THE MANGA GUIDE™ TO

RELATIVITY

COMICS INSIDE!

HIDEO NITTA
MASAFUMI YAMAMOTO
KEITA TAKATSU
TREND-PRO CO., LTD.

Ohmsha

no starch press

Praise for the *Manga Guide* Series

"Highly recommended."
—CHOICE MAGAZINE ON *THE MANGA GUIDE TO DATABASES*

"Stimulus for the next generation of scientists."
—SCIENTIFIC COMPUTING ON *THE MANGA GUIDE TO MOLECULAR BIOLOGY*

"A great fit of form and subject. Recommended."
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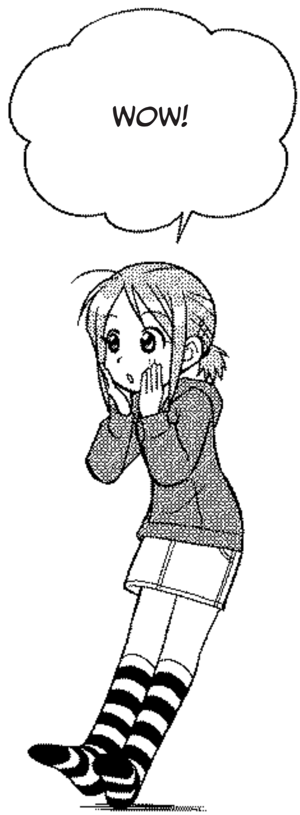

"I found the cartoon approach of this book so compelling and its story so endearing that I recommend that every teacher of introductory physics, in both high school and college, consider using it."
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—SACRAMENTO BOOK REVIEW

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—GEEKDAD BLOG, WIRED.COM

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—MAKE ON *THE MANGA GUIDE TO STATISTICS*

"This book does exactly what it is supposed to: offer a fun, interesting way to learn calculus concepts that would otherwise be extremely bland to memorize."
—DAILY TECH ON *THE MANGA GUIDE TO CALCULUS*

"The art is fantastic, and the teaching method is both fun and educational."
—ACTIVE ANIME ON *THE MANGA GUIDE TO PHYSICS*

"An awfully fun, highly educational read."
—FRAZZLEDDAD ON *THE MANGA GUIDE TO PHYSICS*

"Makes it possible for a 10-year-old to develop a decent working knowledge of a subject that sends most college students running for the hills."
—SKEPTICBLOG ON *THE MANGA GUIDE TO MOLECULAR BIOLOGY*

"This book is by far the best book I have read on the subject. I think this book absolutely rocks and recommend it to anyone working with or just interested in databases."
—GEEK AT LARGE ON *THE MANGA GUIDE TO DATABASES*

"The book purposefully departs from a traditional physics textbook and it does it very well."
—DR. MARINA MILNER-BOLOTIN, RYERSON UNIVERSITY ON *THE MANGA GUIDE TO PHYSICS*

"Kids would be, I think, much more likely to actually pick this up and find out if they are interested in statistics as opposed to a regular textbook."
—GEEK BOOK ON *THE MANGA GUIDE TO STATISTICS*

"*The Manga Guide to Statistics* offers a visualization of statistics that can't be found in any mere textbook."
—ANIME 3000

"A great introduction for readers of any age, and an exemplar of technical communication."
—LINUX USERS OF VICTORIA ON *THE MANGA GUIDE TO ELECTRICITY*

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PREFACE

Welcome to the world of relativity!

Everyone wonders what relativity is all about. Because the theory of relativity predicts phenomena that seem unbelievable in our everyday lives (such as the slowing of time and the contraction of the length of an object), it can seem like mysterious magic.

Despite its surprising, counterintuitive predictions, Einstein's theory of relativity has been confirmed many times over with countless experiments by modern physicists. Relativity and the equally unintuitive quantum mechanics are indispensable tools for understanding the physical world.

In Newton's time, when physicists considered velocities much smaller than the speed of light, it was not a problem to think that the measurement of motion, that is, space and time, were independent, permanent, and indestructible absolutes. However, by the end of the 19th century, precise measurements of the speed of light combined with developments in the study of electromagnetism had set the stage for the discovery of relativity. As a result, time and space, which had always been considered to be independent and absolute, had to be reconsidered.

That's when Einstein arrived on the scene. Einstein proposed that time and space were in fact relative. He discarded the idea that space and time were absolute and considered that they vary together, so that the speed of light is always constant.

This radical insight created a controversy just as Galileo's claim that Earth orbited the Sun (and not vice versa) shocked his peers. However, once we ventured into space, it was obvious that Earth was indeed moving.

In a similar way, relativity has given us a more accurate understanding of concepts regarding the space-time in which we are living. In other words, relativity is the result of asking what is *actually* happening in our world rather than saying our world *should be* a particular way.

Although this preface may seem a little difficult, I hope you will enjoy the mysteries of relativity in a manga world together with Minagi and his teacher, Miss Uraga. Finally, I'd like to express my deep gratitude to everyone in the development bureau at Ohmsha; re_akino, who toiled over the scenario; and Mr. Keita Takatsu, who converted it into such an interesting manga.

Well, then. Let's jump into the world of relativity.

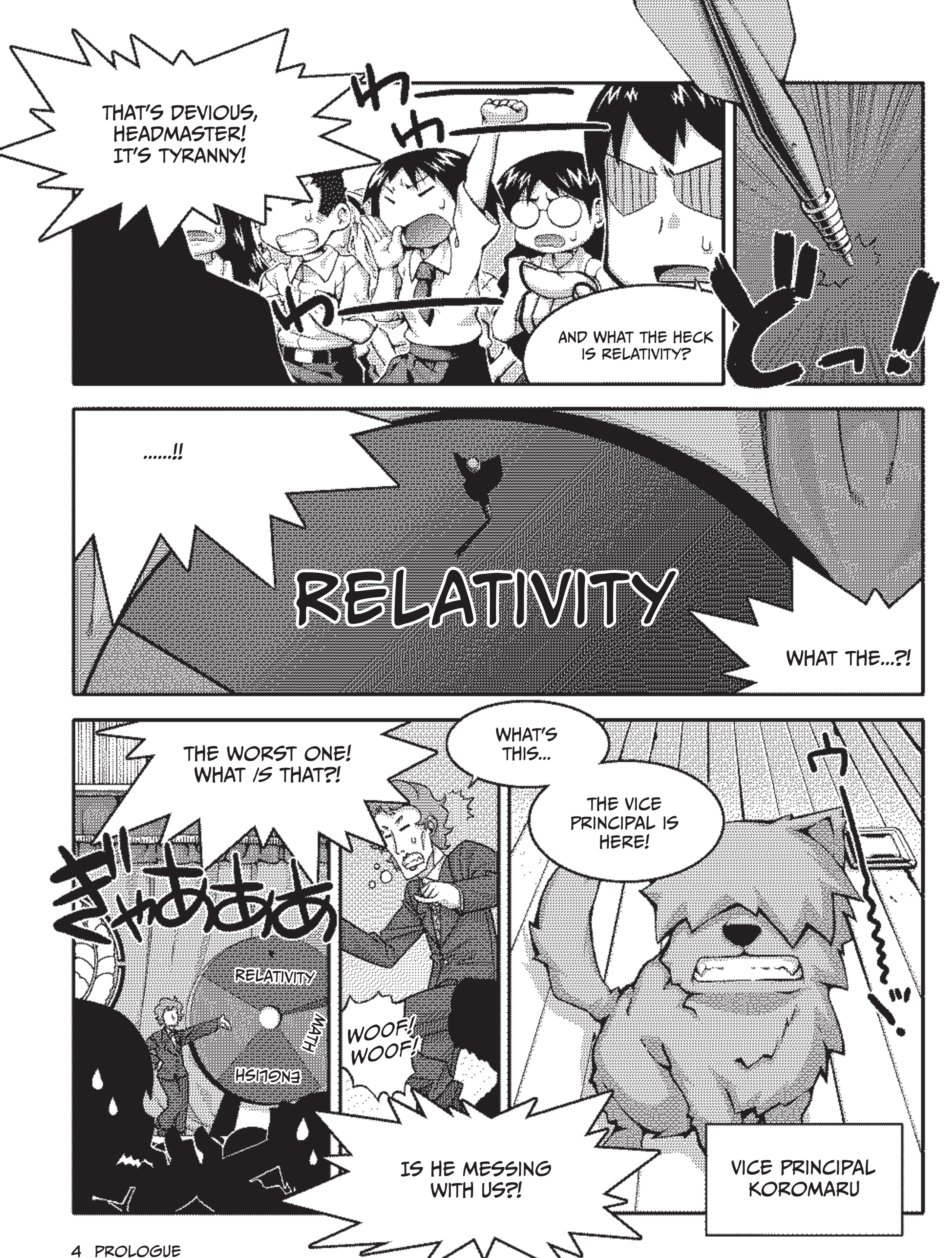
MASAFUMI YAMAMOTO
JUNE 2009

PROLOGUE

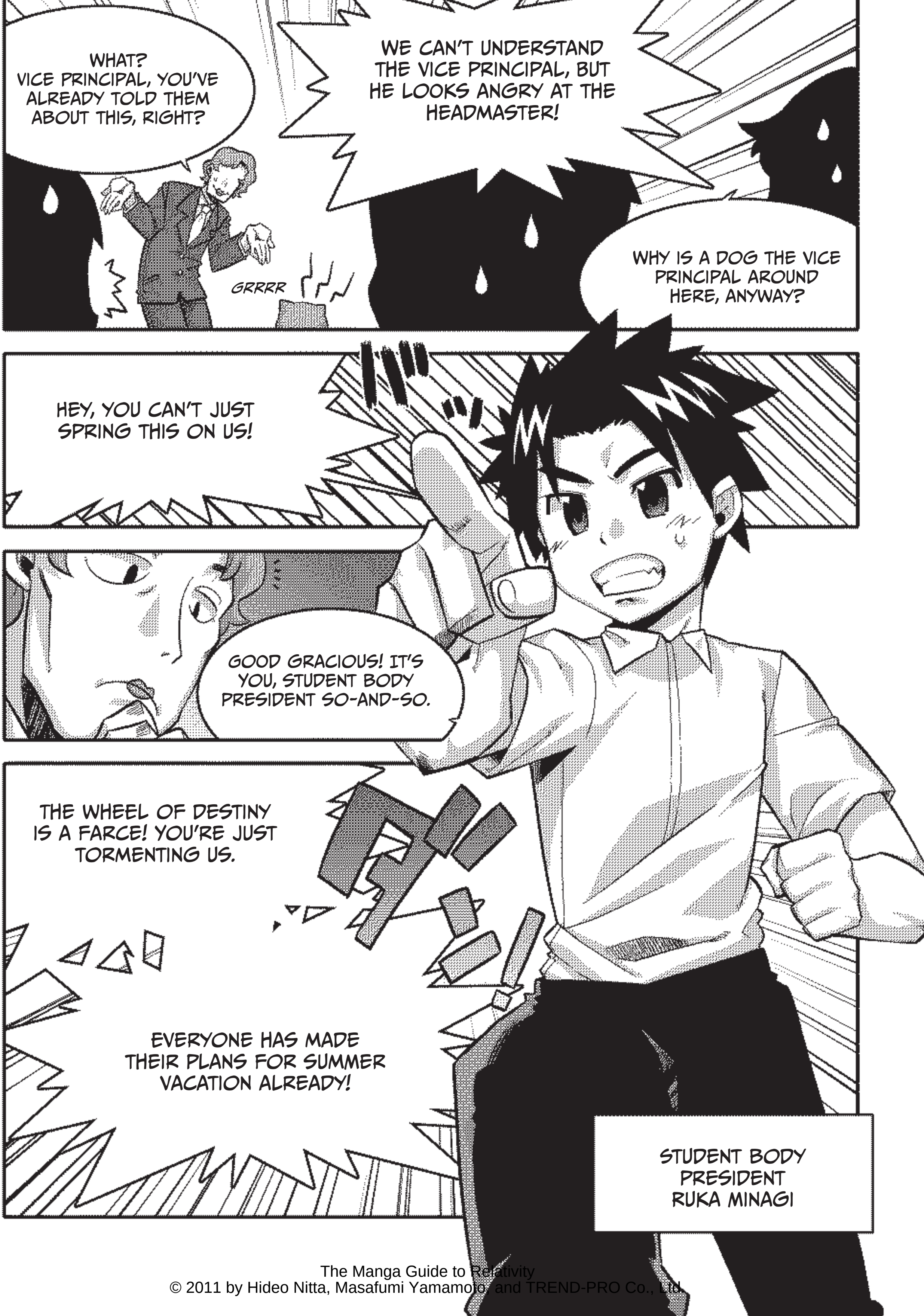
OUTRAGEOUS CLOSING CEREMONY

TAIGAI ACADEMY, THE LAST DAY OF SCHOOL BEFORE SUMMER
THE SCHOOL IS BRIMMING WITH ANTICIPATION OF SWIMMING AT THE BEACH...CAMPING... AND SUMMER LOVE.
I REALIZE THAT YOUR SUMMER VACATION IS ABOUT TO BEGIN...
THEREFORE!
I'M GIVING THIS YEAR'S JUNIOR CLASS A SPECIAL PRESENT SO THAT YOU CAN ENJOY YOUR SUMMER BREAK EVEN MORE.
HEADMASTER RASE IYAGA
WHAT'S HE TRYING TO PULL? THE HEADMASTER IS SUCH A CREEP!


THE WHEEL OF DESTINY!
AND HERE IT IS!
YOUR FATE IS NOW IN MY HANDS!
ENGLISH
FREE RESEARCH
MATH
NONE
RELATIVITY
WHAT?!
OKAY. HERE WE GO!
THIS SUMMER, YOU'LL HAVE THE *OPPORTUNITY* TO STUDY WHICHEVER SUBJECT THE DART HITS. DO YOU UNDERSTAND?
HE'S GOT TO BE KIDDING!
N-NO WAY! WAIT!
WHIRRR
HEH HEH
IF YASHIKI THROWS, HE'LL SURELY HIT "NONE" SINCE HE'S THE STAR PITCHER.
THAT'S RIGHT! SURELY, IF "WINDMILL YASHIKI" DOES IT...
OH NO! HEADMASTER IYAGA WILL BE DOING IT!



THAT'S DEVIOUS, HEADMASTER! IT'S TYRANNY!
AND WHAT THE HECK IS RELATIVITY?
......!!
RELATIVITY
WHAT THE...?!
THE WORST ONE! WHAT IS THAT?!
WHAT'S THIS...
THE VICE PRINCIPAL IS HERE!
RELATIVITY
MATH
ENGLISH
WOOF! WOOF!
IS HE MESSING WITH US?!
VICE PRINCIPAL KOROMARU


WHAT?
VICE PRINCIPAL, YOU'VE ALREADY TOLD THEM ABOUT THIS, RIGHT?
WE CAN'T UNDERSTAND THE VICE PRINCIPAL, BUT HE LOOKS ANGRY AT THE HEADMASTER!
GRRRR
WHY IS A DOG THE VICE PRINCIPAL AROUND HERE, ANYWAY?
HEY, YOU CAN'T JUST SPRING THIS ON US!
GOOD GRACIOUS! IT'S YOU, STUDENT BODY PRESIDENT SO-AND-SO.
THE WHEEL OF DESTINY IS A FARCE! YOU'RE JUST TORMENTING US.
EVERYONE HAS MADE THEIR PLANS FOR SUMMER VACATION ALREADY!
STUDENT BODY PRESIDENT RUKA MINAGI

OH REALLY? WELL, SHALL I HAVE YOU STUDY RELATIVITY AS A REPRESENTATIVE OF ALL STUDENTS?
WHA-WHAT?!
IF YOU DO THAT, THEN THE OTHER STUDENTS WILL BE EXEMPT.
BU-BUT WHAT IS RELATIVITY?!
HEY, IF WE LEAVE IT UP TO MINAGI, WE'LL BE ABLE TO ENJOY SUMMER VACATION, WON'T WE?
CAN WE REALLY DO SUCH A TERRIBLE THING TO HIM?
IT SEEMS LIKE YOU'VE GOT A HEAD START!
ACK!
ACK!
ER...UM...
IF IT'S OKAY WITH YOU, I COULD TEACH YOU ABOUT RELATIVITY.


YOU PROBABLY WOULDN'T WANT ME TO TEACH HIM, WOULD YOU...?
PHYSICS TEACHER ALISA URAGA
WELL, WHAT WILL IT BE, MINAGI?
WE CAN DISCUSS THAT LATER!
COME ON MAN! SAVE US!
WE BEG YOU, MAN.
DO IT!
BLEH.
OKAY, I ACCEPT YOUR CHALLENGE!!!
おお〜〜!!


AND IF YOU CAN'T DO IT...
IF I CAN'T DO IT...?
I DON'T CARE IF MISS URAGA TEACHES YOU, BUT YOU HAVE TO WRITE THE REPORT YOURSELF!
ALL RIGHT THEN. WHEN SUMMER VACATION IS OVER, SUBMIT A REPORT ON RELATIVITY.
OKAY!
YOU'LL SPEND YOUR SENIOR YEAR AS MY...
PERSONAL SECRETARY!
NO FREAKING WAY!
THIS IS CRAZY! I'LL DO IT!
WOOF!
TEE HEE

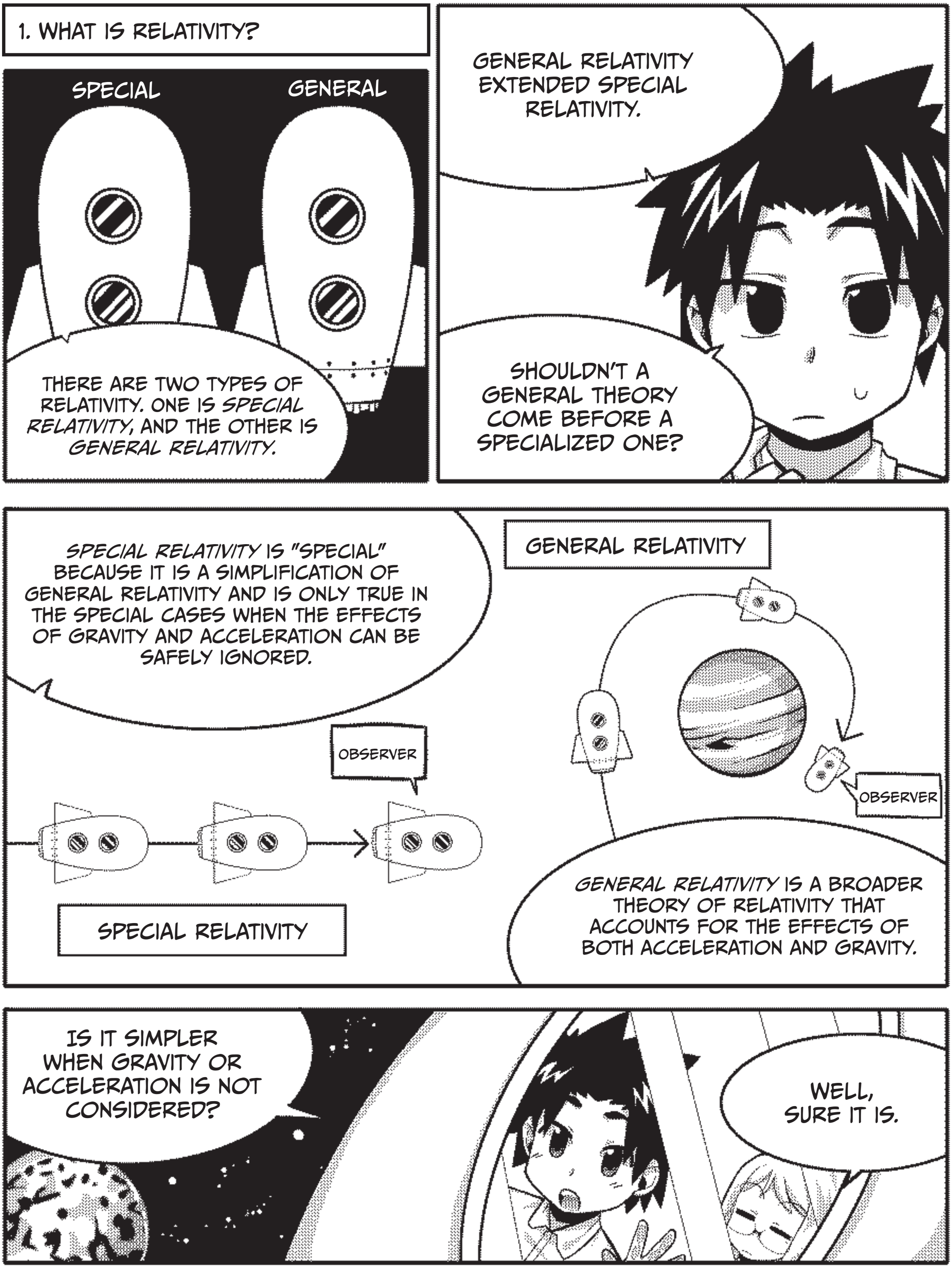
WHAT IS RELATIVITY?


