Image: Dark Energy Camera, from http://darkenergydetectives.org

The Large Scale Structure of the Universe 2: Theory and Measurement

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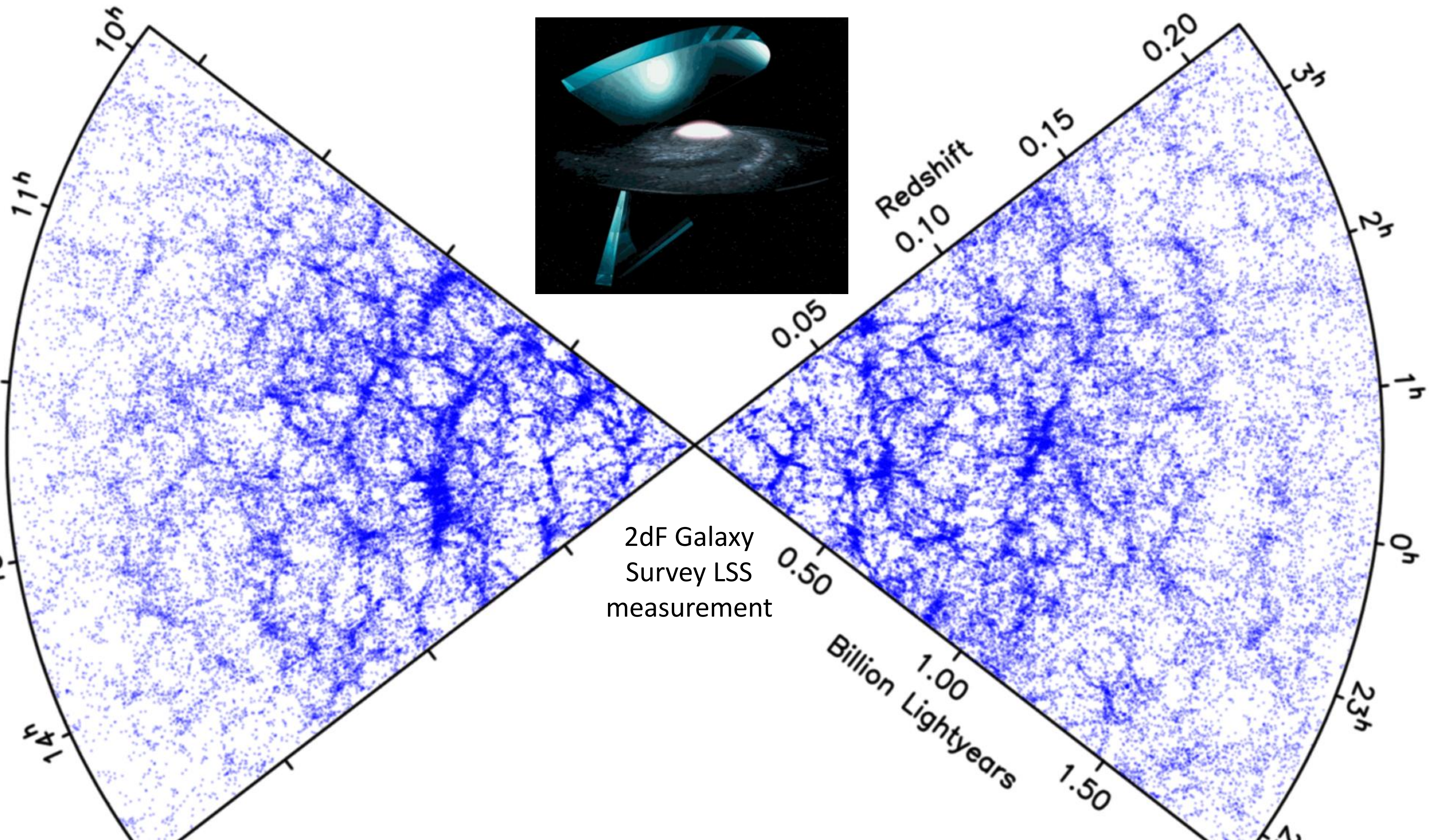
The LSS: Theory and Measurement

Gravitational collapse
Matter Perturbations in the Universe and their growth
The CMB: Brief desciption and results
Non-linear growth: Spherical collapse
Simulations

How to measure LSS,
Correlation function, power spectrum
Galaxy bias
Observational effects. Random samples and other effects

We have treated the Universe as smooth.
But it contains structures!

Galaxies are not randomly distributed, but clustered



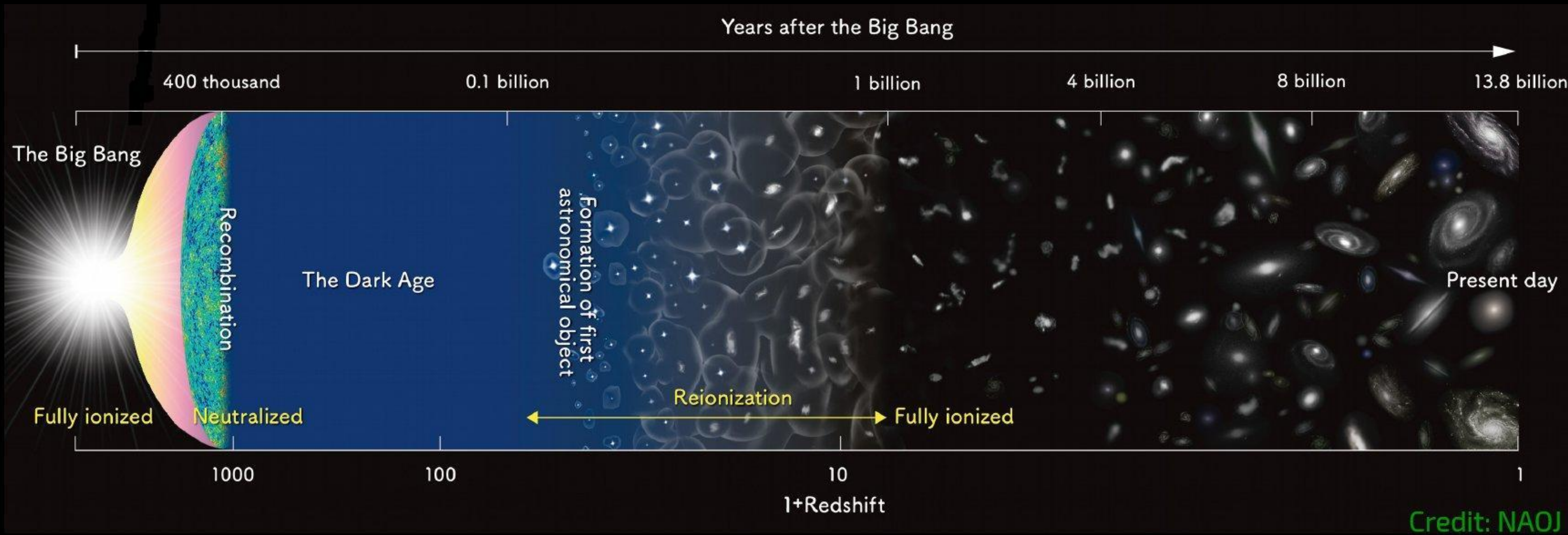
We are able to explain how this structure formed and evolved

LSS Formation and Evolution: General Idea

Generation of fluctuations: Quantum fluctuations of the inflaton become classic due to the wild inflationary expansión. At the end of inflation the inflaton field decays into particles

Quantum fluctuations of the field → fluctuations in the number of particles → fluctuations in the energy density

Inflation generates initial conditions, (Gaussian) i. e. seeds for the LSS: Gravity does the rest



The properties of the initial fluctuations determine the properties of the LSS

Important point: Inflaton is a quantum field → We cannot predict the specific value of the fluctuations, but only their statistical properties → Our predictions for the LSS are statistical

Summary of the formation and evolution of structure in the Universe

10^{-35} s

Quantum Fluctuations during inflation

$V(\phi)$

$\sim 10^5$ years

Perturbation Growth: Pressure. vs. Gravity

Ω_M, Ω_r, Ω_b, f_ν

Photons freestream: Inhomogeneities turn into anisotropies

z_{reion}, Ω_Λ, w

Matter perturbations grow into non-linear structures observed today

Fluctuations are small. We can use perturbation theory

$$g_{\mu\nu}(\eta, \boldsymbol{x}) = \bar{g}_{\mu\nu}(\eta) + \delta g_{\mu\nu}(\eta, \boldsymbol{x})$$

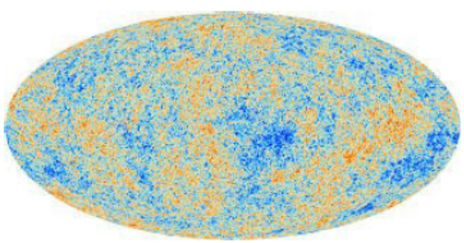
$$T_{\mu\nu}(\eta, \boldsymbol{x}) = \bar{T}_{\mu\nu}(\eta) + \delta T_{\mu\nu}(\eta, \boldsymbol{x})$$

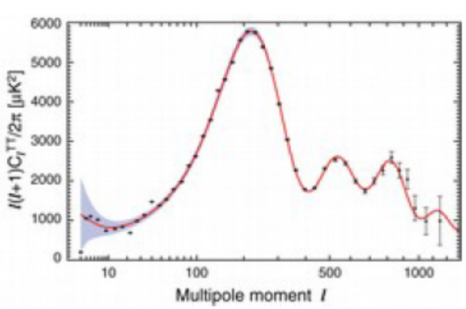
$$\Rightarrow \Phi, \delta\rho_m, \delta\rho_r$$

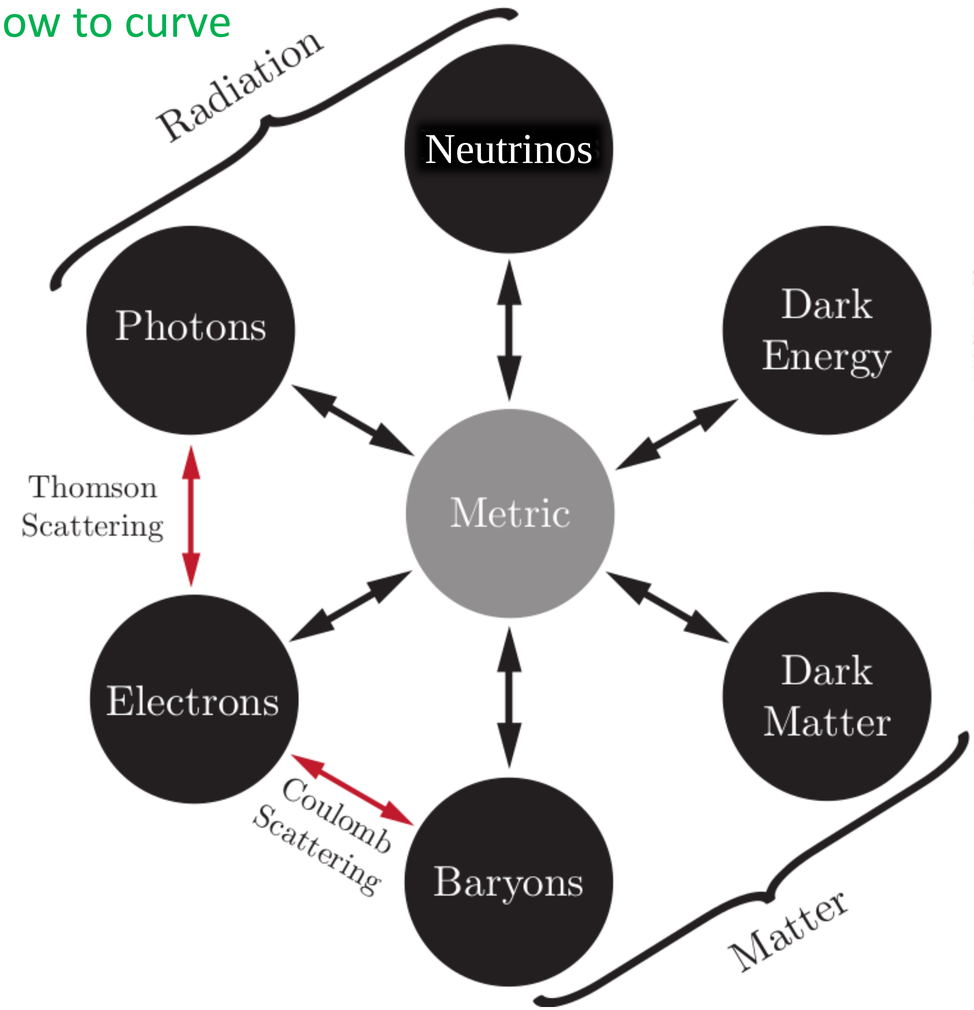
2 types of perturbations: metric perturbations, density perturbations

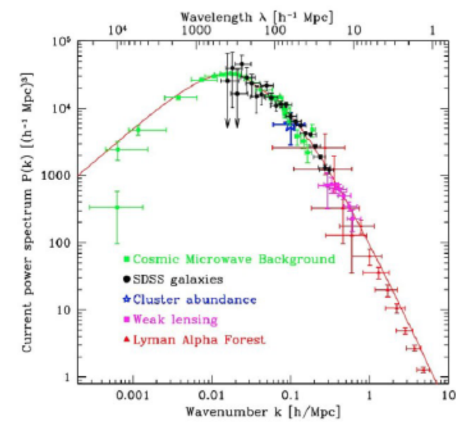
Remember: Spacetime tells matter how to move, matter tells spacetime how to curve

CMB $\Leftarrow$









$\Rightarrow$ LSS

Matter perturbations

Use newtonian gravity → Good approximation of general relativity in cosmology on scales well inside the Hubble radius and when describing non-relativistic matter (for which the pressure P is much less than the energy density ρ).

Continuity
$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0$$

Euler
$$\frac{\partial v}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} = -\frac{1}{\rho}\nabla p - \nabla \phi$$

Poisson
$$\nabla^2 \phi = 4\pi G \rho.$$

These equatons are used in all cosmological N-body simulations of the non-linear growth of structure

3 regimes:

δ << 1: linear theory

δ ~ 1: need specific assumptions (i. e. spherical symmetry)

δ >> 1: non-linear regime. Solve numerically, simulations (also higher order perurbations)

In general: Universe is lumpy on small scales and smoother on large scales – consider inhomogeneities as a perturbation to the homogeneous solution

Matter perturbations

$$\rho \to \bar{\rho}(t) + \delta\rho \equiv \bar{\rho}(t)(1+\delta)$$

$$P \to \bar{P}(t) + \delta P$$

$$\mathbf{u} \to a(t)H(t)\mathbf{x} + \mathbf{v}$$

$$\Phi \to \bar{\Phi}(\mathbf{x}, t) + \phi\,.$$

Linearizing the equation:

$$\ddot{\delta} + 2H\dot{\delta} = 4\pi G\rho_0\delta + \frac{c_s^2}{a^2}\nabla_c^2\delta$$

Using the Fourier transform, we can write eqs. For the Fourier modes:

$$\delta(\boldsymbol{x}, t) = \sum_{\boldsymbol{k}} \delta_{\boldsymbol{k}}(t)\, e^{i\boldsymbol{k}\cdot\boldsymbol{x}}$$

$$\delta_{\boldsymbol{k}}(t) = \frac{1}{V}\int \delta(\boldsymbol{x}, t)\, e^{-i\boldsymbol{k}\cdot\boldsymbol{x}}\, d^3x$$

$$\ddot{\delta}_k + 2H\dot{\delta}_k = \left(4\pi G\rho_0(t) - \frac{k^2 c_s^2}{a^2}\right)\delta_k.$$ For baryonic matter

$$\ddot{\delta}_k + 2H\dot{\delta}_k - 4\pi G\rho_m(t)\delta_k = 0$$ For dark matter

Matter perturbations

We can linearize this equation because δ is very small . The linear regime is very important:

- On all scales, primordial fluctuations were extremelly small, $\delta << 1$. The seeds of structure formation were linear
- The linear stage of structure formation is a relatively long lasting one.
- One may always find large scales where the density and velocity perturbations are still linear. Today, scales larger than ~10 h^{-1} Mpc behave linearly
- CMB measurements have established the linear density fluctuations at the recombination era. By studying the linear structure growth, we are able to translate these into the amplitude of fluctuations at the current epoch, and compare these predictions against the measured LSS in the Galaxy distribution

$$\delta(\mathbf{x}, t) = \frac{\rho(\mathbf{x}, t) - \rho_0(t)}{\rho_0(t)}$$

Matter perturbations

in a matter dominated Universe $$H = \frac{2}{3t}$$

$$\ddot{\delta} + \frac{4}{3t}\dot{\delta} - \frac{2}{3t^2}\delta = 0$$

$$\delta = A(x)t^{2/3} + B(x)t^{-1}$$

linear growth

$$\delta = \delta_0(x)a$$

Matter perturbations

in a lambda dominated Universe

$$H^2 = \frac{\Lambda c^2 \pi G}{3}$$

$$\ddot{\delta} + 2H\dot{\delta} = 0$$

$$\delta = A(x) + B(x)e^{-2Ht}$$

frozen fluctuations

Matter perturbations

In a radiation dominated universe

$$\ddot{\delta}_{\mathbf{k}} + 2H\dot{\delta}_{\mathbf{k}} - 4\pi G\bar{\rho}_m\delta_{\mathbf{k}} = 0$$

$$a \propto t^{1/2} \qquad \Rightarrow \qquad H = \frac{1}{2t} = \sqrt{\frac{8\pi G}{3}\bar{\rho}}$$

$$H^{-2}\ddot{\delta}_{\boldsymbol{k}} + 2H^{-1}\dot{\delta}_{\boldsymbol{k}} = \cancelto{0}{\frac{3}{2}\frac{\bar{\rho}_m}{\bar{\rho}}}\delta_{\boldsymbol{k}} \qquad \bar{\rho}_m \ll \bar{\rho}$$

At most logarithmic growth during radiation domination.

1) The increased expansión rate due to the presence of a smooth component slows down the growth of perturbations
2) There is no significant growth during the radiation dominated period

$$\delta_{\boldsymbol{k}}(t) = A + B\ln t$$

Matter perturbations

- Matter dominated case

$$\delta = A(x)t^{2/3} + B(x)t^{-1}$$

Linear growth

- Radiation dominated case

$$\delta_{\boldsymbol{k}}(t) = A + B\ln t$$

No significant growth

- Lambda dominated case

$$\delta = A(x) + B(x)e^{-2Ht}$$

Frozen fluctuations

- general case

$$\delta = \delta_0(x)ag(a,\Omega_{m0})$$

g is constant at early times and scales as 1/a at late times

for our cosmology, the action ended around z=0.5

Matter perturbations

Baryon photon fluid
Jeans length and scales for collapse

$$\ddot{\delta}_k + 2H\dot{\delta}_k = \left(4\pi G\rho_0(t) - \frac{k^2 c_s^2}{a^2}\right)\delta_k.$$

GRAVITY PRESSURE

Jeans Length: Both effects are equal

$\frac{c_s^2 k^2}{a^2} > 4\pi\rho_0$ → Oscillating solution

$\frac{c_s^2 k^2}{a^2} < 4\pi\rho_0$ → Perturbations grow

$$k_J = \frac{a}{a_0}\frac{\sqrt{4\pi G\bar{\rho}}}{c_s}$$ Jeans wavenumber

$$\lambda_J = \frac{2\pi}{k_J}$$ Jeans wavelength

the perturbations grow exponentially (if no expansion) with time or oscillate as sound waves depending on whether their wave number is greater than or less than the Jeans wave number

For $k > k_J$ we have sound waves, for $k < k_J$ we have collapse. The expansion adds a sort of friction term on the left-hand side: The expansion of the universe slows the growth of perturbations down.

Matter perturbations

CMB shows that at z~1100, perturbations are of the order 10^{-5}. If they grow as $\delta \sim t^{2/3}$, then for z=0 they grow a factor of 1000, becoming of the order 1%

Dark matter provides a solution. Perturbations in dark matter do not couple to radiation and can be much larger tan gas (baryons) perturbations without perturbing the CMB. By starting with much larger perturbations, we can reach the $\delta \sim 1$ regime much earlier, allowing to form the observed structures → **LSS formation NEEDS dark matter!**

It is useful to express the perturbation as $\delta(z) = \delta_0 D_+(z)$

linear growth factor, $D_+(z)$

$$D_+(z) = \frac{1}{1+z}\frac{5\Omega_{\rm m}}{2}\int_0^1 \frac{{\rm d}a}{a^3 H(a)^3}$$

Matter perturbations: Comparing the theory to the observations

The current standard model of cosmology includes the inflation as primordial perturbations generator. In any case, the initial perturbations are Gaussian.

The density contrast δ is a homogeneous, isotropic Gaussian random field (Fourier modes are uncorrelated)

Its statistical properties are completely determined by 2 numbers: mean and variance.

The variance is described in terms of a function called the **POWER SPECTRUM**

$$\left\langle \hat{\delta}(\vec{k})\hat{\delta}^*(\vec{k}')\right\rangle \equiv (2\pi)^3 P(k)\delta_{\rm D}\left(\vec{k}-\vec{k}'\right)$$

The initial power spectrum has the Harrison-Zel´dovich form:

$P(k) \propto k^{n_s}$, $n_s \sim 1$ Spectral index

Matter perturbations

These assumptioins have been very precisely verified in the CMB.
Spectral index can be related to some parameters of the inflationary field

The power spectrum is splitted into a linear and a non-linear part

$$P(k) = P_L(k) + P_{NL}(k)$$

The linear power spectrum corresponds to the linear overdensity field and is given by

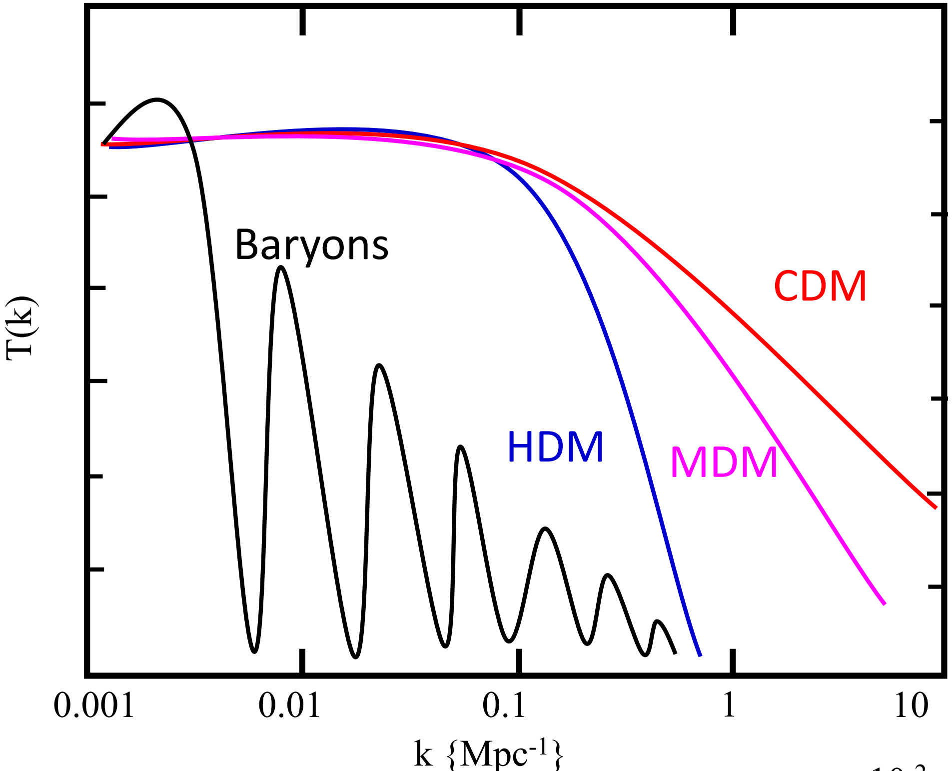
$$P_L(t,k) = A_0\, k^{n_s} T^2(k) D_+^2(t)$$

Where D+(t) is the growth factor and T(k) is the transfer function and takes into account the transformation from the density fluctuations from the primordial spectrum

Through the radiation domination epoch
Through the recombination epoch
To the post recombination power spectrum

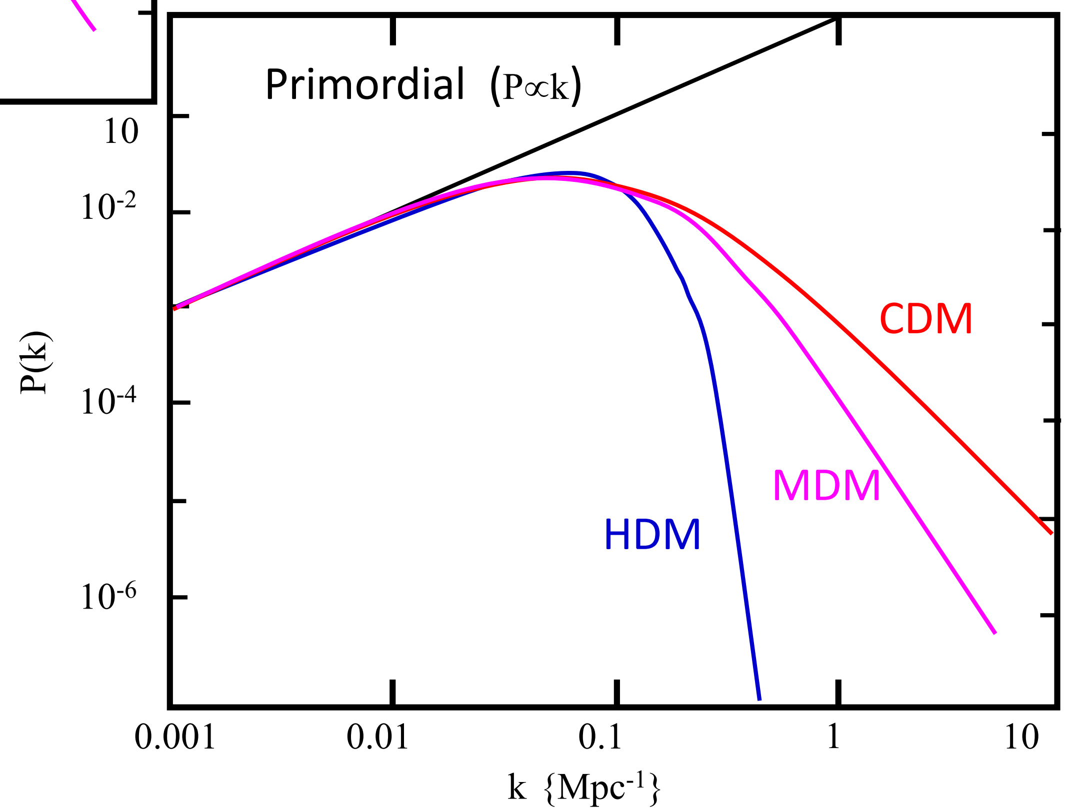
And contains the messy physics of the evolution of density perturbations

It is computed by solving the Boltzmann equation for the primordial multicomponent cosmic plasma numerically (for example, using CAMB (Lewis & Challinor 2011)).



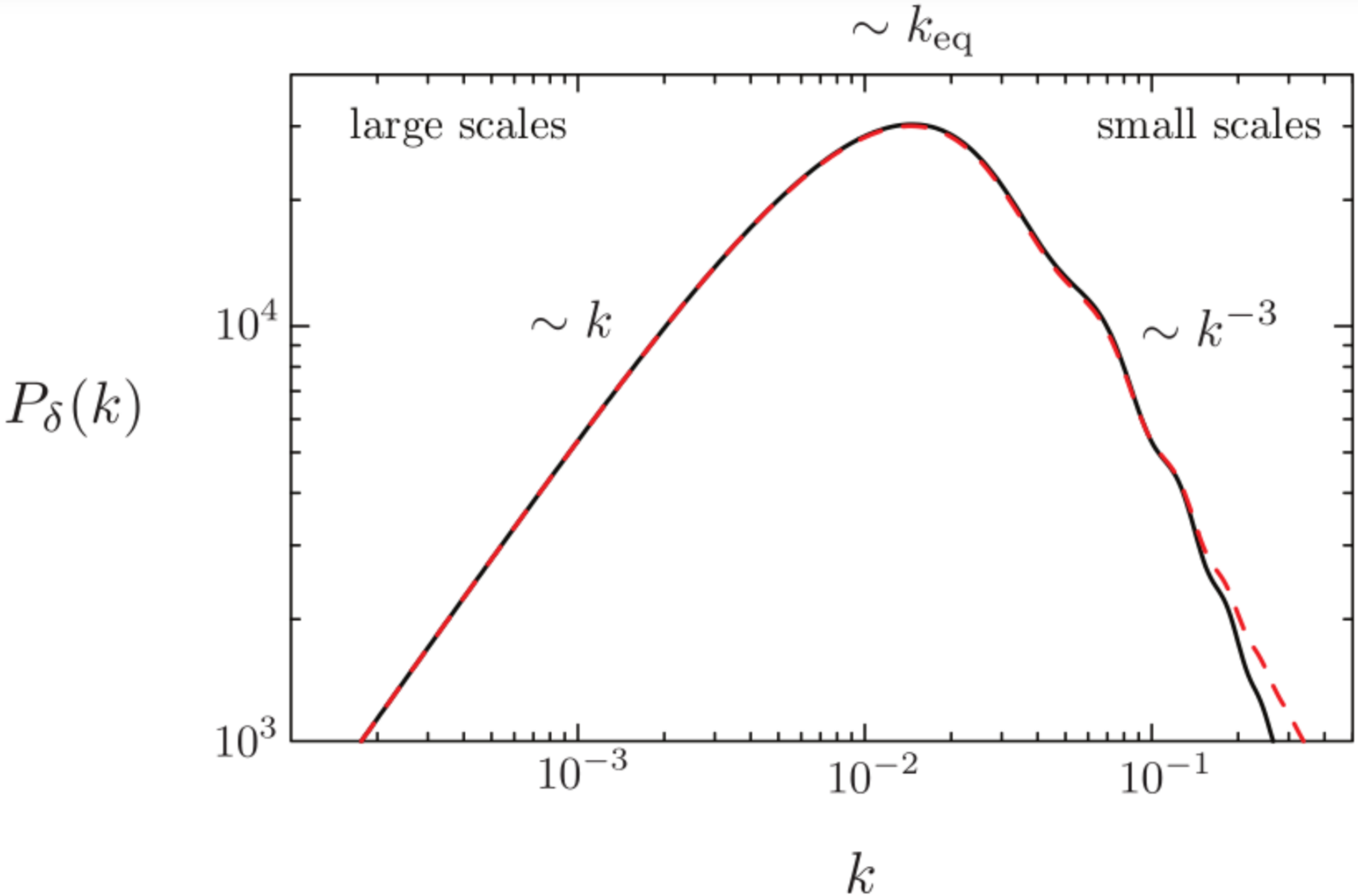
Transfer function and power spectrum for several models:

CDM: Cold dark matter
HDM: Hot dark matter
MDM: mixed dark matter (cold+hot)



