

The Ontario Curriculum  
Grades 11 and 12

# Mathematics



2000

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This publication is available on the Ministry of Education's website at <http://www.edu.gov.on.ca>.



# Introduction

*The Ontario Curriculum, Grades 11 and 12: Mathematics, 2000* will be implemented in Ontario secondary schools starting in September 2001 for students in Grade 11 and in September 2002 for students in Grade 12. This document replaces the parts of the following curriculum guidelines that relate to the senior grades:

- *Mathematics, Part One: Grades 9 and 10, Basic Level; Grades 11 and 12, Basic Level; Intermediate and Senior Divisions, 1985*
- *Mathematics, Part Two: Grades 7 and 8; Grades 9 and 10, General Level; Grades 11 and 12, General Level; Intermediate and Senior Divisions, 1985*
- *Mathematics, Part Three: Grades 7 and 8; Grades 9 and 10, Advanced Level; Grades 11 and 12, Advanced Level; Ontario Academic Courses; Intermediate and Senior Divisions, 1985.*

This document is designed for use in conjunction with *The Ontario Curriculum, Grades 9 to 12: Program Planning and Assessment, 2000*, which contains information relevant to all disciplines represented in the curriculum. The planning and assessment document is available both in print and on the ministry's website, at <http://www.edu.gov.on.ca>.

## The Place of Mathematics in the Curriculum

Unprecedented changes that are taking place in today's world will profoundly affect the futures of today's students. To meet the demands of the world in which they will live, students will need to adapt to changing conditions and to learn independently. They will require the ability to use technology effectively and the skills for processing large amounts of quantitative information. Today's mathematics curriculum must prepare students for their tomorrows. It must equip them with essential mathematics knowledge and skills; with skills of reasoning, problem solving, and communication; and, most importantly, with the ability and the incentive to continue learning on their own.

Mathematical knowledge becomes meaningful and powerful in application. Other disciplines are a ready source of effective contexts for the study of mathematics. Rich problem-solving situations can be drawn from closely related disciplines, such as computer science, physics, or technology, as well as from subjects historically thought of as distant from mathematics, such as political science or art. It is important that these links between disciplines be carefully explored, analysed, and discussed to emphasize for students the pervasiveness of mathematical knowledge and mathematical thinking in all subject areas.

Subject matter from any course in mathematics can be combined with subject matter from one or more courses in other disciplines to create an interdisciplinary course. The policies and procedures regarding the development of interdisciplinary courses are outlined in the interdisciplinary studies curriculum policy document.

The development of mathematical knowledge is a gradual process. A coherent and continuous program is necessary to help students see the "big pictures" or underlying principles of mathematics. The Grade 11 and 12 courses in this curriculum build on the knowledge of concepts and the skills developed in the earlier grades, extending students' knowledge in new areas and

requiring them to solve more complex problems. Students in Grades 11 and 12 will continue to develop key skills and make connections through the exploration of applications. The understanding of abstract mathematics is central to the curriculum in Grades 11 and 12, and its development may occur in various ways. Skill acquisition is an important part of the program; wherever possible, skills should be introduced as they are needed, in the contexts offered by various topics in the mathematics program.

The importance of communication in mathematics is a highlight of the secondary school curriculum. In all courses, expectations are included that require of students the clear and concise communication of reasoning or of findings. It is the expectation that students, over the course of their secondary school experience, will learn to write about their use of mathematics, effectively incorporating mathematical forms such as calculations, equations, graphs, or tables. This curriculum assumes a classroom environment in which students are called upon to explain their reasoning in writing, or orally to the teacher, to the class, or to other students in a group.

# The Program in Mathematics

## Overview

The material in each course is arranged by strands, which are major content organizers. It is expected that in developing detailed courses of study from this document, teachers will weave together related expectations from different strands, in order to create an overall program that integrates and balances concept development, skill acquisition, and applications.

An important part of every course in the mathematics program is the process of inquiry, in which students develop a systematic method for exploring new problems or unfamiliar situations. Knowing how to learn mathematics is the underlying expectation that every student in every course needs to achieve. An important stage in the inquiry process is that of *modelling*, or taking the conditions of a real situation and describing them in mathematical form. A mathematical model can appear in many different ways – as an actual physical model, or as a diagram, a graph, a table of values, an equation, or a computer model. It is important that students understand the relationships between the various models of a given situation. To do so requires that their experiences with mathematical models increase in sophistication as they progress through secondary school. The process of inquiry is highlighted throughout the grades, but the problems, and the models that represent them, become more complex as the students advance through the grades.

The choice of some specific concepts and skills in the curriculum has taken into consideration the development of sophisticated yet easily used calculators and computer software. The curriculum has been designed to integrate appropriate technologies into the learning and doing of mathematics, while equipping students with the manipulation skills necessary to understand other aspects of the mathematics that they are learning, to solve meaningful problems, and to continue to learn mathematics with success in the future. Technology is not used to replace skill acquisition. Rather, technology is required when its use represents either the only way or the most effective way to achieve an expectation.

**A Note About Credits.** Courses in Grades 11 and 12 are designed to be offered as full-credit courses. However, half-credit courses may be developed for specialized programs, such as school-work transition and apprenticeship programs, as long as the original course is not designated as a requirement for entry into a university program. Individual universities will identify the courses that are prerequisites for admission to specific programs. Such courses must be offered as full-credit courses, to ensure that students meet admission requirements.

In Grades 9-12, half-credit courses, which require a minimum of fifty-five hours of scheduled instructional time, must adhere to the following conditions:

- The two half-credit courses created from a full course must together contain all of the expectations of the full course, drawn from all of the strands of that course and divided in a manner that best enables students to achieve the required knowledge and skills in the allotted time.

- A course that is a prerequisite for another course in the secondary curriculum may be offered as two half-credit courses, but students must successfully complete both parts of the course to fulfil the prerequisite. (Students are not required to complete both parts unless the course is a prerequisite for another course that they wish to take.)
- The title of each half-credit course must include the designation Part 1 or Part 2. A half-credit (0.5) will be recorded in the credit-value column of both the report card and the Ontario Student Transcript.

Boards will ensure that all half-credit courses comply with the conditions described above, and will report all half-credit courses to the ministry annually in the School September Report.

