopted because it sounded better (it didn’t then and it still doesn’t, despite two hundred years of cultural conditioning) or because composers and theorists were unaware of the possibility of Just Intonation. The adoption of twelve-tone equal temperament was strictly a matter of expediency. Equal temperament allowed composers to explore increasingly complex chromatic harmonies and remote modulations without increasing the complexity of instrument design or the difficulty of playing techniques. These benefits, as we shall see, were not without costs.

Throughout the baroque and classical eras, while music, at least on keyboard instruments, was dominated first by meantone temperament, then by equal temperament, theorists continued to explain musical consonance as the product of simple, whole-number ratios. Considerable advances were made in the scientific understanding of sound production by musical instruments and of the human auditory mechanism during this period. Ironically, Mersenne, who played such a significant role in the popularization of twelve-tone equal temperament, also first detected and described the presence of the harmonic series in the composite tone of a vibrating string and in the natural tones of the trumpet. Mersenne was also the first theorist to attribute consonance to ratios involving 7, the next step up the harmonic series from Zarlino’s senario. Later theorists, most notably Jean Philippe Rameau (1683 – 1784), appropriated the harmonic series as further support from “nature” for the primacy of whole-number ratios as the source of consonance. It apparently did not strike most of the theorists of the seventeenth and eighteenth centuries as problematic that, although they formed the theoretical basis for the whole of contemporary harmonic practice, simple-ratio intervals were gradually being purged from musical practice in favor of tempered approximations.

In the nineteenth century, a vigorous attack on equal temperament was mounted by Hermann von Helmholtz

(1821 – 1894), surgeon, physicist, and physiologist, and father of modern scientific acoustics and psychoacoustics. Helmholtz considerably advanced scientific understanding of the production and perception of musical sound, and proposed the first truly scientific theory of consonance and dissonance. He was a strong advocate of Just Intonation and deplored the effect that equal temperament had on musical practice, particularly with regard to singing. Contemporary with Helmholtz's studies there was a good deal of interest in the invention of experimental keyboards for Just Intonation (primarily organs or harmoniums), particularly in Great Britain. Among those engaged in this activity, the most notable were General Perronet Thompson, Colin Brown, and R.H.M. Bosanquet. Unfortunately, the proposals of Helmholtz and the other intonational reformers of the nineteenth century appear to have had no detectable effect on contemporary musical practice, although Helmholtz's work, in particular, was to have a significant influence on musicians of subsequent generations. Nineteenth century composers were still enamored of the facility for modulation and for the building of increasingly complex harmonies that equal temperament provided, and it was not until these resources were exhausted that any alternative was seriously considered.

The End of Common Practice

Initially, the effect of equal temperament on Western music was probably beneficial. Composers obtained the ability to modulate freely and to build complex chromatic harmonies that had been impossible under the meantone system. As a result, abstract instrumental music flourished as never before, yielding what is generally considered the "golden age" of Western music. Like a plant stimulated by chemical fertilizers and growth hormones, music based on equal temperament grew rapidly and luxuriously for a short period — then collapsed. If equal temperament played a prominent role in stimulating the growth of harmonic music in the common-practice era, it played an equally large part in its rapid demise as a vital compositional style. Twelve-tone equal temperament is a limited and closed system. Once you have modulated around the so-called circle of fifths, through its twelve major and twelve minor keys, and once you have stacked up every combination of tones that can reasonably be considered a chord, there is nowhere left to go in search of new resources.

This is essentially where Western composers found themselves at the beginning of the twentieth century. Everything that could be done with the equally tempered scale and the principles of tonal harmony had been tried, and the system was breaking down. This situation led many composers to the erroneous conclusion that consonance, tonality, and even pitch had been exhausted as organizing principles. What was really exhausted was merely the very limited resources of the tempered scale. By substituting twelve equally spaced tones for a vast universe of subtle intervallic relationships, the composers and theorists of the eighteenth and nineteenth centuries effectively painted Western music into a corner from which it has not, as yet, extricated itself. Twentieth century composers have tried in vain to invent or discover new organizing principles as powerful as the common-practice tonal system. Instead, they have created a variety of essentially arbitrary systems, which, although they may seem reasonable in the minds of their creators, fail to take into account the capabilities and limitations of the human auditory system. These systems have resulted in music that the great majority of the population find incomprehensible and unlistenable.

