The Lunar Distance Method in the Nineteenth Century.

A simulation of Joshua Slocum's observation on June 16, 1896.

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Abstract

The history and practice of the lunar distance method are described, with special emphasis on its use in the nineteenth century. It is only in the first half of the last century that lunars were widely practised. We describe in some detail the story of Captain Joshua Slocum, the first solitary circumnavigator, whose lunar observation in 1896 was made in its original form, with nothing but the moon as a clock. A simulation of his observations and their reduction by the means available to the nineteenth century navigator is described, and a short review of these methods is presented.

History and prehistory of the lunar distance method.

From the early beginning of voyages across the oceans the determination of latitude has not been a problem: the Portuguese had introduced the marine astrolabe, which permitted altitudes to be taken at meridian passage with an accuracy of half a degree and the sun's declination tables [1] were accurate within a few minutes.

The situation for longitude was different. Ships often found themselves more than ten degrees off from their dead-reckoned positions and sometimes much more. The only time they knew was the local time, which they could tell from the sun. But in addition the time at some standard meridian was needed as a reference to find the longitude. This problem would be solved if only one had a clock that could be regulated to keep the time at some standard meridian. The time of the sun's meridian passage, local noon, read off on such a clock would then tell the longitude.

The notion that the moon could be used as a clock must have already existed among sailors at that time: the first attempt to find the longitude by the lunar distance is said to have been made by Amerigo Vespucci in 1499. He was aboard with Columbus on his third voyage to America as cartographer. Possibly also Magelhães tried it, during his voyage around the world (1519-1521).

Whether this is truth or merely saga, we do not know: there are no records kept. But even if such observations would have been made the mariners of around 1500 could not have deduced their longitude because of a lack of the required mathematical background.

In the beginning of the sixteenth century, mathematicians, astronomers and cartographers, notably Gemma Frisius (1508-1555) [2], had advanced the mathematics of spherical triangulation to the point that they could realistically suggest obtaining the time and thus the longitude at sea from a measurement of the distance between the moon and the sun or a planet or a fixed star.

The moon loses a full circle to the sun in 29.5 days. In the navigator's geocentric world their directions are like the hands of a giant clock, the angle between them changing by 30.5"/min. If the positions of the moon could be predicted well enough and sufficiently in advance, the angle between it and the sun, the *lunar distance*, might be tabulated in the time of some standard meridian. The moon would then be a perfect, never failing clock. In those days the motion of the moon was not well enough understood and neither did the navigational instruments have sufficient accuracy. Yet considerable effort was put in establishing lunar tables for nautical use. A decisive step was made when Isaac Newton established the law of gravitation [3]. It gave the necessary scientific background to the calculation of the motion of celestial bodies, which before had been merely phenomenology. Still it was only in the second half of the eighteenth century, mostly through the work of German astronomers like Johann Tobias Mayer of Göttingen (1723-1762), that lunar distances could be predicted with errors no larger than 1 arcmin. By that time the sextant was already in use. It was Nevil Maskelyne who published the first systematic tabulation of lunar distances, which was based on Mayers tables [4].

Eight years earlier, in 1759, John Harrison had succeeded in making the first useful marine chronometer, for which he was rewarded with £20,000 by theBritish Government. Not long after him, the French clockmaker Berthoud was also able to produce a reliable timekeeper.

And so, within a few years, two methods had become available by which longitude at sea could be obtained. The problem of longitude had finally been solved. A good survey of the history of time measurement is given by Derek Howse [5].

In the beginning chronometers could not be produced in large quantities and in the following years the lunar distance was the more widely used method. James Cook, on his voyage with the *Endeavour*, was one of the first users. For doing the elaborate calculations necessary to deduce the time from the observed lunar distance, he had the help of an astronomer, appointed to this task by the Government.

Principal and practice of the lunar distance method.

Deducing from a lunar observation, or *lunar* for short, the time and the longitude is a complicated procedure. The scheme of the solution is, however, simple and is illustrated in Figure 1:

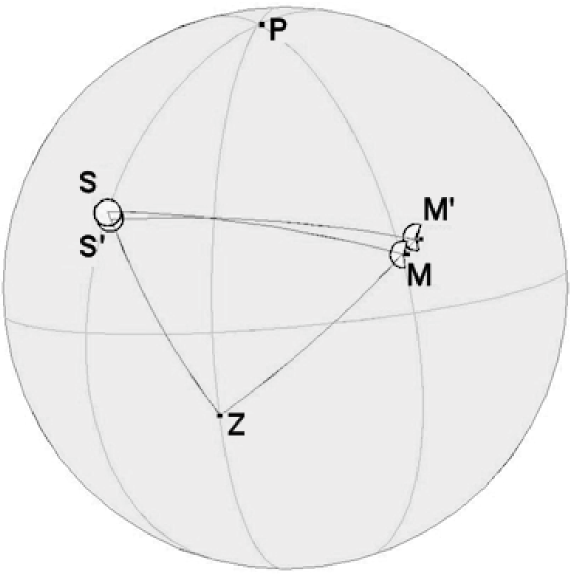

Fig. 1: The situation at the time of the Slocum's lunar distance observation near the Marquesas on June 16, 1896. Positions are shown, projected onto the surface of the earth, in an outward-in view. P = *(North) pole,* S' = *apparent sun,* M' = *apparent moon,* S = *true sun,* M = *true moon,* Z = *the observer's zenith.*

1) From his position (Z) the observer measures the apparent lunar distance between the bright limbs (*d''*), the lower-limb altitude (*H''*) of the sun and the lower-limb altitude (*h''*) of the moon. The altitudes must be reduced to their values at the observation time of the lunar distance, either by calculation or by interpolation of several observations. Ideally the three measurements should be taken simultaneously by three different observers, at the moment indicated by the person who takes the lunar distance, while a fourth person reads the chronometer. For one single observer Bowditch [6] recommends taking both altitudes twice, first preceding the observation of the lunar distance and thereafter once more and in reverse order, noting the chronometer time of each observation. Raper [7] makes a further recommendation that the altitudes of the object farthest from the meridian should be taken as the first and the last.

The measured quantities are first corrected for semidiameters. The altitudes are also corrected for dip to give the apparent altitudes *H'* and *h'*. The apparent zenith distances ZS' = 90º- *H'* and ZM' = 90º - *h'*, together with the apparent lunar distance *d'* = S'M' fix the triangle ZS'M'.

2) The apparent altitudes must be further corrected for refraction and for to give the true altitudes *H* and *h*. These are both "vertical" corrections, which do not influence the enclosed azimuthal angle, so that $Z_{SM} = Z_{S'M'}$. This is the crux that makes the lunar distance method work: the spherical triangle ZSM is fixed and the true lunar distance, *d* = SM follows.

3) With *d* known, the Greenwich mean time (GMT) is found by interpolation in the lunar distance tables.