AH, I WAS A BIG BRAGGART, BUT NOW I'VE GOT TO DO IT...
CHIRP
CHIRP
AT LEAST I'LL BE GETTING PRIVATE LESSONS FROM THE INTRIGUING MISS URAGA!
PHYSICS ROOM
HMMM... OH, HERE IT IS.
YOU'RE LATE!!!
WHAM!

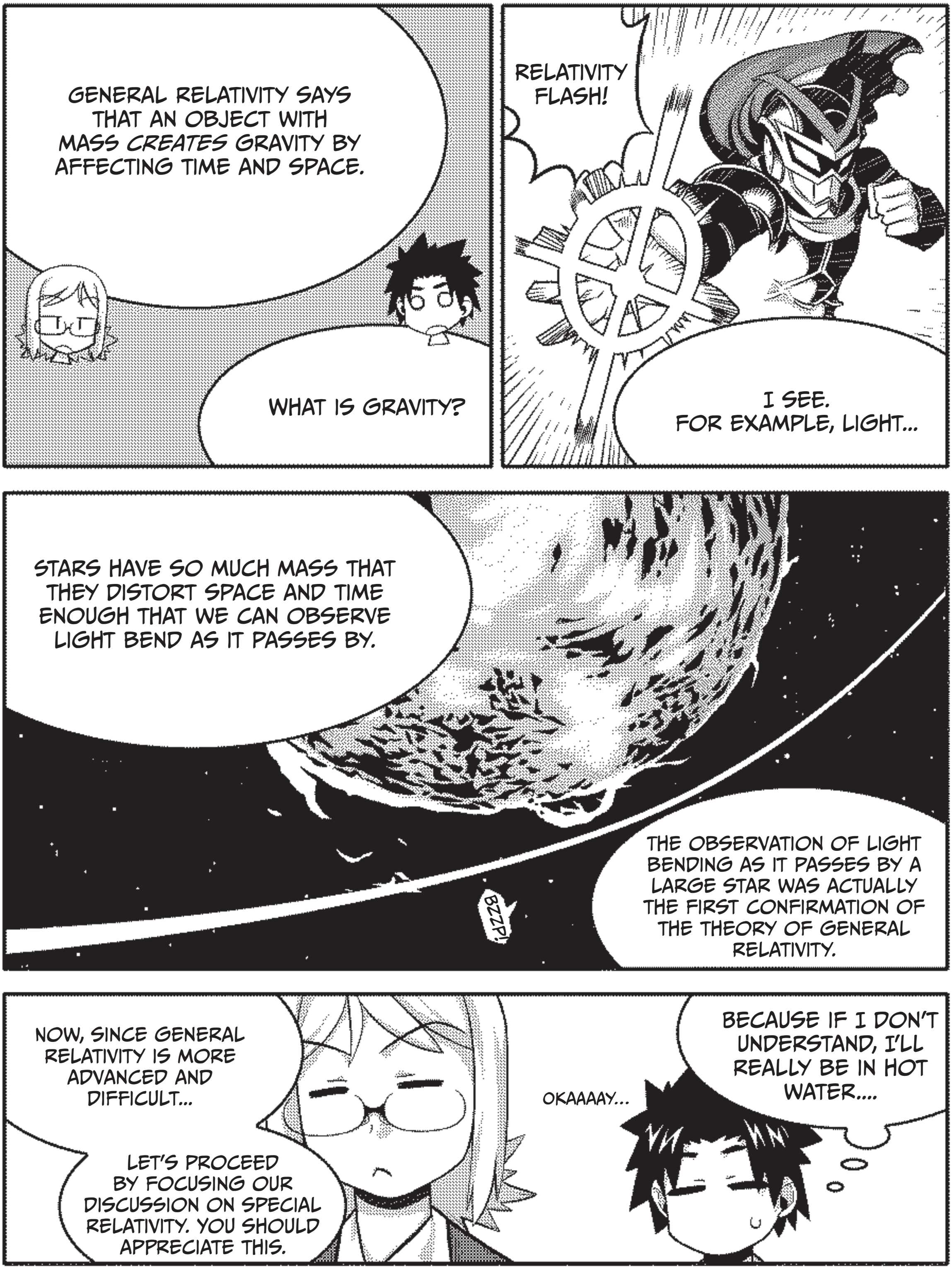

!?
MI. NA. GI.
IT'S OUTRAGEOUS THAT YOU ARRIVED LATER THAN I DID. DO YOU HAVE THE FLEXIBLE HOURS OF AN EXECUTIVE?
SLAP
SLAP
MISS URAGA...?!
IT'S PROBABLY NORMAL, BUT YOUR EYES...ARE THEY BLACK HOLES?
WH-WH-*WHAT?*
PSH, I'M SORRY, I DIDN'T MEAN TO BE SO SEVERE.
SHE'S KIDDING?!
SERIOUSLY, THOUGH, PLEASE DON'T COME LATE AGAIN.


THAT WAS MY DISCIPLINARIAN FACE! ADULTS MUST USE IT EVERY SO OFTEN.
OUR IMAGE OF YOU...
THAT FREAK OF A HEADMASTER GAVE ME AN IMPOSSIBLE TASK.
THIS IS AWFUL. IT'S SUMMER VACATION...
NOW, DON'T BE SO PESSIMISTIC.
HONESTLY, WHEN HE STARTED TALKING ABOUT RELATIVITY, I THOUGHT THAT THIS WAS MY BIG OPPORTUNITY!
オホホの
ホッ!
WHAT DO YOU MEAN BY OPPORTUNITY?

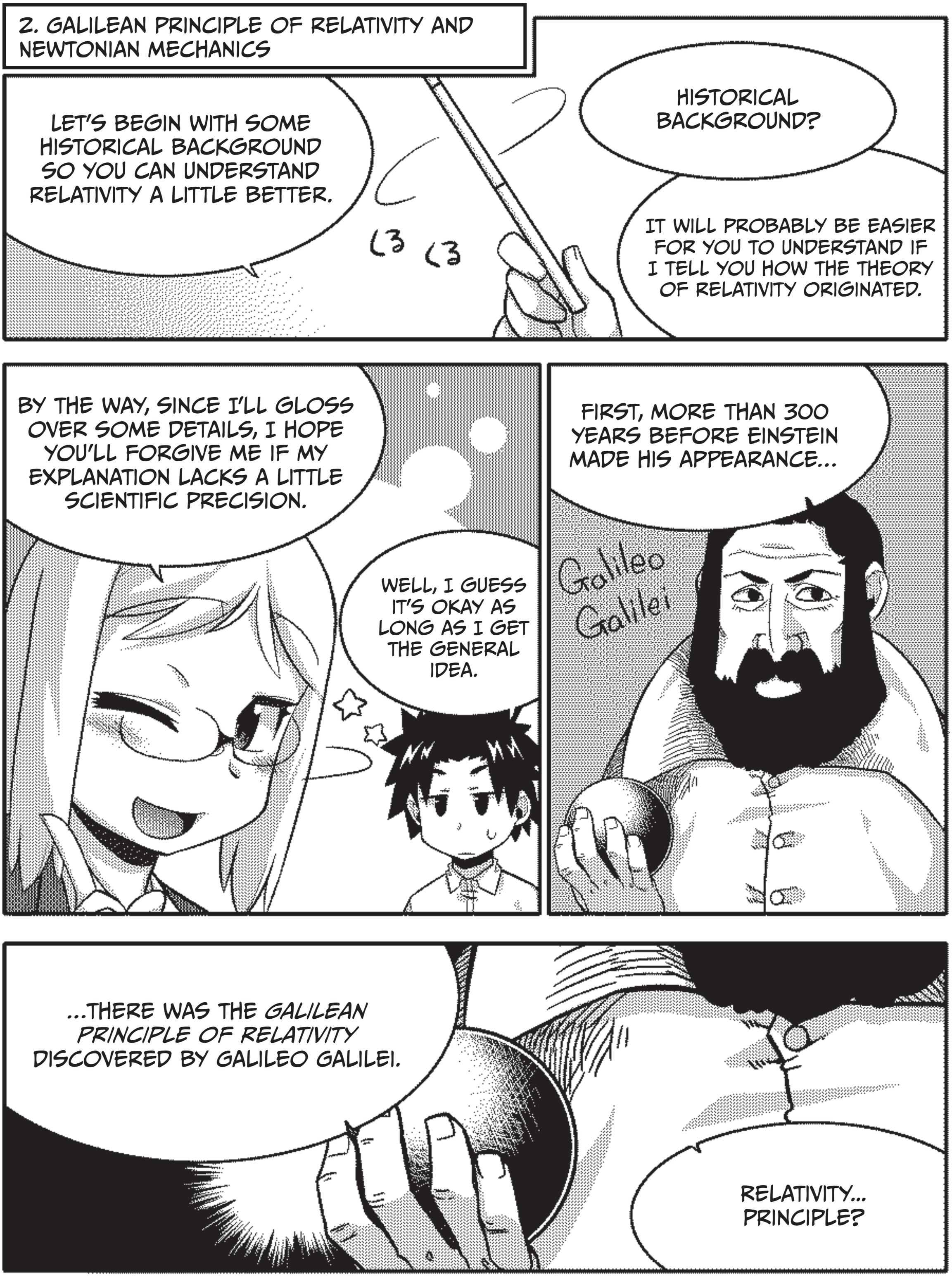

IT'S MY FIELD OF SPECIALTY. I THOUGHT I COULD USE THIS SITUATION TO RACK UP POINTS WITH THE HEADMASTER.
I'M JUST YOUR STEPPING STONE, HUH?
AND IF I GOT A HIGH EVALUATION FOR BEING "MISS URAGA WHO ENTHUSIASTICALLY PROVIDES GUIDANCE TO STUDENTS," I'D EVENTUALLY BECOME THE NEXT HEADMISTRESS.
THAT'S A BIT FAR-FETCHED, ISN'T IT?
WELL, LET'S GET STARTED! FIRST, SINCE YOU'VE ASKED WHAT RELATIVITY IS...
OKAY.
ぱああ

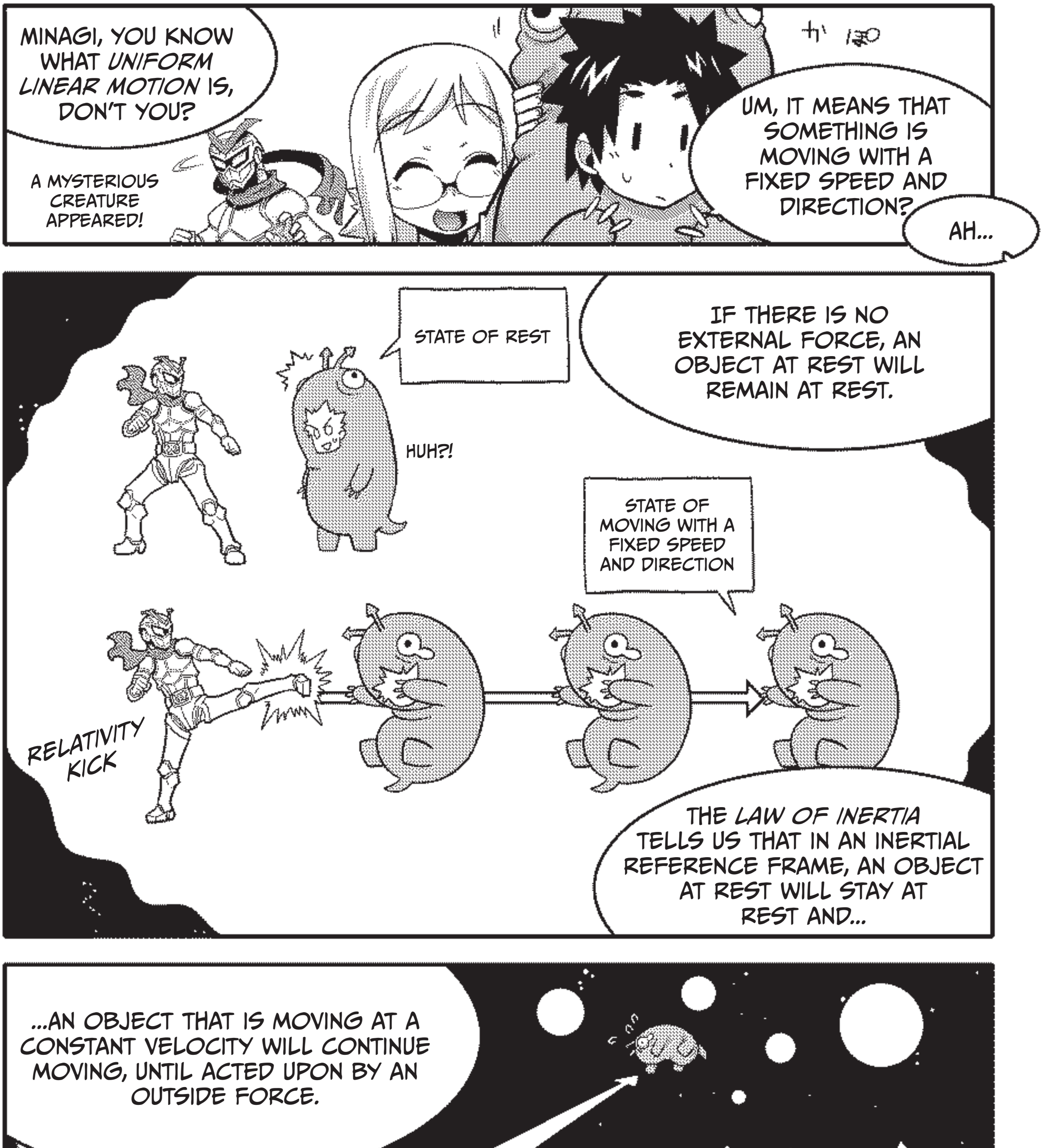

1. WHAT IS RELATIVITY?
SPECIAL
GENERAL
THERE ARE TWO TYPES OF RELATIVITY. ONE IS *SPECIAL RELATIVITY*, AND THE OTHER IS *GENERAL RELATIVITY*.
GENERAL RELATIVITY EXTENDED SPECIAL RELATIVITY.
SHOULDN'T A GENERAL THEORY COME BEFORE A SPECIALIZED ONE?
SPECIAL RELATIVITY IS "SPECIAL" BECAUSE IT IS A SIMPLIFICATION OF GENERAL RELATIVITY AND IS ONLY TRUE IN THE SPECIAL CASES WHEN THE EFFECTS OF GRAVITY AND ACCELERATION CAN BE SAFELY IGNORED.
GENERAL RELATIVITY
OBSERVER
OBSERVER
SPECIAL RELATIVITY
GENERAL RELATIVITY IS A BROADER THEORY OF RELATIVITY THAT ACCOUNTS FOR THE EFFECTS OF BOTH ACCELERATION AND GRAVITY.
IS IT SIMPLER WHEN GRAVITY OR ACCELERATION IS NOT CONSIDERED?
WELL, SURE IT IS.

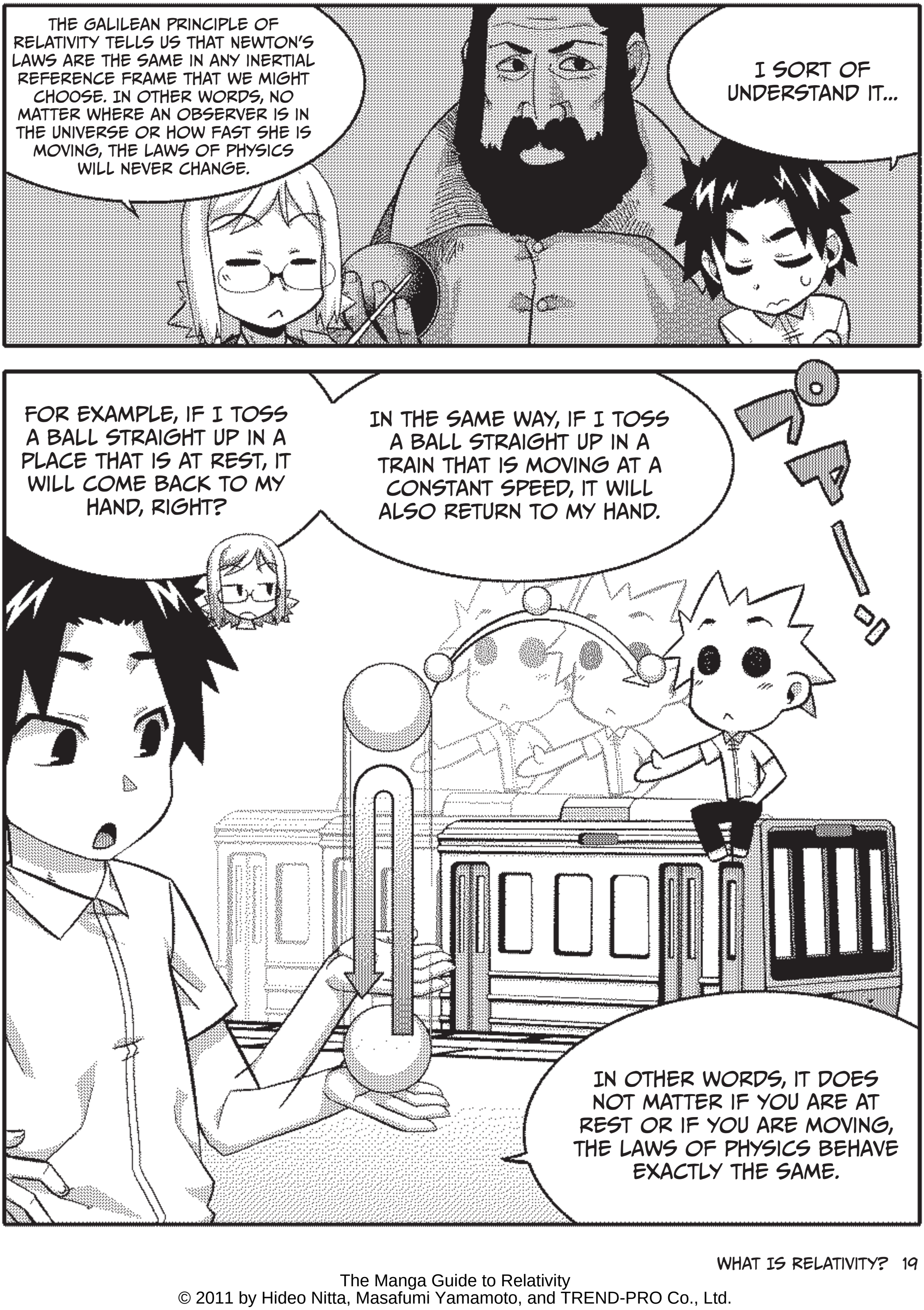

THE THEORY OF RELATIVITY SAYS THAT THE PASSAGE OF TIME, DISTANCE, AND MASS DEPEND ON THE MOTION OF WHOEVER MAKES THE OBSERVATION. IN SPECIAL RELATIVITY, WE ONLY CONSIDER OBSERVERS AT REST OR WHO MOVE AT A CONSTANT VELOCITY. WE CALL THIS VANTAGE POINT AN *INERTIAL REFERENCE FRAME.*
WHEN OBSERVATIONS ARE MADE BY AN OBSERVER UNDERGOING ACCELERATION, THAT IS CALLED MAKING OBSERVATIONS FROM A *NON-INERTIAL REFERENCE FRAME*, AND WE MUST USE GENERAL RELATIVITY. LET ME EXPLAIN WHAT THESE THEORIES BROADLY MEAN.
SPECIAL RELATIVITY SAYS THAT FOR OBJECTS IN MOTION...
TIME SLOWS DOWN, LENGTH CONTRACTS, AND ***MASS INCREASES.***
SLOWS DOWN, CONTRACTS?! DO THOSE THINGS REALLY OCCUR?!
CONTRACT! INCREASE!
TRANSMORPHERS, GO!!
...IT'S NOT LIKE YOU ARE IMAGINING IT.
THE EFFECTS OF RELATIVITY ONLY BECOME NOTICEABLE AT SPEEDS CLOSE TO THE SPEED OF LIGHT.
THESE SPEEDS ARE EXTREMELY FAST, AND SO WE VERY RARELY OBSERVE RELATIVISTIC EFFECTS ON EARTH.
WHAT THE HECK?
ガッ!