Given that equal temperament had only been in general use for about 150 years at the time, it may seem strange that so few of the composers of the early twentieth century recognized that the cure for music's ills lay, at least in part, in the replacement of its inadequate tuning system. (Some theorists and composers did, in fact, advocate the adoption of new, microtonal tuning systems, but most of these proposals were for microtonal equal temperaments, such as quarter tones, third tones, sixth tones, eighth tones, or the like, which merely divided the existing twelve-tone scale into smaller, arbitrary intervals, and which made no improvement in the tuning of Western music's most fundamental intervals.) However, despite its fairly recent origin, equal temperament had already become quite deeply entrenched in Western musical thought and practice. There were several reasons for this. One was the industrial revolution. The nineteenth century saw the redesign and standardization of many instruments, particularly the orchestral woodwinds and brass. Strictly speaking, only fixed-pitch instruments (the piano, organ, harp, tuned percussion, and fretted strings) require temperament, the others being sufficiently flexible as to adjust pitch as musical context requires. Nevertheless, brass and woodwind instruments were also standardized to play a chromatic scale such that the "centers" of their pitches corresponded as closely as possible to the pitches of twelve-tone equal temperament. Another reason for the persistence of equal temperament was the repertory of the common-practice period. The previous 150 years had witnessed the development of the orchestra as we know it, along with its repertory, and the concert system that supported it. It had also seen the evolution of the piano, the preeminent equally tempered instrument, as the predominant instrument for both solo performance and accompaniment, and as the most important tool in musical education. The orchestra, the piano, and their players, trained to perform the works of eighteenth and nineteenth century composers, were the resources that turn-of-the-twentieth-century composers had to use if they wished to have their music performed. And all of these resources were dedicated to music that assumed equal temperament. It was little wonder, then, that few composers were willing to challenge this massive establishment in order to work in some new, untested tuning system.

The Twentieth-Century Just Intonation Revival

Although most composers were sufficiently intimidated by the weight of eighteenth and nineteenth century musical practice, fortunately a few were not. The first twentieth century composer to make a serious commitment to Just Intonation and the person primarily responsible for the revival of Just Intonation as a viable musical resource was Harry Partch (1901 – 1974), the iconoclastic American composer, theorist, instrument builder, dramatist, and musical polemicist. When Partch began his compositional career, no one, to the best of my knowledge, was making music in Just Intonation. Beginning with tentative experiments in the mid-1920s and continuing over a span of fifty years, Partch developed a system of Just Intonation with forty-three tones to the octave, built a large ensemble of predominantly stringed and percussion instruments to play in this tuning system, composed and staged six major musical theater pieces, along with numerous lesser works, and produced and distributed his own records. In 1947, Partch published the first edition of his *Genesis of a Music*, an account of his musical theories, instruments, and compositions that became the bible for subsequent generations of Just Intonation composers.

Whereas in previous centuries the goal of most intonational theorists was to find the ideal or most practical tuning for a culturally predominant scale, such as a major, minor, or chromatic scale, the approach of twentieth century composers and theorists working with Just Intonation, as exemplified by Partch, has been quite different. The goal of these artists has been, in most cases, to discover or create tunings that best served their own particular musical goals, whether for a single composition or for a lifetime's work, rather than one that could serve the needs of the culture as a whole.

From when he began work in the mid-1920s until the mid-1950s, Partch was the only composer in the United States doing significant work in Just Intonation. In the 1950s, Partch was joined by Lou Harrison (b. 1917) and Ben Johnston (b. 1926). Harrison first learned about Just Intonation from Partch's *Genesis of a Music.* He composed his first major work in Just Intonation, *Four Strict Songs for Eight Baritones and Orchestra*, in 1954.[8] Although, unlike Partch, he does not work exclusively in Just Intonation, Harrison has written a large body of work in various just tunings. He is probably best known for the creation, in conjunction with his companion, the late William Colvig, of a number of justly tuned American gamelan (Indonesian-style tuned percussion ensembles) and for the body of music he has composed for this medium, but he has also composed just music for a great variety of instrumental and vocal ensembles, often mixing elements from European and Asian musical traditions. Through his teaching at San Jose State University and Mills College in California and his extensive lecturing, he has introduced many younger composers to Just Intonation.