4) GMT being known, the declinations of the sun and the moon can be looked up from the *Almanac*. With the latitude adopted from the last meridian passage and dead-reckoning , their local hour angles are found from the triangles PSZ and PMZ, respectively, whereafter the longitude follows as *LONG = GHA-LHA.*

Until the year 1834 the *Almanac* gave all quantities in *apparent time*, which is immediately obtained from the sun's position. In order to make the *Almanac* also suitable for astronomical use, the ephemerides were given in *astronomical mean time* from 1834 on. The beginning of the astronomical day was taken to be the Greenwich meridian passage of the mean sun, while the *civil day* started twelve hours earlier, at midnight. In 1925 the astronomical day was redefined to coincide with the civil day.

In the appendix, the interested reader will find a simulation of Joshua Slocum's [8] observation on June 16, 1896, using the methods that were available to the late-eighteenth and nineteenth century navigator. Part of this simulation has appeared in an earlier article [9].

Before the end of the eighteenth century different mathematical reduction procedures were introduced by Lyons, Dunthorne, Maskelyne, Krafft and De Borda. De Borda's method has long been considered as the best. During the first half of the nineteenth century many more methods were introduced which aimed at reducing the calculational burden by the use of tables. For example, one finds four different procedures in the 1849 edition of Nathaniël Bowditch's famous handbook [6].

During the nineteenth century, chronometers became generally available. It became common practice to use the lunar distance method to find the chronometer's correction and rate, thus keeping it regulated at the standard time. The obvious advantage was that the fourth step, the determination of the local time, might then be done at any other instant.

Already by the middle of the century ships would generally have been equipped with several chronometers. Because the time at sea between ports also became shorter, the chronometer time became more reliable than what could be achieved from a lunar distance observation.This meant that lunars were only seldom used in the second half of the nineteenth century. In his handbook [10], published for the first time in 1881 and reprinted almost yearly, Captain Lecky (1838-1902) stated: "The writer of these pages, during a long experience at sea in all manner of vessels...., has not fallen in with a dozen men who had themselves taken Lunars, or even had seen them taken."

In 1902 a review of the history of lunar observation by E. Guyou, member of the French Bureau de Longitude, appeared in *La Revue Maritime* [11]. In this article it was announced that the publication of lunar distance tables in the *Connaissance du Temps* would be stopped from 1905. The *Nautical Almanac* continued to publish lunar distances until 1907. It is, therefore, all the more interesting that maybe the best known lunar observation, if not in history then in literature, was made as late as 1896.

Joshua Slocum's observation on June 16, 1896.

On April 24, 1895, Captain Joshua Slocum set out on his voyage that was to become the first solitary circumnavigation of the globe. His account of this enterprise [8], has become a classic. Figure 2 shows a portrait of Slocum from the 1949 edition.

CAPTAIN JOSHUA SLOCUM

Fig. 2: Captain Joshua Slocum. Pen drawing by A.E. Berbank from Sailing alone around the world*, The Reprint Society, London (1949).*

His book is also interesting because it tells the story of a very keen self-made navigator and literate man, but with no formal education in navigation. There were many like him in the second half of the nineteenth century, captain-owners of sailing ships. Crews were small in those times, especially because of the increasing competition of engine-driven vessels, and often these captains would be the only ones aboard with navigational knowledge. This forms a contrast with earlier times when trade across the oceans was the exclusive domain of larger companies, like for instance the Dutch East- and West Indian Companies, which saw to it that crews should count among them a sufficient number of men with a proper education.

Slocum has become the Adam figure for yachtsmen and even today we are still fascinated by the question of how he navigated. From his time as captain-owner of a moderately large sailing ship he had kept a chronometer. However, it needed repairing which would have cost $15 an amount Slocum was reluctant to spend. Nevertheless: "In our newfangled notions of navigation it is supposed that a mariner cannot find his way without one; and I had myself drifted into this way of thinking." He finds a compromise and buys an old tin clock, discounted from $1,50 to $1,00.

About his navigation on the Atlantic crossings, Slocum gives little detail. It appears that he has limited himself to meridian altitudes: "On September 10 the *Spray* passed the island of St. Antonio, the northwesternmost of the Cape Verdes. The landfall was wonderfully true, considering that no observations for longitude had been made." However, "...the steamship *South Wales* spoke to the *Spray* and unsolicited gave her the longitude by chronometer[1] as 48º W, 'as nearly as I can make it,' the captain said. The *Spray*, with her tin clock, had exactly the same reckoning." Evidently, his clock worked well at that time. However, it is clear that it gradually loses its reliability, although Slocum does not mention this explicitly.

Then, in the Pacific- it is 1896 now - Slocum describes in some detail how he regained the time by making a lunar distance observation. This observation can be dated accurately: he leaves the island Juan Fernández on May 5 and "on the forty-third day from land-a long time to be alone,- the sky being beautifully clear and the moon being 'in distance' with the sun, I threw up my sextant for sights. I found from the result of three observations, after long wrestling with lunar tables, that her longitude agreed within five miles of that by dead-reckoning."

The day is June 16. The moon is close to first quarter and the observation must have been made in the (local) afternoon. We know his position as well because he sights the southernmost island of the Marquesas on the same day.

It is interesting to construct a simulation of his observations and work them out by using the *Nautical Almanac* tables and the reduction methods that were available to him. In this way we can see what is involved and get an impression of his "wrestling". We can also get an idea of the accuracy that can be achieved. This simulation is given in the last section, together with a survey of some mathematical methods. We assume that Slocum used the *Nautical Almanac*. In Montevideo or in Buenos Aires and maybe already in Gibraltar he had had the opportunity to purchase the volume for 1896 (price: 2 shilling and sixpence).

[1] Addendum to the original article: Slocum seems to have done some observations for longitude after all. "Longitude by chronometer" means that the local hour angle is found from an observed solar height and a guessed latitude. With the sun's GHA at the time of the observation, taken from the *Almanac*, the longitude follows as GHA-LHA.

The distances from the moon to the sun and to a number of prominent planets and stars that are close to the ecliptic (the path of the sun) were tabulated in the *Nautical Almanac* for every third hour. About these, the *Almanac* says in its *Explanations*:

Lunar Distances.-These pages contain, for every third hour of Greenwich mean time, the angular distances, available for the determination of the longitude, of the apparent center of the moon from the sun, the larger planets and certain stars as they would appear from the center of the Earth. When a Lunar Distance has been observed, and reduced to the center of the Earth, by clearing it from the effects of Parallax and refraction, the numbers in these pages enable us to ascertain the exact Greenwich mean time at which the objects would have the same distance.