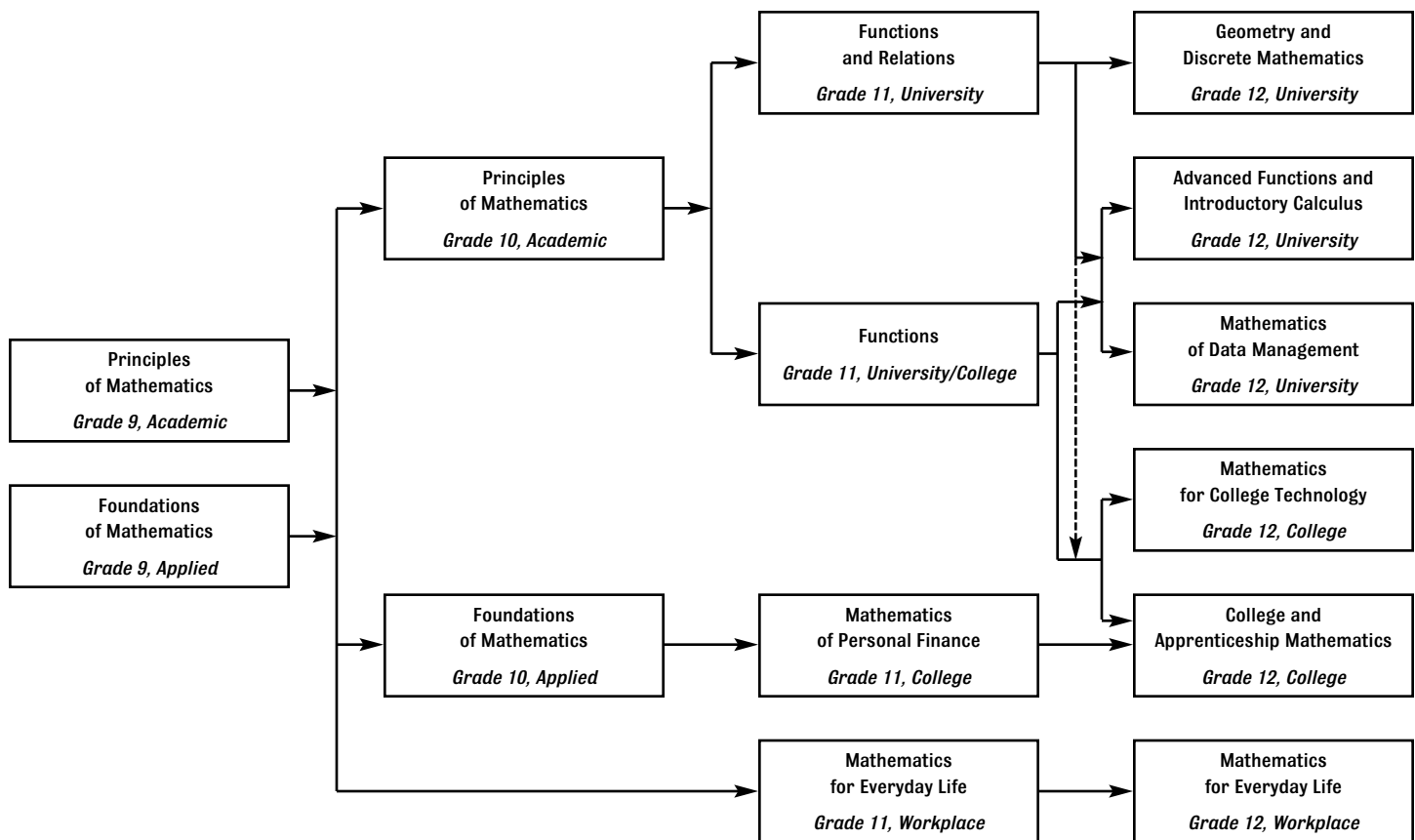
### **Courses in Mathematics, Grades 11 and 12**

<b>Grade</b>	<b>Course Name</b>	<b>Course Type</b>	<b>Course Code</b>	<b>Prerequisites</b>
11	Functions and Relations	University	MCR3U	Grade 10 Principles of Mathematics, Academic
11	Functions	University/ College	MCF3M	Grade 10 Principles of Mathematics, Academic
11	Mathematics of Personal Finance	College	MBF3C	Grade 10 Foundations of Mathematics, Applied
11	Mathematics for Everyday Life	Workplace	MEL3E	Grade 9 Mathematics, Academic or Applied
12	Advanced Functions and Introductory Calculus	University	MCB4U	Grade 11 Functions and Relations, University, or Grade 11 Functions, University/College
12	Geometry and Discrete Mathematics	University	MGA4U	Grade 11 Functions and Relations, University
12	Mathematics of Data Management	University	MDM4U	Grade 11 Functions and Relations, University, or Grade 11 Functions, University/College
12	College and Apprenticeship Mathematics	College	MAP4C	Grade 11 Mathematics of Personal Finance, College, or Grade 11 Functions, University/College (or Grade 11 Functions and Relations, University)
12	Mathematics for College Technology	College	MCT4C	Grade 11 Functions, University/College (or Grade 11 Functions and Relations, University)
12	Mathematics for Everyday Life	Workplace	MEL4E	Grade 11 Mathematics for Everyday Life, Workplace

*Note:* Each of the courses listed above is worth one credit.

## Prerequisite Chart for Mathematics, Grades 9–12

This chart maps out all the courses in the discipline and shows the links between courses and the possible prerequisites for them. It does not attempt to depict all possible movements from course to course.



*Note:* The dotted line indicates that the Grade 11 university preparation course Functions and Relations may also serve as a prerequisite for the Grade 12 college preparation courses.

## Teaching Approaches

Students in a mathematics class typically demonstrate diversity in the way they best learn. It is important, therefore, that students have opportunities to learn in a variety of ways. They should explore concepts individually and cooperatively; independently and with teacher direction; through hands-on activities; and through the study of examples followed by practice. As well, the subject of mathematics varies in terms of the type of knowledge (concepts, skills, processes) that it contains, and competence in each type may be accomplished through various teaching and learning strategies. There is no single, correct way to teach or to learn mathematics. The strategies used should vary according to the object of the learning and the needs of the students.

Learning is enhanced when embedded in a context. Well-chosen contexts for learning are those that are broad enough to allow students to explore and develop initial understandings, to identify and develop relevant supporting skills, and to gain experience with interesting applications of the new knowledge. Such rich environments open the door for students to see the “big ideas” of mathematics – the major underlying principles, such as pattern or relationship. This ability to understand key principles will encourage students to continue using mathematical reasoning throughout their lives.



