Chapter 47
The NPAR1WAY Procedure

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Chapter 47
The NPAR1WAY Procedure

Overview

The NPAR1WAY procedure performs nonparametric tests for location and scale differences across a one-way classification. PROC NPAR1WAY also provides a standard analysis of variance on the raw data and statistics based on the empirical distribution function.

PROC NPAR1WAY performs tests for location and scale differences based on the following scores of a response variable: Wilcoxon, median, Van der Waerden, Savage, Siegel-Tukey, Ansari-Bradley, Klotz, and Mood Scores. Additionally, PROC NPAR1WAY provides tests using the raw input data as scores. When the data are classified into two samples, tests are based on simple linear rank statistics. When the data are classified into more than two samples, tests are based on one-way ANOVA statistics. Both asymptotic and exact p-values are available for these tests.

PROC NPAR1WAY also calculates the following empirical distribution function (EDF) statistics: the Kolmogorov-Smirnov statistic, the Cramer-von Mises statistic, and, when the data are classified into only two samples, the Kuiper statistic. These statistics test whether the distribution of a variable is the same across different groups. Only asymptotic p-values are available for these tests.

Getting Started

This example illustrates how you can use PROC NPAR1WAY to perform a one-way nonparametric analysis. The data[*] consist of weight gain measurements for five different levels of gossypol additive. Gossypol is a substance contained in cottonseed shells, and these data were collected to study the effect of gossypol on animal nutrition.

The following DATA step statements create the SAS data set Gossypol:

```
data Gossypol;
   input Dose n;
   do i=1 to n;
      input Gain @@;
      output;
   end;
   datalines;
 0 16
   228 229 218 216 224 208 235 229 233 219 224 220 232 200 208 232
```

[*]reported by Halverson and Sherwood (1932)

```
.04 11
   186 229 220 208 228 198 222 273 216 198 213
.07 12
   179 193 183 180 143 204 114 188 178 134 208 196
.10 17
   130 87 135 116 118 165 151 59 126 64 78 94 150 160 122 110 178
.13 11
   154 130 130 118 118 104 112 134 98 100 104
;
```

The data set Gossypol contains the variable Dose, which represents the amount of gossypol additive, and the variable Gain, which represents the weight gain.

Researchers are interested in whether there is a difference in weight gain among the different dose levels of gossypol. The following statements invoke the NPAR1WAY procedure to perform a nonparametric analysis of this problem.

```
proc npar1way data=Gossypol;
   class Dose;
   var Gain;
run;
```

The variable Dose is the CLASS variable, and the VAR statement specifies the variable Gain is the response variable. The CLASS statement is required, and you must name only one CLASS variable. You may name one or more analysis variables in the VAR statement. If you omit the VAR statement, PROC NPAR1WAY analyzes all numeric variables in the data set except for the CLASS variable, the FREQ variable, and the BY variables.

Since no analysis options are specified in the PROC NPAR1WAY statement, the WILCOXON, MEDIAN, VW, SAVAGE, and EDF options are invoked by default. The results of these analyses are shown in the following tables.

```
                    The NPAR1WAY Procedure

              Analysis of Variance for Variable Gain
                    Classified by Variable Dose

              Dose             N              Mean
              ------------------------------------
                 0            16           222.187500
              0.04           11           217.363636
              0.07           12           175.000000
               0.1           17           120.176471
              0.13           11           118.363636


       Source    DF    Sum of Squares    Mean Square    F Value    Pr > F
       --------------------------------------------------------------------
       Among      4     140082.986077     35020.74652    55.8143    <.0001
       Within    62      38901.998997       627.45160

                 Average scores were used for ties.
```

These tables are produced with the ANOVA option. For each level of the CLASS variable Dose, PROC NPAR1WAY displays the number of observations and the mean of the analysis variable Gain. PROC NPAR1WAY displays a standard analysis of variance on the raw data. This gives the same results as the GLM and ANOVA procedures. The p-value for the F test is <.0001, which indicates that Dose accounts for a significant portion of the variability in the dependent variable Gain.

```
                       The NPAR1WAY Procedure

              Wilcoxon Scores (Rank Sums) for Variable Gain
                        Classified by Variable Dose

                       Sum of      Expected       Std Dev         Mean
      Dose      N      Scores      Under H0       Under H0        Score
      ---------------------------------------------------------------------
         0     16      890.50        544.0       67.978966     55.656250
      0.04     11      555.00        374.0       59.063588     50.454545
      0.07     12      395.50        408.0       61.136622     32.958333
       0.1     17      275.50        578.0       69.380741     16.205882
      0.13     11      161.50        374.0       59.063588     14.681818

                   Average scores were used for ties.


                        Kruskal-Wallis Test

                  Chi-Square          52.6656
                  DF                        4
                  Pr > Chi-Square     <.0001
```

The WILCOXON option produces these tables. PROC NPAR1WAY first provides a summary of the Wilcoxon scores for the analysis variable Gain by class level. For each level of the CLASS variable Dose, PROC NPAR1WAY displays the following information: number of observations, sum of the Wilcoxon scores, expected sum under the null hypothesis of no difference among class levels, standard deviation under the null hypothesis, and mean score.

Next PROC NPAR1WAY displays the one-way ANOVA statistic, which for Wilcoxon scores is known as the Kruskal-Wallis test. The statistic equals 52.6656, with four degrees of freedom, which is the number of class levels minus one. The p-value, or probability of a larger statistic under the null hypothesis, is <.0001. This leads to rejection of the null hypothesis that there is no difference in location for Gain among the levels of Dose. This p-value is asymptotic, computed from the asymptotic chi-square distribution of the test statistic. For certain data sets it may also be useful to compute the exact p-value; for example, for small data sets, or data sets that are sparse, skewed, or heavily tied. You can use the EXACT statement to request exact p-values for any of the location or scale tests available in PROC NPAR1WAY.

```
                        The NPAR1WAY Procedure


            Median Scores (Number of Points Above Median) for Variable Gain
                           Classified by Variable Dose

                         Sum of       Expected       Std Dev          Mean
        Dose       N     Scores       Under H0       Under H0        Score
        ----------------------------------------------------------------------
           0      16       16.0       7.880597       1.757902         1.00
        0.04      11       11.0       5.417910       1.527355         1.00
        0.07      12        6.0       5.910448       1.580963         0.50
         0.1      17        0.0       8.373134       1.794152         0.00
        0.13      11        0.0       5.417910       1.527355         0.00


                         Average scores were used for ties.



                            Median One-Way Analysis

                            Chi-Square         54.1765
                            DF                       4
                            Pr > Chi-Square     <.0001
```

```
                        The NPAR1WAY Procedure


               Van der Waerden Scores (Normal) for Variable Gain
                           Classified by Variable Dose

                         Sum of       Expected       Std Dev          Mean
        Dose       N     Scores       Under H0       Under H0        Score
        ----------------------------------------------------------------------
           0      16   16.116474          0.0       3.325957      1.007280
        0.04      11    8.340899          0.0       2.889761      0.758264
        0.07      12   -0.576674          0.0       2.991186     -0.048056
         0.1      17  -14.688921          0.0       3.394540     -0.864054
        0.13      11   -9.191777          0.0       2.889761     -0.835616


                         Average scores were used for ties.



                        Van der Waerden One-Way Analysis

                            Chi-Square         47.2972
                            DF                       4
                            Pr > Chi-Square     <.0001
```

```
                       The NPAR1WAY Procedure

              Savage Scores (Exponential) for Variable Gain
                      Classified by Variable Dose

                       Sum of      Expected      Std Dev        Mean
      Dose      N       Scores      Under H0      Under H0      Score
      ----------------------------------------------------------------
         0     16     16.074391         0.0      3.385275     1.004649
      0.04     11      7.693099         0.0      2.941300     0.699373
      0.07     12     -3.584958         0.0      3.044534    -0.298746
       0.1     17    -11.979488         0.0      3.455082    -0.704676
      0.13     11     -8.203044         0.0      2.941300    -0.745731

                    Average scores were used for ties.


                       Savage One-Way Analysis

                    Chi-Square          39.4908
                    DF                        4
                    Pr > Chi-Square      <.0001
```

These tables display the analyses produced by the MEDIAN, VW, and SAVAGE
options. For each score type, PROC NPAR1WAY provides a summary of scores and
the one-way ANOVA statistic, as previously described for Wilcoxon scores. Other
score types available in PROC NPAR1WAY are Siegel-Tukey, Ansari-Bradley, Klotz,
and Mood, which are used to test for scale differences. Additionally, you can request
the SCORES=DATA option, which uses the raw data as scores. This option gives
you the flexibility to construct any scores for your data with the DATA step and then
analyze these scores with PROC NPAR1WAY.

```
                    The NPAR1WAY Procedure

           Kolmogorov-Smirnov Test for Variable Gain
                   Classified by Variable Dose

                            EDF at      Deviation from Mean
         Dose       N       Maximum        at Maximum
         -------------------------------------------------
          0        16      0.000000         -1.910448
         0.04      11      0.000000         -1.584060
         0.07      12      0.333333         -0.499796
          0.1      17      1.000000          2.153861
         0.13      11      1.000000          1.732565
         Total     67      0.477612


           Maximum Deviation Occurred at Observation 36
                Value of Gain at Maximum = 178.0

           Kolmogorov-Smirnov Statistics (Asymptotic)
                 KS   0.457928     KSa   3.748300


           Cramer-von Mises Test for Variable Gain
                   Classified by Variable Dose

                                     Summed Deviation
         Dose               N           from Mean
         ------------------------------------------
          0                16           2.165210
         0.04              11           0.918280
         0.07              12           0.348227
          0.1              17           1.497542
         0.13              11           1.335745


           Cramer-von Mises Statistics (Asymptotic)
                 CM   0.093508     CMa   6.265003
```

These tables display the empirical distribution function statistics, comparing the distribution of Gain for the different levels of Dose. These tables are produced by the EDF option, and they include Kolmogorov-Smirnov statistics and Cramer-von Mises statistics.