Since 1907 lunar distances have no longer been tabulated. One can, however, always construct them from the declination- and hour-angle tables via the spherical triangle PSM in Figure 1:

$$\cos(d) = \sin(DEC_S)\sin(DEC_M) + \cos(DEC_S)\cos(DEC_M)\cos(GHA_S - GHA_M) \qquad (1)$$

Modern computer programs on celestial mechanics exist nowadays in PC versions [12] and they allow one to look back in time and verify the tables of the *Almanac*. That is necessary, because Slocum writes that he has discovered an error in them: “The first set of sights... put her many hundred miles west of my reckoning by account...Then I went in search of a discrepancy in the tables, and I found it.”
Tables from the *Nautical Almanac* for June 1896 are shown in Figure 3. When checking these tables against the computer, all values reproduce very accurately, except for the times for the moon’s right ascension and declination, which appear to be shifted by 12 hours. Only this can be the “error” that Slocum mentions. However, the tables of the lunar distances count the hours starting at noon, rather then at midnight. The discrepancy in the moon’s tables disappears when also here the times are understood as hours after noon. In the *Almanac’s* chapter *Explanations*, it is found that this is indeed the way in which the tables are organized: “Thus, suppose the Right Ascension of the Moon were required at $9^h\ 40^m$ A.M. mean civil time on April 22, 1896, or April 21, $21^h\ 40^m$ mean astronomical time....” All times are thus astronomical times and *Mean Noon* is counted as 0 hours, whereas it is 12 hours in civil time. In the lunar distance tables Noon and Midnight are indicated explicitly and no confusion is possible. But for the tables of the moon's right ascension and declination, the place where the change of the date is indicated, misleadingly suggests the use of civil time. Slocum “corrected” this 12 hour shift and sailed on “with his tin clock fast asleep.”
In the Indian Ocean, the tin clock loses its minute-hand and even has to be boiled to make it run again. On this long passage, Slocum again finds the longitude from the sun's meridian transit. But the lunar distance method must have served to retrieve the time after the clock had stopped.
In Cape Town he meets an astronomer, Dr. David Gill, and they discuss the determination of the standard time at sea by the lunar distance method. He even presents a talk about it at Gill’s Institute. This is an amusing episode: Gill was a famous man. His elaborate photographs of the southern skies formed the basis on which the Dutch astronomer J.C. Kapteyn could base his model of the Milky Way.
Astronomers in those days knew very well that the standard time could be obtained from lunar distances. They practised these methods themselves with an accuracy far beyond that of marine navigators. One can almost picture Gill and his students, being kind to this old sailor, who rediscovers methods that were introduced more than a century before his time and that were already becoming obsolete.
Rediscover, indeed: chronometers had long been standard equipment aboard ships and they were good enough to serve on an ocean crossing without the need of checking them by a lunar. In most ports their error could be established by time signals. By leaving behind his chronometer, Slocum had put himself back almost one century, to the time that the lunar observation had to be worked out to give not only the mean time but also the local time.
Slocum must have used the lunar distance method during his long career as captain on his own ship. Most probably he used only the first half of the method to find the Greenwich mean time, and therewith the chronometer error. Meridian passages were then good enough to him for finding the longitude. How else could it be that he makes mistakes in his first attempts, which he blames on the *Nautical Almanac*? And why would he mention his observation at all if it would have been routine to him?
Yes, Slocum rediscovered how to do the lunar distance method in its original form, with no other clock than the moon. It is not without a certain Don Quixotry when he writes that he feels his vanity "tickled" when his observations of June 16, 1896, come out so nicely. But we should give him the credit that he deserves: it was a great achievement to re-master this almost extinct art.

Conclusions

Finding the time and the longitude at sea by the lunar distance method developed over a period starting in the early sixteenth century to the end of the eighteenth century. When finally lunar distances of sufficient accuracy could be calculated in advance and the *Nautical Almanac* began their yearly publication, the chronometer had likewise advanced to the perfection that was needed for marine purposes. Thus the lunar distance method could blossom for no more than half a century. To this, we have the testimony of Captain

Lecky, author of *Wrinkles in Practical Navigation*, who states that he met no more than a dozen men who had ever taken a lunar or seen one taken. Lecky was born in 1838 and went to sea in the 1850s.

Around the turn of the century there were vivid discussions as to whether or not it would be wise to keep up the knowledge of lunars. The arguments in favor can be found in issues of the *Nautical Magazine* of the years 1900-1905. These arguments were certainly not free of nostalgia, but, in the words of Lord Dunraven, cited in *Wrinkles*: “You never need work one at sea unless it amuses you to do so.”

This situation was similar to what we see today: should the practice of finding ones position by use of a sextant be kept up? Does Dunraven's remark fit here just like it did one hundred years ago, or is the step to abandon the sextant more drastic than that of giving up the lunar distance method? A hundred years ago, the lunar was a backup for the case that the chronometer would fail. Especially if a ship carried more than one chronometer the likelihood of losing the time seems less than that of a failure to receive satellite signals. We shall not take a position in this discussion, but be content with the statement that astronavigation by sextant is indeed amusing.

MEAN TIME.

THE MOON'S RIGHT ASCENSION AND DECLINATION.