GENERAL RELATIVITY SAYS THAT AN OBJECT WITH MASS *CREATES* GRAVITY BY AFFECTING TIME AND SPACE.
WHAT IS GRAVITY?
RELATIVITY FLASH!
I SEE. FOR EXAMPLE, LIGHT...
STARS HAVE SO MUCH MASS THAT THEY DISTORT SPACE AND TIME ENOUGH THAT WE CAN OBSERVE LIGHT BEND AS IT PASSES BY.
THE OBSERVATION OF LIGHT BENDING AS IT PASSES BY A LARGE STAR WAS ACTUALLY THE FIRST CONFIRMATION OF THE THEORY OF GENERAL RELATIVITY.
BZZP!
NOW, SINCE GENERAL RELATIVITY IS MORE ADVANCED AND DIFFICULT...
LET'S PROCEED BY FOCUSING OUR DISCUSSION ON SPECIAL RELATIVITY. YOU SHOULD APPRECIATE THIS.
OKAAAAY...
BECAUSE IF I DON'T UNDERSTAND, I'LL REALLY BE IN HOT WATER....

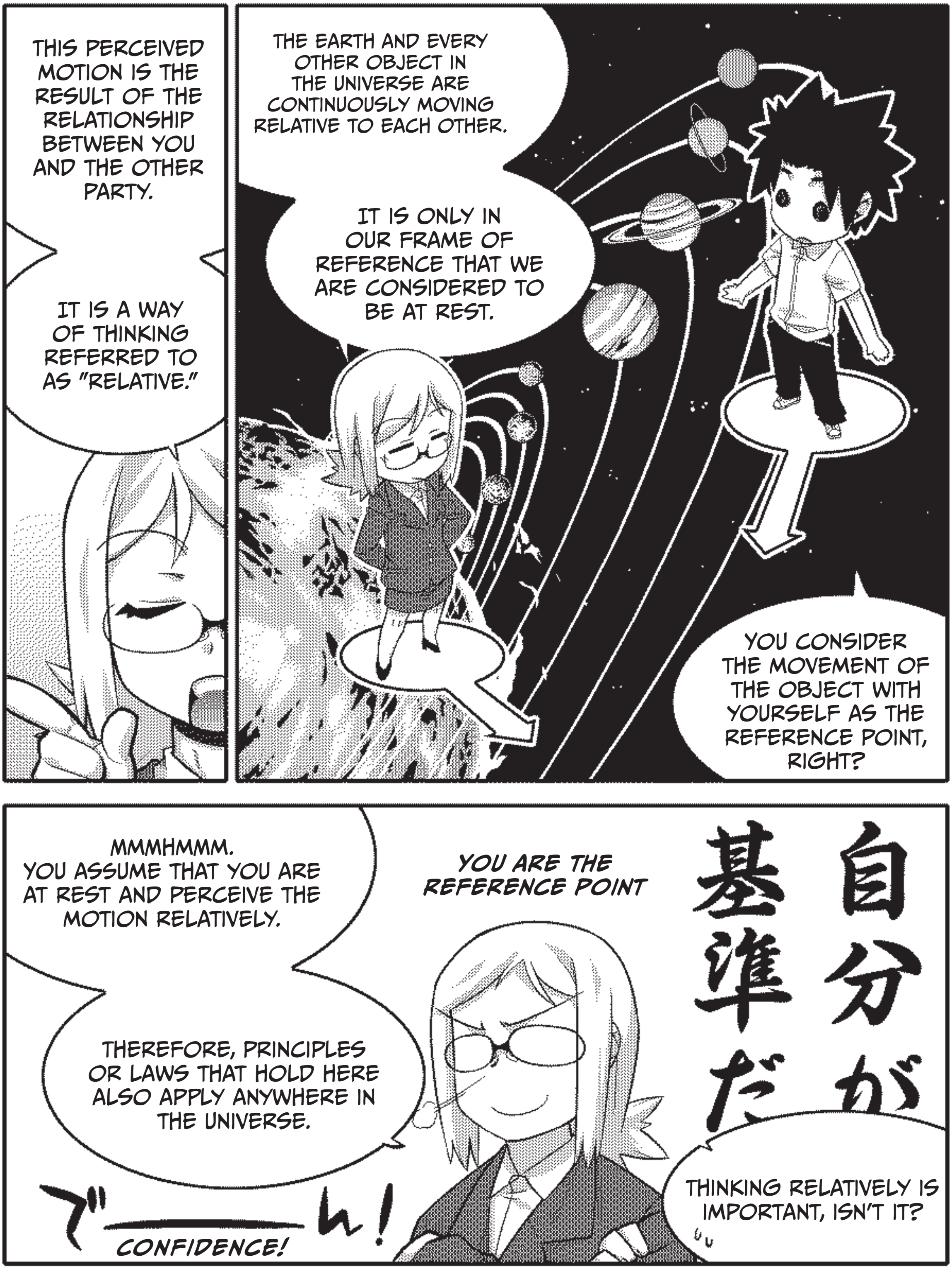
2. Galilean Principle of Relativity and Newtonian Mechanics



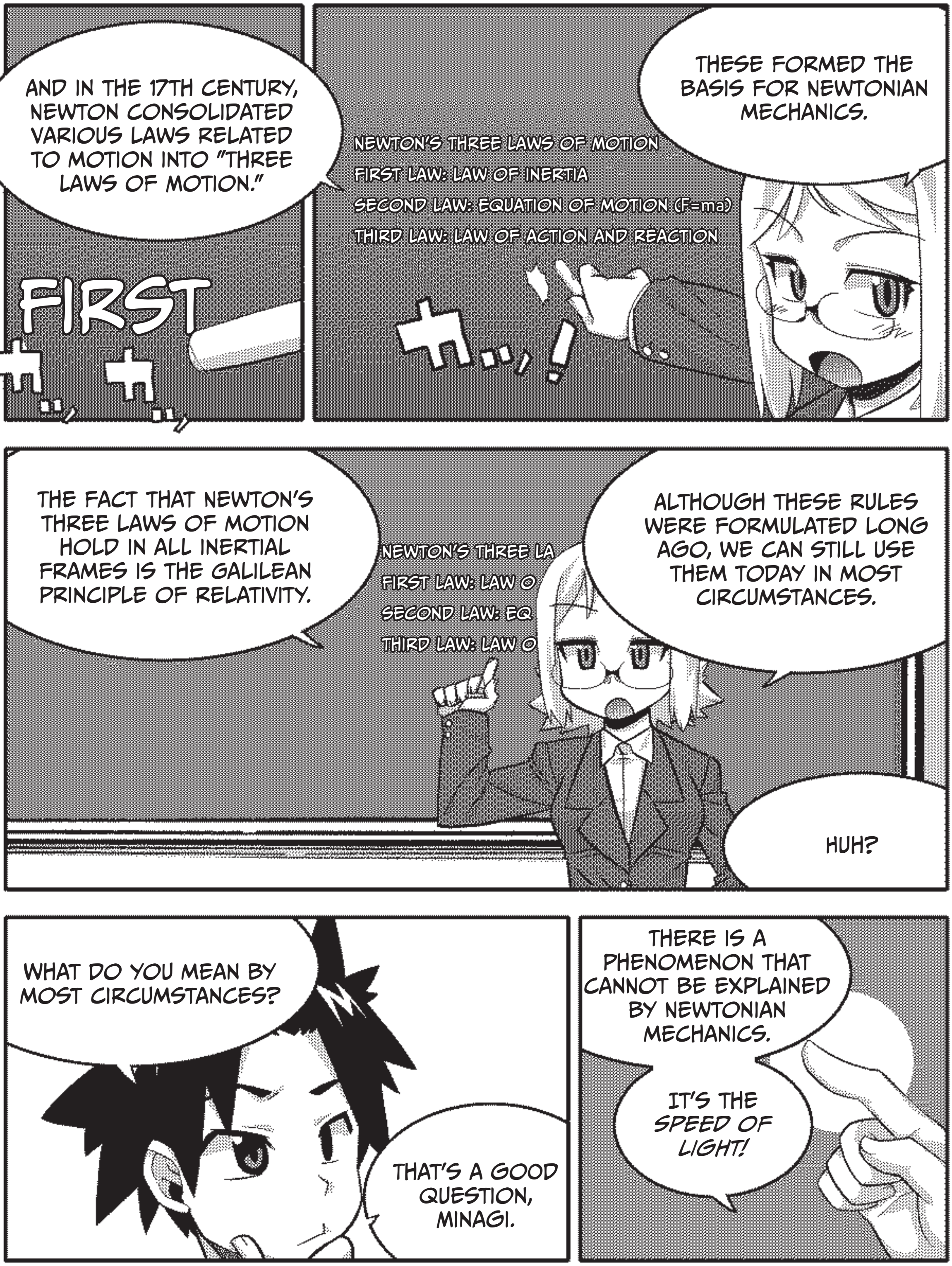

MINAGI, YOU KNOW WHAT *UNIFORM LINEAR MOTION* IS, DON'T YOU?
A MYSTERIOUS CREATURE APPEARED!
UM, IT MEANS THAT SOMETHING IS MOVING WITH A FIXED SPEED AND DIRECTION?
AH...
STATE OF REST
IF THERE IS NO EXTERNAL FORCE, AN OBJECT AT REST WILL REMAIN AT REST.
HUH?!
STATE OF MOVING WITH A FIXED SPEED AND DIRECTION
RELATIVITY KICK
THE *LAW OF INERTIA* TELLS US THAT IN AN INERTIAL REFERENCE FRAME, AN OBJECT AT REST WILL STAY AT REST AND...
...AN OBJECT THAT IS MOVING AT A CONSTANT VELOCITY WILL CONTINUE MOVING, UNTIL ACTED UPON BY AN OUTSIDE FORCE.
HELP MEEEeeee!


THE GALILEAN PRINCIPLE OF RELATIVITY TELLS US THAT NEWTON'S LAWS ARE THE SAME IN ANY INERTIAL REFERENCE FRAME THAT WE MIGHT CHOOSE. IN OTHER WORDS, NO MATTER WHERE AN OBSERVER IS IN THE UNIVERSE OR HOW FAST SHE IS MOVING, THE LAWS OF PHYSICS WILL NEVER CHANGE.
I SORT OF UNDERSTAND IT...
FOR EXAMPLE, IF I TOSS A BALL STRAIGHT UP IN A PLACE THAT IS AT REST, IT WILL COME BACK TO MY HAND, RIGHT?
IN THE SAME WAY, IF I TOSS A BALL STRAIGHT UP IN A TRAIN THAT IS MOVING AT A CONSTANT SPEED, IT WILL ALSO RETURN TO MY HAND.
IN OTHER WORDS, IT DOES NOT MATTER IF YOU ARE AT REST OR IF YOU ARE MOVING, THE LAWS OF PHYSICS BEHAVE EXACTLY THE SAME.

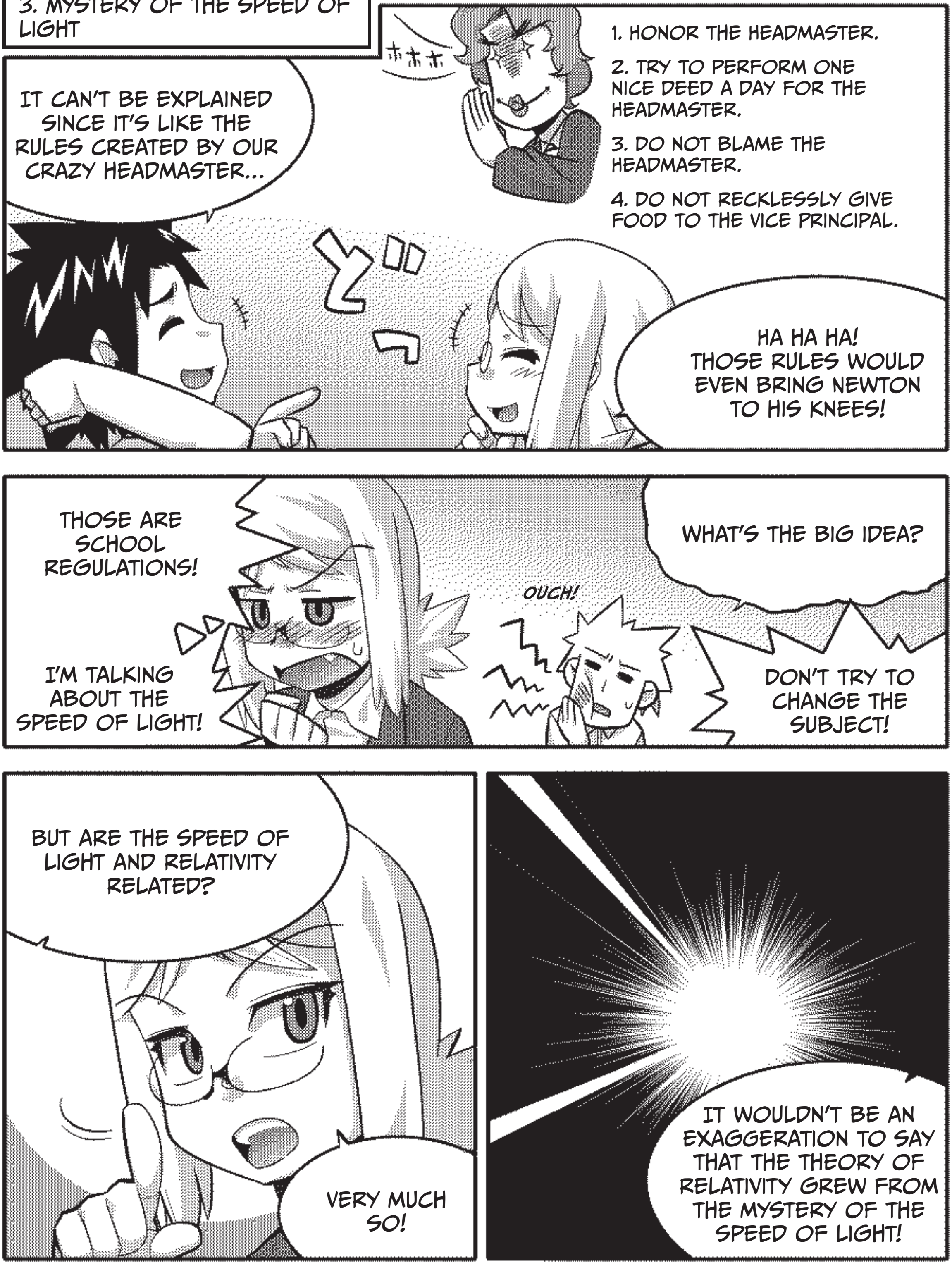
BUT IF EARTH ITSELF IS IN MOTION THROUGH OUR GALAXY, DOES THAT MEAN THAT EARTH IS A *MOVING REFERENCE FRAME*?
THAT'S RIGHT! BECAUSE IN RELATIVITY, NO REFERENCE FRAME IS PREFERABLE TO ANY OTHER REFERENCE FRAME, EVEN EARTH ITSELF CANNOT BE CONSIDERED AN "ABSOLUTE" FRAME OF REFERENCE.
WHAT DO YOU MEAN?
CLACKITY CLACK
CLACK CLACK
DON'T YOU FEEL THIS WHEN YOU'RE RIDING A TRAIN?
CLACKITY CLACK
FOR EXAMPLE, WHEN TRAIN A, WHICH IS MOVING AT 50 KM/H, PASSES BY TRAIN B, WHICH IS MOVING AT 50 KM/H IN THE OPPOSITE DIRECTION...
A PERSON ON THE OTHER TRAIN SEEMS TO YOU TO BE MOVING AT 100 KM/H.
ゴオッ
I SEE. THAT'S JUST THE WAY IT SEEMS.



