# The Lone-Chooser Method <br> Lecture 16 <br> Section 3.4 

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(9) The Lone-Chooser Method
(2) Example-3 Players
(3) Example - 4 Players

4 Assignment

## Outline

# (9) The Lone-Chooser Method 

(2) Example-3 Players
(3) Example-4 Players
4) Assignment

## The Lone-Chooser Method

## Definition (The Lone-Chooser Method)

In the lone-chooser method,

- One player is designated to be the chooser.
- The other players are the dividers. They divide the assets among themselves (details to follow).
- Then each of the dividers divides his share into equal subshares.
- The chooser then chooses one subshare from each of the dividers.
- The dividers keep the subshares that are left.


## The Lone-Chooser Method

- This is normally done in a recursive manner.
- For example, if there are 4 players $A, B, C$, and $D$.


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- Then $A$ divides them into 2 equal shares. $B$ chooses one of them.


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- For example, if there are 4 players $A, B, C$, and $D$.
- A starts off with all the assets.
- Then $A$ divides them into 2 equal shares. $B$ chooses one of them.
- Then $A$ and $B$ each divide their shares into 3 equal subshares. Player $C$ chooses one subshare from each.


## The Lone-Chooser Method

- This is normally done in a recursive manner.
- For example, if there are 4 players $A, B, C$, and $D$.
- A starts off with all the assets.
- Then $A$ divides them into 2 equal shares. $B$ chooses one of them.
- Then $A$ and $B$ each divide their shares into 3 equal subshares. Player $C$ chooses one subshare from each.
- Then $A, B$, and $C$ each divide their shares into 4 equal subshares. Player $D$ chooses one subshare from each.


## The Lone-Chooser Method



A
B
C D

A starts with all the assets

## The Lone-Chooser Method



A
B
C D

## $A$ divides them into 2 equal shares

## The Lone-Chooser Method



A


B
B

D

## $B$ chooses one of the shares

## The Lone-Chooser Method



A
B
C
D
$A$ and $B$ divide their assets into 3 equal subshares

## The Lone-Chooser Method



A


B


C
$C$ chooses one subshare from each

## The Lone-Chooser Method



A
B
C
D
$A, B$, and $C$ divide their assets into 4 equal subshares

## The Lone-Chooser Method



A


B


C


D
$D$ chooses one subshare from each

## Outline

## (1) The Lone-Chooser Method

## (2) Example-3 Players

(3) Example - 4 Players
4) Assignment

## Example

## Example (The Lone-Chooser Method - 3 Players)

- Andy, Bob, and Chuck are dividing 4 pies: apple, cherry, lemon, and pecan.
- The value systems of the players are as follows.

|  | Apple | Cherry | Lemon | Pecan |
| :--- | :---: | :---: | :---: | :---: |
| Andy | 12 | 6 | 10 | 8 |
| Bob | 7 | 2 | 8 | 4 |
| Chuck | 6 | 4 | 2 | 6 |

- Chuck is the lone-chooser.
- But in the meantime, Andy and Bob divide the assets between the two of them, with Andy the divider and Bob the lone chooser.


## Example

## Example (The Lone-Chooser Method - 3 Players)

|  | Apple | Cherry | Lemon | Pecan |
| :--- | :---: | :---: | :---: | :---: |
| Andy | 12 | 6 | 10 | 8 |
| Bob | 7 | 2 | 8 | 4 |
| Chuck | 6 | 4 | 2 | 6 |

- How should Andy divide all the assets into two equal shares?


## Example

## Example (The Lone-Chooser Method - 3 Players)

|  | Apple | Cherry | Lemon | Pecan |
| :--- | :---: | :---: | :---: | :---: |
| Andy | 12 | 6 | 10 | 8 |
| Bob | 7 | 2 | 8 | 4 |
| Chuck | 6 | 4 | 2 | 6 |

- How should Andy divide all the assets into two equal shares? $\mathrm{S}_{1}$ : $A+C(\$ 18.00) ; S_{2}: L+P(\$ 18.00)$


## Example

## Example (The Lone-Chooser Method - 3 Players)

|  | Apple | Cherry | Lemon | Pecan |
| :--- | :---: | :---: | :---: | :---: |
| Andy | 12 | 6 | 10 | 8 |
| Bob | 7 | 2 | 8 | 4 |
| Chuck | 6 | 4 | 2 | 6 |

- How should Andy divide all the assets into two equal shares? $\mathrm{S}_{1}$ : $A+C(\$ 18.00) ; S_{2}: L+P(\$ 18.00)$
- Which share should Bob choose?