In the preceding example, the CLASS variable Dose has five levels, and the analyses examines possible differences among these five levels, or samples. The following statements invoke the NPAR1WAY procedure to perform a nonparametric analysis of the two lowest levels of Dose.

```
proc npar1way data=Gossypol;
   where Dose <= .04;
   class Dose;
   var Gain;
run;
```

The following tables show the results.

```
                        The NPAR1WAY Procedure

                Analysis of Variance for Variable Gain
                       Classified by Variable Dose

                 Dose              N               Mean
                 ------------------------------------------
                    0              16          222.187500
                 0.04             11          217.363636


     Source    DF    Sum of Squares    Mean Square    F Value    Pr > F
     -----------------------------------------------------------------
     Among      1        151.683712      151.683712     0.5587    0.4617
     Within    25       6786.982955      271.479318

               Average scores were used for ties.
```

```
                        The NPAR1WAY Procedure

              Wilcoxon Scores (Rank Sums) for Variable Gain
                       Classified by Variable Dose

                        Sum of      Expected      Std Dev         Mean
     Dose       N       Scores      Under H0      Under H0       Score
     -----------------------------------------------------------------
        0      16       253.50        224.0      20.221565    15.843750
     0.04      11       124.50        154.0      20.221565    11.318182

                 Average scores were used for ties.


                       Wilcoxon Two-Sample Test

                  Statistic              124.5000

                  Normal Approximation
                  Z                       -1.4341
                  One-Sided Pr <  Z        0.0758
                  Two-Sided Pr > |Z|       0.1515

                  t Approximation
                  One-Sided Pr <  Z        0.0817
                  Two-Sided Pr > |Z|       0.1635

              Z includes a continuity correction of 0.5.


                        Kruskal-Wallis Test

                  Chi-Square               2.1282
                  DF                            1
                  Pr > Chi-Square          0.1446
```

These tables are produced by the WILCOXON option. PROC NPAR1WAY provides a summary of the Wilcoxon scores for the analysis variable Gain for each of the two class levels. Since there are only two levels, PROC NPAR1WAY displays the two-sample test, based on the simple linear rank statistic with Wilcoxon scores. The normal approximation includes a continuity correction. To remove this, you can specify the CORRECT=NO option. PROC NPAR1WAY also gives a *t* approximation for the

Wilcoxon two-sample test. And as for the multisample analysis, PROC NPAR1WAY
computes a one-way ANOVA statistic, which for Wilcoxon scores is known as the
Kruskal-Wallis test. All these p-values show no difference in Gain for the two Dose
levels at the .05 level of significance.

```
                      The NPAR1WAY Procedure

    Median Scores (Number of Points Above Median) for Variable Gain
                   Classified by Variable Dose

                    Sum of      Expected      Std Dev         Mean
     Dose      N    Scores      Under H0      Under H0        Score
    -------------------------------------------------------------------
        0     16       9.0      7.703704      1.299995     0.562500
     0.04     11       4.0      5.296296      1.299995     0.363636

                  Average scores were used for ties.


                      Median Two-Sample Test

                 Statistic              4.0000
                 Z                     -0.9972
                 One-Sided Pr <  Z      0.1593
                 Two-Sided Pr > |Z|     0.3187


                     Median One-Way Analysis

                 Chi-Square             0.9943
                 DF                          1
                 Pr > Chi-Square        0.3187
```

```
                      The NPAR1WAY Procedure

          Van der Waerden Scores (Normal) for Variable Gain
                   Classified by Variable Dose

                    Sum of      Expected      Std Dev         Mean
     Dose      N    Scores      Under H0      Under H0        Score
    -------------------------------------------------------------------
        0     16  3.346520           0.0      2.320336     0.209157
     0.04     11 -3.346520           0.0      2.320336    -0.304229

                  Average scores were used for ties.


                  Van der Waerden Two-Sample Test

                 Statistic             -3.3465
                 Z                     -1.4423
                 One-Sided Pr <  Z      0.0746
                 Two-Sided Pr > |Z|     0.1492


                 Van der Waerden One-Way Analysis

                 Chi-Square             2.0801
                 DF                          1
                 Pr > Chi-Square        0.1492
```

```
                    The NPAR1WAY Procedure

            Savage Scores (Exponential) for Variable Gain
                    Classified by Variable Dose

                     Sum of     Expected      Std Dev        Mean
     Dose      N      Scores     Under H0      Under H0      Score
     -------------------------------------------------------------
        0     16     1.834554        0.0      2.401839     0.114660
     0.04     11    -1.834554        0.0      2.401839    -0.166778

                  Average scores were used for ties.


                       Savage Two-Sample Test

                  Statistic            -1.8346
                  Z                    -0.7638
                  One-Sided Pr <  Z     0.2225
                  Two-Sided Pr > |Z|    0.4450


                      Savage One-Way Analysis

                  Chi-Square            0.5834
                  DF                         1
                  Pr > Chi-Square       0.4450
```

These tables display the two-sample analyses produced by the MEDIAN, VW, and SAVAGE options.

```
                      The NPAR1WAY Procedure

            Kolmogorov-Smirnov Test for Variable Gain
                  Classified by Variable Dose

                           EDF at     Deviation from Mean
        Dose        N     Maximum         at Maximum
        -------------------------------------------------
          0        16     0.250000        -0.481481
         0.04      11     0.545455         0.580689
        Total      27     0.370370

           Maximum Deviation Occurred at Observation 4
                Value of Gain at Maximum = 216.0

         Kolmogorov-Smirnov Two-Sample Test (Asymptotic)
              KS   0.145172    D         0.295455
              KSa  0.754337    Pr > KSa  0.6199


           Cramer-von Mises Test for Variable Gain
                  Classified by Variable Dose

                                     Summed Deviation
          Dose             N            from Mean
          ---------------------------------------
            0             16            0.098638
           0.04           11            0.143474

         Cramer-von Mises Statistics (Asymptotic)
              CM  0.008967    CMa  0.242112


                 Kuiper Test for Variable Gain
                  Classified by Variable Dose

                                    Deviation
          Dose             N        from Mean
          -----------------------------
            0             16        0.090909
           0.04           11        0.295455

          Kuiper Two-Sample Test (Asymptotic)
        K  0.386364    Ka  0.986440    Pr > Ka  0.8383
```

These tables display the empirical distribution function statistics, comparing the distribution of Gain for the two levels of Dose. The *p*-value for the Kolmogorov-Smirnov two-sample test is 0.6199, which indicates no rejection of the null hypothesis that the Gain distributions are identical for the two levels of Dose.

Syntax

The following statements are available in PROC NPAR1WAY.

> **PROC NPAR1WAY** < *options* > **;**
>   **BY** *variables* **;**
>   **CLASS** *variable* **;**
>   **EXACT** *statistic-options* < */ computation-options* > **;**
>   **FREQ** *variable* **;**
>   **OUTPUT** < **OUT=***SAS-data-set* > < *options* > **;**
>   **VAR** *variables* **;**

Both the PROC NPAR1WAY statement and the CLASS statement are required for the NPAR1WAY procedure. The rest of this section gives detailed syntax information for the BY, CLASS, EXACT, FREQ, OUTPUT, and VAR statements in alphabetical order after the description of the PROC NPAR1WAY statement. Table 47.1 summarizes the basic function of each PROC NPAR1WAY statement.

Table 47.1. Summary of PROC NPAR1WAY Statements

Statement	Description
BY	provides separate analyses for each BY group
CLASS	identifies the classification variable
EXACT	requests exact tests
FREQ	identifies a frequency variable
OUTPUT	requests an output data set
VAR	identifies analysis variables

PROC NPAR1WAY Statement

> **PROC NPAR1WAY** < *options* > **;**

The PROC NPAR1WAY statement invokes the procedure and optionally identifies the input data set or requests particular analyses. By default, the procedure uses the most recently created SAS data set and omits missing values from the analysis. If you do not specify any analysis options, PROC NPAR1WAY performs an analysis of variance (option ANOVA), tests for location differences (options WILCOXON, MEDIAN, SAVAGE, and VW), and performs empirical distribution function tests (option EDF).

The following table lists the options available with the PROC NPAR1WAY statement. Descriptions follow in alphabetical order.

Table 47.2. PROC NPAR1WAY Statement Options

Task	Options
Specify the input data set	DATA=
Include missing CLASS values	MISSING
Suppress all displayed output	NOPRINT
Specify analyses	AB
	ANOVA
	EDF
	KLOTZ
	MEDIAN
	MOOD
	SAVAGE
	SCORES=DATA
	ST
	VW
	WILCOXON
Suppress continuity correction	CORRECT=NO

You can specify the following options in the PROC NPAR1WAY statement.

AB

requests an analysis using Ansari-Bradley scores. See the section "Ansari-Bradley Scores" on page 2529 for more information.

ANOVA

requests a standard analysis of variance on the raw data.

CORRECT=NO

suppresses the continuity correction for the Wilcoxon two-sample test and the Siegel-Tukey two-sample test. See the section "Simple Linear Rank Tests for Two-Sample Data" on page 2525 for more information.

DATA=*SAS-data-set*

names the SAS data set to be analyzed by PROC NPAR1WAY. If you omit the DATA= option, the procedure uses the most recently created SAS data set.

EDF

requests statistics based on the empirical distribution function. These include the Kolmogorov-Smirnov and Cramer-von Mises statistics and, if there are only two classification levels, the Kuiper statistic. See the section "Statistics Based on the Empirical Distribution Function" on page 2529 for more information.

KLOTZ

requests an analysis using Klotz scores. See the section "Klotz Scores" on page 2529 for more information.

MEDIAN

requests an analysis using median scores. When there are two classification levels, or two samples, this option produces the two-sample median test. When there are more than two samples, this option produces the multisample median test, which is also

known as the Brown-Mood test. See the section "Median Scores" on page 2528 for more information.

MISSING

interprets missing values of the CLASS variable as a valid class level.

MOOD

requests an analysis using Mood scores. See the section "Mood Scores" on page 2529 for more information.

NOPRINT

suppresses the display of all output. You can use the NOPRINT option when you only want to create an output data set. Note that this option temporarily disables the Output Delivery System (ODS). For more information, see Chapter 15, "Using the Output Delivery System."

SAVAGE

requests an analysis using Savage scores. See the section "Savage Scores" on page 2528 for more information.

SCORES=DATA

requests an analysis using raw input data as scores. This option gives you the flexibility to construct any scores for your data with the DATA step and then analyze these scores with PROC NPAR1WAY. See the section "Scores for Linear Rank and One-Way ANOVA Tests" on page 2527 for more information.

ST

requests an analysis using Siegel-Tukey scores. See the section "Siegel-Tukey Scores" on page 2529 for more information.

VW

requests an analysis using Van der Waerden scores. See the section "Van der Waerden Scores" on page 2528 for more information.

WILCOXON

requests an analysis using Wilcoxon scores. When there are two classification levels, or two samples, this option produces the Wilcoxon rank-sum test. For any number of classification levels, this option produces the Kruskal-Wallis test. See the section "Wilcoxon Scores" on page 2528 for more information.

BY Statement

BY *variables* **;**

You can specify a BY statement with PROC NPAR1WAY to obtain separate analyses on observations in groups defined by the BY variables. When a BY statement appears, the procedure expects the input data set to be sorted in order of the BY variables.

If your input data set is not sorted in ascending order, use one of the following alternatives.