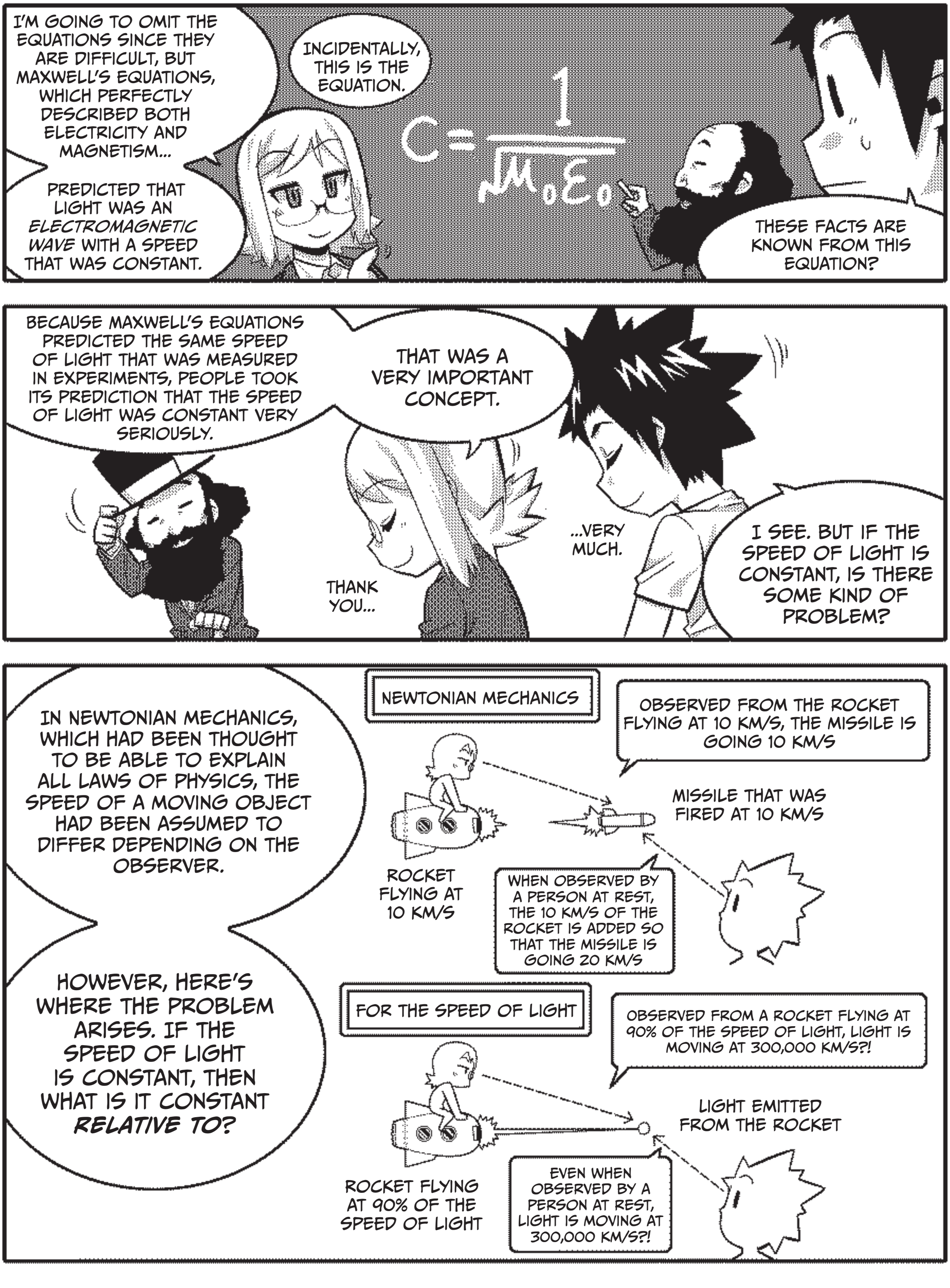
This perceived motion is the result of the relationship between you and the other party.
It is a way of thinking referred to as "relative."
The earth and every other object in the universe are continuously moving relative to each other.
It is only in our frame of reference that we are considered to be at rest.
You consider the movement of the object with yourself as the reference point, right?
自分が基準だ
You are the reference point
Mmmhmmm. You assume that you are at rest and perceive the motion relatively.
Therefore, principles or laws that hold here also apply anywhere in the universe.
Thinking relatively is important, isn't it?
でーーん!
Confidence!

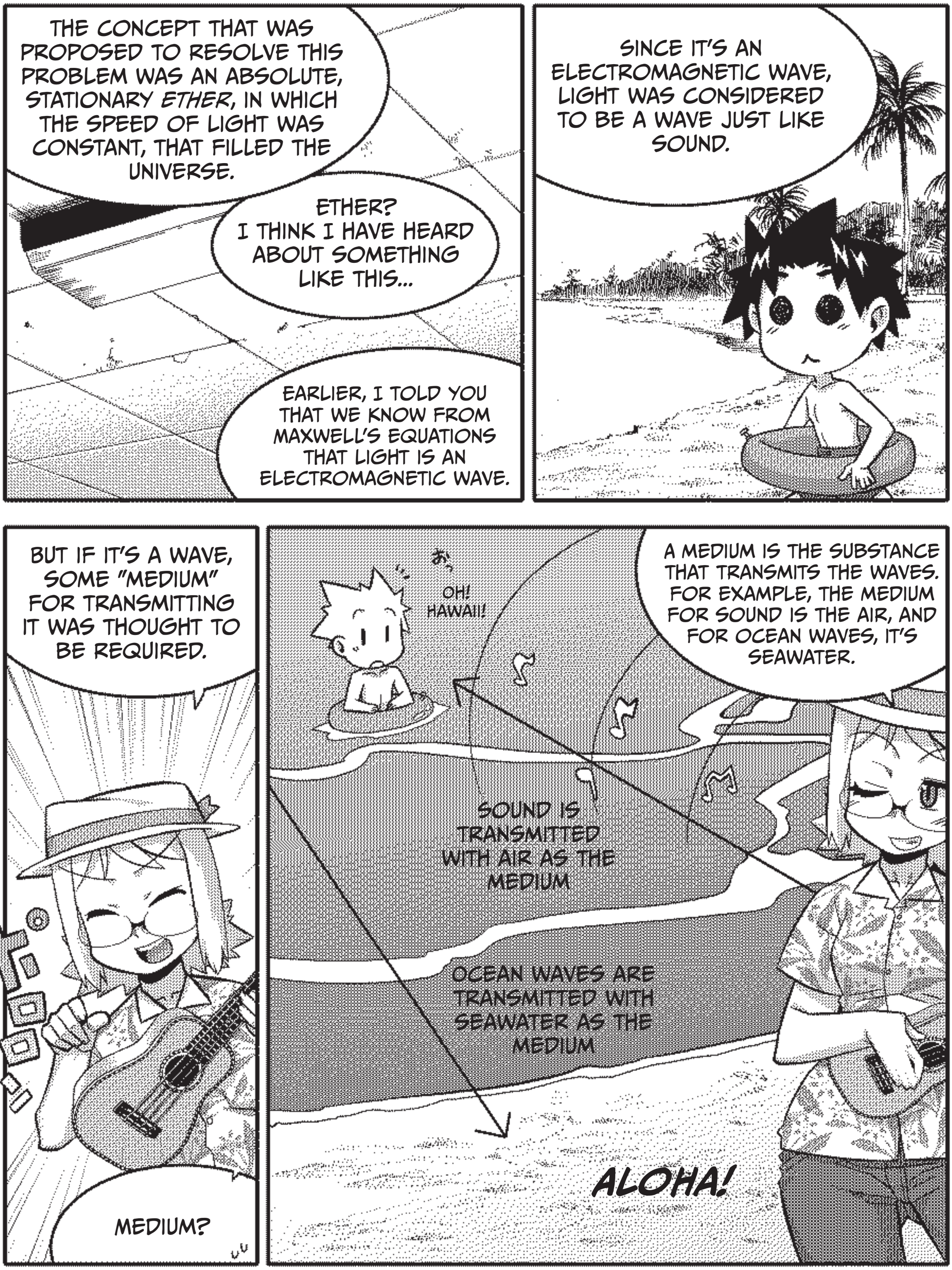

AND IN THE 17TH CENTURY, NEWTON CONSOLIDATED VARIOUS LAWS RELATED TO MOTION INTO "THREE LAWS OF MOTION."
FIRST
NEWTON'S THREE LAWS OF MOTION
FIRST LAW: LAW OF INERTIA
SECOND LAW: EQUATION OF MOTION (F=ma)
THIRD LAW: LAW OF ACTION AND REACTION
THESE FORMED THE BASIS FOR NEWTONIAN MECHANICS.
THE FACT THAT NEWTON'S THREE LAWS OF MOTION HOLD IN ALL INERTIAL FRAMES IS THE GALILEAN PRINCIPLE OF RELATIVITY.
ALTHOUGH THESE RULES WERE FORMULATED LONG AGO, WE CAN STILL USE THEM TODAY IN MOST CIRCUMSTANCES.
HUH?
WHAT DO YOU MEAN BY MOST CIRCUMSTANCES?
THAT'S A GOOD QUESTION, MINAGI.
THERE IS A PHENOMENON THAT CANNOT BE EXPLAINED BY NEWTONIAN MECHANICS.
IT'S THE SPEED OF LIGHT!

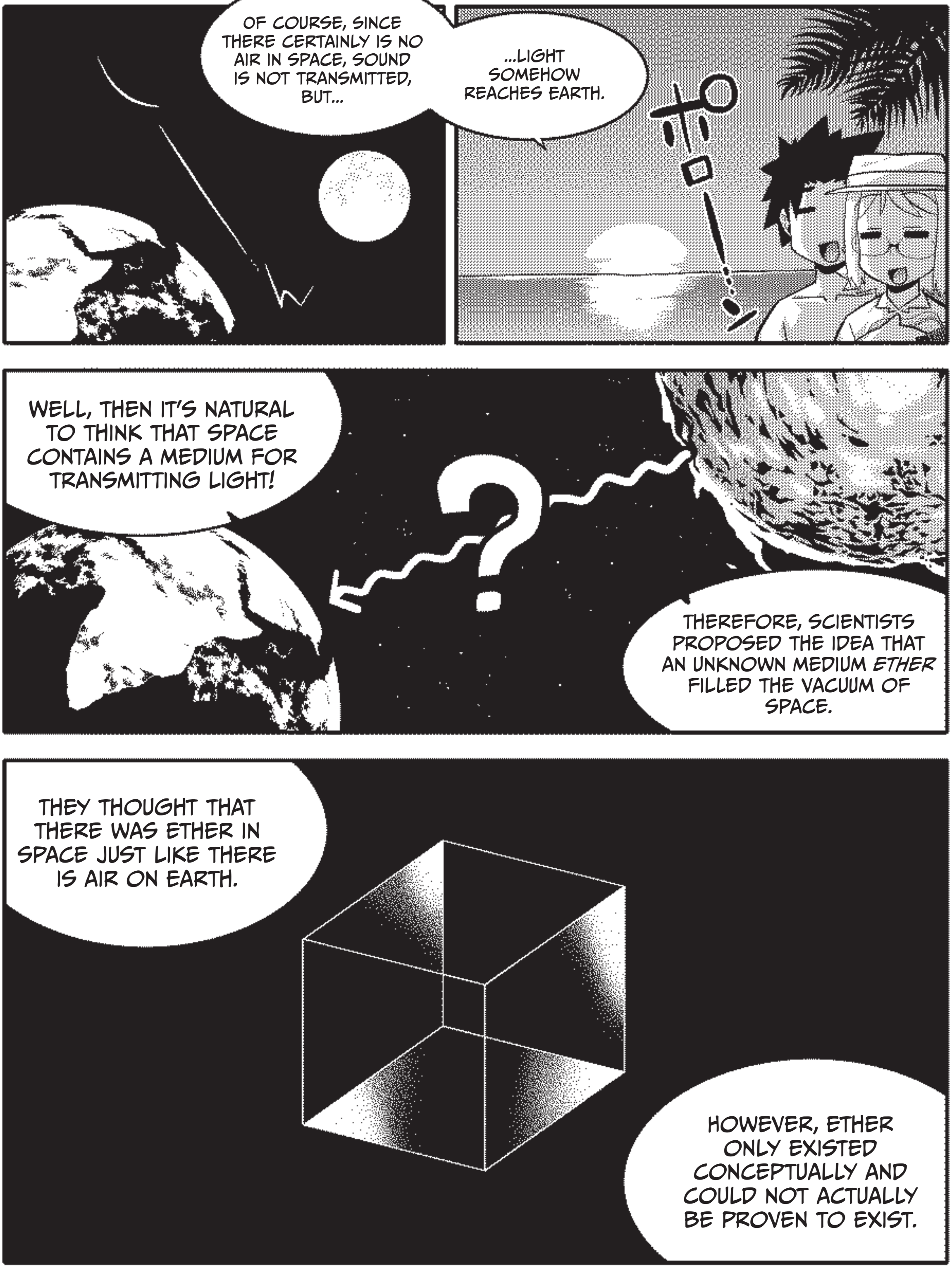
3. MYSTERY OF THE SPEED OF LIGHT

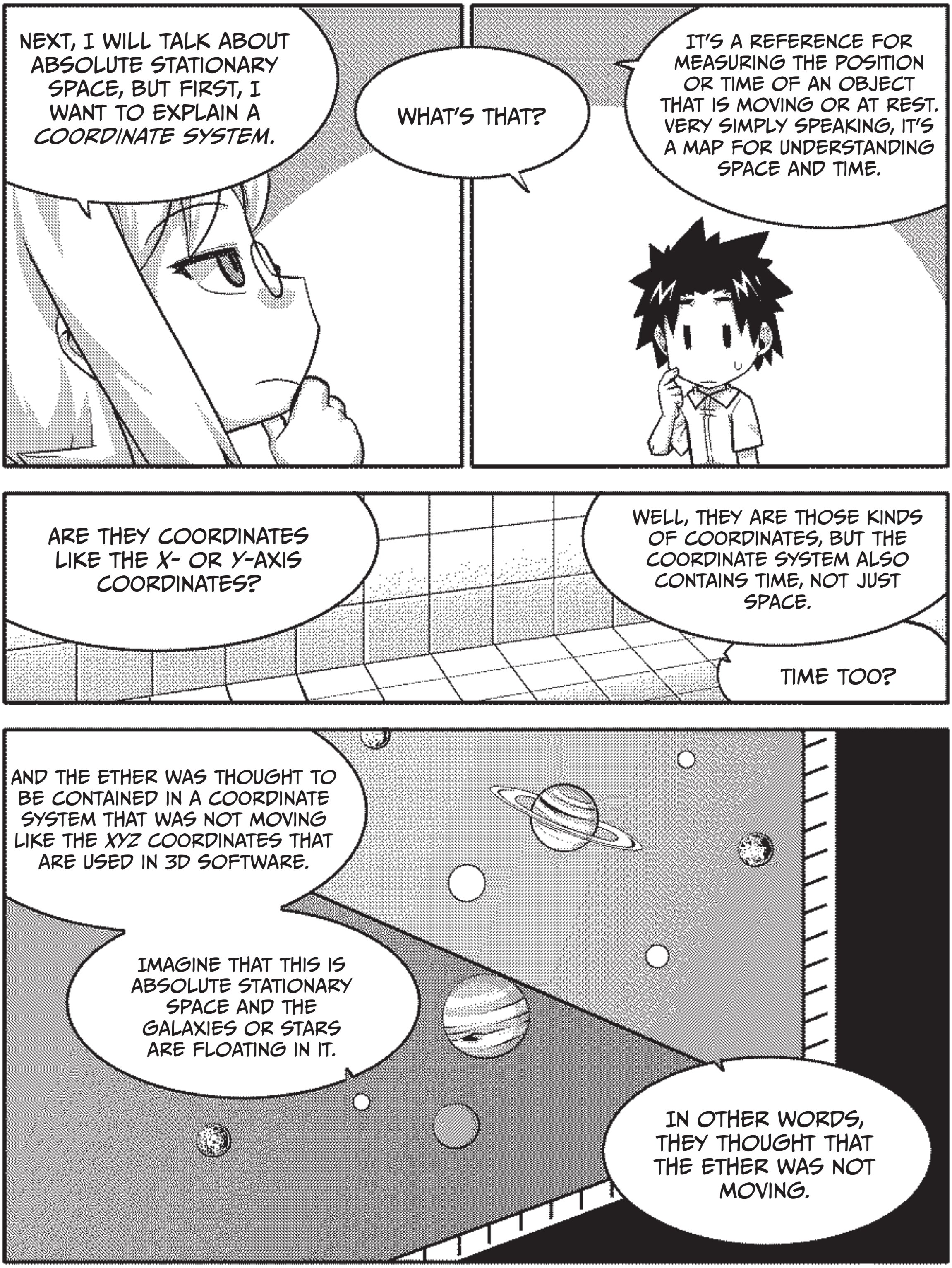


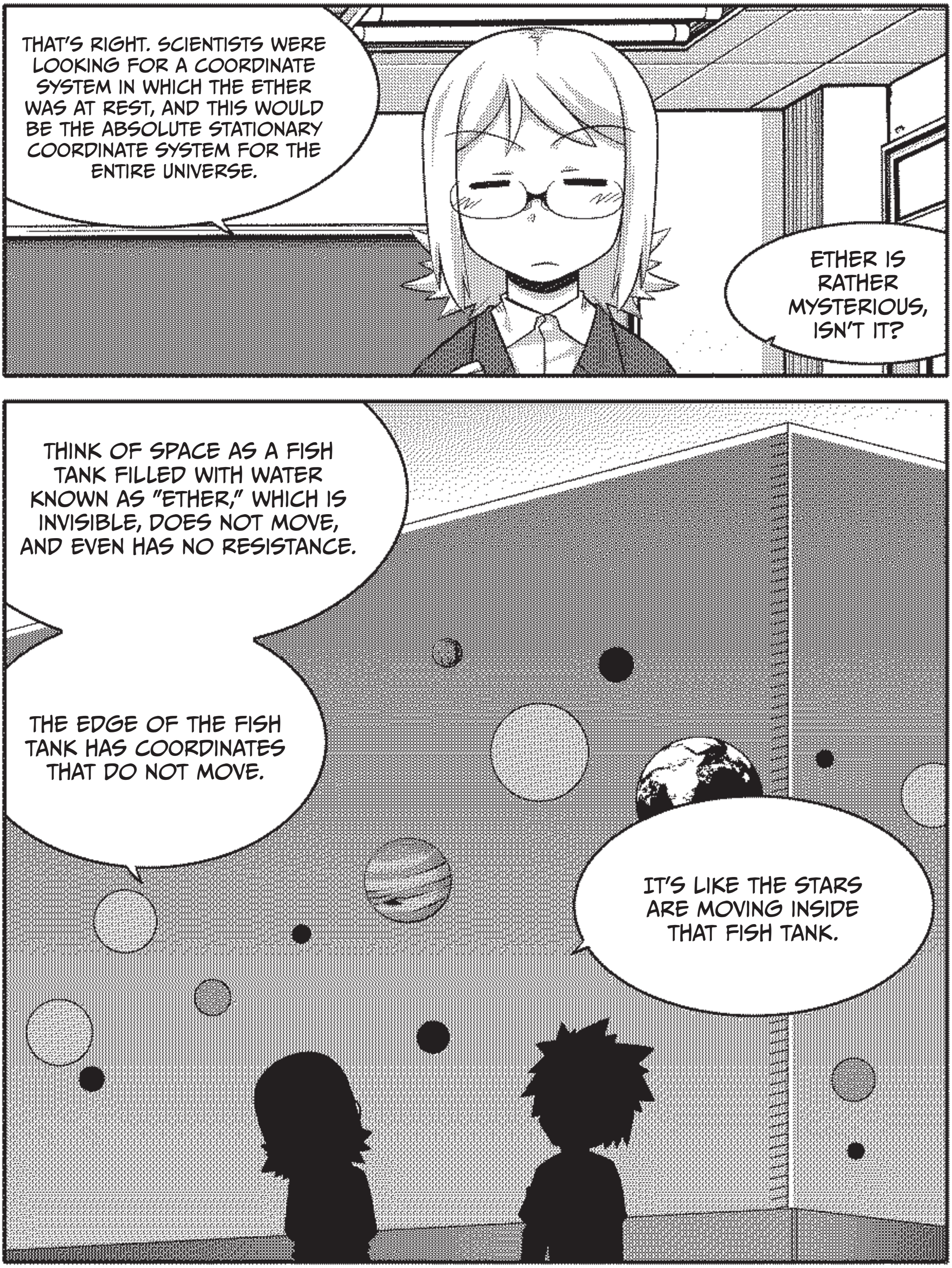

AROUND THE TIME WHEN EINSTEIN WAS BORN IN 1879, THE SPEED OF LIGHT WAS KNOWN TO BE APPROXIMATELY 300,000 KILOMETERS PER SECOND FROM VARIOUS EXPERIMENTS.
APPROXIMATELY 380,000 KM FROM THE EARTH TO THE MOON
AT THE SPEED OF LIGHT, IT WILL ARRIVE IN APPROXIMATELY 1.3 SECONDS
ALTHOUGH WE HAVE THE IMPRESSION THAT LIGHT IS TRANSMITTED IN AN INSTANT, IT HAS A PRECISE SPEED.
EVEN THOUGH THAT WAS ALL PEOPLE KNEW, IT WAS REVOLUTIONARY AT THE TIME. BUT THERE WAS AN EVEN MORE ASTONISHING DISCOVERY.
TA DA!
AHEM
IN 1864, A MAN NAMED MAXWELL FORMULATED WHAT IS KNOWN AS *MAXWELL'S EQUATIONS*, WHICH ENABLE ELECTRICITY AND MAGNETISM TO BE CONSIDERED UNIFIED.
DID YOU SAY HE UNIFIED ELECTRICITY AND MAGNETISM?
James Clerk Maxwell (1831~1879)
HI THERE!