## Example

## Example (The Lone-Chooser Method - 3 Players)

|  | Apple | Cherry | Lemon | Pecan |
| :--- | :---: | :---: | :---: | :---: |
| Andy | 12 | 6 | 10 | 8 |
| Bob | 7 | 2 | 8 | 4 |
| Chuck | 6 | 4 | 2 | 6 |

- How should Andy divide all the assets into two equal shares? $\mathrm{S}_{1}$ : A + C (\$18.00); $S_{2}: L+P(\$ 18.00)$
- Which share should Bob choose? $\mathrm{S}_{2}$


## Example

## Example (The Lone-Chooser Method - 3 Players)

|  | Apple | Cherry | Lemon | Pecan |
| :--- | :---: | :---: | :---: | :---: |
| Andy | 12 | 6 | 10 | 8 |
| Bob | 7 | 2 | 8 | 4 |
| Chuck | 6 | 4 | 2 | 6 |

- How should Andy divide all the assets into two equal shares? $\mathrm{S}_{1}$ : A + C (\$18.00); $S_{2}: L+P(\$ 18.00)$
- Which share should Bob choose? $\mathrm{S}_{2}$
- Division so far: Andy has A + C; Bob has L + P.


## Example

## Example (The Lone-Chooser Method - 3 Players)

- Now Andy and Bob each divide their shares each into 3 equal subshares.
- Andy's share, Apple and Cherry, to be divided 3 ways and Chuck chooses:

|  | Apple | Cherry |
| :--- | :---: | :---: |
| Andy | 12 | 6 |
| Chuck | 6 | 4 |

## Example

## Example (The Lone-Chooser Method - 3 Players)

- Now Andy and Bob each divide their shares each into 3 equal subshares.
- Andy's share, Apple and Cherry, to be divided 3 ways and Chuck chooses:

|  | Apple | Cherry |
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| Andy | 12 | 6 |
| Chuck | 6 | 4 |

- What are Andy's 3 subshares?


## Example

## Example (The Lone-Chooser Method - 3 Players)

- Now Andy and Bob each divide their shares each into 3 equal subshares.
- Andy's share, Apple and Cherry, to be divided 3 ways and Chuck chooses:

|  | Apple | Cherry |
| :--- | :---: | :---: |
| Andy | 12 | 6 |
| Chuck | 6 | 4 |

- What are Andy's 3 subshares? $\mathrm{S}_{1}: \frac{1}{2} \mathrm{~A}(\$ 6.00) ; \mathrm{S}_{2}: \frac{1}{2} \mathrm{~A}(\$ 6.00)$; $\mathrm{S}_{3}$ : C (\$6.00)


## Example

## Example (The Lone-Chooser Method - 3 Players)

- Now Andy and Bob each divide their shares each into 3 equal subshares.
- Andy's share, Apple and Cherry, to be divided 3 ways and Chuck chooses:

|  | Apple | Cherry |
| :--- | :---: | :---: |
| Andy | 12 | 6 |
| Chuck | 6 | 4 |

- What are Andy's 3 subshares? $\mathrm{S}_{1}: \frac{1}{2} \mathrm{~A}(\$ 6.00) ; \mathrm{S}_{2}: \frac{1}{2} \mathrm{~A}(\$ 6.00)$; $\mathrm{S}_{3}$ : C (\$6.00)
- Which one does Chuck choose?


