The Math Resource for Instruction for

North Carolina Math 2

NC Mathematics
Department of Public Instruction

PUBLIC SCHOOLS OF NORTH CAROLINA
State Board of Education
Department of Public Instruction

Link for: Feedback for NC's Math Resource for Instruction

Link to: Suggest Resources for NC's Math Resource for Instruction

North Carolina Course of Study - Math 2 Standards

Number

The real number system

Extended the properties of exponents to rational exponents

NC.M2.N-RN.1
NC.M2.N-RN.2

Use properties of rational and irrational numbers

NC.M2.N-RN.3

The complex number system

Defining complex numbers

NC.M2.N-CN.1

Algebra

Overview

Seeing structure in expressions

Interpret the structure of expressions

NC.M2.A-SSE.1a
NC.M2.A-SSE.1b
NC.M2.A-SSE.3

Perform arithmetic operations on polynomials

Perform arithmetic operations on polynomials

NC.M2.A-APR.1

Creating equations

Create equations that describe numbers or relationships

NC.M2.A-CED.1
NC.M2.A-CED.2
NC.M2.A-CED.3
NC.M2.A-CED.4

Reasoning with equations and inequalities

Understand solving equations as a process of reasoning and explain the reasoning

NC.M2.A-REI.1
NC.M2.A-REI.2

Solve equations and inequalities in one variable

NC.M2.A-REI.4a
NC.M2.A-REI.4b

Solve systems of equations

NC.M2.A-REI.7

Represent and solve equations and inequalities graphically

NC.M2.A-REI.11

Functions

Overview

Interpreting functions

Understand the concept of a function and use function notation

NC.M2.F-IF.1
NC.M2.F-IF.2

Interpret functions that arise in applications in terms of a context

NC.M2.F-IF.4

Analyze functions using different representations

NC.M2.F-IF.7
NC.M2.F-IF.8
NC.M2.F-IF.9

Building functions

Build a function that models a relationship between two quantities

NC.M2.F-BF.1

Build new functions from existing functions

NC.M2.F-BF.3

Geometry

Overview

Congruence

Experiment with transformations in the plane

NC.M1.G-CO.2
NC.M1.G-CO.3
NC.M1.G-CO.4
NC.M1.G-CO.5

Understand congruence in terms of rigid motions

NC.M1.G-CO.6
NC.M1.G-CO.7
NC.M1.G-CO.8

Prove geometric theorems

NC.M1.G-CO.9
NC.M1.G-CO.10

Similarity, right triangles, and trigonometry

Understand similarity in terms of similarity transformations

NC.M1.G-SRT.1a
NC.M1.G-SRT.1b
NC.M1.G-SRT.1c
NC.M1.G-SRT.1d
NC.M1.G-SRT.2a
NC.M1.G-SRT.2b
NC.M1.G-SRT.3

Prove theorems involving similarity

NC.M1.G-SRT.4

Define trigonometric ratios and solve problems involving right triangles

NC.M1.G-SRT.6
NC.M1.G-SRT.8
NC.M1.G-SRT.12

Statistics & Probability

Overview

Making Inference and Justifying Conclusions

Understand and evaluate random processes underlying statistical experiments

NC.M1.S-IC.2

Conditional probability and the rules for probability

Understand independence and conditional probability and use them to interpret data

NC.M1.S-CP.1
NC.M1.S-CP.3a
NC.M1.S-CP.3b
NC.M1.S-CP.4
NC.M1.S-CP.5

Use the rules of probability to compute probabilities of compound events in a uniform probability model

NC.M1.S-CP.6
NC.M1.S-CP.7
NC.M1.S-CP.8

Number – The Real Number System

NC.M2.N-RN.1

Extend the properties of exponents to rational exponents.

Explain how expressions with rational exponents can be rewritten as radical expressions.

Concepts and Skills	The Standards for Mathematical Practices
Pre-requisite • Rewrite algebraic expressions using the properties of exponents (NC.M1.N-RN.1) **Connections** • Rewrite expressions with radicals and rational exponents using the properties of exponents (NC.M2.N-RN.2) • Justify the step in a solving process (NC.M2.A-REI.1)	**Connections** *Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.* 6 – Attend to precision 7 – Look for and make use of structure 7 – Look for and express regularity in repeated reasoning **Disciplinary Literacy** *As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication* Students should be able to explain with mathematical reasoning how expressions with rational exponents can be rewritten as radical expressions.

Mastering the Standard

Comprehending the Standard	Assessing for Understanding
The meaning of an exponent relates the frequency with which a number is used as a factor. So 5^3 indicates the product where 5 is a factor 3 times. Extend this meaning to a rational exponent, then $125^{1/3}$ indicates one of three equal factors whose product is 125. Students recognize that a fractional exponent can be expressed as a radical or a root. ***For example***, an exponent of $\frac{1}{3}$ is equivalent to a cube root; an exponent of $\frac{1}{4}$ is equivalent to a fourth root. Students extend the use of the power rule, $(b^n)^m = b^{nm}$ from whole number exponents i.e., $(7^2)^3 = 7^6$ to rational exponents. They compare examples, such as $\left(7^{1/2}\right)^2 = 7^{1/2*2} = 7^1 = 7$ to $(\sqrt{7})^2 = 7$ to establish a connection between radicals and rational exponents: $7^{1/2} = \sqrt{7}$ and, in general, $b^{1/2} = \sqrt{b}$. Students can then extend their understanding to exponents where the numerator of the rational exponent is a number greater than 1. For example $7^{\frac{1}{2}*3} = 7^{\frac{3}{2}} = \sqrt{7^3} = (\sqrt{7})^3$.	Students should be able to use their understanding of rational exponents to solve problems. **Example**: Determine the value of *x* a. $64^{\frac{1}{2}} = 8^x$ b. $(12^5)^x = 12$ Students should be able to explain their reasoning when rewriting expressions with rational exponents. **Examples**: a. Write $x^{\frac{1}{5}}$ as a radical expression. b. Write $(x^2y)^{\frac{1}{2}}$ as a radical expression. c. Explain how the power rule of exponents, $(b^n)^m = b^{mn}$, can be used to justify why $(\sqrt[3]{b})^3 = b$. d. Explain why $x^{\frac{2}{3}}$ is equivalent to $\sqrt[3]{x^2}$ and $(\sqrt[3]{x})^2$.

Instructional Resources	
Tasks	**Additional Resources**

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Number – The Real Number System

NC.M2.N-RN.2

Extend the properties of exponents to rational exponents.

Rewrite expressions with radicals and rational exponents into equivalent expressions using the properties of exponents.

Concepts and Skills

Pre-requisite

- Rewrite algebraic expressions using the properties of exponents (NC.M1.N-RN.1)
- Explain how expressions with rational expressions can be written as radical expressions (NC.M2.N-RN.1)

Connections

- Operations with polynomials (NC.M2.A-APR.1)
- Solve one variable square root equations (NC.M2.A-REI.2)
- Solve quadratic equations in one variable (NC.M2.A-REI.4a, NC.M2.A-REI.b)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

6 – Attend to precision

7 – Look for and make use of structure

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Students should be able to explain their reasoning while simplifying expressions with rational exponents and radicals.

Mastering the Standard

Comprehending the Standard

Students should be able to simplify expressions with radicals and with rational exponents. Students should be able to rewrite expressions involving rational exponents as expressions involving radicals and simplify those expressions.

Students should be able to rewrite expressions involving radicals as expressions using rational exponents and use the properties of exponents to simplify the expressions.

Students should be able to explain their reasoning while simplifying expressions with rational exponents and radicals.

Assessing for Understanding

Students should be able to rewrite expressions with rational expression into forms that are more simple or useful.

Example: Using the properties of exponents, simplify

a. $\left(\sqrt[4]{32^3}\right)^2$

b. $\dfrac{\sqrt[5]{b^3}}{b^{\frac{4}{3}}}$

Example: Write $\sqrt[3]{27x^2y^6z^3}$ as an expression with rational exponents.

Example: Write an equivalent exponential expression for $8^{\frac{2}{3}}$? Explain how they are equivalent.

Solution: $8^{\frac{2}{3}} = (8^2)^{\frac{1}{3}} = \left(8^{\frac{1}{3}}\right)^2 = 2^2$ *In the first expression, the base number is 8 and the exponent is* $2/3$. *This means that the expression represents* 2 *of the* 3 *equal factors whose product is* 8, *thus the value is 4, since* $(2 \times 2 \times 2) = 8$; *there are three factors of 2; and two of these factors multiply to be 4. In the second expression, there are 2 equal factors of 8 or 64. The exponent* $1/3$ *represents* 1 *of the* 3 *equal factors of* 64. *Since* $4 \times 4 \times 4 = 64$ *then one of the three factors is 4. The last expression there is* 1 *of* 3 *equal factors of* 8 *which is 2 since* $2 \times 2 \times 2 = 8$. *Then there are* 2 *of the equal factors of* 2, *which is* 4.

Example: Given $81^{\frac{3}{4}} = \sqrt[4]{81^3} = \left(\sqrt[4]{81}\right)^3$, which form would be easiest to calculate without using a calculator. Justify your answer?

Mastering the Standard	
Comprehending the Standard	**Assessing for Understanding**
	Example: Determine whether each equation is true or false using the properties of exponents. If false, describe at least one way to make the math statement true. a. $\sqrt{32} = 2^{\frac{5}{2}}$ b. $16^{\frac{3}{2}} = 8^2$ c. $4^{\frac{1}{2}} = \sqrt[4]{64}$ d. $2^8 = \left(\sqrt[3]{16}\right)^6$ e. $\left(\sqrt{64}\right)^{\frac{1}{3}} = 8^{\frac{1}{6}}$

Instructional Resources	
Tasks	**Additional Resources**

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NC.M2.N-RN.3

Use properties of rational and irrational numbers.

Use the properties of rational and irrational numbers to explain why:

- the sum or product of two rational numbers is rational;
- the sum of a rational number and an irrational number is irrational;
- the product of a nonzero rational number and an irrational number is irrational.

Concepts and Skills	The Standards for Mathematical Practices
Pre-requisite	**Connections**
• Understand rational numbers (8.NS.1)	*Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.* 2 – Reason abstractly and quantitatively 3 – Construct viable arguments and critique the reasoning of others
Connections	**Disciplinary Literacy**
• These concepts close out the learning about the real number system.	*As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication*

Mastering the Standard	
Comprehending the Standard	**Assessing for Understanding**
Students know and justify that when • adding or multiplying two rational numbers the result is a rational number. • adding a rational number and an irrational number the result is irrational. • multiplying of a nonzero rational number and an irrational number the result is irrational. Note: Since every difference is a sum and every quotient is a product, this includes differences and quotients as well. Explaining why the four operations on rational numbers produce rational numbers can be a review of students understanding of fractions and negative numbers. Explaining why the sum of a rational and an irrational number is irrational, or why the product is irrational, includes reasoning about the inverse relationship between addition and subtraction and the relationship between multiplication and addition.	Students should be able to explain the properties of rational and irrational numbers. **Example:** Explain why the number 2π must be irrational. ***Sample Response:*** *If 2π were rational, then half of 2π would also be rational, so π would have to be rational as well.* **Example:** Explain why the sum of $3 + 2\pi$ must be irrational. **Example:** Explain why the product of $3 \cdot \sqrt{2}$ must be irrational. **Example:** Given one rational number $\frac{a}{b}$ and another rational number $\frac{r}{s}$, find the product of $\frac{a}{b} \cdot \frac{r}{s}$. Use this product to justify why the product of two rational numbers must be a rational number. Include in your justification why the number $\frac{a}{b}$ or $\frac{r}{s}$ could represent any rational number.

Instructional Resources	
Tasks	**Additional Resources**
	FAL: Evaluating Statements About Rational and Irrational Numbers (Mathematics Assessment Project)

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Number – The Complex Number System

NC.M2.N-CN.1

Defining complex numbers.

Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ where a and b are real numbers.

Concepts and Skills

Pre-requisite

- The understanding of number systems is developed through middle school (8.NS.1)

Connections

- Solve quadratic equations in one variable (NC.M2.A-REI.4b)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

6 – Attend to precision

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication

Complex Number

Imaginary

Students should be able to define a complex number and identify when they are likely to use them.

Mastering the Standard

Comprehending the Standard

When students solve quadratic equations they should understand that there is a solution to an equation when a negative appears in the radicand. This solution does not produce x-intercepts for the function and is not included in the real number system. This means that it is now time to introduce students to a broader classification of numbers so that we have a way to express these solutions.

Students should know that every number can be written in the form $a + bi$,where a and b are real numbers and $i = \sqrt{-1}$,
are classified as complex numbers. If $a = 0$, then the number is a pure imaginary number. If $b = 0$ the number is a real number. This means that all real numbers are included in the complex number system

Assessing for Understanding

Students should be able to rewrite expressions using what they know about complex numbers.

Example: Simplify.

a. i^2

b. $\sqrt{-36}$

c. $2\sqrt{-49}$

d. $-3\sqrt{-10}$

e. $5\sqrt{-7}$

f. $\frac{-3+\sqrt{9-4*2*5}}{4}$

Answers

Problem	Solution
i^2	$i^2 = (\sqrt{-1})^2 = (-1)^{\frac{1}{2}*2} = -1$
$\sqrt{-36}$	$\sqrt{-36} = \sqrt{-1}\cdot\sqrt{36} = 6i$
$2\sqrt{-49}$	$2\sqrt{-49} = 2\sqrt{-1}\cdot\sqrt{49} = 2\cdot 7i = 14i$
$-3\sqrt{-10}$	$-3\sqrt{-10} = -3\sqrt{-1}\cdot\sqrt{10} = -3\cdot i\cdot\sqrt{10} = -3i\sqrt{10}$
$5\sqrt{-7}$	$5\sqrt{-7} = 5\sqrt{-1}\cdot\sqrt{7} = 5\cdot i\cdot\sqrt{7} = 5i\sqrt{7}$
$\frac{-3+\sqrt{9-4*2*5}}{4}$	$\frac{-3+\sqrt{9-4*2*5}}{4} = \frac{-3+\sqrt{-31}}{4} = \frac{-3+i\sqrt{31}}{4}$ Which can be written in the form $a + bi$ as $\frac{-3}{4} + \frac{\sqrt{31}}{4}i$

Mastering the Standard	
Comprehending the Standard	**Assessing for Understanding**
and that the square root of a negative number is a complex number. Students should connect what they have learned regarding properties of exponents to understand that $(\sqrt{-1})^2 = (-1)^{\frac{1}{2}*2} = -1$. Students should be able to express solutions to a quadratic equation as a complex number.	

Instructional Resources	
Tasks	**Additional Resources**

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Algebra, Functions & Function Families

NC Math 1	NC Math 2	NC Math 3
Functions represented as graphs, tables or verbal descriptions in context		
Focus on comparing properties of linear function to *specific* non-linear functions and rate of change. • Linear • Exponential • Quadratic	**Focus on properties of quadratic functions and an introduction to inverse functions through the inverse relationship between quadratic and square root functions.** • Quadratic • Square Root • Inverse Variation	**A focus on more complex functions** • Exponential • Logarithm • Rational functions w/ linear denominator • Polynomial w/ degree $\leq$ three • Absolute Value and Piecewise • Intro to Trigonometric Functions

A Progression of Learning of Functions through Algebraic Reasoning

The conceptual categories of Algebra and Functions are inter-related. Functions describe situations in which one quantity varies with another. The difference between the Function standards and the Algebra standards is that the Function standards focus more on the characteristics of functions (e.g. domain/range or max/min points), function definition, etc. whereas the Algebra standards provide the computational tools and understandings that students need to explore specific instances of functions. As students progress through high school, the coursework with specific families of functions and algebraic manipulation evolve. Rewriting algebraic expressions to create equivalent expressions relates to how the symbolic representation can be manipulated to reveal features of the graphical representation of a function.

Note: The Numbers conceptual category also relates to the Algebra and Functions conceptual categories. As students become more fluent with their work within particular function families, they explore more of the number system. For example, as students continue the study of quadratic equations and functions in Math 2, they begin to explore the complex solutions. Additionally, algebraic manipulation within the real number system is an important skill to creating equivalent expressions from existing functions.

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Algebra – Seeing Structure in Expressions

NC.M2.A-SSE.1a

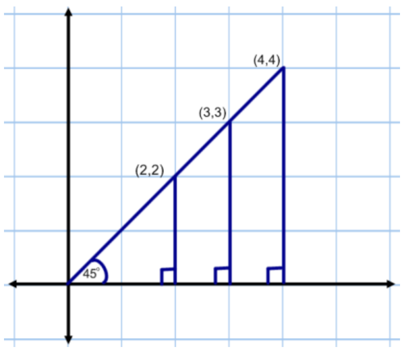
Interpret the structure of expressions.

Interpret expressions that represent a quantity in terms of its context.

a. Identify and interpret parts of a quadratic, square root, inverse variation, or right triangle trigonometric expression, including terms, factors, coefficients, radicands, and exponents.

Concepts and Skills	The Standards for Mathematical Practices
Pre-requisite • Interpreting parts of expressions in context (NC.M1.A-SSE.1a, NC.M1.A-SSE.1b)	**Connections** *Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.* 2 – Reason abstractly and quantitatively. 4 – Model with mathematics 7 – Look for and make use of structure.
Connections • Creating equation to solve, graph, and make systems (NC.M2.A-CED.1, NC.M2.A-CED.2, NC.M2.A-CED.3) • Solve and interpret one variable inverse variation and square root equations (NC.M2.A-REI.2) • Interpreting functions (NC.M2.F-IF.4, NC.M2.F-IF.7, NC.M2.F-IF.9) • Understand the effect of transformations on functions (NC.M2.F-BF.3)	**Disciplinary Literacy** *As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication* New Vocabulary: inverse variation, right triangle trigonometry

Mastering the Standard	
Comprehending the Standard	**Assessing for Understanding**
When given an expression with a context, students should be able to explain how the parts of the expression relate to the context of the problem. Students should be able to write equivalent forms of an expression to be able to identify parts of the expression that can relate to the context of the problem. The parts of expressions that students should be able to interpret include any terms, factors, coefficients, radicands, and exponents. Students should be given contexts that can be modeled with quadratic, square root, inverse	Students should be able to identify and interpret parts of an expression in its context. **Example:** The expression $-4.9t^2 + 17t + 0.6$ describes the height in meters of a basketball t seconds after it has been thrown vertically into the air. Interpret the terms and coefficients of the expression in the context of this situation. **Example:** The area of a rectangle can be represent by the expression $x^2 + 8x + 12$. What do the factors of this expression represent in the context of this problem? **Example:** The stopping distance in feet of a car is directly proportional to the square of its speed. The formula that relates the stopping distance and speed of the car is $D = k \cdot V^2$, where D represents the stopping distance in feet, k represents a constant that depends on the frictional force of the pavement on the wheels of a specific car, and V represents the speed the car was traveling in miles per hour. When there is a car accident it is important to figure out how fast the cars involved were traveling. The expression $\sqrt{\frac{D}{k}}$ can be evaluated to find the speed that a car was traveling. What does the radicand represent in this expression?

Mastering the Standard	
Comprehending the Standard	**Assessing for Understanding**
variation, or right triangle trigonometric expressions.	**Example:** Ohm's Law explains the relationship between current, resistance, and voltage. To determine the current passing through a conductor you would need to evaluate the expression $\frac{V}{R}$, where V represents voltage and R represents resistance. If the resistance is increased, what must happen to the voltage so that the current passing through the conductor remains constant? **Example:** The tangent ratio is $\frac{y}{x}$ where (x, y) is a coordinate on the terminal side of the angle in standard position. Use the diagram to justify why the tangent of 45° is always 1. Then, expand that reasoning to justify why every individual angle measure has exactly one value for tangent. Use similar reasoning to justify why every angle has exactly one value of sine and one value of cosine. 

Instructional Resources	
Tasks	**Additional Resources**
The Physics Professor (Illustrative Mathematics) Quadrupling leads to Halving (Illustrative Mathematics)	

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Algebra – Seeing Structure in Expressions

NC.M2.A-SSE.1b
Interpret the structure of expressions.
Interpret expressions that represent a quantity in terms of its context.

b. Interpret quadratic and square root expressions made of multiple parts as a combination of single entities to give meaning in terms of a context.

Concepts and Skills

Pre-requisite

- Interpreting parts of expressions in context (NC.M1.A-SSE.1a, NC.M1.A-SSE.1b)

Connections

- Use completing the square to write equivalent form of quadratic expressions to reveal extrema (NC.M2.A-SSE.3)
- Creating equation to solve, graph, and make systems (NC.M2.A-CED.1, NC.M2.A-CED.2, NC.M2.A-CED.3)
- Solve and interpret one variable inverse variation and square root equations (NC.M2.A-REI.2)
- Interpreting functions (NC.M2.F-IF.4, NC.M2.F-IF.7, NC.M2.F-IF.9)
- Understand the effect of transformations on functions (NC.M2.F-IF.2, NC.M2.F-BF.3)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.
2 – Reason abstractly and quantitatively.
4 – Model with mathematics
7 – Look for and make use of structure.

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication

Students should be able to describe their interpretation of an expression.