I'M GOING TO OMIT THE EQUATIONS SINCE THEY ARE DIFFICULT, BUT MAXWELL'S EQUATIONS, WHICH PERFECTLY DESCRIBED BOTH ELECTRICITY AND MAGNETISM...
INCIDENTALLY, THIS IS THE EQUATION.
$C = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$
PREDICTED THAT LIGHT WAS AN *ELECTROMAGNETIC WAVE* WITH A SPEED THAT WAS CONSTANT.
THESE FACTS ARE KNOWN FROM THIS EQUATION?
BECAUSE MAXWELL'S EQUATIONS PREDICTED THE SAME SPEED OF LIGHT THAT WAS MEASURED IN EXPERIMENTS, PEOPLE TOOK ITS PREDICTION THAT THE SPEED OF LIGHT WAS CONSTANT VERY SERIOUSLY.
THAT WAS A VERY IMPORTANT CONCEPT.
THANK YOU...
...VERY MUCH.
I SEE. BUT IF THE SPEED OF LIGHT IS CONSTANT, IS THERE SOME KIND OF PROBLEM?
IN NEWTONIAN MECHANICS, WHICH HAD BEEN THOUGHT TO BE ABLE TO EXPLAIN ALL LAWS OF PHYSICS, THE SPEED OF A MOVING OBJECT HAD BEEN ASSUMED TO DIFFER DEPENDING ON THE OBSERVER.
NEWTONIAN MECHANICS
OBSERVED FROM THE ROCKET FLYING AT 10 KM/S, THE MISSILE IS GOING 10 KM/S
MISSILE THAT WAS FIRED AT 10 KM/S
ROCKET FLYING AT 10 KM/S
WHEN OBSERVED BY A PERSON AT REST, THE 10 KM/S OF THE ROCKET IS ADDED SO THAT THE MISSILE IS GOING 20 KM/S
HOWEVER, HERE'S WHERE THE PROBLEM ARISES. IF THE SPEED OF LIGHT IS CONSTANT, THEN WHAT IS IT CONSTANT *RELATIVE TO*?
FOR THE SPEED OF LIGHT
OBSERVED FROM A ROCKET FLYING AT 90% OF THE SPEED OF LIGHT, LIGHT IS MOVING AT 300,000 KM/S?!
LIGHT EMITTED FROM THE ROCKET
ROCKET FLYING AT 90% OF THE SPEED OF LIGHT
EVEN WHEN OBSERVED BY A PERSON AT REST, LIGHT IS MOVING AT 300,000 KM/S?!

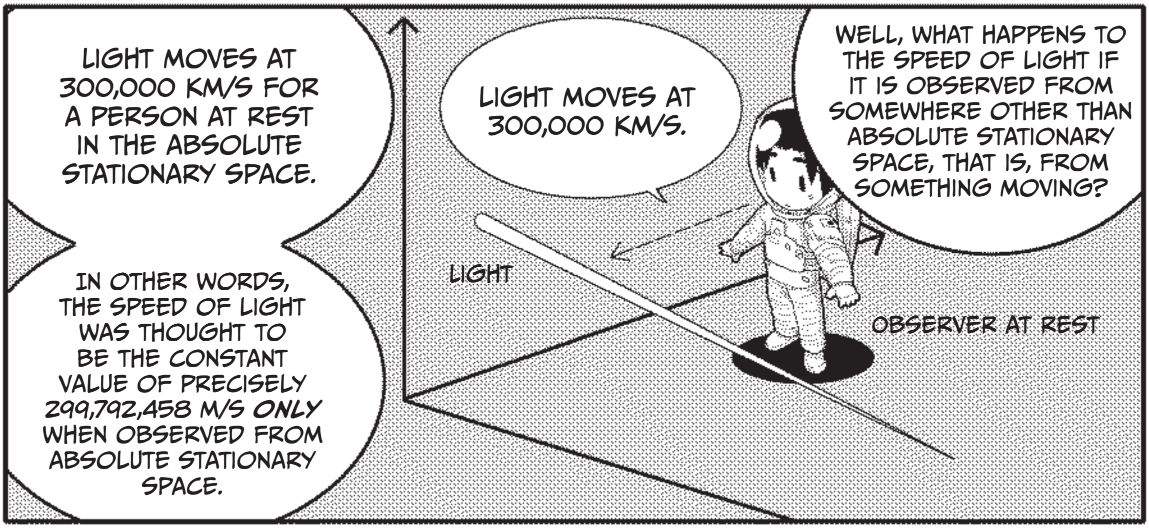

THE CONCEPT THAT WAS PROPOSED TO RESOLVE THIS PROBLEM WAS AN ABSOLUTE, STATIONARY *ETHER*, IN WHICH THE SPEED OF LIGHT WAS CONSTANT, THAT FILLED THE UNIVERSE.
ETHER? I THINK I HAVE HEARD ABOUT SOMETHING LIKE THIS...
EARLIER, I TOLD YOU THAT WE KNOW FROM MAXWELL'S EQUATIONS THAT LIGHT IS AN ELECTROMAGNETIC WAVE.
SINCE IT'S AN ELECTROMAGNETIC WAVE, LIGHT WAS CONSIDERED TO BE A WAVE JUST LIKE SOUND.
BUT IF IT'S A WAVE, SOME "MEDIUM" FOR TRANSMITTING IT WAS THOUGHT TO BE REQUIRED.
MEDIUM?
A MEDIUM IS THE SUBSTANCE THAT TRANSMITS THE WAVES. FOR EXAMPLE, THE MEDIUM FOR SOUND IS THE AIR, AND FOR OCEAN WAVES, IT'S SEAWATER.
OH! HAWAII!
SOUND IS TRANSMITTED WITH AIR AS THE MEDIUM
OCEAN WAVES ARE TRANSMITTED WITH SEAWATER AS THE MEDIUM
ALOHA!

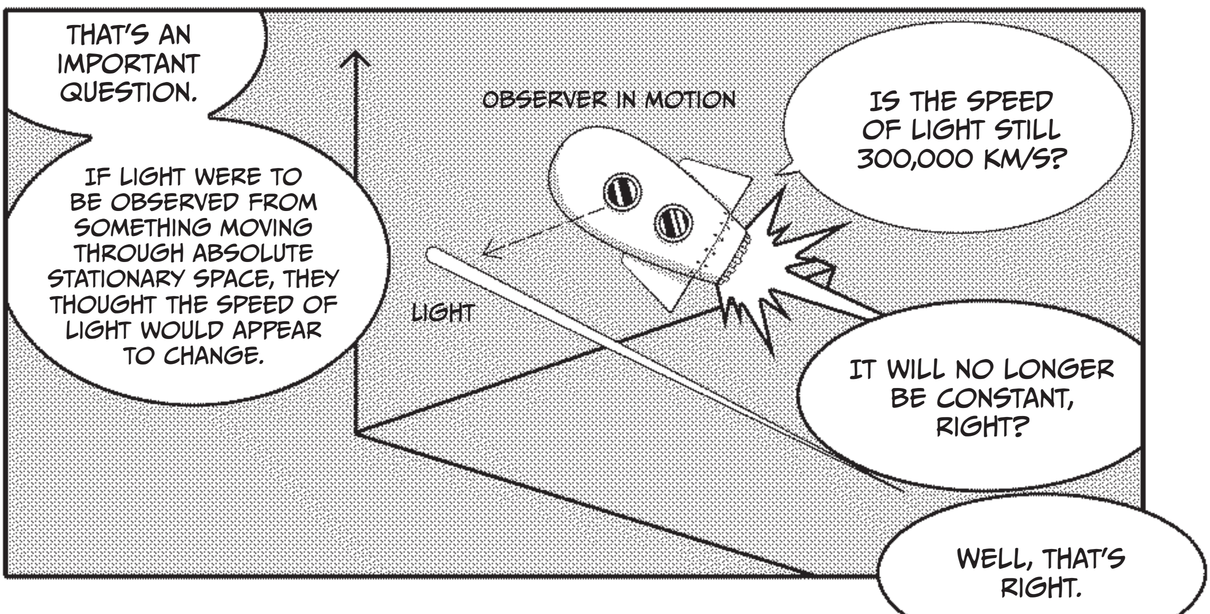

OF COURSE, SINCE THERE CERTAINLY IS NO AIR IN SPACE, SOUND IS NOT TRANSMITTED, BUT...
...LIGHT SOMEHOW REACHES EARTH.
ポロー
WELL, THEN IT'S NATURAL TO THINK THAT SPACE CONTAINS A MEDIUM FOR TRANSMITTING LIGHT!
THEREFORE, SCIENTISTS PROPOSED THE IDEA THAT AN UNKNOWN MEDIUM *ETHER* FILLED THE VACUUM OF SPACE.
THEY THOUGHT THAT THERE WAS ETHER IN SPACE JUST LIKE THERE IS AIR ON EARTH.
HOWEVER, ETHER ONLY EXISTED CONCEPTUALLY AND COULD NOT ACTUALLY BE PROVEN TO EXIST.

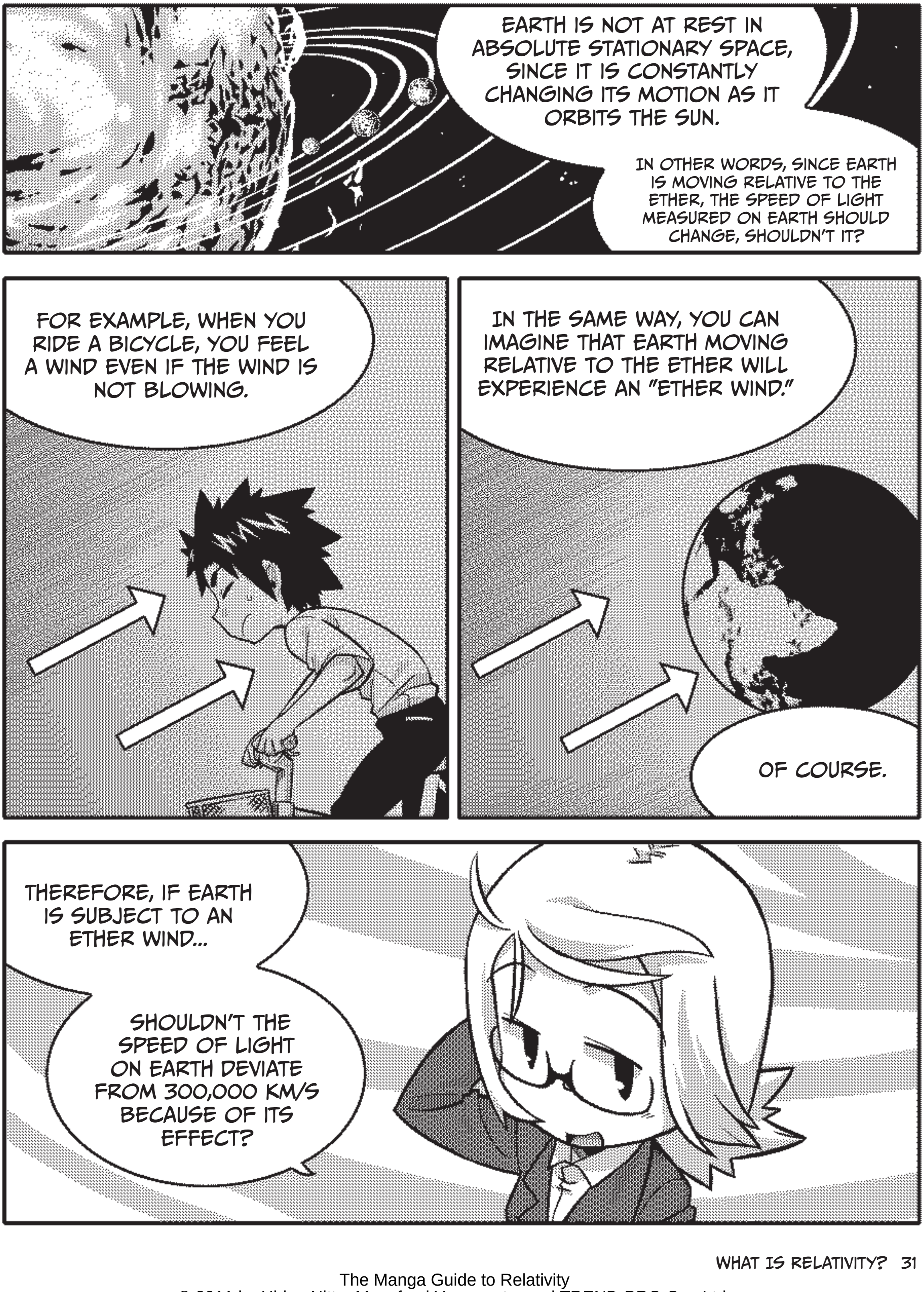

NEXT, I WILL TALK ABOUT ABSOLUTE STATIONARY SPACE, BUT FIRST, I WANT TO EXPLAIN A *COORDINATE SYSTEM.*
WHAT'S THAT?
IT'S A REFERENCE FOR MEASURING THE POSITION OR TIME OF AN OBJECT THAT IS MOVING OR AT REST. VERY SIMPLY SPEAKING, IT'S A MAP FOR UNDERSTANDING SPACE AND TIME.
ARE THEY COORDINATES LIKE THE *X*- OR *Y*-AXIS COORDINATES?
WELL, THEY ARE THOSE KINDS OF COORDINATES, BUT THE COORDINATE SYSTEM ALSO CONTAINS TIME, NOT JUST SPACE.
TIME TOO?
AND THE ETHER WAS THOUGHT TO BE CONTAINED IN A COORDINATE SYSTEM THAT WAS NOT MOVING LIKE THE *XYZ* COORDINATES THAT ARE USED IN 3D SOFTWARE.
IMAGINE THAT THIS IS ABSOLUTE STATIONARY SPACE AND THE GALAXIES OR STARS ARE FLOATING IN IT.
IN OTHER WORDS, THEY THOUGHT THAT THE ETHER WAS NOT MOVING.


THAT'S RIGHT. SCIENTISTS WERE LOOKING FOR A COORDINATE SYSTEM IN WHICH THE ETHER WAS AT REST, AND THIS WOULD BE THE ABSOLUTE STATIONARY COORDINATE SYSTEM FOR THE ENTIRE UNIVERSE.
ETHER IS RATHER MYSTERIOUS, ISN'T IT?
THINK OF SPACE AS A FISH TANK FILLED WITH WATER KNOWN AS "ETHER," WHICH IS INVISIBLE, DOES NOT MOVE, AND EVEN HAS NO RESISTANCE.
THE EDGE OF THE FISH TANK HAS COORDINATES THAT DO NOT MOVE.
IT'S LIKE THE STARS ARE MOVING INSIDE THAT FISH TANK.

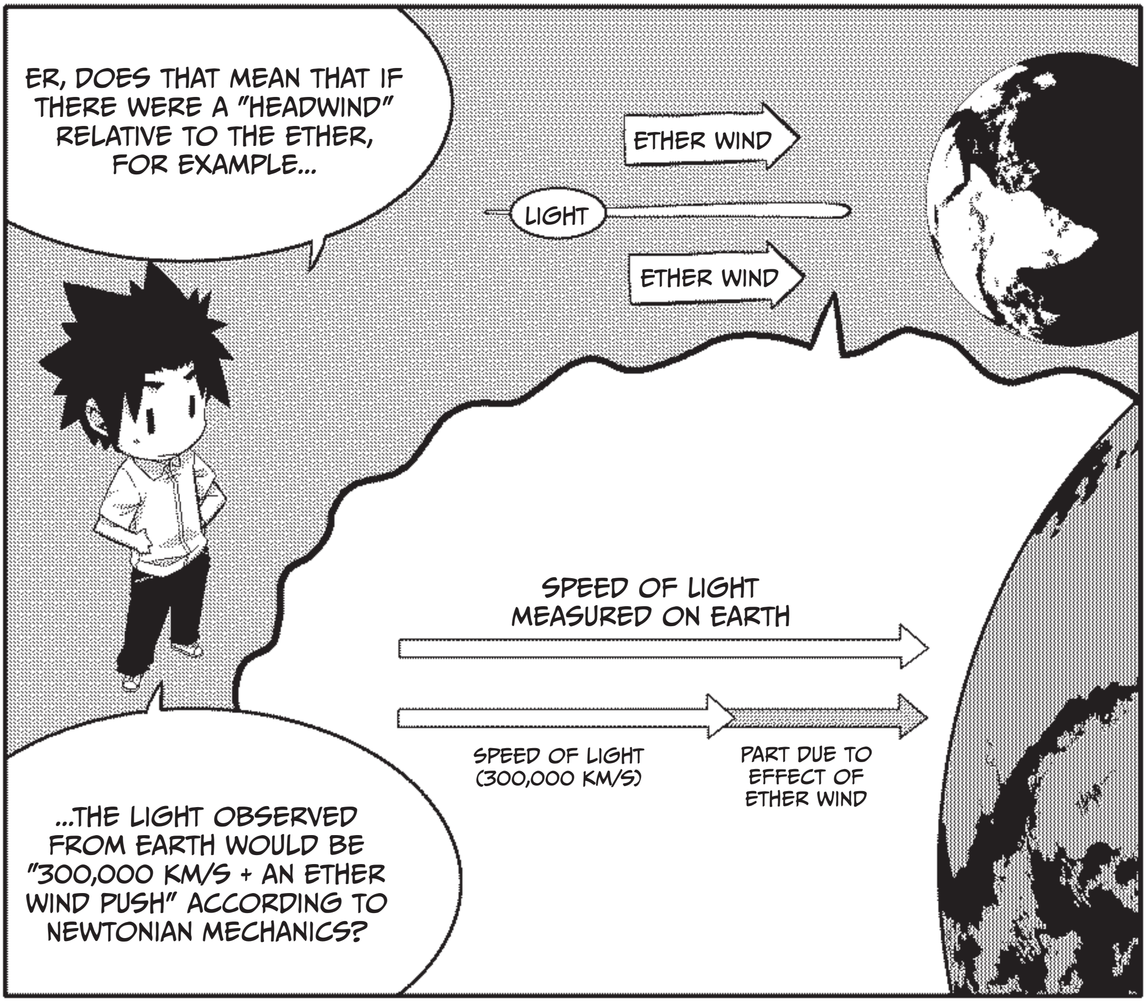

ER, HOW IS THIS RELATED TO SAYING THAT THE SPEED OF LIGHT IS CONSTANT?
HMMM. IT'S BASED ON THE FOLLOWING...