Matter perturbations

The full power spectrum shape

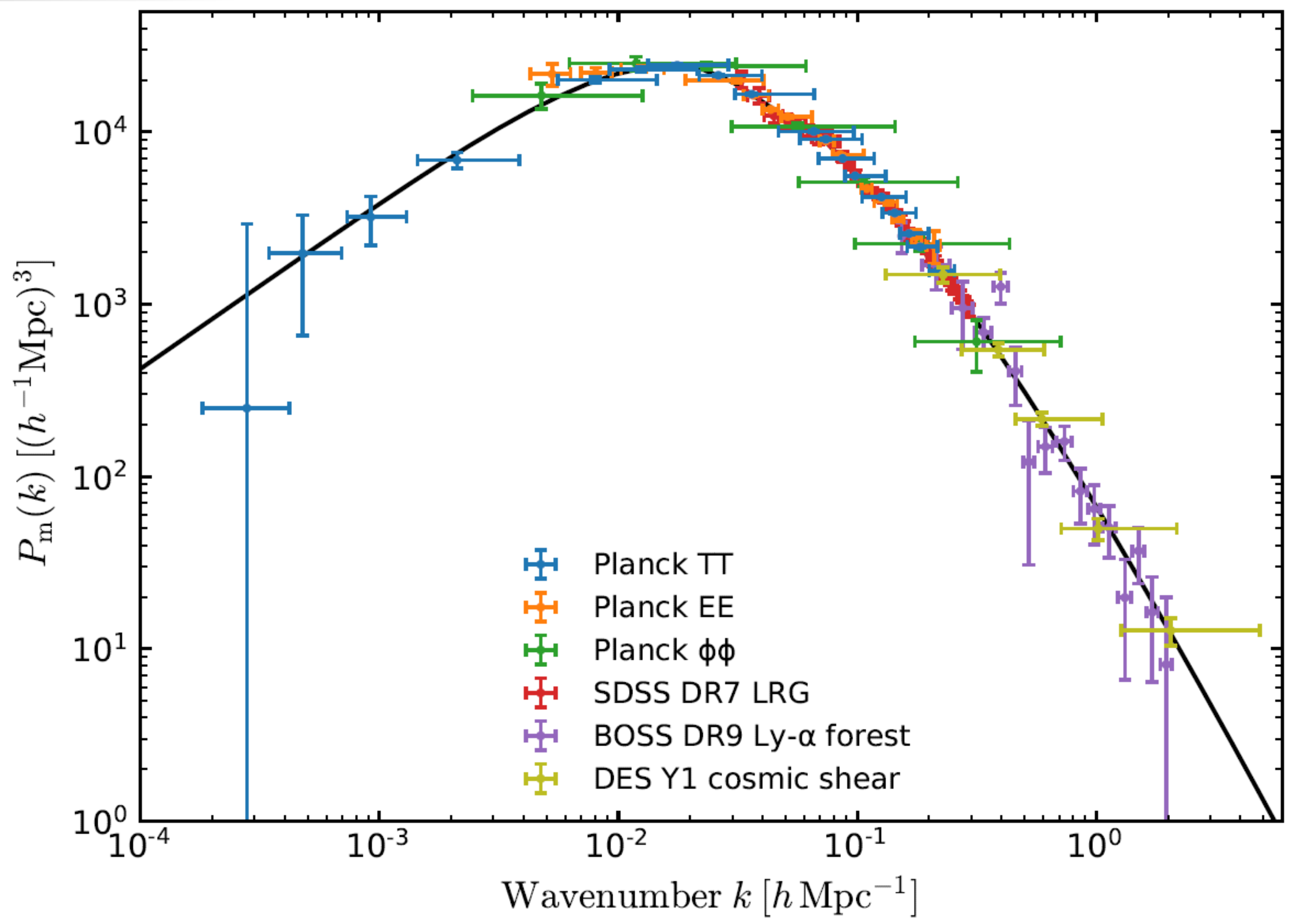


Matter perturbations

Measured power spectrum for different cosmological tracers

Linear approximation

ΛCDM is a good description

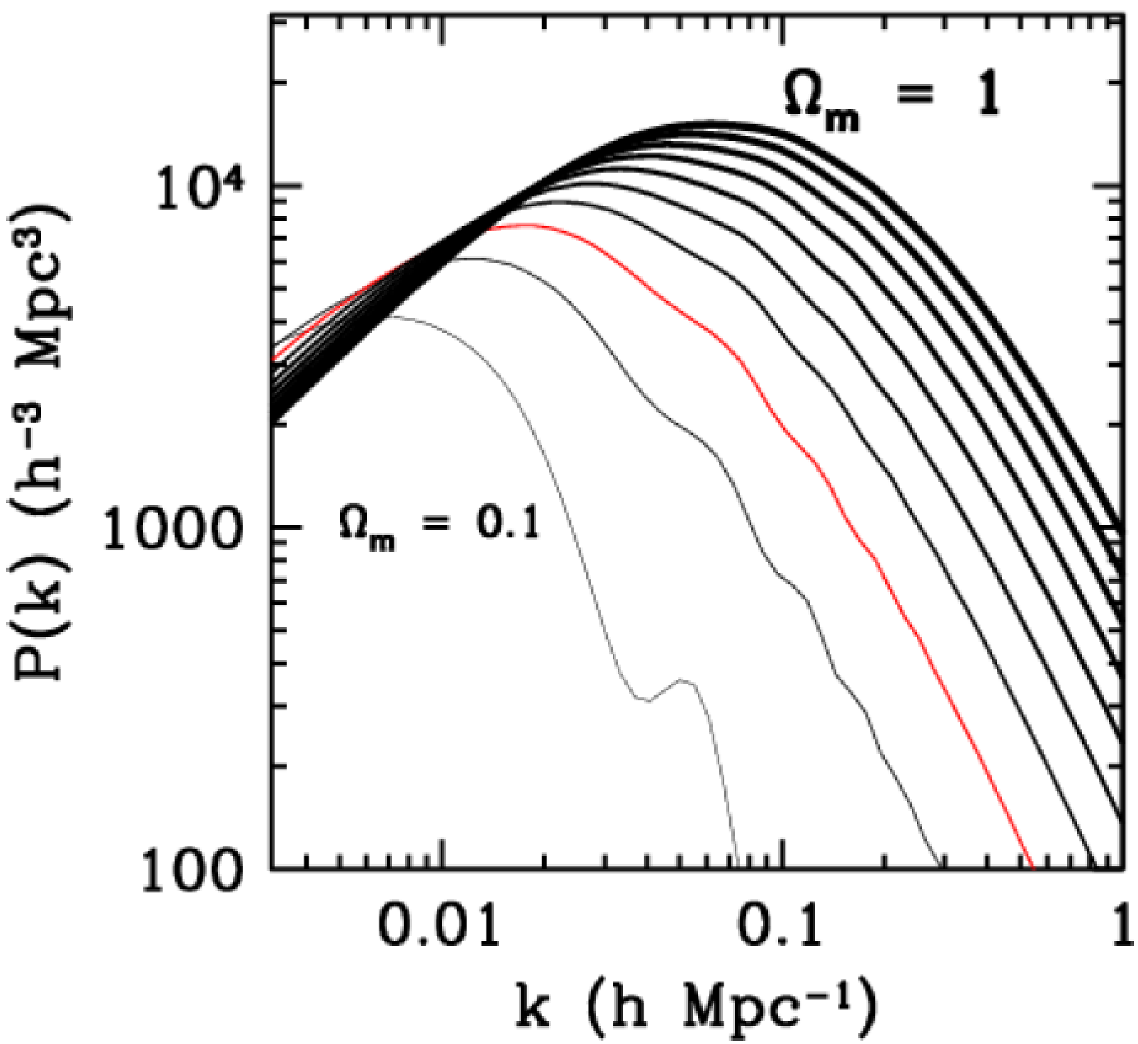


Shape of the matter power spectrum

$$P(k) \propto k^n T^2(k) \propto \begin{cases} k^n & \text{Large scales} \\ k^{n-3}\ln^2(k) & \text{Small scales} \end{cases}$$

Log since structure grows slightly during radiation era when potential decays

The turnover scale is the one that enters the horizon at the epoch of matter-radiation equality



$$k_{EQ} = 0.073\Omega_m h^2 \mathrm{Mpc}^{-1}$$

The shape of the power spectrum is sensitive to the matter density on the Universe through the position of the turnover scale

Neutrinos affect the large scale structure

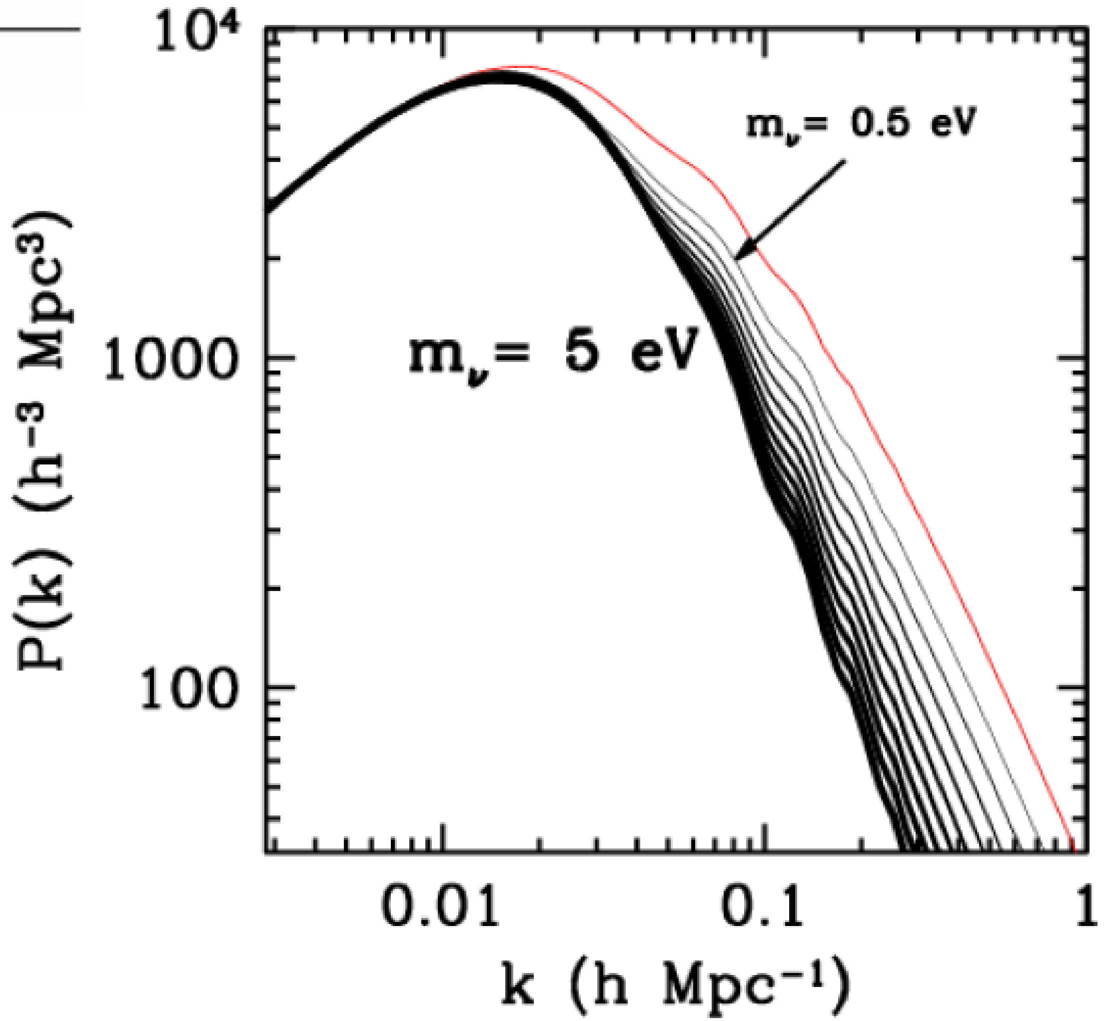
They do not participate in collapse for scales smaller than
The freestreaming scale

$$\Omega_\nu = 0.02 \frac{m_\nu}{1\,\mathrm{eV}}$$

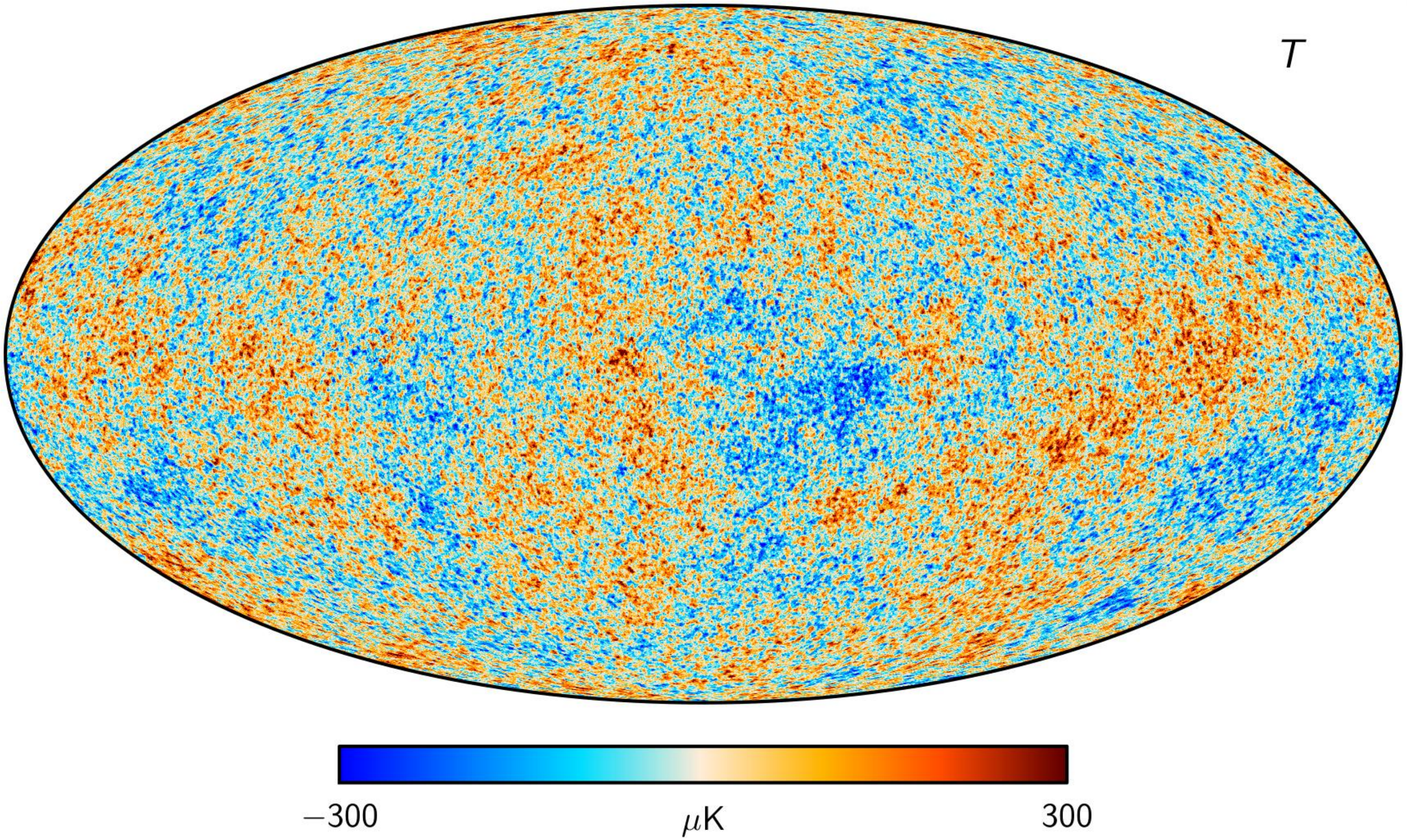
$$k_{\rm fs}^{-1} \simeq \frac{vt}{a} \simeq \frac{(T/m)H^{-1}}{a}$$

This scale is ~0.02 Mpc^{-1} for a 1 eV neutrino. Power on smaller scales is suppressed

Even for a small neutrino mass, a large impact on structure. The power spectrum is an excellent probe of neutrino masses



The Cosmic Microwave Background radiation (CMB)



One of the first and most important application of the perturbation theory and fluctuations description is the study of the CMB anisotropies.

One of the main sources of information about cosmology

Measurements of the CMB provide the most precise results of the comological parameters up to date (Galaxy surveys start to reach a similar precisión level now)

Fluctuations in the baryon-photon plasma

Equation of motion for the baryon-photon fluid

$$\boxed{\delta_\gamma'' + \frac{\mathcal{H}R}{1+R}\,\delta_\gamma' + \underset{\substack{\uparrow\\ \text{pressure}}}{c_s^2 k^2\,\delta_\gamma} \;=\; \underset{\substack{\uparrow\\ \text{gravity}}}{-\frac{4}{3}k^2\Phi} + 4\Phi'' + \frac{4R'}{1+R}\Phi'} \qquad \mathcal{H} = aH$$

Until "decoupling" at z~1100, there is a strong coupling between electrons and photons, which behave like a single fluid.

Baryon to photon ratio

$$R \equiv \frac{3}{4}\frac{\bar{\rho}_b}{\bar{\rho}_\gamma} = 0.6\left(\frac{\Omega_b h^2}{0.02}\right)\left(\frac{a}{10^{-3}}\right)$$

Sound speed

$$c_s^2 \equiv \frac{1}{3(1+R)}$$

For sub-horizon modes

before MRE $a \ll a_{\rm eq}$

- gravitational potential small

$$\delta_r'' - \frac{1}{3}\nabla^2\delta_r \approx 0$$

after MRE $a \gg a_{\rm eq}$

- grav. potential as external force

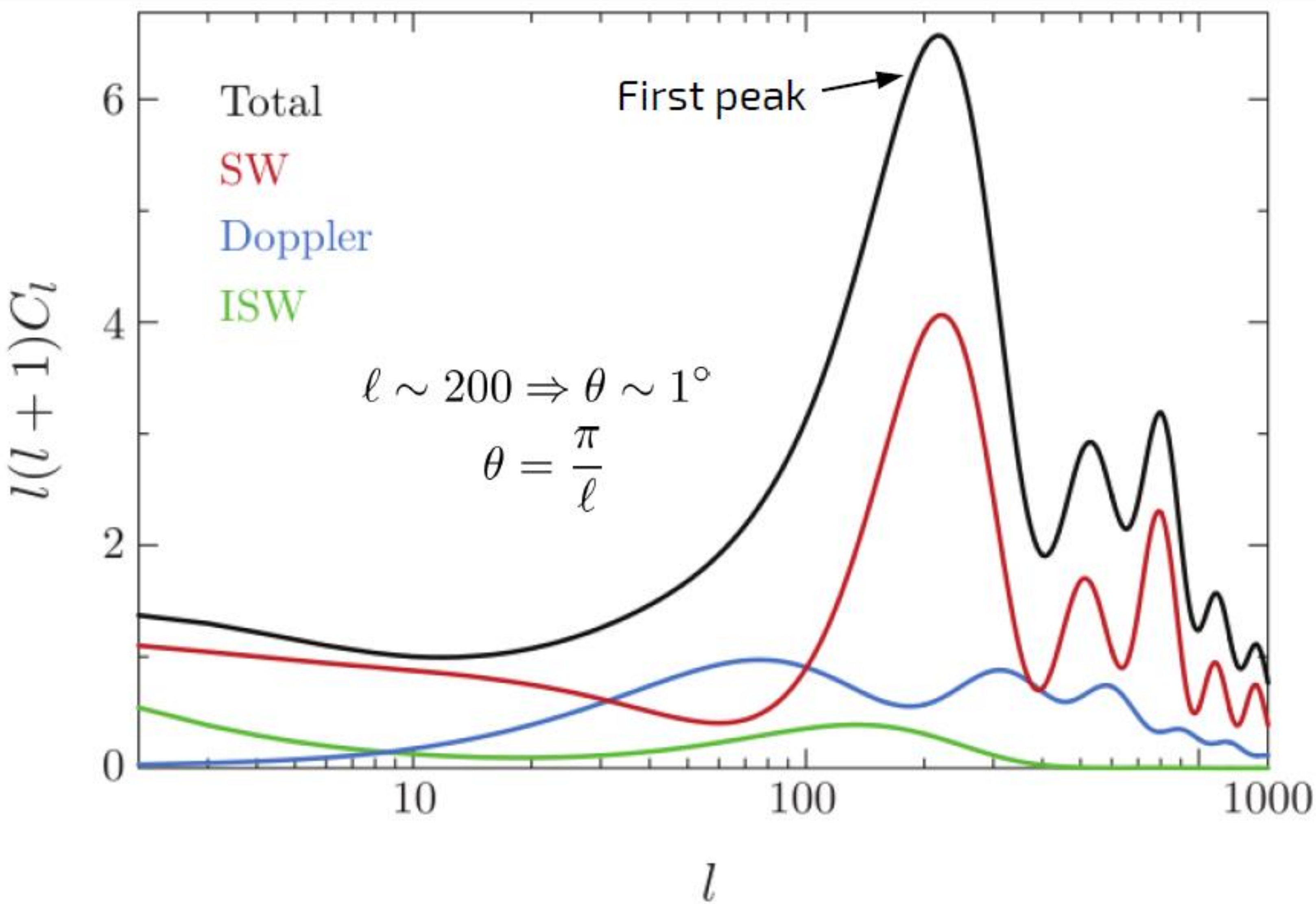
$$\delta_r'' - \frac{1}{3}\nabla^2\delta_r = \frac{4}{3}\nabla^2\Phi = const$$

Calculation more complicated.

Need to take into account all the physics of the photon-electron plasma

Well-known physics → allows a precise prediction of the CMB power spectrum

The shape of the power spectrum has a lot of cosmological information

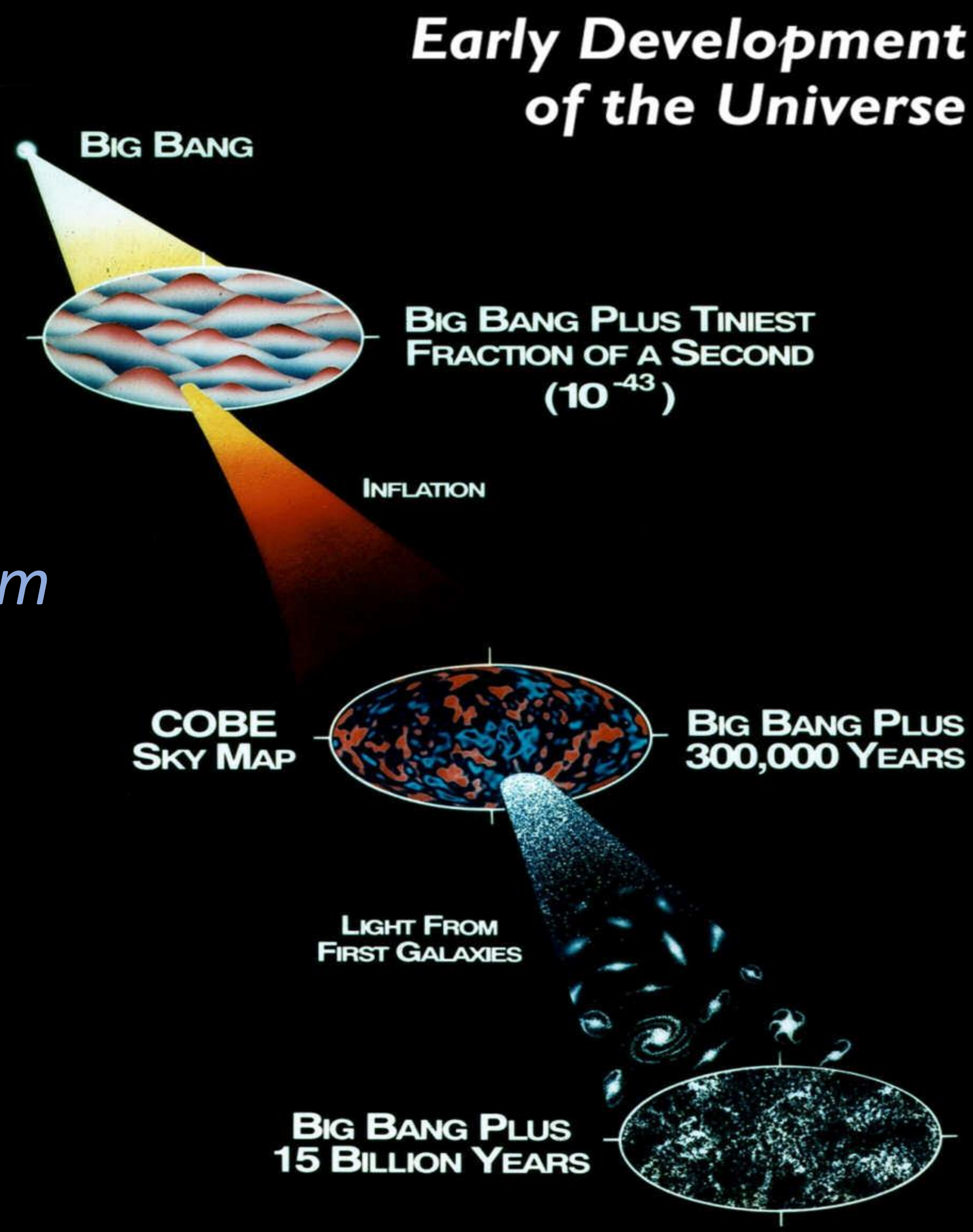


Before recombination

- Early Universe
- High temperature
  - *Electrons are free*
  - *Light interacts with them*

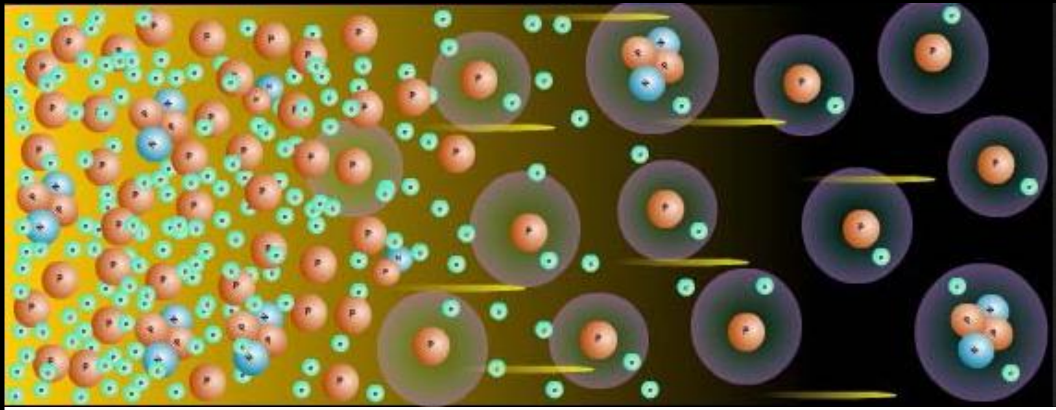
Recombination

- Late Universe
- Lower temperature
  - *e- y p+ form hydrogen*
  - *Light travels freely*

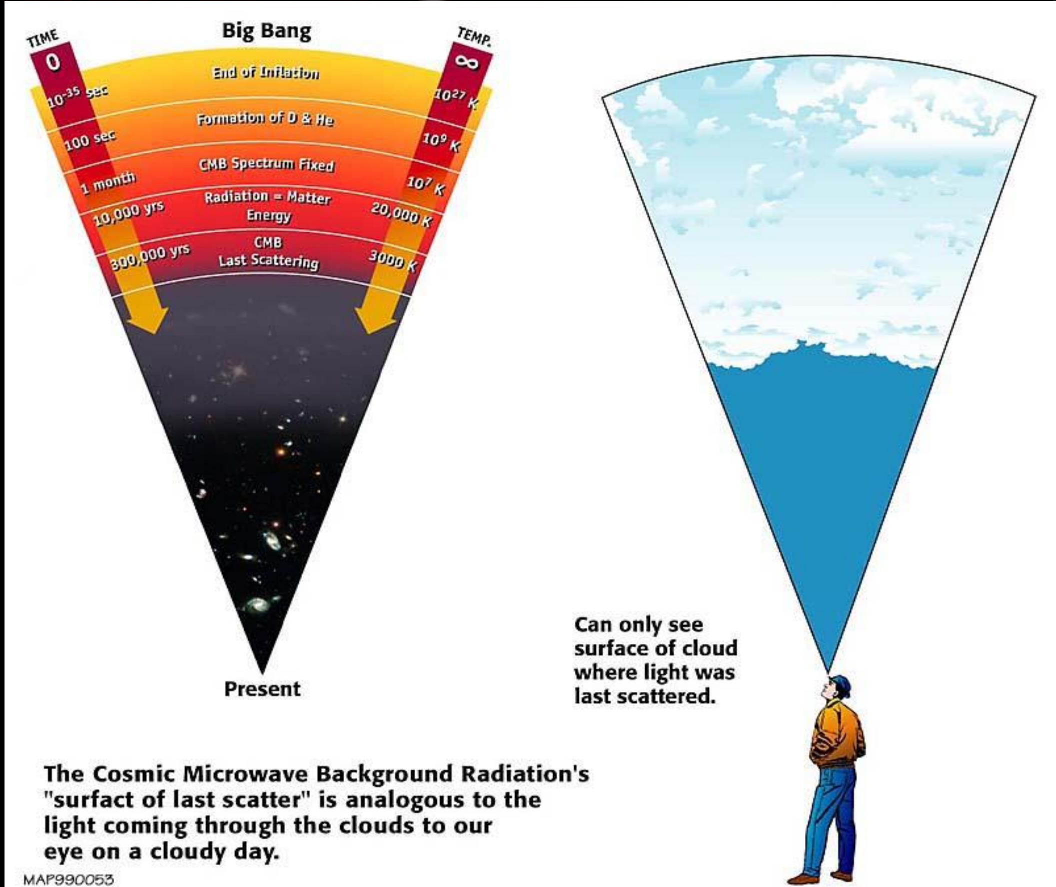


Cosmic Microwave Background (CMB)

Thermal radiation from the atom formation period ~380000 years after the BB or…. 13800 Myears ago!!



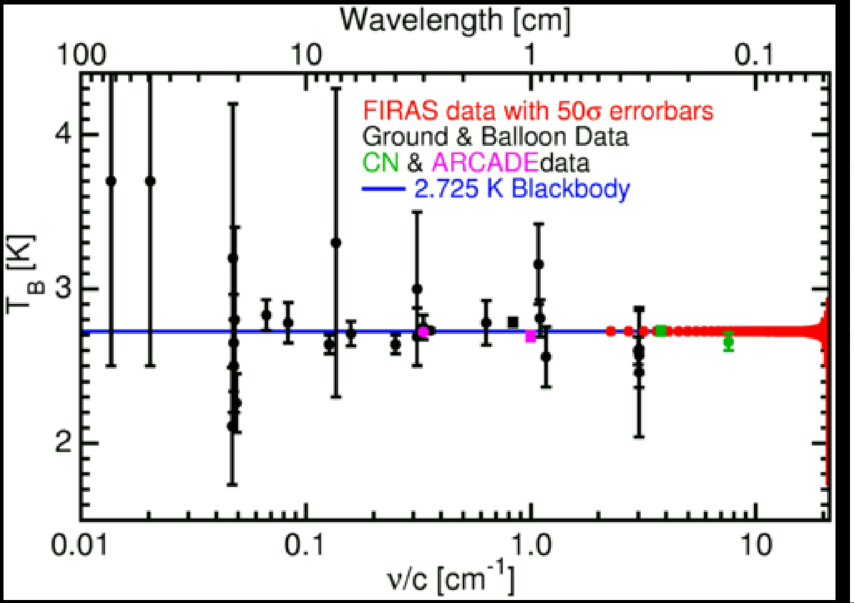
Discovered in 1965
In 1992 Discovery of its non-uniformity. Its small anisotropies are the imprint of the seeds of all the structure of the Universe.

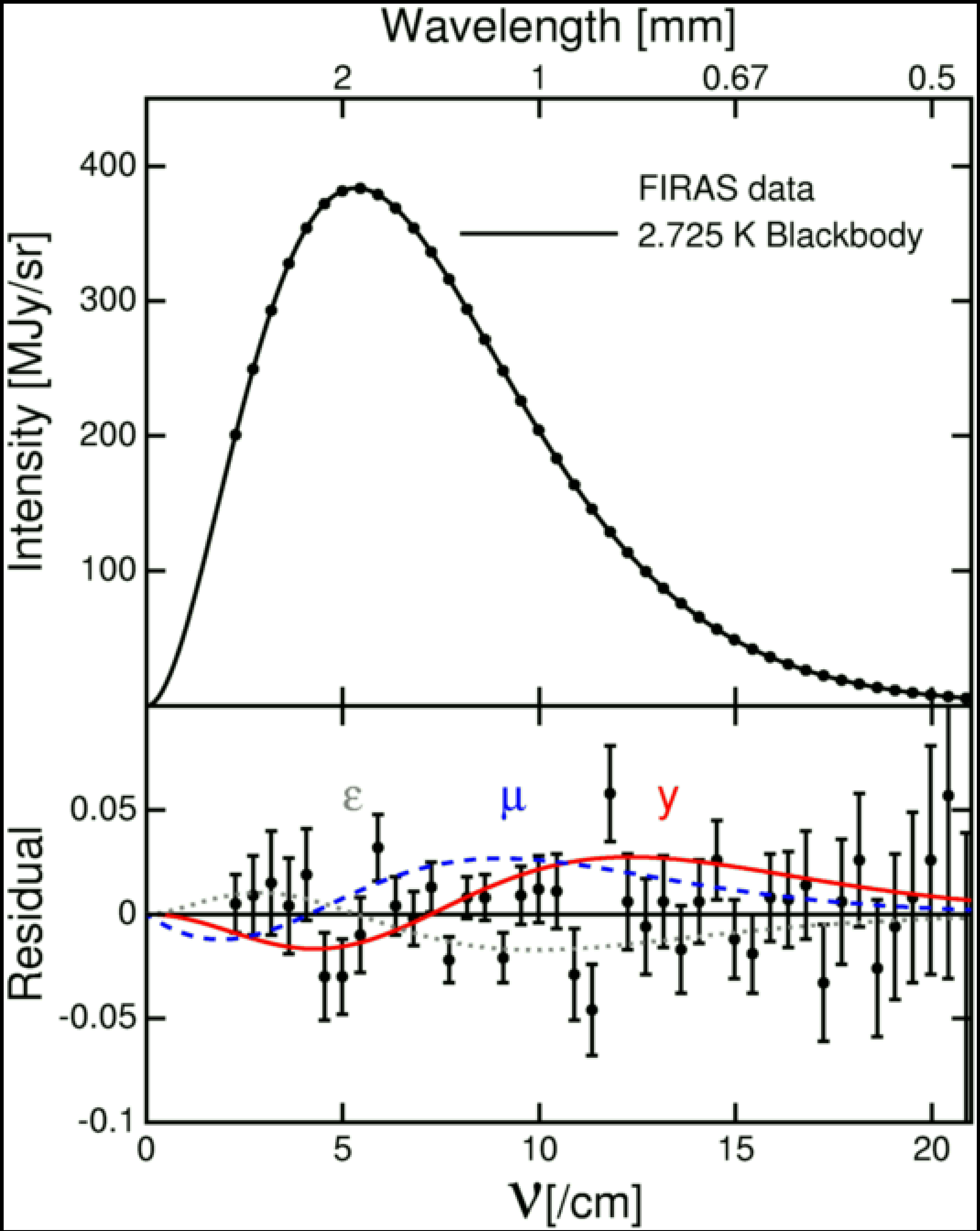


The most precise measurement of the cosmological parameters come from the CMB.