## Example

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- Now Andy and Bob each divide their shares each into 3 equal subshares.
- Andy's share, Apple and Cherry, to be divided 3 ways and Chuck chooses:

|  | Apple | Cherry |
| :--- | :---: | :---: |
| Andy | 12 | 6 |
| Chuck | 6 | 4 |

- What are Andy's 3 subshares? $\mathrm{S}_{1}: \frac{1}{2} \mathrm{~A}(\$ 6.00) ; \mathrm{S}_{2}: \frac{1}{2} \mathrm{~A}(\$ 6.00)$; $\mathrm{S}_{3}$ : C (\$6.00)
- Which one does Chuck choose? $\mathrm{S}_{3}$


## Example

## Example (The Lone-Chooser Method - 3 Players)

- Now Andy and Bob each divide their shares each into 3 equal subshares.
- Andy's share, Apple and Cherry, to be divided 3 ways and Chuck chooses:

|  | Apple | Cherry |
| :--- | :---: | :---: |
| Andy | 12 | 6 |
| Chuck | 6 | 4 |

- What are Andy's 3 subshares? $\mathrm{S}_{1}: \frac{1}{2} \mathrm{~A}(\$ 6.00) ; \mathrm{S}_{2}: \frac{1}{2} \mathrm{~A}(\$ 6.00)$; $\mathrm{S}_{3}$ : C (\$6.00)
- Which one does Chuck choose? $\mathrm{S}_{3}$
- Now Andy has Apple and Chuck has Cherry.


## Example

## Example (The Lone-Chooser Method - 3 Players)

- Bob's share, Lemon and Pecan, to be divided 3 ways and Chuck chooses:

|  | Lemon | Pecan |
| :--- | :---: | :---: |
| Bob | 8 | 4 |
| Chuck | 2 | 6 |

## Example

## Example (The Lone-Chooser Method - 3 Players)

- Bob's share, Lemon and Pecan, to be divided 3 ways and Chuck chooses:

|  | Lemon | Pecan |
| :--- | :---: | :---: |
| Bob | 8 | 4 |
| Chuck | 2 | 6 |

- What are Bob's 3 subshares?


## Example

## Example (The Lone-Chooser Method - 3 Players)

- Bob's share, Lemon and Pecan, to be divided 3 ways and Chuck chooses:

|  | Lemon | Pecan |
| :--- | :---: | :---: |
| Bob | 8 | 4 |
| Chuck | 2 | 6 |

- What are Bob's 3 subshares? $\mathrm{S}_{1}: \frac{1}{2} \mathrm{~L}(\$ 4.00) ; \mathrm{S}_{2}: \frac{1}{2} \mathrm{~L}(\$ 4.00)$; $\mathrm{S}_{3}$ : P (\$4.00)


## Example

## Example (The Lone-Chooser Method - 3 Players)

- Bob's share, Lemon and Pecan, to be divided 3 ways and Chuck chooses:

|  | Lemon | Pecan |
| :--- | :---: | :---: |
| Bob | 8 | 4 |
| Chuck | 2 | 6 |

- What are Bob's 3 subshares? $\mathrm{S}_{1}: \frac{1}{2} \mathrm{~L}(\$ 4.00) ; \mathrm{S}_{2}: \frac{1}{2} \mathrm{~L}(\$ 4.00)$; $\mathrm{S}_{3}$ : P (\$4.00)
- Which one does Chuck choose?


## Example

## Example (The Lone-Chooser Method - 3 Players)

- Bob's share, Lemon and Pecan, to be divided 3 ways and Chuck chooses:

|  | Lemon | Pecan |
| :--- | :---: | :---: |
| Bob | 8 | 4 |
| Chuck | 2 | 6 |

- What are Bob's 3 subshares? $\mathrm{S}_{1}: \frac{1}{2} \mathrm{~L}(\$ 4.00) ; \mathrm{S}_{2}: \frac{1}{2} \mathrm{~L}(\$ 4.00)$; $\mathrm{S}_{3}$ : P (\$4.00)
- Which one does Chuck choose? $\mathrm{S}_{3}$


## Example

## Example (The Lone-Chooser Method - 3 Players)

- Bob's share, Lemon and Pecan, to be divided 3 ways and Chuck chooses:

|  | Lemon | Pecan |
| :--- | :---: | :---: |
| Bob | 8 | 4 |
| Chuck | 2 | 6 |

- What are Bob's 3 subshares? $\mathrm{S}_{1}: \frac{1}{2} \mathrm{~L}(\$ 4.00) ; \mathrm{S}_{2}: \frac{1}{2} \mathrm{~L}(\$ 4.00)$; $\mathrm{S}_{3}$ : P (\$4.00)
- Which one does Chuck choose? $\mathrm{S}_{3}$
- Final division: Andy gets Apple (\$6.00), Bob gets Lemon (\$8.00), Chuck gets Cherry and Pecan (\$10.00).