Mastering the Standard

Comprehending the Standard	Assessing for Understanding
When given an expression with a context that has multiple parts, students should be able to explain how combinations of those parts of the expression relate to the context of the problem. Students should be able to write equivalent forms of an expression to be able to identify combinations of parts of the expression that can represent a quantity in the context of the problem. Students should be given contexts that can be modeled with quadratic and square root expressions.	Students should be able to see parts of an expression as a single quantity that has a meaning based on context. **Example:** If the volume of a rectangular prism is represented by $x(x+3)(x+2)$, what can $(x+3)(x+2)$ represent? **Example:** Sylvia is organizing a small concert as a charity event at her school. She has done a little research and found that the expression $-10x+180$ represents the number of tickets that will sell, given that x represents the price of a ticket. Explain why the income for this event can be represented by the expression $-10x^2+180x$. If all of the expenses will add up to $150, explain why the expression $-10x^2+180x-150$ represents the profit. **Example:** When calculating the standard deviation of a population you must first find the mean of the data, subtract the mean from each value in the data set, square each difference, add all of the squared differences together, divide by the number of terms in the data set and then take the square root. The expression used for calculating standard deviation of a population is $\sqrt{\frac{\Sigma(x-\mu)^2}{n}}$. Given the above description of the process of calculating standard deviation and what you have learned in a previous course about standard deviation being a measure of spread, answer the following questions.

Mastering the Standard	
Comprehending the Standard	**Assessing for Understanding**
	a. Describe what you are finding when you calculate $x - \mu$. b. Describe how the formula for standard deviation is similar to the formula for finding mean. c. What part of the radicand would have to increase so that the value of the standard deviation would also increase: the numerator $(\sum (x-\mu)^2)$ or the denominator (n)? Justify your answer.

Instructional Resources	
Tasks	**Additional Resources**

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Algebra – Seeing Structure in Expressions

NC.M2.A-SSE.3

Interpret the structure of expressions.

Write an equivalent form of a quadratic expression by completing the square, where a is an integer of a quadratic expression, $ax^2 + bx + c$, to reveal the maximum or minimum value of the function the expression defines.

Concepts and Skills	The Standards for Mathematical Practices
Pre-requisite • Rewrite quadratic expression to reveal zeros and solutions (NC.M1.A-SSE.3) • Interpret parts of a function as single entities in context (NC.M2.A-SSE.1b)	**Connections** *Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.* 2 – Reason abstractly and quantitatively 4 – Model with mathematics 7 – Look for and make use of structure
Connections • Understand the relationship between the quadratic formula and the process of completing the square (NC.M2.A-REI.4a) • Find and compare key features of quadratic functions (NC.M2.F-IF.7, NC.M2.F-IF.8, NC.M2.F-IF.9)	**Disciplinary Literacy** *As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication* Students should be able to explain when the process of completing the square is necessary. New Vocabulary: completing the square

Mastering the Standard

Comprehending the Standard

When given an equation in the form $ax^2 + bx + c$ students should be able to complete the square to write a quadratic equation in vertex form: $a(x-h)^2 + k$.

Students should be able to determine that if $a > 0$ there is a minimum and if $a < 0$ there is a maximum.

Students should be able to identify the maximum or minimum point (h, k) from an equation in vertex form.

Algebra Tiles are a great way to demonstrate this process. You can demonstrate the reasoning for all of the steps in the process.
This process also links previous learning of the area model for multiplication.

Assessing for Understanding

Students should be able to reveal the vertex of a quadratic expression using the process of completing the square.

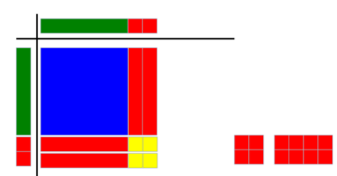
Example: Write each expression in vertex form and identify the minimum or maximum value of the function.

a) $x^2 - 4x + 5$
b) $x^2 + 5x + 8$
c) $2x^2 + 12x - 18$
d) $3x^2 - 12x - 1$
e) $2x^2 - 15x + 3$

Example: The picture at the right demonstrates the process of completing the square using algebra tiles. Looking at the picture, why might this process be called "completing the square"?

Note: There are at least two good answers to this question. First the product must form a square, so you must arrange and complete this missing parts using zero pairs to make the square. The second, completing the square is about finding the "new C" which in the process will be a square as seen in the yellow blocks in this picture.

Change to vertex form: $x^2 - 4x - 8$



$(x-2)(x-2) - 12$

$(x-2)^2 - 12$

Instructional Resources	
Tasks	**Additional Resources**
Seeing Dots (Illustrative Mathematics)	

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NC.M2.A-APR.1

Perform arithmetic operations on polynomials.

Extend the understanding that operations with polynomials are comparable to operations with integers by adding, subtracting, and multiplying polynomials.

Concepts and Skills

Pre-requisite

- Operations with polynomials (NC.M1.A-APR.1)
- Rewrite expressions with radicals and rational exponents using the properties of exponents (NC.M2.N-RN.2)

Connections

- Solving systems of linear and quadratic equations (NC.M2.A-REI.7)
- Use equivalent expression to develop completing the square (NC.M2.F-IF.8)
- Understand the effect of transformations on functions (NC.M2.F-BF.3)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

6 – Attend to precision

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication

Students should be able to describe their process to multiply polynomials.

Mastering the Standard

Comprehending the Standard

The primary strategy for this cluster is to make connections between arithmetic of integers and arithmetic of polynomials. In order to understand this standard, students need to work toward both understanding and fluency with polynomial arithmetic. Furthermore, to talk about their work, students will need to use correct vocabulary, such as integer, monomial, binomial, trinomial, polynomial, factor, and term.

Assessing for Understanding

Students should be able to rewrite polynomials into equivalent forms through addition, subtraction and multiplication.

Example: Simplify and explain the properties of operations apply.

a) $(x^3 + 3x^2 - 2x + 5)(x - 7)$
b) $4b(cb - zd)$
c) $(4x^2 - 3y^2 + 5xy) - (8xy + 3y^2)$
d) $(4x^2 - 3y^2 + 5xy) + (8xy + 3y^2)$
e) $(x + 4)(x - 2)(3x + 5)$

Instructional Resources

Tasks	Additional Resources

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Algebra – Creating Equations

NC.M2.A-CED.1

Create equations that describe numbers or relationships.

Create equations and inequalities in one variable that represent quadratic, square root, inverse variation, and right triangle trigonometric relationships and use them to solve problems.

Concepts and Skills	The Standards for Mathematical Practices
Pre-requisite	**Connections**
• Create and solve equations in one variable (NC.M1.A-CED.1) • Interpret parts of an expression in context (NC.M2.A-SSE.1a, NC.M2.A-SSE.1b) • Justify solving methods and each step (NC.M2.A-REI.1)	*Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.* 1 – Make sense of problems and persevere in solving them 2 – Reason abstractly and quantitatively 4 – Model with mathematics 5 – Use appropriate tools strategically
Connections	**Disciplinary Literacy**
• Solve inverse variation, square root and quadratic equations (NC.M2.A-REI.2, NC.M2.A-REI.4a, NC.M2.A-REI.4b) • Use trig ratios to solve problems (NC.M2.G-SRT.8) • Solve systems of equations (NC.M2.A-REI.7) • Write a system of equations as an equation or write an equations as a system of equations to solve (NC.M2.A-REI.11)	*As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication* Students should be able to explain their reasoning behind their created equation. New Vocabulary: inverse variation, right triangle trigonometry

Mastering the Standard

Comprehending the Standard	Assessing for Understanding
Students should be able to determine a correct equation or inequality to model a given context and use the model to solve problems. Focus on contexts that can be modeled with quadratic, square root, inverse variation, and right triangle trigonometric equations and inequalities. Students need to be familiar with algebraic, tabular, and graphic methods of solving equations and inequalities.	Students should be able to create one variable equations from multiple representations, including from functions. **Example:** Lava ejected from a caldera in a volcano during an eruption follows a parabolic path. The formula to find the height of the lava can be found by combining three terms that represent the different forces effecting the lava. The first term is the original height of the volcano. The second term concerns the speed at which the lava is ejected. The third term is the effect of gravity on the lava. $height(t) = orignal\ height + (initial\ speed\ of\ the\ lava) \cdot t + \frac{1}{2}(effects\ of\ gravity) \cdot t^2$ The original height of the caldera is $936ft$. The lava was ejected at a speed of $64ft/s$. The effect of gravity on any object on earth is approximately $-32ft/s^2$. Write and solve an equation that will find how long (in seconds) it will take for the lava to reach a height of 1000ft. **Example:** The function $h(x) = 0.04x^2 - 3.5x + 100$ defines the height (in feet) of a major support cable on a suspension bridge from the bridge surface where x is the horizontal distance (in feet) from the left end of the bridge. Write an inequality or equation for each of the following problems and then find the solutions. a. Where is the cable less than 40 feet above the bridge surface? b. Where is the cable at least 60 feet above the bridge surface?

Mastering the Standard

Comprehending the Standard	Assessing for Understanding
	See below

Assessing for Understanding

Example: Jamie is selling key chains that he has made to raise money for school trip. He has done a little research and found that the expression $-20x + 140$ represents the number of keychains that he will be able to sell, given that x represents the price of one keychain. Each key chain costs Jamie \$.50 to make. Write and solve an inequality that he can use to determine the range of prices he could charge make sure that he earns at least \$150 in profit.

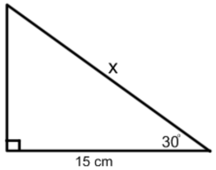
Example: In kickboxing, it is found that the force, *f*, needed to break a board, varies inversely with the length, *l*, of the board. Write and solve an equation to answer the following question:
If it takes 5 lbs. of pressure to break a board 2 feet long, how many pounds of pressure will it take to break a board that is 6 feet long?

Example: To be considered a 'fuel efficient' vehicle, a car must get more than 30 miles per gallon. Consider a test run of 200 miles. How many gallons of fuel can a car use and be considered 'fuel-efficient'?

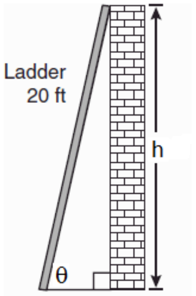
Example: The centripetal force F exerted on a passenger by a spinning amusement park ride is related to the number of seconds t the ride takes to complete one revolution by the equation $t = \sqrt{\frac{155\pi^2}{F}}$. Write and solve an equation to find the centripetal force exerted on a passenger when it takes 12 seconds for the ride to complete one revolution.

Students should be able to create equations using right triangle trigonometry.

Example: Write and solve an equation to find the hypotenuse of the following triangle.



Example: John has a 20-foot ladder leaning against a wall. If the height of the wall that the ladder needs to reach is at least 15ft, create and solve an inequality to find the angle the ladder needs to make with the ground.



Instructional Resources

Tasks	Additional Resources
Throwing a Ball (Illustrative Mathematics)	

Algebra – Creating Equations

NC.M2.A-CED.2

Create equations that describe numbers or relationships.

Create and graph equations in two variables to represent quadratic, square root and inverse variation relationships between quantities.

Concepts and Skills

Pre-requisite

- Create and graph equations in two variables (NC.M1.A-CED.2)
- Interpret parts of an expression in context (NC.M2.A-SSE.1a, NC.M2.A-SSE.1b)

Connections

- Write equations for a system (NC.M2.A-CED.3)
- Solve systems of equations (NC.M2.A-REI.7)
- Write a system of equations as an equation or write an equation as a system of equations to solve (NC.M2.A-REI.11)
- Analyze functions for key features (NC.M2.F-IF.7)
- Build quadratic and inverse variation functions (NC.M2.F-BF.1)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

2 – Reason abstractly and quantitatively

4 – Model with mathematics

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication

New Vocabulary: inverse variation, constant of proportionality

Mastering the Standard

Comprehending the Standard

In this standard students are creating equations and graphs in two variables.

Focus on contexts that can be modeled with quadratic, square root and inverse variation relationships.

This standard needs to be connected with other standards where students interpret functions, generate multiple representations, solve problems, and compare functions.

Assessing for Understanding

Students should be able to create an equation from a context or representation and graph the equation.

Example: The area of a rectangle is 40 in^2. Write an equation for the length of the rectangle related to the width. Graph the length as it relates to the width of the rectangle. Interpret the meaning of the graph.

Example: The formula for the volume of a cylinder is given by $V = \pi r^2 h$, where r represents the radius of the circular cross-section of the cylinder and h represents the height. Given that $h = 10in$…

a. Graph the volume as it relates to the radius.
b. Graph the radius as it relates to the volume.
c. Compare the graphs. Be sure to label your graphs and use an appropriate scale.

Example: Justin and his parents are having a discussion about driving fast. Justin's parents argue that driving faster does not save as much time as he thinks. Justin lives 10 miles from school. Using the formula $r \cdot t = d$, where r is speed in miles per hour and d is the distance from school, rewrite the formula for t and graph. Do Justin's parents have a point?

Instructional Resources	
Tasks	**Additional Resources**
	Marbleslides: Parabolas (Desmos.com) NEW

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Algebra – Creating Equations

NC.M2.A-CED.3

Create equations that describe numbers or relationships.

Create systems of linear, quadratic, square root, and inverse variation equations to model situations in context.

Concepts and Skills	The Standards for Mathematical Practices
Pre-requisite • Create equations for a system of equations in context (NC.M1.A-CED.3) • Interpret parts of an expression in context (NC.M2.A-SSE.1a, NC.M2.A-SSE.1b) • Create equations in two variables (NC.M2.A-CED.2)	**Connections** *Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.* 1 – Make sense of problems and persevere in solving them 2 – Reason abstractly and quantitatively 4 – Model with mathematics
Connections • Solve systems of equations (NC.M2.A-REI.7) • Write a system of equations as an equation or write an equations as a system of equations to solve (NC.M2.A-REI.11)	**Disciplinary Literacy** *As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication* Students should be able to justify their created equations through unit analysis. New Vocabulary: inverse variation, constant of proportionality

Mastering the Standard	
Comprehending the Standard	**Assessing for Understanding**
Students create systems of equations to model situations in contexts. Contexts should be limited to linear, quadratic, square root and inverse variation equations. This standard should be connected with NC.M2.A-REI.7 where students solve and interpret systems and with NC.M2.A-REI.11 where students understand the representation of the solutions of systems graphically.	Students should be able to recognize when a context requires a system of equations and create the equations of that system. **Example:** In making a business plan for a pizza sale fundraiser, students determined that both the income and the expenses would depend on the number of pizzas sold. They predicted that $I(n) = -0.05n^2 + 20n$ and $E(n) = 5n + 250$. Determine values for which $I(n) = E(n)$ and explain what the solution(s) reveal about the prospects of the pizza sale fundraiser. **Example:** The FFA has $2400 in a fund to raise money for a new tractor. They are selling trees and have determined that the number of trees they can buy to sell depends on the price of the tree p, according to the function $n(p) = \frac{2400}{p}$. Also, after allowing for profit, the number of trees that customers will purchase depends on the price which the group purchased the trees with function $c(p) = 300 - 6p$. For what price per tree will the number of trees that can be equal the number of trees that will be sold? **Example:** Susan is designing wall paper that is made of several different sized squares. She is using a drawing tool for the square where she can adjust the area and the computer program automatically adjusts the side length by using the formula $s = \sqrt{A}$. The perimeter of the square can also be inputted into the computer so that the computer will automatically adjust the side length with the formula $s = \frac{P}{4}$. Susan wants to see what the design would look like if the perimeter and area of one of the squares was the same. Create a system of equations that Susan could solve so that she

Mastering the Standard	
Comprehending the Standard	**Assessing for Understanding**
	knows what to input into the computer to see her design. What is the side length that produces the same area and perimeter?

Instructional Resources	
Tasks	**Additional Resources**

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Algebra – Creating Equations

Algebra – Reasoning with Equations and Inequalities

NC.M2.A-REI.1
Understand solving equations as a process of reasoning and explain the reasoning.
Justify a chosen solution method and each step of the solving process for quadratic, square root and inverse variation equations using mathematical reasoning.

Concepts and Skills

Pre-requisite

- Justify a solving method and each step in the process (NC.M1.A-REI.1)
- Explain how expressions with rational exponents can be rewritten as radical expressions (NC.M2.N-RN.1)
- Use equivalent expressions to explain the process of completing the square (NC.M2.F-IF.8)

Connections

- Create and solve one variable equations (NC.M2.A-CED.1)
- Solve inverse variation, square root and quadratic equations (NC.M2.A-REI.2, NC.M2.A-REI.4a, NC.M2.A-REI.4b)
- Use trig ratios to solve problems (NC.M2.G-SRT.8)
- Solve systems of equations (NC.M2.A-REI.7)
- Write a system of equations as an equation or write an equation as a system of equations to solve (NC.M2.A-REI.11)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.
3 – Construct viable arguments and critique the reasoning of others
5 – Use appropriate tools strategically
6 – Attend to precision
7 – Look for and make use of structure

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication

Students should be able to predict the justifications of another student's solving process.
New Vocabulary: inverse variation, constant of proportionality

Mastering the Standard

Comprehending the Standard	Assessing for Understanding
Students need to be able to explain why they choose a specific method to solve an equation. For example, with a quadratic equation, students could choose to factor, use the quadratic formula, take the square root, complete the square to take the square root, solve by graphing or with a table. Students should be able to look at the structure of the quadratic to make this decision. With a square root equation, students could	Students should be able to justify each step in a solving process. **Example:** Explain why the equation $x^2 + 14 = 9x$ can be solved by determining values of x such that $x - 7 = 0$ and $x - 2 = 0$. **Example:** Solve $3x^2 = -4x + 8$. Did you chose to solve by factoring, taking the square root, completing the square, using the quadratic formula, or some other method? Why did you chose that method? Explain each step in your solving process. **Example**: Solve $\frac{2}{x} = x + 1$. Did you chose to solve by factoring, taking the square root, completing the square, using the quadratic formula, or some other method? Why did you chose that method? Explain each step in your solving process. **Example**: Solve $\sqrt{x+3} = 3x - 1$ using algebraic methods and justify your steps. Solve graphically and compare your solutions.

Mastering the Standard	
Comprehending the Standard	**Assessing for Understanding**
choose to square both sides, solve by graphing or with a table. Discussions on the solving processes and the benefits and drawbacks of each method should lead students to not rely on one solving process. Students should make determinations on the solving process based on the context of the problem, the nature and structure of the equation, and efficiency. While solving algebraically, students need to use the properties of equality to justify and explain each step obtained from the previous step, assuming the original equation has a solution. Students need to solve quadratic, square root and inverse variation equations.	**Example:** If a, b, c, and d are real numbers, explain how to solve how to solve $ax^2 + bx + c = d$ in 2 different methods. Discuss the pros and cons of each method. Students should be able to chose and justify solution methods. **Example:** To the right are two methods for solving the equation $5x^2 + 10 = 90$. Select one of the solution methods and construct a viable argument for the use of the method. ***Method A*** $5x^2 + 10 = 90$ $-10 = -10$ $5x^2 = 80$ $\frac{5x^2}{5} = \frac{80}{5}$ $x^2 = 16$ $x = \pm\sqrt{16}$ $x = 4 \text{ or } x = -4$ ***Method B*** $5x^2 + 10 = 90$ $-90 = -90$ $5x^2 - 80 = 0$ $5(x^2 - 16) = 0$ $5(x + 4)(x - 4) = 0$ $x + 4 = 0 \text{ or } x - 4 = 0$ $x = 4 \text{ or } x = -4$ **Example:** To the right are two methods for solving the equation $2x^2 - 3x + 4 = 0$. Select one of the solution methods and construct a viable argument for the use of the method. ***Method A*** $2x^2 - 3x + 4 = 0$ $x = \frac{3 \pm \sqrt{(-3)^2 - 4(2)(4)}}{2(2)}$ $x = \frac{3 \pm \sqrt{-23}}{4}$ $x = \frac{3 \pm i\sqrt{23}}{4}$ $x = \frac{3}{4} \pm \frac{i\sqrt{23}}{4}$ ***Method B*** $2x^2 - 3x + 4 = 0$ $x^2 - \frac{3}{2}x + 2 = 0$ $x^2 - \frac{3}{2}x + \frac{9}{16} = -2 + \frac{9}{16}$ $\left(x - \frac{3}{4}\right)^2 = \frac{-23}{16}$ $x - \frac{3}{4} = \pm\sqrt{\frac{-23}{16}}$ $x = \frac{3}{4} \pm \frac{i\sqrt{23}}{4}$

Instructional Resources	
Tasks	**Additional Resources**

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NC.M2.A-REI.2

Understand solving equations as a process of reasoning and explain the reasoning.

Solve and interpret one variable inverse variation and square root equations arising from a context, and explain how extraneous solutions may be produced.

Concepts and Skills

Pre-requisite

- Solve quadratic equations by taking square roots (NC.M1.A-REI.4)
- Interpret a function in context be relating it domain and range (NC.M1.F-IF.5)
- Rewrite expressions with radicals and rational exponents using the properties of exponents (NC.M2.N-RN.2)
- Interpret parts of an expression in context (NC.M2.A-SSE.1a, NC.M2.A-SSE.1b)

Connections

- Know there is a complex number and the form of complex numbers (NC.M2.N-NC.1)
- Create and solve one variable equations (NC.M2.A-CED.1)
- Justify the solving method and each step in the solving process (NC.M2.A-REI.1)
- Solve quadratic equations (NC.M2.A-REI.4a, NC.M2.A-REI.4b)
- Write a system of equations as an equation or write an equation as a system of equations to solve (NC.M2.A-REI.11)
- Use trig ratios and the Pythagorean Theorem to solve problems (NC.M2.G-SRT.8)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

2 – Reason abstractly and quantitatively
7 – Look for and make use of structure
8 – Look for and express regularity in repeated reasoning

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication

New Vocabulary: inverse variation, extraneous solutions

Mastering the Standard

Comprehending the Standard

Solve one variable inverse variations and square root equations that arise from a context.

