The Linked Geometries of Reims Cathedral’s Nave Section and West Faade

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The west façade of Reims Cathedral ranks as one of the most admired creations of Gothic architectural culture. Crowds of tourists and legions of art history students have gained at least a superficial familiarity with this magnificent structure, whose richly sculpted portals, elegantly detailed buttress tabernacles, and daringly perforated towers combine harmoniously to create an overall effect of sophistication and grandeur. On a more scholarly level, researchers including Robert Branner, Hans Reinhardt, Peter Kurmann, and Alain Villes have fruitfully investigated the sources of the Reims façade design and its place in the larger history of the Reims workshop (Branner 1961; Reinhardt 1963; Kurmann 1987; Villes 2009). None of their studies, however, have carefully considered the geometrical logic of the façade design.

The present essay uses geometrical analysis to shed new light on the Reims façade, and on its relationship to the rest of the cathedral. In methodological terms, it builds on the author’s recent studies of geometry in Gothic architectural drawings (Bork 2011) and buildings (e.g. Bork 2012; Bork and Nussbaum 2014). The goal these projects has been to discover the sequences of geometrical operations that the Gothic designers used in each case to develop their schemes. Although the details of their mental process cannot be unambiguously reconstructed centuries after the fact, careful study of their drawn and built creations, combined with computer-aided geometrical modeling, permits the development and rigorous testing of hypotheses about the logic of their designs. In case after case, it has proven possible to use the progressive unfolding and stacking of regular polygons in the Vectorworks CAD system to create frameworks whose proportions match well with those of the drawings and buildings in question. In each of these cases, moreover, the dimensions the components in the geometrical model are locked together with each other in a single unbroken chain, preventing the kind of opportunistic fine-tuning that skeptics of geometrical research, most notably Konrad Hecht (1979), have criticized. As the weight of convergence evidence from these case studies grows, it becomes possible to gain insight not only into the design strategies used in individual workshops, but also into the relationships between them. Consideration of Reims Cathedral can be particularly rewarding in this regard because its workshop played such an important role in spreading architectural ideas from the heartland of France into Lotharingia and ultimately into the German-speaking world (Schurr 2007; Brachmann 2008). More immediately, consideration of the Reims façade project can reveal a great deal about how designers within a specific workshop engaged with and reacted to the work of their predecessors.

The designers of the Reims west façade, working in the middle of the thirteenth century, clearly took as their point of departure the geometrical givens that had already been established several decades earlier in the construction of the nave. As igure 1 shows, the cross-section of Reims Cathedral was governed by the figure of a large octagon. The bottom facet of this great octagon coincides with the floor of the main nave vessel, which measures 14.65m between the axes of the arcade piers, or 7.32m between the building centerline and the pier axes. The equator of this great octagon lies at height 17.68m, precisely aligned with the top of the arcade zone and the baseline of the triforiumhe lower lateral corners of the octagon fall at height 10.36m, the level at which the capitals spring from the wall piers in the aisles. The top of the octagon rises to height 35.36m, which at first seems to match nothing of significance in the elevation. As Henri Deneux recognized in the middle of the twentieth century, however, the vaults of Reims Cathedral were originally meant to be somewhat shorter than they are today (1948, 126–28). If one continues the curvature of the original vault springers examined by Deneux, as shown in the left-hand portion of fig. 1, then one finds that the originally planned vaults would have reached a height of almost exactly 35.36m, coinciding with the top of the great octagon. It thus seems that the cathedral's arcade was meant to have exactly half the height of the originally planned main vessel. thus the-century draftsman Villard de Honnecourt showed the Reims nave with aisles very nearly half as high as the main vessel on folio 31v of his portfolio (Barnes 2009, 198–200; Villes 2009, 507–36). Together, this evidence suggests that Villard may here have been copying an original design drawing.