- Sort the data using the SORT procedure with a similar BY statement.
- Specify the BY statement option NOTSORTED or DESCENDING in the BY statement for the NPAR1WAY procedure. The NOTSORTED option does not mean that the data are unsorted but rather that the data are arranged in groups (according to values of the BY variables) and that these groups are not necessarily in alphabetical or increasing numeric order.
- Create an index on the BY variables using the DATASETS procedure.

For more information on the BY statement, refer to the discussion in *SAS Language Reference: Concepts*. For more information on the DATASETS procedure, refer to the discussion in the *SAS Procedures Guide*.

CLASS Statement

      CLASS *variable* **;**

The CLASS statement, which is required, names one and only one classification variable. The variable can be character or numeric. The CLASS variable identifies groups (or samples) in the data, and PROC NPAR1WAY provides analyses to examine differences among these groups. There may be two or more groups in the data.

EXACT Statement

      EXACT *statistic-options* < */ computation-options* > **;**

The EXACT statement requests exact tests for the specified statistics. Optionally, PROC NPAR1WAY computes Monte Carlo estimates of the exact *p*-values. The *statistic-options* specify the statistics for which to provide exact tests, and the *computation-options* specify options for the computation of exact statistics.

Caution: For some large problems, computation of exact tests may require a large amount of time and memory. Consider using asymptotic tests for such problems. Alternatively, when asymptotic methods may not be sufficient for such large problems, consider using Monte Carlo estimation of exact *p*-values. See the section "Computational Resources" on page 2533 for more information.

Statistic-Options

The *statistic-options* specify the statistics for which to provide exact tests.

Exact *p*-values are available for all nonparametric tests of location and scale differences produced by PROC NPAR1WAY. These include tests based on the following scores: Wilcoxon, median, Van der Waerden, Savage, Siegel-Tukey, Ansari-Bradley, Klotz, and Mood scores. Additionally, exact *p*-values are available for tests using the

raw input data as scores. The procedure computes exact *p*-values when the data are classified into two levels (two-sample tests) and when the data are classified into more than two levels (multisample tests). Two-sample tests are based on simple linear rank statistics. Multisample tests are based on one-way ANOVA statistics. See the section "Exact Tests" on page 2531 for details.

Table 47.3 lists the available *statistic-options* and the exact tests computed. The option names are identical to the corresponding options in the PROC NPAR1WAY statement and the OUTPUT statement.

If you list no *statistic-options* in the EXACT statement, then PROC NPAR1WAY computes exact *p*-values for all the tests of location and scale differences requested in the PROC NPAR1WAY statement.

Table 47.3. EXACT Statement Statistic-Options

Option	Exact Tests Computed
AB	Ansari-Bradley Test
KLOTZ	Klotz Test
MEDIAN	Median Test
MOOD	Mood Test
SAVAGE	Savage Test
SCORES=DATA	Test Using Raw Data as Scores
ST	Siegel-Tukey Test
WILCOXON	Wilcoxon Test for Two-Sample Data
	Kruskal-Wallis Test for Multisample Data
VW	Van der Waerden Test

Computation-Options

The *computation-options* specify options for computation of exact statistics. You can specify the following *computation-options* in the EXACT statement.

ALPHA=α

specifies the confidence level for the confidence limits for the Monte Carlo *p*-value estimates. The value of the ALPHA= option must be between 0.0001 and 0.9999; the default is 0.01. A confidence level of α results in $100(1-\alpha)$% confidence limits. The default of ALPHA=0.01 results in 99 percent confidence limits. If α is between 0 and 1 but is outside the range of 0.0001 to 0.9999, PROC NPAR1WAY uses the closest range endpoint. For example, if you specify ALPHA=0.000001, PROC NPAR1WAY uses 0.0001 to determine confidence limits. The ALPHA= option invokes the MC option.

MAXTIME=up *value*

specifies the maximum clock time (in seconds) that PROC NPAR1WAY can use to compute an exact *p*-value. If the procedure does not complete the computation within the specified time, the computation terminates. The value of the MAXTIME= option must be a positive number. The MAXTIME= option is valid for Monte Carlo estimation of exact *p*-values, as well as for direct exact *p*-value computation.

See the section "Computational Resources" on page 2533 for more information.

MC

requests Monte Carlo estimation of exact *p*-values, instead of direct exact *p*-value computation. The ALPHA=, N=, or SEED= option also invokes the MC option.

The MC option is available for all the EXACT statement *statistic-options*. Monte Carlo estimation can be useful for large problems that require a great amount of time and memory for exact computations but for which asymptotic approximations may not be sufficient. See the section "Monte Carlo Estimation" on page 2534 for more information.

N=_n_

specifies the number of samples for Monte Carlo estimation. The value of the N= option must be a positive integer, and the default is 10000 samples. Larger values of *n* produce more precise estimates of exact *p*-values. Because larger values of *n* generate more samples, the computation time increases. The N= option invokes the MC option.

SEED=_number_

specifies the initial seed for random number generation for Monte Carlo estimation. The value of the SEED= option must be a positive integer. If you do not specify the SEED= option, PROC NPAR1WAY uses the time of day from the computer's clock to obtain the initial seed. The SEED= option invokes the MC option.

FREQ Statement

FREQ *variable* **;**

The FREQ statement names a variable that provides frequencies for each observation in the DATA= data set. Specifically, if *n* is the value of the FREQ variable for a given observation, then that observation is used *n* times. The analysis produced using a FREQ statement reflects the expanded number of observations.

If the value of the FREQ variable is missing or is less than one, PROC NPAR1WAY does not use the observation in the analysis. If the value is not an integer, PROC NPAR1WAY uses only the integer portion.

OUTPUT Statement

OUTPUT< **OUT=**_SAS-data-set_ > < *options* > **;**

The OUTPUT statement creates a SAS data set containing statistics computed by PROC NPAR1WAY. You specify which statistics to store in the output data set, using options identical to those used in the PROC NPAR1WAY statement. The output data set contains one observation for each analysis variable named in the VAR statement. For more information on the contents of the output data set, see the section "Output Data Set" on page 2535.

Note that you can use the Output Delivery System (ODS) to create a SAS data set from any piece of PROC NPAR1WAY output. For more information, see Chapter 15, "Using the Output Delivery System."

You can specify the following options in the OUTPUT statement.

OUT=SAS-data-set
> names the output data set. If you omit the OUT= option, the data set is named DATA*n*, where *n* is the smallest integer that makes the name unique.

options
> specifies the statistics you want in the new data set. The options are identical to those used in the PROC NPAR1WAY statement to request analyses. Table 47.4 shows the available options. When you specify one of these options in the OUTPUT statement, the output data set contains statistics from that analysis. See the section "Output Data Set" on page 2535 for a list of the output data set variables corresponding to each option.
>
> If you do not specify any statistics options in the OUTPUT statement, then the output data set includes statistics from all analyses specified in the PROC NPAR1WAY statement.

Table 47.4. Output Statement Options

Option	Output Data Set Statistics
AB	Ansari-Bradley Test
ANOVA	Standard analysis of variance
EDF	Kolmogorov-Smirnov Test
	Cramer-von Mises Test
	Kuiper Test for Two-Sample Data
KLOTZ	Klotz Test
MEDIAN	Median Test
MOOD	Mood Test
SAVAGE	Savage Test
SCORES=DATA	Test Using Raw Data as Scores
ST	Siegel-Tukey Test
WILCOXON	Wilcoxon Test for Two-Sample Data
	Kruskal-Wallis Test
VW	Van der Waerden Test

VAR Statement

> **VAR** *variables* **;**

The VAR statement names the response or dependent variables to be analyzed. These variables must be numeric. If the VAR statement is omitted, the procedure analyzes all numeric variables in the data set except for the CLASS variable, the FREQ variable, and the BY variables.

Details

Missing Values

If an observation has a missing value for a response variable, PROC NPAR1WAY excludes that observation from the analysis.

By default, PROC NPAR1WAY excludes observations with missing values of the CLASS variable. If you specify the MISSING option, PROC NPAR1WAY treats missing values of the CLASS variable as a valid class level and includes these observations in the analysis.

PROC NPAR1WAY treats missing BY variable values like any other BY variable value. The missing values form a separate BY group. When a value of the FREQ variable is missing, PROC NPAR1WAY excludes the observation from the analysis.

Tied Values

Tied values occur when two are more observations are equal, whether the observations occur in the same sample or in different samples. In theory, nonparametric tests were developed for continuous distributions where the probability of a tie is zero. In practice, however, ties often occur. PROC NPAR1WAY uses the same method to handle ties for all score types. The procedure computes the scores as if there were no ties, averages the scores for tied observations, and assigns this average score to each observation with the same value.

When there are tied values, PROC NPAR1WAY first sorts the observations in ascending order and assigns ranks as if there were no ties. Then the procedure computes the scores based on these ranks, using the formula for the specified score type. The procedure averages the scores for tied observations and assigns this average score to each of the tied observations. Thus, all equal data values have the same score value. PROC NPAR1WAY then computes the test statistic from these scores.

Note that the asymptotic tests may be less accurate when the distribution of the data is heavily tied. For such data, it may be appropriate to use the exact tests provided by PROC NPAR1WAY, as described in the "Exact Tests" section on page 2531.

When computing empirical distribution function statistics for data with ties, PROC NPAR1WAY uses the formulas given in the "Statistics Based on the Empirical Distribution Function" section on page 2529. No special handling of ties is necessary.

Statistical Computations

Simple Linear Rank Tests for Two-Sample Data

Statistics of the form

$$S \;=\; \sum_{j=1}^{n} c_j a(R_j)$$

are called *simple linear rank statistics*, where

R_j is the rank of the observation j

$a(R_j)$ is the score based on that rank

c_j is an indicator variable denoting the class to which the jth observation belongs

n is the total number of observations

For two-sample data (where the observations are classified into two levels), PROC NPAR1WAY calculates simple linear rank statistics for the scores that you specify. The "Scores for Linear Rank and One-Way ANOVA Tests" section on page 2527 describes the available scores, which you can use to test for differences in location and differences in scale.

To compute S, PROC NPAR1WAY sums the scores of the observations in the smaller of the two samples. If both samples have the same number of observations, PROC NPAR1WAY sums those scores for the sample that appears first in the input data set.

For each score that you specify, PROC NPAR1WAY computes an asymptotic test of the null hypothesis of no difference between the two classification levels. Exact tests are also available for these two-sample linear rank statistics. PROC NPAR1WAY computes exact tests for each score type that you specify in the EXACT statement. See the "Exact Tests" section on page 2531 for details on exact tests.

