The Lone-Chooser Method Lecture 16 Section 3.4

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Wed, Oct 3, 2018

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The Lone-Chooser Method

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Outline

The Lone-Chooser Method

2 Example – 3 Players

3 Example – 4 Players

Assignment

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Definition (The Lone-Chooser Method)

In the lone-chooser method,

- One player is designated to be the chooser.
- The other players are the dividers. They divide the assets among themselves (details to follow).
- Then each of the dividers divides his share into equal subshares.
- The chooser then chooses one subshare from each of the dividers.
- The dividers keep the subshares that are left.

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- This is normally done in a recursive manner.
- For example, if there are 4 players A, B, C, and D.

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 - A starts off with all the assets.

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 - A starts off with all the assets.
 - Then A divides them into 2 equal shares. B chooses one of them.

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- For example, if there are 4 players A, B, C, and D.
 - A starts off with all the assets.
 - Then A divides them into 2 equal shares. B chooses one of them.
 - Then *A* and *B* each divide their shares into 3 equal subshares. Player *C* chooses one subshare from each.

- This is normally done in a recursive manner.
- For example, if there are 4 players A, B, C, and D.
 - A starts off with all the assets.
 - Then A divides them into 2 equal shares. B chooses one of them.
 - Then *A* and *B* each divide their shares into 3 equal subshares. Player *C* chooses one subshare from each.
 - Then *A*, *B*, and *C* each divide their shares into 4 equal subshares. Player *D* chooses one subshare from each.

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A divides them into 2 equal shares

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B C D B chooses one of the shares

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A B C D A and B divide their assets into 3 equal subshares

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ABCDA, B, and C divide their assets into 4 equal subshares









3 Example – 4 Players

4 Assignment

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Example (The Lone-Chooser Method – 3 Players)

- Andy, Bob, and Chuck are dividing 4 pies: apple, cherry, lemon, and pecan.
- The value systems of the players are as follows.

	Apple	Cherry	Lemon	Pecan
Andy	12	6	10	8
Bob	7	2	8	4
Chuck	6	4	2	6

- Chuck is the lone-chooser.
- But in the meantime, Andy and Bob divide the assets between the two of them, with Andy the divider and Bob the lone chooser.

	Apple	Cherry	Lemon	Pecan
Andy	12	6	10	8
Bob	7	2	8	4
Chuck	6	4	2	6

• How should Andy divide all the assets into two equal shares?

	Apple	Cherry	Lemon	Pecan
Andy	12	6	10	8
Bob	7	2	8	4
Chuck	6	4	2	6

 How should Andy divide all the assets into two equal shares? S₁: A + C (\$18.00); S₂: L + P (\$18.00)

	Apple	Cherry	Lemon	Pecan
Andy	12	6	10	8
Bob	7	2	8	4
Chuck	6	4	2	6

• How should Andy divide all the assets into two equal shares? S_1 : A + C (\$18.00); S_2 : L + P (\$18.00)

• Which share should Bob choose?

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	Apple	Cherry	Lemon	Pecan
Andy	12	6	10	8
Bob	7	2	8	4
Chuck	6	4	2	6

 How should Andy divide all the assets into two equal shares? S₁: A + C (\$18.00); S₂: L + P (\$18.00)

Which share should Bob choose? S₂

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	Apple	Cherry	Lemon	Pecan
Andy	12	6	10	8
Bob	7	2	8	4
Chuck	6	4	2	6

- How should Andy divide all the assets into two equal shares? S₁: A + C (\$18.00); S₂: L + P (\$18.00)
- Which share should Bob choose? S₂
- Division so far: Andy has A + C; Bob has L + P.

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Example (The Lone-Chooser Method – 3 Players)

- Now Andy and Bob each divide their shares each into 3 equal subshares.
- Andy's share, Apple and Cherry, to be divided 3 ways and Chuck chooses:

	Apple	Cherry
Andy	12	6
Chuck	6	4

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Example (The Lone-Chooser Method – 3 Players)

- Now Andy and Bob each divide their shares each into 3 equal subshares.
- Andy's share, Apple and Cherry, to be divided 3 ways and Chuck chooses:

	Apple	Cherry
Andy	12	6
Chuck	6	4

What are Andy's 3 subshares?