Further details of the Reims nave cross-section can be found by constructing octagons concentric with the original one. or example, a small octagon of this type framed by the arcade axes fills the space between heights 10.36m and 25.00m. Its upper lateral corners fall at height 20.71m, the level at which the small columns in the triforium terminate. The midpoints of the small octagon's upper diagonal facets fall at height 22.86m, which marks the top edge of the triforium. When these diagonal facets are unfolded, they reach up to height 26.78m, the level from which the main vaults spring from the tops of their capitals. The geometrical center of the originally planned vault curvature, moreover, is a point at this level located directly above the corner of the small octagon; this point is picked out by a small open circle in fig. 1. The radius of curvature in the originally planned vault corresponds to the distance between this center point and the midpoint of the great octagon's upper facet, as the sloping line between these two points

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The section of the Reims nave, like many other Gothic designs, incorporates geometrical relationships based on the inscribing of octagons and circles within each other. It is convenient to call this practice "octature," by analogy to the better-known practice of "quadrature," or square rotation, which is familiar to many students of Gothic architecture from medieval documents such as the portfolio of Villard de Honnecourt and th-century pinnacle design booklet of Mathias Roriczer (Barnes 2009; Shelby 1977). By repeated use of this octature scheme, it is possible to construct sequences of progressively smaller octagons nested within each other, just as one can do with squares using quadrature (Bork 2011, 25–26).

The shading in ig 1 shows how this system of nested octagons was used to help establish the cross-section of the Reims Cathedral nave. The red shading shows the circle circumscribed around the original great octagon, whose lower facet corresponds to the floor of the main nave vessel. As noted previously, the lower lateral corners of this great octagon fall at height 10.36m, aligning with the bottom edges of the capitals on the wall piers in the aisles. When a larger octagon is circumscribed about this one octature, its lower lateral corners fall at height 9.75m, aligning with the bottom edges of the main arcade piers. As arcs on the outer margins of fig. 1 show, the next octature step outwards from the red circle gives the full width of the cathedral's crosssection, measured across its outer buttress surfaces at ground level. Moving inwards, the first set of arcs within the original shows that the next smaller octagon will have corners at height 10.91, aligning with the top of the aisle capitals; in the vertical dimension, meanwhile, these same corner points align precisely with the plane of the aisle windows. Moving inwards by two more octature steps, one finds the corner points at height 12.21m, the level marking the top edge of the diagonal setbacks on the nave buttresses; these same corners align in the vertical dimension with the inner faces of the engaged wall piers. It is clear, therefore, that a system of nested octagons governed the section of the Reims nave in both the horizontal and vertical dimensions, at least in the zone up to the base of the triforium.

Extensions of this octagon-based system appear to have governed the design of the upper nave zone as well, although the modification of the vault heights seems to have introduced some slight inconsistencies into the system. or example, the present vaults rise almost exactly to 36.82m height of the red circle around the original great octagon. In fact, they rise higher because their curvature was set by a simple construction based on the equilateral triangle; the distance from the springing of the vault to its vertex equals the free span between the springers. The interval between the great octagon and its circumscribing circle nevertheless seems to have played a role in the layout of the upper nave since a doubling of this interval above the octagon reaches height 38.27m, where a molding on the outer wall marks the top of the main vessel proper a quadrupling of the interval reaches height 41.19m, a level that coincides with the top level of the stone wall structure and with the tips of the pinnacles on the outer pier buttresses. This height also corresponds quite closely with the width of the structure measured at ground level, suggesting that the whole composition was meant to fit neatly within a square.

Even the roof proportions Reims Cathedral seem to have been determined by this basic geometrical framework. The shape of the original roof can still be read in the west façade gable, despite the destruction of the original roof in a fir and its late medieval replacement in WWI.The height of the roofseemsto have been roughly 57m, which is to say that it was greater than the side length of the 35.36m square by a factor very closely approximating the so-called Golden Section, . In modern mathematical terms, satisfies the equation =1/ (-1) and has value (1+√5)/2 = 1.618…. In the geometrical terms with which a medieval designer would have been more familiar, can be found by unfolding the half-diagonal of the square, as the green arc in fig. 1 shows. It is not yet clear whether designers in other workshops used this strategy to set the height of their roof structures, but smaller-scale constructions based on the Golden ection were evidently employed in the thirteenth century at Amiens and Salisbury athedrals, so this possibility deserves further investigation.