Students should be familiar with direct variation, learned in 7th and 8th grades. Direct variations occur when two quantities are divided to produce a constant, $k = \frac{y}{x}$. This is why direct variation is linked to proportional reasoning. Indirect variations occur when two quantities are multiplied to produce a constant, $k = y \cdot x$.

Assessing for Understanding

Students should be able to solve inverse variation equations.

Example: Tamara is looking to purchase a new outdoor storage shed. She sees an advertisement for a custom built shed that fits into her budget. In this advertisement, the builder offers a 90 square foot shed with any dimensions. Tamara would like the shed to fit into her a corner of her backyard, but the width will be restricted by a tree. She remembers the formula for the area of a rectangle is $l \cdot w = a$ and solves for the width to get $w = \frac{a}{l}$. She then measures the restricted width to be 12 feet. What can be the dimensions of the shed?

Mastering the Standard	
Comprehending the Standard	**Assessing for Understanding**
Students should understand that the process of algebraically solving an equation can produce extraneous solutions. Students study this in Math 2 in connection mainly to square root functions. When teaching this standard, it will be important to link to the concept of having a limited domain, not only by the context of a problem, but also by the nature of the equation. Interpret solutions in terms of the context.	**Example:** The relationship between rate, distance and time can be calculated with the equation $r = \frac{d}{t}$, where r is the rate (speed), d represents the distance traveled, and t represents the time. If the speed of a wave from a tsunami is 150 m/s and the distance from the disturbance in the ocean to the shore is 35 kilometers, how long will it take for the wave to reach the shore? Students should be able to solve square root equations and identify extraneous solutions. **Example:** Solve algebraically: $\sqrt{x-1} = x - 7$ a) Now solve by graphing. b) What do you notice? c) Check the solutions in the original equation. d) Why was an "extra" answer produced? **Example:** The speed of a wave during a tsunami can be calculated with the formula $s = \sqrt{9.81d}$ where s represents speed in meters per second, d represents the depth of the water in meters where the disturbance (for example earthquake) takes place, and 9.81 m/s^2 is the acceleration due to gravity. If the speed of the wave is 150 m/s, what is depth of the water where the disturbance took place?

Instructional Resources	
Tasks	**Additional Resources**

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Algebra – Reasoning with Equations and Inequalities

NC.M2.A-REI.4a

Solve equations and inequalities in one variable.

Solve for all solutions of quadratic equations in one variable.

a. Understand that the quadratic formula is the generalization of solving $ax^2 + bx + c$ by using the process of completing the square.

Concepts and Skills

Pre-requisite

- Rewrite expressions with radicals and rational exponents using the properties of exponents (NC.M2.N-RN.2)
- Use completing the square to write equivalent form of quadratic expressions to reveal extrema (NC.M2.A-SSE.3)
- Justify the solving method and each step in the solving process (NC.M2.A-REI.1)

Connections

- Create and solve one variable equations (NC.M2.A-CED.1)
- Solve inverse variation and square root equations (NC.M2.A-REI.2)
- Explain that quadratic equations have complex solutions (NC.M2.A-REI.4b)
- Solve systems of equations (NC.M2.A-REI.7)
- Write a system of equations as an equation or write an equation as a system of equations to solve (NC.M2.A-REI.11)
- Analyze and compare functions (NC.M2.F-IF.7, NC.M2.F-IF.9)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

2 – Reason abstractly and quantitatively
7 – Look for and make use of structure
8 – Look for and express regularity in repeated reasoning

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication

Students should be able to discuss the relationship between the quadratic formula and the process of completing the square.
New Vocabulary: completing the square, quadratic formula

Mastering the Standard

Comprehending the Standard

Students have used the method of completing the square to rewrite a quadratic expression in standard NC.M2.A-SSE.3. In this standard students are extending the method to solve a quadratic equation.

Some students may set the quadratic equal to zero, rewrite into vertex form $a\,(x-h)^{\,2} + k = 0$, and then begin solving to get the equation into the form $(x-h)^{\,2} = q$ where $q = \frac{-k}{a}$. Other students may adapt the method (i.e. not having to start with the quadratic equal to 0) to get the equation into the same form.

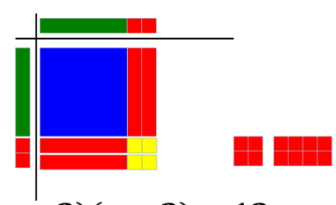
Assessing for Understanding

Students should be able to explain the process of completing the square and be able to generalize it into the quadratic formula.

Example: Solve $-2x^2 - 16x = 20$ by completing the square and the quadratic formula. How are the two methods related?

Example: We often see the need to create a formula when the same steps are repeated in the same type of problems. This is true for completing the square. Recall the steps for completing the square using a visual model, like algebra tiles. A completed example is provided to the right.
To make a formula, we need to generalize the process. To do this, we replace each coefficient with a variable and then solve with those variables in place and we treat those variables same as a numbers.
Below are two columns. In the left is an example, similar to those you have been asked to solve. On the right is a generalized form of the problem. For the

Complete the square $: x^2 - 4x - 8$



$$(x-2)(x-2) - 12$$
$$(x-2)^2 - 12$$

Mastering the Standard

Comprehending the Standard	Assessing for Understanding

Students who write vertex form first

$$-2x^2 - 16x - 20 = 0$$
$$-2(x^2 - 8x) - 20 = 0$$
$$-2(x^2 - 8x + 16) - 20 - 32 = 0$$
$$-2(x-4)^2 - 52 = 0$$
$$-2(x-4)^2 = 52$$
$$(x-4)^2 = 26$$
$$x - 4 = \pm\sqrt{26}$$
$$x = 4 \pm \sqrt{26}$$

Students who adapts method

$$-2(x^2 - 8x) = 20$$
$$-2(x^2 - 8x + 16) = 20 + 32$$
$$-2(x-4)^2 = 52$$
$$(x-4)^2 = 26$$
$$x - 4 = \pm\sqrt{26}$$
$$x = 4 \pm \sqrt{26}$$

This standard is about understanding that the quadratic formula is derived from the process of completing the square. Students should become very familiar with this process before introducing the quadratic formula. Students should understand completing the square both visually and symbolically. Algebra titles are a great way for students to understand the reasoning behind the process of completing the square.
It is not the expectation for students to memorize the steps in deriving the quadratic formula. (Remember that students have no experience with rational expressions which is required as part of completing the derivation on their own!)

left column, provide a mathematical reason for each step as you have done before. (Refer back to a visual model as needed.) One the right side, identify how you can see that mathematical reasoning in the generalized form. When complete, try out the new formula with the example problem from the left column.

Completing the Square
(*Example*)

$$3x^2 + 5x + 4 = 0$$
$$x^2 + \frac{5}{3}x + \frac{4}{3} = 0$$
$$x^2 + \frac{5}{3}x + \frac{5^2}{2^2 \cdot 3^2} = \frac{5^2}{2^2 \cdot 3^2} - \frac{4}{3}$$
$$x^2 + \frac{5}{3}x + \frac{25}{36} = \frac{25}{36} - \frac{4}{3}$$
$$x^2 + \frac{5}{3}x + \frac{25}{36} = \frac{25}{36} - \frac{4}{3} \cdot \frac{12}{12}$$
$$x^2 + \frac{5}{3}x + \frac{25}{36} = \frac{-23}{36}$$
$$\left(x + \frac{5}{6}\right)^2 = \frac{-23}{36}$$
$$x + \frac{5}{6} = \pm\sqrt{\frac{-23}{36}}$$
$$x = \frac{-5}{6} \pm \frac{\sqrt{-23}}{6}$$
$$x = \frac{-5 \pm i\sqrt{23}}{6}$$

Completing the Square
(*Generalized*)

$$ax^2 + bx + c = 0$$
$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$
$$x^2 + \frac{b}{a}x + \frac{b^2}{2^2 \cdot a^2} = \frac{b^2}{2^2 \cdot a^2} - \frac{c}{a}$$
$$x^2 + \frac{b}{a}x + \frac{b^2}{4 \cdot a^2} = \frac{b^2}{4 \cdot a^2} - \frac{c}{a}$$
$$x^2 + \frac{b}{a}x + \frac{b^2}{4 \cdot a^2} = \frac{b^2}{4 \cdot a^2} - \frac{c}{a} \cdot \frac{4a}{4a}$$
$$x^2 + \frac{b}{a}x + \frac{b^2}{4 \cdot a^2} = \frac{b^2 - 4ac}{4 \cdot a^2}$$
$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4 \cdot a^2}$$
$$x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4 \cdot a^2}}$$
$$x = \frac{-b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Instructional Resources

Tasks	Additional Resources

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Algebra – Reasoning with Equations and Inequalities

NC.M2.A-REI.4b

Solve equations and inequalities in one variable.

Solve for all solutions of quadratic equations in one variable.

b. Explain when quadratic equations will have non-real solutions and express complex solutions as $a \pm bi$ for real numbers a and b.

Concepts and Skills

Pre-requisite

- Rewrite expressions with radicals and rational exponents using the properties of exponents (NC.M2.N-RN.2)
- Know there is a complex number and the form of complex numbers (NC.M2.N-NC.1)
- Solve quadratic equations (NC.M2.A-REI.4a)

Connections

- Create and solve one variable equations (NC.M2.A-CED.1)
- Justify the solving method and each step in the solving process (NC.M2.A-REI.1)
- Solve inverse variation and square root equations (NC.M2.A-REI.2)
- Solve systems of equations (NC.M2.A-REI.7)
- Analyze and compare functions (NC.M2.F-IF.7, NC.M2.F-IF.9)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

2 – Reason abstractly and quantitatively
5 – Use appropriate tools strategically
6 – Attend to precision

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication

Students should be able to identify the number of real number solutions of a quadratic equation and justify their assertion.
New Vocabulary: complex solutions

Mastering the Standard

Comprehending the Standard

Students recognize when the quadratic formula gives complex solutions and are able to write them as $a \pm bi$.
Students relate the value of the discriminant to the type of roots expected. A natural extension would be to relate the type of solutions to $ax^2 + bx + c = 0$ to the behavior of the graph of $y = ax^2 + bx + c$.
Students are not required to use the word discriminant, but should be familiar with the concepts of the discriminant.
Students should develop these concepts through experience and reasoning.

Value of Discriminant	Nature of Roots	Nature of Graph
$b^2 - 4ac = 0$	1 real root	Intersects x-axis once
$b^2 - 4ac > 0$	2 real roots	Intersects x-axis twice

Assessing for Understanding

Students should be able to identify the number and type of solution(s) of a quadratic equation.

Example: How many real roots does $2x^2 + 5 = 2x$ have? Find all solutions of the equation.

Example: What is the nature of the roots of $x^2 + 6x + 10 = 0$? How do you know?

Examples: Solve each quadratic using the method indicated and explain when in the solving process you knew the nature of the roots.

a) Square root $3x^2 + 9 = 72$
b) Quadratic formula $4x^2 + 13x - 7 = 0$
c) Factoring $6x^2 + 13x = 5$
d) Complete the square $x^2 + 12x - 2 = 0$

Mastering the Standard	
Comprehending the Standard	**Assessing for Understanding**
<table><tr><td>$b^2 - 4ac < 0$</td><td>2 complex solutions</td><td>Does not intersect x-axis</td></tr></table>	**Example:** Ryan used the quadratic formula to solve an equation and his result was $x = \frac{8 \pm \sqrt{(-8)^2 - 4(1)(-2)}}{2(1)}$. a) Write the quadratic equation Ryan started with in standard form. b) What is the nature of the roots? c) What are the x-intercepts of the graph of the corresponding quadratic function? **Example:** Solve $x^2 + 8x = -17$ for x.

Instructional Resources	
Tasks	**Additional Resources**

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Algebra – Reasoning with Equations and Inequalities

NC.M2.A-REI.7

Solve systems of equations.

Use tables, graphs, and algebraic methods to approximate or find exact solutions of systems of linear and quadratic equations, and interpret the solutions in terms of a context.

Concepts and Skills

Pre-requisite

- Use tables, graphs and algebraic methods to find solutions to systems of linear equations (NC.M1.A-REI.6)
- Operations with polynomials (NC.M2.A-APR.1)
- Justify the solving method and each step in the solving process (NC.M2.A-REI.1)
- Solve quadratic equations (NC.M2.A-REI.4a, NC.M2.A-REI.4b)

Connections

- Create equations (NC.M2.A-CED.1, NC.M2.A-CED.2, NC.M2.A-CED.3)
- Write a system of equations as an equation or write an equation as a system of equations to solve (NC.M2.A-REI.11)
- Analyze and compare functions (NC.M2.F-IF.7, NC.M2.F-IF.9)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

2 – Reason abstractly and quantitatively

5 – Use appropriate tools strategically

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication

Students should be able to discuss the number of solutions possible in a system with a linear and quadratic function and a system with two quadratic functions.

Mastering the Standard

Comprehending the Standard

Students solve a system containing a linear equation and a quadratic equation in two-variables. Students solve graphically and algebraically.

Students interpret solutions of a system of linear and quadratic equations in terms of a context.

Assessing for Understanding

Students should be able to efficiently solve systems of equations with various methods.

Example: In a gymnasium a support wire for the overhead score board slopes down to a point behind the basket. The function $w(x) = -\frac{1}{5}x + 38$ describes the height of the wire above the court, $w(x)$, and the distance in feet from the edge of the score board, x. During a game, a player must shoot a last second shot while standing under the edge of score board. The trajectory of the shot is $b(x) = -.08x^2 + 3x + 6$, where $b(x)$ is the height of the basketball and x is the distance from the player. Describe what could have happened to the shot. (All measurements are in feet.)

Example: The area of a square can be calculated with the formula $Area = s^2$ and the perimeter can be calculated with the formula $Perimeter = 4s$ where s is the length of a side of the square. If the area of the square is the same as its perimeter, what is the length of the side? Demonstrate how you can find the side length using algebraic methods, a table and with a graph.

Example: The student council is planning a dance for their high school. They did some research and found that the relationship between the ticket price and income that they will receive from the dance can be modeled by the function $f(x) = -100(x-4)^2 + 1500$. They also calculated their expenses and found that their expenses can be modeled by the function $g(x) = 300 + 10x$. What ticket price(s) could the student council charge for the dance if they wanted to break-even (the expenses are equal to the income)? Demonstrate how you can find the answer using algebraic methods, a table and with a graph.

Instructional Resources	
Tasks	**Additional Resources**

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Algebra – Reasoning with Equations and Inequalities

NC.M2.A-REI.11

Represent and solve equations and inequalities graphically

Extend the understanding that the x-coordinates of the points where the graphs of two square root and/or inverse variation equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$ and approximate solutions using graphing technology or successive approximations with a table of values.

Concepts and Skills

Pre-requisite

- Understand the mathematical reasoning behind the methods of graphing, using tables and technology to solve systems and equations (NC.M1.A-REI.11)
- Create equations (NC.M2.A-CED.1, NC.M2.A-CED.3)

Connections

- Solve systems of equations (NC.M2.A-REI.7)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.
4 – Model with mathematics

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication

Students should be able to discuss how technology impacts their ability to solve more complex equations or unfamiliar equation types.
New Vocabulary: inverse variation, constant of proportionality

Mastering the Standard

Comprehending the Standard

Students understand that they can solve a system of equations by graphing and finding the point of intersection of the graphs. At this point of intersection the outputs $f(x)$ and $g(x)$ are the same when both graphs have the same input, x.

Students also understand why they can solve any equation by graphing both sides separately and looking for the point of intersection.

In addition to graphing, students can look at tables to find the value of x that makes $f(x) = g(x)$.

Assessing for Understanding

Students should be able to solve complex equations and systems of equations.

Example: Given the following equations determine the x-value that results in an equal output for both functions.

$$f(x) = \sqrt{3x - 2}$$
$$g(x) = \sqrt{x + 2}$$

Example: Solve for x by graphing or by using a table of values.

$$\frac{1}{x} = \sqrt{2x + 3}$$

Instructional Resources

Tasks	Additional Resources

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Algebra, Functions & Function Families

NC Math 1	NC Math 2	NC Math 3
Functions represented as graphs, tables or verbal descriptions in context		
Focus on comparing properties of linear function to *specific* non-linear functions and rate of change. • Linear • Exponential • Quadratic	**Focus on properties of quadratic functions and an introduction to inverse functions through the inverse relationship between quadratic and square root functions.** • Quadratic • Square Root • Inverse Variation	**A focus on more complex functions** • Exponential • Logarithm • Rational functions w/ linear denominator • Polynomial w/ degree $\leq$ three • Absolute Value and Piecewise • Intro to Trigonometric Functions

A Progression of Learning of Functions through Algebraic Reasoning

The conceptual categories of Algebra and Functions are inter-related. Functions describe situations in which one quantity varies with another. The difference between the Function standards and the Algebra standards is that the Function standards focus more on the characteristics of functions (e.g. domain/range or max/min points), function definition, etc. whereas the Algebra standards provide the computational tools and understandings that students need to explore specific instances of functions. As students progress through high school, the coursework with specific families of functions and algebraic manipulation evolve. Rewriting algebraic expressions to create equivalent expressions relates to how the symbolic representation can be manipulated to reveal features of the graphical representation of a function.

Note: The Numbers conceptual category also relates to the Algebra and Functions conceptual categories. As students become more fluent with their work within particular function families, they explore more of the number system. For example, as students continue the study of quadratic equations and functions in Math 2, they begin to explore the complex solutions. Additionally, algebraic manipulation within the real number system is an important skill to creating equivalent expressions from existing functions.

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Functions – Interpreting Functions

NC.M2.F-IF.1

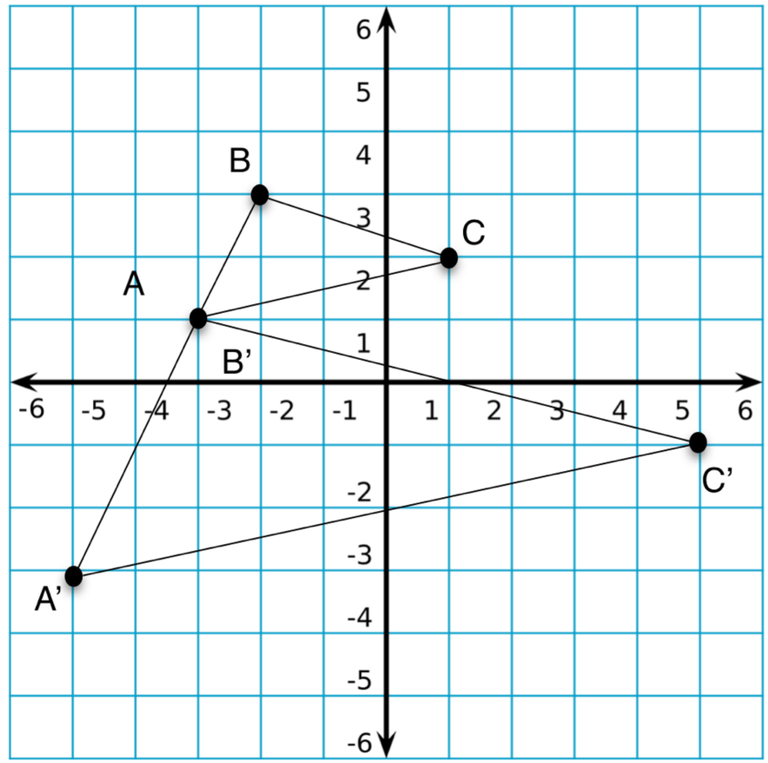
Understand the concept of a function and use function notation.

Extend the concept of a function to include geometric transformations in the plane by recognizing that:

- the domain and range of a transformation function *f* are sets of points in the plane;
- the image of a transformation is a function of its pre-image.

Concepts and Skills	The Standards for Mathematical Practices
Pre-requisite • Formally define a function (NC.M1.F-IF.1)	**Connections** *Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.* 6 – Attend to precision
Connections • Extend the use of a function to express transformed geometric figures (NC.M2.F-IF.2) • Understand the effects of transformations on functions (NC.M2.F-BF.3) • Experiment with transformations on the plane (NC.M2.G-CO.2)	**Disciplinary Literacy** *As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication* Students should discuss how an ordered pair can be the domain of a function. New Vocabulary: preimage, image

Mastering the Standard	
Comprehending the Standard	**Assessing for Understanding**
Students need to understand that coordinate transformations are functions that have a domain and range that are points on the coordinate plane. The domain consists of the points of the pre-image and the range consists of points from the transformed image. This means that the transformed image is a function of its pre-image.	In previous courses, the x-coordinates were the domain and the y-coordinates were the range. As the students understanding is extended, students should be able to view an entire ordered pair as the domain and another ordered pair as the range. **Example:** If the domain of a function that is reflected over the x-axis is (3,4), (2,-1), (-1,2), what is the range? **Example:** If the domain of the coordinate transformation $f(x,y) = (y+1, -x-4)$ is $(1,4), (-3,2), (-1,-1)$, what is the range? **Example:** If the range of the coordinate transformation $f(x,y) = (-2x, -3y+1)$ is $(10,-2), (8,-5), (-2,4)$, what is the domain?