To compute an asymptotic test for a linear rank sum statistic, PROC NPAR1WAY uses a standardized test statistic z, which has an asymptotic standard normal distribution under the null hypothesis. The standardized test statistic is computed as

$$z \;=\; \frac{S - E_0(S)}{\sqrt{var_0(S)}}$$

where $E_0(S)$ is the expected value of S under the null hypothesis, and $Var_0(S)$ is the variance under the null hypothesis. As shown in Randles and Wolfe (1979),

$$E_0(S) \;=\; \frac{n_1}{n} \sum_{j=1}^{n} a(R_j)$$

where n_1 is the number of observations in the first (smaller) class level, and

$$Var_0(S) = \frac{1}{(n-1)} \frac{n_1 \cdot n_2}{n} \left[\sum_{j=1}^{n} (a(R_j) - \bar{a})^2 \right]$$

where

$$\bar{a} = (1/n) \sum_{j=1}^{n} a(R_j)$$

PROC NPAR1WAY computes one-sided and two-sided asymptotic p-values for each two-sample linear rank test. When the test statistic z is greater than its null hypothesis expected value of zero, PROC NPAR1WAY computes the right-sided p-value, which is the probability of a larger value of the statistic occurring under the null hypothesis. When the test statistic is less than or equal to zero, PROC NPAR1WAY computes the left-sided p-value, which is the probability of a smaller value of the statistic occurring under the null hypothesis. The one-sided p-value P_1 can be expressed as

$$P_1 = \text{Prob}\ (Z > z) \quad \text{if}\ \ z > 0$$

$$P_1 = \text{Prob}\ (Z < z) \quad \text{if}\ \ z \leq 0$$

where Z has a standard normal distribution. The two-sided p-value P_2 is computed as

$$P_2 = \text{Prob}\ (|Z| > |z|)$$

For Wilcoxon scores and Siegel-Tukey scores, PROC NPAR1WAY incorporates a continuity correction when computing the standardized test statistic z, unless you specify CORRECT=NO. PROC NPAR1WAY applies the continuity correction by subtracting 0.5 from the numerator $S - E_0(S)$ if it is greater than zero. If the numerator is less than zero, PROC NPAR1WAY adds 0.5. Some sources recommend a continuity correction for nonparametric tests that use a continuous distribution to approximate a discrete distribution. Refer to Sheskin (1997). If you specify CORRECT=NO, PROC NPAR1WAY does not use a continuity correction for any test.

One-Way ANOVA Tests

PROC NPAR1WAY computes a one-way ANOVA test for each score type that you specify. Under the null hypothesis of no difference among class levels (or samples), this test statistic has an asymptotic chi-square distribution with $r - 1$ degrees of freedom, where r is the number of class levels. For Wilcoxon scores, this test is known as the Kruskal-Wallis test.

Exact one-way ANOVA tests are also available for multisample data (where the data are classified into more than two levels). For two-sample data, exact simple linear rank tests are available. PROC NPAR1WAY computes exact tests for each score type

that you specify in the EXACT statement. See the "Exact Tests" section on page 2531 for details on exact tests.

PROC NPAR1WAY computes the one-way ANOVA test statistic as

$$C = \left(\sum_{i=1}^{r} (T_i - E_0(T_i))^2 / n_i \right) / S^2$$

where T_i is the total of scores for the class level i, $E_0(T_i)$ is the expected total for level i under the null hypothesis of no difference among levels, n_i is the number of observations in level i, and S^2 is the sample variance of the scores.

$$T_i = \sum_{j=1}^{n} c_{ij} a(R_j)$$

where $a(R_j)$ is the score for observation j, and c_{ij} indicates whether observation j is in level i.

$$E_0(T_i) = n_i \bar{a}$$

where $\bar{a}$ is the average score,

$$\bar{a} = \sum_{j=1}^{n} a(R_j) / n$$

$$S^2 = \sum_{j=1}^{n} (a(R_j) - \bar{a})^2 / (n-1)$$

Scores for Linear Rank and One-Way ANOVA Tests

For each score type that you specify, PROC NPAR1WAY computes a one-way ANOVA statistic and also a linear rank statistic for two-sample data. The following score types are used primarily to test for differences in location: Wilcoxon, median, Van der Waerden, and Savage. The following scores types are used to test for scale differences: Siegel-Tukey, Ansari-Bradley, Klotz and Mood. This section gives formulas for the score types. For further information on the formulas and the applicability of each score, refer to Randles and Wolfe (1979), Gibbons and Chakraborti (1992), Conover (1980), and Hollander and Wolfe (1973).

In addition to the score types described in this section, you can specify the SCORES=DATA option to use the input data observations as scores. This enables you to produce a very wide variety of tests. You can construct any scores using the DATA step, and then PROC NPAR1WAY computes the corresponding linear rank and one-way ANOVA tests. You can also analyze the raw data with the SCORES=DATA option; for two-sample data, this permutation test is known as Pitman's test.

Wilcoxon Scores

Wilcoxon scores are the ranks of the observations.

$$a(R_j) = R_j$$

Using Wilcoxon scores in the linear rank statistic for two-sample data produces the rank sum statistic of the Mann-Whitney-Wilcoxon test. Using Wilcoxon scores in the one-way ANOVA statistic produces the Kruskal-Wallis test. Wilcoxon scores are locally most powerful for location shifts of a logistic distribution.

When computing the asymptotic Wilcoxon two-sample test, PROC NPAR1WAY uses a continuity correction by default, as described in the "Simple Linear Rank Tests for Two-Sample Data" section on page 2525. If you specify CORRECT=NO in the PROC NPAR1WAY statement, the procedure does not use a continuity correction.

Median Scores

Median scores equal 1 for observations greater than the median, and 0 otherwise.

$$a(R_j) = \begin{cases} 1 & \text{if } R_j > \frac{n+1}{2} \\ 0 & \text{if } R_j \leq \frac{n+1}{2} \end{cases}$$

Using median scores in the linear rank statistic for two-sample data produces the two-sample median test. The one-way ANOVA statistic with median scores is equivalent to the Brown-Mood test. Median scores are particularly powerful for distributions that are symmetric and heavy-tailed.

Van der Waerden Scores

Van der Waerden scores are the quantiles of a standard normal distribution. These scores are also known as *quantile normal scores*.

$$a(R_j) = \Phi^{-1}\left(\frac{R_j}{n+1}\right)$$

where Φ is the cumulative distribution function of a standard normal distribution. These scores are powerful for normal distributions.

Savage Scores

Savage scores are expected values of order statistics from the exponential distribution, with 1 subtracted to center the scores around 0.

$$a(R_j) = \sum_{i=1}^{R_j} \frac{1}{n-i+1} - 1$$

Savage scores are powerful for comparing scale differences in exponential distributions or location shifts in extreme value distributions (Hajek 1969, p. 83).

Siegel-Tukey Scores

Siegel-Tukey scores are computed as

$$a(1) = 1, \quad a(n) = 2, \qquad a(n-1) = 3, \quad a(2) = 4,$$
$$a(3) = 5, \quad a(n-2) = 6, \quad a(n-3) = 7, \quad a(4) = 8, \dots$$

where the score values continue to increase in this pattern towards the middle ranks until all observations have been assigned a score.

Ansari-Bradley Scores

Ansari-Bradley scores are similar to Siegel-Tukey scores, but Ansari-Bradley assigns the same scores to corresponding extreme ranks. (Siegel Tukey scores are just a permutation of the ranks 1, 2, ..., n.)

$$a(1) = 1, \quad a(n) = 1,$$
$$a(2) = 2, \quad a(n-1) = 2, \dots$$

Equivalently, Ansari-Bradley scores are defined as

$$a(R_j) = \frac{n+1}{2} - |R_j - \frac{n+1}{2}|$$

Klotz Scores

Klotz scores are the squares of the Van der Waerden (or quantile normal) scores.

$$a(R_j) = \left[\Phi^{-1} \left(\frac{R_j}{n+1} \right) \right]^2$$

where Φ is the cumulative distribution function of a standard normal distribution.

Mood Scores

Mood scores are computed as the square of the difference between each rank and the average rank.

$$a(R_j) = [R_j - (n+1)/2]^2$$

Statistics Based on the Empirical Distribution Function

If you specify the EDF option, PROC NPAR1WAY computes statistics based on the empirical distribution function. These include Kolmogorov-Smirnov and Cramer-von Mises statistics, and also Kuiper statistics for two-sample data. This section gives formulas for these statistics. For further information on the formulas and the interpretation of EDF statistics, refer to Hollander and Wolfe (1973) and Gibbons and Chakraborti (1992). For details on the k-sample analogues of the Kolmogorov-Smirnov and Cramer-von Mises statistics used by NPAR1WAY, refer to Kiefer (1959).

The *empirical distribution function* (EDF) of a sample $\{x_j\}$, $j = 1, 2, \ldots, n$, is defined as the following function:

$$F(x) = \frac{1}{n}(\text{number of } x_j \leq x) = \frac{1}{n}\sum_{j=1}^{n} I(x_j \leq x)$$

where $I(\cdot)$ is the indicator function. PROC NPAR1WAY uses the subsample of values within the ith class level to generate an EDF, F_i. The EDF for the pooled sample can also be expressed as

$$F = \frac{1}{n}\sum_{i} n_i F_i$$

where n_i is the number of observations in the ith class level, and n is the total number of observations.

Kolmogorov-Smirnov Statistics

The Kolmogorov-Smirnov statistic measures the maximum deviation of the EDF within the classes from the pooled EDF. PROC NPAR1WAY computes the Kolmogorov-Smirnov statistic as

$$\text{KS} = \max_{j} \sqrt{\frac{1}{n}\sum_{i} n_i \left(F_i(x_j) - F(x_j)\right)^2} \quad \text{where } j = 1, 2, \ldots, n$$

The asymptotic Kolmogorov-Smirnov statistic is computed as

$$\text{KS}_a = \text{KS}\sqrt{n}$$

In addition to the overall Kolmogorov-Smirnov statistic and the asymptotic statistic, PROC NPAR1WAY displays the values of the F_i at the maximum deviation from F, the values $(F_i - F)\sqrt{n_i}$ at the maximum deviation from F, the value of F at the maximum deviation, and the point where this maximum deviation occurs.

If there are only two class levels, PROC NPAR1WAY computes the two-sample Kolmogorov statistic as

$$D = \max_{j} |F_1(x_j) - F_2(x_j)| \quad \text{where } j = 1, 2, \ldots, n$$

PROC NPAR1WAY also computes the asymptotic probability of observing a larger test statistic. The quality of this approximation has been studied by Hodges (1957). For tables of the exact distribution of D when the two sample sizes are equal, refer to Lehmann (1975, p. 413). For tables of the exact distribution for unequal sample sizes, refer to Kim and Jennrich (1970, pp. 79–170).