It would have been very surprising if the geometry of the Reims nave design had been forgotten or ignored by the designers of the cathedral's west façade. In general terms, the Reims workshop was marked by a high degree of continuity, which helps to explain the cathedral striking formal coherence. This continuity can be seen, for example, in the builders' consistent use of six-petal rosettes in the aisle and clerestory tracery, and in the way that pinnacled tabernacles wrap consistently around all of the cathedral's major buttresses, from the choir and transept faades through the nave and west façade. This is not to say that the cathedral's design was entirely uniform, however. The transept faades differ from other, for example, and both are far simpler than the west façade. As Kurmann (1987) and Villes (2009) have noted, moreover, the lower lateral buttresses of the façade block are both simpler and more massive than those of the western face, and they step back dramatically at the level of the triforium, suggesting that the west façade design was modified in the course of the construction process. Geometrical analysis helps to clarify the relationship between the current Reims façade and the older parts of the building.

The designers of the current Reims façade clearly paid very close attention to the geometry of the cathedral's nave, even as they developed new permutations on its formal and geometrical logic. The most obvious and fundamental geometrical link between the two designs was their decision to place the axes of the main façade buttresses 7.32m from the building centerline, aligned with the axes of the nave piers. As fig. 2 shows, a red square framed by these buttress axes rises to height 14.65m, the level from which the sloping sides of the portal gables depart. The diagonals of this square can be unfolded as red arcs, which meet the ground 13.39m away from the building centerline, defining the outer edges of the lateral portals. These same arcs rise to height 20.71m, a level already familiar from the nave section, where it marked the top of the small columns in the triforium. In the façade, similarly, this level marks the top of the small shafts just beneath the triads of gablets on the main buttresses. Orange diagonals drawn up from the trumeau base intersect this level 20.71m out from the building centerline; verticals dropped from these intersection points define the outer margins of the forward-facing façade buttresses. The outer margins of the lateral buttresses, , similarly, stand 22.86m out from the building centerline; this dimension equals the height from the nave floor to the top of the triforium, as already established in the nave section. The lower section of the façade thus fits into a pair of horizontallyoriented double squares, the inner rising to 20.71m, and the outer to 22.86m.

The geometry of the lower portal zone unfolds naturally from the generating square framed by the inner buttress axes. The top of the lintel over the doors falls at height 7.32ma height equal to half the nave width. The centerlines of the lateral portals can be found 12.06m out from the building centerline, where the previously described red arcs through the corners of the generating square cut the horizontal at height 7.32m; these portal centerlines are yellow in fig. 2. The lower edge of the lintel is at height 6.07m, as a diagonal struck back from the doorjamb to the buttress axis shows13.39m – 7.32m = 6.07m. The outer edge of the side portals can be found by symmetry 16.79m from the building centerline, as the green arcs around the portal centerlines show. The gables over the three central portals all have60-degree slope he center portal, therefore, has the proportions of an equilateral triangle placed atop a square. The two smaller gables on the framing buttresses have steeper pitch since they fit into the available space between heights 14.65 and 20.71m, and between the widths 16.79m and 20.71m from the building centerline. Lines with 60-degree slope departing from the base of the central trumeau cut the inner buttress axes at height 12.69m, locating the level of the gargoyles on the edge of the façade; these lines continue upward to define the sculpted inner fields of the aisle gables.[1] The 60-degree lines departing from the trumeau base also cut the bottom of the lintels at points 3.50m from the building centerline; this defines width of the main portals and the radius of the rose window in the glazed tympanum immediately above. The center of the rose thus falls at height 10.82m, which is 3.50m higher than the 7.32m to the top of the lintel.

Above the portal zone, the geometry of the façade undergoes a series of subtle but important shifts, with the triforium serving as a kind of geometrical fulcrum between the two

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Their intersections with the vertical portal centerline are about 16cm higher than the horizontal at height 20.71m, but the mathematical discrepancy might not have been evident to a medieval draftsman; this is certainly true for the upper gable vertices, which are 2cm below the 22.86m height to the top of the triforium.

zones. In the façade as in the nave, the triforium occupies a belt between heights 17.68 and 22.68m. The first of these levels corresponds to the equator of the great red generating octagon seen in , whose lower facet corresponds to the floor of the nave vessel. The second, as in the nave, aligns with the center points in the upper diagonal facets of an octagon concentric with thred starting octagon, and framed by the main red arcade axes. The intersection of this 22.86m level with the vertical centerline of the façade marks the precise location where Christ crowns Mary in the central gable; at Reims, therefore, the iconographical focus of the sculptural program coincides with the geometrical focus of the upper façade.