Hour.	Right Ascension.	Var. in 10m.	Declination.	Var. in 10m.	Hour.	Right Ascension.	Var. in 10m.	Declination.	Var. in 10m.
	Saturday 13.					**Monday 15.**			
	h m s	s	° ′ ″	″		h m s	s	° ′ ″	″
0	7 24 5·54	24·090	N. 25 9 50·0	70·95	0	9 15 15·63	22·155	N. 16 53 41·7	131·31
1	7 26 29·99	24·059	25 2 39·8	72·46	1	9 17 28·44	22·114	16 40 31·0	132·26
2	7 28 54·25	24·028	24 55 20·5	73·96	2	9 19 41·00	22·074	16 27 14·6	133·19
3	7 31 18·32	23·995	24 47 52·3	75·44	3	9 21 53·33	22·035	16 13 52·7	134·12
4	7 33 42·19	23·961	24 40 15·2	76·92	4	9 24 5·42	21·996	16 0 25·2	135·03
5	7 36 5·85	23·927	24 32 29·2	78·40	5	9 26 17·28	21·957	15 46 52·3	135·93
6	7 38 29·31	23·892	24 24 34·4	79·87	6	9 28 28·90	21·918	15 33 14·0	136·82
7	7 40 52·55	23·857	24 16 30·8	81·33	7	9 30 40·30	21·881	15 19 30·4	137·70
8	7 43 15·59	23·821	24 8 18·5	82·77	8	9 32 51·47	21·843	15 5 41·6	138·56
9	7 45 38·40	23·783	23 59 57·6	84·20	9	9 35 2·42	21·806	14 51 47·7	139·40
10	7 48 0·99	23·747	23 51 28·1	85·63	10	9 37 13·14	21·769	14 37 48·8	140·23
11	7 50 23·36	23·708	23 42 50·1	87·04	11	9 39 23·65	21·733	14 23 44·9	141·05
12	7 52 45·49	23·669	23 34 3·6	88·45	12	9 41 33·94	21·698	14 9 36·2	141·85
13	7 55 7·39	23·631	23 25 8·7	89 85	13	9 43 44·02	21·663	13 55 22·7	142·65
14	7 57 29·06	23·592	23 16 5·4	91·24	14	9 45 53·89	21·628	13 41 4·4	143·43
15	7 59 50·49	23·552	23 6 53·8	92·62	15	9 48 3·56	21·595	13 26 41·6	144·18
16	8 2 11·68	23·512	22 57 34·0	93·98	16	9 50 13·03	21·561	13 12 14·2	144·94
17	8 4 32·63	23·471	22 48 6·1	95·33	17	9 52 22·29	21·528	12 57 42·3	145·68
18	8 6 53·33	23·430	22 38 30·1	96·68	18	9 54 31·37	21·497	12 43 6·0	146·40
19	8 9 13·79	23·389	22 28 46·0	98·01	19	9 56 40·25	21·464	12 28 25·5	147·11
20	8 11 34·00	23·348	22 18 54·0	99·33	20	9 58 48·94	21·433	12 13 40·7	147·81
21	8 13 53·96	23·305	22 8 54·1	100·64	21	10 0 57·45	21·403	11 58 51·8	148·48
22	8 16 13·66	23·263	21 58 46·3	101·94	22	10 3 5·78	21·373	11 43 58·9	149·15
23	8 18 33·11	23·221	N. 21 48 30·8	103·22	23	10 5 13·93	21·344	N. 11 29 2·0	149·81
	Sunday 14.					**Tuesday 16.**			
0	8 20 52·31	23·178	N. 21 38 7·7	104·48	0	10 7 21·91	21·316	N. 11 14 1·2	150·45
1	8 23 11·25	23·135	21 27 37·0	105·75	1	10 9 29·72	21·288	10 58 56·6	151·08
2	8 25 29·93	23·093	21 16 58·7	107·01	2	10 11 37·37	21·262	10 43 48·3	151·69
3	8 27 48·36	23·050	21 6 12·9	108·24	3	10 13 44·86	21·236	10 28 36·3	152·29
4	8 30 6·53	23·006	20 55 19·8	109·46	4	10 15 52·20	21·210	10 13 20·8	152·88
5	8 32 24·43	22·963	20 44 19·4	110·68	5	10 17 59·38	21·184	9 58 1·8	153·44
6	8 34 42·08	22·920	20 33 11·7	111·88	6	10 20 6·41	21·161	9 42 39·5	154·00
7	8 36 59·47	22·877	20 21 56·8	113·07	7	10 22 13·31	21·138	9 27 13·8	154·55
8	8 39 16·60	22·833	20 10 34·9	114·24	8	10 24 20·06	21·114	9 11 44·9	155·08
9	8 41 33·46	22·789	19 59 5·9	115·41	9	10 26 26·68	21·092	8 56 12·8	155·60
10	8 43 50·07	22·747	19 47 30·0	116·56	10	10 28 33·17	21·071	8 40 37·7	156·10
11	8 46 6·42	22·703	19 35 47·2	117·70	11	10 30 39·53	21·050	8 24 59·6	156·59
12	8 48 22·51	22·660	19 23 57·6	118·82	12	10 32 45·77	21·031	8 9 18·6	157·07
13	8 50 38·34	22·617	19 12 1·3	119·93	13	10 34 51·90	21·012	7 53 34·8	157·53
14	8 52 53·91	22·573	18 59 58·4	121·03	14	10 36 57·91	20·993	7 37 48·3	157·98
15	8 55 9·22	22·531	18 47 49·0	122·12	15	10 39 3·82	20·977	7 21 59·1	158·42
16	8 57 24·28	22·488	18 35 33·0	123·19	16	10 41 9·63	20·960	7 6 7·3	158·83
17	8 59 39·08	22·445	18 23 10·7	124·25	17	10 43 15·34	20·944	6 50 13·1	159·24
18	9 1 53·62	22·403	18 10 42·0	125·30	18	10 45 20·96	20·929	6 34 16·4	159·64
19	9 4 7·92	22·362	17 58 7·1	126·33	19	10 47 26·49	20·915	6 18 17·4	160·02
20	9 6 21·96	22·319	17 45 26·1	127·35	20	10 49 31·94	20·903	6 2 16·2	160·38
21	9 8 35·75	22·277	17 32 38·9	128·36	21	10 51 37·32	20·890	5 46 12·8	160·74
22	9 10 49·29	22·236	17 19 45·8	129·35	22	10 53 42·62	20·878	5 30 7·3	161·08
23	9 13 2·58	22·195	17 6 46·7	130·34	23	10 55 47·85	20·868	5 13 59·9	161·40
24	9 15 15·63	22·155	N. 16 53 41·7	131·31	24	10 57 53·03	20·858	N. 4 57 50·5	161·72

AT MEAN NOON.

Day of the Week.	Day of the Month.	THE SUN'S Apparent Right Ascension.	THE SUN'S Apparent Declination.	THE SUN'S Semi-diameter.*	Equation of Time, to be added to / subt. from Mean Time.	Sidereal Time.
		h m s	° ′ ″	′ ″	m s	h m s
Mon.	1	4 39 31·82	N. 22 9 57·8	15 47·5	2 20·15	4 41 51·97
Tues.	2	4 43 37·80	22 17 38·2	15 47·3	2 10·73	4 45 48·53
Wed.	3	4 47 44·17	22 24 55·3	15 47·2	2 0·92	4 49 45·08
Thur.	4	4 51 50·91	22 31 48·9	15 47·1	1 50·73	4 53 41·64
Frid.	5	4 55 58·01	22 38 18·9	15 46·9	1 40·19	4 57 38·20
Sat.	6	5 0 5·45	22 44 25·1	15 46·8	1 29·31	5 1 34·76
Sun.	7	5 4 13·20	22 50 7·4	15 46·7	1 18·12	5 5 31·32
Mon.	8	5 8 21·24	22 55 25·6	15 46·6	1 6·64	5 9 27·88
Tues.	9	5 12 29·55	23 0 19·6	15 46·5	0 54·89	5 13 24·43
Wed.	10	5 16 38·10	23 4 49·3	15 46·4	0 42·89	5 17 20·99
Thur.	11	5 20 46·88	23 8 54·7	15 46·3	0 30·67	5 21 17·55
Frid.	12	5 24 55·85	23 12 35·6	15 46·2	0 18·26	5 25 14·11
Sat.	13	5 29 4·98	23 15 51·9	15 46·1	0 5·69	5 29 10·67
Sun.	14	5 33 14·25	23 18 43·6	15 46·0	0 7·02	5 33 7·23
Mon.	15	5 37 23·63	23 21 10·6	15 46·0	0 19·84	5 37 3·79
Tues.	16	5 41 33·09	23 23 12·8	15 45·9	0 32·74	5 41 0·34
Wed.	17	5 45 42·60	23 24 50·3	15 45·8	0 45·70	5 44 56·90
Thur.	18	5 49 52·14	23 26 3·0	15 45·8	0 58·68	5 48 53·46
Frid.	19	5 54 1·69	23 26 50·9	15 45·7	1 11·67	5 52 50·02
Sat.	20	5 58 11·22	23 27 14·0	15 45·7	1 24·64	5 56 46·58
Sun.	21	6 2 20·72	23 27 12·3	15 45·6	1 37·58	6 0 43·14
Mon.	22	6 6 30·17	23 26 45·9	15 45·6	1 50·47	6 4 39·70
Tues.	23	6 10 39·54	23 25 54·7	15 45·6	2 3·28	6 8 36·25
Wed.	24	6 14 48·81	23 24 38·8	15 45·5	2 16·00	6 12 32·81
Thur.	25	6 18 57·97	23 22 58·4	15 45·5	2 28·60	6 16 29·37
Frid.	26	6 23 7·00	23 20 53·3	15 45·5	2 41·07	6 20 25·93
Sat.	27	6 27 15·88	23 18 23·6	15 45·4	2 53·39	6 24 22·49
Sun.	28	6 31 24·58	23 15 29·4	15 45·4	3 5·53	6 28 19·05
Mon.	29	6 35 33·09	23 12 10·8	15 45·4	3 17·48	6 32 15 ·1
Tues.	30	6 39 41·38	23 8 27·7	15 45·4	3 29·21	6 36 12 ·6
Wed.	31	6 43 49·44	N. 23 4 20·4	15 45·4	3 40·72	6 40 ·72

* The Semidiameter for *Apparent* Noon may be assumed the same as that for *Mean* Noon.