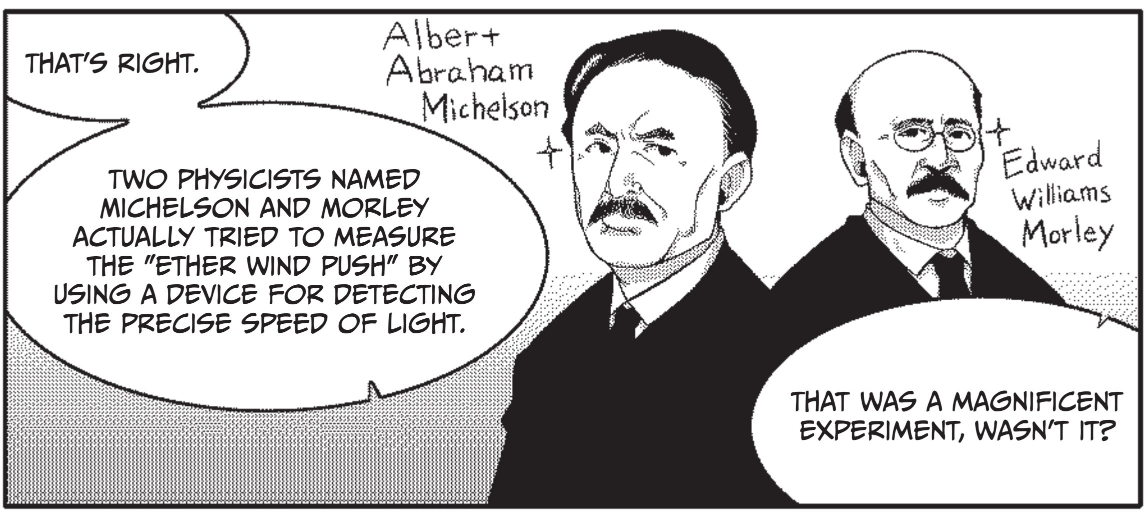

LIGHT MOVES AT 300,000 KM/S FOR A PERSON AT REST IN THE ABSOLUTE STATIONARY SPACE.
LIGHT MOVES AT 300,000 KM/S.
WELL, WHAT HAPPENS TO THE SPEED OF LIGHT IF IT IS OBSERVED FROM SOMEWHERE OTHER THAN ABSOLUTE STATIONARY SPACE, THAT IS, FROM SOMETHING MOVING?
LIGHT
OBSERVER AT REST
IN OTHER WORDS, THE SPEED OF LIGHT WAS THOUGHT TO BE THE CONSTANT VALUE OF PRECISELY 299,792,458 M/S *ONLY* WHEN OBSERVED FROM ABSOLUTE STATIONARY SPACE.

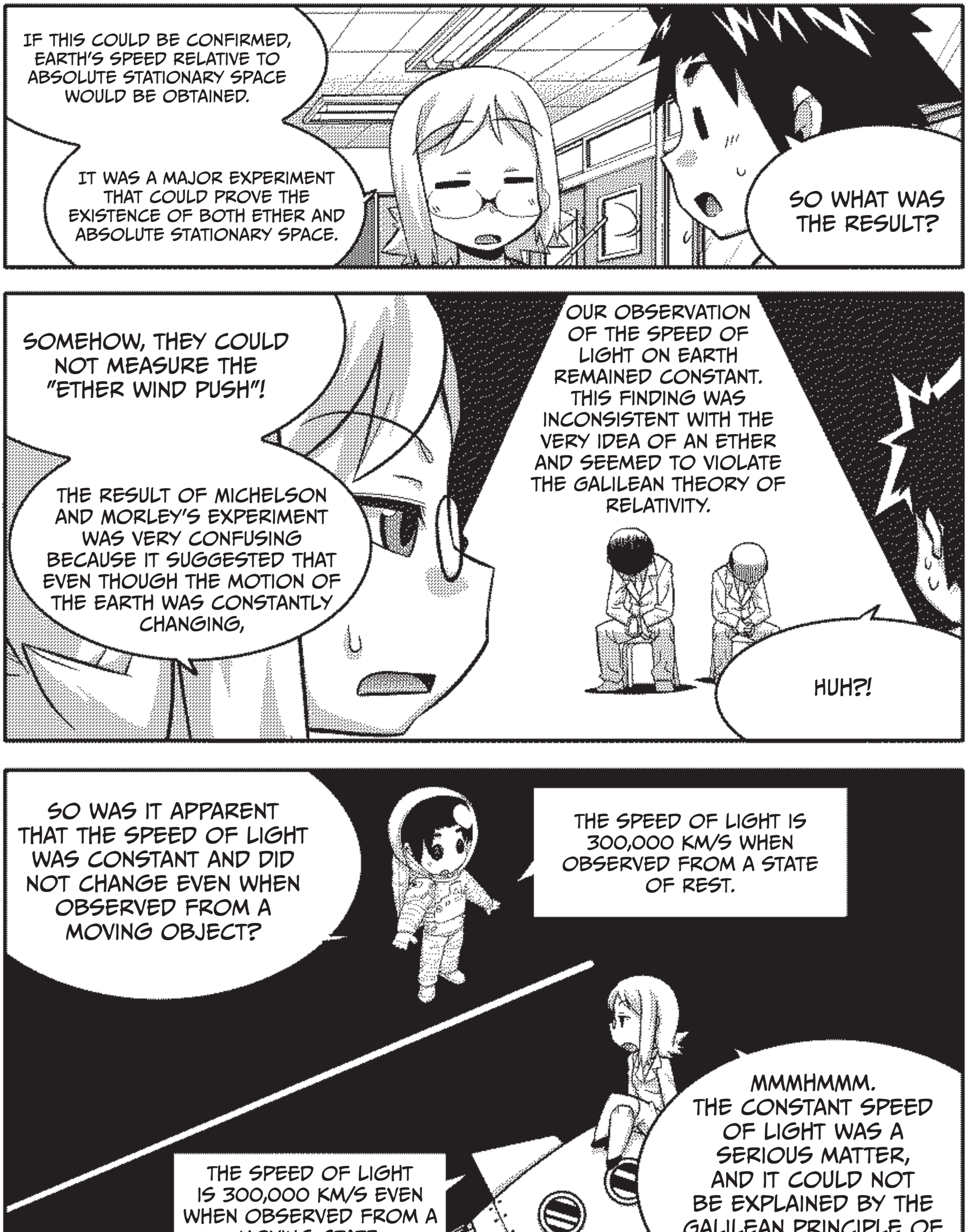

THAT'S AN IMPORTANT QUESTION.
OBSERVER IN MOTION
IS THE SPEED OF LIGHT STILL 300,000 KM/S?
IF LIGHT WERE TO BE OBSERVED FROM SOMETHING MOVING THROUGH ABSOLUTE STATIONARY SPACE, THEY THOUGHT THE SPEED OF LIGHT WOULD APPEAR TO CHANGE.
LIGHT
IT WILL NO LONGER BE CONSTANT, RIGHT?
WELL, THAT'S RIGHT.



EARTH IS NOT AT REST IN ABSOLUTE STATIONARY SPACE, SINCE IT IS CONSTANTLY CHANGING ITS MOTION AS IT ORBITS THE SUN.
IN OTHER WORDS, SINCE EARTH IS MOVING RELATIVE TO THE ETHER, THE SPEED OF LIGHT MEASURED ON EARTH SHOULD CHANGE, SHOULDN'T IT?
FOR EXAMPLE, WHEN YOU RIDE A BICYCLE, YOU FEEL A WIND EVEN IF THE WIND IS NOT BLOWING.
IN THE SAME WAY, YOU CAN IMAGINE THAT EARTH MOVING RELATIVE TO THE ETHER WILL EXPERIENCE AN "ETHER WIND."
OF COURSE.
THEREFORE, IF EARTH IS SUBJECT TO AN ETHER WIND...
SHOULDN'T THE SPEED OF LIGHT ON EARTH DEVIATE FROM 300,000 KM/S BECAUSE OF ITS EFFECT?

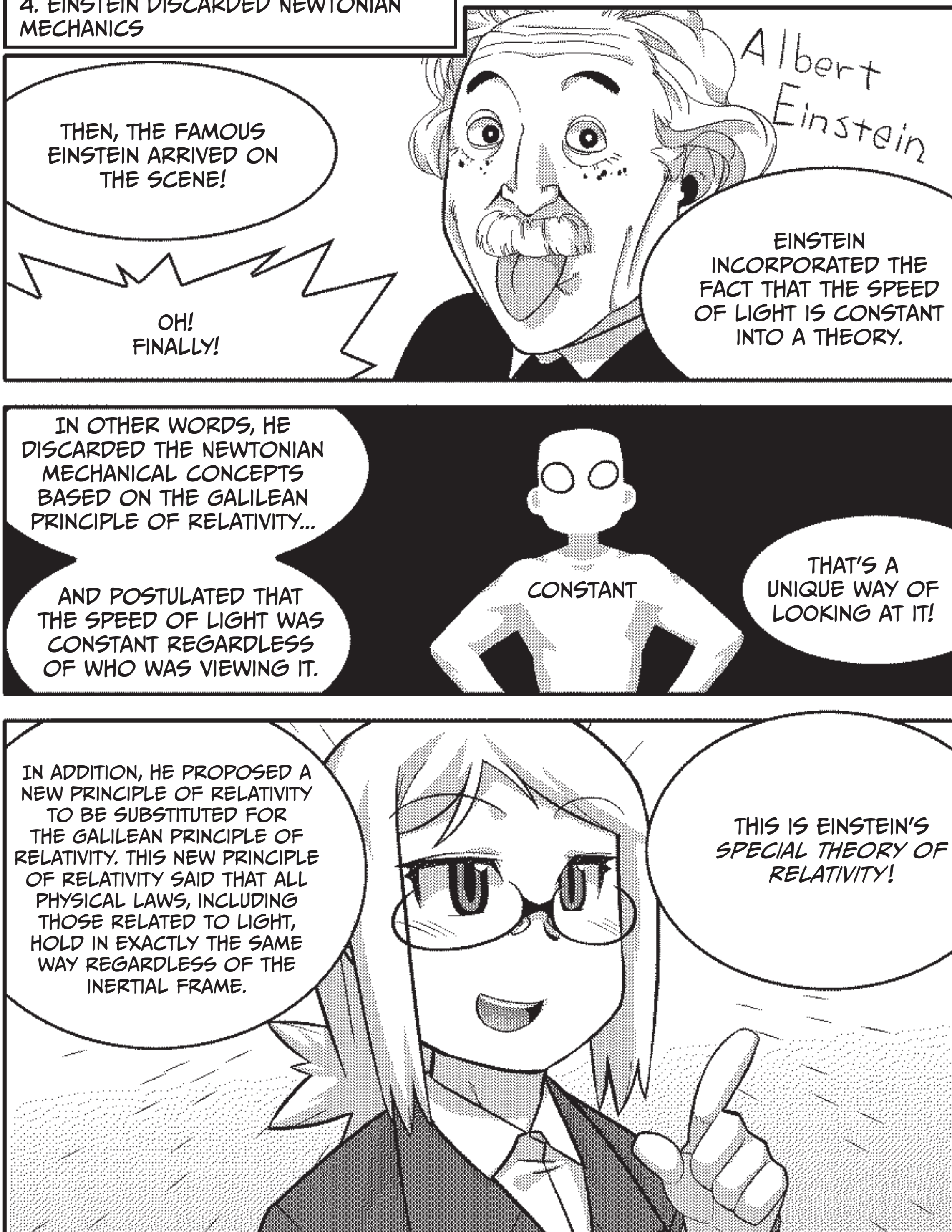

ER, DOES THAT MEAN THAT IF THERE WERE A "HEADWIND" RELATIVE TO THE ETHER, FOR EXAMPLE...
ETHER WIND
LIGHT
ETHER WIND
SPEED OF LIGHT MEASURED ON EARTH
SPEED OF LIGHT (300,000 KM/S)
PART DUE TO EFFECT OF ETHER WIND
...THE LIGHT OBSERVED FROM EARTH WOULD BE "300,000 KM/S + AN ETHER WIND PUSH" ACCORDING TO NEWTONIAN MECHANICS?

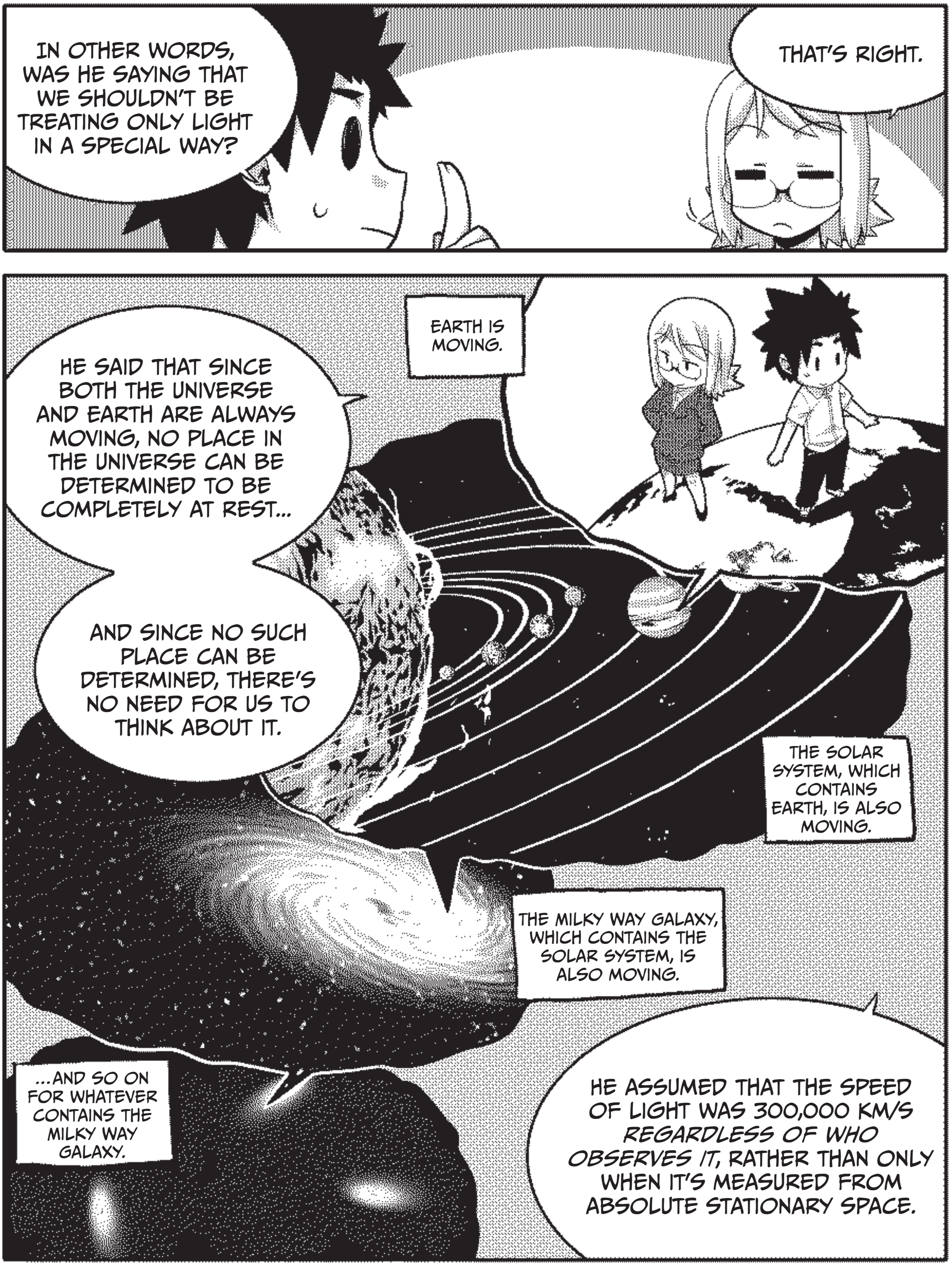

THAT'S RIGHT.
Albert Abraham Michelson
Edward Williams Morley
TWO PHYSICISTS NAMED MICHELSON AND MORLEY ACTUALLY TRIED TO MEASURE THE "ETHER WIND PUSH" BY USING A DEVICE FOR DETECTING THE PRECISE SPEED OF LIGHT.
THAT WAS A MAGNIFICENT EXPERIMENT, WASN'T IT?

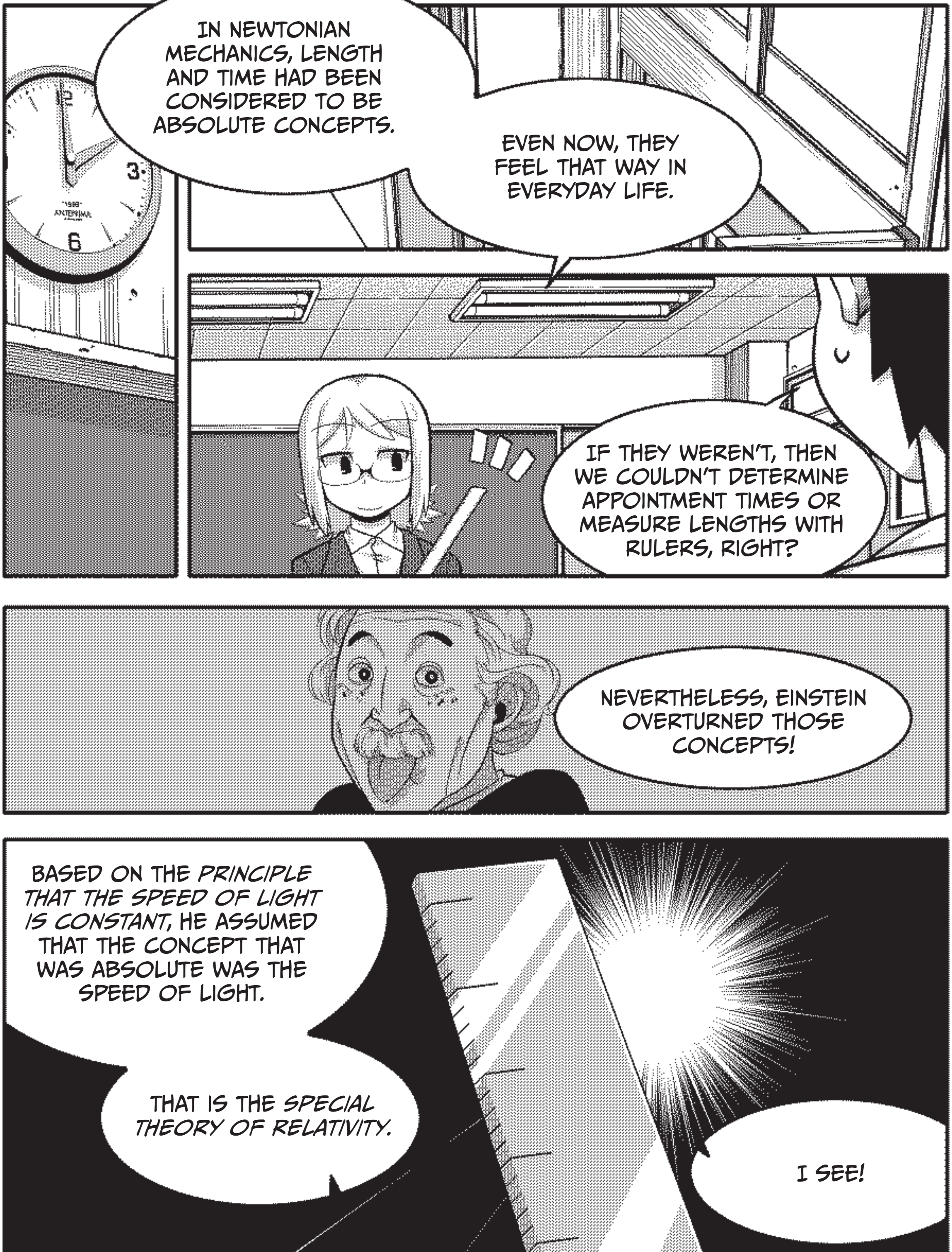


IF THIS COULD BE CONFIRMED, EARTH'S SPEED RELATIVE TO ABSOLUTE STATIONARY SPACE WOULD BE OBTAINED.
IT WAS A MAJOR EXPERIMENT THAT COULD PROVE THE EXISTENCE OF BOTH ETHER AND ABSOLUTE STATIONARY SPACE.
SO WHAT WAS THE RESULT?
SOMEHOW, THEY COULD NOT MEASURE THE "ETHER WIND PUSH"!
THE RESULT OF MICHELSON AND MORLEY'S EXPERIMENT WAS VERY CONFUSING BECAUSE IT SUGGESTED THAT EVEN THOUGH THE MOTION OF THE EARTH WAS CONSTANTLY CHANGING,
OUR OBSERVATION OF THE SPEED OF LIGHT ON EARTH REMAINED CONSTANT. THIS FINDING WAS INCONSISTENT WITH THE VERY IDEA OF AN ETHER AND SEEMED TO VIOLATE THE GALILEAN THEORY OF RELATIVITY.
HUH?!
SO WAS IT APPARENT THAT THE SPEED OF LIGHT WAS CONSTANT AND DID NOT CHANGE EVEN WHEN OBSERVED FROM A MOVING OBJECT?
THE SPEED OF LIGHT IS 300,000 KM/S WHEN OBSERVED FROM A STATE OF REST.
THE SPEED OF LIGHT IS 300,000 KM/S EVEN WHEN OBSERVED FROM A MOVING STATE.
MMMHMMM. THE CONSTANT SPEED OF LIGHT WAS A SERIOUS MATTER, AND IT COULD NOT BE EXPLAINED BY THE GALILEAN PRINCIPLE OF RELATIVITY.