Ben Johnston discovered the possibility of Just Intonation early in life, when he attended a lecture on Helmholtz at age eleven. Later, he, like Harrison, discovered Partch's *Genesis of a Music.* Johnston contacted Partch and for a six month period in 1950 was his student and apprentice in the remote California coastal town of Gualala. Johnston began composing seriously in Just Intonation in 1959. Unlike Partch and Harrison, Johnston's work in Just Intonation employs mainly Western musical forms and instrumental combinations. His earlier work, through the early 1970s, generally combines extended microtonal Just Intonation with serial techniques. His later work tends to be simpler and more tonal, but still uses serialism at least occasionally. Johnston's works include eight string quartets in Just Intonation and numerous vocal and chamber ensemble pieces. He is also the inventor of a system of notation for extended Just Intonation that is used in this primer.

In the 1960s and 1970s, interest in Just Intonation continued to slowly increase. La Monte Young (b. 1935) began working with Just Intonation in the early 1960s in the context of his instrumental/vocal performance group, *The Theater of Eternal Music.* In this ensemble, Young developed the practice of performing long, static compositions based on selected tones from the harmonic series, played on various combinations of amplified instruments and voices. In 1964, Young began work on his semi-improvisational, justly tuned piano composition, *The Well-Tuned Piano*, which can be from five to seven hours in duration and which continues to evolve at the time of this writing. Young is also known for *The Dream House*, a living environment in which a number of electronically generated, harmonically related tones are sustained over a period of months or years.

Terry Riley (b. 1935), who was a member of Young's *Theater of Eternal Music* at various times in the early 1960s, is known primarily as a keyboard composer/improviser. He is perhaps best known as the composer of the early minimalist piece *In C* (1964), which is not explicitly a Just Intonation piece, although it has sometimes been performed this way. In the 1970s, Riley performed extensively on a modified electronic organ tuned in Just Intonation and accompanied by tape delays. More recently, he has been performing his work on justly tuned piano and digital synthesizers, and composing for other ensembles, especially the string quartet.

In the late 1970s and early 1980s the number of composers working with Just Intonation began to increase significantly, due in part to the development of affordable electronic instruments with programmable tuning capabilities and in part to the coming of age of the post–World War II generation of composers. The achievements of Partch, Harrison, Johnston, Young, and Riley made it evident to these younger composers that Just Intonation was a valuable resource for composers of diverse styles and tastes, and the availability of electronic instruments with programmable tuning made it possible for the first time for composers to experiment with a variety of different tuning systems without having to invent and build novel instruments or to train performers in unusual playing techniques. Changing the pitches available on a digital synthesizer simply means changing the data values in a tuning table or switching to a different table. If the instrument and its operating software have been designed to facilitate such changes, either of these functions can be performed virtually instantaneously by a computer running appropriate software. Hence, a conventional keyboard can be used to play a virtually unlimited number of different pitches. This capability has, for all intents and purposes, eliminated the condition that first brought temperament into being: the necessity of restricting the number of pitches used in music to the number of keys available on an affordable, playable keyboard.

Among the many composers currently doing significant work in Just Intonation are William C. Alves, Lydia Ayers, Jon Catler, David Canright, Dean Drummond, Cris Forster, Glenn Frantz, Ellen Fullman, Kraig Grady, Michael Harrison, Ralph David Hill, David Hykes, Douglas Leedy, Norbert Oldani, Larry Polansky, Robert Rich, Daniel Schmidt, Carter Scholz, James Tenney, and Erling Wold. The variety of musical styles represented by this group is extremely diverse, and the use of Just Intonation may be the only feature they all share. Although more than half work primarily or exclusively with electronic media, they also include exponents of Partch's tradition of acoustic instrument building (Drummond and Grady), Lou Harrison's American gamelan movement (Schmidt), Young's and Riley's improvisational keyboard styles (M. Harrison), a harmonic singer (Hykes), and even a justly tuned rock guitarist (Catler).