Mastering the Standard	
Comprehending the Standard	**Assessing for Understanding**
	Example: Using the graph below, if this transformation was written as a function, identify the domain and range. 

Instructional Resources	
Tasks	**Additional Resources**

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Functions – Interpreting Functions

NC.M2.F-IF.2

Understand the concept of a function and use function notation.

Extend the use of function notation to express the image of a geometric figure in the plane resulting from a translation, rotation by multiples of 90 degrees about the origin, reflection across an axis, or dilation as a function of its pre-image.

Concepts and Skills

Pre-requisite

- Describe the effects of dilations, translations, rotations, and reflections on geometric figure using coordinates (8.G.3)
- Interpret parts of a function as single entities in context (NC.M2.A-SSE.1b)
- Extend the concept of functions to include geometric transformations (NC.M2.F-IF.1)

Connections

- Interpret key features of functions from graphs, tables, and descriptions (NC.M2.F-IF.4)
- Understand the effects of the transformation of functions on other representations (NC.M2.F-BF.3)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

8 – Look for and express regularity in repeated reasoning

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication

Students should explain with mathematical reasoning how a dilation, rotation, reflection, and translation can be represented as a function.

Mastering the Standard

Comprehending the Standard

Students use function notation to express a geometric transformation when performing the following operations:

- Translation $f(x, y) = (x + h, y + k)$, where h is a horizontal translation and k is a vertical translation.
- Rotation 90° counterclockwise or 270° clockwise $f(x, y) = (-y, x)$
- Rotation 180° $f(x, y) = (-x, -y)$
- Rotation 90° clockwise or 270° counterclockwise $f(x, y) = (y, -x)$
- Reflection over the x-axis $f(x, y) = (x, -y)$
- Reflection over the y-axis $f(x, y) = (-x, y)$
- Dilation $f(x, y) = (kx, ky)$ where k is the scale factor

Students should also continue to use function notation with all functions introduced in this course and Math 1.

Assessing for Understanding

Students should be able to identify the type of transformation through the function notation.

Example: Evaluate the function $f(x, y) = (-x, -y)$ for the coordinates (4,5), (3,1), and (-1,4). Graph the image of the transformation and describe the transformation with words.

Students should be able to use function notation to describe a geometric transformation.

Example: Write a function rule using function notation that will transform a geometric figure by rotating the figure 90° counterclockwise.

Example: Write a function rule using function notation that will translate a geometric figure 3 units to the right and 4 units down.

Instructional Resources	
Tasks	**Additional Resources**
	Transformations (Geogebra) NEW

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Functions – Interpreting Functions

NC.M2.F-IF.4

Interpret functions that arise in applications in terms of the context.

Interpret key features of graphs, tables, and verbal descriptions in context to describe functions that arise in applications relating two quantities, including: domain and range, rate of change, symmetries, and end behavior.

Concepts and Skills

Pre-requisite

- Interpret key features of graphs, tables and verbal descriptions (NC.M1.F-IF.4)
- Interpret parts of an expression in context (NC.M2.A-SSE.1a, NC.M2.A-SSE.1b)
- Extend the use of function notation to geometric transformations (NC.M2.F-IF.2)

Connections

- Analyze and compare functions (NC.M2.F-IF.7, 8, 9)
- Build a quadratic and inverse variation function given a graph, description, or ordered pairs (NC.M2.F-BF.1)
- Understand the effects of transformations on functions (NC.M2.F-BF.3)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

2 – Reason abstractly and quantitatively

4 – Model with mathematics

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication

Students should be able to describe how they identified key features of graph, table, or verbal description and interpret those key features in context.

Mastering the Standard

Comprehending the Standard

When given a table, graph, or verbal descrition of a function that models a real-life situation, explain the meaning of the key features in the context of the problem.

Key features include: domain and range, rate of change, symmetries, and end behavior.

When interpreting rate of change students should be able to describe the rate at which the function is increasing or decreasing. For example, a linear function with a positive slope is increasing at a constant rate. A quadratic with a maximum point is increasing at a decreasing rate, reaching the maximum, and then decreasing at an increasing rate. An inverse variation function in the first quadrant is decreasing at a decreasing rate.

Assessing for Understanding

Students should be able to interpret key features of a function from a verbal description.

Example: Jason kicked a soccerball that was laying on the ground. It was in the air for 3 seconds before it hit the ground again. While the soccer ball was in the air it reached a height of aproximately 30ft. Assuming that the soccer balls height (in feet) is a function of time (in seconds), interpret the domain, range, rate of change, line of symmertry, and end behavior in this context.

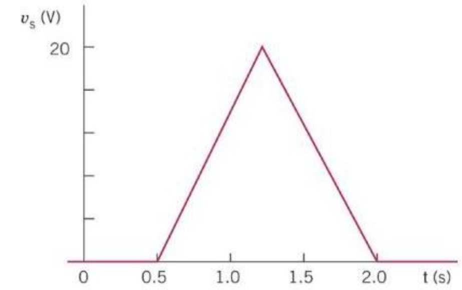
Students should be able to interpret key features of a function from a table.

Example: Julia was experimenting with a toy car and 4ft ramp. She found that as she increased the height of one end of the ramp, the time that the car took to reach the end of the ramp decreased. She collected data to try to figure out the relationship between ramp height and time and came up with the following table.

Height (ft)	.25	.5	.75	1	1.25
Time (sec)	3.9	2.1	1.4	1.1	.9

Assuming that time is a function of height, interpret the domain, range, rate of change, and end behavior in terms of this context.

Students should be able to interpret key features of a function from a graph.

Mastering the Standard	
Comprehending the Standard	**Assessing for Understanding**
Connect this standard with NC.M2.F-IF.7. This standard focuses on interpretation from various representations whereas NC.M2.F-IF.7 focuses on generating different representations. Also, this standard is not limited by function type and can include functions that students do not have the algebraic skills to manipulate. NC.M2.F-IF.7 lists specific function types for which students can use algebra to analyze key features of the function.	**Example:** The graph to the right is the voltage, *v*, in a given circuit as a function of the the time (in seconds). What was the maximum voltage and for how long did it take to complete the circuit? 

Instructional Resources	
Tasks	**Additional Resources**

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Functions – Interpreting Functions

NC.M2.F-IF.7

Analyze functions using different representations.

Analyze quadratic, square root, and inverse variation functions by generating different representations, by hand in simple cases and using technology for more complicated cases, to show key features, including: domain and range; intercepts; intervals where the function is increasing, decreasing, positive, or negative; rate of change; maximums and minimums; symmetries; and end behavior.

Concepts and Skills

Pre-requisite

- Interpret parts of an expression in context (NC.M2.A-SSE.1a, NC.M2.A-SSE.1b)
- Use completing the square to write equivalent form of quadratic expressions to reveal extrema (NC.M2.A-SSE.3)
- Solve quadratic equations (NC.M2.A-REI.4a, NC.M2.A-REI.4b)
- Interpret key features of functions from graphs, tables, and descriptions (NC.M2.F-IF.4)

Connections

- Create and graph two variable equations (NC.M2.A-CED.2)
- Analyze quadratic functions rewritten into vertex form (NC.M2.F-IF.8)
- Compare functions (NC.M2.F-IF.8)
- Build a quadratic and inverse variation function given a graph, description, or ordered pairs (NC.M2.F-BF.1)
- Understand the effects of transformations on functions (NC.M2.F-BF.3)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

2 – Reason abstractly and quantitatively
4 – Model with mathematics
7 – Look for and make use of structure

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication

Students should explain which key features are necessary to find given the context of the problem.
New Vocabulary: inverse variation, constant of proportionality

Mastering the Standard

Comprehending the Standard	Assessing for Understanding
Students need to be able to represent a function with an equation, table, graph, and verbal/written description. When given one representation students need to be able to generate the other representations and use those representations to identify key features. Key features include: domain and range; intercepts; intervals where the function is increasing, decreasing, positive, or negative; rate	Students should be able to find the appropriate key feature to solve problems by analyzing the given function. **Example:** The distance a person can see to the horizon can be found using the function $d(h) = \sqrt{\frac{3h}{2}}$, where $d(h)$ represents the distance in miles and h represents the height the person is above sea level. Create a table and graph to represent this function. Use a table, graph, and the equation to find the domain and range, intercepts, end behavior and intervals where the function is increasing, decreasing, positive, or negative. **Example:** Represent the function $f(x) = 2(x+3)^2 - 2$ with a table and graph. Identify the following key features: domain and range; intercepts; intervals where the function is increasing, decreasing, positive, or negative; rate of change; maximums and minimums; symmetries; and end behavior.

Mastering the Standard	
Comprehending the Standard	**Assessing for Understanding**
of change; maximums and minimums; symmetries; and end behavior. In Math 2 students should focus on quadratic, square root, and inverse variation functions.	**Example:** Represent the function $f(x) = \frac{2}{x}$ with a table and graph. Identify the following key features: domain and range; intercepts; intervals where the function is increasing, decreasing, positive, or negative; rate of change; maximums and minimums; symmetries; and end behavior.

Instructional Resources	
Tasks	**Additional Resources**
Egg Launch Contest NEW	Card Sort: Parabolas (Desmos.com) NEW

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Functions – Interpreting Functions

NC.M2.F-IF.8

Analyze functions using different representations.

Use equivalent expressions to reveal and explain different properties of a function by developing and using the process of completing the square to identify the zeros, extreme values, and symmetry in graphs and tables representing quadratic functions, and interpret these in terms of a context.

Concepts and Skills

Pre-requisite

- Rewrite a quadratic function to reveal key features (NC.M1.F-IF.8a)
- Interpret parts of an expression in context (NC.M2.A-SSE.1a, NC.M2.A-SSE.1b)
- Use completing the square to write equivalent form of quadratic expressions to reveal extrema (NC.M2.A-SSE.3)

Connections

- Creating and graphing equations in two variables (NC.M2.A-CED.2)
- Interpret key features of functions from graphs, tables, and descriptions (NC.M2.F-IF.4)
- Analyze and compare functions for key features (NC.M2.F-IF.7, NC.M2.F-IF.9)
- Build a quadratic and inverse variation function given a graph, description, or ordered pairs (NC.M2.F-BF.1)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

7 – Look for and make use of structure

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication

Students should be able to explain which key features can be found from each form of a quadratic function.

New Vocabulary: completing the square

Mastering the Standard

Comprehending the Standard

Students look at equivalent expressions of functions to identify key features on the graph and in a table of the function.

For example, students should factor quadratics to identify the zeros, complete the square to reveal extreme values and the line of symmetry, and look at the standard form of the equation to reveal the y-intercept.

Students could also argue that by factoring and finding the zeros they could easily find the line of symmetry by finding the midpoint between the zeros.

Once identifying the key features students should interpret them in terms of the context.

Assessing for Understanding

Students should be able use the process of completing the square to identify key features of the function.

Example: Coyote was chasing roadrunner, seeing no easy escape, Roadrunner jumped off a cliff towering above the roaring river below. Molly Mathematician was observing the chase and obtained a digital picture of this fall. Using her mathematical knowledge, Molly modeled the Road Runner's fall with the following quadratic functions:

$$h(t) = -16t^2 + 32t + 48$$
$$h(t) = -16(t+1)(t-3)$$
$$h(t) = -16(t-1)^2 + 64$$

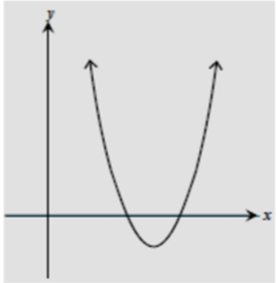
a) How can Molly have three equations?

b) Which of the rules would be most helpful in answering each of these questions? Explain.

  i. What is the maximum height the Road Runner reaches and when will it occur?

  ii. When would the Road Runner splash into the river?

  iii. At what height was the Road Runner when he jumped off the cliff?

Mastering the Standard	
Comprehending the Standard	**Assessing for Understanding**
	Students should be able to identify the key features able to be found in each form of a quadratic function. **Example:** Which of the following equations could describe the function of the given graph to the right? Explain. $f_1(x) = (x+12)^2 + 4$ $\quad$ $f_5(x) = -4(x+2)(x+3)$ $f_2(x) = -(x-2)^2 - 1$ $\quad$ $f_6(x) = (x+4)(x-6)$ $f_3(x) = (x+18)^2 - 40$ $\quad$ $f_7(x) = (x-12)(-x+18)$ $f_4(x) = (x+12)^2 + 4$ $\quad$ $f_8(x) = (20-x)(30-x)$ 

Instructional Resources	
Tasks	**Additional Resources**
Throwing Horseshoes (Illustrative Mathematics) Profit of a Company (Illustrative Mathematics)	FAL: Representing Quadratics Graphically (Mathematics Assessment Project)

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NC.M2.F-IF.9

Analyze functions using different representations.

Compare key features of two functions (linear, quadratic, square root, or inverse variation functions) each with a different representation (symbolically, graphically, numerically in tables, or by verbal descriptions).

Concepts and Skills

Pre-requisite

- Compare key features of two functions (NC.M1.F-IF.9)
- Interpret parts of an expression in context (NC.M2.A-SSE.1a, NC.M2.A-SSE.1b)
- Use completing the square to write equivalent form of quadratic expressions to reveal extrema (NC.M2.A-SSE.3)
- Solve quadratic equations (NC.M2.A-REI.4a, NC.M2.A-REI.4b)
- Interpret key features of functions from graphs, tables, and descriptions (NC.M2.F-IF.4)
- Analyze functions for key features (NC.M2.F-IF.7, NC.M2.F-IF.8)
- Build a quadratic and inverse variation function given a graph, description, or ordered pairs (NC.M2.F-BF.1)
- Understand the effects of transformations on functions (NC.M2.F-BF.3)

Connections

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

1 – Make sense of problems and persevere in solving them

7 – Look for and make use of structure

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication

New Vocabulary: inverse variation, constant of proportionality

Mastering the Standard

Comprehending the Standard

Students need to compare characteristics of two functions. The representations of the functions should vary: table, graph, algebraically, or verbal description.

In this standard students are comparing any two of the following functions:

- Linear
- Quadratic
- Square root

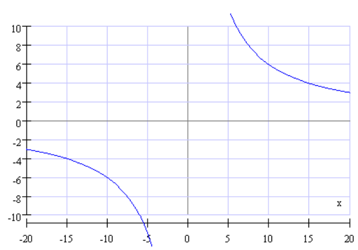
Assessing for Understanding

Students should be able to compare key features of two functions in different representations.

Example: Compare the constant of proportionality for each of the following inverse variation models and list them in order from least to greatest.

$$y = \frac{90}{x}$$

x	y
5	36
10	18
15	12
20	9
25	7.2



Mastering the Standard

Comprehending the Standard

- Inverse variation

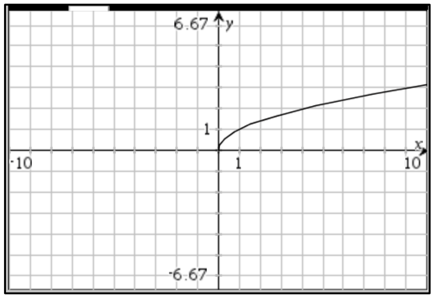
This means that students need to be able to compare functions that are in the same function family (for example quadratic vs quadratic) and functions that are in different function families (for example square root vs inverse variation).

The representations of the functions that are being compared needs to be different. For example compare a graph of one function to an equation of another.

Assessing for Understanding

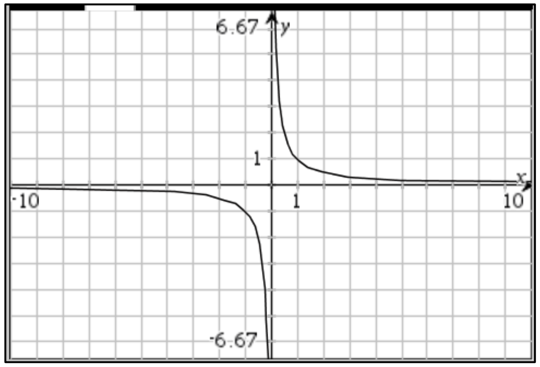
Example: Compare and contrast the domain and range, rate of change and intercepts of the two functions below represented below.

Meredith runs at a constant rate of 6 miles per hour when she runs on her treadmill. The distance that she runs on her treadmill is a function of the time that she is runs.



Example: Compare and contrast the end behavior and symmetries of the two functions represented below.

x	$f(x)$
-2	4
-1	1
0	0
1	1
2	4



Instructional Resources

Tasks	Additional Resources
Throwing Baseballs (Illustrative Mathematics)	

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Functions – Building Functions

NC.M2.F-BF.1

Build a function that models a relationship between two quantities.

Write a function that describes a relationship between two quantities by building quadratic functions with real solution(s) and inverse variation functions given a graph, a description of a relationship, or ordered pairs (include reading these from a table).

Concepts and Skills

Pre-requisite

- Build linear and exponential functions from tables, graphs, and descriptions (NC.M1.F-BF.1a)
- Creating and graphing equations in two variables (NC.M2.A-CED.2)
- Interpret key features of functions from graphs, tables, and descriptions (NC.M2.F-IF.4)

Connections

- Analyze and compare functions for key features (NC.M2.F-IF.7, NC.M2.F-IF.8, NC.M2.F-IF.9)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

2 – Reason abstractly and quantitatively
4 – Model with mathematics
5 – Use appropriate tools strategically

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication

Students should be able to justify their chosen model with mathematical reasoning.
New Vocabulary: inverse variation, constant of proportionality

Mastering the Standard

Comprehending the Standard

Given a graph, ordered pairs (including a table), or description of a relationship, students need to be able to write an equation of a function that describes a quadratic or inverse variation relationship.
Make sure that quadratic functions have real solutions. (Operations with complex numbers are **not** part of the standards.)

Student should realize that in an inverse variation relationship they can multiply the x and y coordinates of an ordered pair together to get the constant of proportionality.

When given the x-intercepts and a point on a quadratic students can solve the equation $f(x) = a(x - m)(x - n)$ for a after

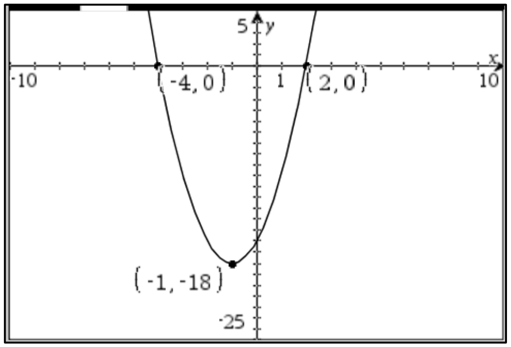
Assessing for Understanding

Students should be able to build functions that model a given situation using the context and information available from various representations.

Example: Write an equation of the function given the table.

x	-3	-2	-1	0	1	2	3
$f(x)$	-4	-6	-12	undefined	12	6	4

Example: Write an equation to represent the following relationship: y varies inversely with x. When $x = 3$ then $y = 5$.

Mastering the Standard	
Comprehending the Standard	**Assessing for Understanding**
substituting the x-intercpets for m and n, and the x and y coordinates from the point for x and $f(x)$. Once the student has solved for a they can plug $a, m,$ and n into the equation so that their equation is written in factored form. When given a maximum or minimum point on a quadratic and another point students can use the equation $f(x) = a(x-h)^2 + k$ to solve for a so that their function equation is written in vertex form.	**Example:** Write an equation of the function given the graph. 

Instructional Resources	
Tasks	**Additional Resources**

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Functions – Building Functions

NC.M2.F-BF.3

Build new functions from existing functions.

Understand the effects of the graphical and tabular representations of a linear, quadratic, square root, and inverse variation function f with $k \cdot f(x)$, $f(x) + k$, $f(x + k)$ for specific values of k (both positive and negative).

Concepts and Skills

Pre-requisite

- Interpret parts of an expression in context (NC.M2.A-SSE.1a, NC.M2.A-SSE.1b)
- Operations with polynomials (NC.M2.A-APR.1)
- Extend the concept of functions to include geometric transformations (NC.M2.F-IF.1)

Connections

- Extend the use of function notation to express the transformation of geometric figures (NC.M2.F-IF.2)
- Interpret key features of functions from graphs, tables, and descriptions (NC.M2.F-IF.4)
- Analyze and compare functions for key features (NC.M2.F-IF.7, NC.M2.F-IF.9)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

7 – Look for and make sense of structure

8 – Look for and express regularity in repeated reasoning

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication

Students should be able to compare and contrast the transformation of geometric figures and two variable equations expressed as functions.

New Vocabulary: inverse variation, constant of proportionality, vertical compression, vertical stretch

Mastering the Standard

Comprehending the Standard

It is important to note that this standard is under the domain of building functions. The functions are being built for a purpose, to solve a problem or to offer insight.

Students should conceptually understand the transformations of functions and refrain from blindly memorizing patterns of functions.

Students should be able to explain why $f(x + k)$ moves the graph of the function left or right depending on the value of k.