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- Now Andy and Bob each divide their shares each into 3 equal subshares.
- Andy's share, Apple and Cherry, to be divided 3 ways and Chuck chooses:

	Apple	Cherry
Andy	12	6
Chuck	6	4

What are Andy's 3 subshares? S₁: ¹/₂A (\$6.00); S₂: ¹/₂A (\$6.00); S₃: C (\$6.00)

Example (The Lone-Chooser Method – 3 Players)

- Now Andy and Bob each divide their shares each into 3 equal subshares.
- Andy's share, Apple and Cherry, to be divided 3 ways and Chuck chooses:

	Apple	Cherry
Andy	12	6
Chuck	6	4

- What are Andy's 3 subshares? S₁: ¹/₂A (\$6.00); S₂: ¹/₂A (\$6.00); S₃: C (\$6.00)
- Which one does Chuck choose?

Example (The Lone-Chooser Method – 3 Players)

- Now Andy and Bob each divide their shares each into 3 equal subshares.
- Andy's share, Apple and Cherry, to be divided 3 ways and Chuck chooses:

	Apple	Cherry
Andy	12	6
Chuck	6	4

- What are Andy's 3 subshares? S₁: ¹/₂A (\$6.00); S₂: ¹/₂A (\$6.00); S₃: C (\$6.00)
- Which one does Chuck choose? S₃

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Example (The Lone-Chooser Method – 3 Players)

- Now Andy and Bob each divide their shares each into 3 equal subshares.
- Andy's share, Apple and Cherry, to be divided 3 ways and Chuck chooses:

	Apple	Cherry
Andy	12	6
Chuck	6	4

- What are Andy's 3 subshares? S₁: ¹/₂A (\$6.00); S₂: ¹/₂A (\$6.00); S₃: C (\$6.00)
- Which one does Chuck choose? S₃
- Now Andy has Apple and Chuck has Cherry.

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Example (The Lone-Chooser Method – 3 Players)

 Bob's share, Lemon and Pecan, to be divided 3 ways and Chuck chooses:

	Lemon	Pecan
Bob	8	4
Chuck	2	6

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Example (The Lone-Chooser Method – 3 Players)

 Bob's share, Lemon and Pecan, to be divided 3 ways and Chuck chooses:

	Lemon	Pecan
Bob	8	4
Chuck	2	6

• What are Bob's 3 subshares?

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Example (The Lone-Chooser Method – 3 Players)

 Bob's share, Lemon and Pecan, to be divided 3 ways and Chuck chooses:

	Lemon	Pecan
Bob	8	4
Chuck	2	6

What are Bob's 3 subshares? S₁: ¹/₂L (\$4.00); S₂: ¹/₂L (\$4.00); S₃: P (\$4.00)

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 Bob's share, Lemon and Pecan, to be divided 3 ways and Chuck chooses:

	Lemon	Pecan
Bob	8	4
Chuck	2	6

- What are Bob's 3 subshares? S₁: ¹/₂L (\$4.00); S₂: ¹/₂L (\$4.00); S₃: P (\$4.00)
- Which one does Chuck choose?

 Bob's share, Lemon and Pecan, to be divided 3 ways and Chuck chooses:

	Lemon	Pecan
Bob	8	4
Chuck	2	6

- What are Bob's 3 subshares? S₁: ¹/₂L (\$4.00); S₂: ¹/₂L (\$4.00); S₃: P (\$4.00)
- Which one does Chuck choose? S₃

• Bob's share, Lemon and Pecan, to be divided 3 ways and Chuck chooses:

	Lemon	Pecan
Bob	8	4
Chuck	2	6

- What are Bob's 3 subshares? S₁: ¹/₂L (\$4.00); S₂: ¹/₂L (\$4.00); S₃: P (\$4.00)
- Which one does Chuck choose? S₃
- Final division: Andy gets Apple (\$6.00), Bob gets Lemon (\$8.00), Chuck gets Cherry and Pecan (\$10.00).