MEAN TIME.

LUNAR DISTANCES.

Day.	Star's Name and Position.		*Midnight.*	P.L. of diff.	XVh.	P.L. of diff.	XVIIIh.	P.L. of diff.	XXIh.	P.L. of diff.
			° ′ ″		° ′ ″		° ′ ″		° ′ ″	
13	Saturn	E.	107 23 21	2522	105 42 39	2514	104 1 45	2505	102 20 39	2498
14	Sun	W.	44 55 10	2781	46 30 3	2772	48 5 7	2763	49 40 23	2756
	Spica	E.	73 14 21	2443	71 31 47	2436	69 49 3	2429	68 6 9	2423
	Saturn	E.	93 52 30	2460	92 10 21	2454	90 28 3	2448	88 45 36	2441
15	Sun	W.	57 39 18	2717	59 15 35	2710	60 52 1	2704	62 28 36	2697
	Spica	E.	59 29 28	2392	57 45 42	2387	56 1 48	2381	54 17 46	2375
	Saturn	E.	80 11 6	2411	78 27 47	2406	76 44 21	2401	75 0 48	2396
	Antares	E.	105 21 15	2388	103 37 23	2382	101 53 23	2376	100 9 14	2371
16	Sun	W.	70 33 40	2666	72 11 6	2661	73 48 39	2655	75 26 19	2649
	Jupiter	W.	28 32 48	2405	30 16 16	2400	31 59 51	2394	33 43 34	2389
	Spica	E.	45 35 41	2351	43 50 55	2346	42 6 2	2341	40 21 2	2337
	Saturn	E.	66 21 20	2374	64 37 8	2371	62 52 51	2367	61 8 29	2364
	Antares	E.	91 26 32	2344	89 41 36	2340	87 56 34	2334	86 11 24	2329
17	Sun	W.	83 36 30	2624	85 14 52	2619	86 53 21	2615	88 31 55	2610
	Jupiter	W.	42 23 52	2366	44 8 16	2362	45 52 45	2357	47 37 21	2354
	Regulus	W.	22 33 26	2309	24 19 13	2304	26 5 7	2300	27 51 7	2296
	Spica	E.	31 34 37	2319	29 49 5	2315	28 3 28	2313	26 17 48	2312
	Saturn	E.	52 25 40	2353	50 40 57	2352	48 56 13	2352	47 11 29	2352
	Antares	E.	77 23 52	2307	75 38 2	2302	73 52 6	2298	72 6 4	2294
18	Sun	W.	96 46 13	2591	98 25 20	2588	100 4 31	2585	101 43 47	2582
	Jupiter	W.	56 21 42	2336	58 6 49	2333	59 52 1	2329	61 37 18	2326
	Regulus	W.	36 42 34	2277	38 29 7	2274	40 15 45	2271	42 2 27	2268
	Saturn	E.	38 28 7	2363	36 43 39	2369	34 59 19	2375	33 15 9	2384
	Antares	E.	63 14 32	2277	61 27 58	2274	59 41 20	2270	57 54 37	2268
19	Sun	W.	110 1 1	2570	111 40 37	2569	113 20 15	2568	114 59 54	2566
	Jupiter	W.	70 24 38	2315	72 10 16	2313	73 55 56	2312	75 41 38	2311
	Regulus	W.	50 56 56	2257	52 43 59	2255	54 31 5	2253	56 18 14	2252
	Saturn	E.	24 38 51	2478	22 57 7	2513	21 16 12	2559	19 36 20	2619
	Antares	E.	49 0 5	2256	47 13 1	2255	45 25 55	2253	43 38 46	2252
	α Aquilæ	E.	101 42 19	1976	100 11 36	2966	98 40 41	2957	97 9 35	2951
20	Sun	W.	123 18 28	2566	124 58 10	2566	126 37 52	2568	128 17 31	2569
	Jupiter	W.	84 30 30	2308	86 16 18	2308	88 2 6	2309	89 47 53	2310
	Regulus	W.	65 14 18	2249	67 1 33	2249	68 48 47	2249	70 36 1	2251
	Antares	E.	34 42 42	2249	32 55 27	2249	31 8 13	2250	29 21 0	2251
	α Aquilæ	E.	89 32 32	2938	88 1 2	2939	86 29 33	2942	84 58 8	2947
21	Jupiter	W.	98 36 20	2318	100 21 53	2321	102 7 22	2324	103 52 46	2327
	Regulus	W.	79 31 42	2259	81 18 42	2262	83 5 38	2265	84 52 29	2268
	Spica	W.	25 33 26	2275	27 20 3	2276	29 6 38	2277	30 53 12	2279
	Antares	E.	20 25 22	2260	18 38 23	2262	16 51 28	2266	15 4 38	2270
	α Aquilæ	E.	77 23 9	2993	75 52 48	3008	74 22 45	3023	72 53 1	3042
22	Regulus	W.	93 45 24	2290	95 31 38	2296	97 17 44	2301	99 3 42	2307
	Spica	W.	39 44 57	2297	41 31 1	2302	43 16 57	2307	45 2 46	2313
	α Aquilæ	E.	65 30 48	3165	64 3 57	3198	62 37 45	3232	61 12 14	3270
	Fomalhaut	E.	88 26 41	2606	86 47 54	2613	85 9 17	2621	83 30 50	2629

Fig. 3—Tables from the Nautical Almanac *for June 1896 (The tables give lunar distances and the right ascensions and declinations for the sun and the moon. Note that the times are in* Mean Astronomical Time *and that the date changes at 12 h.)*

APPENDIX: Mathematical basis and practice of the Lunar Distance Method.

Introduction

This appendix describes in detail the different steps that are involved in deducing the longitude from a lunar distance observation. As an illustration, Table 1 presents a simulated set of observations, such as Joshua Slocum could have made on June 16, 1896, just South-East of the Marquesas Islands and we shall work them out with the methods available to him at that time.

Consider the spherical triangles M'ZS' and in Figure 1, where Z is the zenith of the observer. The arcs ZM' and ZM are the apparent and true zenith distances of the moon and likewise ZS' and ZS are those of the sun.We shall speak here of the sun, but it is understood that the following holds equally if S is to denote a star or planet.

Let the measured lunar distance and the altitudes, after reduction to a common time, be given by:

h' = 90º - ZM', the apparent altitude of the moon's center, corrected for dip.
H' = 90º - ZS', the apparent altitude of the sun's center, corrected for dip.
d' = the apparent distance of the centers.