Process and content, together, are important aspects of learning mathematics. An emphasis on one over the other tends to reduce the usefulness and the effectiveness of the learning. It is the integration of various aspects of mathematical knowledge that provides a powerful tool for reasoning and problem solving. This curriculum reflects a meaningful blend of both process and content.

### **Curriculum Expectations**

The expectations identified for each course describe the knowledge and skills that students are expected to develop and demonstrate in their class work, on tests, and in various other activities on which their achievement is assessed and evaluated.

Two sets of expectations are listed for each *strand*, or broad curriculum area, of each course. The *overall expectations* describe in general terms the knowledge and skills that students are expected to demonstrate by the end of each course. The *specific expectations* describe the expected knowledge and skills in greater detail. The specific expectations are organized under subheadings that correspond to the overall expectations. The organization of expectations in strands and subgroupings does not imply that the expectations in a particular strand or group are to be achieved independently of those in any other strand or group.

For some expectations, additional information is given in parentheses. Where the information begins with the abbreviation *i.e.*, it represents another way of expressing the expectation or provides a complete list of the details involved in the expectation. Where the information begins with the abbreviation *e.g.*, it comprises examples that are meant to illustrate the kind of skill, the specific area of learning, the depth of learning, and/or the level of complexity that the expectation entails. These examples are intended as a guide for teachers rather than as an exhaustive or a mandatory list. Also provided in parentheses following some expectations are sample problems.

### **Strands**

Because all but two courses in mathematics in Grades 11 and 12 have different strands (two Grade 11 courses share three strands), the focus of the strands is briefly described below for all courses.

#### **University Preparation and University/College Preparation Courses**

Two Grade 11 courses – the university preparation course, Functions and Relations, and the university/college preparation course, Functions – prepare students for the Grade 12 university preparation course Advanced Functions and Introductory Calculus. For this reason, the two courses have a common core, as reflected in the three strands Financial Applications of Sequences and Series, Trigonometric Functions, and Tools for Operating and Communicating with Functions. In the strand Financial Applications of Sequences and Series, students explore and solve problems involving applications of sequences to compound interest, annuities, and related financial decision making. In the strand Trigonometric Functions, students investigate and apply the properties of trigonometric functions, and develop facility in related manipulation skills. In the strand Tools for Operating and Communicating with Functions, students develop facility in operating with polynomials, rational expressions, and exponential expressions; develop an understanding of inverses and transformations of functions; and develop facility in using function notation and in communicating reasoning. The Grade 11 university

preparation course, Functions and Relations, is also the prerequisite to the Grade 12 university preparation course Geometry and Discrete Mathematics. For this reason, Functions and Relations has a fourth strand, Investigations in Loci and Conics, in which students extend investigations of loci into a study of the conics.

The Grade 12 university preparation course Advanced Functions and Introductory Calculus is designed for students intending to study university programs that will involve calculus. The course is focused on the properties, applications, and differential calculus of polynomial, rational, exponential, and logarithmic functions. It contains three strands: Advanced Functions, in which students explore the properties and applications of polynomial, exponential, and logarithmic functions; Underlying Concepts of Calculus, in which students develop an understanding of the basic concepts of calculus by analysing the rates of change involved in applications; and Derivatives and Applications, in which students develop, consolidate, and apply to graphing and problem solving the rules and properties of differentiation.

The Grade 12 university preparation course Geometry and Discrete Mathematics is designed for students planning to study university programs that are highly focused on mathematics, such as engineering, computer science, mathematics, and physical science. The course involves students in the exploration of the nature of proof and logical thinking through a variety of perspectives, and in the development of a broad range of tools for problem solving. In the Geometry strand, students operate with geometric and algebraic vectors, determine equations of lines and planes in three-space, solve systems of equations using matrices, and determine intersections of lines and planes in three-space. In the Proof and Problem Solving strand, students apply deductive, algebraic, and vector methods to prove some properties of plane figures. The strand also describes expectations of students regarding problem solving and working independently. In the Discrete Mathematics strand, students solve problems involving counting techniques, and relate the knowledge gained to values for the binomial coefficients. Students use mathematical induction to prove the binomial theorem and to prove the formulas for the sums of series.

In the Grade 12 university preparation course Mathematics of Data Management, each strand highlights a different aspect of the theme of the course; together, the strands assist in preparing students to deal with the increasingly technological nature of both studying and living. In the strand called Organization of Data for Analysis, students examine, use, and develop methods for organizing large amounts of data. In the Counting and Probability strand and the Statistics strand, students explore very powerful bodies of mathematical knowledge, which deal with analysing and interpreting large amounts of data. The fourth strand, Integration of the Techniques of Data Management, describes the expectation that students undertake a culminating project dealing with a significant issue that will require the application of skills from all strands of the course.

### **College Preparation Courses**

The Grade 11 college preparation course, Mathematics of Personal Finance, includes a powerful blend of topics that will equip students not only for many college destinations but also for their adult lives. In the strand called Models of Exponential Growth, students explore applications and properties of exponential functions. In the Applications of Compound

Interest and Annuities strand, students undertake an in-depth study of the effect of compound interest on saving, borrowing, and investing. In the Personal Financial Decisions strand, students explore mathematical decision making related to choices about vehicle purchase, accommodation, and careers.

The Grade 12 college preparation course College and Apprenticeship Mathematics consists of three strands. In the Applications of Statistics strand, students acquire critical skills in the area of information collection, analysis, and evaluation. In the strand called Applications of Geometry, Measurement, and Trigonometry, students apply geometric properties to the creation of plans and scale models, solve measurement problems in both the metric and the imperial systems, and solve problems involving trigonometry in triangles. Finally, in the strand called Analysis of Mathematical Models, students consolidate their skills in analysing and interpreting graphical models and in manipulating key algebraic expressions.

The Grade 12 college preparation course Mathematics for College Technology is intended for students planning to study mathematics-focused programs at college, such as diploma technology programs. Its three strands build on the function base established in Grade 11. In the strand called Polynomial Functions and Inverse Proportionality, students investigate and apply properties of polynomial functions, solve problems involving applications of inverse proportionality, and investigate the properties of reciprocal functions. In the strand Exponential and Logarithmic Functions, students investigate and apply properties of exponential and logarithmic functions. Finally, in the strand Applications and Consolidation, students analyse models of a variety of functions, explore applications of piecewise-defined functions, solve linear-quadratic systems, and consolidate key manipulation and communication skills.