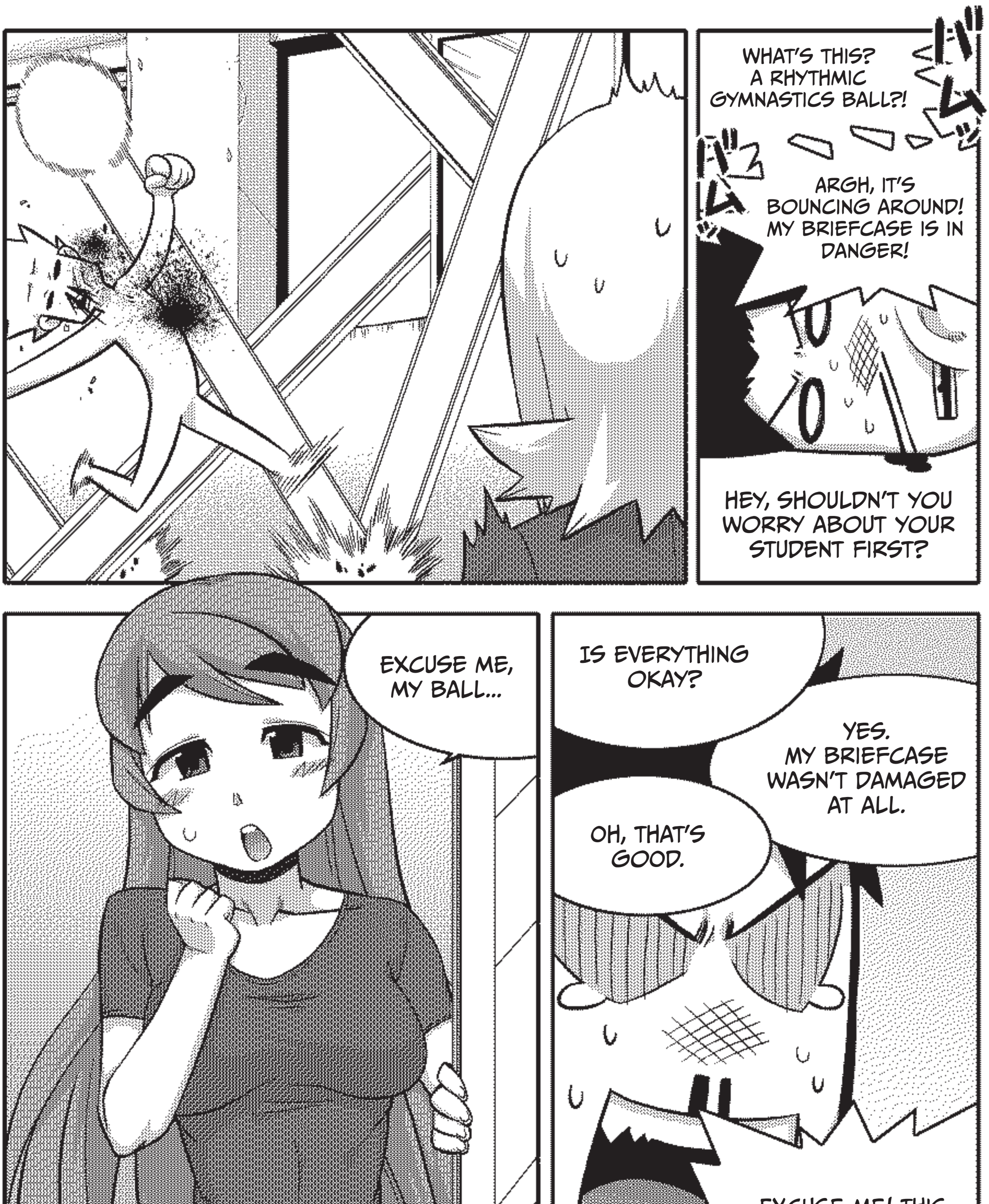
4. EINSTEIN DISCARDED NEWTONIAN MECHANICS

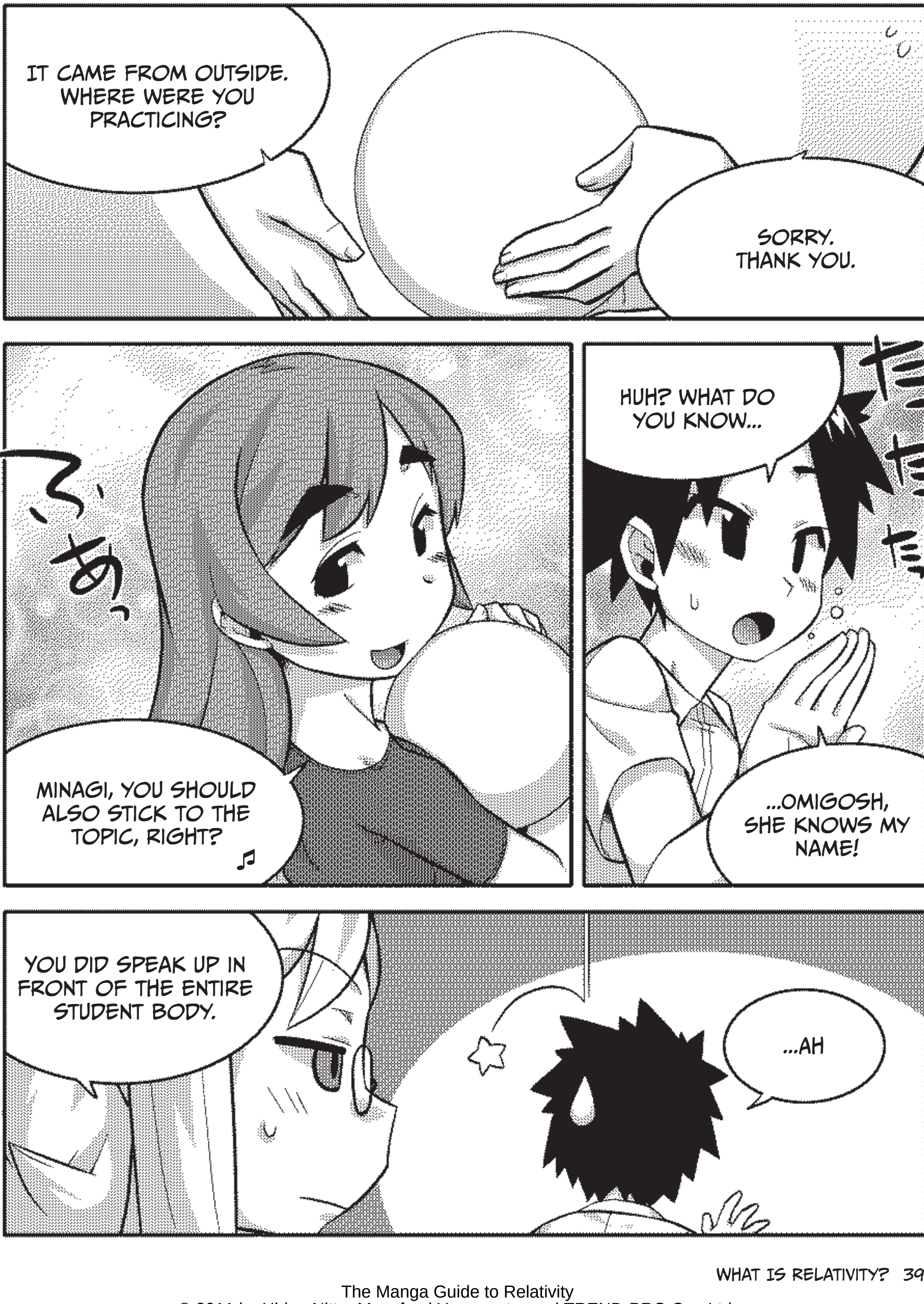



IN OTHER WORDS, WAS HE SAYING THAT WE SHOULDN'T BE TREATING ONLY LIGHT IN A SPECIAL WAY?
THAT'S RIGHT.
HE SAID THAT SINCE BOTH THE UNIVERSE AND EARTH ARE ALWAYS MOVING, NO PLACE IN THE UNIVERSE CAN BE DETERMINED TO BE COMPLETELY AT REST...
EARTH IS MOVING.
AND SINCE NO SUCH PLACE CAN BE DETERMINED, THERE'S NO NEED FOR US TO THINK ABOUT IT.
THE SOLAR SYSTEM, WHICH CONTAINS EARTH, IS ALSO MOVING.
THE MILKY WAY GALAXY, WHICH CONTAINS THE SOLAR SYSTEM, IS ALSO MOVING.
...AND SO ON FOR WHATEVER CONTAINS THE MILKY WAY GALAXY.
HE ASSUMED THAT THE SPEED OF LIGHT WAS 300,000 KM/S *REGARDLESS OF WHO OBSERVES IT*, RATHER THAN ONLY WHEN IT'S MEASURED FROM ABSOLUTE STATIONARY SPACE.


IN NEWTONIAN MECHANICS, LENGTH AND TIME HAD BEEN CONSIDERED TO BE ABSOLUTE CONCEPTS.
EVEN NOW, THEY FEEL THAT WAY IN EVERYDAY LIFE.
IF THEY WEREN'T, THEN WE COULDN'T DETERMINE APPOINTMENT TIMES OR MEASURE LENGTHS WITH RULERS, RIGHT?
NEVERTHELESS, EINSTEIN OVERTURNED THOSE CONCEPTS!
BASED ON THE *PRINCIPLE THAT THE SPEED OF LIGHT IS CONSTANT*, HE ASSUMED THAT THE CONCEPT THAT WAS ABSOLUTE WAS THE SPEED OF LIGHT.
THAT IS THE *SPECIAL THEORY OF RELATIVITY*.
I SEE!


SPEED IS DISTANCE TRAVELED ÷ TIME, RIGHT?
THEREFORE, SINCE THE SPEED OF LIGHT IS CONSTANT IN ANY REFERENCE FRAME, DISTANCE AND TIME VARY DEPENDING ON THE MOTION OF THE OBSERVER. THIS IS A MAJOR PREMISE OF SPECIAL RELATIVITY!
EVEN THOUGH YOU SAY THIS, IT FEELS STRANGE, BUT THIS REALLY HAPPENS, RIGHT?
"TIME" AND "SPACE," WHICH HAD BEEN THOUGHT TO BE SEPARATE THINGS IN NEWTONIAN MECHANICS...
...WERE NOW CONSIDERED TOGETHER IN THE FORM OF A NEW, AMAZING COORDINATE SYSTEM CALLED SPACE-TIME.
WHEN YOU SAY THAT, IT SOUNDS KIND OF AMAZING.
IT REALLY IS AMAZING! NOW I'LL TELL YOU WHAT SPECIAL RELATIVITY IS ALL ABOUT.
OKAY! PLEASE DO!
I'M SUDDENLY OVERFLOWING WITH AMBITION!
HE'S GETTING A LITTLE TOO EXCITED...


WHAT'S THIS? A RHYTHMIC GYMNASTICS BALL?!
ARGH, IT'S BOUNCING AROUND! MY BRIEFCASE IS IN DANGER!
HEY, SHOULDN'T YOU WORRY ABOUT YOUR STUDENT FIRST?
EXCUSE ME, MY BALL...
IS EVERYTHING OKAY?
YES. MY BRIEFCASE WASN'T DAMAGED AT ALL.
OH, THAT'S GOOD.
EXCUSE ME! THIS WASN'T SOME VICTIMLESS CRIME!


IT CAME FROM OUTSIDE. WHERE WERE YOU PRACTICING?
SORRY. THANK YOU.
MINAGI, YOU SHOULD ALSO STICK TO THE TOPIC, RIGHT? ♪
HUH? WHAT DO YOU KNOW...
...OMIGOSH, SHE KNOWS MY NAME!
YOU DID SPEAK UP IN FRONT OF THE ENTIRE STUDENT BODY.
...AH

What Is Light?

Maxwell's equations tell us that light is an electromagnetic wave. The color of light is determined by the wavelength of the electromagnetic wave. Red light has a wavelength of 630 nm, and blue light has a shorter wavelength of approximately 400 nm, where one nanometer (1 nm) = one billionth of a meter (10^{-9} m). Electromagnetic radiation at different wavelengths takes many forms, such as radio waves, X-rays, and gamma (γ) rays (see Figure 1-1).

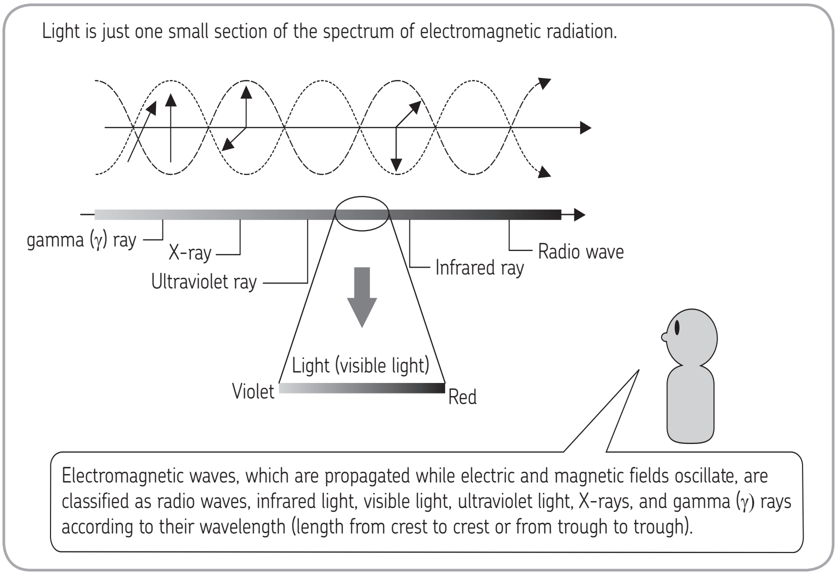


Figure 1-1: Light is an electromagnetic wave.

Although light may seem common enough—it is all around us, after all—it is fundamental to both relativity and quantum theory, the cornerstones of modern physics.

But before we delve into light's true nature, let's introduce the properties of light that have been known for a long time.

First, you know that light is *reflected* by a mirror or the surface of water. You also know about the *refraction* of light—you only need to look at your feet the next time you take a bath or see how your straw "bends" when you put it in a glass of water. Any change in medium changes a wave's direction, due to a change in the wave's speed through that medium.

Some mediums refract light of different wavelengths different amounts. In other words, light of different colors is bent to different degrees, a property known as *dispersion*. This causes white light, which consists of light of all colors, to be spread out into a spectrum of light from red to violet. We can see the seven colors of a rainbow because of dispersion.

These properties of reflection, refraction, and dispersion have been used to create precision camera lenses and telescopes. Figure 1-2 shows what happens to light when it is reflected, refracted, or dispersed.

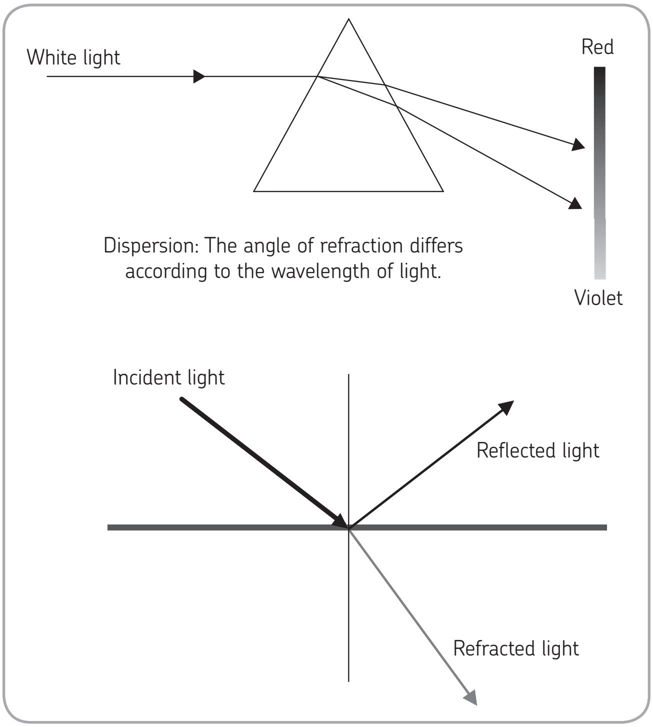


Figure 1-2: Dispersion, reflection, and refraction

Next, more subtle phenomena called *interference* and *diffraction* can be observed. These phenomena stem from the fact that light is a wave. Interference describes what happens when two light waves come together. When the two waves come together, the result is either *constructive interference*, where the waves' amplitudes are added together, or *destructive interference*, where one wave's amplitude is subtracted from the other's. Figure 1-3 shows the different kinds of interference.

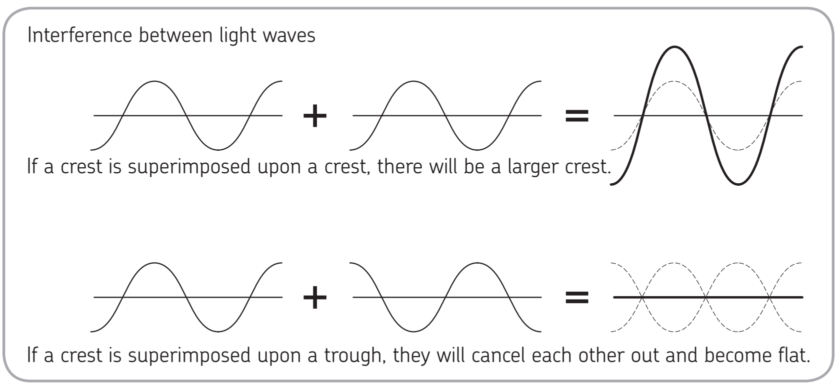


Figure 1-3: Interference can make waves stronger or weaker.