The Purpose of this Publication

Although the technical barriers to the composition and performance of significant music in Just Intonation have been considerably reduced in recent years, barriers of another type remain largely in place, namely the weight of custom and the lack of accessible information on principles of Just Intonation. The colleges, universities, and conservatories continue to teach a curriculum based on music of the common-practice era, in which alternate tunings are unlikely to receive more than a passing mention. With the exception of the fortunate few who find themselves in institutions with a microtonal composer or theorist on the faculty, students who develop an interest in these matters are unlikely to receive much support or encouragement, much less practical instruction, from the academic establishment. Such students, if they persist, generally find it necessary to educate themselves, and in the process often have to reinvent or rediscover principles and structures that are well known to more experienced composers.

In an attempt to remedy this situation, in the fall of 1984, I and my associates in the experimental music ensemble Other Music, in consultation with a number of other West Coast Just Intonation composers and theorists, founded the Just Intonation Network. The Just Intonation Network is a nonprofit group fostering communication among composers, musicians, instrument designers, and theorists working with Just Intonation. Its primary goal is to make information about the theory and practice of composition in Just Intonation available to all who want or need it. The primary method for distributing this information is the network's journal, **1/1**, the only current periodical devoted primarily or exclusively to Just Intonation. For the past eighteen years I have served as editor of this publication.

A survey of Just Intonation Network members taken several years ago revealed that more than half were newcomers to the study of Just Intonation who found a significant portion of the articles in **1/1** over their heads. It was with the goal of assisting these readers that the *Just Intonation Primer* was conceived. The *Just Intonation Primer*, as its title indicates, is not intended to provide a complete or comprehensive course in the theory and practice of Just Intonation, let alone tuning in general or other aspects of composition. Its purpose, rather, is to provide the reader with the basic information and skills necessary to read and comprehend intermediate and advanced texts such as articles in **1/1** or Harry Partch's *Genesis of a Music*, and to prepare the reader to begin independent study and composition.

The *Primer* is intended for readers with at least an elementary knowledge of common-practice Western music theory, including the basic terminology of intervals, chords, and scales, and the fundamentals of harmony. The reader is not assumed to have any prior knowledge of Just Intonation or of alternative tunings in general, nor is the reader expected to be a mathematician or number theorist. The only math required to understand this book is basic arithmetic, in combination with some simple procedures explained in Chapter Three. An inexpensive scientific calculator will prove useful for comparing the sizes of intervals.

INDEX

A

Added-second chord 57
Aeolian scale 39
Alves, William C. 7
American gamelan 6, 7, 66, 72
Anomalies 33. *See also* great diesis, Pythagorean comma, septimal comma, syntonic comma, etc.
Ars Antiqua period 38
Ayers, Lydia 7

B

Bach, J.S. 4
Barbershop quartet 35
Barbour, J. Murray 2
Baroque era 3, 4
Basilar membrane 14, 18
Bassoon 74
Beating harmonics. *See* Harmonics: beating
Beats. *See* Harmonics: beating; Interference beats
Beat frequency (f_B) 13
  due to the inharmonicity of piano strings 68
Benade, Arthur H.
  experiment to identify "special relationships" 22–24
Bethea, Rob 76
Blues 35, 51
Blue notes 35, 55
Boethius, Anicius Manlius Severinus 2
Bosanquet, R.H.M. 5, 70
Bowed string instruments 75
Branca, Glenn 65
Brass instruments 19, 74
Brown, Colin 5, 70

C

Canright, David 7, 71
Carlos, Wendy 73
Catler, Jon 7
Cents 15
  calculation 26
  defined 4
Chalmers, John H.
  tritriadic scales 64
Cheng 71
Chords 30–32. *See also* Triads, other chord names
  condissonant 31
  consonant 31, 46
  dissonant 31
  eleven- and thirteen-limit 62–64
  five-limit 46–48
    condissonant 47–48
  seven-limit 56–59
    condissonant 58–59
    subharmonic 57–58
Chromatic scale 45. *See also* Harmonic duodene
Chromelodeon 70
Circle of fifths 37
Clarinet 19
Classical era 4
Clavichord 67
Clusters. *See* Tone clusters
Cochlea 14
Cocktail-party effect 16
Colvig, William 6, 72
Combination tones. *See* Difference tones; Summation tones
Comma. *See* Pythagorean comma, septimal comma, syntonic comma
Comma of Didymus. *See* Syntonic comma
Common-practice period 6
Common-practice theory 35, 51
Complex tones 19–20
Continuous-pitch instruments 66, 74–76
Critical band 13, 51