### **Workplace Preparation Courses**

The Grade 11 and 12 workplace preparation courses, entitled Mathematics for Everyday Life, present a blend of topics dealing with personal finance and with areas of mathematics that may be useful in a variety of jobs. All of the strands in both the Grade 11 and Grade 12 workplace preparation courses lend themselves to the use of real data gathered in the community, and to hands-on projects and experiences.

The focus of the Grade 11 course is personal finance. In the strand called Earning, Paying Taxes, and Purchasing, students gather information and solve problems in the three areas described. In the Saving, Investing, and Borrowing strand, students make calculations involving simple and compound interest and apply them in investigating various types of investment and credit. Finally, in the Transportation and Travel strand, students investigate the purchase or lease of a vehicle and carry out projects that examine the costs involved in various forms of travel.

In the Grade 12 course, there is one strand involving personal finance – Everyday Financing – in which students gather data to investigate the costs involved in various types of accommodation, and design household budgets. In the Statistics and Probability strand, students interpret graphs and statements found in the media, and gain first-hand experience in gathering data and carrying out probability experiments. In the strand called Applications of Measurement and Geometry, students solve problems and complete projects involving measurement, transformation geometry, and design.

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## Functions and Relations, Grade 11, University Preparation

(MCR3U)

This course introduces some financial applications of mathematics, extends students' experiences with functions, and introduces second-degree relations. Students will solve problems in personal finance involving applications of sequences and series; investigate properties and applications of trigonometric functions; develop facility in operating with polynomials, rational expressions, and exponential expressions; develop an understanding of inverses and transformations of functions; and develop facility in using function notation and in communicating mathematical reasoning. Students will also investigate loci and the properties and applications of conics.

**Prerequisite:** Principles of Mathematics, Grade 10, Academic

## Financial Applications of Sequences and Series

### Overall Expectations

By the end of this course, students will:

- solve problems involving arithmetic and geometric sequences and series;
- solve problems involving compound interest and annuities;
- solve problems involving financial decision making, using spreadsheets or other appropriate technology.

### Specific Expectations

#### *Solving Problems Involving Arithmetic and Geometric Sequences and Series*

By the end of this course, students will:

- write terms of a sequence, given the formula for the  $n$ th term or given a recursion formula;
- determine a formula for the  $n$ th term of a given sequence (e.g., the  $n$ th term of the sequence  $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \dots$  is  $\frac{n}{n+1}$ );
- identify sequences as arithmetic or geometric, or neither;
- determine the value of any term in an arithmetic or a geometric sequence, using the formula for the  $n$ th term of the sequence;
- determine the sum of the terms of an arithmetic or a geometric series, using appropriate formulas and techniques.

#### *Solving Problems Involving Compound Interest and Annuities*

By the end of this course, students will:

- derive the formulas for compound interest and present value, the amount of an ordinary annuity, and the present value of an ordinary annuity, using the formulas for the  $n$ th term of a geometric sequence and the sum of the first  $n$  terms of a geometric series;
- solve problems involving compound interest and present value;

- solve problems involving the amount and the present value of an ordinary annuity;
- demonstrate an understanding of the relationships between simple interest, arithmetic sequences, and linear growth;
- demonstrate an understanding of the relationships between compound interest, geometric sequences, and exponential growth.

#### *Solving Problems Involving Financial Decision Making*

By the end of this course, students will:

- analyse the effects of changing the conditions in long-term savings plans (e.g., altering the frequency of deposits, the amount of deposit, the interest rate, the compounding period, or a combination of these) (*Sample problem:* Compare the results of making an annual deposit of \$1000 to an RRSP, beginning at age 20, with the results of making an annual deposit of \$3000, beginning at age 50);
- describe the manner in which interest is calculated on a mortgage (i.e., compounded semi-annually but calculated monthly) and compare this with the method of interest compounded monthly and calculated monthly;
- generate amortization tables for mortgages, using spreadsheets or other appropriate software;

- analyse the effects of changing the conditions of a mortgage (e.g., the effect on the length of time needed to pay off the mortgage of changing the payment frequency or the interest rate);
- communicate the solutions to problems and the findings of investigations with clarity and justification.

## Trigonometric Functions

### Overall Expectations

By the end of this course, students will:

- solve problems involving the sine law and the cosine law in oblique triangles;
- demonstrate an understanding of the meaning and application of radian measure;
- determine, through investigation, the relationships between the graphs and the equations of sinusoidal functions;
- solve problems involving models of sinusoidal functions drawn from a variety of applications.

### Specific Expectations

#### *Solving Problems Involving the Sine Law and the Cosine Law in Oblique Triangles*

By the end of this course, students will:

- determine the sine, cosine, and tangent of angles greater than  $90^\circ$ , using a suitable technique (e.g., related angles, the unit circle), and determine two angles that correspond to a given single trigonometric function value;
- solve problems in two dimensions and three dimensions involving right triangles and oblique triangles, using the primary trigonometric ratios, the cosine law, and the sine law (including the ambiguous case).

#### *Understanding the Meaning and Application of Radian Measure*

By the end of this course, students will:

- define the term *radian measure*;
- describe the relationship between radian measure and degree measure;
- represent, in applications, radian measure in exact form as an expression involving  $\pi$  (e.g.,  $\frac{\pi}{3}$ ,  $2\pi$ ) and in approximate form as a real number (e.g., 1.05);
- determine the exact values of the sine, cosine, and tangent of the special angles  $0$ ,  $\frac{\pi}{6}$ ,  $\frac{\pi}{4}$ ,  $\frac{\pi}{3}$ ,  $\frac{\pi}{2}$  and their multiples less than or equal to  $2\pi$ ;

- prove simple identities, using the Pythagorean identity,  $\sin^2 x + \cos^2 x = 1$ , and the quotient relation,

$$\tan x = \frac{\sin x}{\cos x};$$

- solve linear and quadratic trigonometric equations (e.g.,  $6 \cos^2 x - \sin x - 4 = 0$ ) on the interval  $0 \leq x \leq 2\pi$ ;
- demonstrate facility in the use of radian measure in solving equations and in graphing.