The frequency spectrum is a perfect black body at 2.725K

The Universe was in thermodynamic equilibrium before the recombination:
The collision rate was much smaller than the expansion rate





On the CN non-discovery

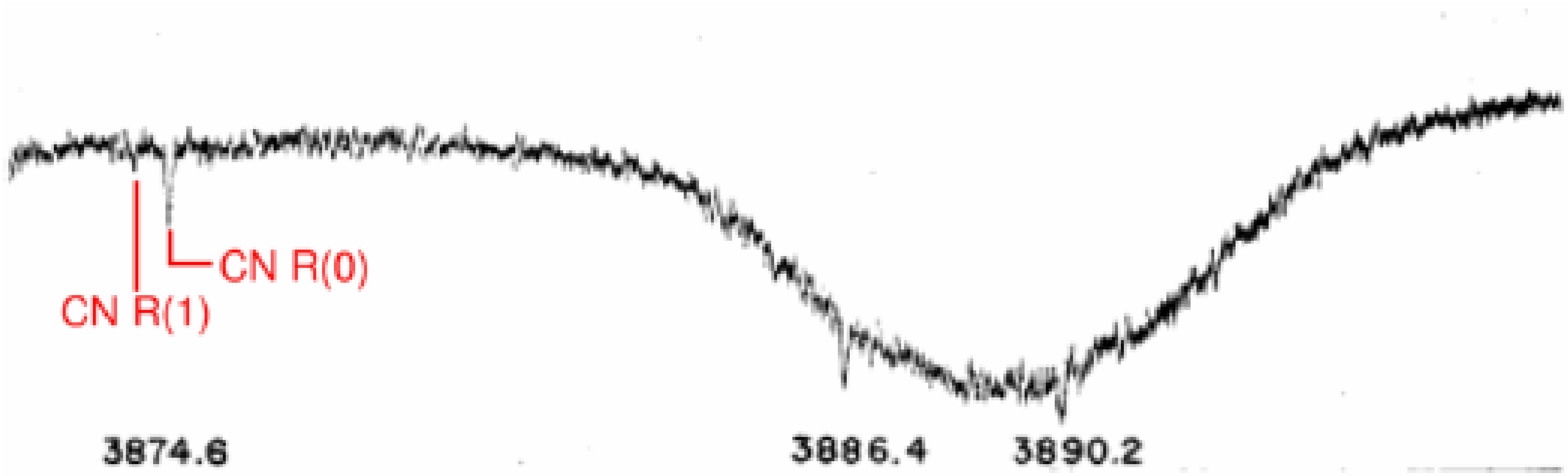


Plate 3 of Adams (1941, ApJ, 93, 11-23)

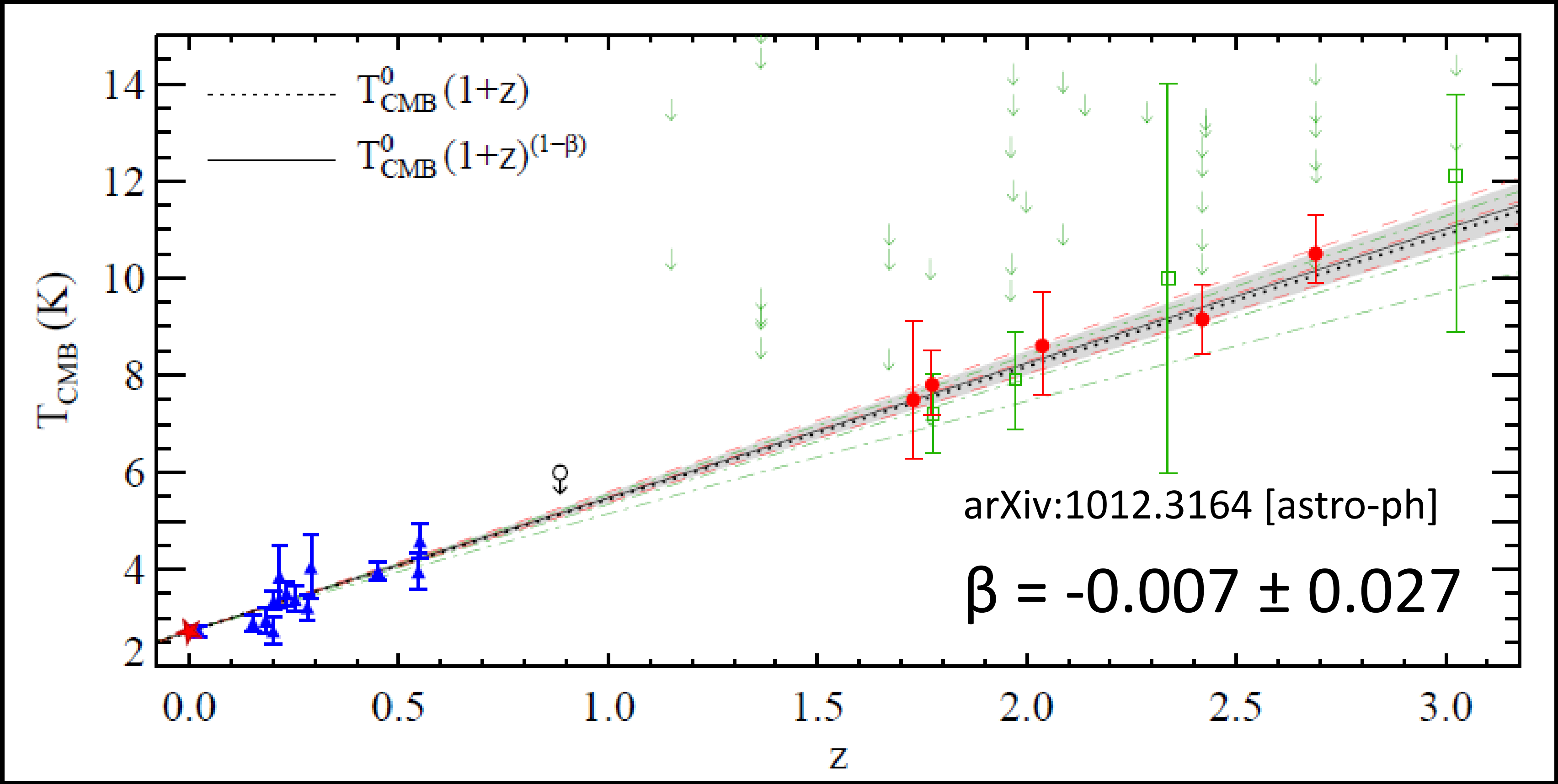
Herzberg (1950) in *Spectra of Diatomic Molecules*, p 496:

"From the intensity ratio of the lines with K=0 and K=1 a rotational temperature of 2.3° K follows, which has of course only a **very restricted meaning**."

There went Herzberg's [second] Nobel Prize.

Slide from Ned Wright

CMB Temperature . vs . z



COBE

SZ Effect

CO Molecule lines

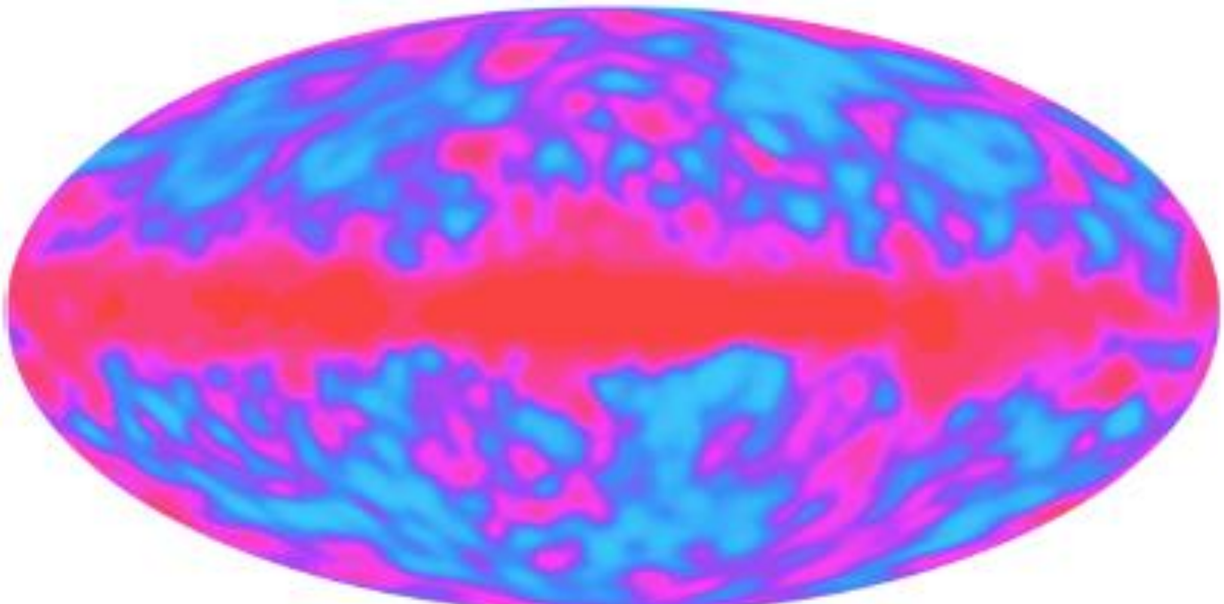
C atom lines

A Big Media Splash in 1992:

THE TIMES

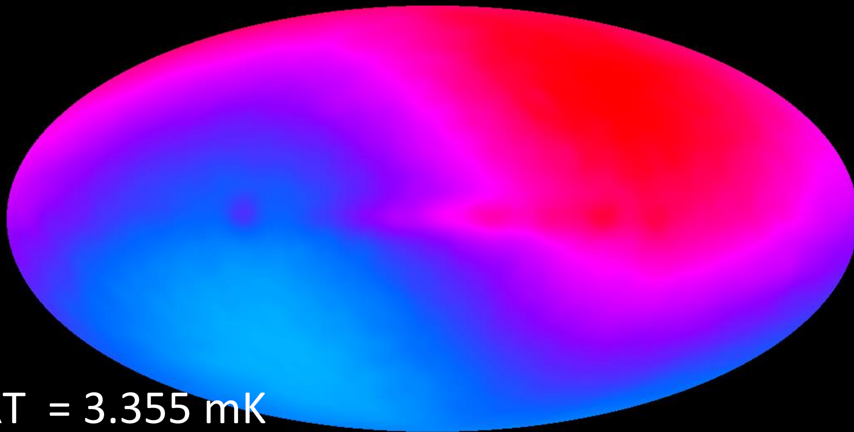
25 April 1992

Prof. Stephen Hawking of Cambridge University, not usually noted for overstatement, said: “It is the discovery of the century, if not of all time.”

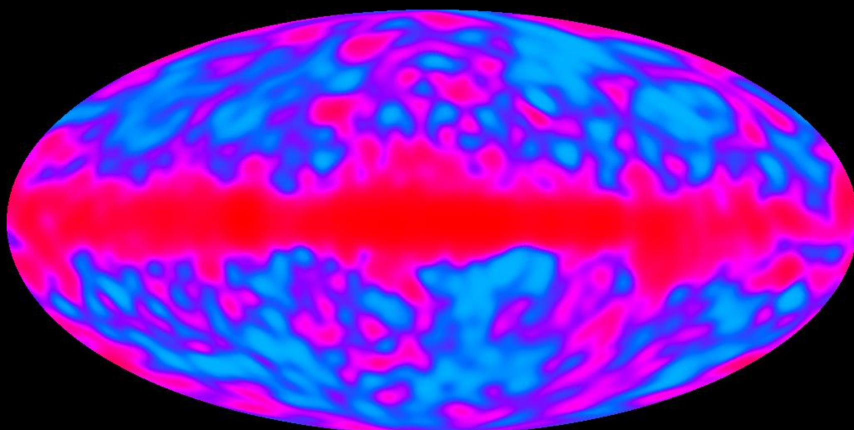


Slide from Ned Wright

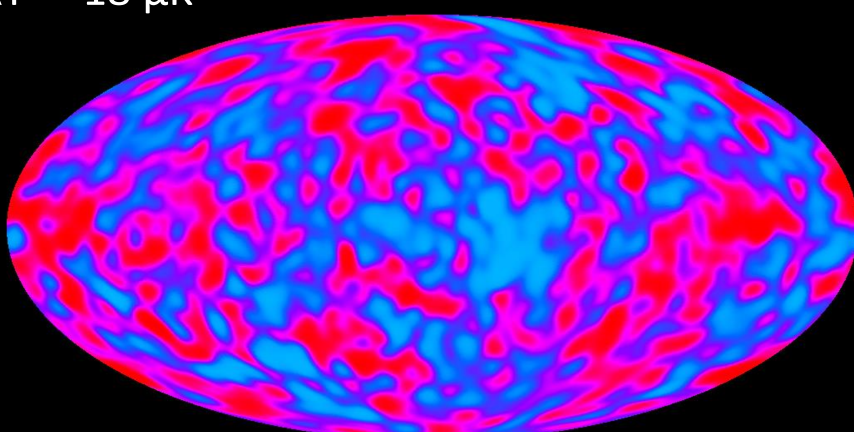
Dipole anisotropy from the Earth movement



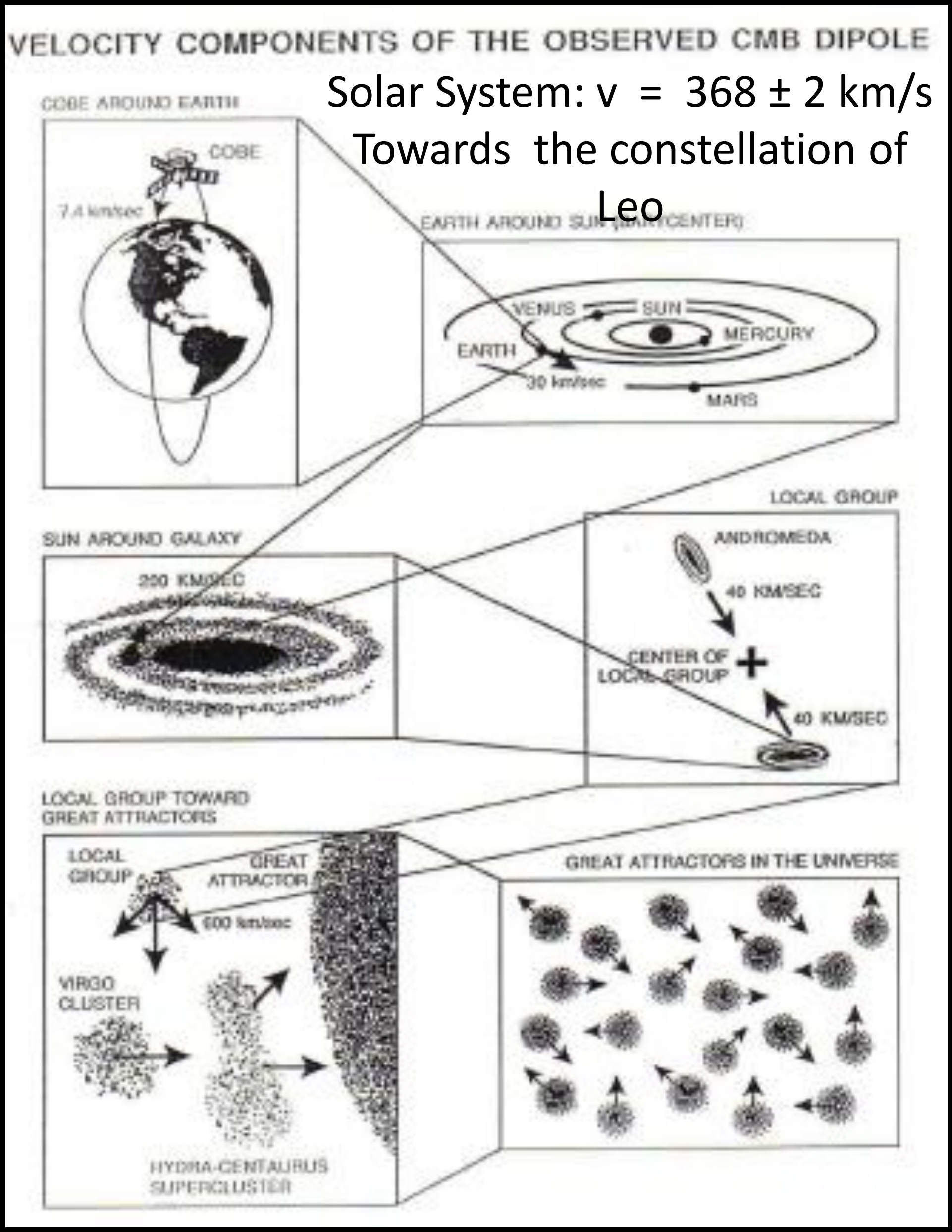
ΔT = 3.355 mK



ΔT = 18 μK

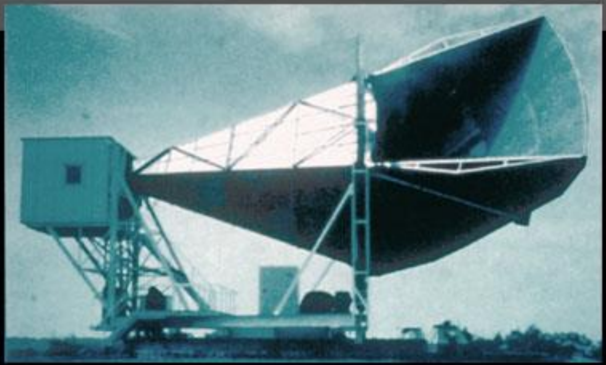


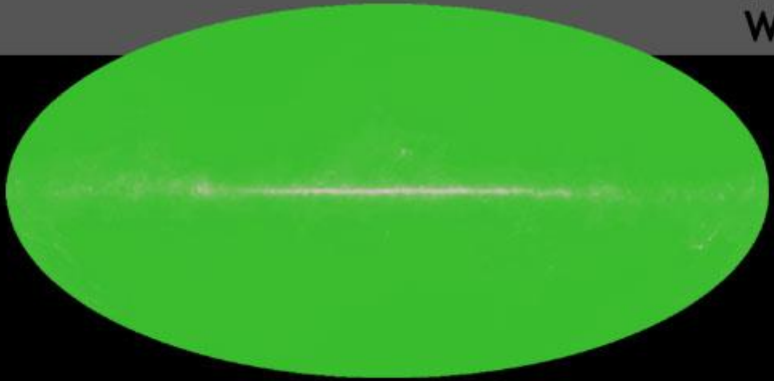
Solar System: v = 368 ± 2 km/s
Towards the constellation of Leo



1965

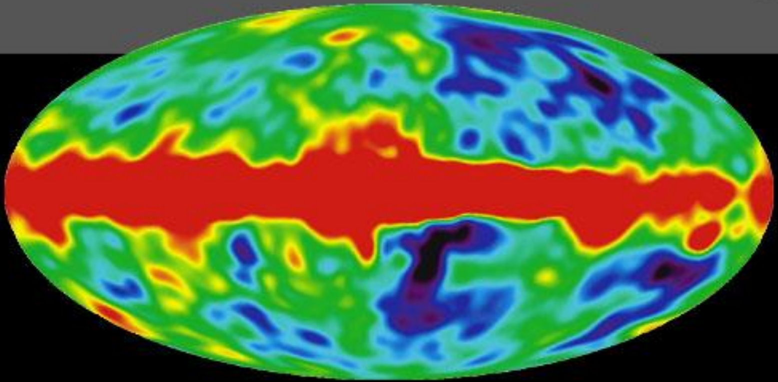
Penzias and Wilson





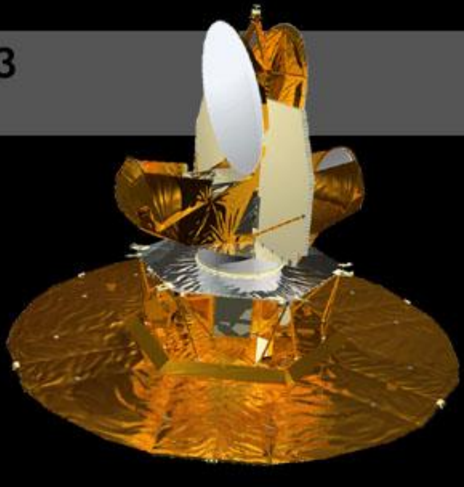
1992

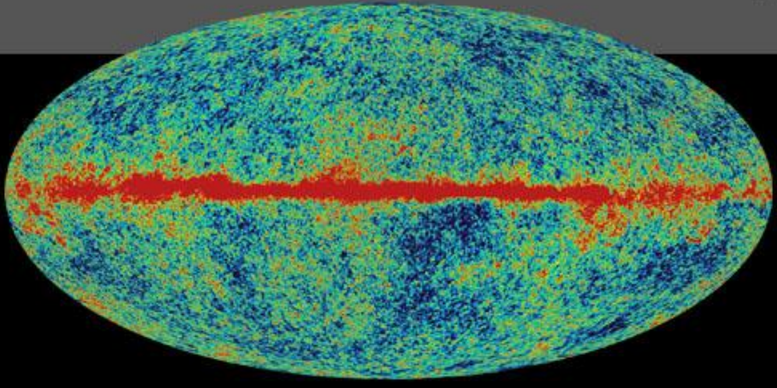
COBE



2003

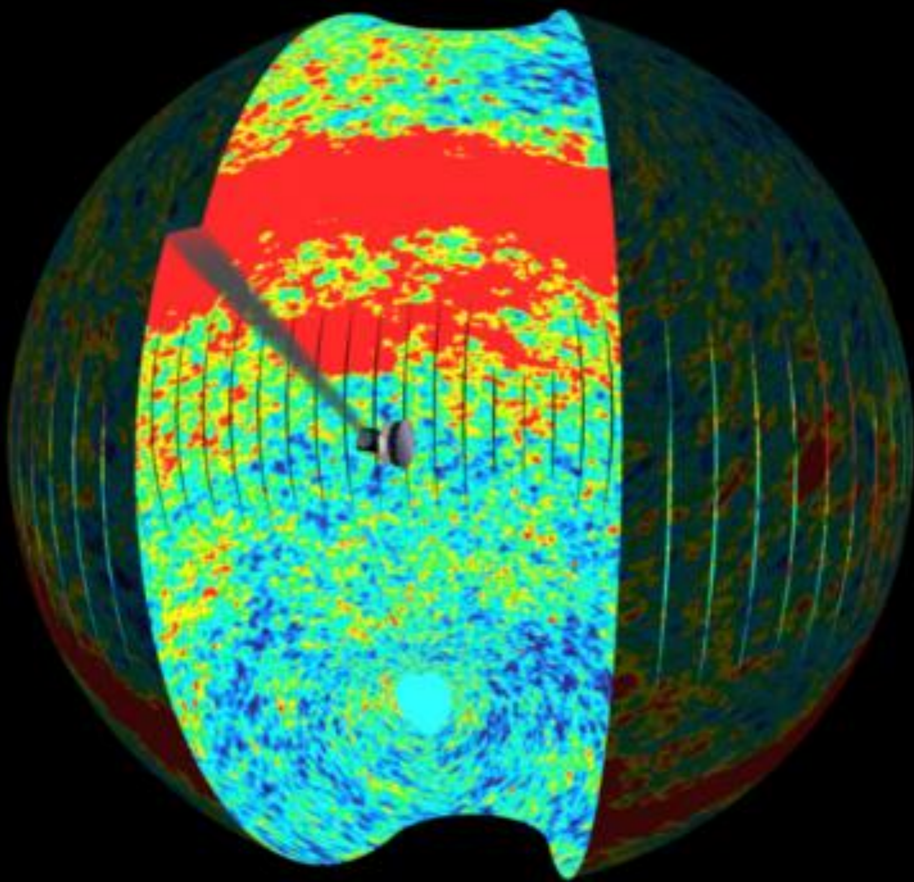
WMAP

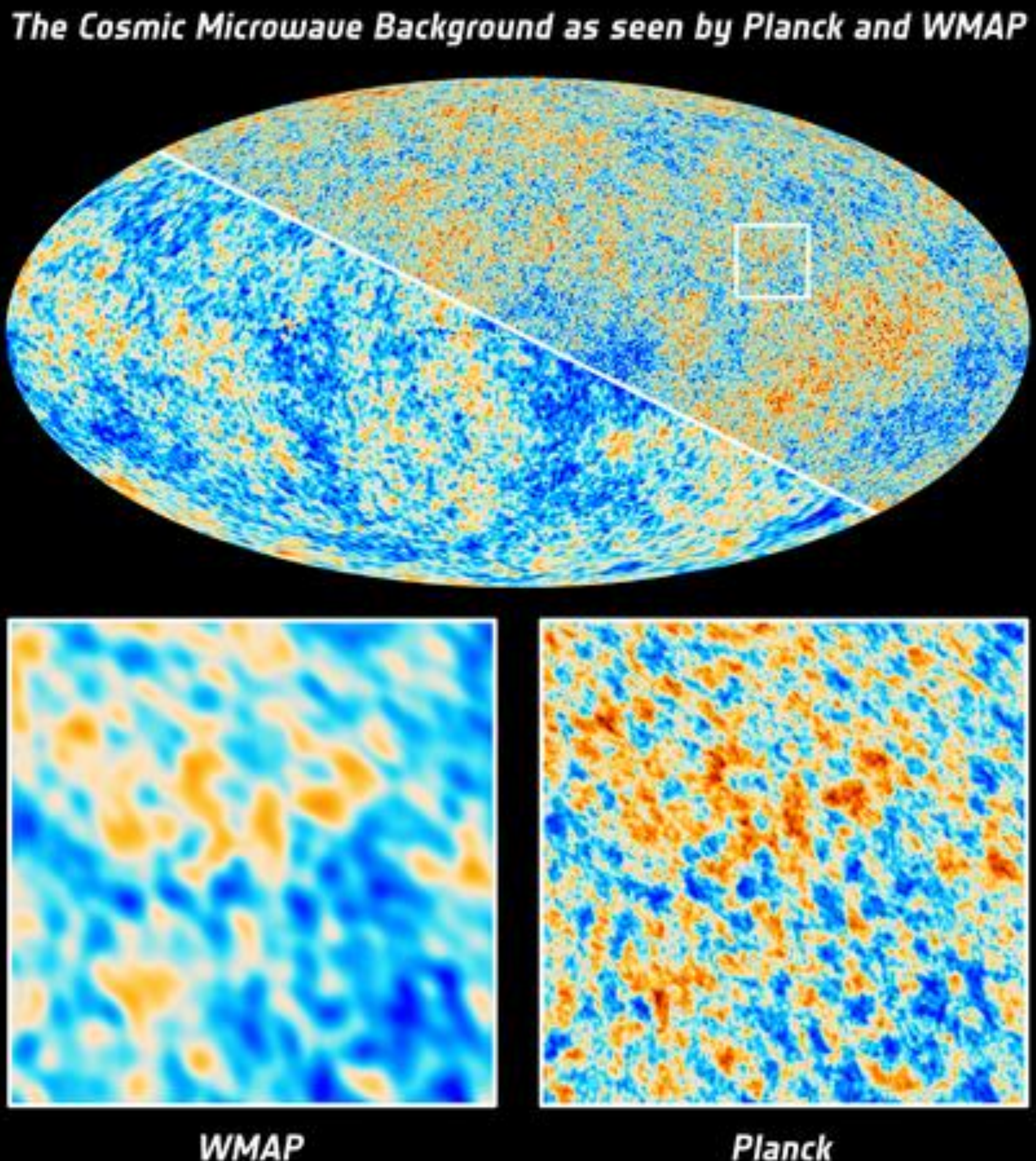




Planck: The most recent satellite

From May 2009 to October 2013
Much more precise tan previous
Able to measure polarization
Arrived at L2 in July 2009.
Final results in july 2018.