Cramer-von Mises Statistics

The Cramer-von Mises statistic is defined as

$$
\mathrm{CM} = \frac{1}{n^2} \sum_i \left(n_i \sum_{j=1}^{p} t_j \left(F_i(x_j) - F(x_j) \right)^2 \right)
$$

where t_j is the number of ties at the jth distinct value and p is the number of distinct values. CM measures the integrated deviation of the EDF within the classes to the pooled EDF. PROC NPAR1WAY displays the contribution of each class to the sum together with the sum, which is the asymptotic value formed by multiplying the Cramer-von Mises statistic by the number of observations.

Kuiper Statistics

For data with two class levels, PROC NPAR1WAY computes the Kuiper statistic, its scaled value for the asymptotic distribution, and the asymptotic p-value. The Kuiper statistic is computed as

$$
K = \max_j \left(F_1(x_j) - F_2(x_j) \right) - \min_j \left(F_1(x_j) - F_2(x_j) \right) \quad \text{where } j = 1, 2, \ldots, n
$$

The asymptotic value is

$$
K_a = K \sqrt{\frac{n_1 n_2}{n}}
$$

The p-value is the probability of observing a larger value of K_a under the null hypothesis of no difference between the two classes.

Exact Tests

PROC NPAR1WAY provides exact p-values for tests for location and scale differences based on the following scores: Wilcoxon, median, van der Waerden, Savage, Siegel-Tukey, Ansari-Bradley, Klotz, and Mood scores. Additionally, PROC NPAR1WAY provides exact p-values for tests using the raw data as scores. Exact tests are available for two-sample and multisample data. When the data are classified into two samples, tests are based on simple linear rank statistics. When the data are classified into more than two samples, tests are based on one-way ANOVA statistics.

Exact tests can be useful in situations where the asymptotic assumptions are not met and the asymptotic p-values are not close approximations for the true p-values. Standard asymptotic methods involve the assumption that the test statistic follows a particular distribution when the sample size is sufficiently large. When the sample size is not large, asymptotic results may not be valid, with the asymptotic p-values differing perhaps substantially from the exact p-values. Asymptotic results may also be unreliable when the distribution of the data is sparse, skewed, or heavily tied. Refer to Agresti (1996) and Bishop, Fienberg, and Holland (1975). Exact computations are based on the statistical theory of exact conditional inference for contingency tables, reviewed by Agresti (1992).

In addition to computation of exact p-values, PROC NPAR1WAY provides the option of estimating exact p-values by Monte Carlo simulation. This can be useful for problems that are so large that exact computations require a great amount of time and memory, but for which asymptotic approximations may not be sufficient.

The following sections summarize the exact computational algorithms, define the exact p-values that PROC NPAR1WAY computes, discuss the computational resource requirements, and describe the Monte Carlo estimation option.

Computational Algorithms

PROC NPAR1WAY computes exact p-values using the network algorithm developed by Mehta and Patel (1983). This algorithm provides a substantial advantage over direct enumeration, which can be very time consuming and feasible only for small problems. Refer to Agresti (1992) for a review of algorithms for computation of exact p-values, and refer to Mehta, Patel, and Tsiatis (1984) and Mehta, Patel, and Senchaudhuri (1991) for information on the performance of the network algorithm.

PROC NPAR1WAY constructs a contingency table from the input data, with rows formed by the levels of the classification variable and columns formed by the response variable values. The reference set for a given contingency table is the set of all contingency tables with the observed marginal row and column sums. Corresponding to this reference set, the network algorithm forms a directed acyclic network consisting of nodes in a number of stages. A path through the network corresponds to a distinct table in the reference set. The distances between nodes are defined so that the total distance of a path through the network is the corresponding value of the test statistic. At each node, the algorithm computes the shortest and longest path distances for all the paths that pass through that node. For the two-sample linear rank statistics, which can be expressed as a linear combination of cell frequencies multiplied by increasing row and column scores, PROC NPAR1WAY computes shortest and longest path distances using the algorithm given in Agresti, Mehta, and Patel (1990). For the multisample one-way test statistics, PROC NPAR1WAY computes an upper bound for the longest path and a lower bound for the shortest path, following the approach of Valz and Thompson (1994).

The longest and shortest path distances or bounds for a node are compared to the value of the test statistic to determine whether all paths through the node contribute to the p-value, none of the paths through the node contribute to the p-value, or neither of these situations occur. If all paths through the node contribute, the p-value is incremented accordingly, and these paths are eliminated from further analysis. If no paths contribute, these paths are eliminated from the analysis. Otherwise, the algorithm continues, still processing this node and the associated paths. The algorithm finishes when all nodes have been accounted for.

In applying the network algorithm, PROC NPAR1WAY uses full precision to represent all statistics, row and column scores, and other quantities involved in the computations. Although it is possible to use rounding to improve the speed and memory requirements of the algorithm, PROC NPAR1WAY does not do this since it can result in reduced accuracy of the p-values.

Definition of p-Values

For two-sample linear rank tests, PROC NPAR1WAY computes exact one-sided and two-sided p-values for each test specified in the EXACT statement. For the one-sided test, PROC NPAR1WAY displays the right-sided p-value when the observed value of the test statistic is greater than its expected value. The right-sided p-value is the sum of probabilities for those tables having a test statistic greater than or equal to the observed test statistic. Otherwise, when the test statistic is less than or equal to its expected value, PROC NPAR1WAY displays the left-sided p-value. The left-sided p-value is the sum of probabilities for those tables having a test statistic less than or equal to the one observed. The one-sided p-value P_1 can be expressed as

$$P_1 \quad = \quad \text{Prob (Test Statistic} \geq S) \qquad \text{if} \;\; S > Mean$$

$$P_1 \quad = \quad \text{Prob (Test Statistic} \leq S) \qquad \text{if} \;\; S \leq Mean$$

where S is the observed value of the test statistic and $Mean$ is the expected value of the test statistic under the null hypothesis. PROC NPAR1WAY computes the two-sided p-value as the sum of the one-sided p-value and the corresponding area in the opposite tail of the distribution of the statistic, equidistant from the expected value. The two-sided p-value P_2 can be expressed as

$$P_2 \quad = \quad \text{Prob } (\,|\,\text{Test Statistic} - Mean\,| \geq |\,S - Mean\,|\,)$$

For multisample data, the tests are based on one-way ANOVA statistics. For a test of this form, large values of the test statistic indicate a departure from the null hypothesis; the test is inherently two-sided. The exact p-value is the sum of probabilities for those tables having a test statistic greater than or equal to the value of the observed test statistic.

Computational Resources

PROC NPAR1WAY uses relatively fast and efficient algorithms for exact computations. These recently developed algorithms, together with improvements in computer power, make it feasible now to perform exact computations for data sets where previously only asymptotic methods could be applied. Nevertheless, there are still large problems that may require a prohibitive amount of time and memory for exact computations, depending on the speed and memory available on your computer. For large problems, consider whether exact methods are really needed or whether asymptotic methods might give results quite close to the exact results while requiring much less computer time and memory. When asymptotic methods may not be sufficient for such large problems, consider using Monte Carlo estimation of exact p-values, as described in the section "Monte Carlo Estimation" on page 2534.

A formula does not exist that can predict in advance how much time and memory are needed to compute an exact p-value for a certain problem. The time and memory required depend on several factors, including which test is being performed, the total sample size, the number of rows and columns, and the specific arrangement of the observations into table cells. Generally, larger problems (in terms of total sample size, number of rows, and number of columns) tend to require more time and memory.

Additionally, for a fixed total sample size, time and memory requirements tend to increase as the number of rows and columns increase, since this corresponds to an increase in the number of tables in the reference set. Also for a fixed sample size, time and memory requirements increase as the marginal row and column totals become more homogeneous. Refer to Agresti, Mehta, and Patel (1990) and Gail and Mantel (1977).

At any time while PROC NPAR1WAY is computing exact p-values, you can terminate the computations by pressing the system interrupt key sequence (refer to the *SAS Companion* for your system) and choosing to stop computations. After you terminate exact computations, PROC NPAR1WAY completes all other remaining tasks. The procedure produces the requested output and reports missing values for any exact p-values not computed by the time of termination.

You can also use the MAXTIME= option in the EXACT statement to limit the amount of time PROC NPAR1WAY uses for exact computations. You specify a MAXTIME= value that is the maximum amount of time (in seconds) that PROC NPAR1WAY can use to compute an exact p-value. If PROC NPAR1WAY does not finish computing an exact p-value within that time, it terminates the computation and completes all other remaining tasks.

Monte Carlo Estimation

If you specify the option MC in the EXACT statement, PROC NPAR1WAY computes Monte Carlo estimates of the exact p-values instead of directly computing the exact p-values. Monte Carlo estimation can be useful for large problems that require a great amount of time and memory for exact computations but for which asymptotic approximations may not be sufficient. To describe the precision of each Monte Carlo estimate, PROC NPAR1WAY provides the asymptotic standard error and $100(1 - \alpha)\%$ confidence limits. The confidence level α is determined by the ALPHA= option in the EXACT statement, which, by default, equals 0.01, and produces 99% confidence limits. The N=n option in the EXACT statement specifies the number of samples PROC NPAR1WAY uses for Monte Carlo estimation; the default is 10000 samples. You can specify a larger value for n to improve the precision of the Monte Carlo estimates. Because larger values of n generate more samples, the computation time increases. Or you can specify a smaller value of n to reduce the computation time.

To compute a Monte Carlo estimate of an exact p-value, PROC NPAR1WAY generates a random sample of tables with the same total sample size, row totals, and column totals as the observed table. PROC NPAR1WAY uses the algorithm of Agresti, Wackerly, and Boyett (1979), which generates tables in proportion to their hypergeometric probabilities conditional on the marginal frequencies. For each sample table, PROC NPAR1WAY computes the value of the test statistic and compares it to the value for the observed table. When estimating a right-sided p-value, PROC NPAR1WAY counts all sample tables for which the test statistic is greater than or equal to the observed test statistic. Then the p-value estimate equals the number of these tables divided by the total number of tables sampled.

$$\hat{P}_{\text{MC}} \quad = \quad M \ / \ N$$
$$M \quad = \quad \text{number of samples with (Test Statistic} \geq t)$$
$$N \quad = \quad \text{total number of samples}$$
$$t \quad = \quad \text{observed Test Statistic}$$

PROC NPAR1WAY computes left-sided and two-sided p-value estimates in a similar manner. For left-sided p-values, PROC NPAR1WAY evaluates whether the test statistic for each sampled table is less than or equal to the observed test statistic. For two-sided p-values, PROC NPAR1WAY examines the sample test statistics according to the expression for P_2 given in the section "Definition of p-Values" on page 2533.