The main generating figure for the upper façade is a great octagon concentric with the Coronation of the Virgin. Since the center of this octagon lies at height 22.86m, its top facet reaches 45.73m and its upper lateral corners at height 32.33m, defining the top of the buttress tabernacles and the bases of their crowning pinnacles. A large orange circle circumscribed about this new octagon has radius 24.75m, which equals the width between the building centerline and the outer face of the lateral façade buttress measured at ground level. The tops of the gables in the tower bays and the tip of the framing arch over the rose window lie at height 40.36, the level where orange diagonals emanating from the Coronation cut this large circle. Yellow verticals through these intersection points, which lie 17.50m from the building centerline, define the axes of the outer buttress tabernacles. These tabernacles, in other words, stand frther outboard than the buttress axes at ground level, which were only 16.79m from the centerline. The inner buttress tabernacles, similarly, are displaced slightly outwards compared to the buttress axes at ground level; they lie 7.63m instead of 7.32m out from the building centerline. The tabernacle axes can be found by striking verticals through the centers of the small circles at height 17.68m, where they fit between the faces of the green octagon and its yellow circumscribing circle. These inner tabernacle axes cut the large orange circle at height 46.41m, the level where the kings stand in their gallery, which also marks where the tabernacle pinnacles terminate. The outboard faces of the inner tabernacles are located by verticals from the points where the large red circle cuts the rays to the octagon corners at height 43.99m. The top edge of the balustrade beneath the arcade of kings also lies at this level. A green circle concentric with the coronation and rising to this same 43.99 level will cut the red rays of the octagon at height 42.38m, locating the base of the foliate stringcourse that terminates the main body of the façade.

An important geometrical baseline for the façade superstructure lies at height 43.18m, where the verticals begin in the gallery of kings; this is halfway between the previously established heights 42.38m and 43.99m. In the vertical dimension the gallery reaches to height 53.06m, a level that can be found by striking diagonals upward from the 43.18m baseline between the yellow uprights of the tabernacle axes. The width of the geometrical framework governing the tower composition, meanwhile, can be found by striking diagonals up from the Coronation to the gallery baseline at height 43.18m; the intersection points will be located 20.32m away from the building centerline. The blue vertical rising through this intersection point defines the outer edge of the final gabled niche in the gallery of kings. The center axis of the tower bay, meanwhile, will be 12.56m from the centerline,halfway between the yellow axes 7.63m and 17.50m from the centerline 12.56= (7.63+17.50)/2= 25.13/2. Since the central axis is 12.56m from the centerlineand the outer face of the tower composition 20.32m from the centerline, at the level of the gallery of kings, the inner face of the tower framework must be 4.81m from the centerline.

To make sense of the tower elevation, it helps to consider the plan of the towers in the horizontal plane. At the top right of ig 2, therefore, a plan of the towers has been aligned with the elevation, while a more detailed plan of this zone appears in the bottom half of ig 3. Careful examination of these plan reveals that the central axes of the slender octagonal turrets flanking the tower do not coincide with the centerlines of the buttress tabernacles below; instead, they are stepped subtly in towards the building centerline. This is because the tower plan has its own logic, as the schematic at the top left of fig. 2 shows. The basic proportions of the tower plan develop within a square framed by verticals 4.81 and 20.32m from the building centerline. Within this square, rotated squares and octagons can readily be inscribed, in accord with the principle of quadrature. The diameter of the unshaded central octagon, the span across the blueshaded strips, and the span between the blueframing uprights thus fall in a 1:√2:2 ratio. The centerlines of the shaded strips cross at the centerpoints of the small satellite octagons describing the plans of the turrets, and the facets of the turrets have the same width as the shaded strips. As the top half of ig 3 shows, this simple and elegant geometrical scheme precisely matches that of the Laon tower plan drawn by Villard de Honnecourt, a relationship that attests to the close ties between the Laon and Reims cathedral workshops in the thirteenth century (Bork 2011, 41).