## Example

## Example (The Lone-Chooser Method - 3 Players)

- What if Chuck were the first divider, Bob the first lone chooser, and then Chuck and Bob the subdividers and Andy the second lone chooser?


## Example

## Example (The Lone-Chooser Method - 3 Players)

- What if Chuck were the first divider, Bob the first lone chooser, and then Chuck and Bob the subdividers and Andy the second lone chooser?
- What if Bob were the first divider, Andy the first lone chooser, and then Bob and Andy the subdividers and Chuck the second lone chooser?


## Example

## Example (The Lone-Chooser Method - 3 Players)

- What if Chuck were the first divider, Bob the first lone chooser, and then Chuck and Bob the subdividers and Andy the second lone chooser?
- What if Bob were the first divider, Andy the first lone chooser, and then Bob and Andy the subdividers and Chuck the second lone chooser?
- In general, which role would you prefer to be in: first divider, first lone chooser, or second lone chooser?


## Outline

## (1) The Lone-Chooser Method

(2) Example-3 Players
(3) Example-4 Players
(4) Assignment

## Example

## Example (The Lone-Chooser Method - 4 Players)

- Andy, Bob, Chuck, and Dave are dividing 3 pies.
- Their value systems are as follows.

|  | Apple | Cherry | Lemon |
| :--- | :---: | :---: | :---: |
| Andy | 12 | 6 | 6 |
| Bob | 4 | 8 | 4 |
| Chuck | 6 | 8 | 6 |
| Dave | 9 | 8 | 3 |

- First, Andy divides into equal halves and Bob chooses.
- Second, Andy and Bob divide into equal thirds and Chuck chooses.
- Finally, Andy, Bob, and Chuck divide into equal fourths and Dave chooses.


## Example

## Example (The Lone-Chooser Method - 4 Players)

|  | Apple | Cherry | Lemon |
| :--- | :---: | :---: | :---: |
| Andy | 12 | 6 | 6 |
| Bob | 4 | 8 | 4 |
| Chuck | 6 | 8 | 6 |
| Dave | 9 | 8 | 3 |

- How does Andy divide the assets into two equal shares?


## Example

## Example (The Lone-Chooser Method - 4 Players)

|  | Apple | Cherry | Lemon |
| :--- | :---: | :---: | :---: |
| Andy | 12 | 6 | 6 |
| Bob | 4 | 8 | 4 |
| Chuck | 6 | 8 | 6 |
| Dave | 9 | 8 | 3 |

- How does Andy divide the assets into two equal shares? $S_{1}$ : $A$; $S_{2}: C+L$


## Example

## Example (The Lone-Chooser Method - 4 Players)

|  | Apple | Cherry | Lemon |
| :--- | :---: | :---: | :---: |
| Andy | 12 | 6 | 6 |
| Bob | 4 | 8 | 4 |
| Chuck | 6 | 8 | 6 |
| Dave | 9 | 8 | 3 |

- How does Andy divide the assets into two equal shares? $\mathrm{S}_{1}$ : A ; $\mathrm{S}_{2}: \mathrm{C}+\mathrm{L}$
- Which share does Bob choose?