#### *Investigating the Relationships Between the Graphs and the Equations of Sinusoidal Functions*

By the end of this course, students will:

- sketch the graphs of  $y = \sin x$  and  $y = \cos x$ , and describe their periodic properties;
- determine, through investigation, using graphing calculators or graphing software, the effect of simple transformations (e.g., translations, reflections, stretches) on the graphs and equations of  $y = \sin x$  and  $y = \cos x$ ;
- determine the amplitude, period, phase shift, domain, and range of sinusoidal functions whose equations are given in the form  $y = a \sin(kx + d) + c$  or  $y = a \cos(kx + d) + c$ ;
- sketch the graphs of simple sinusoidal functions [e.g.,  $y = a \sin x$ ,  $y = \cos kx$ ,  $y = \sin(x + d)$ ,  $y = a \cos kx + c$ ];

- write the equation of a sinusoidal function, given its graph and given its properties;
- sketch the graph of  $y = \tan x$ ; identify the period, domain, and range of the function; and explain the occurrence of asymptotes.

***Solving Problems Involving***

***Models of Sinusoidal Functions***

By the end of this course, students will:

- determine, through investigation, the periodic properties of various models (e.g., the table of values, the graph, the equation) of sinusoidal functions drawn from a variety of applications;
- explain the relationship between the properties of a sinusoidal function and the parameters of its equation, within the context of an application, and over a restricted domain;
- predict the effects on the mathematical model of an application involving sinusoidal functions when the conditions in the application are varied;
- pose and solve problems related to models of sinusoidal functions drawn from a variety of applications, and communicate the solutions with clarity and justification, using appropriate mathematical forms.



## Tools for Operating and Communicating with Functions

### Overall Expectations

By the end of this course, students will:

- demonstrate facility in manipulating polynomials, rational expressions, and exponential expressions;
- demonstrate an understanding of inverses and transformations of functions and facility in the use of function notation;
- communicate mathematical reasoning with precision and clarity throughout the course.

### Specific Expectations

#### *Manipulating Polynomials, Rational Expressions, and Exponential Expressions*

By the end of this course, students will:

- solve first-degree inequalities and represent the solutions on number lines;
- add, subtract, and multiply polynomials;
- determine the maximum or minimum value of a quadratic function whose equation is given in the form  $y = ax^2 + bx + c$ , using the algebraic method of completing the square;
- identify the structure of the complex number system and express complex numbers in the form  $a + bi$ , where  $i^2 = -1$  (e.g.,  $4i$ ,  $3 - 2i$ );
- determine the real or complex roots of quadratic equations, using an appropriate method (e.g., factoring, the quadratic formula, completing the square), and relate the roots to the  $x$ -intercepts of the graph of the corresponding function;
- add, subtract, multiply, and divide complex numbers in rectangular form;
- add, subtract, multiply, and divide rational expressions, and state the restrictions on the variable values;
- simplify and evaluate expressions containing integer and rational exponents, using the laws of exponents;
- solve exponential equations (e.g.,  $4^x = 8^{x+3}$ ,  $2^{2x} - 2^x = 12$ ).

#### *Understanding Inverses and Transformations and Using Function Notation*

By the end of this course, students will:

- define the term *function*;
- demonstrate facility in the use of function notation for substituting into and evaluating functions;
- determine, through investigation, the properties of the functions defined by  $f(x) = \sqrt{x}$  [e.g., domain, range, relationship to  $f(x) = x^2$ ] and  $f(x) = \frac{1}{x}$  [e.g., domain, range, relationship to  $f(x) = x$ ];
- explain the relationship between a function and its inverse (i.e., symmetry of their graphs in the line  $y = x$ ; the interchange of  $x$  and  $y$  in the equation of the function; the interchanges of the domain and range), using examples drawn from linear and quadratic functions, and from the functions  $f(x) = \sqrt{x}$  and  $f(x) = \frac{1}{x}$ ;
- represent inverse functions, using function notation, where appropriate;
- represent transformations (e.g., translations, reflections, stretches) of the functions defined by  $f(x) = x$ ,  $f(x) = x^2$ ,  $f(x) = \sqrt{x}$ ,  $f(x) = \sin x$ , and  $f(x) = \cos x$ , using function notation;
- describe, by interpreting function notation, the relationship between the graph of a function and its image under one or more transformations;

- state the domain and range of transformations of the functions defined by  $f(x) = x$ ,  $f(x) = x^2$ ,  $f(x) = \sqrt{x}$ ,  $f(x) = \sin x$ , and  $f(x) = \cos x$ .

***Communicating Mathematical Reasoning***

By the end of this course, students will:

- explain mathematical processes, methods of solution, and concepts clearly to others;
- present problems and their solutions to a group, and answer questions about the problems and the solutions;
- communicate solutions to problems and to findings of investigations clearly and concisely, orally and in writing, using an effective integration of essay and mathematical forms;
- demonstrate the correct use of mathematical language, symbols, visuals (e.g., diagrams, graphs), and conventions;
- use graphing technology effectively (e.g., use appropriate menus and algorithms; set the graph window to display the appropriate section of a curve).

## Investigations of Loci and Conics

### Overall Expectations

By the end of this course, students will:

- represent loci, using various models (e.g., a verbal description, a diagram, a dynamic model, an equation);
- determine the equation and the key features of a conic;
- solve problems involving applications of the conics.

### Specific Expectations

#### *Representing Loci*

By the end of this course, students will:

- construct a geometric model (e.g., a diagram created by hand, a diagram created by using dynamic geometry software) to represent a described locus of points; determine the properties of the geometric model; and use the properties to interpret the locus (e.g., the locus of points equidistant from two fixed points is the right bisector of the line segment joining the two fixed points);
- explain the process used in constructing a geometric model of a described locus;
- determine an equation to represent a described locus [e.g., determine the equation of the locus of points equidistant from  $(-2, 7)$  and  $(5, 4)$ ];
- construct geometric models to represent the locus definitions of the conics;
- determine equations for conics from their locus definitions, by hand for simple particular cases [e.g., determine the equation of the locus of points the sum of whose distances from  $(-3, 0)$  and  $(3, 0)$  is 10].