After further correcting the altitudes for refraction and parallax, we have:

h = 90º - ZM, the moon's true altitude.
H = 90º - ZS, the sun's true altitude.
d = the required true distance of the centers.

We further denote:

Z = the azimuthal angle MZS, which equals M'ZS'.

Table 1: Simulation of Slocum's lunar observation on June 16, 1896

TIME	OBSERVATION		
T-6^m	$H_1'' =$	$41° \; 42'.4$	Sun, Lower Limb
T-3^m	$h_1'' =$	$48° \; 7'.2$	Moon, Lower Limb
T	$d'' =$	$70° \; 14'.6$	LD, Nearest Limbs
T+3^m	$h_2'' =$	$49° \; 25'.4$	Moon, Lower Limb
T+6^m	$H_2'' =$	$39° \; 36'.4$	Sun, Lower Limb

CORRECTIONS

MOON		SUN		LUNAR DIST.	
$h'' = \frac{1}{2}(h_1'' + h_2'') =$	$48° \; 46'.3$	$H'' = \frac{1}{2}(H_1'' + H_2'') =$	$40° \; 39'.4$	$d'' =$	$70° \; 14'.6$
dip $=$	-$2'.8$	dip $=$	-$2'.8$	s.d.$_S =$	$15'.8$
s.d.$_M =$	$16'.1$	s.d.$_S =$	$15'.8$	s.d.$_M =$	$16'.1$
$h' =$	$48° \; 59'.6$	$H' =$	$40° \; 52'.4$	$d' =$	$70° \; 46'.5$
Refr. $=$	-$0'.8$	Refr. $=$	-$1'.2$		
Par. $=$	$38'.6$	Par. $=$	$0'.1$		
$h =$	$49° \; 37'.4$	$H =$	$40° \; 51'.3$		

The fact that the azimuthal angle *Z* is common for the triangles M'ZS' and and MZS is the key to all lunar distance reduction-schemes that have been put into practice. The relations between *Z*, *d'*, *h'* and *H'* and between *Z*, *d*, *h* and *H* can be written in different ways. Today we would choose the cosine formula:

$$\cos(d') = \sin(h')\sin(H') + \cos(h')\cos(H')\cos(Z) \qquad (2a)$$
$$\cos(d) = \sin(h)\sin(H) + \cos(h)\cos(H)\cos(Z) \qquad (2b)$$

and use a pocket calculator or a computer to obtain cos(*Z*) from the first equation, and insert it in the second to obtain cos(*d*).

However, before the advent of pocket calculators which is after all very recent, this scheme was unpractical because it involves not only addition and subtraction but also multiplication and division. Our minds are trained to do the former operations quickly, but not the latter. Therefore the relation that expresses the required true lunar distance d in terms of *h'*, *H'*, *d'*, *h* and *H* must be of product form so that the procedure is reduced to addition and subtraction by taking the logarithms of the different factors.

De Borda's rigorous method of finding the true lunar distance.

An ingenious and rigorous scheme for deducing the true lunar distance was developed by Jean de Borda. It was the most widely used method during the first half of the nineteenth century and has stayed in use as long as lunar distances were measured. We present it here in the formulation as given by William Chauvenet [13]. A complete model for reducing a lunar distance observation following De Borda's method, can be found in a handbook on the subject by J.H. van Swinden [14].

The method uses the fact that the relation between the angles *Z*, *h*', *H*' and *d*' may alternatively be written as:

$$\cos^2(\frac{1}{2}Z) = \frac{\cos[\frac{1}{2}(h'+H'+d')]\cos[\frac{1}{2}(h'+H'-d')]}{\cos(h')\cos(H')} \quad (3)$$

Of course the same relation holds for the unprimed angles.Yet another form, valid for the primed and unprimed angles alike, but used here for the unprimed ones, is:

$$\sin^2(\frac{1}{2}d) = \cos^2(\frac{1}{2}[h+H]) - \cos(h)\cos(H)\cos^2(\frac{1}{2}Z) \quad (4)$$

Eliminating the factor $\cos^2(\frac{1}{2}Z)$ from the above equations, and writing for brevity:

$$m = \frac{1}{2}(h'+H'+d')$$

yields:

$$\sin^2(\frac{1}{2}d) = \cos^2(\frac{1}{2}[h+H]) - \frac{\cos(h)\cos(H)}{\cos(h')\cos(H')}\cos(m)\cos(m-d') \quad (5)$$

Defining now an auxiliary angle *M* by:

$$\sin^2(M) = \frac{\cos(h)\cos(H)\cos(m)\cos(m-d')}{\cos(h')\cos(H')\cos^2(\frac{1}{2}[h+H])} \quad (6)$$

leads finally to:

$$\sin(\frac{1}{2}d) = \cos(\frac{1}{2}[h+H])\cos(M) \quad (7)$$

Equations (6) and (7) are of the desired product form. With the help of tables of log cos and log sin, the angle *M* is obtained from eq. (6), whereafter the true lunar distance d follows from the eq. (7).
The derivation of the true lunar distance from Slocum's simulated observation of June 16, 1896, is presented in Table 2.

Table 2: Finding the true lunar distance by De Borda's method

$d' =$	70° 46′.5					
$h' =$	48° 59′.6	log sec	0.18300			
$H' =$	40° 52′.4	log sec	0.12139			
$m =$	80° 19′.25	log cos	9.22565			
$m - d' =$	9° 32′.75	log cos	9.99394			
$h =$	49° 37′.4	log cos	9.81145			
$H =$	40° 51′.3	log cos	9.87873			
			———	add		
			9.21416			
		log $\sqrt{\text{above}}$	9.60708			
$\frac{1}{2}(h+H) =$	45° 14′.35	log cos	9.84766	....	9.84766	
			———	sub		
		log sin M	9.75942	log cos M	9.91296	
					———	add
				log sin($\frac{1}{2}d$)	9.76062	
$\frac{1}{2}d =$	35° 11′.3					
$d =$	70° 22′.6					

Approximative methods for finding the true lunar distance.

Many different ways for clearing the lunar distance from the effects of refraction and parallax have been developed. In particular it was desired to make the method pedagogically transparent by making the corrections additive, so that the procedure would take the form:

$$d = d' + a(h - h') + b((H - H') \qquad (8)$$

This necessarily entails an approximative method, which, however can be made sufficiently accurate for all practical purposes. Bowditch gives four different approximative schemes for deducing the true lunar distance in the 1849 edition of his handbook. The formulas of Bowditch's fourth metod are given and again applied to the simulation of Slocum's observation.

By introducing an auxiliary angle *A*, defined through,

$$\frac{\tan(\frac{1}{2}[h' + H'])}{\tan(\frac{1}{2}[h + H'])} \tan(\frac{1}{2} d') = \tan(A) \qquad (9)$$

the reduction can be cast in the form:

$$d = d' + \frac{\tan(h')}{\tan(A + \frac{1}{2} d')}(h - h') - \frac{\tan(H')}{\tan(A - \frac{1}{2} d')}(H - H') + 3^{rd} \text{ corr.} \qquad (10)$$

where the 3^{rd} correction is always very small and can be looked up in a table.