## Example

## Example (The Lone-Chooser Method - 4 Players)

|  | Apple | Cherry | Lemon |
| :--- | :---: | :---: | :---: |
| Andy | 12 | 6 | 6 |
| Bob | 4 | 8 | 4 |
| Chuck | 6 | 8 | 6 |
| Dave | 9 | 8 | 3 |

- How does Andy divide the assets into two equal shares? $\mathrm{S}_{1}$ : A ; $\mathrm{S}_{2}: \mathrm{C}+\mathrm{L}$
- Which share does Bob choose? $\mathrm{S}_{2}$


## Example

## Example (The Lone-Chooser Method - 4 Players)

|  | Apple | Cherry | Lemon |
| :--- | :---: | :---: | :---: |
| Andy | 12 | 6 | 6 |
| Bob | 4 | 8 | 4 |
| Chuck | 6 | 8 | 6 |
| Dave | 9 | 8 | 3 |

- How does Andy divide the assets into two equal shares? $S_{1}: A$; $\mathrm{S}_{2}$ : C + L
- Which share does Bob choose? $\mathrm{S}_{2}$
- Now Andy has Apple and Bob has Cherry and Lemon.


## Example

## Example (The Lone-Chooser Method - 4 Players)

|  | Apple |
| :--- | :---: |
| Andy | 12 |
| Chuck | 6 |

- Andy has only the Apple pie and now Chuck is the chooer.


## Example

## Example (The Lone-Chooser Method - 4 Players)

|  | Apple |
| :--- | :---: |
| Andy | 12 |
| Chuck | 6 |

- Andy has only the Apple pie and now Chuck is the chooer.
- How does Andy divide his asset into three equal subshares?


## Example

## Example (The Lone-Chooser Method - 4 Players)

|  | Apple |
| :--- | :---: |
| Andy | 12 |
| Chuck | 6 |

- Andy has only the Apple pie and now Chuck is the chooer.
- How does Andy divide his asset into three equal subshares? $S_{1}$ : $\frac{1}{3} A ; S_{2}: \frac{1}{3} A ; S_{3}: \frac{1}{3} A$


## Example

## Example (The Lone-Chooser Method - 4 Players)

|  | Apple |
| :--- | :---: |
| Andy | 12 |
| Chuck | 6 |

- Andy has only the Apple pie and now Chuck is the chooer.
- How does Andy divide his asset into three equal subshares? $S_{1}$ : $\frac{1}{3} A ; S_{2}: \frac{1}{3} A ; S_{3}: \frac{1}{3} A$
- Which subshare does Chuck choose?


## Example

## Example (The Lone-Chooser Method - 4 Players)

|  | Apple |
| :--- | :---: |
| Andy | 12 |
| Chuck | 6 |

- Andy has only the Apple pie and now Chuck is the chooer.
- How does Andy divide his asset into three equal subshares? $S_{1}$ : $\frac{1}{3} A ; S_{2}: \frac{1}{3} A ; S_{3}: \frac{1}{3} A$
- Which subshare does Chuck choose? $\mathrm{S}_{3}$ (Or any other. They are identical.)


## Example

## Example (The Lone-Chooser Method - 4 Players)

|  | Apple |
| :--- | :---: |
| Andy | 12 |
| Chuck | 6 |

- Andy has only the Apple pie and now Chuck is the chooer.
- How does Andy divide his asset into three equal subshares? $S_{1}$ : $\frac{1}{3} A ; S_{2}: \frac{1}{3} A ; S_{3}: \frac{1}{3} A$
- Which subshare does Chuck choose? $\mathrm{S}_{3}$ (Or any other. They are identical.)
- Now Andy has $\frac{2}{3}$ Apple and Chuck has $\frac{1}{3}$ Apple.


## Example

## Example (The Lone-Chooser Method - 4 Players)

|  | Cherry | Lemon |
| :--- | :---: | :---: |
| Bob | 8 | 4 |
| Chuck | 8 | 6 |

- So far, Bob has the Cherry and Lemon pies and Chuck is the chooser.


## Example

## Example (The Lone-Chooser Method - 4 Players)

|  | Cherry | Lemon |
| :--- | :---: | :---: |
| Bob | 8 | 4 |
| Chuck | 8 | 6 |

- So far, Bob has the Cherry and Lemon pies and Chuck is the chooser.
- How does Bob divide his assets into three equal subshares?