D

Difference tones 16–17, 19
  first-order 16
    of condissonant triads 48
    of just major triad 46
    of just minor triad 46–47
    of major-seventh chord 48
    of minor-seventh chord 48
  higher-order 16–17
  of dominant-ninth chord 56
  of dominant-seventh chord 56
  of subharmonic pentad 58
  of tempered major third 17
Diminished-seventh chord 49–50, 64
Diminished major-seventh chord 65
Ditone diatonic. *See* Pythagorean diatonic scale
Divisions of the Tetrachord (book) 34
Dominant-minor-ninth chord 50, 64
Dominant-ninth chord 31, 55, 56, 58, 62
  subsets of 56–57
Dominant-seventh chord 31, 35, 48–49, 51, 55, 56
  subsets of 56–57
Drummond, Dean 7, 72
Duodenarium 46
Duodene. *See* Harmonic duodene

E

Electronic organ 70
Ellis, Alexander J. 26, 31, 45, 64, 70
Enharmonic equivalents 44
Enharmonic spellings 49
Experimental Musical Instruments (periodical) 67

F

False Consonance 54–55
Fender Rhodes Electric Piano 70
Fixed-Pitch Instruments 67–71
Fixed scales 42, 55
  seven-limit 59
Flute 19, 74
Fokker, Adriaan
  method for representing intervals on the lattice 43
Formants 19
Forster, Cris 7, 72
Frantz, Glenn 7
French horn 74
French impressionists 63
Frequency, absolute 11. *See also* Hertz (Hz)
  calculating 28
  of 1/1 36
Frequency ratios 11, 25–29. *See also* Just Intervals
  calculations with 25–29
    addition 25
    complementation 26
    converting to cents 26
    division 26–28
    subtraction 25–26
  converting to harmonic or subharmonic series segment 28–29
  only unambiguous inerval names 32
  rules and conventions for using 25
  superparticular 27
Fretted instruments 3, 71
Fullman, Ellen 7
Fused tone 13, 14

G

Galilei, Vincenzo
  proposed fretting for the lute 64
Genesis of a Music 2, 6, 67, 70
Grady, Kraig 7, 72
Great diesis 44–45, 50
Guitar 71

H

Hair cells 14
Half-diminished-seventh chord 55, 56–57
Harmonics 15, 16. *See also* Harmonic series; Partials: harmonic
  beating 20–24
    of an octave (2:1) 20–21
    of a perfect fifth (3:2) 21–22
    of a unison (1:1) 20
  coincident (matching) 20–24
    formula for 21
    of just intervals (figure) 21
    of just major triad 46
    of just minor triad 46
Harmonic and subharmonic series (figure) 29
Harmonic duodene 45–46
Harmonic series 4, 14–16, 18, 19, 28–30
  defined 14
  fundamental of 15, 19, 31
    missing. *See* Periodicity pitch
Harmonic spectrum 15, 19
  cutoff frequency of 19
  of strings 19
Harmonists (ancient Greek theorists) 2
Harp 70, 72

Harpsichord 67, 72
Harrison, Lou 6, 66, 72, 75
  Concerto for Piano with Javanese Gamelan 69
  Concerto for Piano with Selected Orchestra 69, 76
  Four Strict Songs for Eight Baritones and Orchestra 6, 76
  The Tomb of Charles Ives 76
Harrison, Michael 7, 69
  From Ancient Worlds 68
Heim, Bruce 74
Helmholtz, Hermann von 4–5, 45, 70
Heptad 62, 63
Hertz (Hz) 10, 11. *See also* Periodic vibration: frequency
Hexad 62, 63
Hill, Ralph David 7, 54
Hohner Clavinet 70
Hykes, David 7