On the basis of these results, the tower elevation can be readily determined. In fig. 2, violet verticals descending from the satellite octagons describe the slender corner mullions of the turrets.The tower zone proper starts at height 53.06m, the previously established level where the arches end in the gallery of kings. An “X” of violet diagonals based on this level and framed by the outer violet margins of the turrets will rise to height 66.30m, which can become the center of an octagonal composition framed by the blue uprights. The upper rays to the corners of the octagon will cut the turret margins at height 69.04m, the level of the capitals in the turrets. The midpoint in the upper diagonal facets of the octagon will fall at height 71.78m, where the gablets atop the turrets begin, and the top of the octagon will be at height 74.05m, where the gablets terminate. The diagonal facets of the octagon can be extended to a convergence point at height 77.27m, where decorative collars separated the main gables from the finials above them. Extensions of this system would likely have governed the proportions of the spires and pinnacles originally planned to rise above this level, but too little survives of their incomplete lower courses to permit detailed analysis of their intended geometry.

With these geometrical analyses of Reims Cathedral's nave section and west façade now in hand, it becomes possible to evaluate the history and impact of the Reims workshop in new ways. The first of these analyses has shown, for example, that the overall nave section was established by a neatly interlocking set of nested octagons, inscribed within one another in accord with the system here called octature. The originally planned height of the high vaults, as documented by Deneux's careful investigation of their springers (1948, 126–28), would have matched the height of a great octagon whose lower facet coincided with the nave floor, measured between the arcade axes. In this original scheme, the arcades would have risen to half the total height of the building, as Villard de Honnecourt showed in his elevation drawings of the nave. The impact of this original octagon-based scheme can be seen in many important churches in France and the Holy Roman Empire, including the Liebfrauenkirche in Trier, the Cistercian church of Altenberg, and the cathedrals of Clermont-Ferrand, Cologne, Prague, and Aachen (Bork 2011; Bork 2012; Bork and Nussbaum 2014; Bork forthcoming). Closer to home, similar octagon-based geometries also defined the proportions of the façade designs recorded in the so-called Reims Palimpsest, and extensions of this geometrical system also governed the spectacular west façade of Reims Cathedral itself. Even the large and simple lateral buttresses of the façade block, whose format may have been established early in the façade design process, have proportions governed by octagons extrapolated from this system. The rest of the façade block, with its more elaborate articulation, may well have been designed somewhat later, as Kurmann (1987) and Villes (2009, 443–57) have proposed, but even these portions have an octagon-based geometry that relates intimately to the nave section, testifying to the strong continuity of method and intention within the Reims workshop. The intimate geometrical relationship between the west façade and its towers, moreover, strongly suggests that the towers were designed together with the façade in the mid-thirteenth century, even though their construction dragged on into the fifteenth century. The close geometrical links between these towers and the Laon tower plan drawn by Villard de Honnecourt lends further support to this early dating of the Reims tower design. So, while the full implications of this analysis for the interpretation of Reims Cathedral remain to be worked out, the preliminary results presented begin to suggest the potential of this geometrical approach.

Works Cited

Barnes, Carl F., Jr. 2009. *The Portfolio of Villard de Honnecourt.* Farnham: Ashgate.

Bork, Robert. 2011. *The Geometry of Creation*. Farnham: Ashgate.

———. 2012. "Neue Überlegungen zur Geometrie des Chores, in Norbert Nussbaum and Sabine Lepsky." *Gotische Konstruktion und Baupraxis an der Zisterzienserkirche Altenberg* 2. Bergisch Gladbach; Altenberg Dom-Verein: 75-88.———. Forthcoming. "Geometrie, Proportionen, und Vermessungen in der Liebfrauenkirche," In *Liebfrauen in Trier – Ein Schlüsselbau der europäischen Gotik,* edited by Stefan Heinz. Trier: Universität Trier.