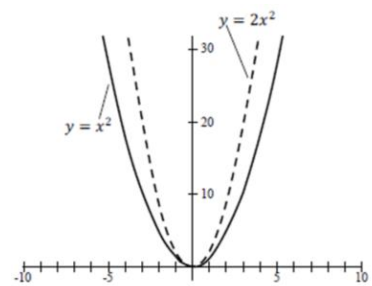
Students should understand how changes in the equation effect changes in graphs and tables of values.

- $k \cdot f(x)$ If $0 < k < 1$ there is a vertical compression meaning that the outputs of the

Assessing for Understanding

Students should be able to describe the effect of transformations on algebraic functions.

Example: Compare the shape and position of the graphs of $f(x) = x^2$ and $g(x) = 2x^2$ and explain the differences in terms of the algebraic expressions for the functions.



Example: Describe the effect of varying the parameters a, h, and k on the shape and position of the graph of the equation $f(x) = a(x-h)^2 + k$. Then compare that to the effect of varying the parameters a, h, and k on the shape and position of the graph of the equation $g(x) = a\sqrt{x-h} + k$.

Mastering the Standard

Comprehending the Standard

function have been reduced since they were multiplied by a number between 0 and 1. If $k > 1$ there is a vertical stretch meaning that the outputs have all been multiplied by the same value. If k is negative, then all of the outputs will change signs and this will result in a reflection over the x-axis.

- $f(x) + k$ If k is positive all of the outputs are being increased by the same value and the graph of the function will move up. If k is negative, all of the outputs are being decreased by the same value and the graph of the function will move down.
- $f(x + k)$ If k is positive then all of the inputs are increasing by the same value. Since they are increasing before they are plugged into the operations of the function, the graph will move to the left. If k is negative, then all of the inputs are decreasing by the same value. Since they are decreasing before they are plugged into the operations of the function the graph will move to the right.

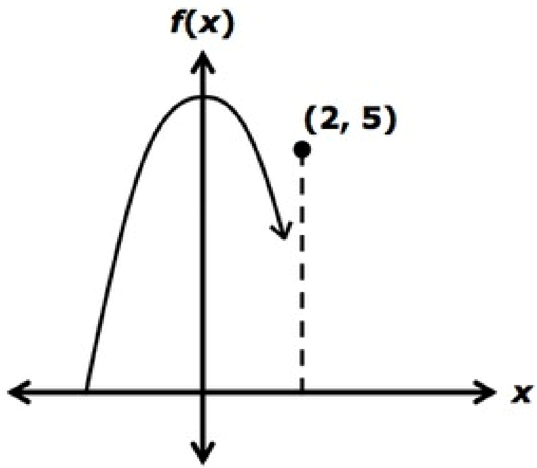
Students should focus on linear, quadratic, square root, and inverse variation functions in this course.

Assessing for Understanding

Example: Describe the transformation that took place with the function transformation where $f(x) = \sqrt{x}$ is transformed to $g(x) = 2\sqrt{x+3} - 4$.

Example: Write an equation for the transformation of $f(x) = \frac{1}{x}$ after it has been translated 3 units to the right and reflected over the x-axis.

Example: A computer game uses functions to simulate the paths of an archer's arrows. The x-axis represents the level ground on which the archer stands, and the coordinate pair (2,5) represents the top of a castle wall over which he is trying to fire an arrow.
In response to user input, the first arrow followed a path defined by the function $f(x) = 6 - x^2$ failing to clear the castle wall.



The next arrow must be launched with the same force and trajectory, so the user must reposition the archer in order for his next arrow to have any chance of clearing the wall.

a) How much closer to the wall must the archer stand in order for the arrow to clear the wall by the greatest possible distance?
b) What function must the user enter in order to accomplish this?
c) If the user can only enter functions of the form $f(x + k)$, what are all the values of k that would result in the arrow clearing the castle wall?

https://www.illustrativemathematics.org/content-standards/HSF/BF/B/3/tasks/695

Instructional Resources

Tasks	Additional Resources
Medieval Archer (Illustrative Mathematics)	

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Geometry

NC Math 1	NC Math 2	NC Math 3
Analytic & Euclidean		
Focus on coordinate geometry • Distance on the coordinate plane • Midpoint of line segments • Slopes of parallel and perpendicular lines • Prove geometric theorems algebraically	**Focus on triangles** • Congruence • Similarity • Right triangle trigonometry ○ Special right triangles	**Focus on circles and continuing the work with triangles** • Introduce the concept of radian • Angles and segments in circles • Centers of triangles • Parallelograms
A Progression of Learning		
Integration of Algebra and Geometry • Building off of what students know from 5^{th} – 8^{th} grade with work in the coordinate plane, the Pythagorean theorem and functions. • Students will integrate the work of algebra and functions to prove geometric theorems algebraically. • Algebraic reasoning as a means of proof will help students to build a foundation to prepare them for further work with geometric proofs.	**Geometric proof and SMP3** • An extension of transformational geometry concepts, lines, angles, and triangles from 7^{th} and 8^{th} grade mathematics. • Connecting proportional reasoning from 7^{th} grade to work with right triangle trigonometry. • Students should use geometric reasoning to prove theorems related to lines, angles, and triangles. *It is important to note that proofs here are not limited to the traditional two-column proof. Paragraph, flow proofs and other forms of argumentation should be encouraged.*	**Geometric Modeling** • Connecting analytic geometry, algebra, functions, and geometric measurement to modeling. • Building from the study of triangles in Math 2, students will verify the properties of the centers of triangles and parallelograms.

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NC.M2.G-CO.2

Experiment with transformations in the plane.

Experiment with transformations in the plane.

- Represent transformations in the plane.
- Compare rigid motions that preserve distance and angle measure (translations, reflections, rotations) to transformations that do not preserve both distance and angle measure (e.g. stretches, dilations).
- Understand that rigid motions produce congruent figures while dilations produce similar figures.

Concepts and Skills

Pre-requisite

- Verify experimentally the properties of rotations, reflections and translations. (8.G.1)
- Understand congruence through rotations, reflections and translations (8.G.2)
- Use coordinates to describe the effects of transformations on 2-D figures (8.G.3)

Connections

- Verify experimentally properties of rigid motions in terms of angles, circles, ⊥ and || lines and line segments (NC.M2.G-CO.4)
- Verify experimentally the properties of dilations given center and scale factor (NC.M2.G-SRT.1)
- Geometric transformations as functions (NC.M2.F-IF.1)
- Using function notation to express transformations (NC.M2.F-IF.2)
- Given a regular polygon, identify reflections/rotations that carry the image onto itself (NC.M2.G-CO.3)
- Given a geometric figure and a rigid motion, find the image of the figure/Given a figure and its image, describe a sequence of rigid motions between preimage and image (NC.M2.G-CO.5)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

1 – Make sense of problems and persevere in solving them
5 – Use appropriate tools strategically
6 – Attend to precision

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication

New Vocabulary: rigid motion, non-rigid motion

Mastering the Standard

Comprehending the Standard

In 8th grade, students understand transformations and their relationship to congruence and similarity through the use of physical models, transparencies, and geometry software.

In Math 2, students begin to formalize these ideas and connect transformations to the algebraic concept of function. A transformation is a new type of function that maps *two* numbers (an ordered pair) to another pair of numbers.

Assessing for Understanding

Students describe and compare function transformations on a set of points as inputs to produce another set of points as outputs.

Example: A plane figure is translated 3 units right and 2 units down. The translated figure is then dilated with a scale factor of 4, centered at the origin.

a. Draw a plane figure and represent the described transformation of the figure in the plane.
b. Explain how the transformation is a function with inputs and outputs.
c. Write a mapping rule for this function.
d. Determine what type of relationship, if any, exists between the pre-image and the image after this series of transformations. Provide evidence to support your thinking.

Mastering the Standard	
Comprehending the Standard	**Assessing for Understanding**
Transformations that are **rigid** (preserve distance and angle measure: reflections, rotations, translations, or combinations of these) and those that are not (stretches, dilations or rigid motions followed by stretches or dilations). Translations, rotations and reflections produce congruent figures while dilations produce similar figures. Note: It is not intended for students to memorize transformation rules and thus be able to identify the transformation from the rule. Students should understand the structure of the rule and how to use it as a function to generate outputs from the provided inputs.	**Example:** Transform ΔABC with vertices $A\ (1,1)$, $B\ (6,3)$ and $C\ (2,13)$ using the function rule $(x, y) \rightarrow (-y, x)$. Describe the transformation as completely as possible.

Instructional Resources	
Tasks	**Additional Resources**
Horizontal Stretch of the Plane (Illustrative Mathematics)	Transforming 2D Figures (Mathematics Assessment Project) Marcellus the Giant (Desmos.com) NEW

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NC.M2.G-CO.3

Experiment with transformations in the plane.

Given a triangle, quadrilateral, or regular polygon, describe any reflection or rotation symmetry i.e., actions that carry the figure onto itself. Identify center and angle(s) of rotation symmetry. Identify line(s) of reflection symmetry. Represent transformations in the plane.

Concepts and Skills

Pre-requisite

- Understand congruence through rotations, reflections and translations (8.G.2)
- Use coordinates to describe the effects of transformations on 2-D figures (8.G.3)

Connections

- Geometric transformations as functions (NC.M2.F-IF.1)
- Using function notation to express transformations (NC.M2.F-IF.2)
- Understand that rigid motions produce congruent figures (NC.M2.G-CO.2)
- Verify experimentally properties of rigid motions in terms of angles, circles and lines (NC.M2.G-CO.4)
- Given a geometric figure and a rigid motion, find the image of the figure/Given a figure and its image, describe a sequence of rigid motions between preimage and image (NC.M2.G-CO.5)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

3 – Construct viable arguments and critique the reasoning of others

6 – Attend to precision

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication

- What kinds of figures have only rotational symmetry? What kinds of figures have only reflection symmetry? What kind have both? Why do you think this happens?

Mastering the Standard

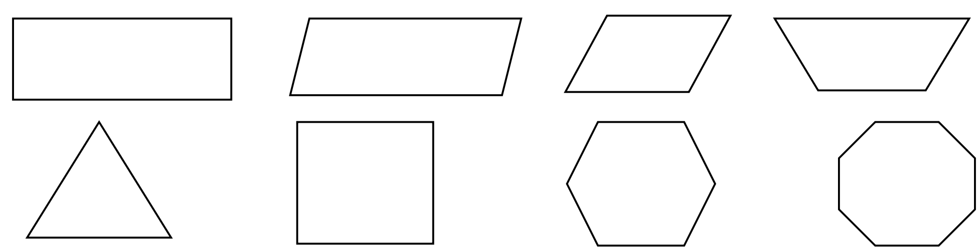
Comprehending the Standard

"The concepts of congruence, similarity, and symmetry can be understood from the perspective of geometric transformation. Fundamental are the rigid motions: translations, rotations, reflections, and combinations of these, all of which are here assumed to preserve distance and angles (and therefore shapes generally). Reflections and rotations each explain a particular type of symmetry, and the symmetries of an object offer insight into its attributes—as when the reflective symmetry of an isosceles triangle assures that its base angles are congruent." (*Intro of HS Geometry strand of the CCSS-M*)

Assessing for Understanding

Students describe and illustrate how figures such as an isosceles triangle, equilateral triangle, rectangle, parallelogram, kite, isosceles trapezoid or regular polygon are mapped onto themselves using transformations.

Example: For each of the following figures, describe and illustrate the rotations and/or reflections that carry the figure onto itself.



Students should make connections between the symmetries of a geometric figure and its properties. In addition to the example of an isosceles triangle noted above, figures with 180° rotation symmetry have opposite sides that are congruent.

Example: What connections can you make between a particular type of symmetry and the properties of a figure?

Mastering the Standard	
Comprehending the Standard	**Assessing for Understanding**
	Students can describe and illustrate the center of rotation and angle(s) of rotation symmetry and line(s) of reflection symmetry.

Instructional Resources	
Tasks	**Additional Resources**
	Transforming 2D Figures (Mathematics Assessment Project)

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NC.M2.G-CO.4

Experiment with transformations in the plane.

Verify experimentally properties of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

Concepts and Skills

Pre-requisite

- Using coordinates to solve geometric problems algebraically (NC.M1.G-GPE.4)
- Using slope to determine parallelism and perpendicularity (NC.M1.G-GPE.5)
- Finding midpoint/endpoint of a line segment, given either (NC.M1.G-GPE.6)

Connections

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

4 – Model with mathematics
5 – Use appropriate tools strategically
6 – Attend to precision

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication

New Vocabulary: rigid motion, non-rigid motion

Mastering the Standard

Comprehending the Standard

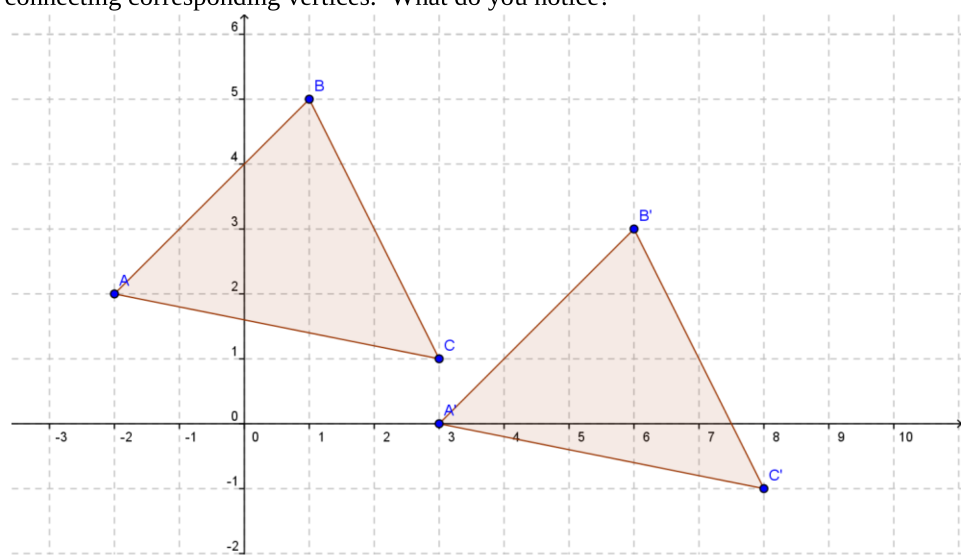
This standard is intended to help students develop the definition of each rigid motion in regards to the characteristics between pre-image and image points through experimentation.

- For *translations*: connecting points on the pre-image to corresponding points on the image produces line segments that are congruent and parallel.
- For *reflections*: the line of reflection is the perpendicular bisector of any line segment joining a point on the pre-image to the corresponding point on the image. Therefore, corresponding points on the pre-image and the image are equidistant from the line of reflection.
- For *rotations*: a point on the pre-image and its corresponding point on the image lie on a circle whose center is the center of rotation. Therefore, line segments connecting corresponding points on the pre-image and the image to the center of

Assessing for Understanding

Students develop the definition of each transformation in regards to the characteristics between pre-image and image points.

Example: Triangle A'B'C' is a translation of triangle ABC. Write the rule for the translation. Draw line segments connecting corresponding vertices. What do you notice?



Productive answers:
$(x, y) \rightarrow (x+5, y-2)$
$\overline{AA'} \parallel \overline{BB'} \parallel \overline{CC'}$
$\overline{AA'} \cong \overline{BB'} \cong \overline{CC'}$

Mastering the Standard

Comprehending the Standard

rotation are congruent and form an angle equal to the angle of rotation.

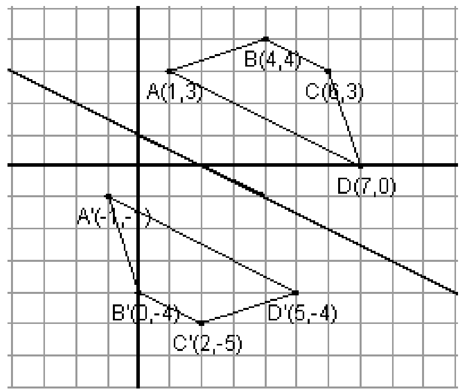
There are two approaches – both that should be used when teaching this standard. First, work with transformations on the coordinate plane. For this, students need to have some reasoning skills with figures on the coordinate plane. Calculating *distances* on the coordinate plane can help achieve this:

- show that the line of symmetry bisects the segment connecting image to preimage for a reflection;
- show that the segments connecting the image to center and preimage to center are the same length and represent the radius of the circle whose central angle is the angle of rotation
- show line segments are parallel for translations
- show line segments are perpendicular for reflection

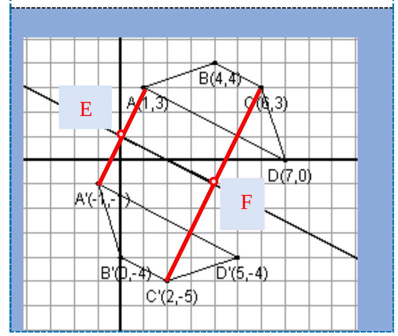
The second approach is to work with the transformations on the Euclidean plane. Students should use tools (patty paper, mirrors, rulers, protractors, string, technology, etc) to measure and reason.

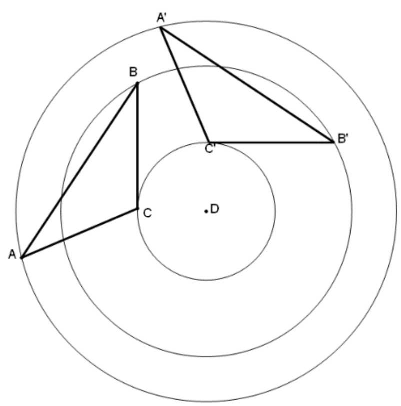
Assessing for Understanding

Example: Quadrilateral A'B'C'D' is a reflection of quadrilateral ABCD across the given line. Draw line segments connecting A to A' and C to C'. Label the points of intersection with the line of reflection as E and F. What do you notice?



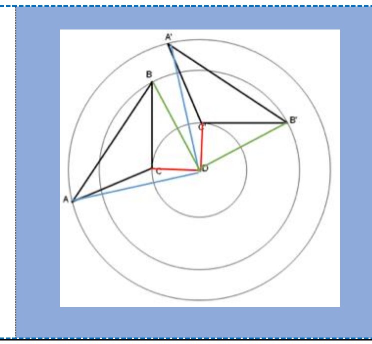
Productive answers:
$\overline{AA'} \parallel \overline{CC'}$
$\overline{AE} \cong \overline{A'E}$
$\overline{CF} \cong \overline{C'F}$
$\overline{AA'} \perp \overline{EF}$
$\overline{CC'} \perp \overline{EF}$
A and A' are equidistant from the line of reflection.
C and C' are equidistant from the line of reflection.





Example: Triangle $A'B'C'$ is a rotation of triangle ABC. Describe the rotation, indicating center, angle, and direction. Draw line segments connecting corresponding vertices to the center. What do you notice?

Triangle ABC is rotated 90° CW around point D.
Corresponding vertices lie on the same circle. The circles all have center D.
$\overline{CD} \cong \overline{C'D}$ and $m\angle CDC' = 90°$.
$\overline{AD} \cong \overline{A'D}$ and $m\angle ADA' = 90°$.
$\overline{BD} \cong \overline{B'D}$ and $m\angle BDB' = 90°$.



Instructional Resources

Tasks	Additional Resources

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Geometry – Congruence

NC.M2.G-CO.5

Experiment with transformations in the plane.

Given a geometric figure and a rigid motion, find the image of the figure. Given a geometric figure and its image, specify a rigid motion or sequence of rigid motions that will transform the pre-image to its image.

Concepts and Skills

Pre-requisite

- Understand congruence through rotations, reflections and translations (8.G.2)

Connections

- Geometric transformations as functions (NC.M2.F-IF.1)
- Using function notation to express transformations (NC.M2.F-IF.2)
- Understand that rigid motions produce congruent figures (NC.M2.G-CO.2)
- Verify experimentally properties of rigid motions in terms of angles, circles and lines (NC.M2.G-CO.4)
- Given a regular polygon, identify reflections/rotations that carry the image onto itself (NC.M2.G-CO.3)
- Determining congruence through a sequence of rigid motions (NC.M2.G-CO.6)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

1 – Make sense of problems and persevere in solving them

4 – Model with mathematics

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication

New Vocabulary: rigid motion, non-rigid motion

Mastering the Standard

Comprehending the Standard

In 8th grade, students build an understanding of congruence through translations, reflections and rotation informally and in terms of coordinates. Students in MS verify that images transformed in the plane with rigid motions keep the same property as the preimage. They also note the effect of the rigid motion on the coordinates of the image and preimage. This standard extends the work in MS by requiring students to give precise descriptions of sequences of rigid motions where they specify exact points, lines and angles with coordinates and/or equations. Analytically, each rigid motion should be specified as follows:

- **<u>For each rotation</u>**, students should specify a point (x, y) and angle.
- **<u>For each translation</u>**, specific pairs of points (x, y) should be identified;
- **<u>For each reflection</u>**, the equation of the line $(y = mx + b)$ should be identified.

These specificities hold true whether working in the coordinate or Euclidean plane. Students must specify all points, lines of reflection/symmetry and angles of rotation.