I

Identities
  as defining characteristics of a chord 30, 31
  defined 30
Idiophones 19, 72–73
Industrial revolution 5
Interference Beats 12–13, 46. *See also* Harmonics: beating
Intermodulation products. *See* Difference tones, Summation tones
Interval 10–11. *See also* Frequency ratios; Just intervals
  conventional names for 10, 32
  defined 10
  harmonic vs subharmonic interpretation 28
Intonation Systems 71

J

Jazz 35, 51
Johnston, Ben 6, 75
  notation for Just Intonation 33
  ratio scales 64
  remarks on 11 and 13 61
  Sonata for Microtonal Piano 70
Just Intervals
  1:1 (unison) 36
  2:1 (octave) 36, 37, 44
  3:1 (perfect twelfth) 24
  3:2 (perfect fifth) 36, 37, 38, 39, 40, 42, 45, 46, 51, 54, 55, 57, 59
  4:1 (double octave) 24
  4:3 (perfect fourth) 33, 36, 37, 38, 40, 42, 54, 55
  5:1 24
  5:2 (major tenth) 24
  5:3 (major sixth) 24, 39
  5:4 (major third) 38, 39, 40, 42, 45, 46, 50, 51, 55, 56, 57, 58
  6:1 24
  6:5 (minor third) 24, 39, 45, 49, 50
  7:1 24
  7:2 24, 51
  7:3 (septimal minor or subminor tenth) 24, 51
  7:4 (harmonic, subminor, or septimal minor seventh) 35, 51
  7:5 (most consonant tritone) 23, 51, 55, 57
  7:6 (subminor or septimal minor third) 23, 51, 55, 57, 58, 59
  8:1 (triple octave) 24
  8:3 24
  8:5 (minor sixth) 24, 39, 40, 41, 42, 50, 55
  8:7 (septimal whole tone; supermajor second) 23, 24, 51, 54, 55, 57
  9:4 (major ninth) 24, 37
  9:5 (acute minor seventh) 40, 49, 56, 57
  9:7 (supermajor third) 55, 56, 57, 58, 59
  9:8 (major whole tone) 33, 37, 38, 40, 55, 64, 65
  10:3 24
  10:7 (septimal tritone) 51, 55
  10:9 (minor whole tone) 40, 55
  11:1, 11:2, 11:3, 11:4, 11:5 24
  11:6 62
  11:8 (primary interval for 11) 61, 62
  11:9 (neutral third) 62
  11:10 62
  12:5 (minor tenth) 24
  12:7 51
  12:11 62
  13:1, 13:2, 13:3, 13:4, 13:5, 13:6 24
  13:8 (primary interval for 13) 61, 62
  13:9 62
  13:10 62
  15:8 (diatonic major seventh) 40
  15:13 62
  15:14 54, 55
  16:5 24
  16:9 (minor seventh) 37, 49
  16:11 62
  16:13 62
  16:15 (diatonic semitone) 40, 45, 55
  17:16 64
  17:16 (primary interval for 17) 64
  18:11 62
  18:17 64
  19:16 (primary interval for 19) 64
  21:16 (septimal subfourth) 54, 55
  21:20 54, 55
  25:16 50
  25:24 (small chromatic semitone) 45, 55
  27:16 (Pythagorean major sixth) 37
  27:20 (acute, imperfect, or wolf fourth) 40
  27:25 (great limma) 45
  28:25 (intermediate septimal whole tone) 54
  28:27 54, 55
  32:21 54, 55
  32:25 (diminished fourth) 45, 50
  32:27 (Pythagorean minor third) 37, 41, 49
  33:32 62
  35:32 (small septimal whole tone) 54
  36:25 (acute diminished fifth) 49
  36:35 (septimal quarter tone or diesis) 54
  40:27 (grave, imperfect, or “wolf” fifth) 40, 41
  49:48 (septimal sixth tone) 54
  50:49 (septimal sixth tone) 54
  64:63. *See* Septimal comma (64:63)
  65:64 62
  75:64 (augmented second) 45
  81:64 (Pythagorean major third, ditone) 36, 37, 38, 40, 41
  81:80. *See* Syntonic comma
  128:81 (Pythagorean minor sixth) 37, 40
  128:125. *See* Great diesis
  135:128 (large limma, large chromatic semitone) 45
  243:128 (Pythagorean major seventh) 40
  256:243 (Pythagorean limma) 37, 40
Just Intonation
  defined 1–2
  discovery 2
  eleven- and thirteen-limit 59–63
  five-limit 35, 38–50, 75
    difficiencies of 48–50
  obtaining satisfactory performances in 79–80
  seven-limit 51–59
    coloristic use of 55
  seventeen- and ninteen-limit 64–65
  three-limit 36–38
  twentieth-century revival 6–7
Just Intonation Network 8, 78