 What if Chuck were the first divider, Bob the first lone chooser, and then Chuck and Bob the subdividers and Andy the second lone chooser?

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- What if Chuck were the first divider, Bob the first lone chooser, and then Chuck and Bob the subdividers and Andy the second lone chooser?
- What if Bob were the first divider, Andy the first lone chooser, and then Bob and Andy the subdividers and Chuck the second lone chooser?

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- What if Chuck were the first divider, Bob the first lone chooser, and then Chuck and Bob the subdividers and Andy the second lone chooser?
- What if Bob were the first divider, Andy the first lone chooser, and then Bob and Andy the subdividers and Chuck the second lone chooser?
- In general, which role would you prefer to be in: first divider, first lone chooser, or second lone chooser?

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Outline

The Lone-Chooser Method

2 Example – 3 Players

3 Example – 4 Players

Assignment

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Example

Example (The Lone-Chooser Method – 4 Players)

- Andy, Bob, Chuck, and Dave are dividing 3 pies.
- Their value systems are as follows.

	Apple	Cherry	Lemon
Andy	12	6	6
Bob	4	8	4
Chuck	6	8	6
Dave	9	8	3

- First, Andy divides into equal halves and Bob chooses.
- Second, Andy and Bob divide into equal thirds and Chuck chooses.
- Finally, Andy, Bob, and Chuck divide into equal fourths and Dave chooses.

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	Apple	Cherry	Lemon
Andy	12	6	6
Bob	4	8	4
Chuck	6	8	6
Dave	9	8	3

• How does Andy divide the assets into two equal shares?

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	Apple	Cherry	Lemon
Andy	12	6	6
Bob	4	8	4
Chuck	6	8	6
Dave	9	8	3

How does Andy divide the assets into two equal shares? S₁: A;
 S₂: C + L

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	Apple	Cherry	Lemon
Andy	12	6	6
Bob	4	8	4
Chuck	6	8	6
Dave	9	8	3

How does Andy divide the assets into two equal shares? S₁: A;
 S₂: C + L

• Which share does Bob choose?

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	Apple	Cherry	Lemon
Andy	12	6	6
Bob	4	8	4
Chuck	6	8	6
Dave	9	8	3

How does Andy divide the assets into two equal shares? S₁: A;
 S₂: C + L

Which share does Bob choose? S₂

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	Apple	Cherry	Lemon
Andy	12	6	6
Bob	4	8	4
Chuck	6	8	6
Dave	9	8	3

- How does Andy divide the assets into two equal shares? S₁: A;
 S₂: C + L
- Which share does Bob choose? S₂
- Now Andy has Apple and Bob has Cherry and Lemon.





Andy has only the Apple pie and now Chuck is the chooer.

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Andy has only the Apple pie and now Chuck is the chooer.

• How does Andy divide his asset into three equal subshares?

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- Andy has only the Apple pie and now Chuck is the chooer.
- How does Andy divide his asset into three equal subshares? S_1 : $\frac{1}{3}A$; S_2 : $\frac{1}{3}A$; S_3 : $\frac{1}{3}A$

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- Andy has only the Apple pie and now Chuck is the chooer.
- How does Andy divide his asset into three equal subshares? S₁: ¹/₃A; S₂: ¹/₃A; S₃: ¹/₃A
- Which subshare does Chuck choose?

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- Andy has only the Apple pie and now Chuck is the chooer.
- How does Andy divide his asset into three equal subshares? S₁: ¹/₃A; S₂: ¹/₃A; S₃: ¹/₃A
- Which subshare does Chuck choose? S₃ (Or any other. They are identical.)

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- Andy has only the Apple pie and now Chuck is the chooer.
- How does Andy divide his asset into three equal subshares? S₁: ¹/₃A; S₂: ¹/₃A; S₃: ¹/₃A
- Which subshare does Chuck choose? S₃ (Or any other. They are identical.)
- Now Andy has $\frac{2}{3}$ Apple and Chuck has $\frac{1}{3}$ Apple.