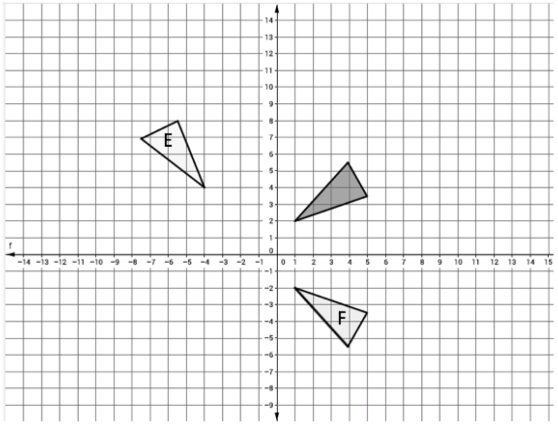
Assessing for Understanding

Students transform a geometric figure given a rotation, reflection, or translation, using graph paper, tracing paper and/or geometry software.

Example: Using the figure on the right:

Part 1: Draw the shaded triangle after:

a. It has been translated −7 units horizontally and +1 units vertically. Label your answer A.
b. It has been reflected over the *x*-axis. Label your answer B.
c. It has been rotated 90° clockwise about the origin. Label your answer C.
d. It has been reflected over the line $y = 6$. Label your answer D.



Students predict and verify the sequence of transformations (a composition) that will map a figure onto another.

Part 2: Describe fully the transformation or sequence of transformations that:

a. Takes the shaded triangle onto the triangle labeled E.
b. Takes the shaded triangle onto the triangle labeled F.

Instructional Resources

Tasks	Additional Resources

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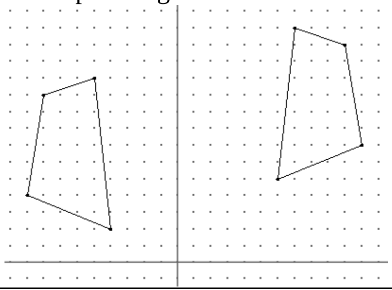
NC.M2.G-CO.6

Understand congruence in terms of rigid motions.

Determine whether two figures are congruent by specifying a rigid motion or sequence of rigid motions that will transform one figure onto the other.

Concepts and Skills	The Standards for Mathematical Practices
Pre-requisite • Given a geometric figure and a rigid motion, find the image of the figure/Given a figure and its image, describe a sequence of rigid motions between preimage and image (NC.M2.G-CO.5) **Connections** • Use the properties of rigid motions to show that two triangles are congruent if their corresponding sides and angles are congruent (NC.M2.G-CO.7)	**Connections** *Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.* 3 – Construct viable arguments and critique the reasoning of others 5 – Use appropriate tools strategically 7 – Look for and make use of structure **Disciplinary Literacy** *As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication* New Vocabulary: rigid motion, non-rigid motion

Mastering the Standard	
Comprehending the Standard	**Assessing for Understanding**
This standard connects to the 8th grade standard where students informally addressed congruency of figures through rigid motions to the formalized HS standard where students specifically defined points, lines, planes and angles of rigid motion transformations. Students recognize rigid transformations preserve size and shape (or distance and angle) and develop the definition of congruence. This standard goes beyond the assumption of mere correspondence of points, lines and angles and thus establishing the properties of congruent figures.	Students use descriptions of rigid motion and transformed geometric figures to predict the effects rigid motion has on figures in the coordinate plane. **Example**: Consider parallelogram ABCD with coordinates $A(2,-2)$, $B(4,4)$, $C(12,4)$ and $D(10,-2)$. Consider the following transformations. Make predictions about how the lengths, perimeter, area and angle measures will change under each transformation below: a. A reflection over the *x*-axis. b. A rotation of 270° counter clockwise about the origin. c. A dilation of scale factor 3 about the origin. d. A translation to the right 5 and down 3. Verify your predictions by performing the transformations. Compare and contrast which transformations preserved the size and/or shape with those that did not preserve size and/or shape. Generalize: which types of transformation(s) will produce congruent figures?

Mastering the Standard	
Comprehending the Standard	**Assessing for Understanding**
	Students determine if two figures are congruent by determining if rigid motions will map one figure onto the other. **Example**: Determine if the figures are congruent. If so, describe and demonstrate a sequence of rigid motions that maps one figure onto the other. 

Instructional Resources	
Tasks	**Additional Resources**

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Geometry – Congruence

NC.M2.G-CO.7

Understand congruence in terms of rigid motions.

Use the properties of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.

Concepts and Skills

Pre-requisite

- Determining congruence through a sequence of rigid motions (NC.M2.G-CO.6)

Connections

- Use and justify criteria to determine triangle congruence (NC.M2.G-CO.8)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

3 – Construct viable arguments and critique the reasoning of others

5 – Use appropriate tools strategically

7 – Look for and make use of structure

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication

New Vocabulary: rigid motion, non-rigid motion

Mastering the Standard

Comprehending the Standard

A rigid motion is a transformation of points in space consisting of a sequence of one or more translations, reflections, and/or rotations. Rigid motions are assumed:

- to map lines to lines, rays to rays, and segments to segments and
- to preserve distances and angle measures.

Two triangles are said to be congruent if one can be exactly superimposed on the other by a rigid motion, and the congruence theorems specify the conditions under which this can occur.

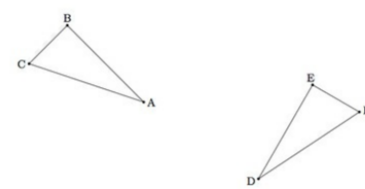
This standard connects the establishment of congruence to congruent triangle proofs based on corresponding sides and angles.

Assessing for Understanding

Students identify corresponding sides and corresponding angles of congruent triangles. Explain that in a pair of congruent triangles, corresponding sides are congruent (distance is preserved) and corresponding angles are congruent (angle measure is preserved). They demonstrate that when distance is preserved (corresponding sides are congruent) and angle measure is preserved (corresponding angles are congruent) the triangles must also be congruent.

Example: Illustrative Mathematics Task – Properties of Congruent Triangles

To the right is a picture of two triangles:



a. Suppose there is a sequence of rigid motions which maps ΔABC to ΔDEF. Explain why corresponding sides and angles of these triangles are congruent.

b. Suppose instead that corresponding sides and angles of ΔABC to ΔDEF are congruent. Show that there is a sequence of rigid motions which maps ΔABC to ΔDEF'

Instructional Resources

Tasks	Additional Resources

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Geometry – Congruence

NC.M2.G-CO.8

Understand congruence in terms of rigid motions.

Use congruence in terms of rigid motion.

Justify the ASA, SAS, and SSS criteria for triangle congruence. Use criteria for triangle congruence (ASA, SAS, SSS, HL) to determine whether two triangles are congruent.

Concepts and Skills	The Standards for Mathematical Practices
Pre-requisite	**Connections**
• Use the properties of rigid motions to show that two triangles are congruent if their corresponding sides and angles are congruent (NC.M2.G-CO.7)	*Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.* 3 – Construct viable arguments and critique the reasoning of others 5 – Use appropriate tools strategically 7 – Look for and make use of structure
Connections	**Disciplinary Literacy**
• Use triangle congruence to prove theorems about lines, angles, and segments for relationships in geometric figures (NC.M2.G-CO.9) • Use triangle congruence to prove theorems about triangles (NC.M2.G-CO.10)	*As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication*

Mastering the Standard	
Comprehending the Standard	**Assessing for Understanding**
Extending from the 7th grade standard where students examine the conditions required to determine a unique triangle, students come to understand the specific characteristics of congruent triangles which lays the groundwork for geometric proof. Proving these theorems helps students to then prove theorems about lines and angles in other geometric figures and other triangle proofs. **<u>Videos of Transformation Proofs:</u>** Animated Proof of SAS (YouTube) Animated Proof of ASA (YouTube)	Students list the sufficient conditions to prove triangles are congruent: ASA, SAS, and SSS. They map a triangle with one of the sufficient conditions (e.g., SSS) onto the original triangle and show that corresponding sides and corresponding angles are congruent. **Example:** Josh is told that two triangles ΔABC and ΔDEF share two sets of congruent sides and one set of congruent angles: $\overline{AB}$ is congruent to $\overline{DE}$, $\overline{BC}$ is congruent to $\overline{EF}$, and $\angle B$ is congruent to $\angle E$. He is asked if these two triangles must be congruent. Josh draws the two triangles marking congruent sides and angles. Then he says, "They are definitely congruent because two pairs of sides are congruent and the angle between them is congruent!" a. Draw the two triangles. Explain whether Josh's reasoning is correct using triangle congruence criteria. b. Given two triangles ΔABC and ΔDEF, give an example of three sets of congruent parts that will not always guarentee that the two triangles are congruent. Explain your thinking.

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Instructional Resources	
Tasks	**Additional Resources**
Why Does SAS Work? (Illustrative Mathematics) Why Does ASA Work? (Illustrative Mathematics) Why Does SSS Work? (Illustrative Mathematics)	

Geometry – Congruence

NC.M2.G-CO.9

Prove geometric theorems.

Prove theorems about lines and angles and use them to prove relationships in geometric figures including:

- Vertical angles are congruent.
- When a transversal crosses parallel lines, alternate interior angles are congruent.
- When a transversal crosses parallel lines, corresponding angles are congruent.
- Points are on a perpendicular bisector of a line segment if and only if they are equidistant from the endpoints of the segment.
- Use congruent triangles to justify why the bisector of an angle is equidistant from the sides of the angle.

Concepts and Skills

Pre-requisite

- Use informal arguments to establish facts about angle sums and exterior angles in triangles and angles created by parallel lines cut by a transversal (8.G.5)
- Verify experimentally properties of rigid motions in terms of angles, circles, ⊥ and // lines and line segments (NC.M2.G-CO.4)
- Use and justify criteria to determine triangle congruence (NC.M2.G-CO.8)

Connections

- Use triangle congruence to prove theorems about triangles (NC.M2.G-CO.10)
- Apply properties, definitions, and theorems of 2-D figures to prove geometric theorems (NC.M3.G-CO.14)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

3 – Construct viable arguments and critique the reasoning of others
5 – Use appropriate tools strategically
6 – Attend to precision
7 – Look for and make use of structure

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication

Mastering the Standard

Comprehending the Standard

In 8th grade, students experimented with the properties of angles and lines. The focus in this standard is on *proving* the properties; not just knowing and applying them.

Students should use transformations and tactile experiences to gain an intuitive understanding of these theorems, before moving to a formal proof. *For example, vertical angles can be shown to be equal using a reflection across a line passing through the vertex or a 180° rotation around the vertex. Alternate interior angles can be matched up using a rotation around a point midway between the parallel lines on the transversal. Corresponding angles can be matched up using a translation.*

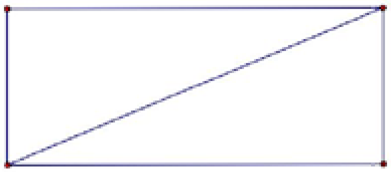
Assessing for Understanding

Students can prove theorems about intersecting lines and their angles.

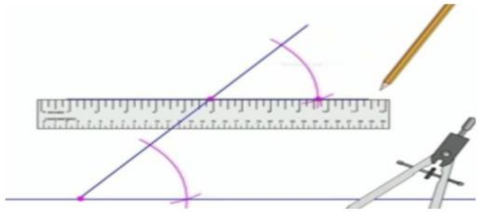
Example: Prove that any point equidistant from the endpoints of a line segment lies on the perpendicular bisector of the line. [Example YouTube Proof: Point equidistant from segment end points is on perpendicular bisector]

Students can prove theorems about parallel lines cut by a transversal and the angles formed by the lines.

Example: A carpenter is framing a wall and wants to make sure the edges of his wall are parallel. He is using a cross-brace as show in the diagram.



a. What are some different ways that he could verify that the edges are parallel?
b. Write a formal argument to show that the walls are parallel.
c. Pair up with another student who created a different argument than yours, and critique their reasoning. Did you modify your diagram as a result of the collaboration? How? Why?

Mastering the Standard	
Comprehending the Standard	**Assessing for Understanding**
Expose students to multiple formats for writing proofs, such as narrative paragraphs, bulleted lists of statements, flow diagrams, two-column format, and using diagrams without words. Students should be encouraged to focus on the validity of the underlying reasoning while exploring a variety of formats for expressing that reasoning. Students should not be required to master all formats, but to be able to read and analyze proofs in different formats, choosing a format (or formats) that best suit their learning style for writing proofs.	**Example:** The diagram below depicts the construction of a parallel line, above the ruler. The steps in the construction result in a line through the given point that is parallel to the given line. Which statement below justifies why the constructed line is parallel to the given line? a. When two lines are each perpendicular to a third line, the lines are parallel. b. When two lines are each parallel to a third line, the lines are parallel. c. When two lines are intersected by a transversal and alternate interior angles are congruent, the lines are parallel. d. When two lines are intersected by a transversal and corresponding angles are congruent, the lines are parallel. 

Instructional Resources	
Tasks	**Additional Resources**
Points equidistant from two points in the plane (Illustrative Mathematics) Congruent angles made by parallel lines and a transverse (Illustrative Mathematics) Proving the Alternate Interior Angles Theorem (CPalms)	**Videos of Angle and Line Proofs:** • Vertical angles are congruent. (Khan Academy) • Alternate interior angles congruent (YouTube) • Corresponding Angle Proof (YouTube) • Corresponding Angle Proofs – by contradiction (YouTube)

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NC.M2.G-CO.10

Prove geometric theorems.

Prove theorems about triangles and use them to prove relationships in geometric figures including:

- The sum of the measures of the interior angles of a triangle is 180º.
- An exterior angle of a triangle is equal to the sum of its remote interior angles.
- The base angles of an isosceles triangle are congruent.
- The segment joining the midpoints of two sides of a triangle is parallel to the third side and half the length.

Concepts and Skills

Pre-requisite

- Verify experimentally properties of rigid motions in terms of angles, circles, ⊥ and // lines and line segments (NC.M2.G-CO.4)
- Use and justify criteria to determine triangle congruence (NC.M2.G-CO.8)
- Use triangle congruence to prove theorems about lines, angles, and segments for relationships in geometric figures (NC.M2.G-CO.9)

Connections

- Verify experimentally, properties of the centers of triangles (NC.M3.G-CO.10)
- Prove theorems about parallelograms (NC.M3.G-CO.11)
- Apply properties, definitions, and theorems of 2-D figures to prove geometric theorems (NC.M3.G-CO.14)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

3 – Construct viable arguments and critique the reasoning of others
5 – Use appropriate tools strategically
6 – Attend to precision
7 – Look for and make use of structure

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication

Mastering the Standard

Comprehending the Standard

Encourage multiple ways of writing proofs, such as *narrative paragraphs* and *flow diagrams*. Students should be encouraged to focus on the validity of the underlying reasoning while exploring a variety of formats for expressing that reasoning.

Geometry is visual and should be taught in ways that leverage this aspect. Sketching, drawing and constructing figures and relationships between and within geometric objects should be central to any geometric study and certainly to proof. The use of transparencies and dynamic geometry software can be important tools for helping students conceptually understand important geometric concepts.

Example Proofs:

Triangle Angle Sum Theorem

Given ΔABC, prove that the $m\angle A + m\angle B + m\angle C = 180°$.

Assessing for Understanding

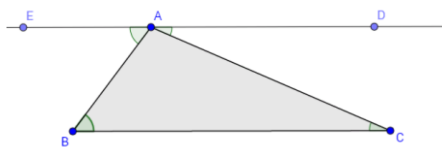
Students can prove theorems about triangles.

Example: Prove the Converse of the Isosceles Triangle Theorem: If two angles of a triangle are congruent, then the sides opposite them are congruent.

Example: Prove that an equilateral triangle is also equiangular.

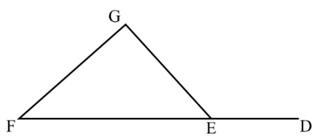
Mastering the Standard

Comprehending the Standard	Assessing for Understanding



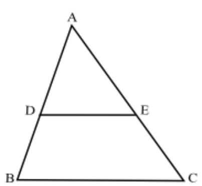
Draw $\overleftrightarrow{ED}$ through point A, parallel to $\overline{BC}$. Since $\overleftrightarrow{ED}$ and $\overleftrightarrow{BC}$ are parallel, alternate interior angles are congruent. Therefore, $\angle DAC \cong \angle ACB$ and $\angle EAB \cong \angle ABC$. By Angle Addition Postulate, $\angle EAB + \angle BAC + \angle DAC = \angle EAD$. Since $\angle EAD$ is a straight angle, its measure is 180°. Therefore $m\angle EAB + m\angle BAC + m\angle DAC = 180°$. Thus, the sum of the measures of the interior angles of a triangle is 180°.

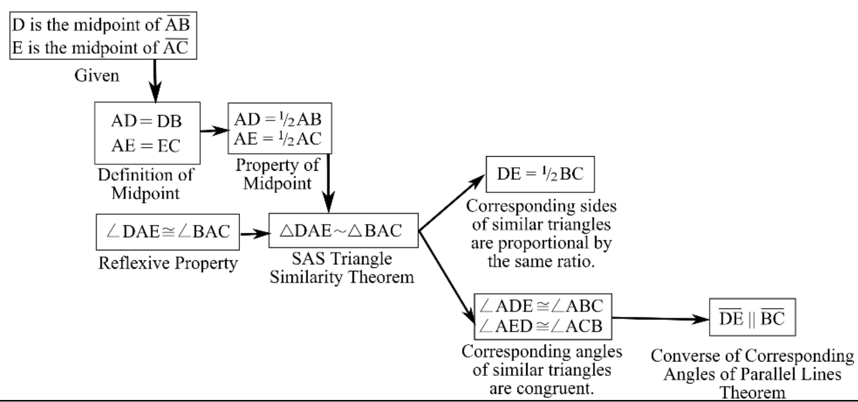
Exterior Angle Theorem
Given the figure on the right, prove m∠EFG + m∠FGE = m∠DEG



Statement	Reason
m∠DEG + m∠GEF = 180º	Two angles that form a straight line are supplementary.
m∠EFG+m∠FGE+m∠GEF=180º	Sum of angles in a triangle is 180º
m∠EFG+m∠FGE+m∠GEF= m∠DEG+m∠GEF	Substitution as both sums equal 180º.
m∠EFG+m∠FGE=m∠DEG	Subtract m∠GEF from both sides of equation

Triangle Midsegment Theorem
Given that D is the midpoint of $\overline{AB}$, and E is the midpoint of $\overline{AC}$, prove $\overline{DE} || \overline{BC}$ and $\overline{DE} = \frac{1}{2}\overline{BC}$.





Instructional Resources

Tasks	Additional Resources
Seven Circles (Illustrative Mathematics)	Exterior Angle Theorem (YouTube video) Base Angles Congruent (Khan Academy Video) Triangle Midsegment Theorem (Proof using dilations)

Geometry – Similarity, Right Triangles, and Trigonometry

NC.M2.G-SRT.1

Understand similarity in terms of similarity transformations.

Verify experimentally the properties of dilations with given center and scale factor:

a. When a line segment passes through the center of dilation, the line segment and its image lie on the same line. When a line segment does not pass through the center of dilation, the line segment and its image are parallel.
b. Verify experimentally the properties of dilations with given center and scale factor: The length of the image of a line segment is equal to the length of the line segment multiplied by the scale factor.
c. The distance between the center of a dilation and any point on the image is equal to the scale factor multiplied by the distance between the dilation center and the corresponding point on the pre-image.
d. Dilations preserve angle measure.

Concepts and Skills

Pre-requisite

- Use coordinates to describe the effects of transformations on 2-D figures (8.G.3)
- Understand similarity through transformations (8.G.4)
- Finding the distance between points in the coordinate plane (8.G.8)
- Using slope to determine parallelism and perpendicularity (NC.M1.G-GPE.5)
- Understand that dilations produce similar figures (NC.M2.G-CO.2)

Connections

- Using coordinates to solve geometric problems algebraically (NC.M1.G-GPE.4)
- Determining similarity by a sequence of transformations; use the properties of dilations to show that two triangles are similar if their corresponding sides proportional and corresponding angles are congruent (NC.M2.G-SRT.2)
- Verify experimentally properties of rigid motions in terms of angles, circles, ⊥ and // lines and line segments (NC.M2.G-CO.4)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

1 – Make sense of problems and persevere in solving them

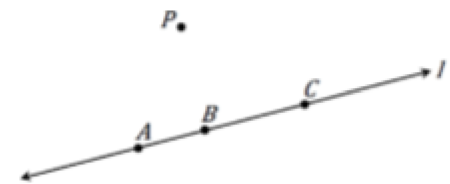
6 – Attend to precision

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication

Mastering the Standard

Comprehending the Standard	Assessing for Understanding
Students use hands-on techniques (graph paper) and/or technology (geometry software) to experiment with dilations. This standard extends to the observation of the basic properties of dilations as they build a deeper understanding of similarity.	Students verify that a side length of the image is equal to the scale factor multiplied by the corresponding side length of the pre-image. **Example:** Given ΔABC with $A\ (-2,-4)$, $B\ (1,2)$ and $C\ (4,-3)$. a. Perform a dilation from the origin using the following function rule $f(x,y) \to (3x,3y)$. What is the scale factor of the dilation?