In working out this method, one needs the socalled *proportional logarithms*, a clever method for making interpolations, that is described in the next subsection.

It will be noted from the example in Table 3 below, that Bowditch's fourth method is by no means less complicated or time-consuming than the rigorous method of de Borda, and neither is any of his other three methods.

Table 3: Finding the true lunar distance by Bowditch's 4th method

$h' =$	$48^\circ\ 59'.6$		$h =$	$49^\circ\ 37'.4$	corr $h =$	$37'.8$	
$H' =$	$40^\circ\ 52'.4$		$H =$	$40^\circ\ 51'.3$	corr $H =$	-$1'.1$	
	————	+					
Sum $=$	$89^\circ\ 52'.0$		$\frac{1}{2}$Sum $=$	$44^\circ\ 56'.0$	log tan	9.99899	
Diff $=$	$8^\circ\ 7'.2$		$\frac{1}{2}$Diff $=$	$4^\circ\ 3'.6$	log cot	11.14887	
$d' =$	$70^\circ\ 46'.5$		$\frac{1}{2}d' =$	$35^\circ\ 23'.25$	log tan	9.85146	
						————	+
Angle $A =$	$84^\circ\ 16'.8$		$\Leftarrow$		log tan	0.99932	
			$A - \frac{1}{2}d' =$	$48^\circ\ 53'.55$	log tan	10.05919	
			$H' =$	$40^\circ\ 52'.4$	log cot	10.06278	
			$-$corr $H =$	$1'.1$	P.L.	2.21388	
						————	+
1st corr $=$	$0'.8$		$\Leftarrow$		P.L.	2.33585	
			$A + \frac{1}{2}d' =$	$119^\circ\ 40'.05$	log tan	10.24440	
			$h' =$	$48^\circ\ 59'.6$	log cot	9.93927	
			corr $h =$	$37'.8$	P.L.	0.67778	
						————	+
2nd corr $=$	$24'.8$		$\Leftarrow$		P.L.	0.86145	

$d' =$	$70^\circ\ 46'.5$		
1st corr $=$	$0'.8$		
	————	+	
	$70^\circ\ 47'.3$		
2nd corr $=$	$24'.8$		
	————	-	
	$70^\circ\ 22'.5$		
3rd corr $=$	$0'.1$		from table XX
	————	+	
$d =$	$70^\circ\ 22'.6$		

Finding the time by use of proportional logarithms.

The interpolation in the lunar distance tables is done with the help of proportional logarithms (P.L.), which are defined by:

$$\text{P.L.}(x) = \log(\frac{3}{x}) \qquad (11)$$

The proportional logarithms can be found in [6]. The tabulation is made for every second between zero and three hours. Since the subdivision of hours in 60 min and of minutes again in 60 s is identical to the subdivision of a degree, the tabulation applies equally to time and angles.

Let d be the deduced true lunar distance, which is found to be in between the tabulated values d_1 and d_2, given in the *Nautical Almanac* at times T_1 and T_2, respectively. The tabulation is given for every third hour; hence $(T_2\text{-}T_1) = 3$ h. Assuming the rate of change of d constant over this time interval, one has

$$\log(\frac{T_2 - T_1}{T - T_1}) = \log(\frac{d_2 - d_1}{d - d_1}) \qquad (12)$$

or, putting explicitly $T_2\text{-}T_1 = 3$ h,

$$\log(\frac{3^h}{T - T_1}) = \log(\frac{3^\circ}{d - d_1}) - \log(\frac{3^\circ}{d_2 - d_1}) \qquad (13)$$

which, by the definition of the proportional logarithm (11) becomes

$$\text{P.L.}(T - T_1) = \text{P.L.}(d - d_1) - \text{P.L.}(d_2 - d_1) \qquad (14)$$

Table 4 provides an illustration of this method.

Table 4: Finding the time by use of proportional logarithms

T =	to be found	$d =$	$70° \, 22' \, 36''$		
T_1=	$9^h \, 00^m \, 00^s$	d_1=	$68° \, 56' \, 23''$		
		$d - d_1$=	$1° \, 26' \, 13''$	P.L.=	0.3197
T_2=	$12^h \, 00^m \, 00^s$	d_2=	$70° \, 33' \, 40''$		
T_1=	$9^h \, 00^m \, 00^s$	d_1=	$68° \, 56' \, 23''$		
		d_2-d_1=	$1° \, 37' \, 17''$	P.L.=	0.2672
				P.L.=	0.0525
				approx. T-T_1 =	$2^h \, 39^m \, 30^s$
				tabular corr. =	2^s
				T_1 =	$9^h \, 00^m \, 00^s$
				T =	$11^h \, 39^m \, 32^s$

Note: times are in mean astronomical time, 12 hours ahead of mean civil time.

Usually this was accurate enough. But for small lunar distances such as could occur when measuring the distance between the moon and a star or a planet, the rate of change can vary sufficiently rapidly to cause errors as large as 1 min. A parabolic interpolation using also the lunar distance differences over the preceding and the following 3 h time intervals is then required. The *Nautical Almanac* provided a table with this additional time correction.

Finding the local hour angle and the longitude.

With the correct mean time established as above from the true lunar distance, the ship's chronometer, and often also its rate, is calibrated. An observation from which some local hour angle is to be determined can then be made at any later instant. However, the altitudes of the moon and the sun have been taken as simultaneous with the lunar distance. If the measurements were made one after the other, as was practice in the case of only one observer, these altitude measurements have been reduced to the same time as the observation of the lunar distance. They are therefore suitable for providing the desired hour angles.
As a preparation for the calculation one had to find the declinations DEC_M and DEC_S of the moon and the sun at the established mean time by interpolation in the *Nautical Almanac* tables. The declination of the moon was tabulated for every hour along with its variation over every 10 min. period. The interpolation within the last 10 min. interval was left to the practitioner. The sun's declination was only given at mean noon. It's daily variation being small, the interpolation was also left to do by heart.
As an aside, it is remarkable that a faster interpolation, with the help of proportional logarithms, suited to find the moon's declination and right ascension, seems never to have been used. For angular values ϕ, that are tabulated for every hour ($T_2 - T_1 = 1$ h), the formula would be

$$\mathrm{P.L.}(T - T_1) - \mathrm{P.L.}(1) = \mathrm{P.L.}(\phi - \phi_1) - \mathrm{P.L.}(\phi_2 - \phi_1) \qquad (15)$$

where P.L.(1) = log(3) = 0.4771 is just a constant.
With a slight modification, the method can even be used for slowly varying angular values that are tabulated in 24^h intervals, such as the sun's declination and right ascension: omitting in the time differences the seconds and reading seconds for the minutes and minutes for the hours, all values are brought within the range of the tabulation range of the proportional logarithm. One then has:

$$\mathrm{P.L.}(T - T_1) - \mathrm{P.L.}(24^h) = \mathrm{P.L.}(\phi - \phi_1) - \mathrm{P.L.}(\phi_2 - \phi_1) \qquad (16)$$

where again P.L.(24^h) = 0.8751 is a constant. The accuracy in time up to 1 min. would be acceptable for nautical, but not for astronomical purposes.
Besides the two declinations, one needed a good guess for the latitude by dead-reckoning from the last meridian passage observation. The local hour angles LHA_M\$ and LHA_S are then found from the spherical triangles PZM and PZS, respectively, where P is the North Pole. Again the relevant formula must have a product form, to make the procedure additive in terms of logarithms. With $z = 90^o - h$ or $z = 90^o - H$, denoting the zenith distance, the most convenient form is:

$$\cos^2\left(\frac{1}{2}LHA\right) = \frac{\cos[\frac{1}{2}(DEC + LAT + z)]\cos[\frac{1}{2}(DEC + LAT - z)]}{\cos(DEC)\cos(LAT)} \qquad (17)$$

Table 5 shows the determination of the hour angle both for the moon and the sun from Slocum's observations, and finally in Table 6 the steps are given that lead from the local hour angle to the longitude.