K

Keyboard instruments 3, 4, 67–70
  Acoustic 67–70
  electroacoustic 70
Koto 71

L

Lattice
  complementation on 44
  five-limit 42–44
    figure 43
  seven-limit 51
    figure 52, 53
  syntonon diatonic (figure) 40
  transposition on 43
Leedy, Douglas 7
Limit of frequency discrimination 13
Lute 71

M

Major-ninth chord 48
Major-seventh chord 48
Major scale
  five-limit
    constructing 39–40
Maxwell, Miles 54
McClain, Ernest G. 2
Mersenne, Marin 4
Microtones. *See also* Just Intervals
  eleven- and thirteen-limit 62
  seven-limit 54
Middle Ages 38
MIDI 76–79
  keyboard controller 78
  pitch bend 76–77
    using with tuning tables 78
  Tuning-Dump Specification 78–79
Minor-major-seventh chord 65
Minor-ninth chord 48
Minor-seventh chord 48

Minor-sixth chord 64
Missing fundamental. *See* Periodicity pitch

N

Neutral intervals 62
Nondominant-ninth chords 47
Nondominant-seventh chords 47
Notation for Just Intonation 32–33
  Ben Johnston's 33, 62
    Table 32
Novatone 71

O

Oboe 74
Odington, Walter 3
Oldani, Norbert 7
Oliveros, Pauline 70
1/1 (Journal of the Just Intonation Network) 8
One (foundation of all just tunings) 36
On the Sensations of Tone (book) 45
Orchestra 6
Other Music 7
Otonalities 29, 47. *See also* Harmonic series
Overtones. *See* Harmonics; Harmonic series; Partials

P

Partch, Harry 2, 6, 47, 58, 61, 62, 66, 67, 70, 71, 72
  tonality diamond 64
Partials 15. *See also* Harmonics; Harmonic series
  harmonic 15, 19–20
  inharmonic 15, 19
    of a free rectangular bar 72
    of a hinged bar 68
    of piano strings 67–69
Periodicity pitch 17–19, 20
  of eleven- and thirteen-limit dyads 61
  of just major triad 46
  of just minor triad 46–47
  of subharmonic pentad 58
  of subminor triad 57
Periodic vibration 9–10, 15
  amplitude 10
  frequency 10
  graph of 9
  period (τ) 10, 16
Phase Relationship 12
Piano 6, 35, 67–70
  single-strung 68
  tuning strategies 69
Pipe organ 70
Pitch 10. *See also* Periodic vibration: frequency
Pitch (periodical) 74
Place theory 18
Polansky, Larry 7
Primary interval
  defined 30
  for eleven (11:8) 61
  for five (5:4) 38
  for ninteen (19:16) 64
  for seventeen (17:16) 64
  for seven (7:4) 51
  for thirteen (13:8) 61
  for three (3:2) 36
  for two (2:1) 36
Prime limit 27, 35
  defined 30
Prime numbers 35
  defined 30
Psaltery 70, 72
Ptolemy, Claudius 2, 3, 39
Ptolemy's syntonon diatonic 33, 59. *See also* Major scale: five-limit
  complementation (figure) 44
  relative minor of 41
  tuning method (figure) 39
Pure tones 10, 15
  superposition of 11–12
Pythagoras of Samos 2, 36
Pythagorean comma 37–38, 44
Pythagorean diatonic scale 36–37, 40
Pythagorean tuning 2–3, 4, 36–38. *See also* Just Intonation, three-limit
  tendency of string players for 75