Bork, Robert, and Norbert Nussbaum. 2014. "Gotischer Baubetrieb am Aachener Münsterchor n *Sie glänzte wie ein kostbarer Edelstein, wie ein kristallklarer Jaspis. 600 Jahre Aachener Chorhalle,* 22–42. Karl-Dombauverein Aachen: Dombauverein

Brachmann, Christoph. 2008. *Um 1300. Vorparlerische Architektur im Elsass, in Lothringen und Südwestdeutschland.* Korb: Didymos-Verlag.

Branner, Robert. 1961. "The North Transept and the First West Facades of Reims Cathedral." *Zeitschrift für Kunstgeschichte* 24: 220–41.

Deneux, Henri. 1948. "Des modifcations apportées à la Cathédrale De Reims au Cours de sa construction du Xiiie au xve Siècle. *Bulletin Monumental* 106: 121–40.

Hecht, Konrad. 1979. *Maß und zahl in der gotischen baukunst*. Hildesheim: Olms.

Kidson, Peter. 1993. "The Historical Circumstance and the Principles of the Design." In *Salisbury Cathedral: Perspectives on the architectural history*, edited by Thomas Cocke and Peter Kidson, 31–97. London: Royal Commission on Historical Monuments.

Kurmann, Peter. 1987. *La façade de la Cathédrale de Reims: architecture et sculpture des portails: étude archéologique et stylistique*. Paris: Payot-CNRS.

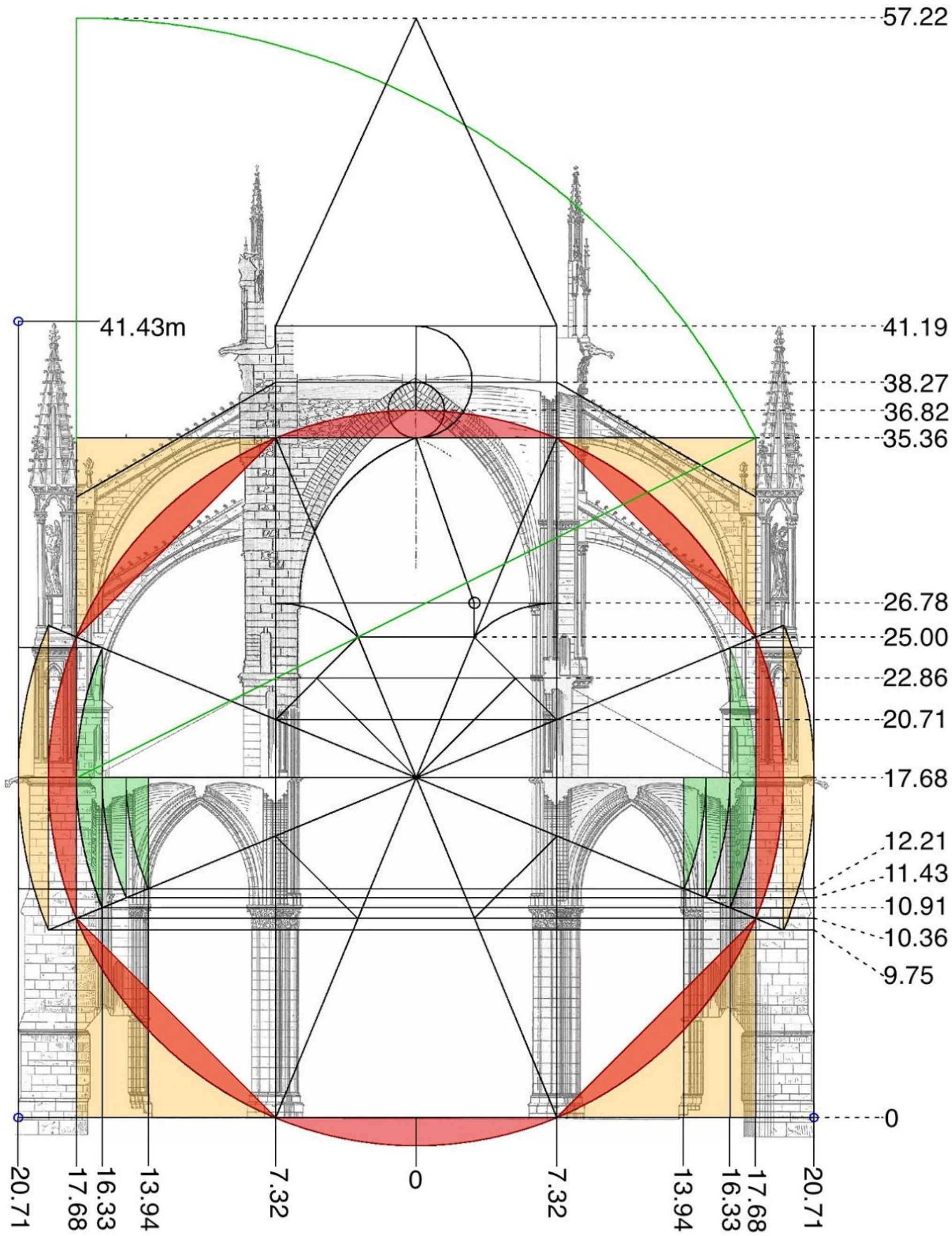
Murray, Stephen, and James Addiss. 1990. "Plan and Space at Amiens Cathedral: With a new plan drawn by James Addiss." *Journal of the Society of Architectural Historians* 49: 44–66.