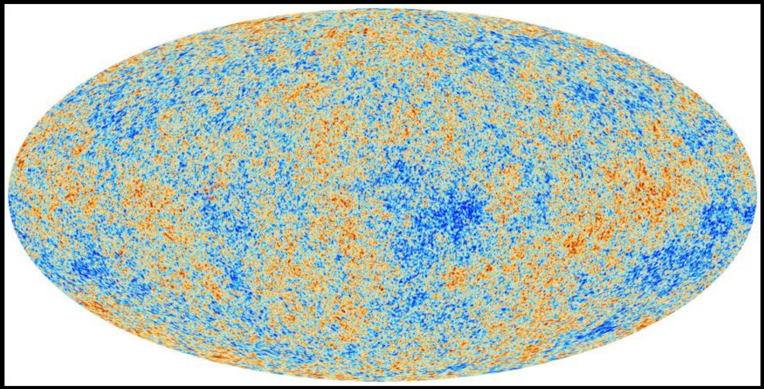




Final results released on 17 july 2018

Highest precisión confirmation of ΛCDM

Statistical Properties

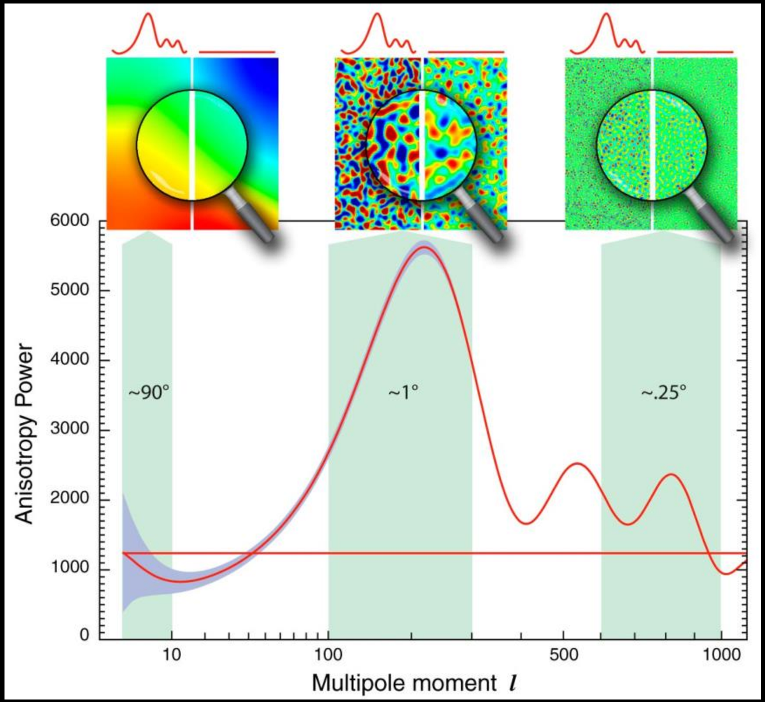


Expansion in spherical harmonics (Fourier transform on the sphere)

Quantifies clustering at different scales

$T_0 = 2.726K$

$\Delta T(\theta,\phi) = T(\theta,\phi) - T_0$



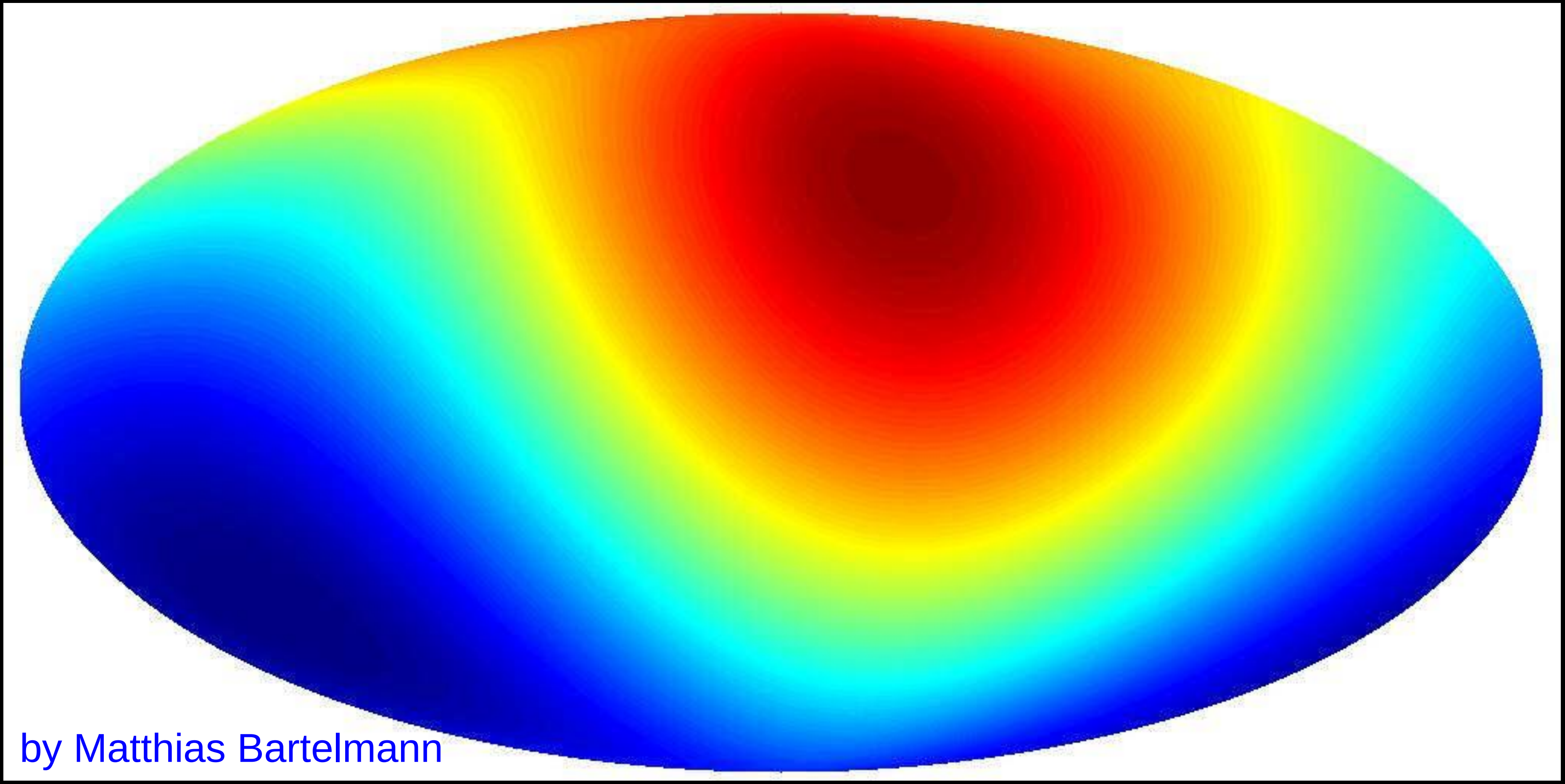
$$\frac{\delta T}{T_0}(\theta,\phi) = \sum a_{\ell m} Y_{\ell m}(\theta,\phi)$$

$$a_{\ell m} = \int Y^*_{\ell m}(\theta,\phi) \frac{\delta T}{T_0}(\theta,\phi) d\Omega$$

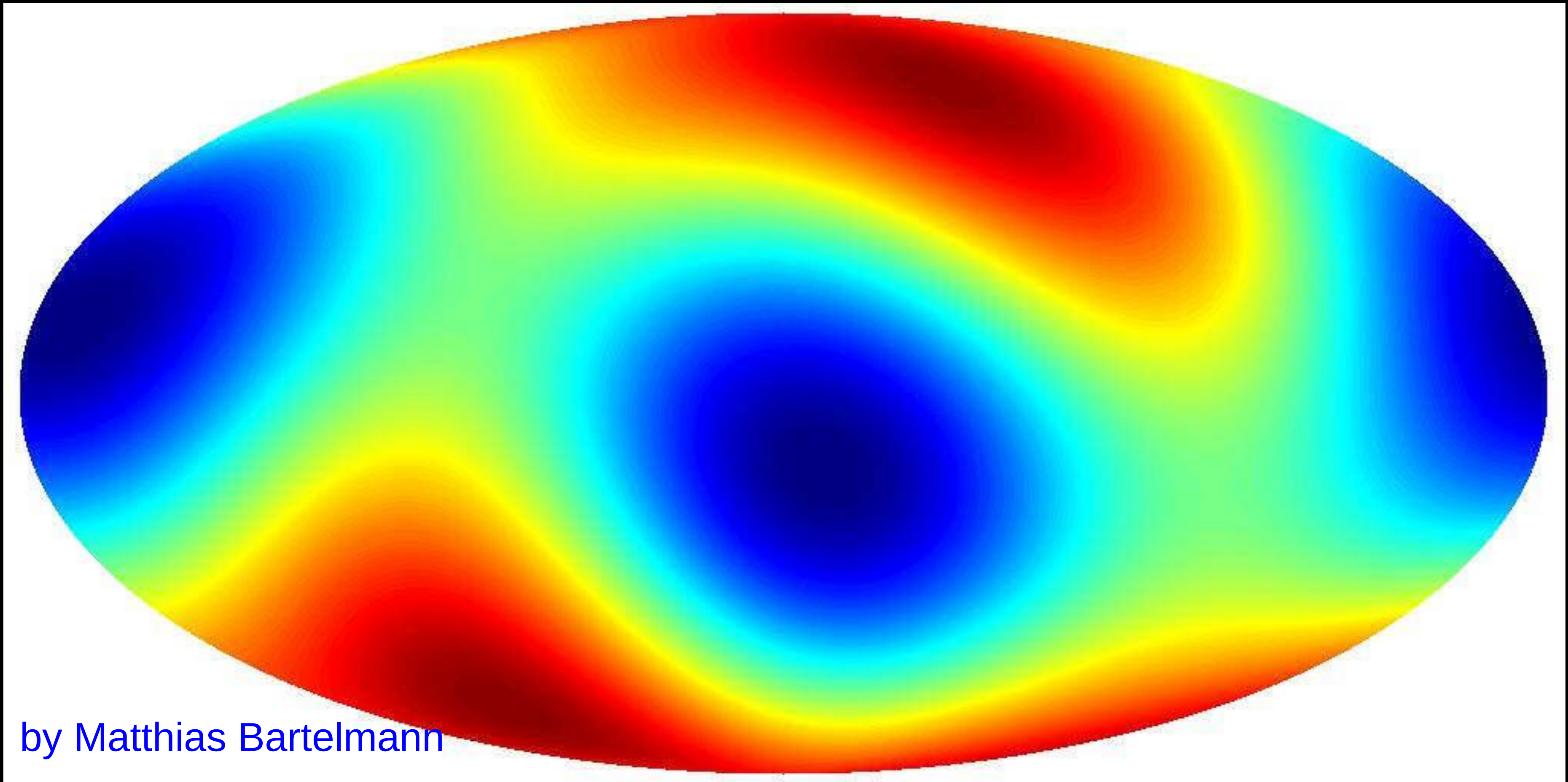
$$C_\ell \equiv \langle |a_{\ell m}|^2 \rangle = \frac{1}{2\ell+1} \sum_m \langle |a_{\ell m}|^2 \rangle$$

Spherical Harmonics:

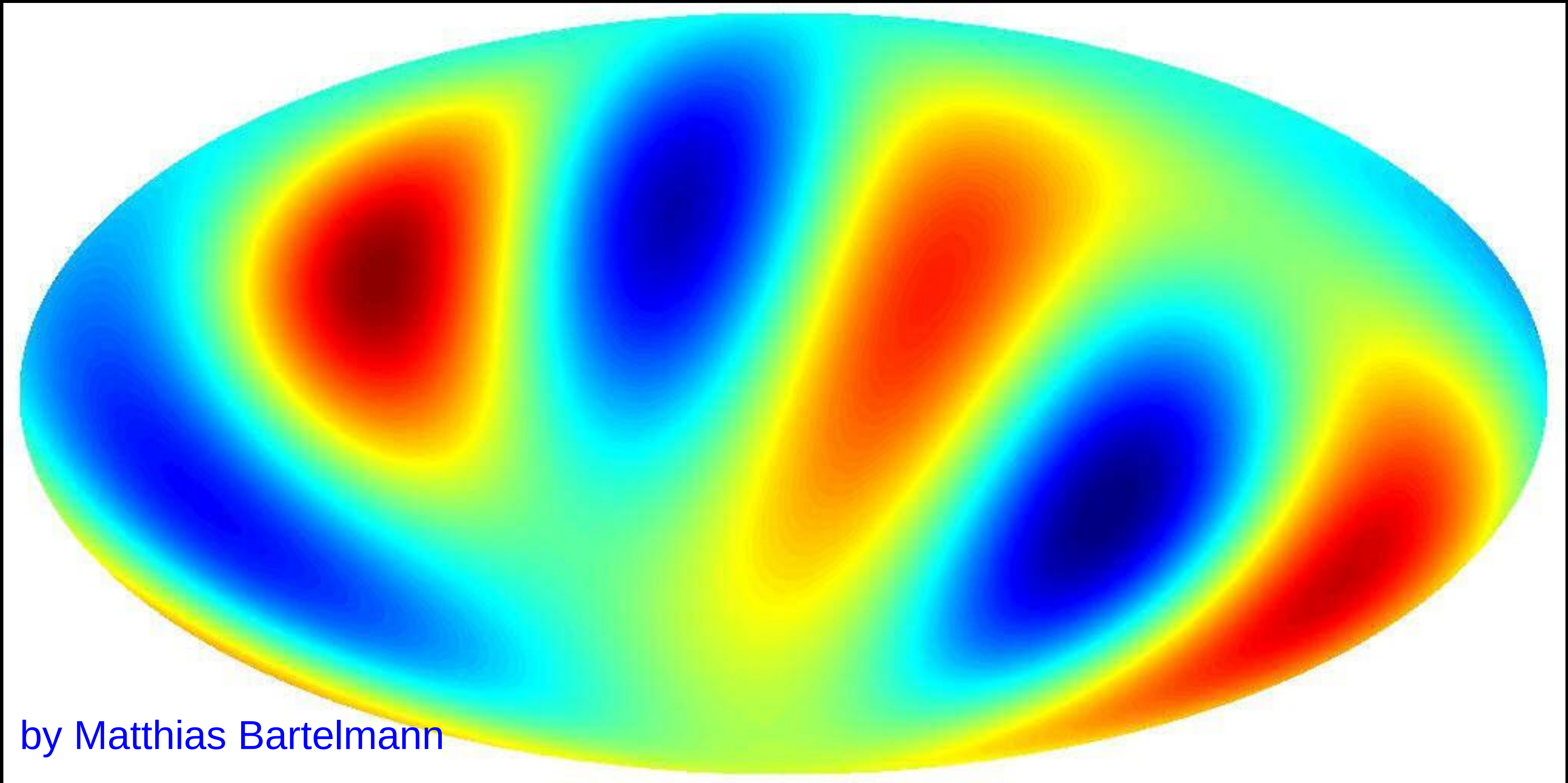
l=1



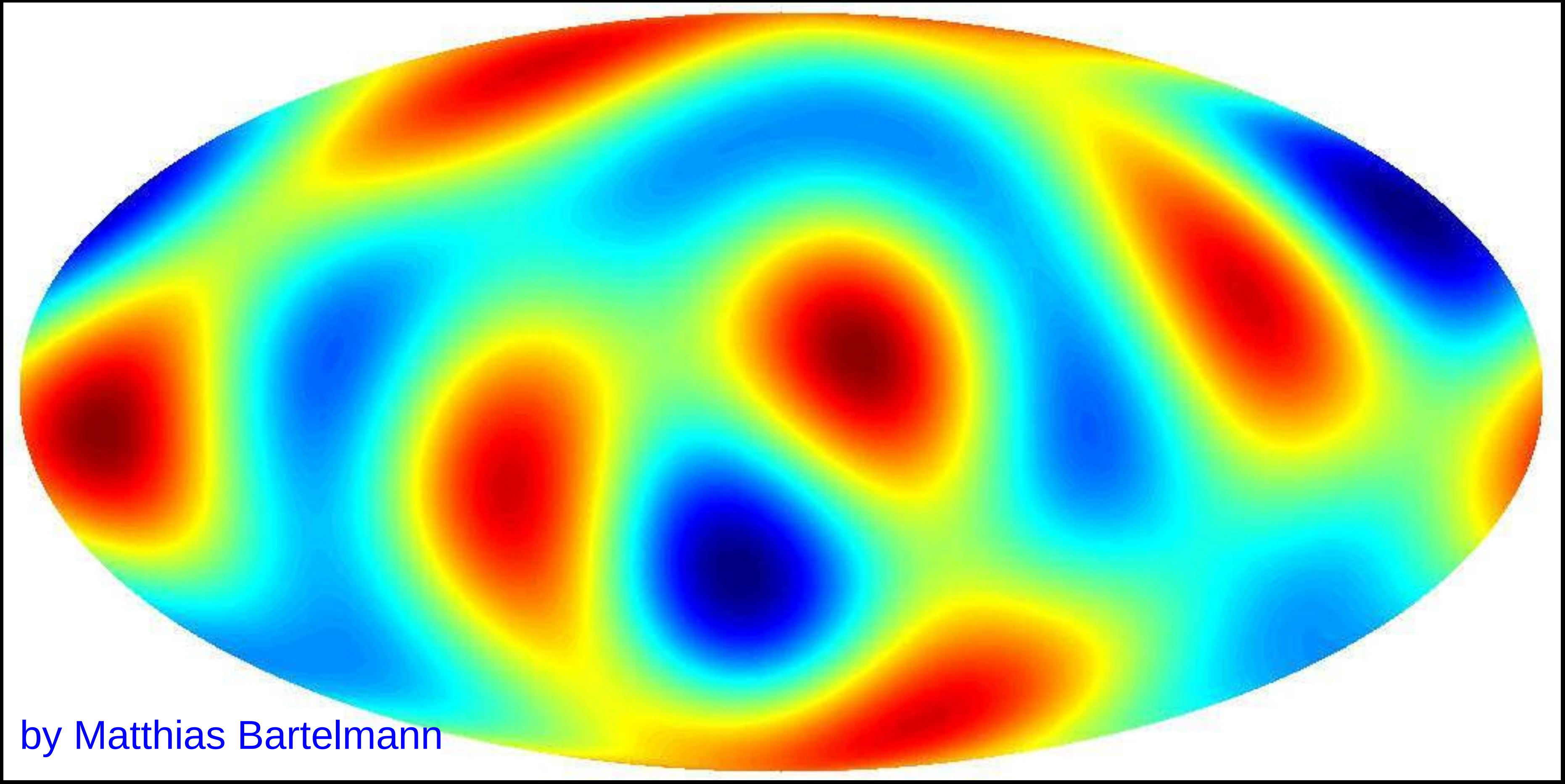
l=2



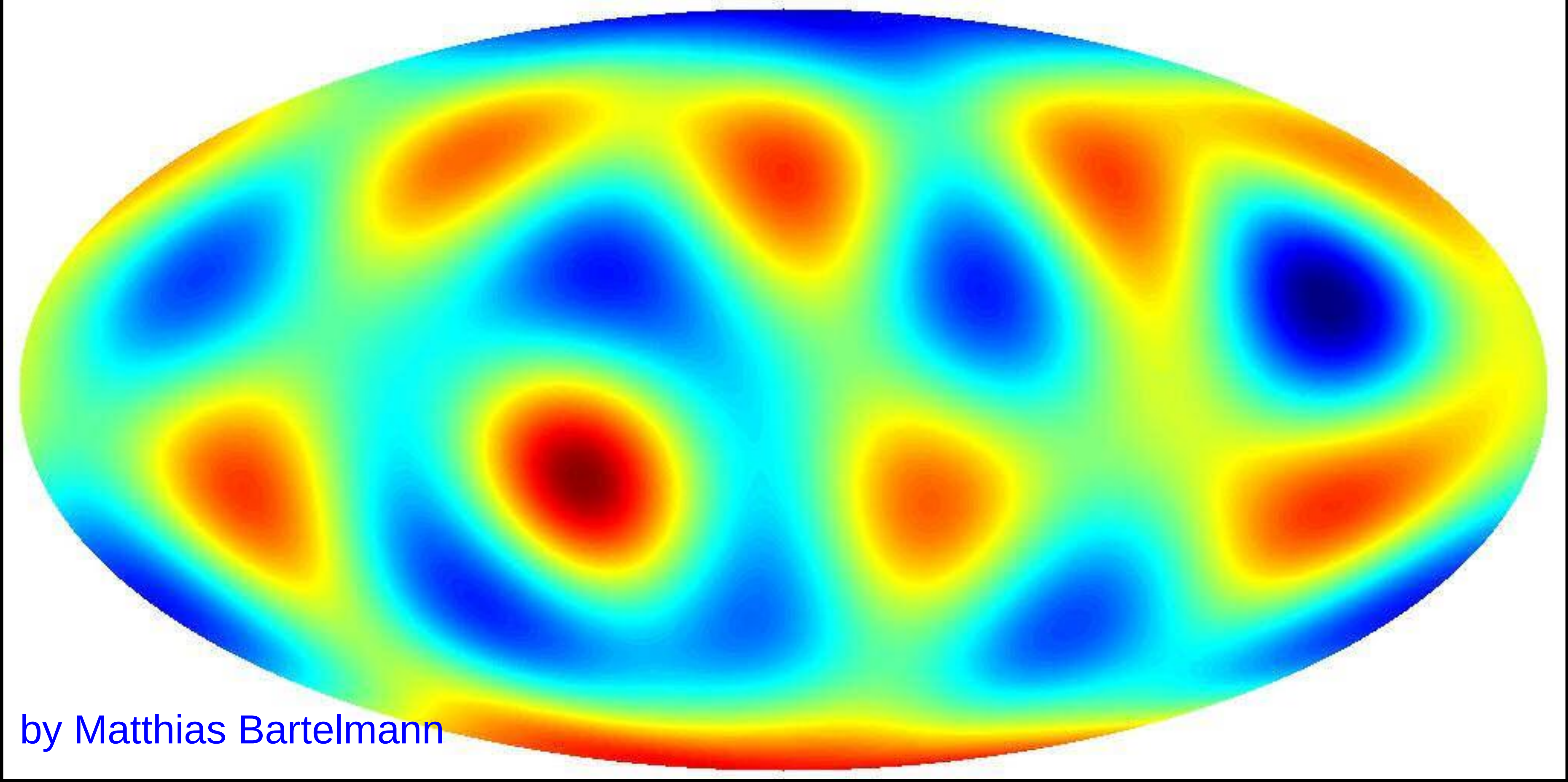
l=3



l=4

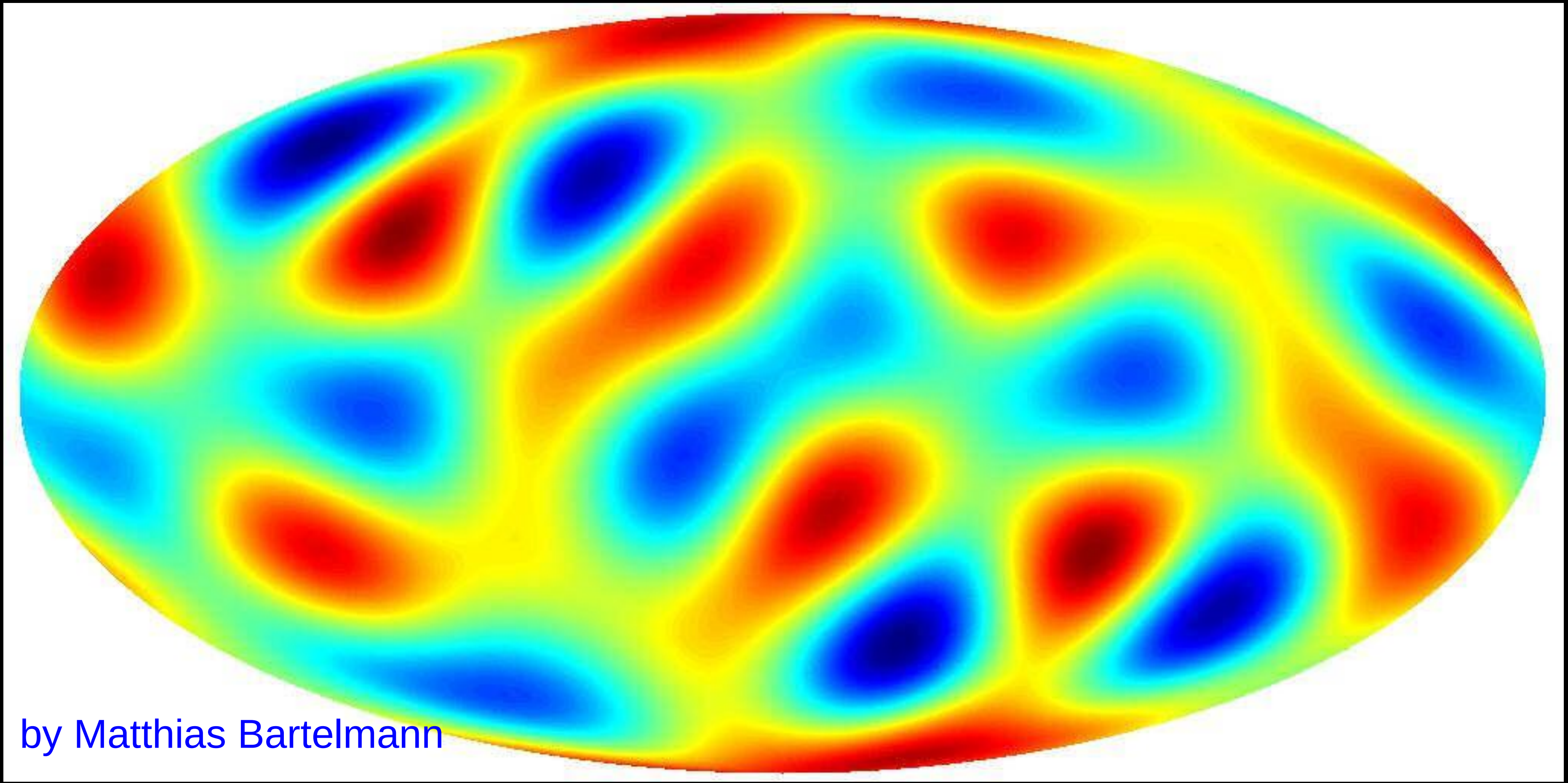


l=5

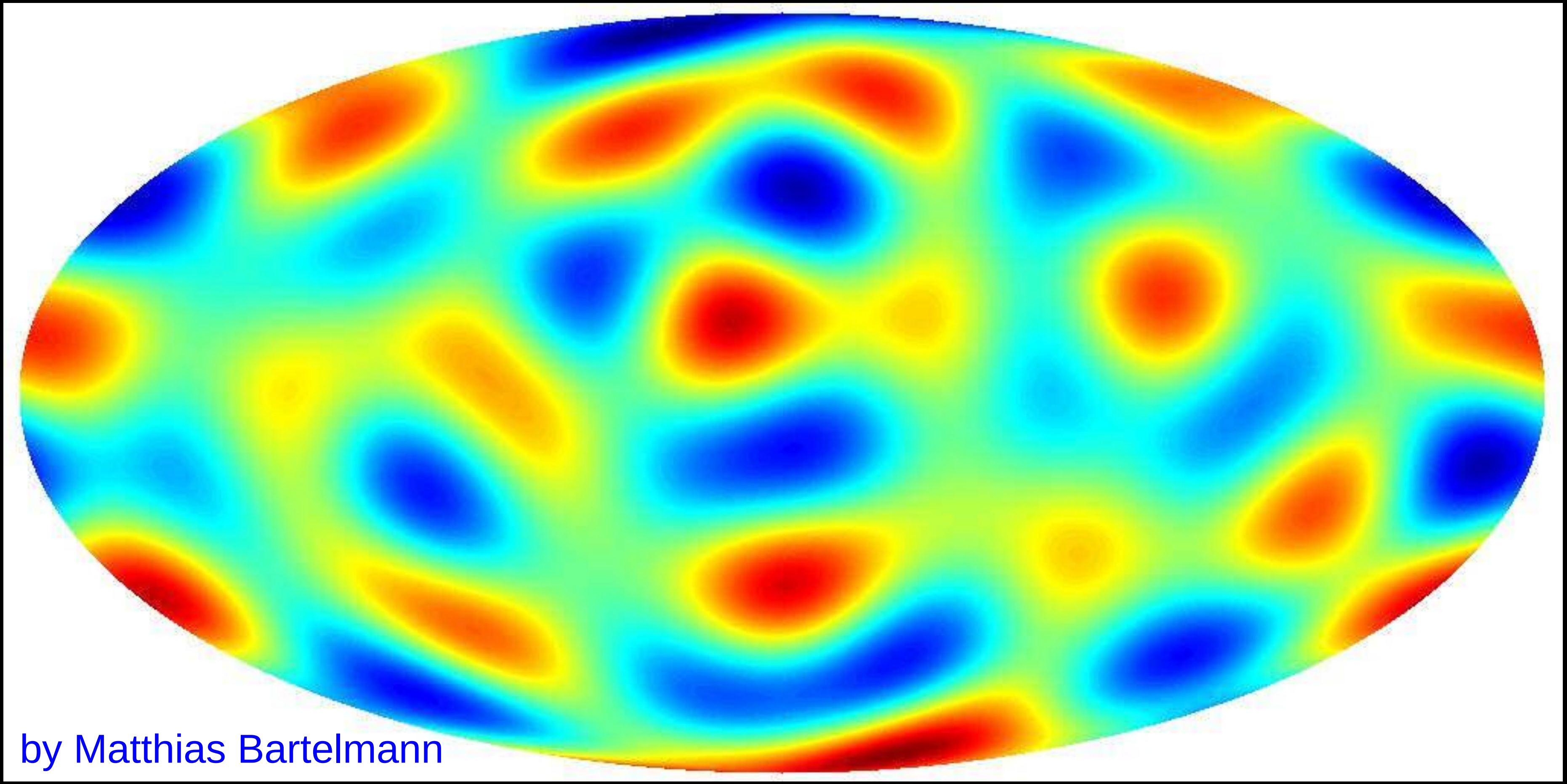


by Matthias Bartelmann

l=6

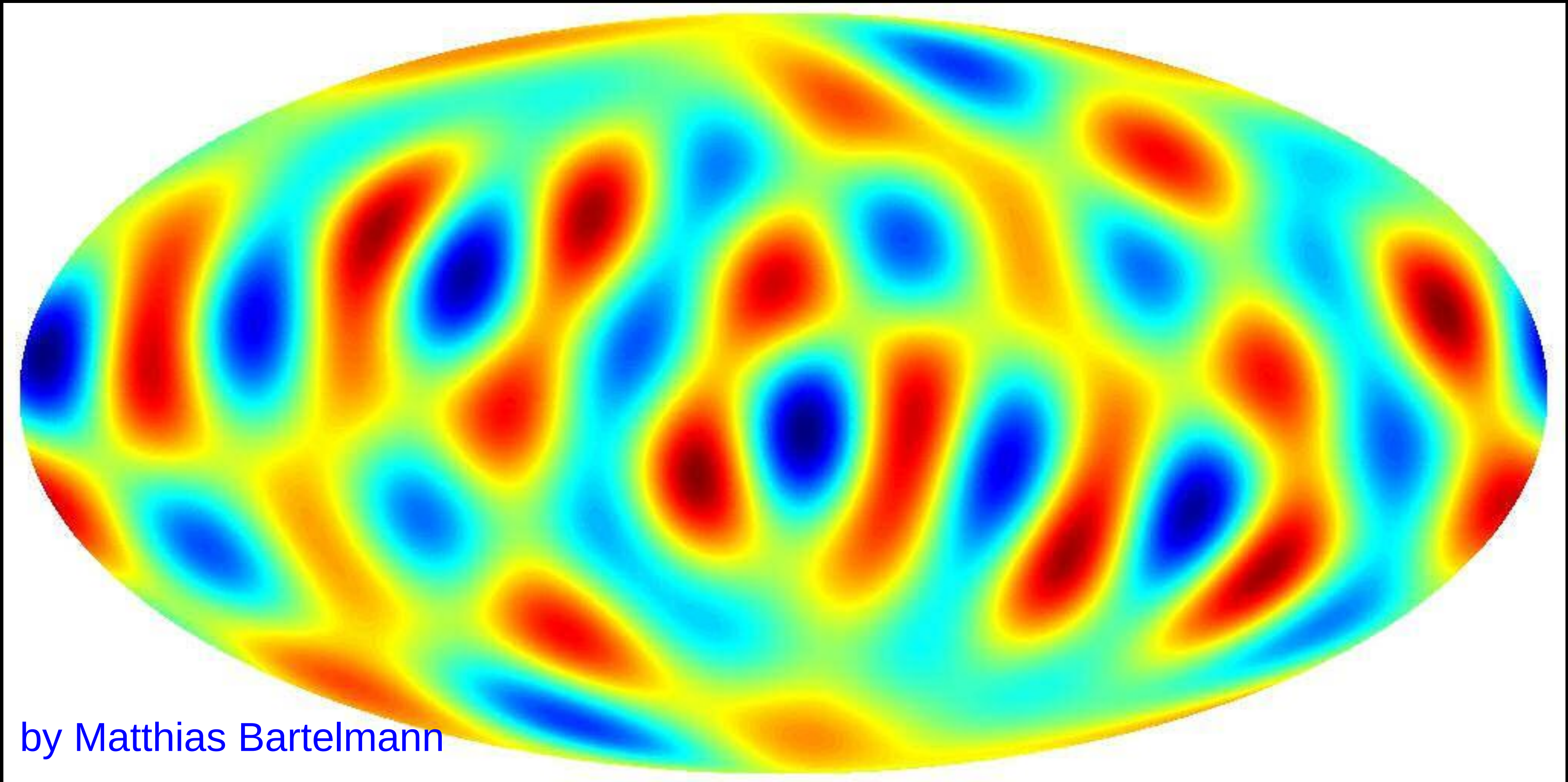


l=7

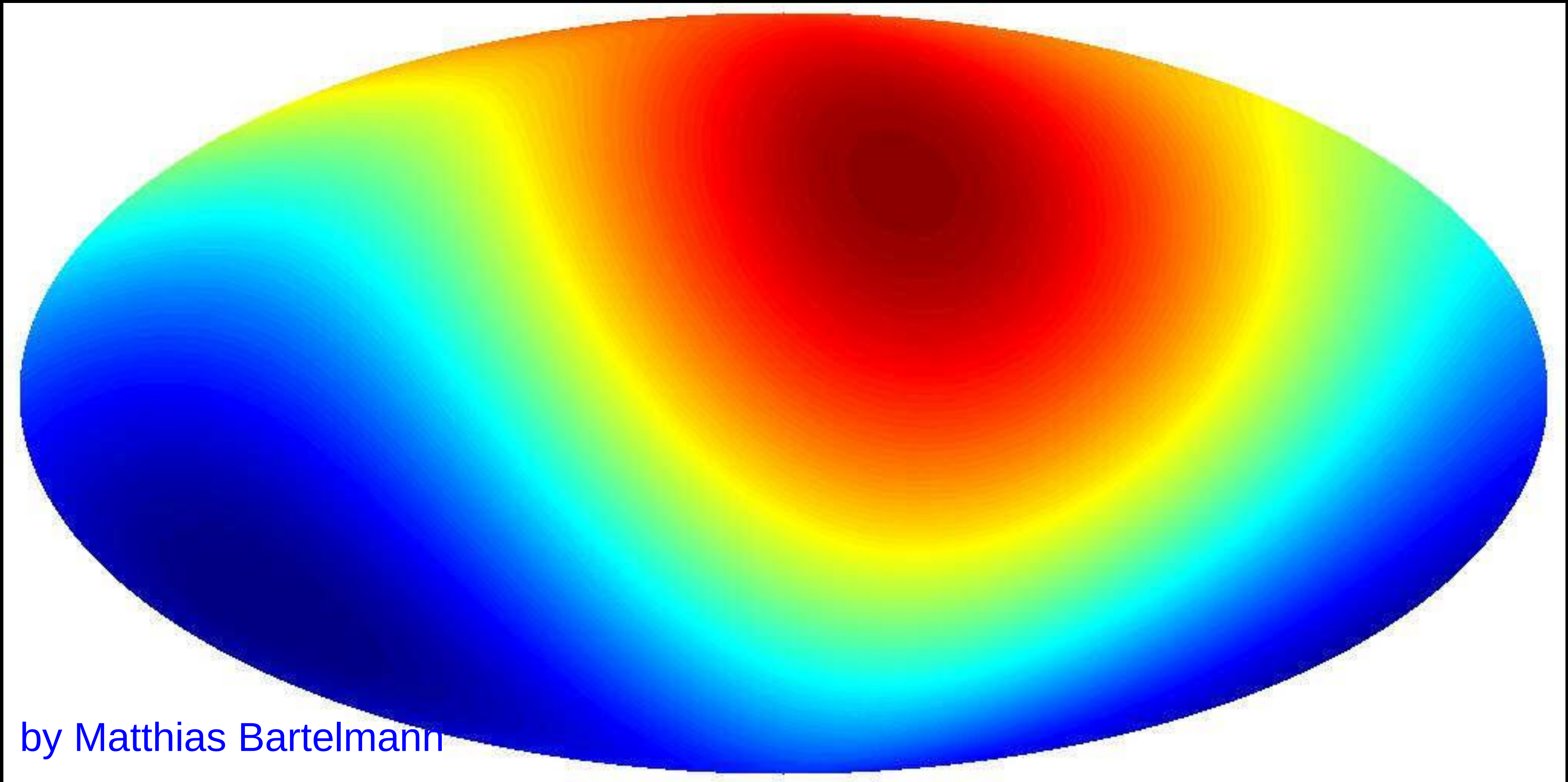


Higher l means smaller scales; l~π/θ

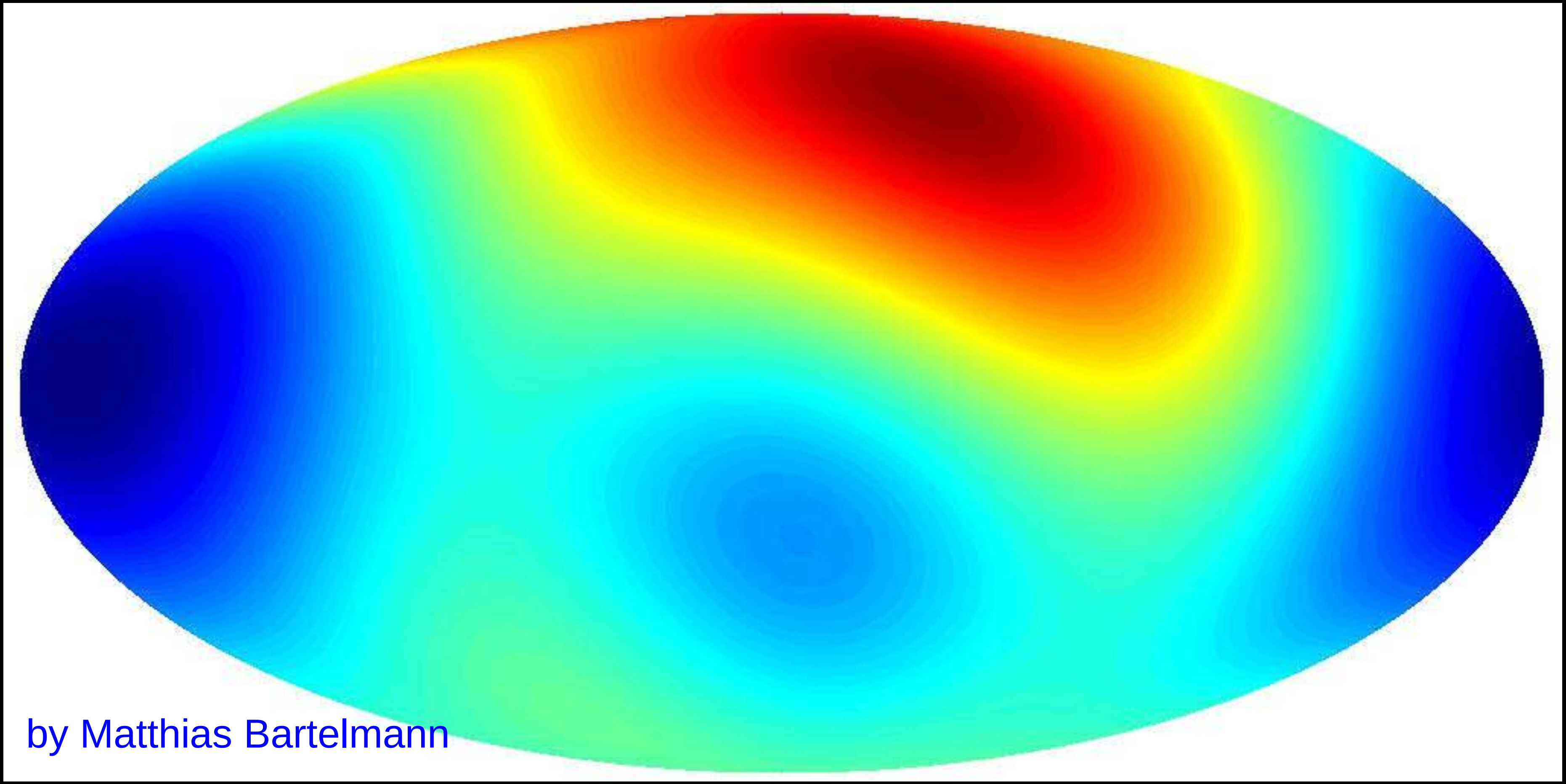
l=8



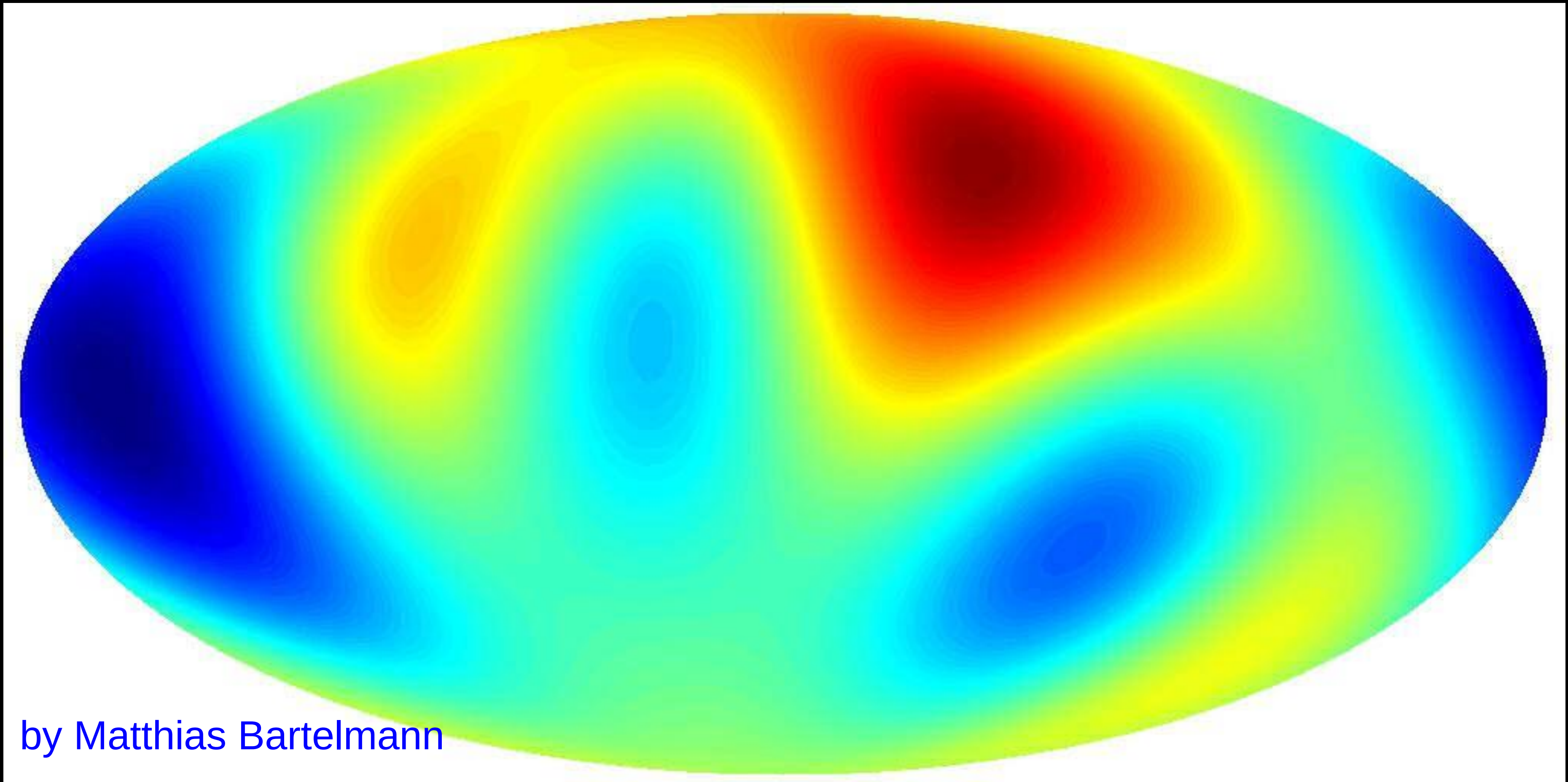
# Example of a map reconstruction
l=1



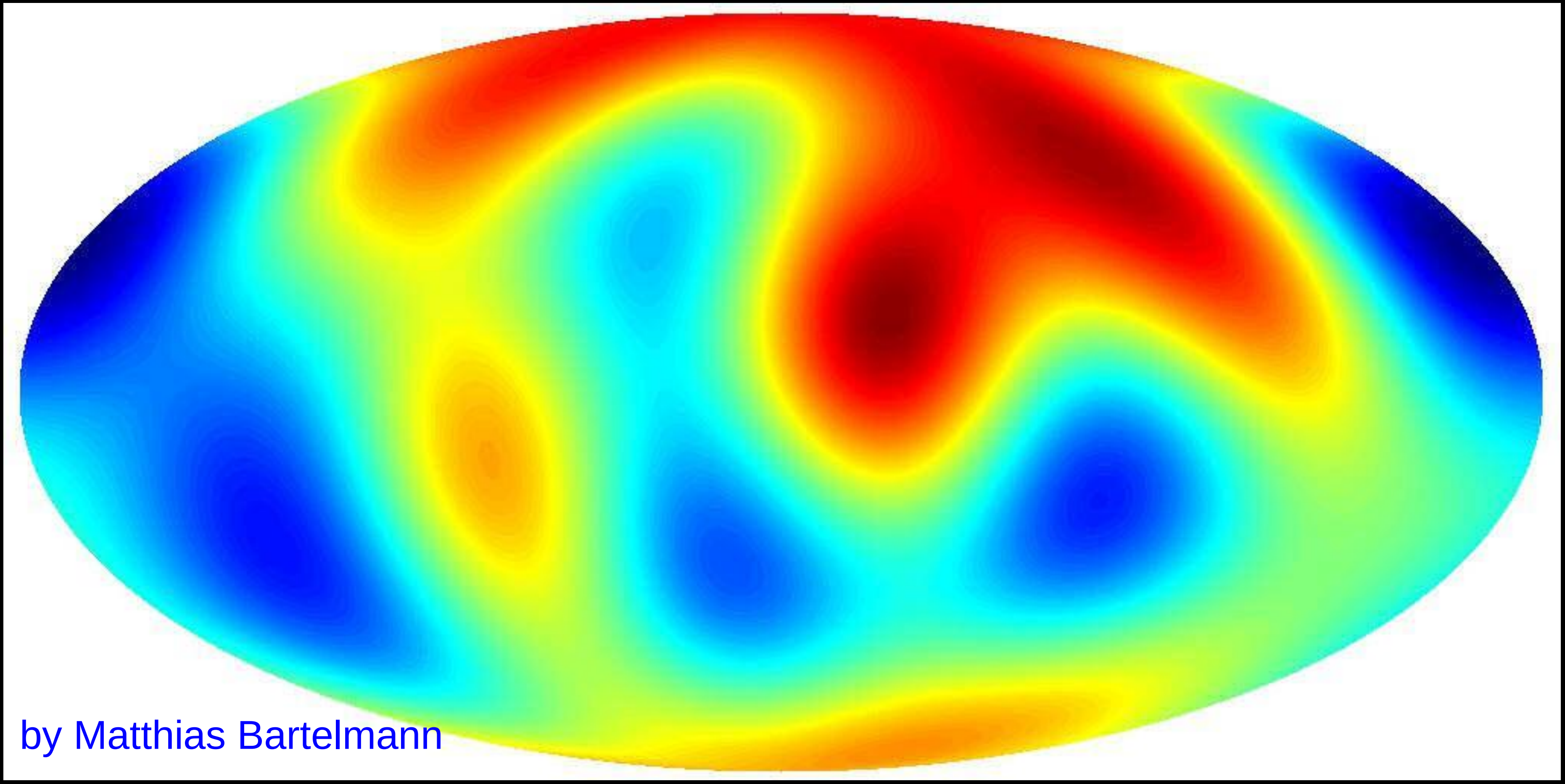
l=1 + l=2



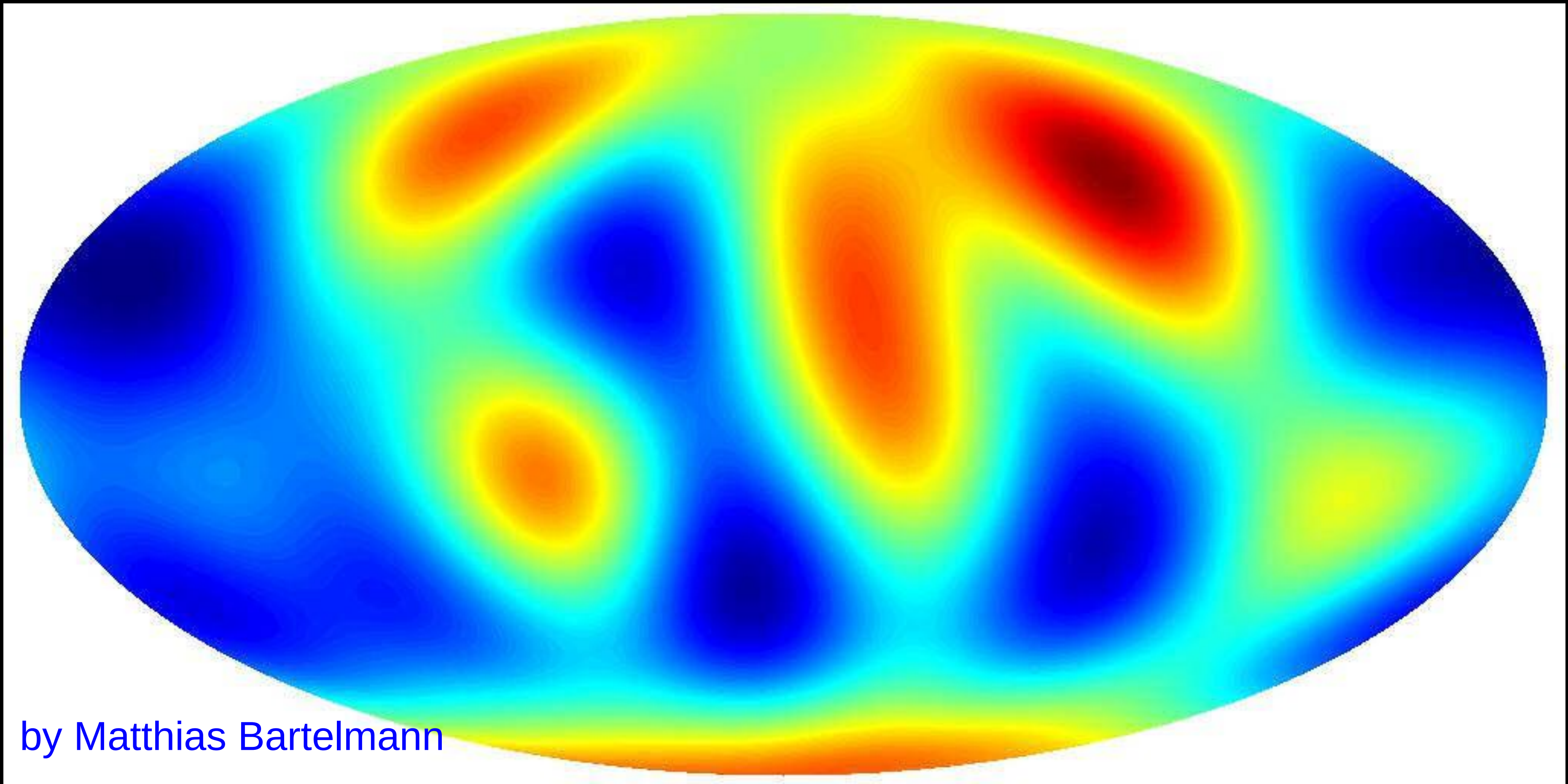
l = 1 - 3



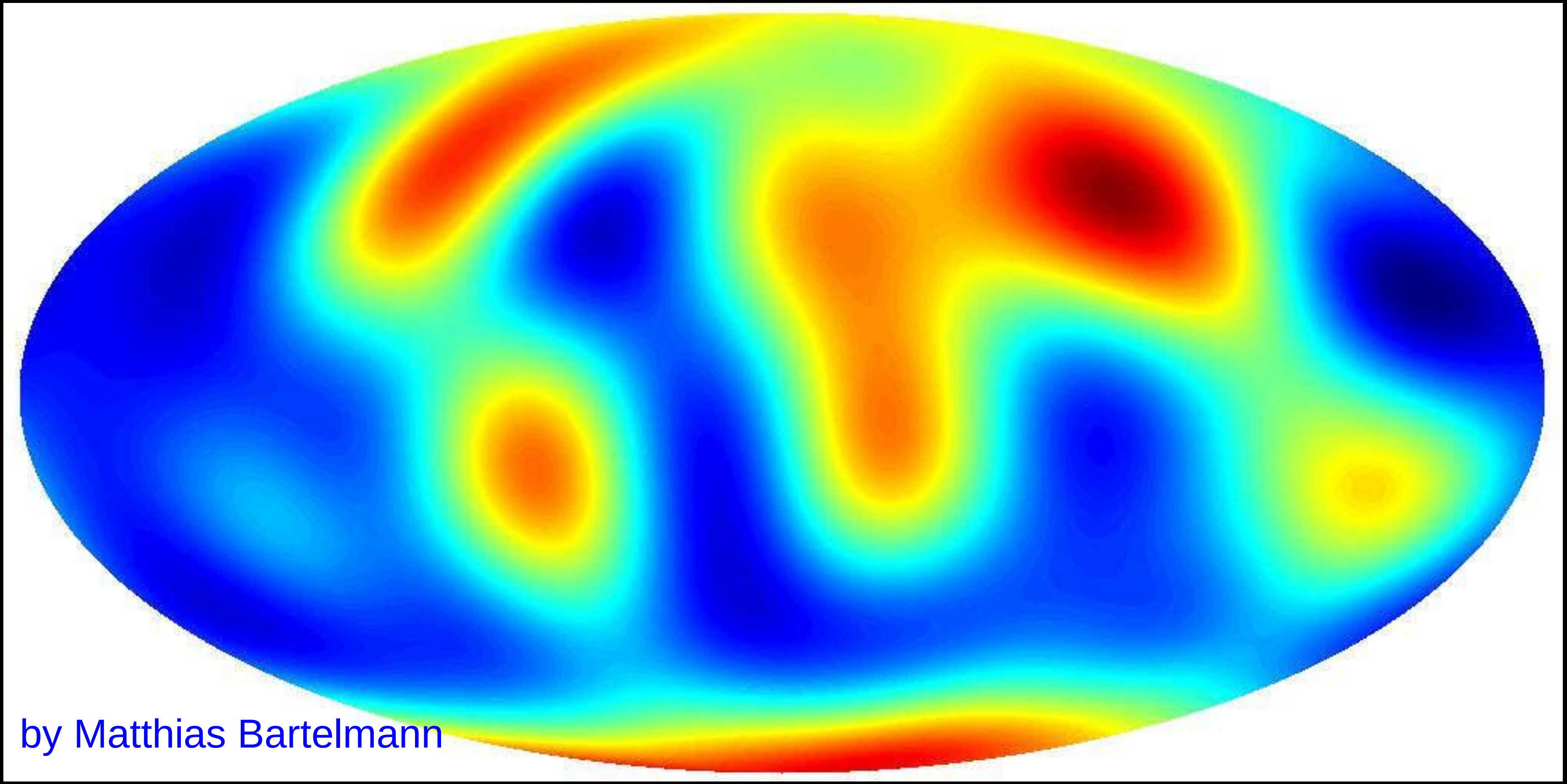
l= 1- 4



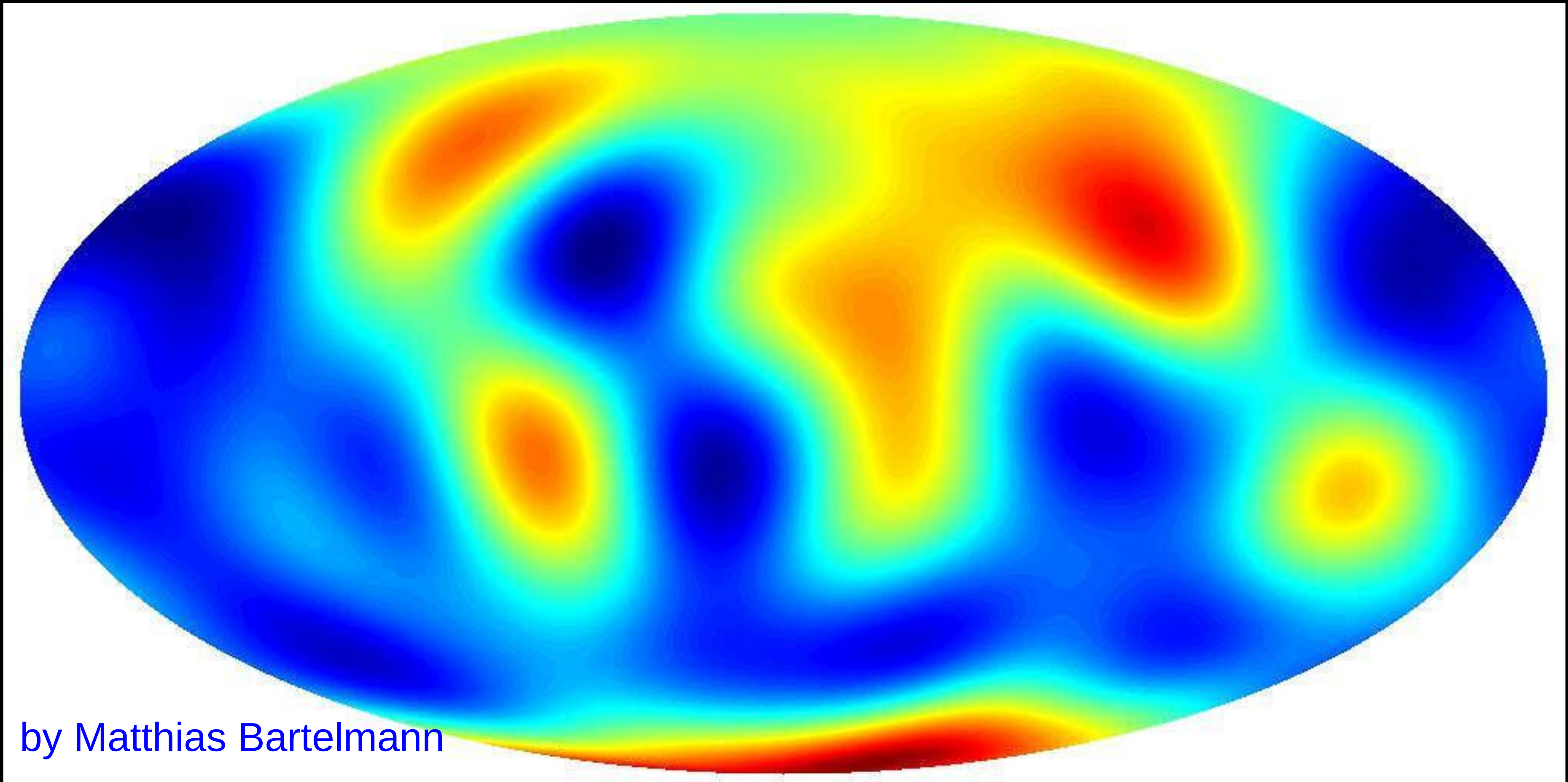
l= 1- 5



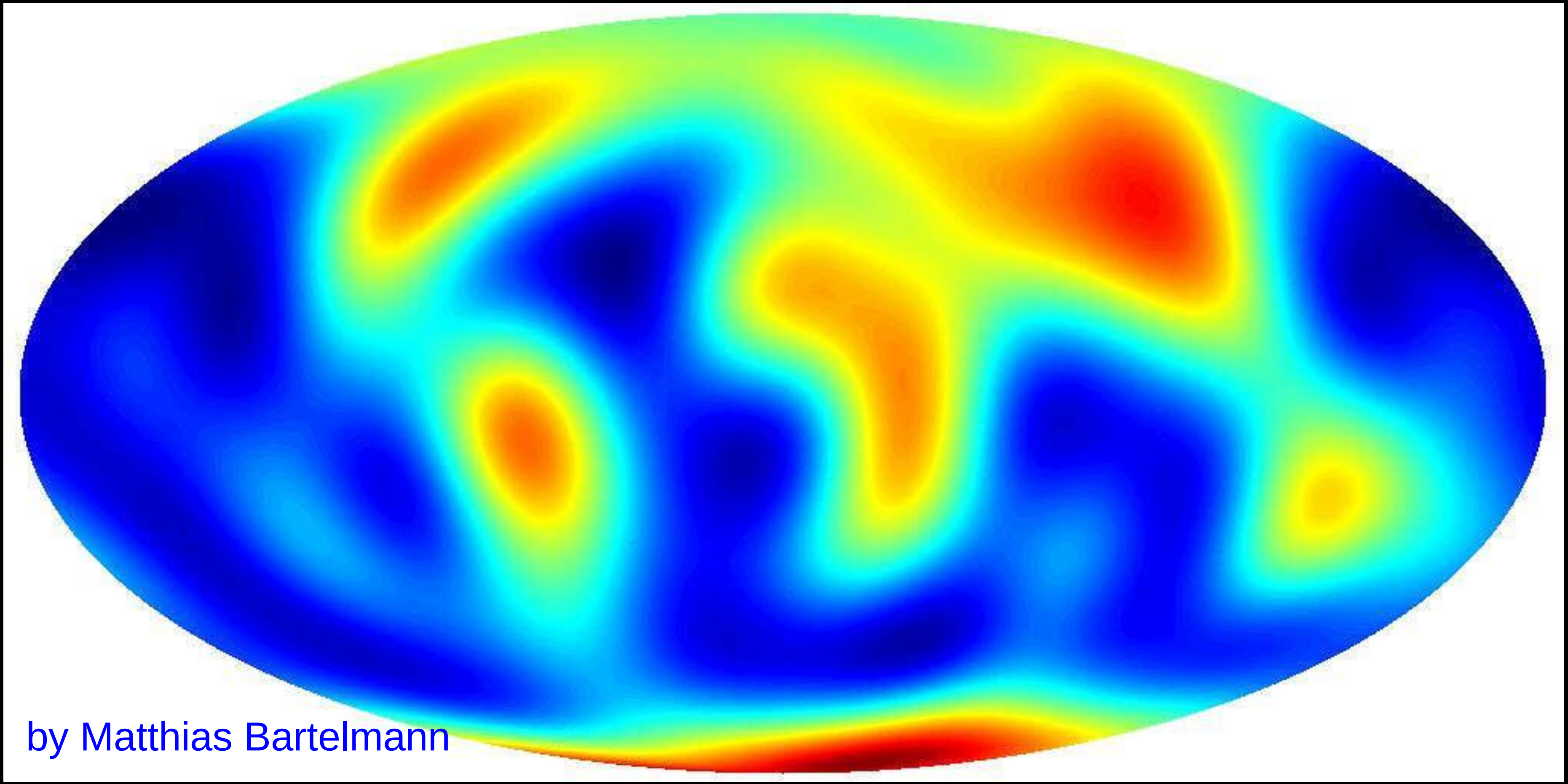
l= 1 - 6



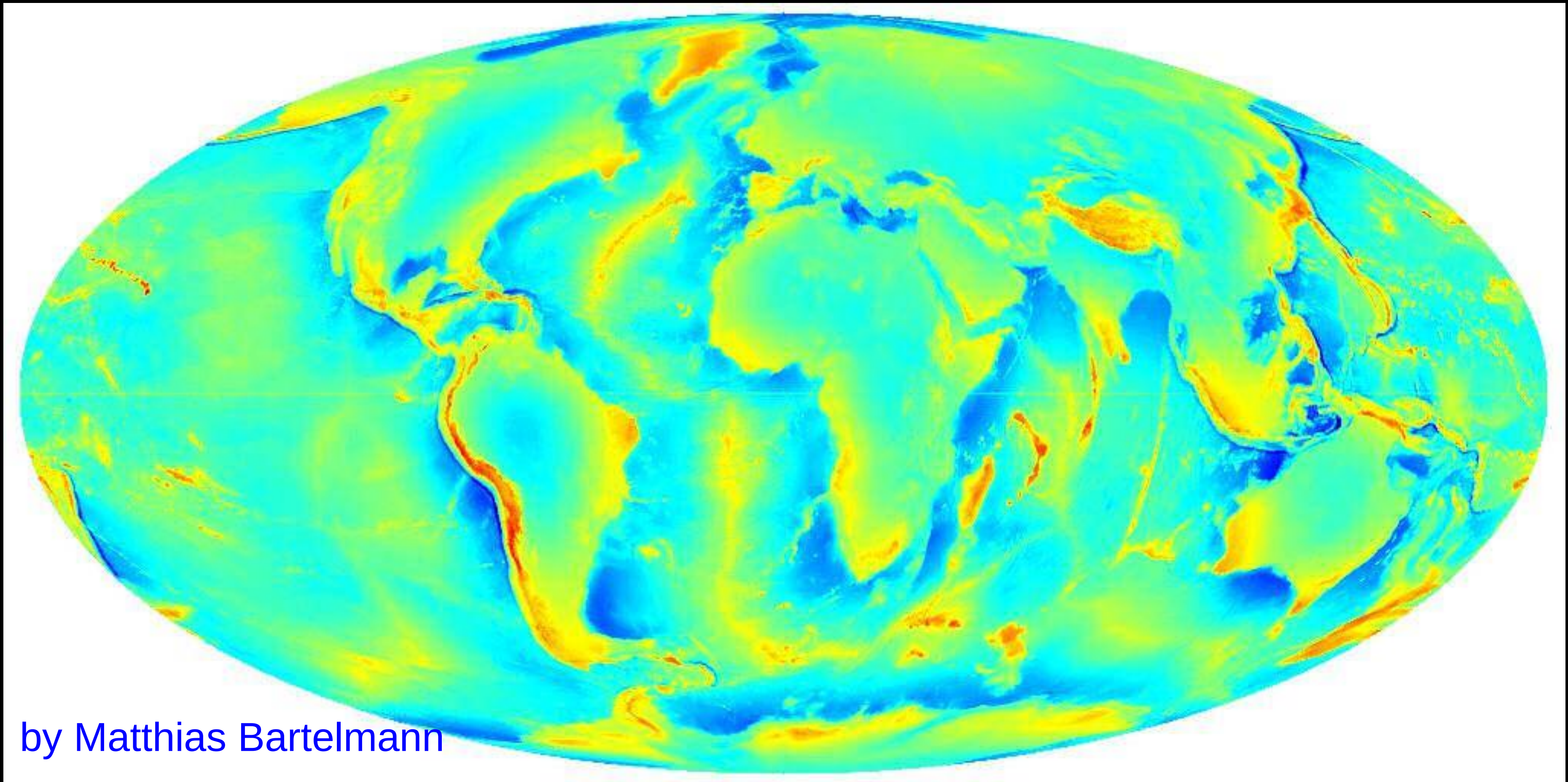
l= 1 – 7



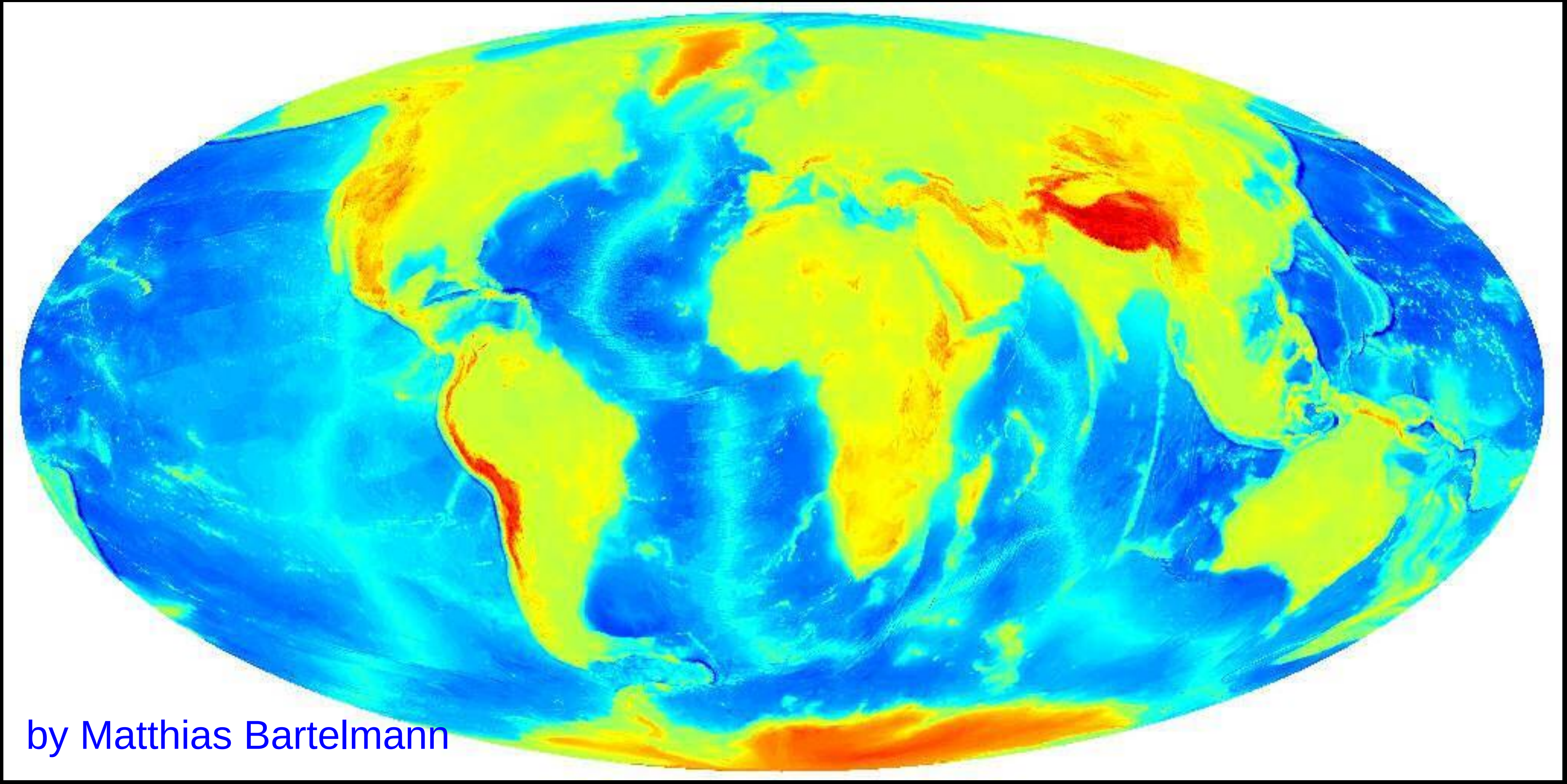
l= 1 - 8



Very high l

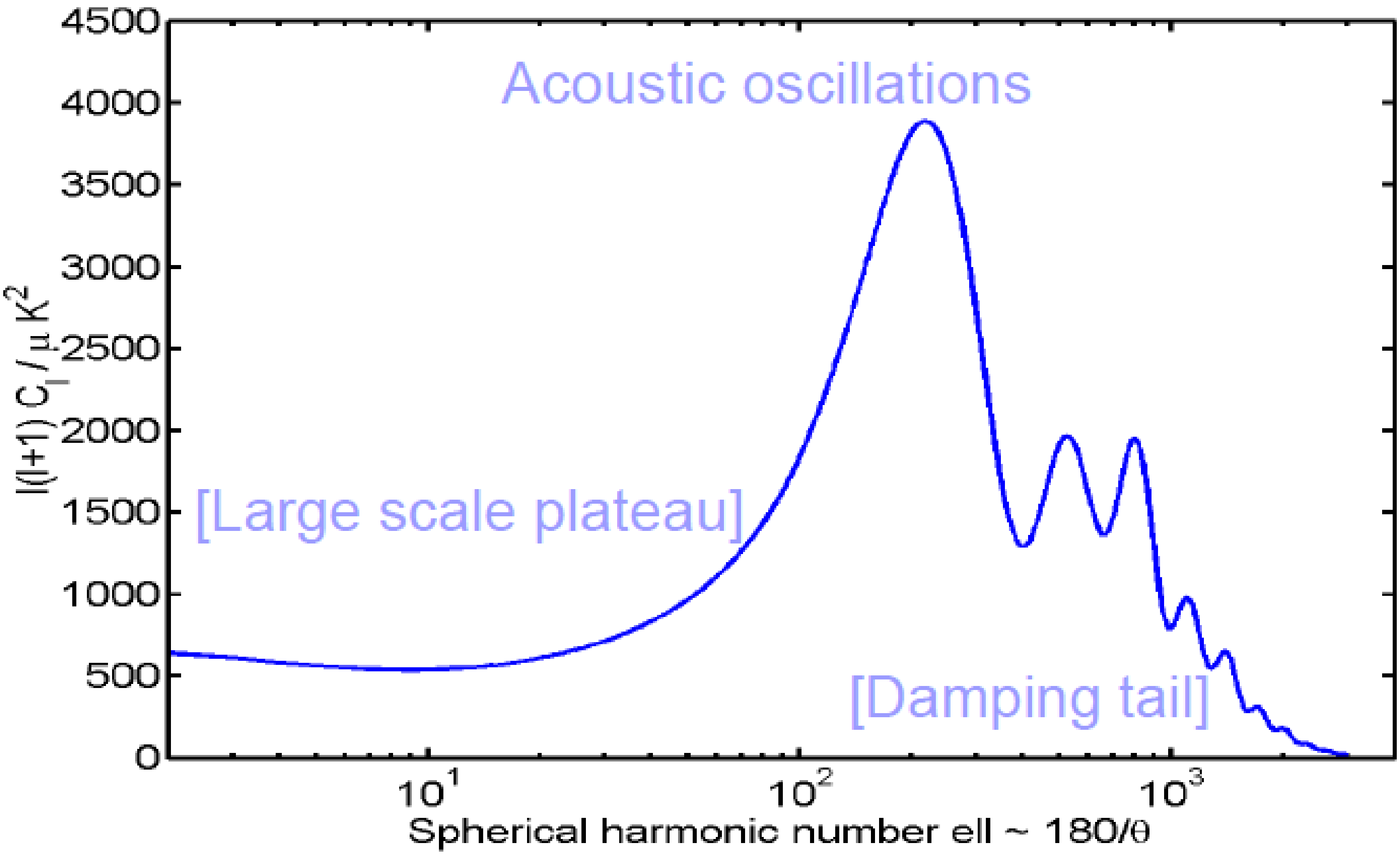


Original map

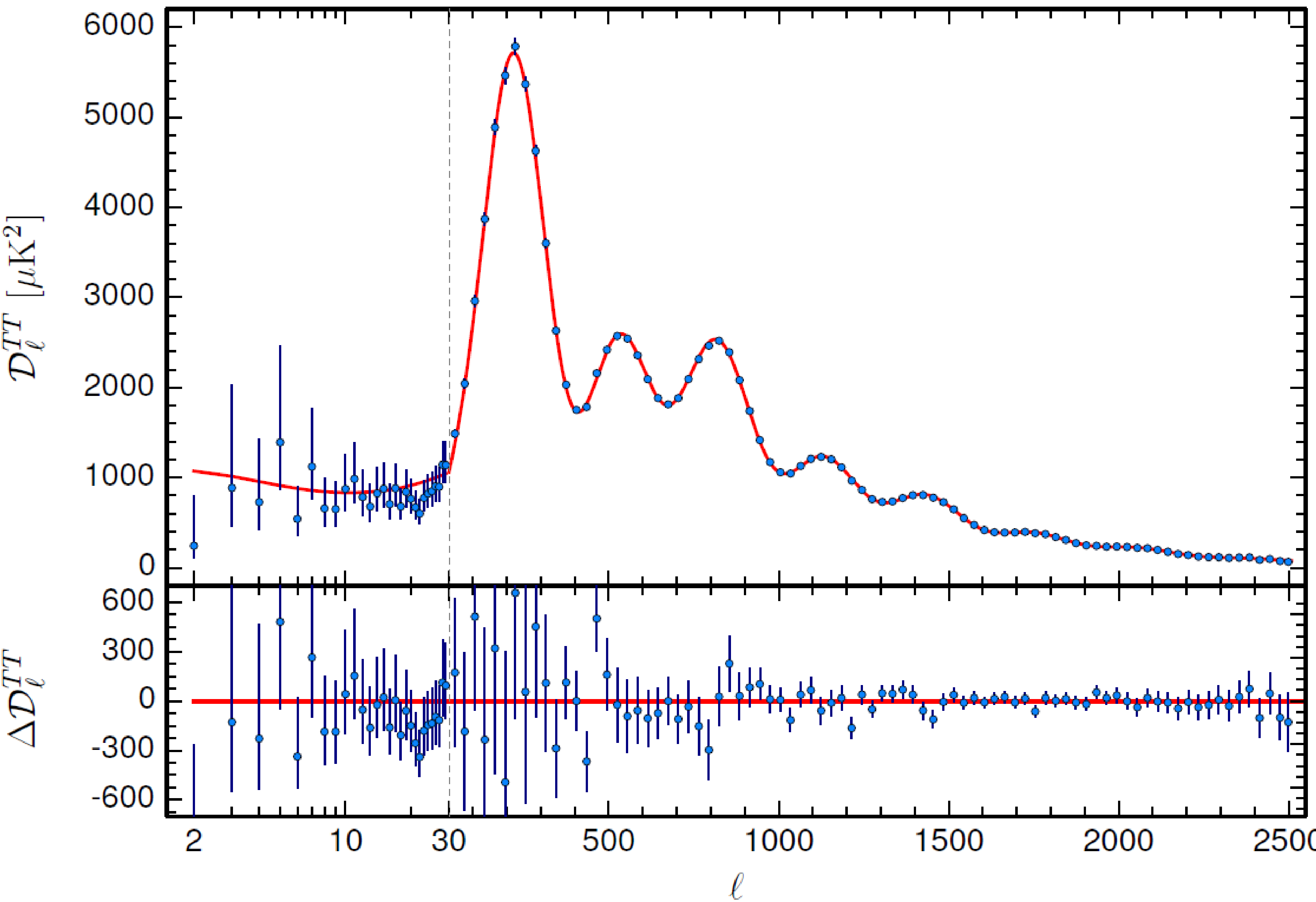


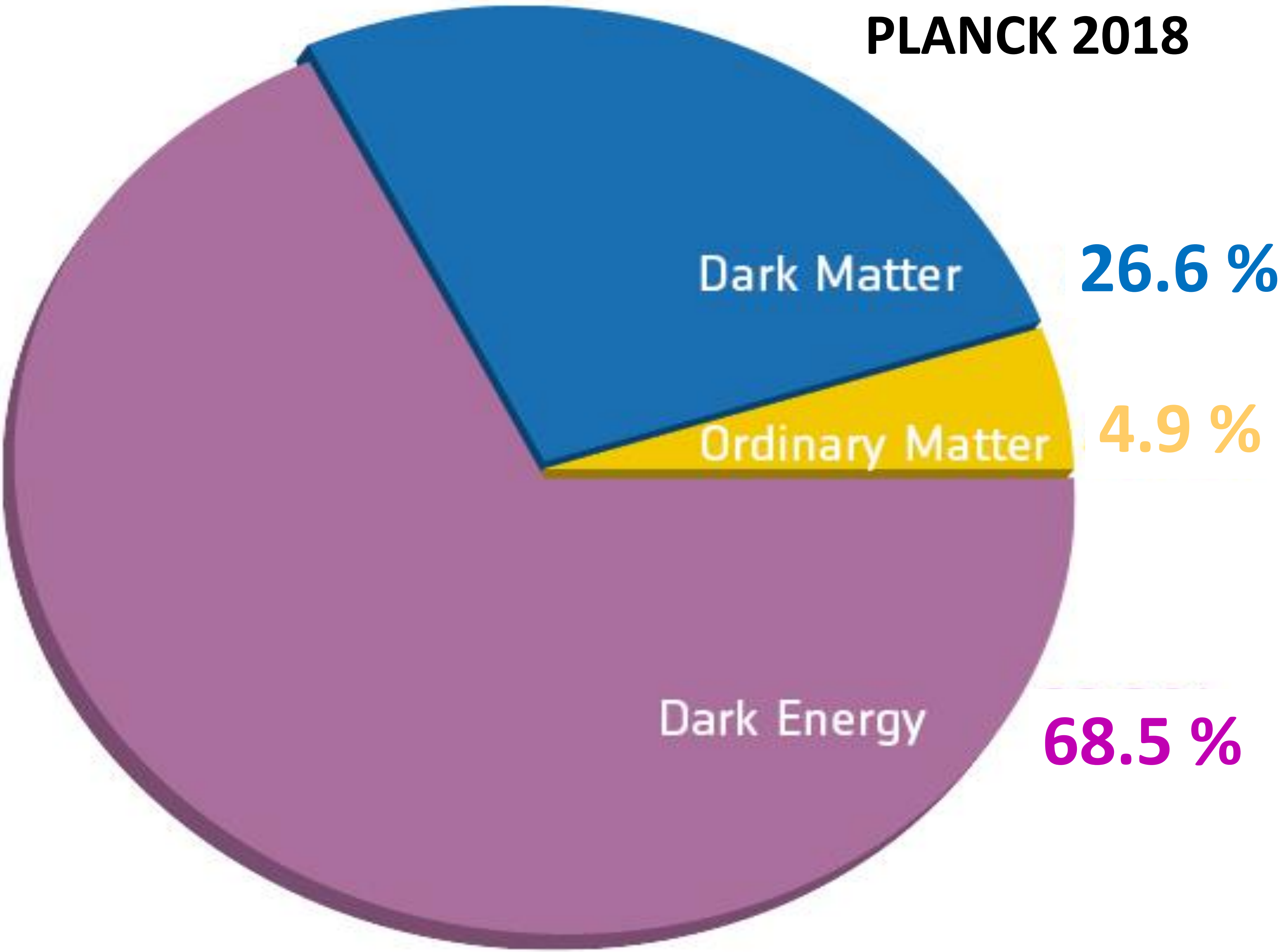
3 zones in the power spectrum

There is a characteristic scale , θ~1°



Planck .vs. ΛCDM

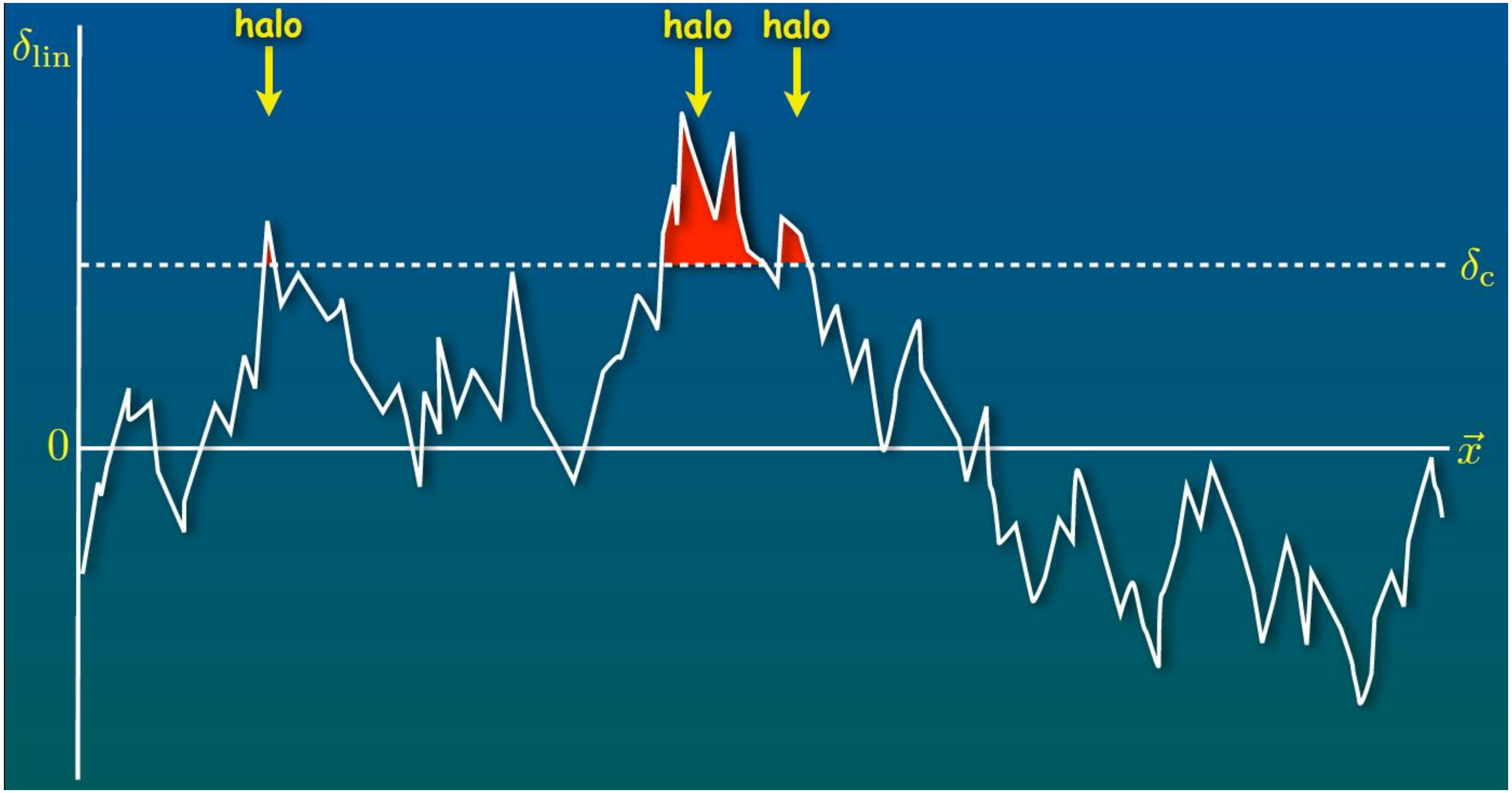



PLANCK 2018
Dark Matter
26.6 %
Ordinary Matter
4.9 %
Dark Energy
68.5 %

Spherical collapse, non-linear evolution

New concept: HALO

Halos are the sef-gravitating systems in the Universe
Peaks in the density field above δ_c
Sites for Galaxy formation (gastrophysics, virialization)
halos are non-linear peaks in the dark matter density field whose selfgravity has overcome the Hubble expansion



Spherical collapse, non-linear evolution

Spherical model: Overdense sphere ←→ closed sub-universo

Friedmann equation in a closed universe

$$\frac{1}{a}\frac{da}{dt} = H_0\left(\Omega_m a^{-3} + (1-\Omega_m)a^{-2}\right)^{1/2}$$

$$r(\theta) = A(1-\cos\theta) \quad A = r_0\Omega_m/2(\Omega_m - 1)$$

$$t(\theta) = B(\theta - \sin\theta) \quad B = H_0^{-1}\Omega_m/2(\Omega_m - 1)^{3/2}$$