Q

Quartertone scale. *See* Temperament: twenty-four-tone equal

R

Rameau, Jean Philippe 4
Ratios. *See* Frequency ratios; Just Intervals
Ratios of eleven
  table 61
Ratios of thirteen
  table 62
Recorder 74
Reed instruments 19
Reed organ 70
Renaissance 3
Residue tone. *See* Periodicity pitch
Resultant tones. *See* Difference tones, Summation tones
Rich, Robert 7, 78
Riley, Terry 7, 69, 70, 75
  In C 7
Root (of a chord) 31
Rosenthal, David 72
Roughness 13, 20. *See also* Interference Beats

S

Sackbut 76
Sawtooth wave 16
Saxophone 74
Schmidt, Daniel 7, 72
Scholz, Carter 7, 78
Senario 3, 4
Septimal comma (64:63) 54, 65
Seven
  consonance of 35
7♭5 chord 64
7♯5 chord 65
Siegel, Jules 77
Simple harmonic motion 9. *See also* Sine wave
Simple tone 10
Sine wave 9, 10
Singing. *See* Voice
Special relationships 22–24
  absence of among eleven- and thirteen-limit dyads 59
  beyond the octave 23–24
  defined 22
  involving higher harmonics 24
  table 23
Sruti box 76
Stone, Tom 71
String players
  "natural tendencies" of 74
  use of vibrato 75
Subharmonic series 28–30
Subjective pitch. *See* Periodicity pitch
Subminor-seventh chord 58
Summation tones 16, 17
  first-order 17
Supermajor-seventh chord 59
Supertonic problem 40–42
Synthesizers. *See also* MIDI
  with user-programmable tuning tables 7, 77–78
  Yamaha DX/TX 77, 79
Syntonic comma 39, 41, 42, 44, 71
Syntonon diatonic. *See* Ptolemy's syntonon diatonic

T

Tamboura 76
Temperament
  defined 3
  meantone 3, 35, 42
  twelve-tone equal 4, 11, 33, 35, 37, 42, 49
    benefits 5
    deviation of harmonic series from 15
  twenty-four-tone equal 62
  well temperament 4, 35
Tempered major third 17, 23
Tenney, James 7
Tetrachordal genera 34, 59
Tetrachordal scales
  equal 34, 37, 40
  mixed 34
  seven-limit 59
    (figure) 60
Tetrachords 33–34, 55
  defined 33
  disjunct 34
  permutations 34
Tetrad. *See also* Dominant-seventh chord, Major-seventh chord, Minor-seventh chord
  subharmonic 58
Thompson, General Perronet 5, 71
Tied frets 71
Timbre 10, 19
Timing theory 18
Tone clusters 63
  eleven- and thirteen-limit 63
  seven-limit 56

Transposition 11
Triads
  augmented 48, 49–50, 65
  condissonant 47–48
  diminished 31, 35, 48, 51, 55, 56
    subharmonic 58
  dominant 39
  5:7:9 57
  4:6:7 57
    subharmonic 58
  major 31, 39, 46
    voicings in C (table) 30
  minor 31, 39, 41, 46
    proposed tuning with 19:16 64–65
    supertonic 41, 42
  open 38
  Pythagorean 38
    major 46
    minor 46
  subdominant 39
  subminor 31, 55, 57
  supermajor 58
  tonic 39
Trombone 76
Tuning
  History 2–7
Tuning and Temperament (book) 2
Twelfth root of two 11

U

Utonalities 29, 47, 58. *See also* Subharmonic series

V

Viol 71
Virtual pitch. *See* Periodicity pitch
Vogt, Walter 71
Voice 76

W

The Well-Tempered Clavier 4
Widener, Ward 64, 65
Wilson, Ervin
  combination product sets 64
Wind instruments 73–74
Wold, Erling 7

Y

Young, La Monte 7, 61, 69
  Dream Chords 38, 64, 65
  The Dream House 7
  5 V 67 6:38 PM NYC 65
  The Four Dreams of China 64, 76
  The Lower Map of The Eleven's Division in The Romantic Symmetry (over a 60 cycle base) in Prime Time from 122 to 119 with 144 65
  The Romantic Symmetry in Prime Time from 112 to 144 with 119 65
  The Second Dream of the High Tension Line Step-Down Transformer 64, 76
  The Subsequent Dreams of China 64
  The Theater of Eternal Music 7
  The Well-Tuned Piano 7, 65, 68

Z

Zarlino, Gioseffe 3
Zither 70