Reinhardt, Hans. 1963. *La cathédrale de Reims*. Paris: Presses universitaires de France.

Schurr, Marc. 2007. *Gotische architektur im mittleren Europa 1220–1340: von Metz bis Wien*. Berlin: Deutscher Kunstverlag.

Shelby, Lon. 1977. *Gothic Design Techniques: The fifteenth-century design booklets of Mathes Roriczer and Hans Schmuttermayer*. Carbondale: Southern Illinois University Press.

Villes, Alain. 2009. *La Cathédrale Notre Dame de Reims: chronologie et campagnes de travaux.* Joue les Tours: La Simarre.


57.22
41.43m
41.19
38.27
36.82
35.36
26.78
25.00
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10.91
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9.75
0
20.71
17.68
16.33
13.94
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0
7.32
13.94
16.33
17.68
20.71

Figure 1. Reims Cathedral, geometry of the nave cross-section (schematics by the author, overlaid on section from Georg Dehio and Gustav von Bezold, *Die Kirchliche Baukunst des Abendlandes*, Stuttgart, 1901).

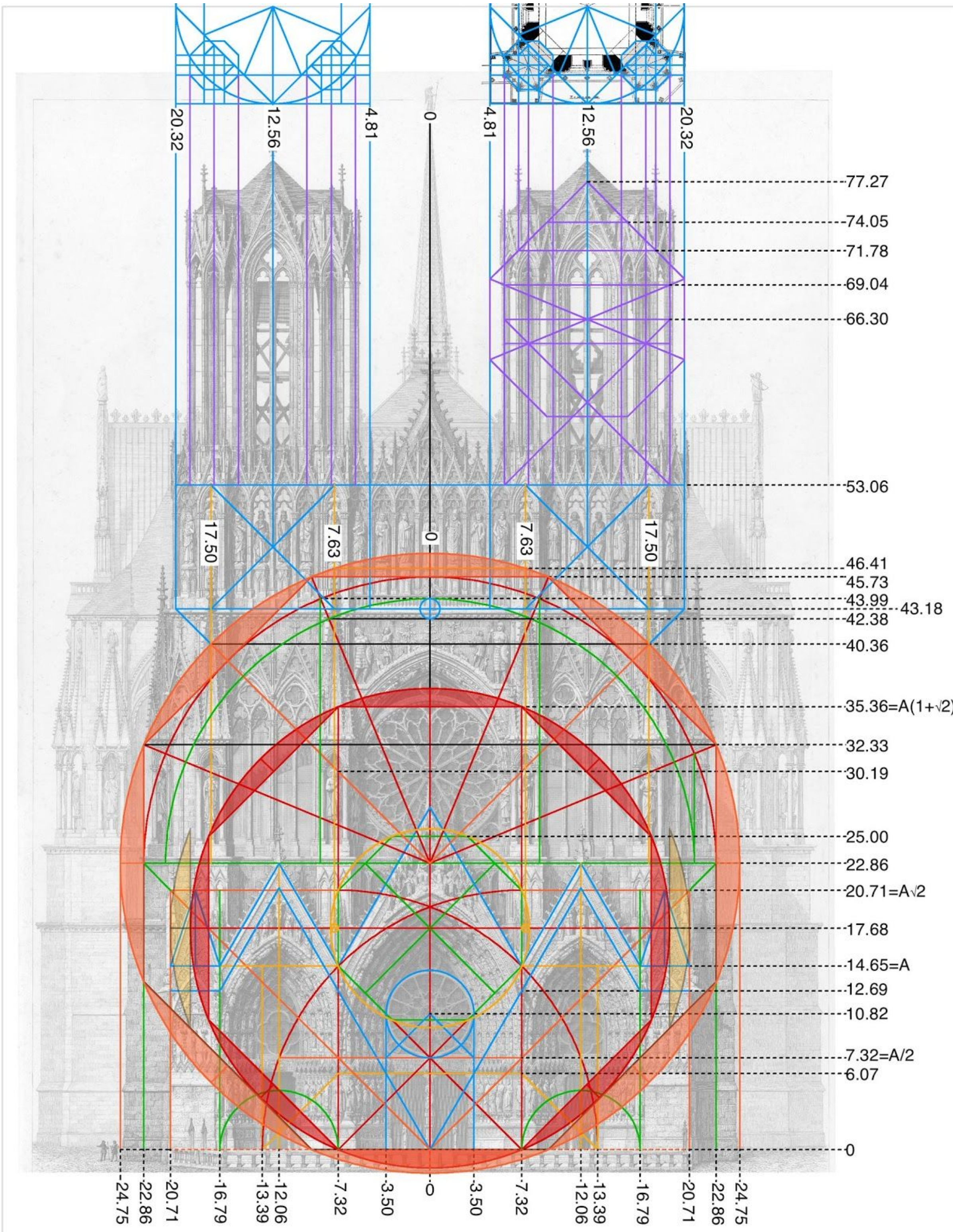

Figure 2. Reims Cathedral, geometry of the west façade (schematics by the author, overlaid on

drawing by Eugène Leblan, engraved by Ribault and Sulpis 1858).

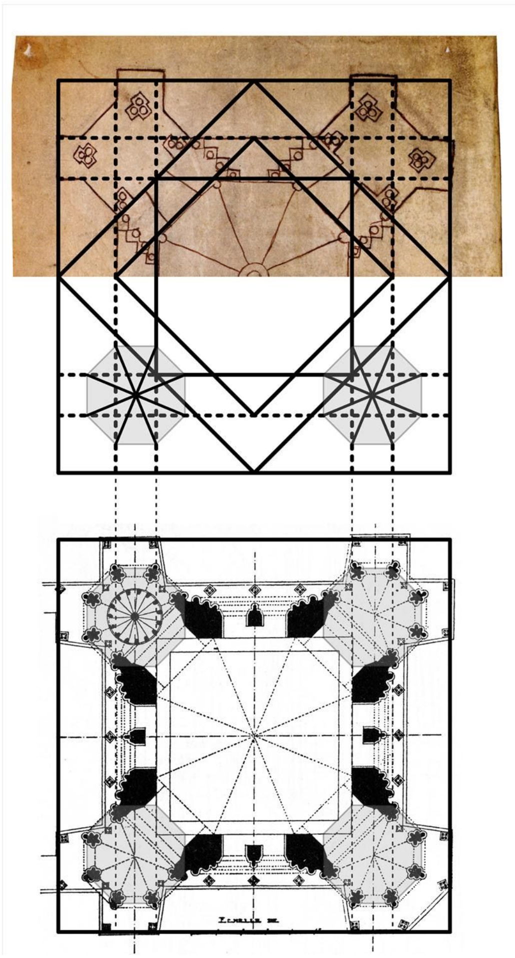

Figure 3. Comparison between plan geometries of west towers at Laon and Reims Cathedrals (schematics by the author, overlaid on folio 9v from Villard de Honnecourt's portfolio, Paris, Bibliothèque nationale de France, MS Fr 19093, above; and on fig. 74 from the article "Clocher," in Eugène Emmanuel Viollet-le-Duc, *Dictionnaire raisonné de l'architecture française*, Paris, 1858-68, below).