Mastering the Standard	
Comprehending the Standard	**Assessing for Understanding**
Students should understand that a dilation is a transformation that moves each point along the ray through the point emanating from a fixed center, and multiplies distances from the center by a common scale factor.	b. Using ΔABC and its image $\Delta A'B'C'$, connect the corresponding pre-image and image points. Describe how the corresponding sides are related. c. Determine the length of each side of the triangle. How do the side lengths compare? How is this comparison related to the scale factor? d. Determine the distance between the origin and point A and the distance between the origin and point A'. Do the same for the other two vertices. What do you notice? e. Determine the angle measures for each angle of ΔABC and $\Delta A'B'C'$. What do you notice? Students perform a dilation with a given center and scale factor on a figure in the coordinate plane. **Example:** Suppose we apply a dilation by a factor of 2, centered at the point P to the figure below. a. In the picture, locate the images A', B', and C' of the points A, B, C under this dilation. b. What is the relationship between $\overleftrightarrow{AC}$ and $\overleftrightarrow{A'C'}$? c. What is the relationship between the length of A'B' and the length of AB? Justify your thinking.  Students verify that when a side passes through the center of dilation, the side and its image lie on the same line and the remaining corresponding sides of the pre-image and images are parallel.

Instructional Resources	
Tasks	**Additional Resources**

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Geometry – Similarity, Right Triangles, and Trigonometry

NC.M2.G-SRT.2

Understand similarity in terms of similarity transformations.

Understand similarity in terms of transformations.

a. Determine whether two figures are similar by specifying a sequence of transformations that will transform one figure into the other.

b. Use the properties of dilations to show that two triangles are similar when all corresponding pairs of sides are proportional and all corresponding pairs of angles are congruent

Concepts and Skills

Pre-requisite

- Given a geometric figure and a rigid motion, find the image of the figure/Given a figure and its image, describe a sequence of rigid motions between preimage and image (NC.M2.G-CO.5)
- Verify experimentally properties of dilations with given center and scale factor (NC.M2.G-SRT.1)

Connections

- Use the properties of dilations to show that two triangles are similar if their corresponding sides proportional and corresponding angles are congruent Determining similarity by a sequence of transformations (NC.M2.G-SRT.2b)
- Use transformations for the AA criterion for triangle similarity (NC.M2.G-SRT.3)
- Verify experimentally that side ratios in similar right triangles are properties of the angle measures and use to define trig ratios (NC.M2.G-SRT.6)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

3 – Construct viable arguments and critique the reasoning of others

4 – Model with Mathematics

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication

Mastering the Standard

Comprehending the Standard

Students use the idea of dilation transformations to develop the definition of similarity. They understand that a similarity transformation is a combination of a rigid motion and a dilation.

Students demonstrate that in a pair of similar triangles, corresponding angles are congruent (angle measure is preserved) and corresponding sides are proportional. They determine that two figures are similar by verifying that angle measure is preserved and corresponding sides are proportional.

Assessing for Understanding

Students use the idea of dilation transformations to develop the definition of similarity.

Example: In the picture to the right, line segments AD and BC intersect at X. Line segments AB and CD are drawn, forming two triangles ΔAXB and ΔCXD.

In each part a-d below, some additional *assumptions* about the picture are given. For each assumption:

I. Determine whether the given assumptions are enough to prove that the two triangles are similar. If so, what is the correct correspondence of vertices. If not, explain why not.

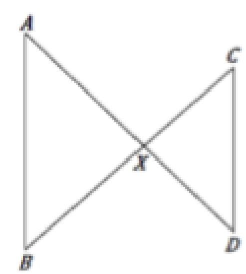
II. If the two triangles must be similar, prove this result by describing a sequence of similarity transformations that maps one variable to the other.

a. The lengths of AX and AD satisfy the equation $2AX = 3XD$.

b. The lengths AX, BX, CX, and DX satisfy the equation $\frac{AX}{BX} = \frac{DX}{CX}$

c. Lines AB and CD are parallel.

d. $\angle XAB$ is congruent to angle $\angle XCD$.



(From Illustrative Mathematics)

Instructional Resources	
Tasks	**Additional Resources**
Similar Triangles (Illustrative Mathematics)	

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NC.M2.G-SRT.3

Understand similarity in terms of similarity transformations.

Understand similarity in terms of transformations.

Use transformations (rigid motions and dilations) to justify the AA criterion for triangle similarity.

Concepts and Skills

Pre-requisite

- Verify experimentally properties of dilations with given center and scale factor (NC.M2.G-SRT.1)
- Determining similarity by a sequence of transformations; use the properties of dilations to show that two triangles are similar if their corresponding sides proportional and corresponding angles are congruent (NC.M2.G-SRT.2)

Connections

- Use similarity to prove The Triangle Proportionality Theorem and the Pythagorean Theorem (NC.M2.G-SRT.4)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

5 – Use appropriate tools strategically

6 – Attend to precision

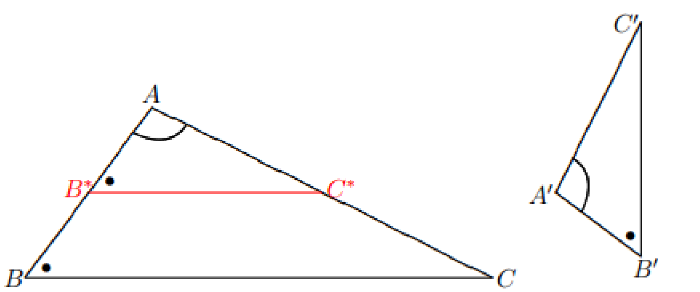
Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication

Mastering the Standard

Comprehending the Standard

Given two triangles for which AA holds, students use rigid motions to map a vertex of one triangle onto the corresponding vertex of the other in such a way that their corresponding sides are in line. Then show that the dilation will complete the mapping of one triangle onto the other. See p. 98 of Dr. Wu, *Teaching Geometry According to the Common Core Standards.*

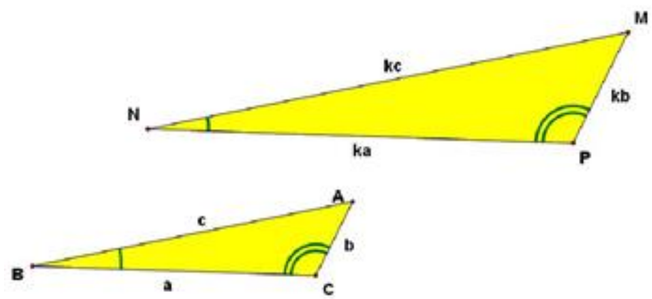


Dynamic geometry software visual of this process. (Geogebra.org)

Assessing for Understanding

Students can use the properties of dialations to show that two triangles are similar based on the AA criterion.

Example: Given that ΔMNP is a dilation of ΔABC with scale factor k, use properties of dilations to show that the AA criterion is sufficient to prove similarity.



Instructional Resources	
Tasks	**Additional Resources**
	Informal Proof of AA Criterion for Similarity (EngageNY) The AA Criterion for Two Triangles to Be Similar (EngageNY)

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NC.M2.G-SRT.4

Prove theorems involving similarity.

Use similarity to solve problems and to prove theorems about triangles. Use theorems about triangles to prove relationships in geometric figures.

- A line parallel to one side of a triangle divides the other two sides proportionally and its converse.
- The Pythagorean Theorem

Concepts and Skills

Pre-requisite

- Use transformations for the AA criterion for triangle similarity (NC.M2.G-SRT.3)

Connections

- Use trig ratios and the Pythagorean Theorem in right triangles (NC.M2.G-SRT.8)
- Derive the equation of a circle given center and radius using the Pythagorean Theorem (NC.M3.G-GPE.1)
- Prove theorems about parallelograms (NC.M3.G-CO.11)
- Apply properties, definitions, and theorems of 2-D figures to prove geometric theorems (NC.M3.G-CO.14)
- Understand apply theorems about circles (NC.M3.G-C.2)
- Use similarity to demonstrate that the length of the arc is proportional to the radius of the circle (NC.M3.G-C.5)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

1 – Make sense of problems and persevere in solving them
2 – Reason abstractly and quantitatively
3 – Construct viable arguments and critique the reasoning of others

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication

Mastering the Standard

Comprehending the Standard

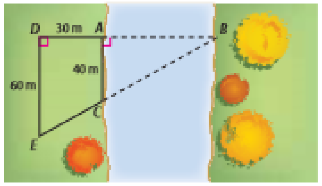
Students use the concept of similarity to solve problem situations (e.g., indirect measurement, missing side(s)/angle measure(s)). Students use the properties of dilations to prove that a line parallel to one side of a triangle divides the other two sides proportionally (often referred to as side-splitter theorem) and its converse.

The altitude from the right angle is drawn to the hypotenuse, which creates three similar triangles. The proportional relationships

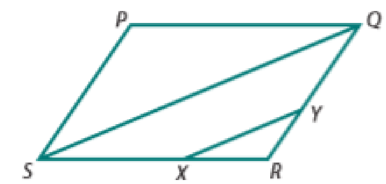
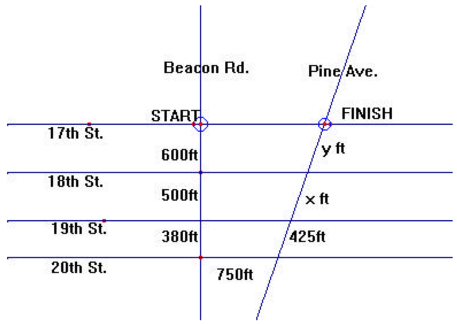
Assessing for Understanding

Students use similarity to prove the Pythagorean Theorem.

Example: Calculate the distance across the river, AB.



Mastering the Standard

Comprehending the Standard	Assessing for Understanding
among the sides of these three triangles can be used to derive the Pythagorean relationship.	Students can use triangle theorems to prove relationships in geometric figures. **Example:** In the diagram, quadrilateral PQRS is a parallelogram, SQ is a diagonal, and SQ ∥ XY. a. Prove that $\Delta XYR \sim \Delta SQR$. b. Prove that $\Delta XYR \sim \Delta QSP$.   **Example:** Parade Route Problem The parade committee has come up with the Beacon County Homecoming Parade route for next year. They want to start at the intersection of 17th Street and Beacon Road. The parade will proceed south on Beacon Road, turning left onto 20th Street. Then the parade will turn left onto Pine Avenue and finish back at 17th Street. For planning purposes, the committee needs to know approximately how long the parade will last. Can you help them? Justify your estimate. What assumptions did you make? (adapted from http://www.math.uakron.edu/amc/Geometry/HSGeometryLessons/SideSplitterTheorem.pdf) **Example:** Use similarity to prove the slope criteria for similar triangles. (https://www.illustrativemathematics.org/content-standards/HSG/SRT/B/5/tasks/1876)

Instructional Resources

Tasks	Additional Resources
[Bank Shot Task](#) (Illustrative Mathematics)	***Example proofs:*** [Proof of Pythagorean Theorem](#) using similar triangles (YouTube video) [Side-Splitter Theorem](#) (YouTube video)

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NC.M2.G-SRT.6

Define trigonometric ratios and solve problems involving right triangles.

Verify experimentally that the side ratios in similar right triangles are properties of the angle measures in the triangle, due to the preservation of angle measure in similarity. Use this discovery to develop definitions of the trigonometric ratios for acute angles.

Concepts and Skills

Pre-requisite

- Determining similarity by a sequence of transformations; use the properties of dilations to show that two triangles are similar if their corresponding sides are proportional and their corresponding angles are congruent (NC.M2.G-SRT.2)

Connections

- Develop properties of special right triangles (NC.M2.G-SRT.12)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

2 – Reason abstractly and quantitatively

6 – Attend to precision

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication

New Vocabulary: sine, cosine, tangent

Mastering the Standard

Comprehending the Standard

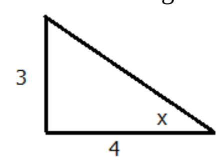
Students establish that the side ratios of a right triangle are equivalent to the corresponding side ratios of *similar* right triangles and are a function of the acute angle(s). Because all right triangles have a common angle, the right angle, if two right triangles have an acute angle in common (i.e. of the same measure), then they are similar by the AA criterion. Therefore, their sides are proportional.

We define the ratio of the length of the side opposite the acute angle to the length of the side adjacent to the acute angle as the tangent ratio. Note that the tangent ratio corresponds to the slope of a line passing through the origin at an angle to the x-axis that equals the measure of the acute angle. For example, in the diagram below, students can see that the tangent of 45° is 1, since the slope of a line passing through the origin at a 45° angle is 1. Using this visual, it is also easy to see that the slope of lines making an angle less than 45° will be less than 1; therefore the tangent ratio for angles between 0° and 45° is less than 1. Similarly, the slope of lines making an angle greater than 45° will be greater than 1; therefore, the tangent ratio for angles between 45° and 90° will be greater than 1.

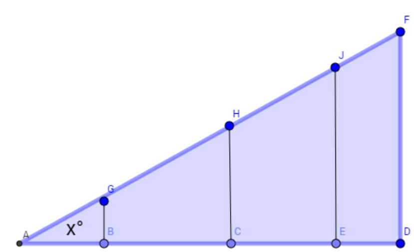
Assessing for Understanding

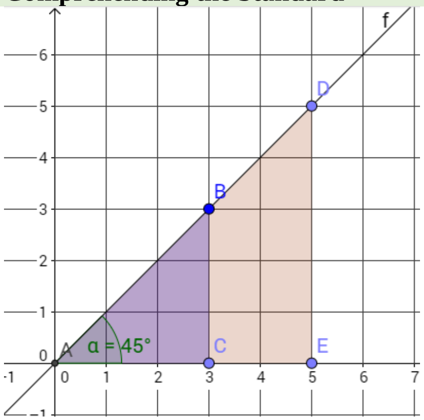
Students can use proportional reasoning to develop definitions of the trigonometric ratios of acute angles.

Example: Find the sine, cosine, and tangent of *x*.



Example: Explain why the sine of $x°$ is the same regardless of which triangle is used to find it in the figure below.



Mastering the Standard	
Comprehending the Standard	**Assessing for Understanding**
 Connect with 8.EE.6 “Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane.” We define the ratio of the length of the side opposite the acute angle to the length of the hypotenuse as the sine ratio. We define the ratio of the length of the side adjacent to the acute angle to the length of the hypotenuse as the cosine ratio.	

Instructional Resources	
Tasks	**Additional Resources**

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Geometry – Similarity, Right Triangles, and Trigonometry

NC.M2.G-SRT.8

Define trigonometric ratios and solve problems involving right triangles.

Use trigonometric ratios and the Pythagorean Theorem to solve problems involving right triangles in terms of a context.

Concepts and Skills

Pre-requisite

- Use similarity to prove The Triangle Proportionality Theorem and the Pythagorean Theorem (NC.M2.G-SRT.4)

Connections

- Develop properties of special right triangles (NC.M2.G-SRT.12)
- Understand apply theorems about circles (NC.M3.G-C.2)
- Build an understanding of trigonometric functions (NC.M3.F-TF.2)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

1 – Make sense of problems and persevere in solving them

4 - Model with mathematics (contextual situations are required)

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication

New Vocabulary: sine, cosine, tangent

Mastering the Standard

Comprehending the Standard	Assessing for Understanding
This standard is an application standard where students use the Pythagorean Theorem, learned in MS, and trigonometric ratios to solve application problems involving right triangles, including angle of elevation and depression, navigation, and surveying.	Students can use trig ratios and the Pythagorean theorem to find side lengths and angle measures in right triangles. **Example:** Find the height of a flagpole to the nearest tenth if the angle of elevation of the sun is 28° and the shadow of the flagpole is 50 feet. **Example:** A new house is 32 feet wide. The rafters will rise at a 36° angle and meet above the centerline of the house. Each rafter also needs to overhang the side of the house by 2 feet. How long should the carpenter make each rafter?

Instructional Resources

Tasks	Additional Resources

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Geometry – Similarity, Right Triangles, and Trigonometry

NC.M2.G-SRT.12

Define trigonometric ratios and solve problems involving right triangles.

Develop properties of special right triangles (45-45-90 and 30-60-90) and use them to solve problems.

Concepts and Skills

Pre-requisite

- Use similarity to prove The Triangle Proportionality Theorem and the Pythagorean Theorem (NC.M2.G-SRT.4)

Connections

- Verify experimentally that side ratios in similar right triangles are properties of the angle measures and use to define trig ratios (NC.M2.G-SRT.6)
- Use trig ratios and the Pythagorean Thm to solve problems (NC.M2.G-SRT.8)
- Understand apply theorems about circles (NC.M3.G-C.2)
- Build an understanding of trigonometric functions (NC.M3.F-TF.2)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

8 – Look for and express regularity in repeated reasoning

Disciplinary Literacy

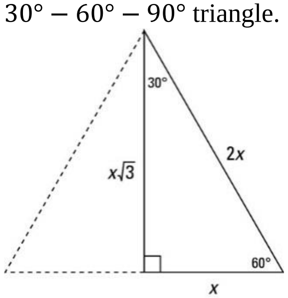
As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication

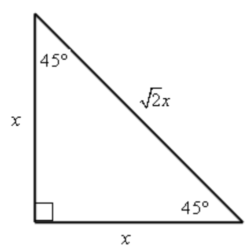
New Vocabulary: sine, cosine, tangent

Mastering the Standard

Comprehending the Standard

By drawing the altitude to one side of an equilateral triangle, students form two congruent $30° - 60° - 90°$ triangles. Starting with an initial side length of $2x$, students use the Pythagorean Theorem to develop relationships between the sides of a $30° - 60° - 90°$ triangle.





Students begin by drawing an isosceles right triangle with leg length of x. Using the Isosceles Triangle Theorem, the Triangle Angle Sum Theorem, and the Pythagorean Theorem students develop and justify relationships between the sides of a $45° - 45° - 90°$triangle.

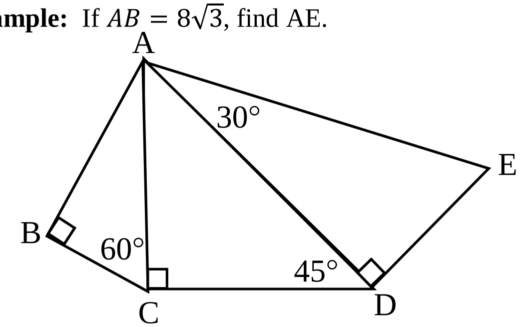
In Math 3, this relationship can be revisited with quadrilaterals by drawing the diagonal of a square to create two congruent $45° - 45° - 90°$ triangles. Using the properties of the diagonal and the Pythagorean Theorem, these relationships can be established in a different manner.

Assessing for Understanding

Students can solve problems involving special right triangles.

Example: The Garden Club at Heritage High wants to build a flower garden near the outdoor seating at the back of the school. The design is a square with diagonal walkways. The length of each side of the garden is 50 ft. How long is each walkway?

Example: If $AB = 8\sqrt{3}$, find AE.



Instructional Resources	
Tasks	**Additional Resources**

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Statistics & Probability

A statistical process is a problem-solving process consisting of four steps:

1. Formulating a statistical question that anticipates variability and can be answered by data.
2. Designing and implementing a plan that collects appropriate data.
3. Analyzing the data by graphical and/or numerical methods.
4. Interpreting the analysis in the context of the original question.

NC Math 1	NC Math 2	NC Math 3
Focus on analysis of univariate and bivariate data • Use of technology to represent, analyze and interpret data • Shape, center and spread of univariate numerical data • Scatter plots of bivariate data • Linear and exponential regression • Interpreting linear models in context.	**Focus on probability** • Categorical data and two-way tables • Understanding and application of the Addition and Multiplication Rules of Probability • Conditional Probabilities • Independent Events • Experimental vs. theoretical probability	**Focus on the use of sample data to represent a population** • Random sampling • Simulation as it relates to sampling and randomization • Sample statistics • Introduction to inference
A Progression of Learning		
• A continuation of the work from middle grades mathematics on summarizing and describing quantitative data distributions of univariate (6th grade) and bivariate (8th grade) data.	• A continuation of the work from 7th grade where students are introduced to the concept of probability models, chance processes and sample space; and 8th grade where students create and interpret relative frequency tables. • The work of MS probability is extended to develop understanding of conditional probability, independence and rules of probability to determine probabilities of compound events.	• Bringing it all back together • Sampling and variability • Collecting unbiased samples • Decision making based on analysis of data

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NC.M2.S-IC.2

Understand and evaluate random processes underlying statistical experiments

Use simulation to determine whether the experimental probability generated by sample data is consistent with the theoretical probability based on known information about the population.

Concepts and Skills

Pre-requisite

- Random sampling can be used to support valid inferences if the sample is representative of the population (7.SP.1)
- Approximate probabilities by collecting data and observing long-run frequencies (7.SP.6)

Connections

- Use simulation to understand how samples are used to estimate population means/proportions and how to determine margin of error (NC.M3.S-IC.4)
- Use simulation to determine whether observed differences between samples indicates actual differences in terms of the parameter of interest (NC.M3.S-IC.5)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

2 – Reason abstractly and quantitatively
4 – Model with Mathematics
5 – Use appropriate tools strategically

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

New vocabulary – simulation, experimental probability, theoretical probability

Mastering the Standard

Comprehending the Standard

This standard is an expansion of MS (7^{th} grade) where students approximate the probability of a chance event by collecting data and observing long-run relative frequencies of chance phenomenon. In the middle grades work, students understand that increasing the size of the trial yields results that are pretty consistent with the theoretical probability model. They also understand that randomization is an important element of sampling and that samples that reflect the population can be used to make inferences about the population.