$$\theta = H_0\eta(\Omega_m - 1)^{1/2}$$

Solve with the development angle (Scaled conformal time η) as the parameter

$$\delta \equiv \frac{\rho_m - \bar{\rho}_m}{\bar{\rho}_m} \approx \frac{3}{20}\left(\frac{6t}{B}\right)^{2/3}$$

density perturbation within the sphere

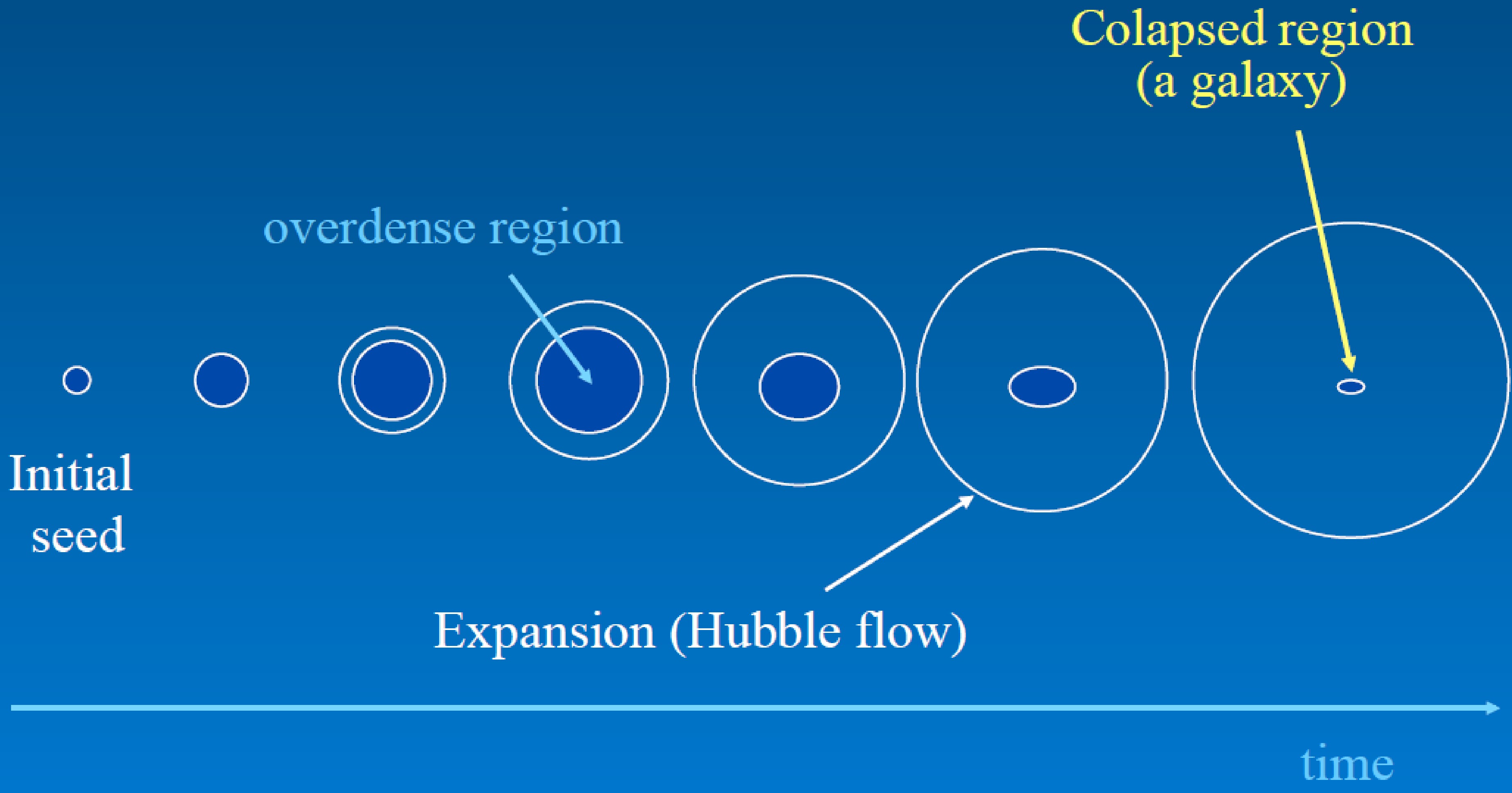
3 epochs:

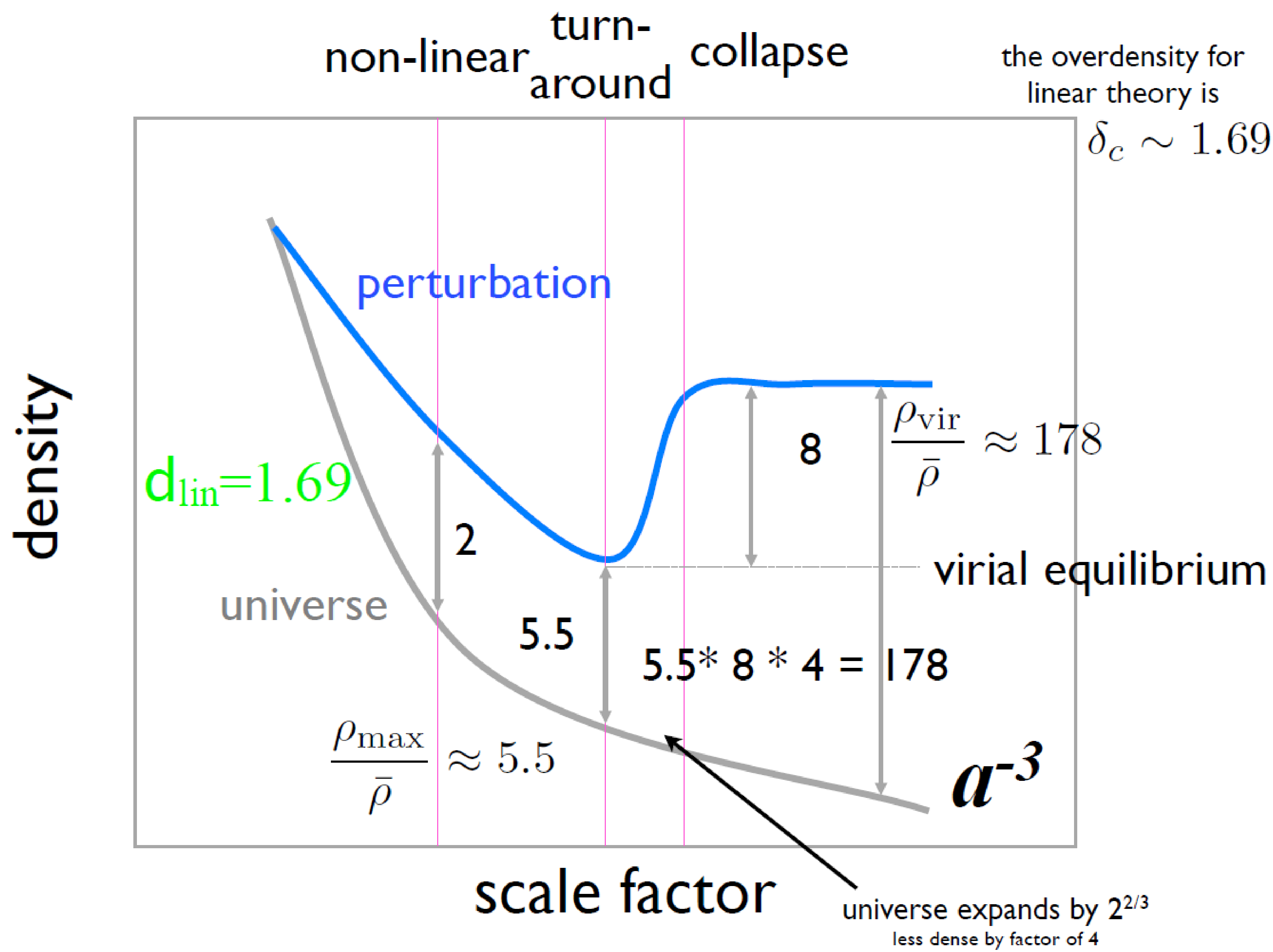
1) Turnaround: Sphere breaks away from the general expansión and reaches a maximum radius (at $\theta = \pi$, $t = \pi B$)
   Density enhancement $\rho/<\rho> \sim 5.55$ and $\delta \sim (3/20)(6\pi)^{2/3} \sim 1.06$

2) Collapse: Sphere will collapse to a singularity at $\theta = 2\pi$ (in reality it virializes due to non-gravitational physics)

3) Virialization: Interactions → Convert kinetic energy of collapse into random motions, V=-2K
   Density enhancement at collapse: $\rho/<\rho> \sim 178$; $\delta_C \sim 1.686$

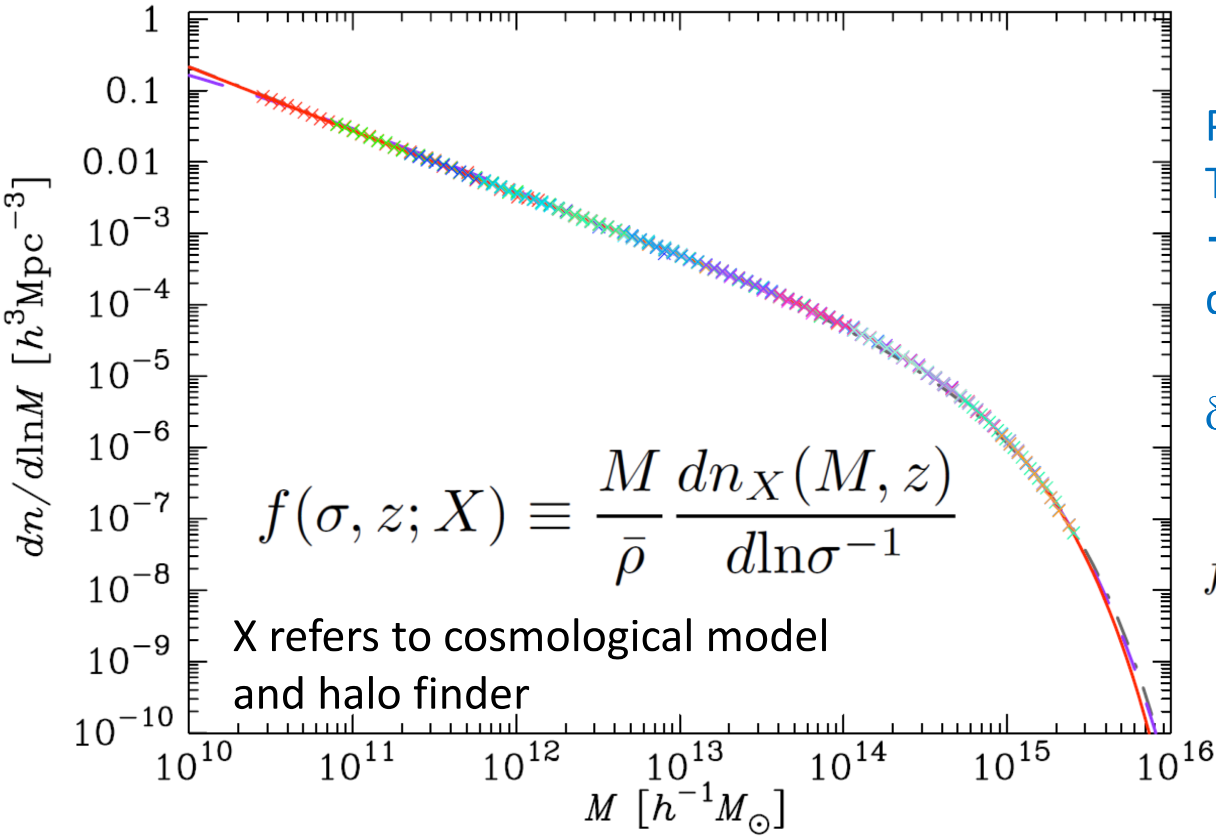
Gravitational collapse





non-linear
turn-around
collapse
the overdensity for linear theory is
$\delta_c \sim 1.69$
perturbation
density
$d_{lin}=1.69$
2
8
$\frac{\rho_{vir}}{\bar{\rho}} \approx 178$
virial equilibrium
universe
5.5
5.5* 8 * 4 = 178
$\frac{\rho_{max}}{\bar{\rho}} \approx 5.5$
a^{-3}
scale factor
universe expands by $2^{2/3}$
less dense by factor of 4

To quantify this distributions, define the mass function: Number of halos with a mass above some threshold



$$f(\sigma, z; X) \equiv \frac{M}{\bar{\rho}} \frac{dn_X(M,z)}{d\ln\sigma^{-1}}$$

X refers to cosmological model and halo finder

Press-Schechter:
The fraction of mass in halos >M
→ the fraction of volume with density above threshold δ_c

$\delta_c = 1.686$

$$f(\sigma, PS) = \sqrt{\frac{2}{\pi}} \frac{\delta_c}{\sigma} \exp(-\frac{\delta_c^2}{2\sigma^2})$$

$$\sigma^2(M,z) = \frac{b^2(z)}{2\pi} \int k^2 P(k) W^2(k; M) dk$$

Mass function is parameterized in terms of fluctuations in the mass field

Many formulae:

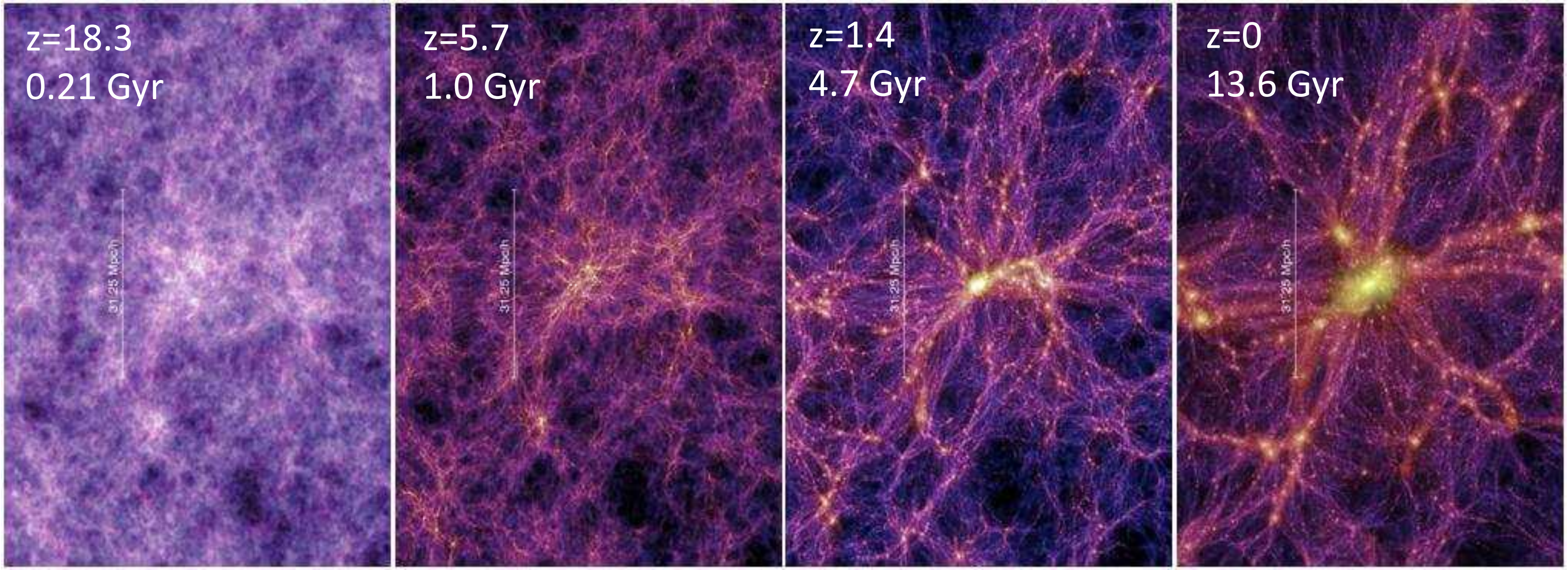
Press & Schechter 1974
Sheth & Tormen 1999, 2001
Jenkins et al 2001
Reed et al 2005
Warren at al 2005

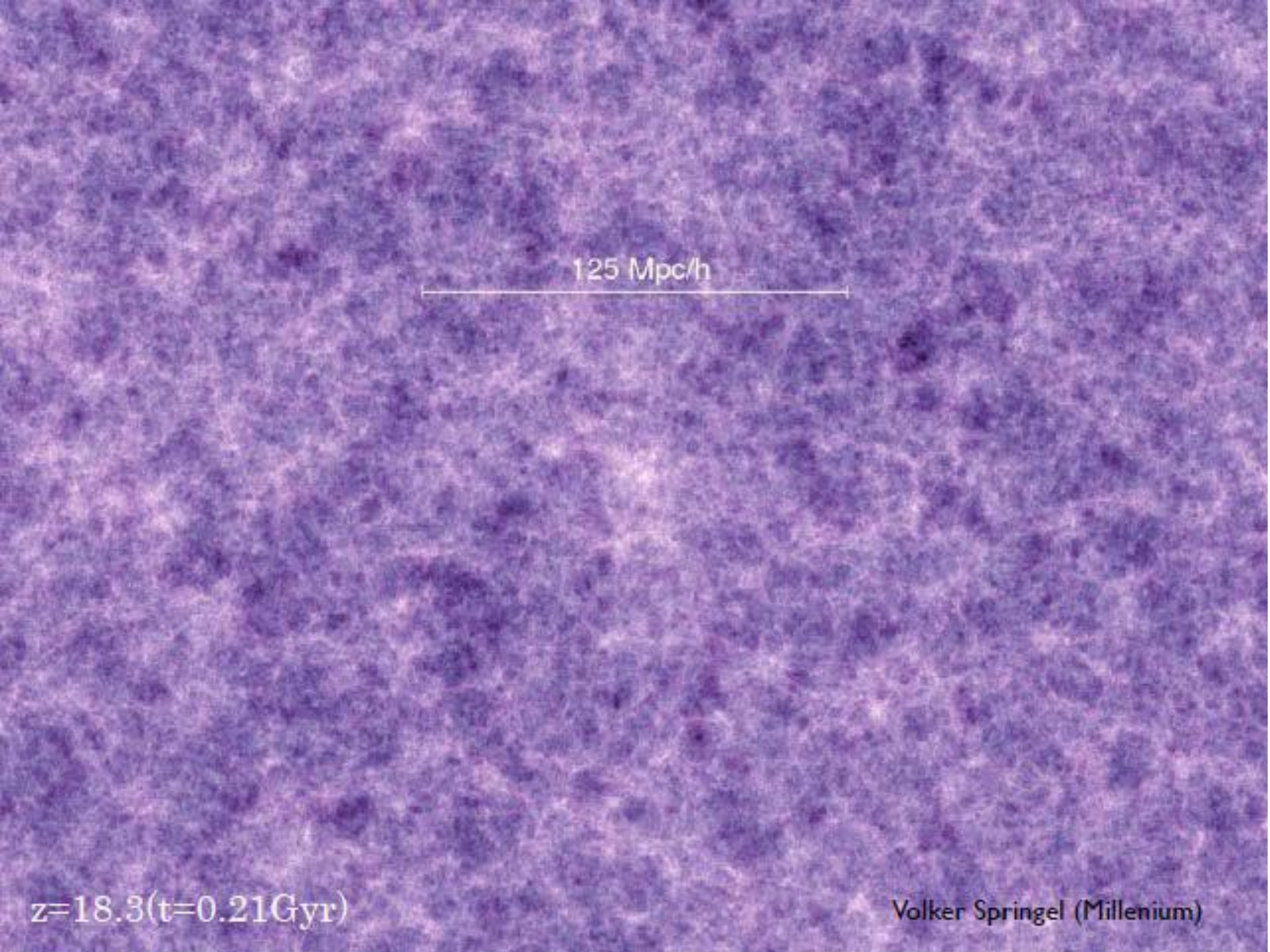
Non- linear growth and N-body simulations

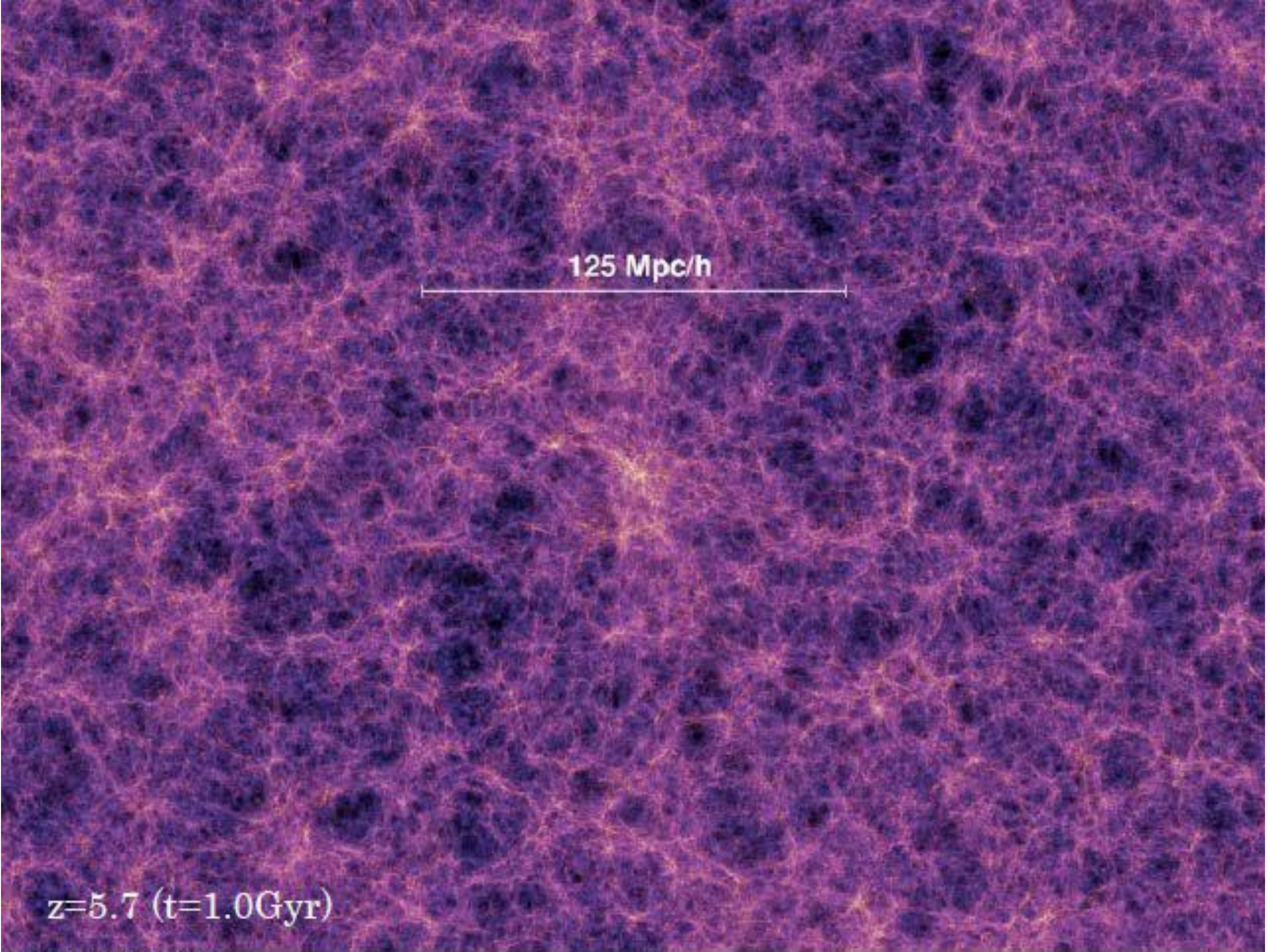
Numerical N-body simulations are the best tool to understand the nature of non-linear dynamics, and to test methods and compare with observations.

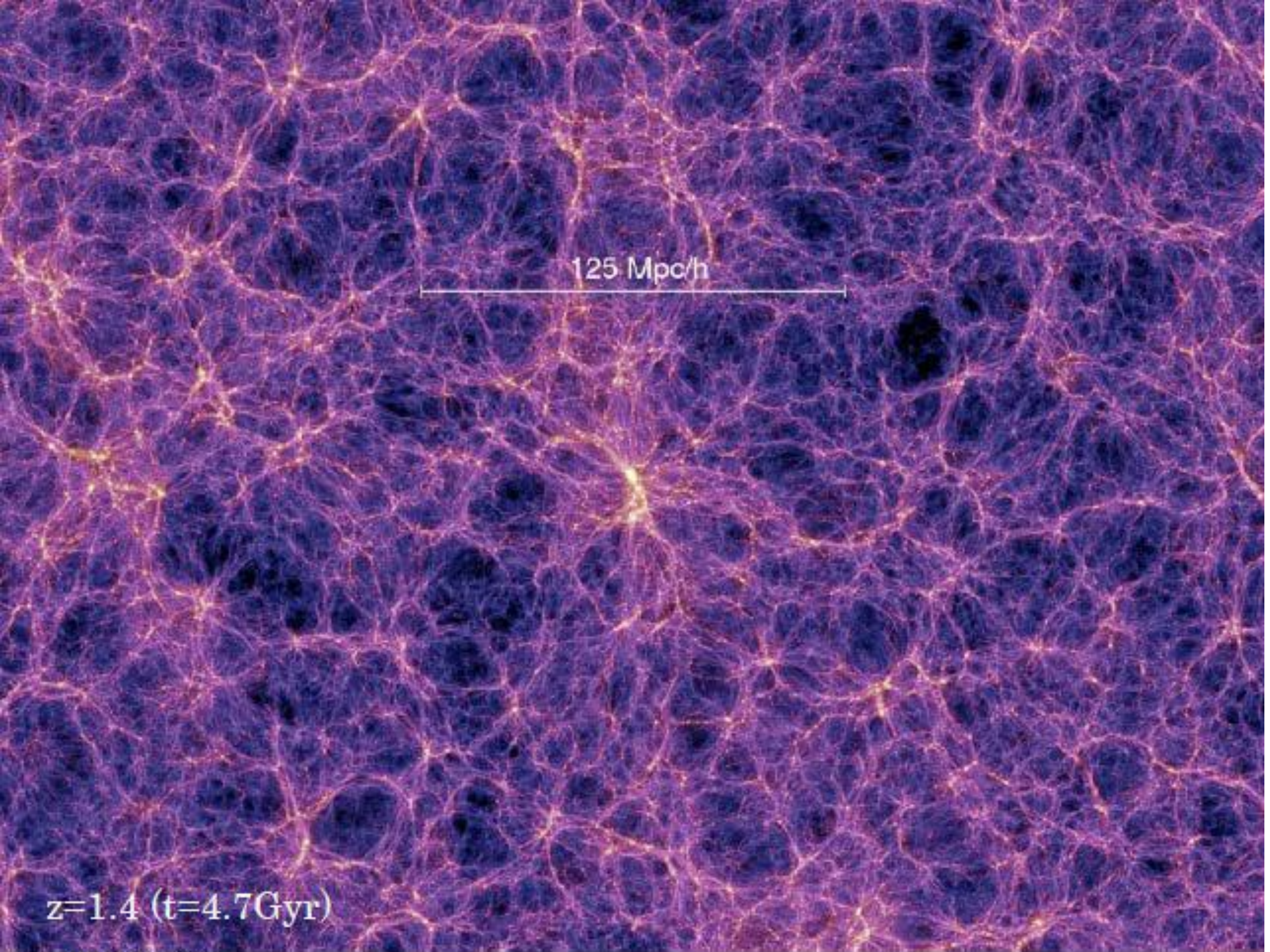
Simulations use dark matter halos and evolve them using only gravity, evolving into a nonlinear gravitational clustering

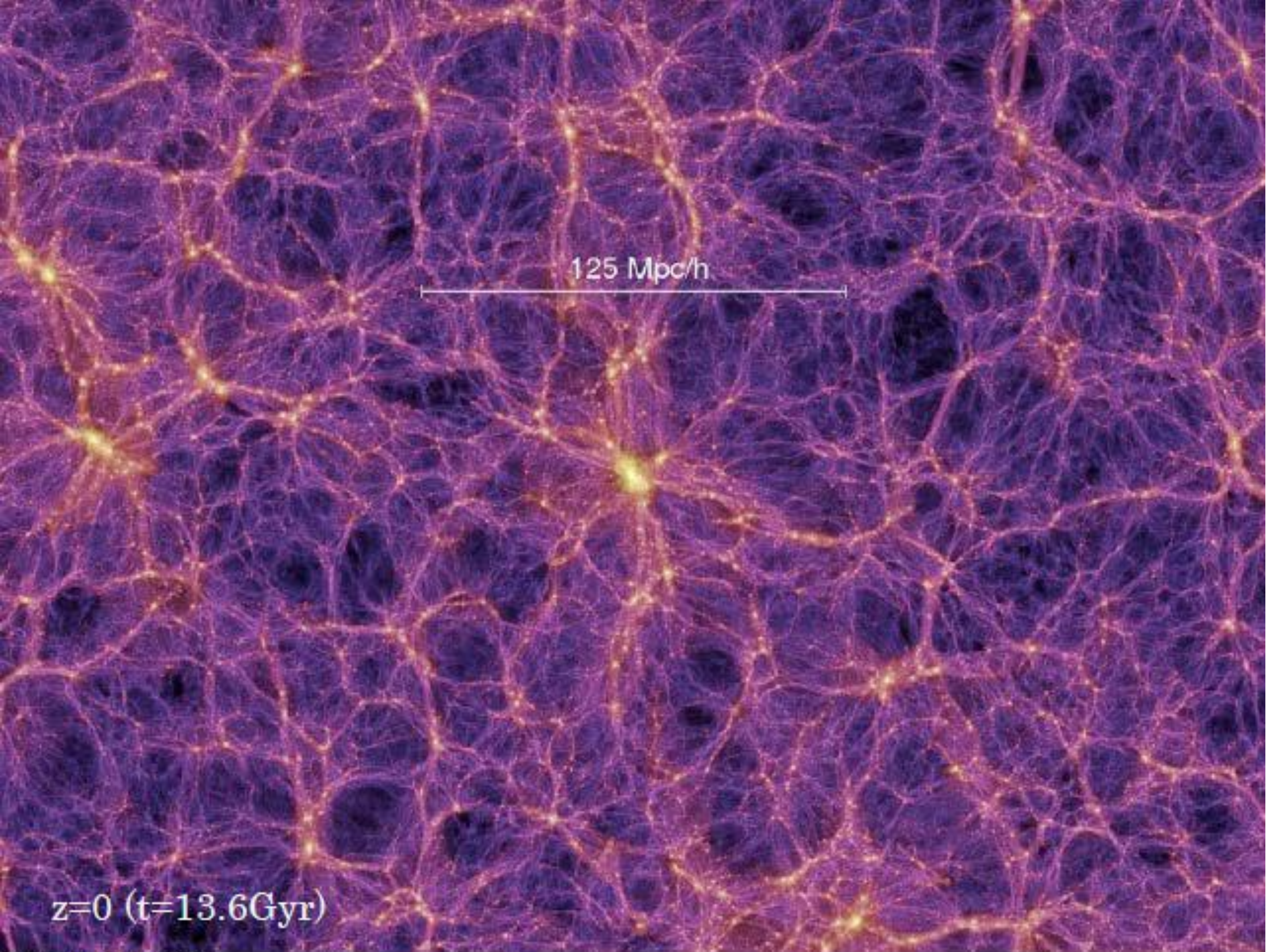
Galaxies are included in dark matter halos using semi-analitical and phenomenological methods, and matching them to observations (reproduce clustering, bias…)



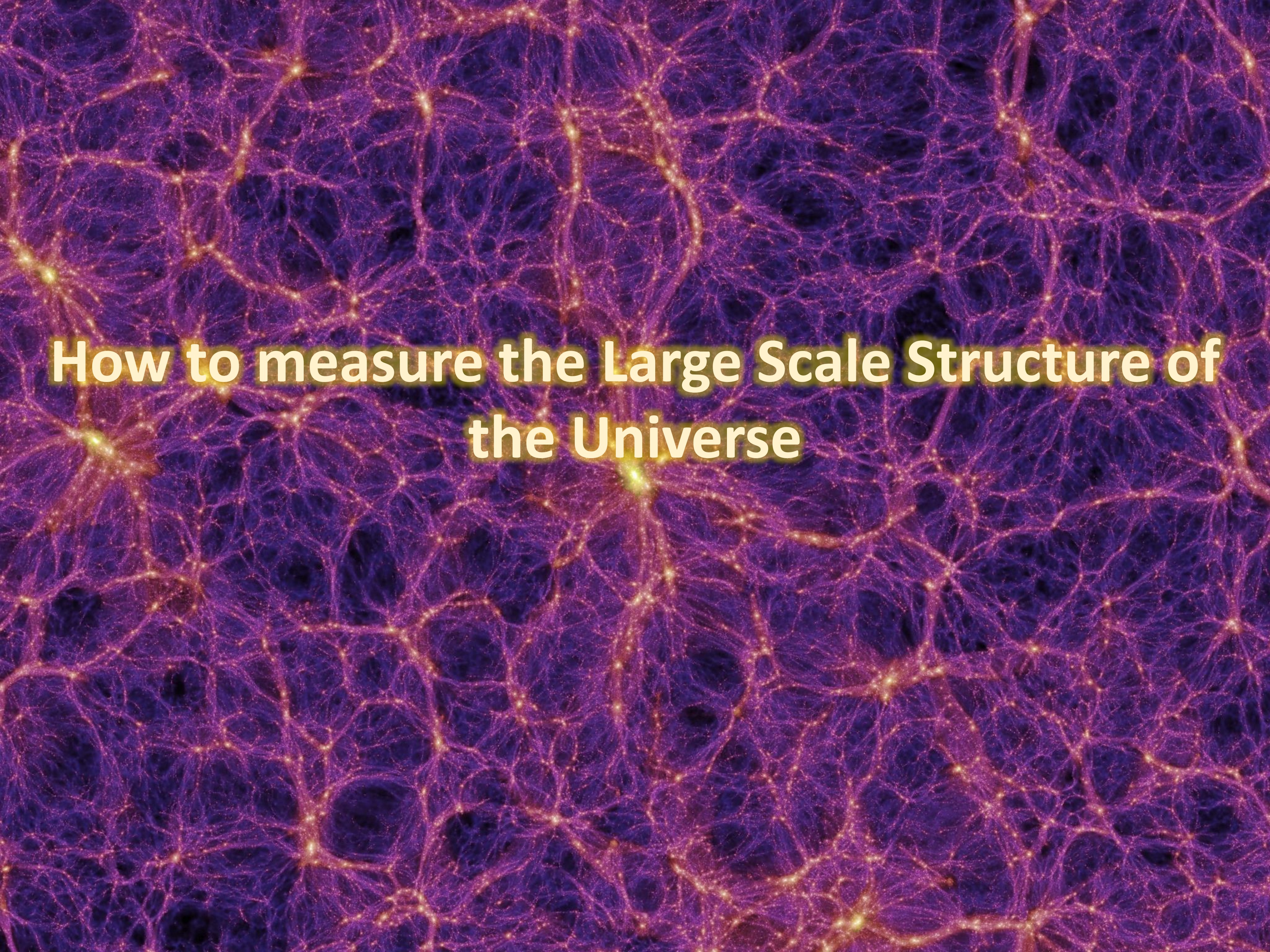

125 Mpc/h
z=18.3(t=0.21Gyr)
Volker Springel (Millenium)


125 Mpc/h
z=5.7 (t=1.0Gyr)


125 Mpc/h
z=1.4 (t=4.7Gyr)


125 Mpc/h
z=0 (t=13.6Gyr)

How to measure the Large Scale Structure of the Universe



How to measure LSS: Characterizing structure

If one Galaxy has comoving coordinate x, then the probability of finding another Galaxy in the vicinity of x is not random. They are correlated.

Consider two comoving points x and y. If <n> is the average number density of galaxies, probability of finding a Galaxy in the volumen element dV around x is

$$P_1 = \langle n \rangle \, dV$$

In practice, asume dV is small so that $P_1 \ll 1$ and the probability of finding >1 galaxies in dV is negligible

The probability of finding a Galaxy in dV around x and finding a Galaxy in dV around y is

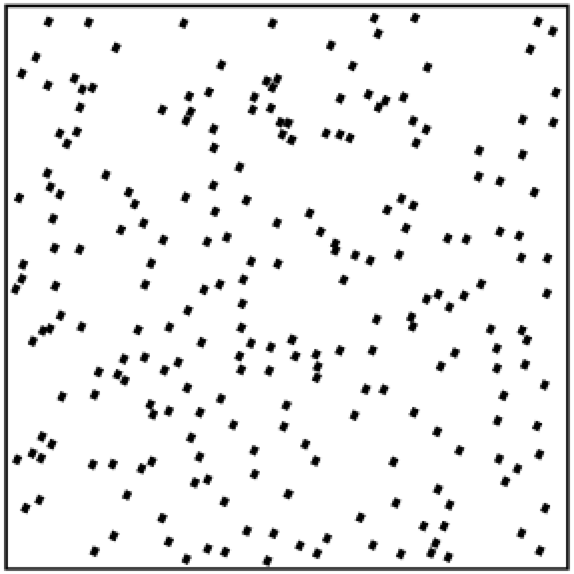
$$P_2 = (\langle n \rangle \, dV)^\wedge 2 \, [1 + \xi_G(x,y)]$$

If the probabilities were uncorrelated, $P_2 = P_1^2$. Because they are correlated include an extra term $\xi_G(x,y)$, which is the correlation function

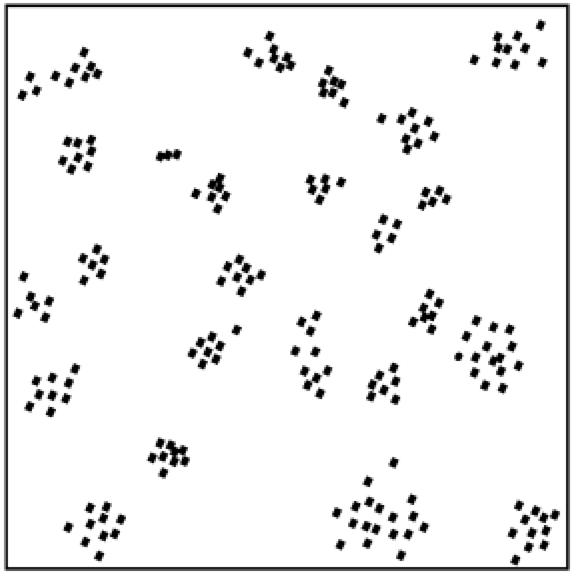
How to measure LSS: Characterizing structure

Many methods:

- The Spatial Correlation Function
- The Angular Correlation Function
- Power spectrum
- Counts in Cells
- Void Probability Functions
- Higher order statistics



Random Distribution



Clustered Distribution

Generally we want to measure how a distribution deviates from the Poisson case

How can we distinguish between a random and a clustered distribution?

2pt-correlation function or power spectrum are the main observables to study the structure of the universe

How to measure LSS: Characterizing structure

2 posible measurements:

Spatial correlation function $\xi(r,z)$ (clustering in 3D)

Angular correlation function $w(\vartheta,z)$ (projected sky)

Excess of probability with respect to a uniform distribution to find two galaxies separated by r or θ

$$dP = n\,(1+\xi(r,z))\,dV \;;\; \xi(r,z) > -1 \;;\; \xi(r,z) \rightarrow 0 \text{ when } r \rightarrow \infty$$

Definition of the correlation function

$$\xi(\mathbf{r1}, \mathbf{r2}) = \langle \delta(\mathbf{r1})\, \delta(\mathbf{r2}) \rangle =$$

$$= \quad \xi(\mathbf{r1}-\mathbf{r2})$$

$$= \quad \xi(|\mathbf{r1}-\mathbf{r2}|)$$

Statistical Homogeneity

Statistical Isotropy

In practice: the correlation function is calculated by counting the number of pairs around galaxies in a sample volume and comparing with a Poisson distribution

How to measure LSS: Characterizing structure

Compare the data with a homogeneous randomly distributed (no clustering) distribution of points, that has the same spatial sampling as galaxies

Estimators of the Correlation Function

$w(\vartheta) = (DD/RR) - 1$ *Natural*

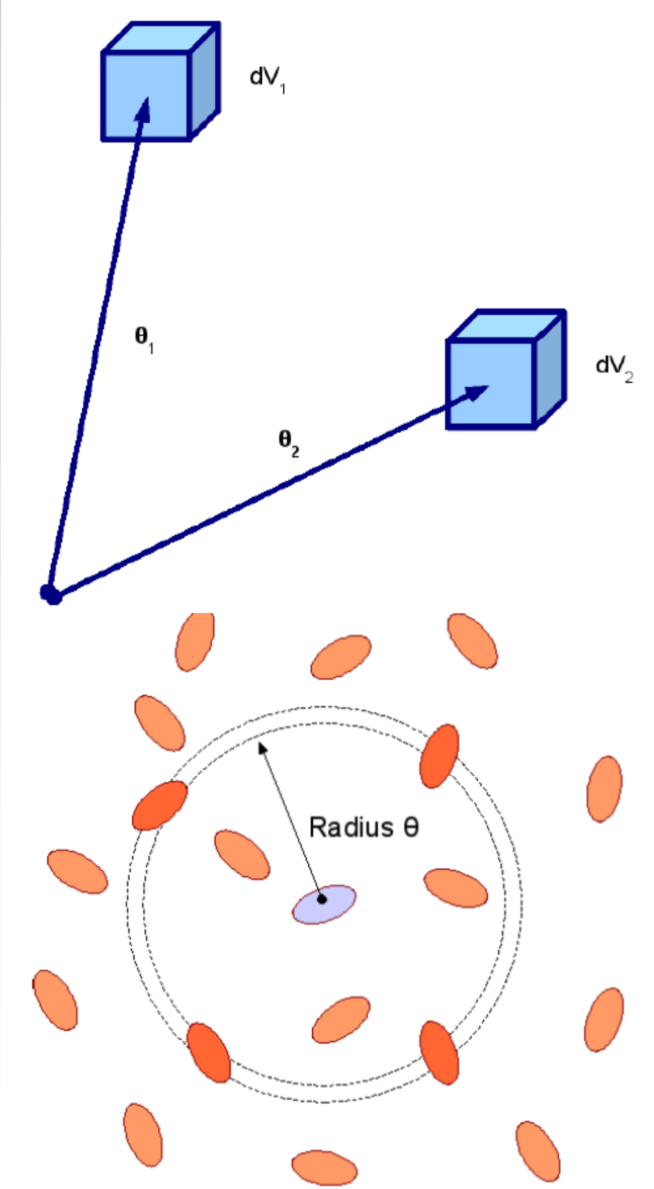
$w(\vartheta) = (2DD/DR) - 1$ *Standard*

$w(\theta) = (DD-2DR+RR)/RR$ **Landy-Szalay**

$w(\vartheta) = 4(DD\times DR)/(DR^2-1)$ *Hamilton*

DD(r) number of pairs data-data
RR(r) number of pairs random-random
DR(r) number of pairs data-random

Using the random sample one can take into account practical difficulties like the partial covering of the sky with observations or the different depth of the observations for different points in the sky



How to measure LSS: Characterizing structure

Comparing measurements to theory: The correlation function is the Fourier transform of the power spectrum

The power spectrum and correlation function contain the same information; accurate measurement of each give the same constraints on cosmological models.

$$\delta(\mathbf{x}) = \sum_{\mathbf{k}} \delta_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{x}} \qquad \delta_{\mathbf{k}} = \frac{1}{V}\int_V \delta(\mathbf{x}) e^{-i\mathbf{k}\cdot\mathbf{x}} d^d x$$

$$\begin{aligned}
\langle \delta^*_{\mathbf{k}} \delta_{\mathbf{k}'} \rangle &= \frac{1}{V^2}\int d^d x e^{i\mathbf{k}\cdot\mathbf{x}} \int d^d x' e^{-i\mathbf{k}'\cdot\mathbf{x}'} \langle \delta(\mathbf{x})\delta(\mathbf{x}')\rangle \\
&= \frac{1}{V^2}\int d^d x e^{i\mathbf{k}\cdot\mathbf{x}} \int d^d r e^{-i\mathbf{k}'\cdot(\mathbf{x}+\mathbf{r})} \langle \delta(\mathbf{x})\delta(\mathbf{x}+\mathbf{r})\rangle \\
&= \frac{1}{V^2}\int d^d r e^{-i\mathbf{k}'\cdot\mathbf{r}} \xi(\mathbf{r}) \int d^d x e^{i(\mathbf{k}-\mathbf{k}')\cdot\mathbf{x}} \\
&= \frac{1}{V}\delta_{\mathbf{k}\mathbf{k}'} \int d^d r e^{-i\mathbf{k}\cdot\mathbf{r}} \xi(\mathbf{r}) \equiv \frac{1}{V}\delta_{\mathbf{k}\mathbf{k}'} P(\mathbf{k}),
\end{aligned}$$

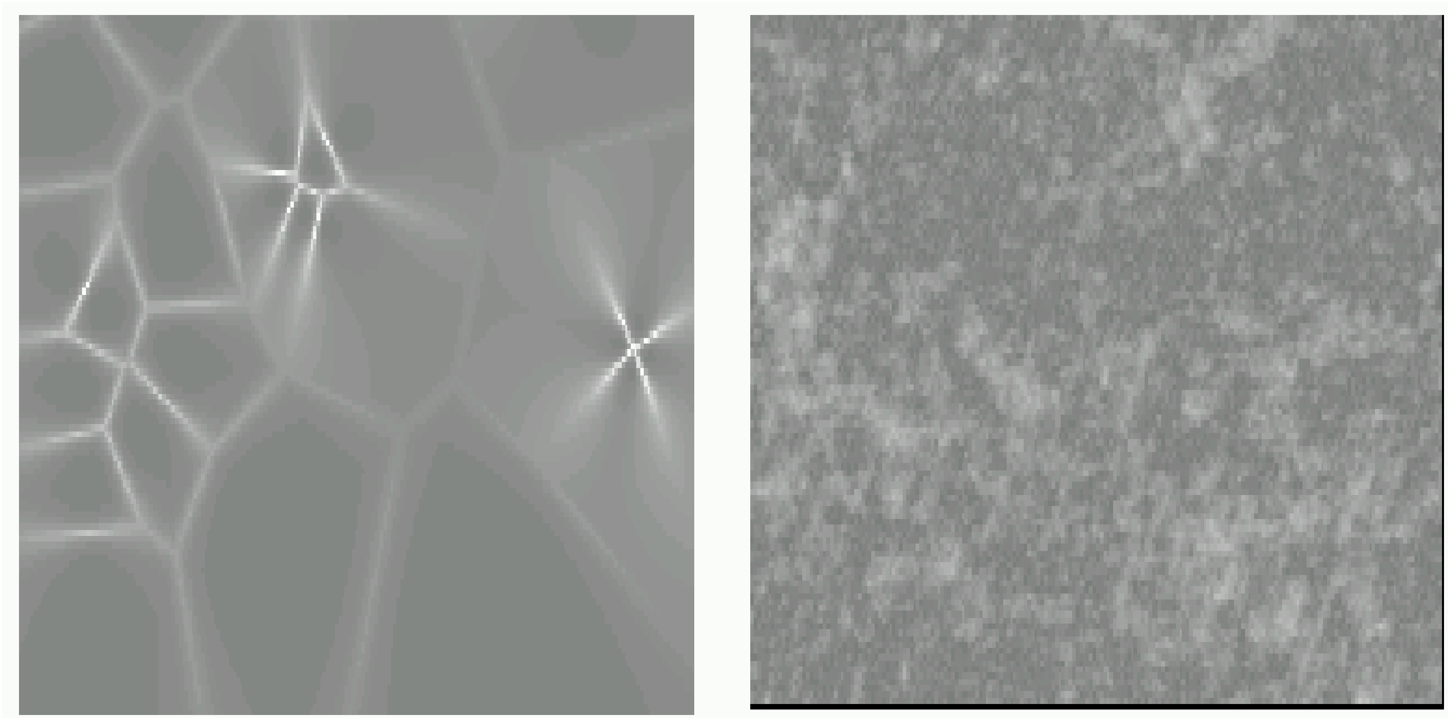
How to measure LSS: Characterizing structure

Since ξ(r) is independent of the r direction, the angular integrals can be calculated:

$$
\begin{aligned}
P_g(k) &= \int_0^{2\pi} d\phi \int_0^{\pi} \sin\theta d\theta \int_0^{\infty} dr r^2 \xi_g(r) e^{-ikr\cos\theta} \\
&= 2\pi \int_0^{\infty} dr r^2 \xi_g(r) \int_0^{\pi} d\theta \sin\theta e^{-ikr\cos\theta} \\
&= 2\pi \int_0^{\infty} dr r^2 \xi_g(r) \frac{1}{ikr} \int_{-ikr}^{ikr} dx e^{-x} \\
&= 4\pi \int_0^{\infty} dr r^2 \xi_g(r) \frac{\sin(kr)}{kr},
\end{aligned}
$$

How to measure LSS: Characterizing structure

Same 2pt, different 3pt functions



The correlation function (or the power spectrum) contains the full statistical information only for Gaussian distributions.