	Cherry	Lemon
Bob	8	4
Chuck	8	6

• So far, Bob has the Cherry and Lemon pies and Chuck is the chooser.

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	Cherry	Lemon
Bob	8	4
Chuck	8	6

- So far, Bob has the Cherry and Lemon pies and Chuck is the chooser.
- How does Bob divide his assets into three equal subshares?

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	Cherry	Lemon
Bob	8	4
Chuck	8	6

- So far, Bob has the Cherry and Lemon pies and Chuck is the chooser.
- How does Bob divide his assets into three equal subshares?
 S₁: ¹/₂C; S₂: ¹/₂C; S₃: L

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	Cherry	Lemon
Bob	8	4
Chuck	8	6

- So far, Bob has the Cherry and Lemon pies and Chuck is the chooser.
- How does Bob divide his assets into three equal subshares?
 S₁: ¹/₂C; S₂: ¹/₂C; S₃: L
- Which subshare does Chuck choose?

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	Cherry	Lemon
Bob	8	4
Chuck	8	6

- So far, Bob has the Cherry and Lemon pies and Chuck is the chooser.
- How does Bob divide his assets into three equal subshares?
 S₁: ¹/₂C; S₂: ¹/₂C; S₃: L
- Which subshare does Chuck choose? S₃

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	Cherry	Lemon
Bob	8	4
Chuck	8	6

- So far, Bob has the Cherry and Lemon pies and Chuck is the chooser.
- How does Bob divide his assets into three equal subshares?
 S₁: ¹/₂C; S₂: ¹/₂C; S₃: L
- Which subshare does Chuck choose? S₃
- Now Andy has $\frac{2}{3}$ Apple, Bob has Cherry (both halves), and Chuck has $\frac{1}{3}$ Apple and the Lemon.

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	$\frac{2}{3}$ Apple
Andy	8
Dave	6

• Andy has only the $\frac{2}{3}$ of the Apple pie.

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- Andy has only the $\frac{2}{3}$ of the Apple pie.
- How does Andy divide his asset into four equal subshares?

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- Andy has only the $\frac{2}{3}$ of the Apple pie.
- How does Andy divide his asset into four equal subshares?
 S₁: ¹/₆A; S₂: ¹/₆A; S₃: ¹/₆A; S₄: ¹/₆A

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- Andy has only the $\frac{2}{3}$ of the Apple pie.
- How does Andy divide his asset into four equal subshares?
 S₁: ¹/₆A; S₂: ¹/₆A; S₃: ¹/₆A; S₄: ¹/₆A
- Which subshare does Dave choose?

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- Andy has only the $\frac{2}{3}$ of the Apple pie.
- How does Andy divide his asset into four equal subshares?
 S₁: ¹/₆A; S₂: ¹/₆A; S₃: ¹/₆A; S₄: ¹/₆A
- Which subshare does Dave choose? S₄ (Or any other. They are identical.)



- Andy has only the $\frac{2}{3}$ of the Apple pie.
- How does Andy divide his asset into four equal subshares?
 S₁: ¹/₆A; S₂: ¹/₆A; S₃: ¹/₆A; S₄: ¹/₆A
- Which subshare does Dave choose? S₄ (Or any other. They are identical.)
- Now Andy has $\frac{1}{2}$ Apple and Dave has $\frac{1}{6}$ Apple.

	Cherry	
Bob	8	
Dave	8	

Now Bob has only the Cherry pie.

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	Cherry	
Bob	8	
Dave	8	

- Now Bob has only the Cherry pie.
- How does Bob divide his asset into four equal subshares?

	Cherry
Bob	8
Dave	8

- Now Bob has only the Cherry pie.
- How does Bob divide his asset into four equal subshares? S₁: ¹/₄C;
 S₂: ¹/₄C; S₃: ¹/₄C; S₄: ¹/₄C

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	Cherry
Bob	8
Dave	8

- Now Bob has only the Cherry pie.
- How does Bob divide his asset into four equal subshares? S₁: ¹/₄C;
 S₂: ¹/₄C; S₃: ¹/₄C; S₄: ¹/₄C
- Which subshare does Chuck choose?