The variable M is a binomially distributed variable with N trials and success probability p. It follows that the asymptotic standard error of the Monte Carlo estimate is

$$se(\hat{P}_{\text{MC}}) \quad = \quad \sqrt{\hat{P}_{\text{MC}}(1 - \hat{P}_{\text{MC}})/(N-1)}$$

PROC NPAR1WAY constructs asymptotic confidence limits for the p-values according to

$$\hat{P}_{\text{MC}} \quad \pm \quad z_{\alpha/2} \ \cdot \ se(\hat{P}_{\text{MC}})$$

where $z_{\alpha/2}$ is the $100(1 - \alpha/2)$ percentile of the standard normal distribution, and the confidence level α is determined by the ALPHA= option in the EXACT statement.

When the Monte Carlo estimate $\hat{P}_{\text{MC}}$ equals 0, then PROC NPAR1WAY computes the confidence limits for the p-value as

$$(\ 0, \ \ 1 - \alpha^{(1/N)}\ )$$

When the Monte Carlo estimate $\hat{P}_{MC}$ equals 1, then PROC NPAR1WAY computes the confidence limits as

$$(\ \alpha^{(1/N)}, \ \ 1\ )$$

Output Data Set

The OUTPUT statement creates a SAS data set that contains statistics computed by PROC NPAR1WAY. You specify which statistics to store in the output data set, using options identical to those used in the PROC NPAR1WAY statement. When you specify one of these options in the OUTPUT statement, PROC NPAR1WAY includes all statistics from that analysis in the output data set.

The output data set contains one observation for each analysis variable within a BY-group. The OUTPUT data set can include the following variables.

- BY variables

- _VAR_, which identifies the analysis variable

- variables containing the specified statistics

The following table lists the variable names and descriptions for all available statistics. Note that some statistics are available only for the two-sample case (where the classification variable groups the data into two classes). Other statistics are available only for the multisample case.

When you request exact p-values for certain analyses using the EXACT statement, PROC NPAR1WAY also includes those p-values in the output data set if you specify the corresponding analysis options in the OUTPUT statement. If you do not request exact p-values, then they do not appear in the output data set.

Table 47.5. Output Data Set Variable Names and Descriptions

Option	Output Variables		Variable Descriptions
ANOVA	_MSA_		ANOVA Effect Mean Square, Among MS
	MSE		ANOVA Error Mean Square, Within MS
	_F_		F Statistic for ANOVA
	P_F		p-value, F Statistic for ANOVA
WILCOXON	_WIL_	*	Two-sample Wilcoxon Statistic
	Z_WIL	*	Wilcoxon Statistic, Standardized
	PL_WIL	*	p-value, Wilcoxon Test (Left-sided)
	PR_WIL	*	p-value, Wilcoxon Test (Right-sided)
	P2_WIL	*	p-value, Wilcoxon Test (Two-sided)
	PTL_WIL	*	p-value, Wilcoxon t Approximation (Left-sided)
	PTR_WIL	*	p-value, Wilcoxon t Approximation, (Right-sided)
	PT2_WIL	*	p-value, Wilcoxon t Approximation, (Two-sided)
	XPL_WIL	*	Exact p-value, Wilcoxon Test (Left-sided)
	XPR_WIL	*	Exact p-value, Wilcoxon Test (Right-sided)
	XP2_WIL	*	Exact p-value, Wilcoxon Test (Two-sided)
	_KW_		Kruskal-Wallis Statistic
	DF_KW		Degrees of Freedom, Kruskal-Wallis Test
	P_KW		p-value, Kruskal-Wallis Test
	XP_KW	**	Exact p-value, Kruskal-Wallis Test
MEDIAN	_MED_	*	Two-sample Median Statistic
	Z_MED	*	Median Statistic, Standardized
	PL_MED	*	p-value, Median Test (Left-sided)
	PR_MED	*	p-value, Median Test (Right-sided)
	P2_MED	*	p-value, Median Test (Two-sided)
	XPL_MED	*	Exact p-value, Median Test (Left-sided)
	XPR_MED	*	Exact p-value, Median Test (Right-sided)
	XP2_MED	*	Exact p-value, Median Test (Two-sided)
	CHMED		Median Chi-square (Brown-Mood Test)
	DF_CHMED		Degrees of Freedom, Median Chi-square
	P_CHMED		p-value, Median Chi-square Test

Table 47.5. (continued)

Option	Output Variables		Variable Descriptions
	XP_CHMED	**	Exact *p*-value, Median Chi-square
VW	_VW_	*	Two-sample Van der Waerden Statistic
	Z_VW	*	Van der Waerden Statistic, Standardized
	PL_VW	*	*p*-value, Van der Waerden Test (Left-sided)
	PR_VW	*	*p*-value, Van der Waerden Test (Right-sided)
	P2_VW	*	*p*-value, Van der Waerden Test (Two-sided)
	XPL_VW	*	Exact *p*-value, Van der Waerden Test (Left-sided)
	XPR_VW	*	Exact *p*-value, Van der Waerden Test (Right-sided)
	XP2_VW	*	Exact *p*-value, Van der Waerden Test (Two-sided)
	CHVW		Van der Waerden Chi-square
	DF_CHVW		Degrees of Freedom, Van der Waerden Chi-square
	P_CHVW		*p*-value, Van der Waerden Chi-square Test
	XP_CHVW	**	Exact *p*-value, Van der Waerden Chi-square
SAVAGE	_SAV_	*	Two-sample Savage Statistic
	Z_SAV	*	Savage Statistic, Standardized
	PL_SAV	*	*p*-value, Savage Test (Left-sided)
	PR_SAV	*	*p*-value, Savage Test (Right-sided)
	P2_SAV	*	*p*-value, Savage Test (Two-sided)
	XPL_SAV	*	Exact *p*-value, Savage Test (Left-sided)
	XPR_SAV	*	Exact *p*-value, Savage Test (Right-sided)
	XP2_SAV	*	Exact *p*-value, Savage Test (Two-sided)
	CHSAV		Savage Chi-square
	DF_CHSAV		Degrees of Freedom, Savage Chi-square
	P_CHSAV		*p*-value, Savage Chi-square Test
	XP_CHSAV	**	Exact *p*-value, Savage Chi-square
ST	_ST_	*	Two-sample Siegel-Tukey Statistic
	Z_ST	*	Siegel-Tukey Statistic, Standardized
	PL_ST	*	*p*-value, Siegel-Tukey Test (Left-sided)
	PR_ST	*	*p*-value, Siegel-Tukey Test (Right-sided)
	P2_ST	*	*p*-value, Siegel-Tukey Test (Two-sided)
	XPL_ST	*	Exact *p*-value, Siegel-Tukey Test (Left-sided)
	XPR_ST	*	Exact *p*-value, Siegel-Tukey Test (Right-sided)
	XP2_ST	*	Exact *p*-value, Siegel-Tukey Test (Two-sided)
	CHST		Siegel-Tukey Chi-square
	DF_CHST		Degrees of Freedom, Siegel-Tukey Chi-square
	P_CHST		*p*-value, Siegel-Tukey Chi-square Test
	XP_CHST	**	Exact *p*-value, Siegel-Tukey Chi-square
AB	_AB_	*	Two-sample Ansari-Bradley Statistic
	Z_AB	*	Ansari-Bradley Statistic, Standardized
	PL_AB	*	*p*-value, Ansari-Bradley Test (Left-sided)
	PR_AB	*	*p*-value, Ansari-Bradley Test (Right-sided)
	P2_AB	*	*p*-value, Ansari-Bradley Test (Two-sided)
	XPL_AB	*	Exact *p*-value, Ansari-Bradley Test (Left-sided)
	XPR_AB	*	Exact *p*-value, Ansari-Bradley Test (Right-sided)

Table 47.5. (continued)

Option	Output Variables		Variable Descriptions
	XP2_AB	*	Exact *p*-value, Ansari-Bradley Test (Two-sided)
	CHAB		Ansari Bradley Chi-square
	DF_CHAB		Degrees of Freedom, Ansari-Bradley Chi-square
	P_CHAB		*p*-value, Ansari-Bradley Chi-square Test
	XP_CHAB	**	Exact *p*-value, Ansari-Bradley Chi-square
KLOTZ	_KLOTZ_	*	Two-sample Klotz Statistic
	Z_K	*	Klotz Statistic, Standardized
	PL_K	*	*p*-value, Klotz Test (Left-sided)
	PR_K	*	*p*-value, Klotz Test (Right-sided)
	P2_K	*	*p*-value, Klotz Test (Two-sided)
	XPL_K	*	Exact *p*-value, Klotz Test (Left-sided)
	XPR_K	*	Exact *p*-value, Klotz Test (Right-sided)
	XP2_K	*	Exact *p*-value, Klotz Test (Two-sided)
	CHK		Klotz Chi-square
	DF_CHK		Degrees of Freedom, Klotz Chi-square
	P_CHK		*p*-value, Klotz Chi-square Test
	XP_CHK	**	Exact *p*-value, Klotz Chi-square
MOOD	_MOOD_	*	Two-sample Mood Statistic
	Z_MOOD	*	Mood Statistic, Standardized
	PL_MOOD	*	*p*-value, Mood Test (Left-sided)
	PR_MOOD	*	*p*-value, Mood Test (Right-sided)
	P2_MOOD	*	*p*-value, Mood Test (Two-sided)
	XPL_MOOD	*	Exact *p*-value, Mood Test (Left-sided)
	XPR_MOOD	*	Exact *p*-value, Mood Test (Right-sided)
	XP2_MOOD	*	Exact *p*-value, Mood Test (Two-sided)
	CHMOOD		Mood Chi-square
	DF_CHMOO		Degrees of Freedom, Mood Chi-square
	P_CHMOOD		*p*-value, Mood Chi-square Test
	XP_CHMOO	**	Exact *p*-value, Mood Chi-square
SCORES=DATA	_DATA_	*	Two-sample Data Scores Statistic
	Z_DATA	*	Data Scores Statistic, Standardized
	PL_DATA	*	*p*-value, Data Scores Test (Left-sided)
	PR_DATA	*	*p*-value, Data Scores Test (Right-sided)
	P2_DATA	*	*p*-value, Data Scores Test (Two-sided)
	XPL_DATA	*	Exact *p*-value, Data Scores Test (Left-sided)
	XPR_DATA	*	Exact *p*-value, Data Scores Test (Right-sided)
	XP2_DATA	*	Exact *p*-value, Data Scores Test (Two-sided)
	CHDATA		Data Scores Chi-square
	DF_CHDAT		Degrees of Freedom, Data Scores Chi-square
	P_CHDATA		*p*-value, Data Scores Chi-square Test
	XP_CHDAT	**	Exact *p*-value, Data Scores Chi-square

Table 47.5. (continued)