This standard is extended to the idea of increasing the number of samples collected and examining the results of more samples opposed to larger sample sizes. This standard uses simulation to build an understanding of how taking more samples of the same size can be used to make predictions about the population of interest.

Simulation can be used to mock real-world experiments. It is time saving and provides a way for students to conceptually understand and explain random phenomenon.

It is suggested at this level for students to conduct simulation using tactile tools and methods. Cards, number cubes, spinners, colored tiles and other common items are excellent tools for performing simulation. Technology can be used to compile and analyze the results, but should not be used to perform simulations at this level.

Assessing for Understanding

Students explain how well and why a sample represents the variable of interest from a population.

> **Example:** Multiple groups flip coins. One group flips a coin 5 times, one group flips a coin 20 times, and one group flips a coin 100 times. Which group's results will most likely approach the theoretical probability?

Instructional Resources	
Tasks	**Additional Resources**

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Statistics and Probability – Conditional Probability and the Rules for Probability

NC.M2.S-CP.1

Understand independence and conditional probability and use them to interpret data.

Describe events as subsets of the outcomes in a sample space using characteristics of the outcomes or as unions, intersections and complements of other events.

Concepts and Skills

Pre-requisite

- Find probabilities of compound events using lists, tables, tree diagrams and simulations (7.SP.8)

Connections

- Develop and understand independence and conditional probability (NC.M2.S-CP.3a, NC.M2.S-CP.3b)
- Use the rules of probability to compute probabilities (NC.M2.S-CP.6, NC.M2.S-CP.7, NC.M2.S-CP.8)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

2 – Reason abstractly and quantitatively

6 – Attend to precision

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

New vocabulary – subset, union, intersections, complements

Mastering the Standard

Comprehending the Standard

In MS (7th grade) students collect data to approximate relative frequencies of probable events. They use the information to understand theoretical probability models based on long-run relative frequency. This allows students to assign probability to simple events, therefore students develop the understanding for sample space as the collection of all possible outcomes. Additionally, MS students develop probability models for compound events using lists tables, tree diagrams and simulations.

This standard builds on the MS work by formalizing probability terminology associated with simple and compound events and using characteristics of the outcomes:

Assessing for Understanding

Students define a sample space and events within the sample space.

Example: Describe the sample space for rolling two number cubes.
For the teacher: *This may be modeled well with a 6x6 table with the rows labeled for the first event and the columns labeled for the second event.*

Example: Describe the sample space for picking a colored marble from a bag with red and black marbles.
For the teacher*: This may be modeled with set notation.*

Example: Andrea is shopping for a new cellphone. She is either going to contract with Verizon (60% chance) or with Sprint (40% chance). She must choose between an Android phone (25% chance) or an IPhone (75% chance). Describe the sample space. ***For the teacher:*** *This may be modeled well with an area model.*

Example: The 4 aces are removed from a deck of cards. A coin is tossed and one of the aces is chosen. Describe the sample space. ***For the teacher:*** *This may be modeled well with a tree diagram.*

Students establish events as subsets of a sample space. An event is a subset of a sample space.

Example: Describe the event of rolling two number cubes and getting evens.

Example: Describe the event of pulling two marbles from a bag of red/black marbles.

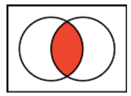
Example: Describe the event that the summing of two rolled number cubes is larger than 7 and even, and contrast it with the event that the sum is larger than 7 or even.

Mastering the Standard

Comprehending the Standard

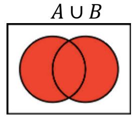
- The **intersection** of two sets A and B is the set of elements that *are common to both* set A and set B. It is denoted by $A \cap B$ and is read "A intersection B"

For sets A and B:
$A \cap B$



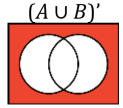
- The **union** of two sets A and B is the set of elements, which are *in A or in B, or in both*. It is denoted by $A \cup B$, and is read "A union B"

For sets A and B:
$A \cup B$



- The **complement** of the set $A \cup B$ is the set of elements that are members of the universal set U but *are not in* $A \cup B$. It is denoted by $(A \cup B)'$

For sets A and B:
$(A \cup B)'$



Assessing for Understanding

Example: If the subset of outcomes for choosing one card from a standard deck of cards is the intersection of two events: {queen of hearts, queen of diamonds}.

a. Describe the sample space for the experiment.
b. Describe the subset of outcomes for the union of two events.

Instructional Resources

Tasks	Additional Resources

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NC.M2.S-CP.3a

Understand independence and conditional probability and use them to interpret data.

Develop and understand independence and conditional probability.

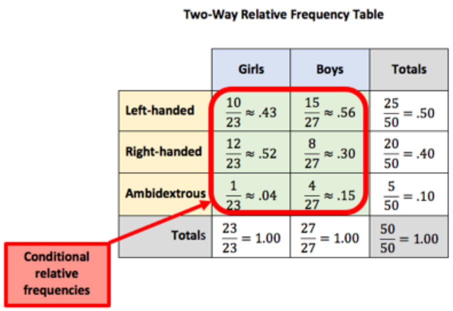
a. Use a 2-way table to develop understanding of the conditional probability of A given B (written P(A|B)) as the likelihood that A will occur given that B has occurred. That is, P(A|B) is the fraction of event B's outcomes that also belong to event A.

Concepts and Skills	The Standards for Mathematical Practices
Pre-requisite • Understand patterns of association from two-way tables in bivariate categorical data (8.SP.4)	**Connections** *Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.* 2 – Reason abstractly and quantitatively 6 – Attend to precision
Connections • Represent data on two categorical by constructing two-way frequency tables of data and use the table to determine independence (NC.M2.S-CP.4) • Recognize and explain the concepts of conditional probability and independence (NC.M2.S-CP.5) • Find conditional probabilities and interpret in context (NC.M2.S-CP.6)	**Disciplinary Literacy** *As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.* New vocabulary – independence, conditional probability

Mastering the Standard

Comprehending the Standard

Students created two-way tables of categorical data and used them to examine patterns of association in MS. They also displayed frequencies (counts) and relative frequencies (percentages) in two-way tables. This standard uses two-way tables to establish an understanding for conditional probability, that is given the occurrence of one event the probability of another event occurs.


Two-Way Relative Frequency Table

	Girls	Boys	Totals
Left-handed	$\frac{10}{23} \approx .43$	$\frac{15}{27} \approx .56$	$\frac{25}{50} = .50$
Right-handed	$\frac{12}{23} \approx .52$	$\frac{8}{27} \approx .30$	$\frac{20}{50} = .40$
Ambidextrous	$\frac{1}{23} \approx .04$	$\frac{4}{27} \approx .15$	$\frac{5}{50} = .10$
Totals	$\frac{23}{23} = 1.00$	$\frac{27}{27} = 1.00$	$\frac{50}{50} = 1.00$

The rows/columns determine the ***condition***. Using the example above, the probability that you select a left-handed person, given that it is a girl is the number of left-handed girls divided by the total number of girls → $P(Left - handed|Girl) = \frac{10}{23} \approx .43$. The ***condition*** in this problem is a ***girl*** therefore, the number of girls represents the total of the conditional probability.

Assessing for Understanding

Students can use two-way tables to find conditional probabilities.

Example: Each student in the Junior class was asked if they had to complete chores at home and if they had a curfew. The table represents the data.

a. What is the probability that a student who has chores also has a curfew?
b. What is the probability that a student who has a curfew also has chores?
c. Are the two events have chores and have a curfew independent? Explain.

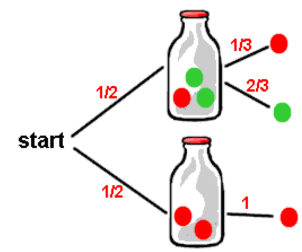
		Curfew		
		Yes	**No**	**Total**
Chores	**Yes**	51	24	75
	No	30	12	42
	Total	81	36	117

Students understand conditional probability as the probability of A occurring given B has occurred.

Example: What is the probability that the sum of two rolled number cubes is 6 given that you rolled doubles?

Example: There are two identical bottles. A bottle is selected at random and a single ball is drawn. Use the tree diagram at the right to determine each of the following:

a. $P\ (red|bottle\ 1)$
b. $P\ (red|bottle\ 2)$



Instructional Resources

Tasks	Additional Resources

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Statistics and Probability – Conditional Probability and the Rules for Probability

NC.M2.S-CP.3b

Understand independence and conditional probability and use them to interpret data.

Develop and understand independence and conditional probability.

b. Understand that event A is independent from event B if the probability of event A does not change in response to the occurrence of event B. That is P(A|B) = P(A).

Concepts and Skills

Pre-requisite

- Understand patterns of association from two-way tables in bivariate categorical data (8.SP.4)

Connections

- Represent data on two categorical by constructing two-way frequency tables of data and use the table to determine independence (NC.M2.S-CP.4)
- Recognize and explain the concepts of conditional probability and independence (NC.M2.S-CP.5)
- Apply the general Multiplication Rule, including when *A* and *B* are independent, and interpret in context (NC.M2.S-CP.8)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

2 – Reason abstractly and quantitatively

6 – Attend to precision

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

New vocabulary – independence, conditional probability

Mastering the Standard

Comprehending the Standard

Assessing for Understanding

Students can use two-way tables to find conditional probabilities.

Example: Each student in the Junior class was asked if they had to complete chores at home and if they had a curfew. The table represents the data. Are the two events have chores and have a curfew independent? Explain

		Curfew		
		Yes	**No**	**Total**
Chores	**Yes**	51	24	75
	No	30	12	42
	Total	81	36	117

Instructional Resources

Tasks

Conditional Probabilities 1 NEW

Conditional Probabilities 2 NEW

Additional Resources

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Statistics and Probability – Conditional Probability and the Rules for Probability

NC.M2.S-CP.4

Understand independence and conditional probability and use them to interpret data.

Represent data on two categorical variables by constructing a two-way frequency table of data. Interpret the two-way table as a sample space to calculate conditional, joint and marginal probabilities. Use the table to decide if events are independent.

Concepts and Skills

Pre-requisite

- Understand patterns of association from two-way tables in bivariate categorical data (8.SP.4)

Connections

- Develop and understand independence and conditional probability (NC.M2.S-CP.3a, NC.M2.S-CP.3b)
- Recognize and explain the concepts of conditional probability and independence (NC.M2.S-CP.5)
- Apply the general Multiplication Rule, including when *A* and *B* are independent, and interpret in context (NC.M2.S-CP.8)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

2 – Reason abstractly and quantitatively

6 – Attend to precision

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

New vocabulary – joint probabilities, marginal probabilities

Mastering the Standard

Comprehending the Standard

This standard builds upon the study of bivariate categorical data from MS. This standard supports data analysis from the statistical process.

The statistical process includes four essential steps:

1. Formulate a question that can be answered with data.
2. Design and use a plan to collect data.
3. **Analyze the data with appropriate methods.**
4. Interpret results and draw valid conclusions.

Students created two-way tables of categorical data and used them to examine patterns of association in 8th grade. They also displayed frequencies (counts) and relative frequencies (percentages) in two-way tables. Additionally, students have determined the sample space of simple and compound events in 7th grade. This standard expands on both of the 7th and 8th

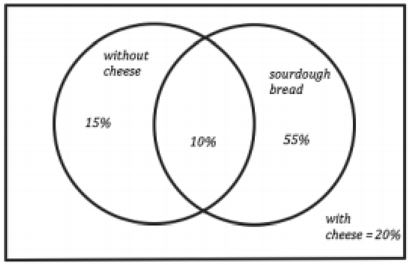
Assessing for Understanding

Students can create a two-way frequency table for data and calculate probabilities from the table.

Example: Collect data from a random sample of students in your school on their favorite subject among math, science, history, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.

Students can use a two-way table to evaluate independence of two variables.

Example: The Venn diagram to the right shows the data collected at a sandwich shop for the last six months with respect to the type of bread people ordered (sourdough or wheat) and whether or not they got cheese on their sandwich. Use the diagram to construct a two-way frequency table and then answer the following questions.



a. *P* (sourdough)

b. *P* (cheese | wheat)

c. *P* (without cheese or sourdough)

d. Are the events "sourdough" and "with cheese" independent events? Justify your reasoning.

Mastering the Standard	
Comprehending the Standard	**Assessing for Understanding**
grade concepts to using the table to determine independence of two events.	**Example:** Complete the two-way frequency table at the right and develop three conditional statements regarding the data. Determine if there are any set of events that independent. Justify your conclusion.

	Ice Cream	Cake	Total
Male		20	
Female	10		60
Total	85		

Instructional Resources	
Tasks	**Additional Resources**
Conditional Probabilities 1 NEW Conditional Probabilities 2 NEW	

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Statistics and Probability – Conditional Probability and the Rules for Probability

NC.M2.S-CP.5

Understand independence and conditional probability and use them to interpret data.

Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.

Concepts and Skills	The Standards for Mathematical Practices
Pre-requisite	**Connections**
•	*Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.* 3 – Construct viable arguments and critique the reasoning of others
Connections	**Disciplinary Literacy**
• Develop and understand independence and conditional probability (NC.M2.S-CP.3a, NC.M2.S-CP.3b) • Find conditional probabilities and interpret in context (NC.M2.S-CP.6) • Apply the general Multiplication Rule, including when *A* and *B* are independent, and interpret in context (NC.M2.S-CP.8)	*As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.*

Mastering the Standard	
Comprehending the Standard	**Assessing for Understanding**
This standard is about helping students make meaning of data and statistical questions. It is about communicating in their own language what the data/graphs/information is "saying." **The statistical process includes four essential steps:** 1. Formulate a question that can be answered with data. 2. Design and use a plan to collect data. 3. Analyze the data with appropriate methods. **4. Interpret results and draw valid conclusions.** This standard supports the idea of helping students to process the information around them presented in different formats or combination of formats (graphs, tables, narratives with percentages, etc.)	Students can use everyday language to determine if two events are dependent. **Example:** Felix is a good chess player and a good math student. Do you think that the events "being good at playing chess" and "being a good math student" are independent or dependent? Justify your answer. **Example:** Juanita flipped a coin 10 times and got the following results: T, H, T, T, H, H, H, H, H, H. Her math partner Harold thinks that the next flip is going to result in tails because there have been so many heads in a row. Do you agree? Explain why or why not. Students can explain conditional probability using everyday language. **Example:** A family that is known to have two children is selected at random from amongst all families with two children. Josh said that the probability of having two boys is $\frac{1}{3}$. Do you agree with Josh? Why or why not? Explain how you arrived at your answer?

Instructional Resources	
Tasks	**Additional Resources**
Conditional Probabilities 1 NEW Conditional Probabilities 2 NEW	

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Statistics and Probability – Conditional Probability and the Rules for Probability

NC.M2.S-CP.6

Use the rules of probability to compute probabilities of compound events in a uniform probability model.

Find the conditional probability of *A* given *B* as the fraction of *B*'s outcomes that also belong to *A*, and interpret the answer in context.

Concepts and Skills

Pre-requisite

- Develop and understand independence and conditional probability (NC.M2.S-CP.3a, NC.M2.S-CP.3b)

Connections

- Recognize and explain the concepts of conditional probability and independence (NC.M2.S-CP.5)
- Apply the general Multiplication Rule, including when *A* and *B* are independent, and interpret in context (NC.M2.S-CP.8)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

2 – Reason abstractly and quantitatively

6 – Attend to precision

Disciplinary Literacy

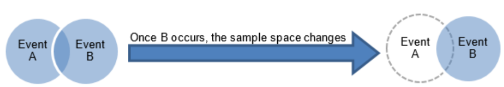
As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

Mastering the Standard

Comprehending the Standard

This standard should build on conditional probability and lead to the introduction of the addition and general multiplication rules of probability. Venn diagrams and/or tables of outcomes should serve as visual aids to build to the rules for computing probabilities of compound events.

The sample space of an experiment can be modeled with a Venn diagram such as:



So, the $P(A|B) = \frac{P(A \text{ and } B)}{P(B)}$

Assessing for Understanding

Students can find the conditional probability of compound events.

Example: If a balanced tetrahedron with faces 1, 2, 3, 4 is rolled twice.

(A): Sum is prime

(B): A 3 is rolled on at least one of the rolls.

a. Create a table showing all possible outcomes (sample space) for rolling the two tetrahedron.
b. What is the probability that the sum is prime (A) of those that show a 3 on at least one roll (B)?
c. Use the table to support the answer to part (b).

Example: Peter has a bag of marbles. In the bag are 4 white marbles, 2 blue marbles, and 6 green marbles. Peter randomly draws one marble, sets it aside, and then randomly draws another marble. What is the probability of Peter drawing out two green marbles? *Note: Students must recognize that this a conditional probability P(green | green).*

Example: A teacher gave her class two quizzes. 30% of the class passed both quizzes and 60% of the class passed the first quiz. What percent of those who passed the first quiz also passed the second quiz?

Instructional Resources	
Tasks	**Additional Resources**
Conditional Probabilities 1 NEW Conditional Probabilities 2 NEW	

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NC.M2.S-CP.7

Use the rules of probability to compute probabilities of compound events in a uniform probability model.

Apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B), and interpret the answer in context.

Concepts and Skills

Pre-requisite

- Describe events as subsets of the outcomes in a sample space based on characteristics of the outcomes or as unions, intersections or complements of other events (NC.M2.S-CP.1)

Connections

- Apply the general Multiplication Rule, including when *A* and *B* are independent, and interpret in context (NC.M2.S-CP.8)

The Standards for Mathematical Practices

Connections

Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.

2 – Reason abstractly and quantitatively

6 – Attend to precision

Disciplinary Literacy

As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.

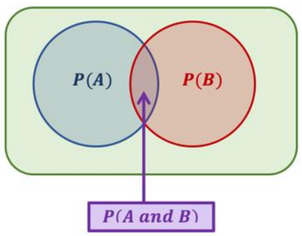
Mastering the Standard

Comprehending the Standard

Students should apply the addition rule for computing probabilities of compound events and interpret them in context. Students should understand $P\ (A\ or\ B)$ **OR** $P(A \cup B)$ to mean all elements of *A* and all elements of *B* excluding all elements shared by *A* and *B*.

The Venn diagram shows that when you include everything in both sets the middle region is included twice, therefore you must subtract the intersection region out once. The probability for calculating **joint** events is…

$$P\ (A\ or\ B) = P(A) + P(B) - P\ (A\ and\ B)$$



Students may recognize that if two events *A* and *B* are mutually exclusive, also called **disjoint,** the rule can be simplified to $P\ (A\ or\ B) = P\ (A)\ +\ P\ (B)$ since for mutually exclusive events $P\ (A\ and\ B) = 0$.

Assessing for Understanding

Students can apply the general addition rule for calculating conditional probabilities.

Example: Given the situation of drawing a card from a standard deck of cards, calculate the probability of the following:

a. Drawing a red card or a king
b. Drawing a ten or a spade
c. Drawing a four or a queen

Example: In a math class of 32 students, 18 boys and 14 are girls. On a unit test, 5 boys and 7 girls made an A grade. If a student is chosen at random from the class, what is the probability of choosing a girl or an A student?

Instructional Resources	
Tasks	**Additional Resources**

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NC.M2.S-CP.8

Use the rules of probability to compute probabilities of compound events in a uniform probability model.

Apply the general Multiplication Rule P (A and B) = P(A)P(B|A) = P(B)P(A|B), and interpret the answer in context. Include the case where A and B are independent: P (A and B) = P(A) P(B).

Concepts and Skills	The Standards for Mathematical Practices
Pre-requisite • Describe events as subsets of the outcomes in a sample space based on characteristics of the outcomes or as unions, intersections or complements of other events (NC.M2.S-CP.1) **Connections** • Apply the Addition Rule and interpret in context (NC.M2.S-CP.7)	**Connections** *Generally, all SMPs can be applied in every standard. The following SMPs can be highlighted for this standard.* 2 – Reason abstractly and quantitatively 6 – Attend to precision **Disciplinary Literacy** *As stated in SMP 6, the precise use of mathematical vocabulary is the expectation in all oral and written communication.*

Mastering the Standard

Comprehending the Standard	Assessing for Understanding
Students should understand $P\ (A\ and\ B)$ **OR** $P(A \cap B)$ to mean all elements of A that are also elements of B excluding all elements shared by A and B. Two events must be ***independent*** to apply the general multiplication rule $P(A\ and\ B) = P(A)P(B\|A) = P(B)P(A\|B)$ The general rule can be explained based on the definitions of independence and dependence. Events are either independent or dependent. • Two events are said to be **independent** if the occurrence of one event does not affect the probability of the occurrence of the other event. • Two events are **dependent** if the occurrence of one event does, in fact, affect the probability of the occurrence of the other event. Sampling with and without replacement are opportunities to model independent and dependent events.	Students can apply the general multiplication rule for computing conditional probabilities. **Example:** You have a box with 3 blue marbles, 2 red marbles, and 4 yellow marbles. You are going to pull out one marble, record its color, put it back in the box and draw another marble. What is the probability of pulling out a red marble followed by a blue marble? **Example:** Consider the same box of marbles as in the previous example. However, in this case, we are going to pull out the first marble, leave it out, and then pull out another marble. What is the probability of pulling out a red marble followed by a blue marble? **Example:** Suppose you are going to draw two cards from a standard deck. What is the probability that the first card is an ace and the second card is a jack (just one of several ways to get "blackjack" or 21)? Students can use the general multiplication rule to determine whether two events are independent.

Instructional Resources	
Tasks	**Additional Resources**

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