## Example

## Example (The Lone-Chooser Method - 4 Players)

|  | Cherry | Lemon |
| :--- | :---: | :---: |
| Bob | 8 | 4 |
| Chuck | 8 | 6 |

- So far, Bob has the Cherry and Lemon pies and Chuck is the chooser.
- How does Bob divide his assets into three equal subshares?
$S_{1}: \frac{1}{2} C ; S_{2}: \frac{1}{2} C ; S_{3}: L$


## Example

## Example (The Lone-Chooser Method - 4 Players)

|  | Cherry | Lemon |
| :--- | :---: | :---: |
| Bob | 8 | 4 |
| Chuck | 8 | 6 |

- So far, Bob has the Cherry and Lemon pies and Chuck is the chooser.
- How does Bob divide his assets into three equal subshares? $\mathrm{S}_{1}: \frac{1}{2} \mathrm{C} ; \mathrm{S}_{2}: \frac{1}{2} \mathrm{C} ; \mathrm{S}_{3}: \mathrm{L}$
- Which subshare does Chuck choose?


## Example

## Example (The Lone-Chooser Method - 4 Players)

|  | Cherry | Lemon |
| :--- | :---: | :---: |
| Bob | 8 | 4 |
| Chuck | 8 | 6 |

- So far, Bob has the Cherry and Lemon pies and Chuck is the chooser.
- How does Bob divide his assets into three equal subshares? $S_{1}: \frac{1}{2} C ; S_{2}: \frac{1}{2} C ; S_{3}: L$
- Which subshare does Chuck choose? $\mathrm{S}_{3}$


## Example

## Example (The Lone-Chooser Method - 4 Players)

|  | Cherry | Lemon |
| :--- | :---: | :---: |
| Bob | 8 | 4 |
| Chuck | 8 | 6 |

- So far, Bob has the Cherry and Lemon pies and Chuck is the chooser.
- How does Bob divide his assets into three equal subshares?
$\mathrm{S}_{1}: \frac{1}{2} \mathrm{C} ; \mathrm{S}_{2}: \frac{1}{2} \mathrm{C} ; \mathrm{S}_{3}: \mathrm{L}$
- Which subshare does Chuck choose? $\mathrm{S}_{3}$
- Now Andy has $\frac{2}{3}$ Apple, Bob has Cherry (both halves), and Chuck has $\frac{1}{3}$ Apple and the Lemon.


## Example

## Example (The Lone-Chooser Method - 4 Players)

|  | $\frac{2}{3}$ Apple |
| :---: | :---: |
| Andy | 8 |
| Dave | 6 |

- Andy has only the $\frac{2}{3}$ of the Apple pie.


## Example

## Example (The Lone-Chooser Method - 4 Players)

|  | $\frac{2}{3}$ Apple |
| :---: | :---: |
| Andy | 8 |
| Dave | 6 |

- Andy has only the $\frac{2}{3}$ of the Apple pie.
- How does Andy divide his asset into four equal subshares?


## Example

## Example (The Lone-Chooser Method - 4 Players)

|  | $\frac{2}{3}$ Apple |
| :---: | :---: |
| Andy | 8 |
| Dave | 6 |

- Andy has only the $\frac{2}{3}$ of the Apple pie.
- How does Andy divide his asset into four equal subshares?
$S_{1}: \frac{1}{6} A ; S_{2}: \frac{1}{6} A ; S_{3}: \frac{1}{6} A ; S_{4}: \frac{1}{6} A$


## Example

## Example (The Lone-Chooser Method - 4 Players)

|  | $\frac{2}{3}$ Apple |
| :---: | :---: |
| Andy | 8 |
| Dave | 6 |

- Andy has only the $\frac{2}{3}$ of the Apple pie.
- How does Andy divide his asset into four equal subshares?
$S_{1}: \frac{1}{6} A ; S_{2}: \frac{1}{6} A ; S_{3}: \frac{1}{6} A ; S_{4}: \frac{1}{6} A$
- Which subshare does Dave choose?


## Example

## Example (The Lone-Chooser Method - 4 Players)

|  | $\frac{2}{3}$ Apple |
| :---: | :---: |
| Andy | 8 |
| Dave | 6 |

- Andy has only the $\frac{2}{3}$ of the Apple pie.
- How does Andy divide his asset into four equal subshares?
$S_{1}: \frac{1}{6} A ; S_{2}: \frac{1}{6} A ; S_{3}: \frac{1}{6} A ; S_{4}: \frac{1}{6} A$
- Which subshare does Dave choose? $\mathrm{S}_{4}$ (Or any other. They are identical.)