Diffraction can be observed when light passes through a tiny hole about the same size as the wavelength of the light. Due to the constructive and destructive interference of different parts of the light wave with itself, passing through a tiny aperture can cause the light to spread out or bend, as shown in Figure 1-4. Diffraction is often what limits the resolution of microscopes.

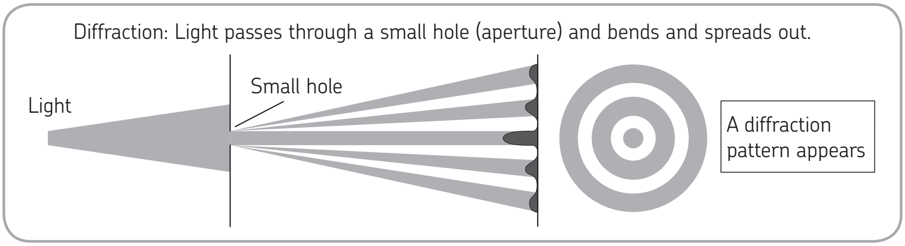


Figure 1-4: Diffraction comes about from interference.

Another property of light is called *polarization*, a property that describes the orientation of the transverse electric and magnetic components of the electromagnetic wave. This property is very useful; it allows special filters to be made (called *polarizing filters*) that allow only light with a specific polarization to pass (see Figure 1-5).

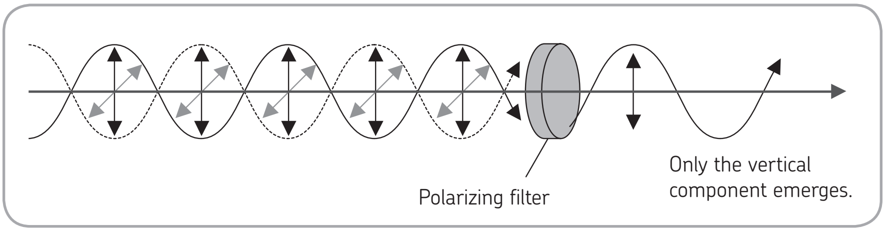


Figure 1-5: Polarization

In *scattering*, light collides with dust and other particles in the air, thereby changing direction (see Figure 1-6). Since blue light (with shorter wavelengths) is scattered by water molecules in the air more than red light (with longer wavelengths), the sky appears blue.

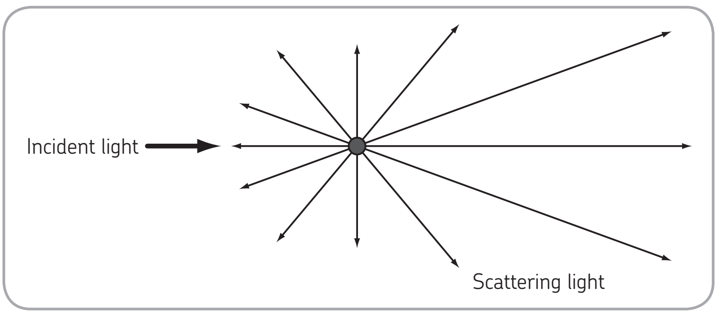


Figure 1-6: Scattering

Light Is Constant (and They Prove It Every Day in a Lab Called SPring-8)

Various tests have been conducted to verify that the speed of light is truly constant. This is important because it is one of the fundamental premises of relativity.

One way that we can test this property is to measure the speed of light coming from an object that is moving very fast. If the speed of light is not constant, the Newtonian notion of “adding” relative velocities predicts that light coming from an object moving towards the observer will be the speed of light plus the speed of the moving object; for example, if the object is moving near the speed of light, then the light from the object should be moving nearly twice the speed of light. If the speed of light is constant, on the other hand, than the light coming from the fast-moving object will just be the speed of light. Measurements confirm that the speed of light is always the same, regardless of the speed of the object from which it comes (see Figure 1-7).

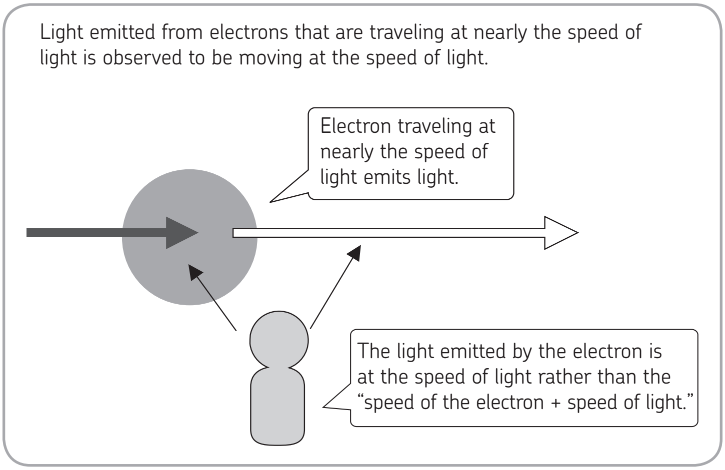


Figure 1-7: Verification that the speed of light is constant at SPring-8

Moving objects near the speed of light for these experiments is extremely difficult, and these experiments are performed at very specialized scientific facilities. SPring-8 is a synchotron radiation facility in Japan’s Hyogo Prefecture that performs experiments by smashing together electrons traveling at extremely fast speeds (99.9999998 percent of the speed of light). Besides verifying that the speed of light is constant, these experiments help scientists uncover the basic building blocks of matter.

What's Simultaneous Depends on Whom You Ask! (Simultaneity Mismatch)

If we consider the principle that "the speed of light is constant," various phenomena appear strange. One of these is the phenomenon called the *simultaneity mismatch*, which means that what is simultaneous for me is not the same as what is simultaneous for you.

I can imagine that you are thinking, "What in the world are you saying?" So let's consider the concept of "simultaneous" again. We will compare the case of Newtonian velocity addition (nonrelativistic addition of velocity) with the case in which the speed of light is constant (relativistic addition of velocity).

Consider Mr. A, who is riding on a rocket flying at a constant velocity, and Mr. B, who is observing Mr. A from a stationary space station. Assume that Mr. A is in the middle of the rocket. Sensors have been placed at the front and back of the rocket. Mr. A throws balls (or emits light) toward the front and back of the rocket. We will observe how those balls (or light beams) hit the sensors at the front and back of the rocket.

Case of Newtonian Velocity Addition (Nonrelativistic Addition)

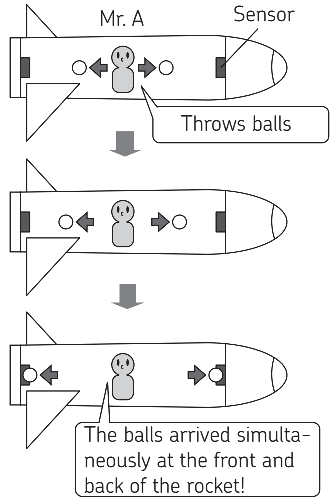
First, we will use the motion of the balls to consider the case in which velocities are added in a Newtonian mechanical manner (before considering relativity).

First, let's look at Mr. A as shown in Figure 1-8. Since from Mr. A's perspective the rocket is not moving, the balls, which are moving at the same velocity from the center toward the sensors at the front and back of the rocket, arrive at the sensors "simultaneously."

Next, when observed by Mr. B from the space station, the rocket advances in the direction of travel. In other words, using the point of departure of the balls (dotted line) as a reference, the front of the ship moves away from the dotted line, and the back of the ship approaches the dotted line. However, since the velocity of the rocket is added to the velocity of the ball in the forward direction, according to normal addition, the ball's velocity increases and it catches up with the front of the ship. On the other hand, the velocity of the ball toward the back of the ship is reduced by the velocity of the rocket (indicated by the short arrow in the figure), and the back of the ship catches up to the ball. Therefore, Mr. B also observes that the balls arrive at the front and back of the ship "simultaneously."

Nonrelativistic addition:

Mr. A observes the motion of the balls inside the rocket.



Nonrelativistic addition:

Mr. B observes the motion of the balls inside the rocket from his space station. Since the balls are moving together with the rocket, the velocity of the ball is increased by the velocity of the rocket toward the front of the rocket and decreased by the velocity of the rocket toward the back of the rocket. Therefore, the balls arrive "simultaneously" (the lengths of the arrows indicate the difference in the velocities of the balls).

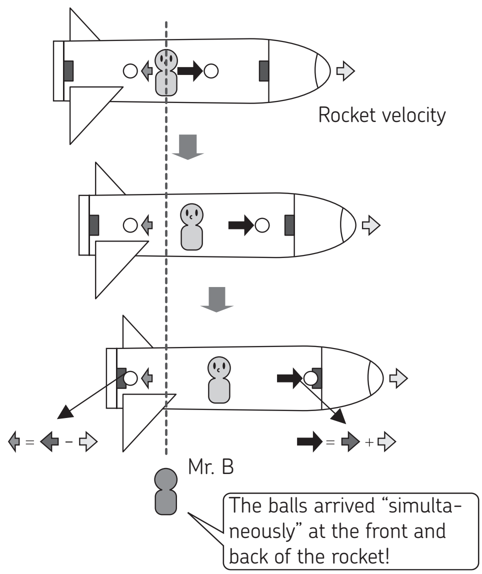


Figure 1-8: Newtonian velocity addition

Case in Which the Speed of Light Is Constant (Relativistic Addition of Velocity)

Now let's consider the case in which the speed of light is constant. Instead of throwing balls, Mr. A will emit light while traveling at nearly the speed of light (see Figure 1-9).

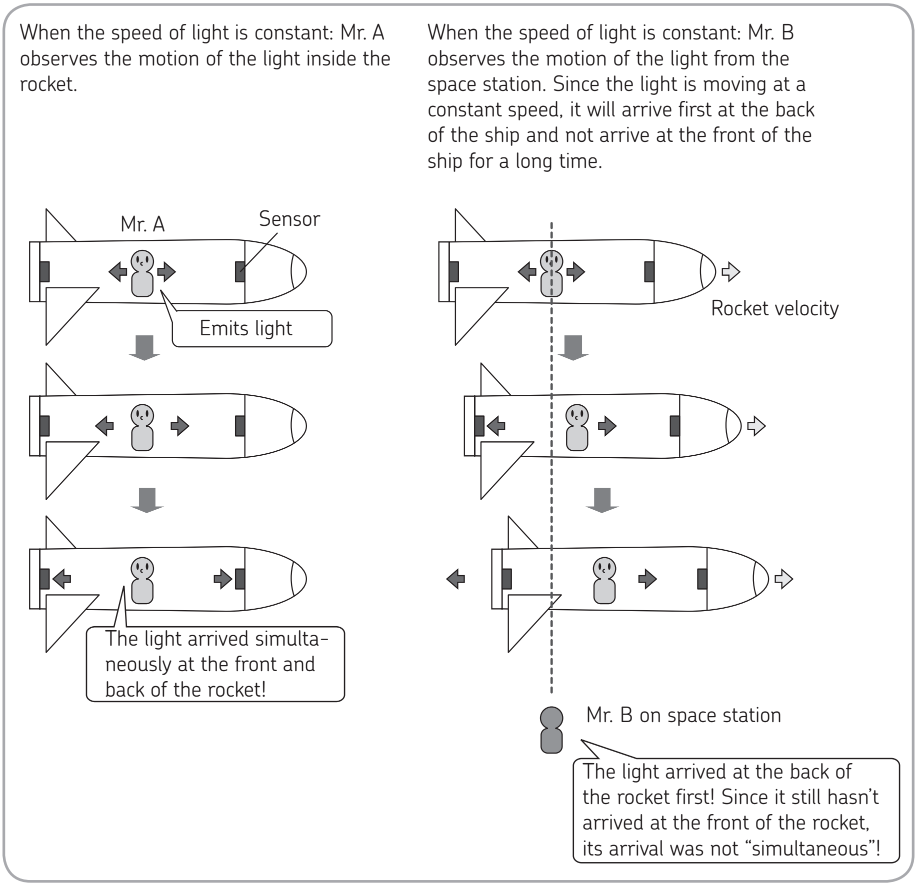


Figure 1-9: Case in which the speed of light is constant (relativistic addition of velocity)

You may have already realized what is at issue: Mr. B's observation will differ from that of Mr. A.

For Mr. A, even when the speed of light is constant, the light will arrive "simultaneously" at the front and back of the rocket.

However, when observed by Mr. B, the light moving towards the front of the ship does not arrive for a long time. It has to overtake the ship, which is moving away at nearly the speed of the light. Therefore, the light arrives at the back of the ship before it reaches the front of the ship.

That's right; when observed by Mr. B, the light does not arrive "simultaneously" at the front and back of the ship.

The simultaneity property of light differs in this way depending on the standpoint of the observer. This is called *simultaneity mismatch*.

GALILEAN PRINCIPLE OF RELATIVITY AND GALILEAN TRANSFORMATION

The Galilean principle of relativity says that "the laws of physics are the same regardless of whether the coordinate system from which the observation is made is at rest or moving at a constant velocity." In other words, Newtonian mechanics (the physical laws that govern motion) are always the same, regardless of whether observations are made in a reference frame that is at rest or one that is moving at a constant velocity. This principle was derived from an experiment in which an iron ball was dropped from the mast of a ship, as shown in Figure 1-10. The iron ball fell directly under the mast whether the ship was moving or at rest.

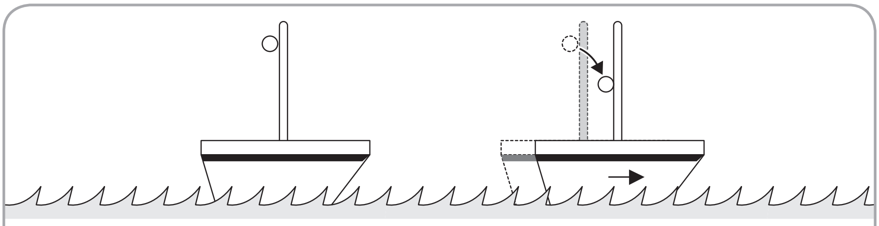


Figure 1-10: Galilean principle of relativity

Since the laws of physics are the same in any reference frame, Galileo arrived at a straightforward way to describe how observations look different depending on which reference frame you are in. Today we use algebraic equations called the *Galilean transformation* to help understand the notion of "adding" relative velocities.

Let's take two coordinate systems, one with the coordinates (x, t) and the other with coordinates (x', t'), where x and x' describe position and t and t' describe time. One can go from one coordinate system to the other, by considering the relative velocity between the two coordinate systems v.

$$x' = x - vt$$
$$t' = t$$

The above equations show the relationship between coordinates from a coordinate system at rest and a coordinate system moving at a constant velocity v relative to the coordinate system at rest. Inertial frames are mutually linked in this way by the Galilean transformation. If we compare them using Newton's equation of motion, we can prove that Newton's equation of motion takes the same form in each inertial frame. In other words, when the Galilean principle of relativity holds, Newtonian mechanics will hold.

DIFFERENCES BETWEEN THE GALILEAN PRINCIPLE OF RELATIVITY AND EINSTEIN'S SPECIAL PRINCIPLE OF RELATIVITY

As just described, the Galilean principle of relativity indicates that Newtonian mechanics apply across inertial frames when linked with the Galilean transformation.

On the other hand, the assumption that the speed of light is constant in any reference frame forced scientists to reformulate the Galilean transformation to be consistent with relativity. This new transformation is called the *Lorentz transformation*.

The Lorentz transformation is shown by the equations below, which show the relationship between coordinates from a coordinate system at rest and a coordinate system moving at a constant velocity v relative to the coordinate system at rest. The variables with the prime symbol (′) attached represent coordinates observed from the coordinate system at rest; the variables without the prime symbol represent coordinates observed from the system in motion. Note that the speed of light c appears in the equations here. Another point to notice is that time t is transformed in a manner similar to that of length; time does not exist independently but must be considered to be unified with space.

$$x' = \frac{x - vt}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$$

$$t' = \frac{t - \frac{v}{c^2}x}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$$

WAIT A SECOND—WHAT HAPPENS WITH THE ADDITION OF VELOCITIES?

When we assume that the speed of light is constant, what happens when velocities are added to the mix?

According to the principle of relativity, when calculated based on the Lorentz transformation, the addition of velocities is indicated by the following equation.

$$w = \frac{u + v}{1 + \frac{vu}{c^2}}$$

This equation describes the resulting addition of velocities of a missile w when the velocity of a rocket is v and the velocity (observed from the rocket) of the missile shot from the rocket is u, as shown in Figure 1-11. The difference is apparent when this equation is compared with the normal addition (nonrelativistic) equation $w = u + v$.

If we enter specific velocities in the above equations, we'll obtain some interesting results.

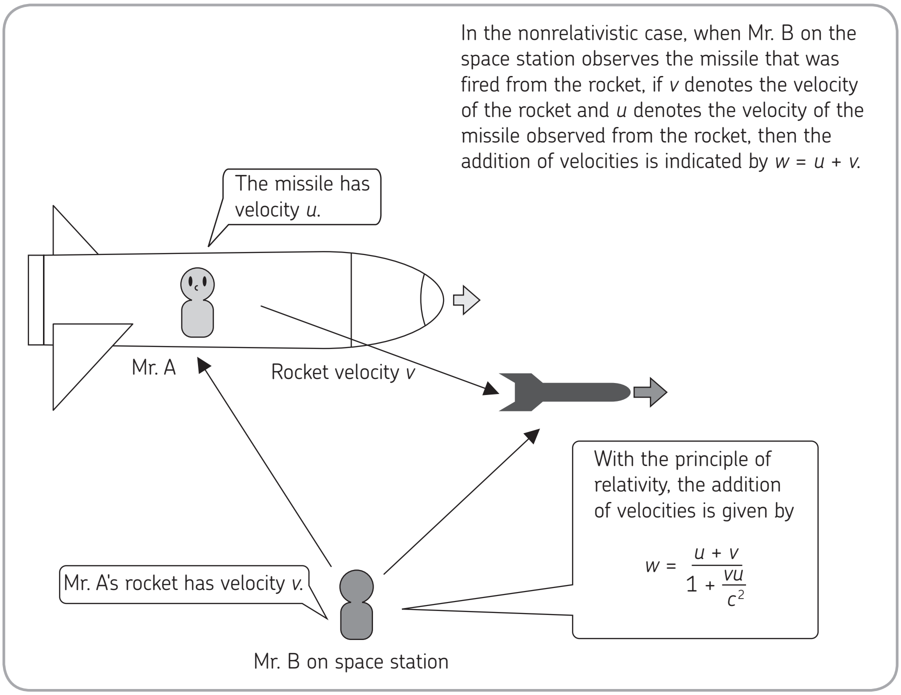


Figure 1-11: Addition of velocities

For example, when the rocket velocity v is 50 percent of the speed of light ($0.5c$) and the missile velocity u observed from the rocket is 50 percent of the speed of light (also $0.5c$), then the missile velocity w observed by Mr. B will be 80 percent of the speed of light ($0.8c$).

$$w = \frac{(0.5c + 0.5c)}{\left(1 + \frac{(0.5c)^2}{c^2}\right)} = \frac{c}{1.25} = 0.8c$$

This equation also yields an interesting result when v and u are their maximum values. If the rocket velocity v is 100 percent of the speed of light (practically speaking, $v = c$ is impossible for an object with mass, like a rocket) and the missile velocity u observed from the rocket is 100 percent of the speed of light, then the missile velocity w observed by Mr. B will be the speed of light.

$$w = \frac{(c + c)}{\left(1 + \frac{c^2}{c^2}\right)} = \frac{2c}{2} = c$$

The speed of light cannot be exceeded under any circumstances!