This is the 2-point correlation function.

Higher order statistics to obtain more information: 3, 4 … points correlations functions (instead of pairs, consider triangles, quadrangles…) → non-Gaussianity

How to measure LSS: Characterizing structure

To measure the correlation function, we need a catalog of objects (usually galaxies)

2 main kinds of catalogs:
Spectroscopic: Obtain the spectrum for a selected group of galaxies. This gives an accurate determination of the redshift, and allows to measure the full spatial distribution

photometric: Obtain images in different colors for all the objects. This gives a not so precise determination of the redshift (photometric redshift or photoz). Measure the angular (projected) distribution of galaxies for several redshift intervals.

We can define angular quantities that behave like the full spatial ones:
Angular correlation function (w(θ,z)), angular power spectrum C_l

$$\omega(\theta) = \int_0^\infty dz_1\,\phi(z_1) \int_0^\infty dz_2\,\phi(z_2)\,\xi(r;\bar{z})$$

$$\bar{z} = (z_1 + z_2)/2 \qquad r = \sqrt{\chi(z_1)^2 + \chi(z_2)^2 - 2\chi(z_1)\chi(z_2)\cos\theta}$$

$$\chi(z) = \frac{c}{H_0} \int_0^z \frac{dz}{\sqrt{\Omega_m(1+z)^3 + \Omega_\Lambda(1+z)^{3(1+w)}}}$$

How to measure LSS: Characterizing structure

Non-trivial to compare observation to theory

The observables, δ_{gal}, are complicated *functionals* of the easy-to-predict linear matter density field, δ_L.

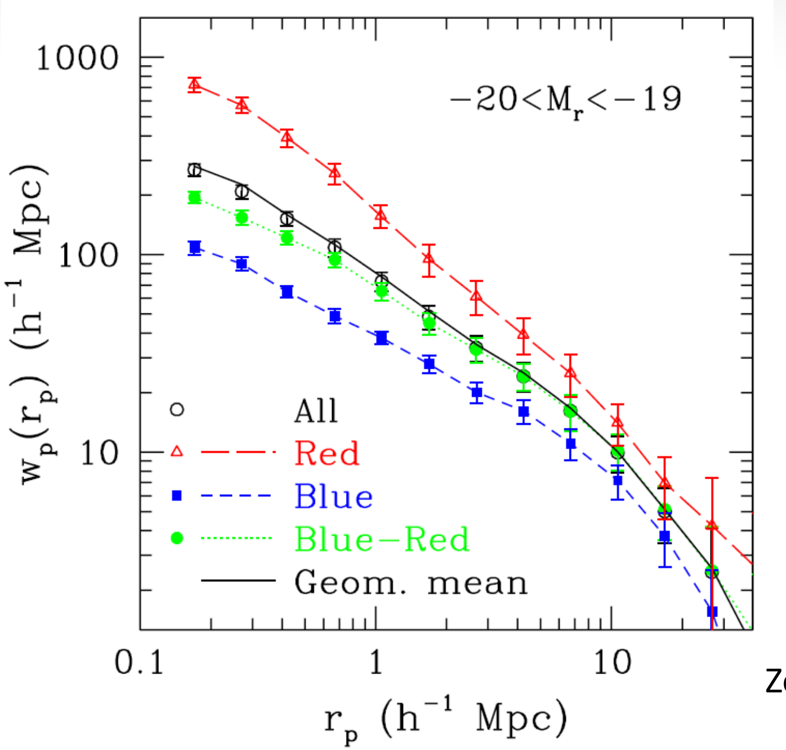
$$\delta_L \longrightarrow \delta_{\mathrm{NL}} \longrightarrow \delta_{\mathrm{gal}}$$

N-Body interactions in Newtonian gravity

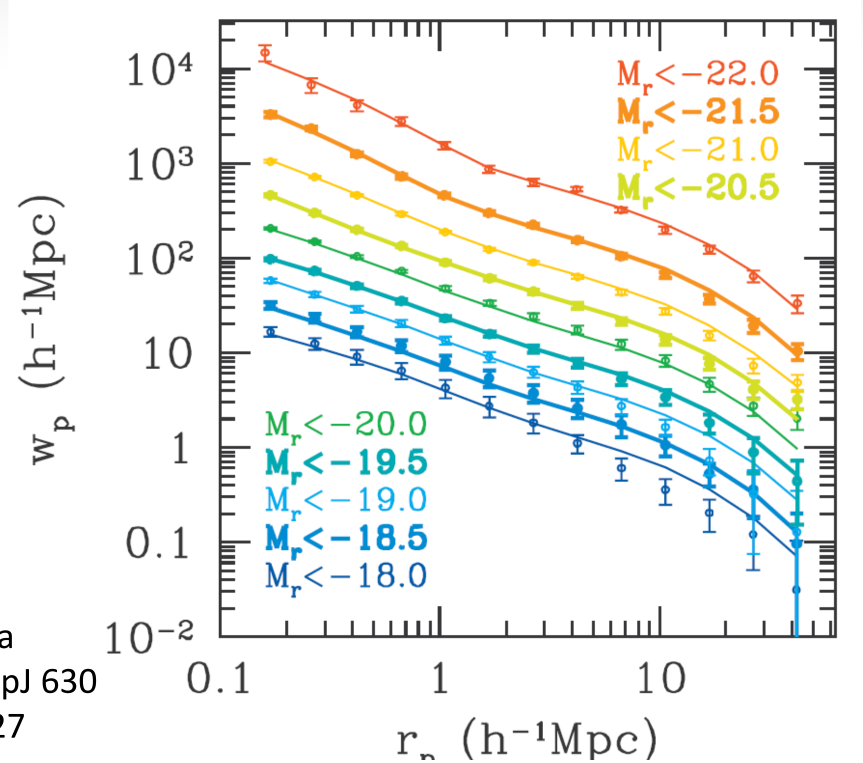
Galaxy formation including hydro, feedback from SN, star formation, …

How to measure LSS: Characterizing structure

Difficulties: Biasing. We observe galaxies, not dark matter.
How well do galaxies trace the underlying perturbations in the matter?



SDSS Data
Zehavi et al., ApJ 630 (2005) 1-27



Correlation function depends on galaxy properties: Brighter, more massive galaxies have a larger bias than fainter, lower mass galaxies

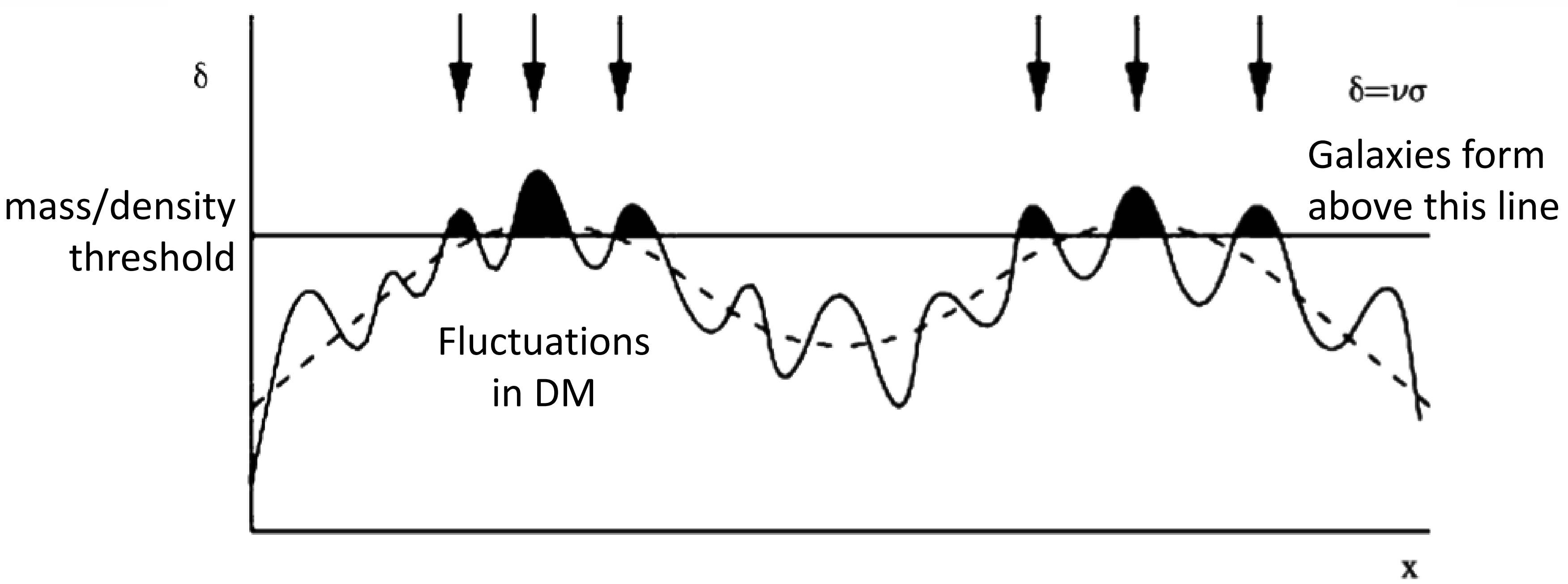
The different clustering properties of these galaxies tell us something about how they form

Use these dependencies in the data analysis to obtain information about bias and control systematic errors

How to measure LSS: Galaxy Bias

The galaxies we observe do not perfectly trace the underlying mass distribution in the universe (*i.e., light does not trace mass*)

Expect galaxies to be found preferentially in the most prominent high-mass peaks



How to measure LSS: Galaxy Bias

Express fluctuations in the number of observed galaxies in terms of fluctuations in the mass density times biasing factor:

$$\delta_{\text{Galaxies}} = b\ \delta_{\text{Matter}}$$

linear bias

(in general,more complicated)

In general, bias b >= 1

Bias depends on the properties of the selected Galaxy sample

How to measure LSS: Characterizing structure

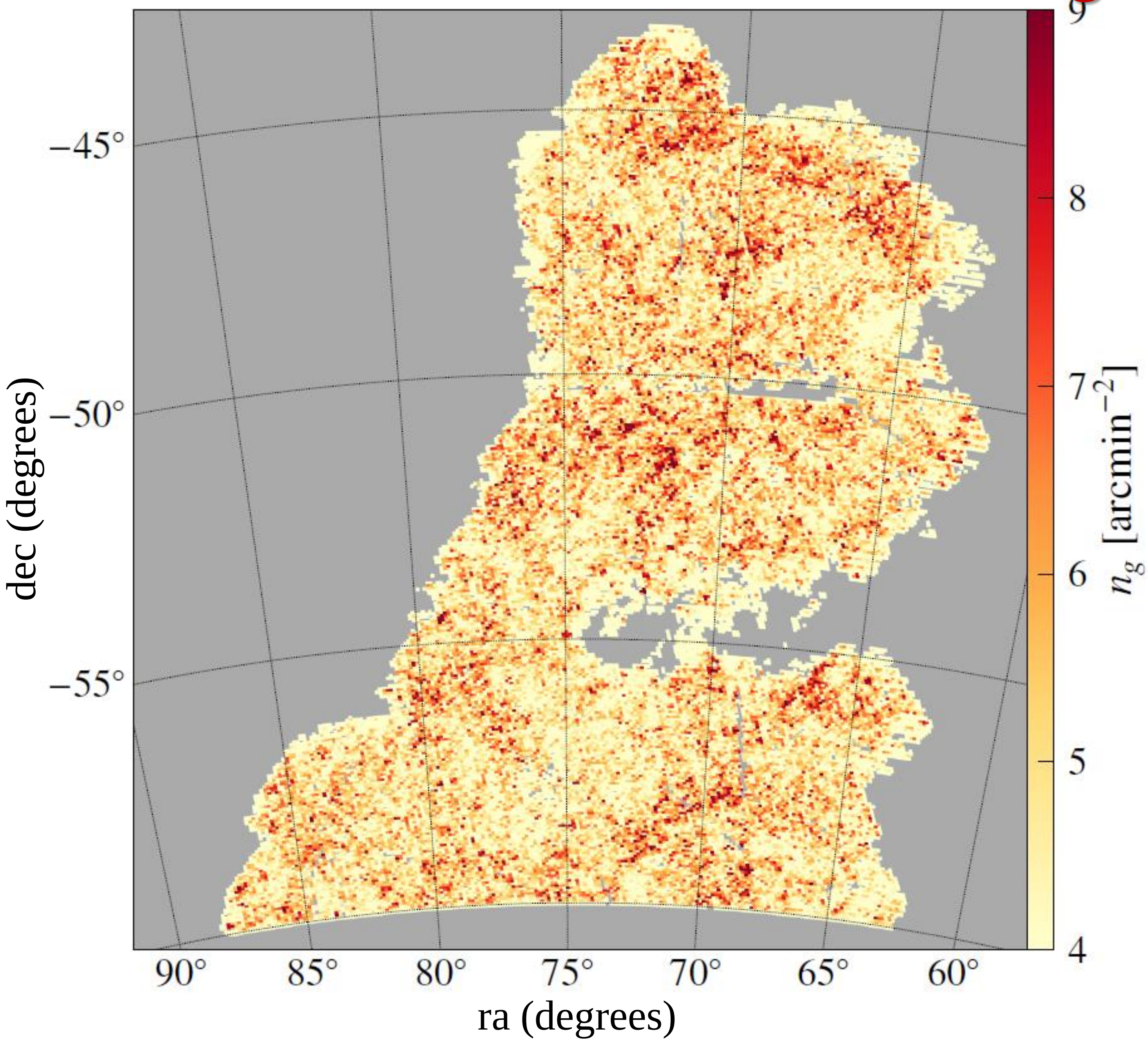
How do we compile these galaxy samples?

I. Obtaining multi-colour images of a large area of the sky

II. Create a catalogue and then select the sources over some range of brightness (and perhaps using some other criteria)

III. Measure redshifts for sources (to add third dimension)

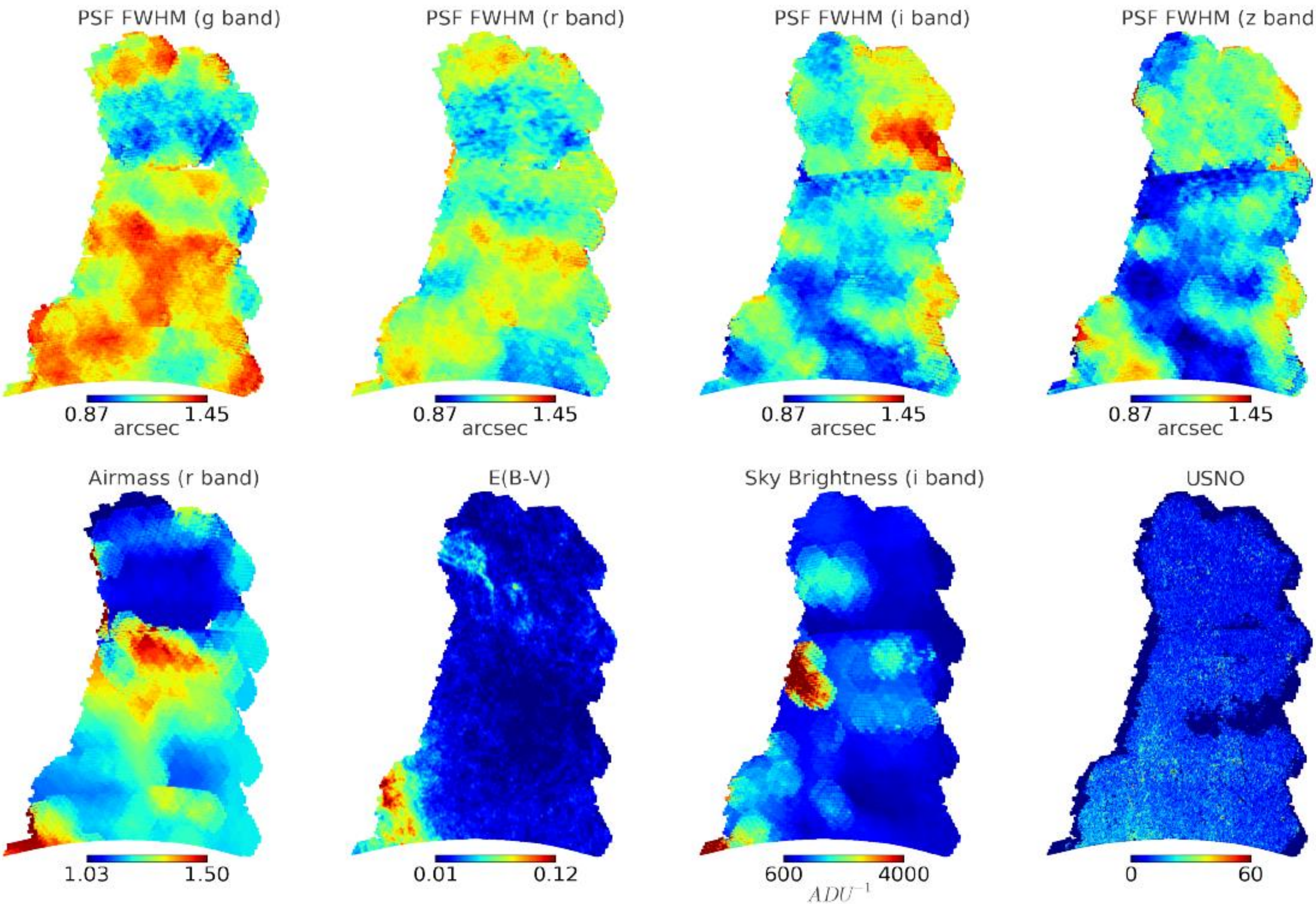
How to measure LSS: Characterizing structure



Example: Galaxy density map for DES Science Verification data (Benchmark Sample)

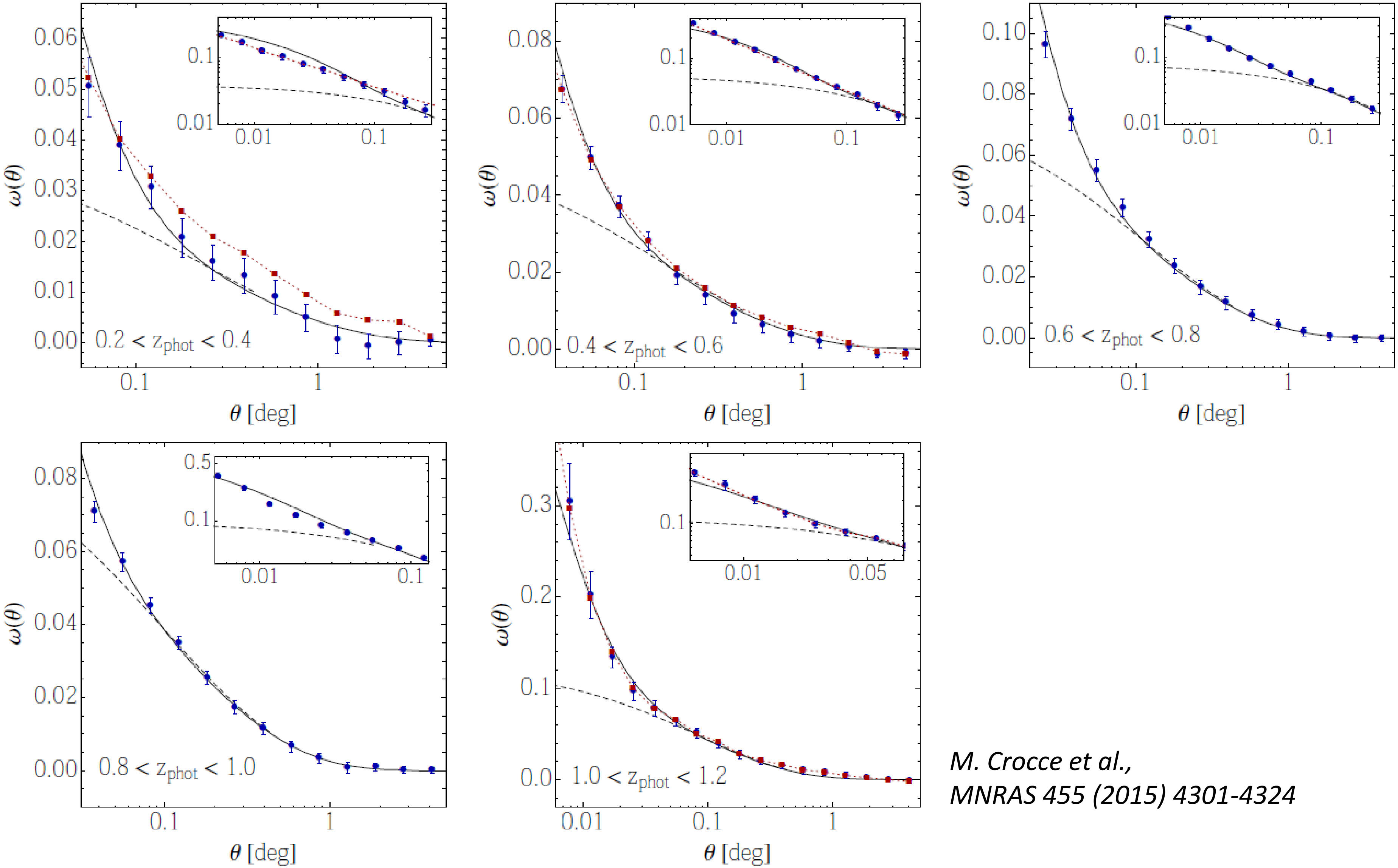
How to measure LSS: Characterizing structure

Survey conditions maps that can affect the clustering of galaxies



How to measure LSS: Characterizing structure

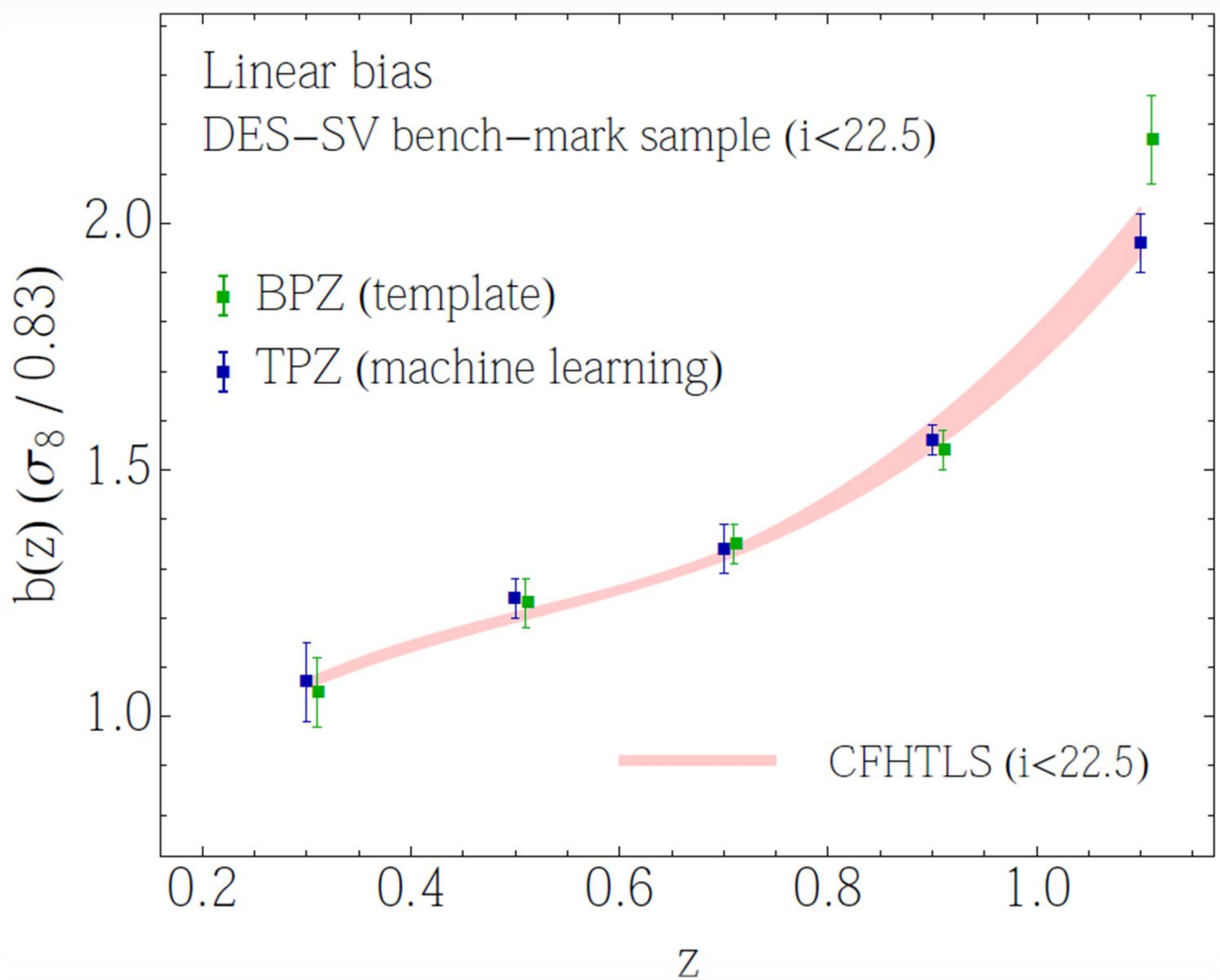
Measured correlation functions



M. Crocce et al.,
MNRAS 455 (2015) 4301-4324

How to measure LSS: Characterizing structure

From the previous analysis one can obtain the Galaxy bias and its evolution with the redshift



How to measure LSS: Other Techniques

Of course, there are other techniques as well for quantifying clustering:

Counts In Cells -- Divide the Space into Discrete Grid Points "Cells" and Calculate the Variation in the # of Sources per Grid Point

Void Probability Function -- Probability of Finding Zero Galaxies in a Volume of Radius R

Higher order statistics – 3pt correlation functions

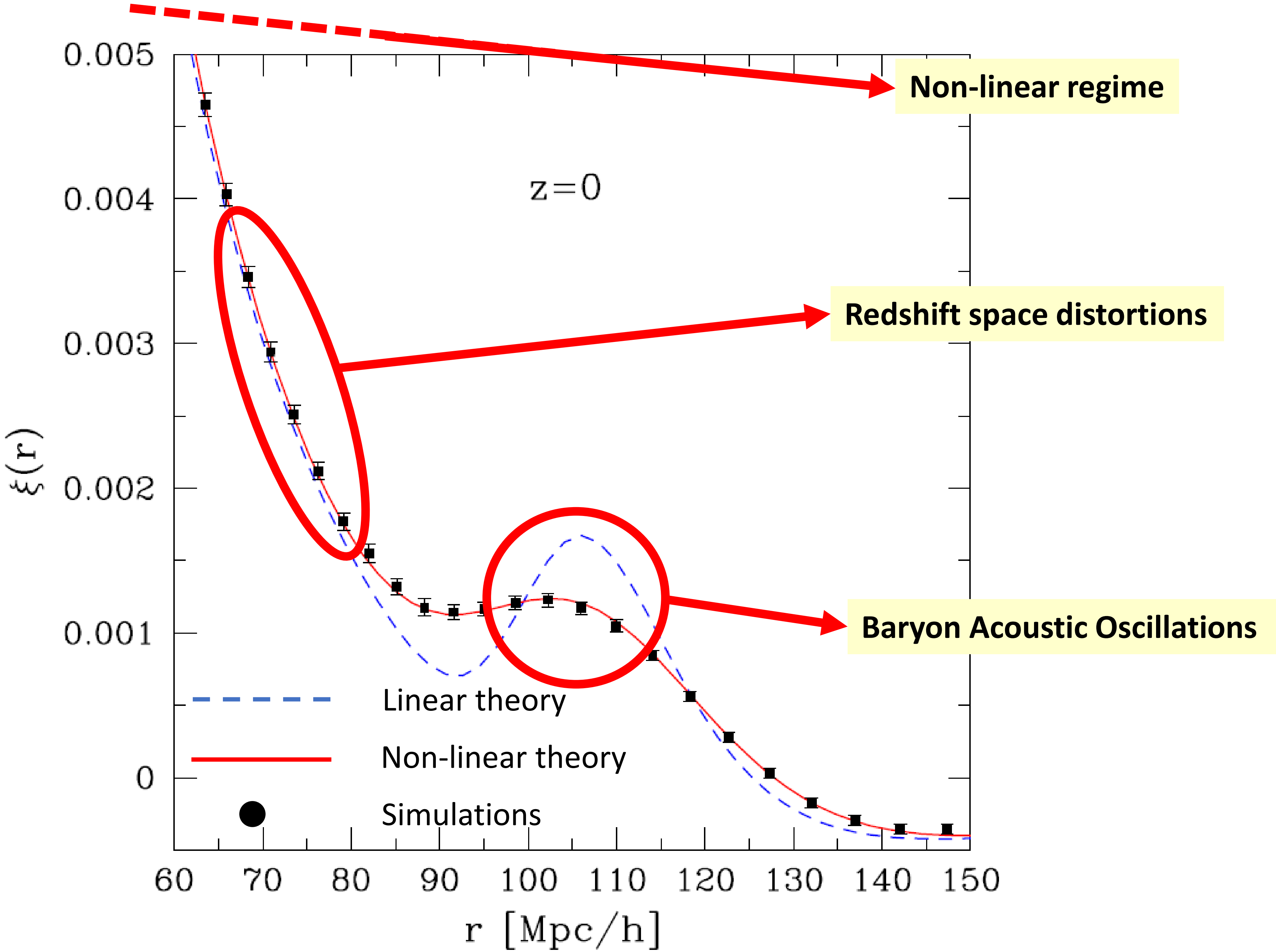
We will not describe them in this course

For next sessions: How to do cosmology with the correlation function

Baryon acoustic oscillations

Redshift space distortions

Other probes and combinations


Non-linear regime
Redshift space distortions
Baryon Acoustic Oscillations
z=0
ξ(r)
0.005
0.004
0.003
0.002
0.001
0
Linear theory
Non-linear theory
Simulations
60
70
80
90
100
110
120
130
140
150
r [Mpc/h]