Table 5: Finding the moon's and the sun's local hour angles

MOON			
$DEC^{a)} =$	8° 14′ 39″	log sec	0.00451
$LAT^{b)} =$	-10° 38′ 00″	log sec	0.00752
$Z =$	40° 22′ 36″		
Sum =	37° 59′ 15″		
$\frac{1}{2}$Sum =	18° 59′ 38″	log cos	9.97571
$\frac{1}{2}$Sum-Z =	-21° 22′ 58″	log cos	9.96903
			——— add
		log cos^2	9.95677
$\frac{1}{2}LHA =$	17° 55′ 38″	⟸ log cos	9.97839
$LHA =$	35° 51′ 16″	East of Meridian	
SUN			
$DEC^{a)} =$	23° 24′ 00″	log sec	0.03727
$LAT^{b)} =$	-10° 38′ 00″	log sec	0.00752
$Z =$	49° 8′ 42″		
Sum =	61° 54′ 42″		
$\frac{1}{2}$Sum =	30° 57′ 21″	log cos	9.93327
$\frac{1}{2}$Sum-Z =	18° 11′ 21″	log cos	9.97775
			——— add
		log cos^2	9.95581
$\frac{1}{2}LHA =$	18° 7′ 18″	⟸ log cos	9.97791
$LHA =$	36° 14′ 36″	West of Meridian	

a) Found by interpolation from the *Nautical Almanac* at the established mean time.
b) The latitude must be guessed on the basis of the last meridian altitude.

Accuracy and sources of error.

It is interesting to investigate how the accuracies in the observations of the lunar distance and the two altitudes affect the final results. This is accomplished by evaluating in the example the change in the established mean time and in the longitudes, both deduced via the local hour angles of the sun and of the moon, upon a 1' error in each of the observed quantities and in the adopted latitude.
A misreading of ±1' in the lunar distance, *d''*, gives a change of ± $1^m\,50^s$ in the mean time, and a change of ± 28' in the longitude, calculated either via the moon or the sun.
The effect of an error in the adopted latitude is small in the present example and the change in the deduced longitudes is in the order of 1'.
A 1' error in either the altitude of the sun, *H''*, or that of the moon, *h''*, causes an error of only 1 s in the time and errors no larger than 1' in the longitude. This result was to be expected. Clearing the lunar distance of the effects of parallax and refraction gives a correction on the lunar distance which is in the order of 1 deg or less, *d* - *d'* = 23'.9 in the example.
Errors in the altitudes which are on the 1:1000 level of the altitudes themselves, affect (*d* - *d'*) also on the 1:1000 level and thus cause changes in the order of 1 arcsec or less.
It follows that the necessity of synchronizing the altitude observations with that of the lunar distance, on which handbooks like Bowditch and Raper lay strong emphasis, is much less if the only interest is in finding the time. To solidify this statement, a sequence of observations *H''*, *d''*, *h''* is analyzed, with *d''* taken at the sought-for mean time, *H''* taken 3 min earlier and *h''* 3 min. later. It is then found that the time comes out only 32 s early. Inverting the order of the observations and taking *h''* first and *H''* last gives a time that is 30 s late.
Of course the effect on the longitude is more severe if one continues the calculation of the local hour angles from the measured altitudes as if they were synchronous with the lunar distance. In the example the resulting errors are then 35' for ZPS and 52' for

ZPM, but other examples can easily be constructed where the error will be much larger.
Yet, such a sequence of only three observations can still give satisfactory results if, as above, the time differences are only ignored in deducing the mean time *but not* in evaluating the local hour angles. In the example, the sun's and the moon's heights were taken 3 min before or after the lunar distance observation. Since the time deduced for the lunar distance observation might have an uncertainty of 0.5 min., the proper times to be used for the height observations will likewise have this uncertainty. The corresponding error in the longitude stays then within 10'.
Slocum took such a sequence of three observations and from his statement that he left his tin clock "asleep", it may be guessed that he estimated the time intervals between them by counting aloud.
In conclusion, it is found that when the different observations are properly synchronized or else the times elapsed between them are duly taken into account, it is the accuracy of the observed lunar distance, *d''*, which is by far the most crucial element in finding the time and the longitude.
The assumed uncertainty of 1' in the reading of *d''* is a typical value to be expected for an experienced observer, equipped with a perfect sextant, and can be worse in high seas and better in fair conditions. The uncertainty in *d''* entails an uncertainty of about 2 min in the mean time or 30' in the longitude.

Table 6: Finding the longitude

SIDERIAL TIME						
Time of obs.	June 16 AT =	$11^h\ 39^m\ 32^s$				
Last transit Ariës	June 15 AT =	$18^h\ 19^m\ 56^s$				
	time diff =	$17^h\ 19^m\ 36^s$				
From table	corr. to ST=	$0^h\ 02^m\ 51^s$				
			add			
	ST =	$17^h\ 22^m\ 27^s$				
MOON (via Sid. Time)						
	ST =	$17^h\ 22^m\ 27^s$				
	moon's r.a.[a] =	$10^h\ 32^m\ 02^s$				
			sub			
	Hour Angle =	$6^h\ 50^m\ 25^s$	$\Longrightarrow$	GHA=	$102°\ 36'\ 15''$	
				LHA(East)=	$35°\ 51'\ 16''$	
						add
				Longitude =	$138°\ 27'\ 31''$	
SUN (via Sid. Time)						
	ST =	$17^h\ 22^m\ 27^s$				
	sun's r.a.[a] =	$5^h\ 43^m\ 34^s$				
			sub			
	Hour Angle =	$11^h\ 38^m\ 53^s$	$\Longrightarrow$	GHA=	$174°\ 43'\ 15''$	
				LHA(West)=	$36°\ 14'\ 36''$	
						sub
				Longitude =	$138°\ 28'\ 39''$	
or equivalently						
SUN (via App. Time)						
	Mean AT =	$11^h\ 39^m\ 32^s$				
	Eq. of Time[a] =	$0^h\ 00^m\ 39^s$				
			sub			
	App. AT =	$11^h\ 38^m\ 53^s$	$\Longrightarrow$	GHA=	$174°\ 43'\ 15''$	
				LHA(West)=	$36°\ 14'\ 36''$	
						sub
				Longitude =	$138°\ 28'\ 39''$	

a) Found by interpolation from the *Nautical Almanac*.

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