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	Cherry
Bob	8
Dave	8

- Now Bob has only the Cherry pie.
- How does Bob divide his asset into four equal subshares? S₁: ¹/₄C;
 S₂: ¹/₄C; S₃: ¹/₄C; S₄: ¹/₄C
- Which subshare does Chuck choose? S₄ (Or any other. They are identical.)

	Cherry
Bob	8
Dave	8

- Now Bob has only the Cherry pie.
- How does Bob divide his asset into four equal subshares? S₁: ¹/₄C;
 S₂: ¹/₄C; S₃: ¹/₄C; S₄: ¹/₄C
- Which subshare does Chuck choose? S₄ (Or any other. They are identical.)
- Now Bob has $\frac{3}{4}$ Cherry and Dave has $\frac{1}{4}$ Cherry.

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	$\frac{1}{3}$ Apple	Lemon
Chuck	2	6
Dave	3	3

• Chuck has $\frac{1}{3}$ Apple pie and the Lemon pie.

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	$\frac{1}{3}$ Apple	Lemon
Chuck	2	6
Dave	3	3

- Chuck has $\frac{1}{3}$ Apple pie and the Lemon pie.
- How does Chuck divide his assets into four equal subshares?

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	$\frac{1}{3}$ Apple	Lemon
Chuck	2	6
Dave	3	3

- Chuck has $\frac{1}{3}$ Apple pie and the Lemon pie.
- How does Chuck divide his assets into four equal subshares? S_1 : $\frac{1}{3}A$; S_2 : $\frac{1}{3}L$; S_3 : $\frac{1}{3}L$; S_4 : $\frac{1}{3}L$

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	$\frac{1}{3}$ Apple	Lemon
Chuck	2	6
Dave	3	3

- Chuck has $\frac{1}{3}$ Apple pie and the Lemon pie.
- How does Chuck divide his assets into four equal subshares? S_1 : $\frac{1}{3}A$; S_2 : $\frac{1}{3}L$; S_3 : $\frac{1}{3}L$; S_4 : $\frac{1}{3}L$
- Which subshare does Dave choose?

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	$\frac{1}{3}$ Apple	Lemon
Chuck	2	6
Dave	3	3

- Chuck has $\frac{1}{3}$ Apple pie and the Lemon pie.
- How does Chuck divide his assets into four equal subshares? S_1 : $\frac{1}{3}A$; S_2 : $\frac{1}{3}L$; S_3 : $\frac{1}{3}L$; S_4 : $\frac{1}{3}L$
- Which subshare does Dave choose? S₁

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	$\frac{1}{3}$ Apple	Lemon
Chuck	2	6
Dave	3	3

- Chuck has $\frac{1}{3}$ Apple pie and the Lemon pie.
- How does Chuck divide his assets into four equal subshares? S_1 : $\frac{1}{3}A$; S_2 : $\frac{1}{3}L$; S_3 : $\frac{1}{3}L$; S_4 : $\frac{1}{3}L$
- Which subshare does Dave choose? S₁
- Now Chuck has Lemon and Dave has $\frac{1}{3}$ Apple.

				Fair	Actual
	Apple	Cherry	Lemon	Share	Share
Andy	12	6	6	6.00	6.00
Bob	4	8	4	4.00	6.00
Chuck	6	8	6	4.50	6.00
Dave	9	8	3	5.00	6.50

• The final division:

- Andy gets $\frac{1}{2}$ of the Apple pie.
- Bob gets $\frac{3}{4}$ of the Cherry pie.
- Chuck gets the Lemon pie.
- Dave gets $\frac{1}{2}$ of the Apple pie and $\frac{1}{4}$ of the Cherry pie.

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Outline

The Lone-Chooser Method

- 2 Example 3 Players
- 3 Example 4 Players



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Assignment

- Chapter 3: Exercises 41, 42.
- Handout exercises.

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