Option	Output Variables		Variable Descriptions
EDF	_KS_		Kolmogorov-Smirnov Statistic
	KSA		Kolmogorov-Smirnov Statistic (Asymptotic)
	_D_	*	Two-sample Kolmogorov-Smirnov Statistic
	P_KSA	*	*p*-value, Two-sample Kolmogorov-Smirnov
	_CM_		Cramer-von Mises Statistic
	CMA		Cramer-von Mises Statistic (Asymptotic)
	_K_	*	Kuiper Two-sample Statistic
	_KA_	*	Kuiper Two-sample Statistic (Asymptotic)
	P_KA	*	*p*-value, Two-sample Kuiper (Asymptotic)

* Statistic included only for two-sample cases

** Statistic included only for multisample cases

Displayed Output

If you specify the ANOVA option, PROC NPAR1WAY displays a Class Means table and an Analysis of Variance table for each response variable. The Class Means table includes the following information for each CLASS variable value, or level:

- N, the number of observations
- the Mean of the response variable

The Analysis of Variance table includes the following information for each Source of variation (Among classes, and Within classes):

- DF, the degrees of freedom associated with the source
- the Sum of Squares
- the Mean Square, the sum of squares divided by the degrees of freedom

The Analysis of Variance table also includes the following:

- the F Value for testing the hypothesis that the group means are equal. This is computed by dividing the Mean Square (Among) by the Mean Square (Within).
- Pr > F, the significance probability corresponding to the F Value

For each score type that you specify, PROC NPAR1WAY displays a Class Scores table. The available score types include Wilcoxon, median, Van der Waerden, Savage, Siegel-Tukey, Ansari-Bradley, Klotz, Mood, and raw data scores. PROC NPAR1WAY assigns the specified scores to the response variable values, and classifies then according to the CLASS variable values. The Class Scores table includes the following information for each class.

- N, the number of observations

- Sum of Scores

- Expected Under H0, the expected sum of scores under the null hypothesis of no difference among classes

- Std Dev Under H0, the standard deviation under the null hypothesis

- Mean Score

When there are only two levels of the CLASS variable, PROC NPAR1WAY displays the following Two-Sample Test results for each analysis of scores:

- Statistic, which is the sum of scores for the class with the smaller sample size

- Z, the standardized test statistic, which has an asymptotic standard normal distribution under the null hypothesis

- One-Sided Pr < Z, or One-Sided Pr > z, the asymptotic one-sided p-value, displayed as Pr < Z or Pr > Z, depending on whether Z is <= 0 or > 0

- Two-Sided Pr > |Z|, the asymptotic two-sided p-value

For Wilcoxon scores, PROC NPAR1WAY also displays a t-approximation for the two-sample test.

If you request an exact test by specifying the score type in the EXACT statement, PROC NPAR1WAY displays the following exact p-values for two-sample data:

- One-Sided Pr <= S, or One-Sided Pr >= S, the one-sided exact p-value, displayed as Pr <= S or Pr >= S, depending on whether S <= Mean or S > Mean, where S is the test statistic and Mean is its expected value under the null hypothesis

- Two-Sided Pr >= |S - Mean|, the two-sided exact p-value

If you request Monte Carlo estimates for the exact test by specifying the MC option in the EXACT statement, PROC NPAR1WAY displays the following information for two-sample data:

- Estimate of One-Sided Pr <= S or One-Sided Pr >=S, the one-sided exact p-value, together with its Lower and Upper Confidence Limits

- Estimate of Two-Sided Pr >= |S - Mean|, the two-sided exact p-value, together with its Lower and Upper Confidence Limits

- Number of Samples used to compute the Monte Carlo estimates

- Iniital Seed used to compute the Monte Carlo estimates

For both two-sample and multisample data, PROC NPAR1WAY displays the following One-Way Analysis for each score type.

- Chi-Square, the one-way ANOVA statistic for testing the null hypothesis of no difference among classes
- DF, the degrees of freedom
- Pr > Chi-Square, the asymptotic p-value

For multisample data, if you request an exact test by specifying the score type in the EXACT statement, PROC NPAR1WAY also displays the exact p-value as follows:

- Exact Pr >= Chi-Square

For multisample data, if you request a Monte Carlo estimate for the exact test by specifying the MC option in the EXACT statement, PROC NPAR1WAY displays the following information:

- Estimate of Pr >= Chi-Square, together with its Lower and Upper Confidence Limits
- Number of Samples used to compute the Monte Carlo estimate
- Iniital Seed used to compute the Monte Carlo estimate

If you specify the EDF option, PROC NPAR1WAY produces tables for the Kolmogorov-Smirnov Test, the Cramer-von Mises Test, and for two-sample data only, the Kuiper Test. The Kolmogorov-Smirnov Test table includes the following information for each CLASS variable value, or level.

- N, the number of observations
- EDF at Maximum, the value of the class EDF (empirical distribution function) at its maximum deviation from the pooled EDF
- Deviation from Mean at Maximum, the value of $\sqrt{n_i}\sqrt{F_i - F}$ at its maximum, where n_i it the class sample size, F_i is the class EDF, and F is the pooled EDF

PROC NPAR1WAY also displays the following Kolmogorov-Smirnov statistics:

- KS, the Kolmogorov-Smirnov statistic
- KSa, the asymptotic Kolmogorov-Smirnov statistic, where KSa = $\sqrt{n}$ KS

For two-sample data, PROC NPAR1WAY also displays the following Kolmogorov-Smirnov statistics:

- Pr > KSa, the asymptotic p-value for KSa
- D, the two-sample Kolmogorov statistic

The Cramer-von Mises Test table includes the following information for each CLASS variable value, or level:

- N, the number of observations
- Summed Deviation from Mean, which is $(n_i/n) \sum_{j=1}^{n} F_i(x_j) - F(x_j)^2$

PROC NPAR1WAY also displays the following Cramer-von Mises statistics:

- CM, the Cramer-von Mises statistic
- CMa, the asymptotic Cramer-von Mises statistic, where CMa $= n \cdot$ CM

For two-sample data, PROC NPAR1WAY displays the Kuiper Test table, which includes the following information for each class:

- N, the number of observations
- Deviation from Mean, which is $\max_j |F_1(x_j) - F_2(x_j)|$

PROC NPAR1WAY also displays the following Kuiper two-sample test statistics:

- K, the Kuiper two-sample test statistic
- Ka, the asymptotic Kuiper two-sample test statistic, where Ka $= $ K $\cdot \sqrt{n_1 n_2/n}$
- Pr > Ka

ODS Table Names

PROC NPAR1WAY assigns a name to each table it creates. You can use these names to reference the table when using the Output Delivery System (ODS) to select tables and create output data sets. These names are listed in the following table. For more information on ODS, see Chapter 15, "Using the Output Delivery System."

The WILCOXON, MEDIAN, VW, SAVAGE, and EDF options are the default if you do not specify any analysis options in the PROC NPAR1WAY statement.

Table 47.6. ODS Tables Produced in PROC NPAR1WAY

ODS Table Name	Description	Statement	Option
ANOVA	Analysis of variance	PROC	ANOVA
ABAnalysis	Ansari-Bradley one-way analysis	PROC	AB
ABMC	Monte Carlo estimates for the Ansari-Bradley exact test	EXACT	AB / MC
ABScores	Ansari-Bradley scores	PROC	AB
ABTest	Ansari-Bradley two-sample test	PROC	AB*
ClassMeans	Class Means	PROC	ANOVA
CVMStats	Cramer-von Mises statistics	PROC	EDF
CVMTest	Cramer-von Mises test	PROC	EDF

Table 47.6. (continued)

ODS Table Name	Description	Statement	Option
DataScores	Data scores	PROC	SCORES=DATA
DataScoresAnalysis	Data scores one-way analysis	PROC	SCORES=DATA
DataScoresMC	Monte Carlo estimates for the exact test based on data scores	EXACT	SCORES=DATA / MC
DataScoresTest	Data scores two-sample test	PROC	SCORES=DATA*
KlotzAnalysis	Klotz one-way analysis	PROC	KLOTZ
KlotzMC	Monte Carlo estimates for the Klotz exact test	EXACT	KLOTZ / MC
KlotzScores	Klotz scores	PROC	KLOTZ
KlotzTest	Klotz two-sample test	PROC	KLOTZ
KolSmir2Stats	Kolmogorov-Smirnov two-sample statistics	PROC	EDF*
KolSmirStats	Kolmogorov-Smirnov statistics	PROC	EDF**
KolSmirTest	Kolmogorov-Smirnov test	PROC	EDF
KruskalWallisMC	Monte Carlo estimates for the Kruskal-Wallis exact test	EXACT	WILCOXON / MC**
KruskalWallisTest	Kruskal-Wallis test	PROC	WILCOXON
KuiperStats	Kuiper two-sample statistics	PROC	EDF*
KuiperTest	Kuiper test	PROC	EDF*
MedianAnalysis	Median one-way analysis	PROC	MEDIAN
MedianMC	Monte Carlo estimates for the median exact test	EXACT	MEDIAN / MC
MedianScores	Median scores	PROC	MEDIAN
MedianTest	Median two-sample test	PROC	MEDIAN*
MoodAnalysis	Mood one-way analysis	PROC	MOOD
MoodMC	Monte Carlo estimates for the Mood exact test	EXACT	MOOD / MC
MoodScores	Mood scores	PROC	MOOD
MoodTest	Mood two-sample test	PROC	MOOD
SavageAnalysis	Savage one-way analysis	PROC	SAVAGE
SavageMC	Monte Carlo estimates for the Savage exact test	EXACT	SAVAGE / MC
SavageScores	Savage scores	PROC	SAVAGE
SavageTest	Savage two-sample test	PROC	SAVAGE*
STAnalysis	Siegel-Tukey one-way analysis	PROC	ST
STMC	Monte Carlo estimates for the Siegel-Tukey exact test	EXACT	ST / MC
STScores	Siegel-Tukey scores	PROC	ST
STTest	Siegel-Tukey two-sample test	PROC	ST*
VWAnalysis	Van der Waerden one-way analysis	PROC	VW
VWMC	Monte Carlo estimates for the Van der Waerden exact test	EXACT	VW / MC
VWScores	Van der Waerden scores	PROC	VW
VWTest	Van der Waerden two-sample test	PROC	VW*

Table 47.6. (continued)

ODS Table Name	Description	Statement	Option
WilcoxonMC	Monte Carlo estimates for the Wilcoxon two-sample exact test	EXACT	WILCOXON / MC*
WilcoxonScores	Wilcoxon scores	PROC	WILCOXON
WilcoxonTest	Wilcoxon two-sample test	PROC	WILCOXON*

* PROC NPAR1WAY produces this table only for two-sample data.