#### *Determining the Equation and the Key Features of a Conic*

By the end of this course, students will:

- identify the standard forms for the equations of parabolas, circles, ellipses, and hyperbolas having centres at  $(0, 0)$  and at  $(h, k)$ ;
- identify the type of conic, given its equation in the form  $ax^2 + by^2 + 2gx + 2fy + c = 0$ ;
- determine the key features (e.g., the centre or the vertex, the focus or foci, the asymptotes, the lengths of the axes) of a conic whose equation is given in the form  $ax^2 + by^2 + 2gx + 2fy + c = 0$ , by hand in simple cases (e.g.,  $x^2 + 9y^2 - 6x + 36y - 36 = 0$ );
- sketch the graph of a conic whose equation is given in the form  $ax^2 + by^2 + 2gx + 2fy + c = 0$ ;
- illustrate the conics as intersections of planes with cones, using concrete materials or technology.

***Solving Problems Involving******Applications of the Conics***

By the end of this course, students will:

- describe the importance, within applications, of the focus of a parabola, an ellipse, or a hyperbola (e.g., all incoming rays parallel to the axis of a parabolic antenna are reflected through the focus; the planets move in elliptical orbits with the sun at one of the foci);
- pose and solve problems drawn from a variety of applications involving conics, and communicate the solutions with clarity and justification (*Sample problem:* A parabolic antenna is 320 m wide at a distance of 50 m above its vertex. Determine the distance above the vertex of the focus of the antenna);
- solve problems involving the intersections of lines and conics.

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## Functions, Grade 11, University/College Preparation (MCF3M)

This course introduces some financial applications of mathematics and extends students' experiences with functions. Students will solve problems in personal finance involving applications of sequences and series; investigate properties and applications of trigonometric functions; develop facility in operating with polynomials, rational expressions, and exponential expressions; develop an understanding of inverses and transformations of functions; and develop facility in using function notation and in communicating mathematical reasoning.

**Prerequisite:** Principles of Mathematics, Grade 10, Academic

## Financial Applications of Sequences and Series

### Overall Expectations

By the end of this course, students will:

- solve problems involving arithmetic and geometric sequences and series;
- solve problems involving compound interest and annuities;
- solve problems involving financial decision making, using spreadsheets or other appropriate technology.

### Specific Expectations

#### *Solving Problems Involving Arithmetic and Geometric Sequences and Series*

By the end of this course, students will:

- write terms of a sequence, given the formula for the  $n$ th term;
- determine a formula for the  $n$ th term of a given sequence (e.g., the  $n$ th term of the sequence  $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \dots$  is  $\frac{n}{n+1}$ );
- identify sequences as arithmetic or geometric, or neither;
- determine the value of any term in an arithmetic or a geometric sequence, using the formula for the  $n$ th term of the sequence;
- determine the sum of the terms of an arithmetic or a geometric series, using appropriate formulas and techniques.

#### *Solving Problems Involving Compound Interest and Annuities*

By the end of this course, students will:

- derive the formulas for compound interest and present value, the amount of an ordinary annuity, and the present value of an ordinary annuity, using the formulas for the  $n$ th term of a geometric sequence and the sum of the first  $n$  terms of a geometric series;
- solve problems involving compound interest and present value;

- solve problems involving the amount and the present value of an ordinary annuity;
- demonstrate an understanding of the relationships between simple interest, arithmetic sequences, and linear growth;
- demonstrate an understanding of the relationships between compound interest, geometric sequences, and exponential growth.

#### *Solving Problems Involving Financial Decision Making*

By the end of this course, students will:

- analyse the effects of changing the conditions in long-term savings plans (e.g., altering the frequency of deposits, the amount of deposit, the interest rate, the compounding period, or a combination of these) (*Sample problem:* Compare the results of making an annual deposit of \$1000 to an RRSP, beginning at age 20, with the results of making an annual deposit of \$3000, beginning at age 50);
- describe the manner in which interest is calculated on a mortgage (i.e., compounded semi-annually but calculated monthly) and compare this with the method of interest compounded monthly and calculated monthly;
- generate amortization tables for mortgages, using spreadsheets or other appropriate software;

- analyse the effects of changing the conditions of a mortgage (e.g., the effect on the length of time needed to pay off the mortgage of changing the payment frequency or the interest rate);
- communicate the solutions to problems and the findings of investigations with clarity and justification.

# Trigonometric Functions

## Overall Expectations

By the end of this course, students will:

- solve problems involving the sine law and the cosine law in oblique triangles;
- demonstrate an understanding of the meaning and application of radian measure;
- determine, through investigation, the relationships between the graphs and the equations of sinusoidal functions;
- solve problems involving models of sinusoidal functions drawn from a variety of applications.

## Specific Expectations

### *Solving Problems Involving the Sine Law and the Cosine Law in Oblique Triangles*

By the end of this course, students will:

- determine the sine, cosine, and tangent of angles greater than  $90^\circ$ , using a suitable technique (e.g., related angles, the unit circle), and determine two angles that correspond to a given single trigonometric function value;
- solve problems in two dimensions and three dimensions involving right triangles and oblique triangles, using the primary trigonometric ratios, the cosine law, and the sine law (including the ambiguous case).

### *Understanding the Meaning and Application of Radian Measure*

By the end of this course, students will:

- define the term *radian measure*;
- describe the relationship between radian measure and degree measure;
- represent, in applications, radian measure in exact form as an expression involving  $\pi$  (e.g.,  $\frac{\pi}{3}$ ,  $2\pi$ ) and in approximate form as a real number (e.g., 1.05);
- determine the exact values of the sine, cosine, and tangent of the special angles  $0$ ,  $\frac{\pi}{6}$ ,  $\frac{\pi}{4}$ ,  $\frac{\pi}{3}$ ,  $\frac{\pi}{2}$  and their multiples less than or equal to  $2\pi$ ;

- prove simple identities, using the Pythagorean identity,  $\sin^2 x + \cos^2 x = 1$ , and the quotient relation,

$$\tan x = \frac{\sin x}{\cos x}$$

- solve linear and quadratic trigonometric equations (e.g.,  $6 \cos^2 x - \sin x - 4 = 0$ ) on the interval  $0 \leq x \leq 2\pi$ ;
- demonstrate facility in the use of radian measure in solving equations and in graphing.