## Example

## Example (The Lone-Chooser Method - 4 Players)

|  | $\frac{2}{3}$ Apple |
| :---: | :---: |
| Andy | 8 |
| Dave | 6 |

- Andy has only the $\frac{2}{3}$ of the Apple pie.
- How does Andy divide his asset into four equal subshares?
$S_{1}: \frac{1}{6} A ; S_{2}: \frac{1}{6} A ; S_{3}: \frac{1}{6} A ; S_{4}: \frac{1}{6} A$
- Which subshare does Dave choose? $\mathrm{S}_{4}$ (Or any other. They are identical.)
- Now Andy has $\frac{1}{2}$ Apple and Dave has $\frac{1}{6}$ Apple.


## Example

## Example (The Lone-Chooser Method - 4 Players)

|  | Cherry |
| :--- | :---: |
| Bob | 8 |
| Dave | 8 |

- Now Bob has only the Cherry pie.


## Example

## Example (The Lone-Chooser Method - 4 Players)

|  | Cherry |
| :--- | :---: |
| Bob | 8 |
| Dave | 8 |

- Now Bob has only the Cherry pie.
- How does Bob divide his asset into four equal subshares?


## Example

## Example (The Lone-Chooser Method - 4 Players)

|  | Cherry |
| :--- | :---: |
| Bob | 8 |
| Dave | 8 |

- Now Bob has only the Cherry pie.
- How does Bob divide his asset into four equal subshares? $\mathrm{S}_{1}: \frac{1}{4} \mathrm{C}$; $\mathrm{S}_{2}: \frac{1}{4} \mathrm{C} ; \mathrm{S}_{3}: \frac{1}{4} \mathrm{C} ; \mathrm{S}_{4}: \frac{1}{4} \mathrm{C}$


## Example

## Example (The Lone-Chooser Method - 4 Players)

|  | Cherry |
| :--- | :---: |
| Bob | 8 |
| Dave | 8 |

- Now Bob has only the Cherry pie.
- How does Bob divide his asset into four equal subshares? $\mathrm{S}_{1}: \frac{1}{4} \mathrm{C}$; $\mathrm{S}_{2}: \frac{1}{4} \mathrm{C} ; \mathrm{S}_{3}: \frac{1}{4} \mathrm{C} ; \mathrm{S}_{4}: \frac{1}{4} \mathrm{C}$
- Which subshare does Chuck choose?


## Example

## Example (The Lone-Chooser Method - 4 Players)

|  | Cherry |
| :--- | :---: |
| Bob | 8 |
| Dave | 8 |

- Now Bob has only the Cherry pie.
- How does Bob divide his asset into four equal subshares? $\mathrm{S}_{1}: \frac{1}{4} \mathrm{C}$; $\mathrm{S}_{2}: \frac{1}{4} \mathrm{C} ; \mathrm{S}_{3}: \frac{1}{4} \mathrm{C} ; \mathrm{S}_{4}: \frac{1}{4} \mathrm{C}$
- Which subshare does Chuck choose? $\mathrm{S}_{4}$ (Or any other. They are identical.)


## Example

## Example (The Lone-Chooser Method - 4 Players)

|  | Cherry |
| :--- | :---: |
| Bob | 8 |
| Dave | 8 |

- Now Bob has only the Cherry pie.
- How does Bob divide his asset into four equal subshares? $\mathrm{S}_{1}: \frac{1}{4} \mathrm{C}$; $\mathrm{S}_{2}: \frac{1}{4} \mathrm{C} ; \mathrm{S}_{3}: \frac{1}{4} \mathrm{C} ; \mathrm{S}_{4}: \frac{1}{4} \mathrm{C}$
- Which subshare does Chuck choose? $\mathrm{S}_{4}$ (Or any other. They are identical.)
- Now Bob has $\frac{3}{4}$ Cherry and Dave has $\frac{1}{4}$ Cherry.