** PROC NPAR1WAY produces this table only for multisample data.

Examples

Example 47.1. Two-Sample Location Tests and EDF Statistics

Fifty-nine female patients with rheumatoid arthritis who participated in a clinical trial were assigned to two groups, active and placebo. The response status (excellent=5, good=4, moderate=3, fair=2, poor=1) of each patient was recorded.

The following SAS statements create the data set Arthritis, which contains the observed status values for all the patients. The variable Treatment denotes the treatment received by a patient, and the variable Response contains the response status of the patient. The variable Freq contains the frequency of the observation, which is the number of patients with the Treatment and Response combination.

```
data Arthritis;
   input Treatment $ Response Freq @@;
   datalines;
Active 5 5 Active 4 11 Active 3 5 Active 2 1 Active 1 5
Placebo 5 2 Placebo 4 4 Placebo 3 7 Placebo 2 7 Placebo 1 12
;
```

PROC NPAR1WAY tests the null hypothesis that there is no difference in the patient response status against an alternative hypothesis that the patient response status differs in the two treatment groups. The WILCOXON option requests the Wilcoxon test for difference in location, and the MEDIAN option requests the median test for difference in location. The EDF option requests empirical distribution function statistics. The variable Treatment is the CLASS variable, and the VAR statement specifies that the variable Response is the response variable.

```
proc npar1way wilcoxon median edf data=Arthritis;
   class Treatment;
   var Response;
   freq Freq;
run;
```

Example 47.1.   Two-Sample Location Tests and EDF Statistics ⋄ 2545

Output 47.1.1.   Wilcoxon Two-Sample Test

```
                        The NPAR1WAY Procedure

               Wilcoxon Scores (Rank Sums) for Variable Response
                      Classified by Variable Treatment

                           Sum of      Expected      Std Dev        Mean
      Treatment      N     Scores      Under H0      Under H0       Score
      -------------------------------------------------------------------
      Active        27      999.0        810.0       63.972744   37.000000
      Placebo       32      771.0        960.0       63.972744   24.093750

                      Average scores were used for ties.


                          Wilcoxon Two-Sample Test

                      Statistic              999.0000

                      Normal Approximation
                      Z                        2.9466
                      One-Sided Pr >  Z        0.0016
                      Two-Sided Pr > |Z|       0.0032

                      t Approximation
                      One-Sided Pr >  Z        0.0023
                      Two-Sided Pr > |Z|       0.0046

                  Z includes a continuity correction of 0.5.


                            Kruskal-Wallis Test

                      Chi-Square               8.7284
                      DF                            1
                      Pr > Chi-Square          0.0031
```

Output 47.1.1 shows the results of the Wilcoxon analysis. The Wilcoxon two-sample test statistic equals 999.0, which is the sum of the Wilcoxon scores for the smaller sample (Active). This sum is greater than 810.0, its expected value under the null hypothesis of no difference between the two samples Active and Placebo. The one-sided *p*-value is 0.0016, which shows that the patient response for the Active treatment is significantly more than for the Placebo group.

Output 47.1.2. Median Two-Sample Test

```
                       The NPAR1WAY Procedure

      Median Scores (Number of Points Above Median) for Variable Response
                    Classified by Variable Treatment

                        Sum of       Expected       Std Dev          Mean
    Treatment      N     Scores       Under H0       Under H0        Score
    -------------------------------------------------------------------------
    Active        27   18.916667     13.271186      1.728195      0.700617
    Placebo       32   10.083333     15.728814      1.728195      0.315104

                     Average scores were used for ties.


                        Median Two-Sample Test

                  Statistic              18.9167
                  Z                       3.2667
                  One-Sided Pr >  Z       0.0005
                  Two-Sided Pr > |Z|      0.0011


                        Median One-Way Analysis

                  Chi-Square             10.6713
                  DF                           1
                  Pr > Chi-Square         0.0011
```

Output 47.1.2 shows the results of the median two-sample test. The statistic equals 18.9167, with a one-sided *p*-value of 0.0005. This shows that the response for the Active treatment is significantly more than for the Placebo group.

Example 47.1.   Two-Sample Location Tests and EDF Statistics ⬥ 2547

Output 47.1.3. Empirical Distribution Function Statistics

```
                    The NPAR1WAY Procedure

        Kolmogorov-Smirnov Test for Variable Response
              Classified by Variable Treatment

                            EDF at      Deviation from Mean
   Treatment        N       Maximum        at Maximum
   -------------------------------------------------------
   Active          27      0.407407         -1.141653
   Placebo         32      0.812500          1.048675
   Total           59      0.627119

        Maximum Deviation Occurred at Observation 3
             Value of Response at Maximum = 3.0

       Kolmogorov-Smirnov Two-Sample Test (Asymptotic)
            KS   0.201818    D          0.405093
            KSa  1.550191    Pr > KSa   0.0164


         Cramer-von Mises Test for Variable Response
              Classified by Variable Treatment

                                   Summed Deviation
       Treatment            N          from Mean
       -----------------------------------------
       Active              27         0.526596
       Placebo             32         0.444316

         Cramer-von Mises Statistics (Asymptotic)
              CM  0.016456    CMa  0.970912


             Kuiper Test for Variable Response
             Classified by Variable Treatment

                               Deviation
           Treatment        N   from Mean
           -------------------------------
           Active          27   0.000000
           Placebo         32   0.405093

         Kuiper Two-Sample Test (Asymptotic)
     K  0.405093    Ka  1.550191    Pr > Ka  0.1409
```

Output 47.1.3 shows empirical distribution function statistics comparing these two samples. The asymptotic *p*-value for the Kolmogorov-Smirnov test is 0.0164. This indicates rejection of the null hypothesis that the distributions are identical for the two groups.

Example 47.2. The Exact Wilcoxon Two-Sample Test

Researchers conducted an experiment to compare the effects of two stimulants. Thirteen randomly selected subjects received the first stimulant and six randomly selected subjects received the second stimulant. The reaction times (in minutes) were measured while the subjects were under the influence of the stimulants.

The following SAS statements create the data set React, which contains the observed reaction times for each stimulant. The variable Stim represents Stimulant 1 or 2. The variable Time contains the reaction times observed for subjects under the stimulant.

```
data React;
   input Stim Time @@;
   datalines;
1 1.94   1 1.94   1 2.92   1 2.92   1 2.92   1 2.92   1 3.27
1 3.27   1 3.27   1 3.27   1 3.70   1 3.70   1 3.74
2 3.27   2 3.27   2 3.27   2 3.70   2 3.70   2 3.74
;
```

PROC NPAR1WAY tests the null hypothesis that there is no difference between the effects of the two stimulants against an alternative hypothesis that stimulant 1 has smaller reaction times than stimulant 2. The WILCOXON option specifies that Wilcoxon scores are to be used. The CLASS statement specifies that the variable Stim determines the classes. The VAR statement identifies Time as the response variable. The EXACT option requests the exact *p*-values. Since the sample size is small, the normal approximation may not be completely accurate, and it is appropriate to compute the exact test.

```
proc npar1way wilcoxon data=React;
   class Stim;
   var Time;
   exact;
run;
```

Example 47.2. *The Exact Wilcoxon Two-Sample Test* ♦ 2549

Output 47.2.1. Results of the Wilcoxon Two-Sample Test

```
                      The NPAR1WAY Procedure

             Wilcoxon Scores (Rank Sums) for Variable Time
                      Classified by Variable Stim

                        Sum of      Expected      Std Dev       Mean
      Stim      N       Scores      Under H0      Under H0      Score
      -------------------------------------------------------------------
      1        13       110.50        130.0      11.004784      8.500
      2         6        79.50         60.0      11.004784     13.250

                    Average scores were used for ties.


                       Wilcoxon Two-Sample Test

                   Statistic (S)                79.5000

                   Normal Approximation
                   Z                             1.7265
                   One-Sided Pr >  Z             0.0421
                   Two-Sided Pr > |Z|            0.0843

                   t Approximation
                   One-Sided Pr >  Z             0.0507
                   Two-Sided Pr > |Z|            0.1014

                   Exact Test
                   One-Sided Pr >=  S            0.0527
                   Two-Sided Pr >= |S - Mean|    0.1054

                Z includes a continuity correction of 0.5.


                          Kruskal-Wallis Test

                     Chi-Square           3.1398
                     DF                        1
                     Pr > Chi-Square      0.0764
```

Output 47.2.1 displays the results of the Wilcoxon two-sample test. The Wilcoxon statistic equals 79.50. Since this value is greater than 60.0, the expected value under the null hypothesis, PROC NPAR1WAY displays the right-sided p-values. The one-sided exact p-value equals 0.0527, which is not significant at the 0.05 level. The normal approximation yields a one-sided p-value of 0.0421, which is significant at the 0.05 level. Thus, the normal approximation may result in rejection of the null hypothesis, while the exact test may result in acceptance.

Example 47.3. The Exact Savage Multisample Test

A researcher conducting a laboratory experiment randomly assigned fifteen mice to receive one of three drugs. The survival time (in days) was then recorded.

The following SAS statements create the data set Mice, which contains the observed survival times for all the mice. The variable Trt denotes the treatment received by a mouse. The variable Days contains the number of days the mouse survived.

```
data Mice;
   input Trt $ Days @@;
   datalines;
1 1 1 1 1 3 1 3 1 4
2 3 2 4 2 4 2 4 2 15
3 4 3 4 3 10 3 10 3 26
;
```

PROC NPAR1WAY tests the null hypothesis that there is no difference in the survival times among the three drugs against an alternative hypothesis of difference among the drugs. The SAVAGE option specifies that Savage scores are to be used. The variable Trt is the CLASS variable, and the VAR statement specifies that the variable Days is the response variable. The EXACT statement requests the exact test.

```
proc npar1way savage data=Mice;
   class Trt;
   var Days;
   exact;
run;
```

Output 47.3.1. Results of the Savage Multisample Test

```
                    The NPAR1WAY Procedure

           Savage Scores (Exponential) for Variable Days
                   Classified by Variable Trt

                   Sum of      Expected      Std Dev         Mean
  Trt      N       Scores      Under H0      Under H0        Score
  -----------------------------------------------------------------
  1        5      -3.367980       0.0        1.634555     -0.673596
  2        5       0.095618       0.0        1.634555      0.019124
  3        5       3.272362       0.0        1.634555      0.654472

                Average scores were used for ties.


                      Savage One-Way Analysis

             Chi-Square                      5.5047
             DF                                   2
             Asymptotic Pr >  Chi-Square     0.0638
             Exact       Pr >= Chi-Square    0.0445
```

Output 47.3.1 shows the results of the Savage test. The exact p-value is 0.0445, which is significant at the 0.05 level. However, the p-value based on the chi-square approximation is 0.0638, which results in nonrejection of the null hypothesis at the 0.05 level.

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