### *Investigating the Relationships Between the Graphs and the Equations of Sinusoidal Functions*

By the end of this course, students will:

- sketch the graphs of  $y = \sin x$  and  $y = \cos x$ , and describe their periodic properties;
- determine, through investigation, using graphing calculators or graphing software, the effect of simple transformations (e.g., translations, reflections, stretches) on the graphs and equations of  $y = \sin x$  and  $y = \cos x$ ;
- determine the amplitude, period, phase shift, domain, and range of sinusoidal functions whose equations are given in the form  $y = a \sin(kx + d) + c$  or  $y = a \cos(kx + d) + c$ ;
- sketch the graphs of simple sinusoidal functions [e.g.,  $y = a \sin x$ ,  $y = \cos kx$ ,  $y = \sin(x + d)$ ,  $y = a \cos kx + c$ ];



- write the equation of a sinusoidal function, given its graph and given its properties;
- sketch the graph of  $y = \tan x$ ; identify the period, domain, and range of the function; and explain the occurrence of asymptotes.

***Solving Problems Involving***

***Models of Sinusoidal Functions***

By the end of this course, students will:

- determine, through investigation, the periodic properties of various models (e.g., the table of values, the graph, the equation) of sinusoidal functions drawn from a variety of applications;
- explain the relationship between the properties of a sinusoidal function and the parameters of its equation, within the context of an application, and over a restricted domain;
- predict the effects on the mathematical model of an application involving sinusoidal functions when the conditions in the application are varied;
- pose and solve problems related to models of sinusoidal functions drawn from a variety of applications, and communicate the solutions with clarity and justification, using appropriate mathematical forms.

## Tools for Operating and Communicating with Functions

### Overall Expectations

By the end of this course, students will:

- demonstrate facility in manipulating polynomials, rational expressions, and exponential expressions;
- demonstrate an understanding of inverses and transformations of functions and facility in the use of function notation;
- communicate mathematical reasoning with precision and clarity throughout the course.

### Specific Expectations

#### *Manipulating Polynomials, Rational Expressions, and Exponential Expressions*

By the end of this course, students will:

- solve first-degree inequalities and represent the solutions on number lines;
- add, subtract, and multiply polynomials;
- determine the maximum or minimum value of a quadratic function whose equation is given in the form  $y = ax^2 + bx + c$ , using the algebraic method of completing the square;
- identify the structure of the complex number system and express complex numbers in the form  $a + bi$ , where  $i^2 = -1$  (e.g.,  $4i$ ,  $3 - 2i$ );
- determine the real or complex roots of quadratic equations, using an appropriate method (e.g., factoring, the quadratic formula, completing the square), and relate the roots to the  $x$ -intercepts of the graph of the corresponding function;
- add, subtract, multiply, and divide rational expressions, and state the restrictions on the variable values;
- simplify and evaluate expressions containing integer and rational exponents, using the laws of exponents;
- solve exponential equations (e.g.,  $4^x = 8^{x+3}$ ,  $2^{2x} - 2^x = 12$ ).

#### *Understanding Inverses and Transformations and Using Function Notation*

By the end of this course, students will:

- define the term *function*;
- demonstrate facility in the use of function notation for substituting into and evaluating functions;
- determine, through investigation, the properties of the functions defined by  $f(x) = \sqrt{x}$  [e.g., domain, range, relationship to  $f(x) = x^2$ ] and  $f(x) = \frac{1}{x}$  [e.g., domain, range, relationship to  $f(x) = x$ ];
- explain the relationship between a function and its inverse (i.e., symmetry of their graphs in the line  $y = x$ ; the interchange of  $x$  and  $y$  in the equation of the function; the interchanges of the domain and range), using examples drawn from linear and quadratic functions, and from the functions  $f(x) = \sqrt{x}$  and  $f(x) = \frac{1}{x}$ ;
- represent inverse functions, using function notation, where appropriate;
- represent transformations (e.g., translations, reflections, stretches) of the functions defined by  $f(x) = x$ ,  $f(x) = x^2$ ,  $f(x) = \sqrt{x}$ ,  $f(x) = \sin x$ , and  $f(x) = \cos x$ , using function notation;

- describe, by interpreting function notation, the relationship between the graph of a function and its image under one or more transformations;
- state the domain and range of transformations of the functions defined by  $f(x) = x$ ,  $f(x) = x^2$ ,  $f(x) = \sqrt{x}$ ,  $f(x) = \sin x$ , and  $f(x) = \cos x$ .

***Communicating Mathematical Reasoning***

By the end of this course, students will:

- explain mathematical processes, methods of solution, and concepts clearly to others;
- present problems and their solutions to a group, and answer questions about the problems and the solutions;
- communicate solutions to problems and to findings of investigations clearly and concisely, orally and in writing, using an effective integration of essay and mathematical forms;
- demonstrate the correct use of mathematical language, symbols, visuals (e.g., diagrams, graphs), and conventions;
- use graphing technology effectively (e.g., use appropriate menus and algorithms; set the graph window to display the appropriate section of a curve).

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## **Mathematics of Personal Finance, Grade 11, College Preparation**

(MBF3C)

This course enables students to broaden their understanding of exponential growth and of important areas of personal finance. Students will investigate properties of exponential functions and develop skills in manipulating exponential expressions; solve problems and investigate financial applications involving compound interest and annuities; and apply mathematics in making informed decisions about transportation, accommodation, and career choices.

**Prerequisite:** Foundations of Mathematics, Grade 10, Applied

## Models of Exponential Growth

### Overall Expectations

By the end of this course, students will:

- demonstrate an understanding of the nature of exponential growth;
- describe the mathematical properties of exponential functions;
- manipulate expressions related to exponential functions.

### Specific Expectations

#### *Understanding the Nature of Exponential Growth*

By the end of this course, students will:

- describe the significance of exponential growth or decay within the context of applications represented by various mathematical models (e.g., tables of values, graphs, equations);
- compare the effects of exponential growth within a context (e.g., interest earned, population size) with the effects of linear or quadratic growth within the same context;
- pose and solve problems related to models of exponential functions drawn from a variety of applications, and communicate the solutions with clarity and justification.

#### *Describing the Mathematical Properties of Exponential Functions*

By the end of this course, students will:

- sketch the graphs of simple exponential functions, given their equations [e.g., those with equations  $y = 2^x$ ,  $y = 10^x$ ,  $y = \left(\frac{1}{2}\right)^x$ ], without using technology;
- compare the rates of change of different types of functions (e.g., those with equations  $y = 2x$ ,  $y = x^2$ ,  $y = 2^x$ );
- identify, through investigations, using graphing calculators or graphing software, the key properties of exponential functions with equations of the form  $y = a^x$  ( $a > 0$ ,  $a \neq 1$ ) and their graphs (e.g., the domain is the set of the real numbers; the range is

the set of the positive real numbers; the function either increases or decreases throughout its domain; the graph has the  $x$ -axis as an asymptote and has  $y$ -intercept = 1).