## Example

## Example (The Lone-Chooser Method - 4 Players)

|  | $\frac{1}{3}$ Apple | Lemon |
| :--- | :---: | :---: |
| Chuck | 2 | 6 |
| Dave | 3 | 3 |

- Chuck has $\frac{1}{3}$ Apple pie and the Lemon pie.


## Example

Example (The Lone-Chooser Method - 4 Players)

|  | $\frac{1}{3}$ Apple | Lemon |
| :--- | :---: | :---: |
| Chuck | 2 | 6 |
| Dave | 3 | 3 |

- Chuck has $\frac{1}{3}$ Apple pie and the Lemon pie.
- How does Chuck divide his assets into four equal subshares?


## Example

Example (The Lone-Chooser Method - 4 Players)

|  | $\frac{1}{3}$ Apple | Lemon |
| :--- | :---: | :---: |
| Chuck | 2 | 6 |
| Dave | 3 | 3 |

- Chuck has $\frac{1}{3}$ Apple pie and the Lemon pie.
- How does Chuck divide his assets into four equal subshares? $S_{1}$ : $\frac{1}{3} A ; S_{2}: \frac{1}{3} L ; S_{3}: \frac{1}{3} L ; S_{4}: \frac{1}{3} L$


## Example

## Example (The Lone-Chooser Method - 4 Players)

|  | $\frac{1}{3}$ Apple | Lemon |
| :--- | :---: | :---: |
| Chuck | 2 | 6 |
| Dave | 3 | 3 |

- Chuck has $\frac{1}{3}$ Apple pie and the Lemon pie.
- How does Chuck divide his assets into four equal subshares? $\mathrm{S}_{1}$ : $\frac{1}{3} A ; S_{2}: \frac{1}{3} L ; S_{3}: \frac{1}{3} L ; S_{4}: \frac{1}{3} L$
- Which subshare does Dave choose?


## Example

## Example (The Lone-Chooser Method - 4 Players)

|  | $\frac{1}{3}$ Apple | Lemon |
| :--- | :---: | :---: |
| Chuck | 2 | 6 |
| Dave | 3 | 3 |

- Chuck has $\frac{1}{3}$ Apple pie and the Lemon pie.
- How does Chuck divide his assets into four equal subshares? $\mathrm{S}_{1}$ : $\frac{1}{3} A ; S_{2}: \frac{1}{3} L ; S_{3}: \frac{1}{3} L ; S_{4}: \frac{1}{3} L$
- Which subshare does Dave choose? $\mathrm{S}_{1}$


## Example

## Example (The Lone-Chooser Method - 4 Players)

|  | $\frac{1}{3}$ Apple | Lemon |
| :--- | :---: | :---: |
| Chuck | 2 | 6 |
| Dave | 3 | 3 |

- Chuck has $\frac{1}{3}$ Apple pie and the Lemon pie.
- How does Chuck divide his assets into four equal subshares? $S_{1}$ : $\frac{1}{3} A ; S_{2}: \frac{1}{3} L ; S_{3}: \frac{1}{3} L ; S_{4}: \frac{1}{3} L$
- Which subshare does Dave choose? $\mathrm{S}_{1}$
- Now Chuck has Lemon and Dave has $\frac{1}{3}$ Apple.


## Example

## Example (The Lone-Chooser Method - 4 Players)

|  | Apple | Cherry | Lemon | Fair <br> Share | Actual <br> Share |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Andy | 12 | 6 | 6 | 6.00 | 6.00 |
| Bob | 4 | 8 | 4 | 4.00 | 6.00 |
| Chuck | 6 | 8 | 6 | 4.50 | 6.00 |
| Dave | 9 | 8 | 3 | 5.00 | 6.50 |

- The final division:
- Andy gets $\frac{1}{2}$ of the Apple pie.
- Bob gets $\frac{3}{4}$ of the Cherry pie.
- Chuck gets the Lemon pie.
- Dave gets $\frac{1}{2}$ of the Apple pie and $\frac{1}{4}$ of the Cherry pie.


## Outline

## (1) The Lone-Chooser Method

(2) Example-3 Players
(3) Example - 4 Players
4) Assignment

## Assignment

## Assignment

- Chapter 3: Exercises 41, 42.
- Handout exercises.