#### *Manipulating Expressions*

By the end of this course, students will:

- demonstrate the quick recall or calculation of simple powers of natural numbers (e.g.,  $2^8$ ,  $6^3$ ,  $5^4$ ,  $20^2$ ), without using technology;
- evaluate simple numerical expressions involving rational exponents, without using technology;
- evaluate numerical expressions involving negative and decimal exponents, using scientific calculators;
- simplify algebraic expressions involving integral exponents, using the laws of exponents;
- solve exponential equations involving common bases (e.g.,  $2^x = 32$ ,  $4^{5x-1} = 4^{x+11}$ ,  $3^{5x+8} = 27^x$ ).

## Applications of Compound Interest and Annuities

### Overall Expectations

By the end of this course, students will:

- solve problems involving arithmetic and geometric sequences and series;
- solve problems involving compound interest and annuities;
- demonstrate an understanding of the effect on investment and borrowing of compounding interest.

### Specific Expectations

#### *Solving Problems Involving Arithmetic and Geometric Sequences and Series*

By the end of this course, students will:

- determine terms that follow three or more given terms in a sequence;
- determine whether a sequence is arithmetic or geometric, or neither;
- solve problems related to the formulas for the  $n$ th term and the sum of  $n$  terms of arithmetic and geometric sequences and series.

#### *Solving Problems Involving Compound Interest and Annuities*

By the end of this course, students will:

- solve problems involving the calculation of any variable in the simple-interest formula ( $I = Prt$ ), using scientific calculators;
- solve problems involving the calculation of the amount ( $A$ ) and the principal ( $P$ ) in the compound-interest formula  $A = P(1 + i)^n$ , using scientific calculators;
- solve problems involving the calculation of the interest rate per period ( $i$ ) and the number of periods ( $n$ ) in the compound-interest formula  $A = P(1 + i)^n$ , using a spreadsheet;
- solve problems involving the calculation of the amount and the regular payment in the formula for the amount of an ordinary annuity, using scientific calculators;

- solve problems involving the calculation of the present value and the regular payment in the formula for the present value of an ordinary annuity, using scientific calculators;
- demonstrate an understanding of the relationships between simple interest, arithmetic sequences, and linear growth;
- demonstrate an understanding of the relationships between compound interest, geometric sequences, and exponential growth.

#### *Understanding the Effect of Compounding*

By the end of this course, students will:

- determine, through investigation, the characteristics of various savings alternatives available from a financial institution (e.g., savings accounts, GICs);
- determine the effect of compound interest on deposits made into savings accounts (e.g., determine the doubling period of a single deposit; demonstrate the effect of saving a small amount on a regular basis; compare the effects of different compounding periods);
- determine, through investigation, the properties of a variety of investment alternatives (e.g., stocks, bonds, mutual funds, real estate), and compare the alternatives from the point of view of risk versus return;

- demonstrate, through calculation, the advantages of early deposits to long-term savings plans (e.g., compare the results of making an annual deposit of \$1000 to an RRSP, beginning at age 20, with the results of making an annual deposit of \$3000, beginning at age 50);
- identify the common terminology and features associated with mortgages;
- describe the manner in which interest is usually calculated on a mortgage (i.e., compounded semi-annually but calculated monthly) and compare this with the method of interest compounded monthly and calculated monthly;
- generate an amortization table for a mortgage, using a spreadsheet or other appropriate software;
- calculate the total amount of interest paid over the life of a mortgage, using a spreadsheet or other appropriate software, and compare the amount with the original principal of the mortgage or value of the property;
- compare the effects of various payment periods, payment amounts, and interest rates on the length of time needed to pay off a mortgage;
- demonstrate, through calculations, using technology, the effect on interest paid of retiring a loan before it is due;
- determine, through investigation, the features of various credit and debit cards;
- demonstrate, using technology, the effects of delayed payment on a credit card balance, on the basis of current credit card rates and regulations;
- calculate the cost of borrowing to purchase a costly item (e.g., a car, a stereo);
- design an effective financial plan to facilitate the achievement of a long-term goal (e.g., attending college, purchasing a car, moving into an apartment, purchasing a house, establishing a small business).

## Personal Financial Decisions

### Overall Expectations

By the end of this course, students will:

- demonstrate an understanding of the costs involved in owning and operating a vehicle;
- determine, through investigation, the relative costs of renting an apartment and buying a house;
- design effective personal and household budgets for individuals and families described in case studies;
- demonstrate the ability to make informed decisions involving life situations;
- apply decision making in the investigation of career opportunities.

### Specific Expectations

#### *Owning and Operating a Vehicle*

By the end of this course, students will:

- identify the procedures, costs, advantages, and disadvantages involved in buying a new vehicle and a used vehicle;
- compare the costs involved in buying versus leasing the same vehicle;
- calculate the fixed and variable costs involved in owning and operating a vehicle (e.g., the licence fee, insurance, maintenance);
- determine, through investigation, the cost of purchasing or leasing a chosen new vehicle or purchasing a chosen used vehicle, including financing.

#### *Renting or Buying Accommodation*

By the end of this course, students will:

- collect, organize, and analyse data involving the costs of various kinds of accommodation in the community;
- compare the costs of maintaining an apartment with the costs of maintaining a house;
- compare the advantages and disadvantages of renting accommodation with the advantages and disadvantages of buying accommodation;
- summarize the findings of investigations in effective presentations, blending written and visual forms.

#### *Designing Budgets*

By the end of this course, students will:

- describe and estimate the living costs involved for different family groupings (e.g., a family of four, including two young children; a single young person; a single parent with one child);
- design a budget suitable for a family described in a given case study, reflecting the current costs of common items (e.g., interest rates, utility rates, rents), using technology (e.g., spreadsheets, budgeting software, the Internet);
- explain and justify budgets, using appropriate mathematical forms (e.g., written explanations, charts, tables, graphs, calculations);
- determine the effect on an overall budget of changing one component, using a spreadsheet or budgeting software.

#### *Making Informed Decisions*

By the end of this course, students will:

- describe a decision involving a choice between alternatives (e.g., Which program should I study at college? What car should I buy? Should I stay at home or rent an apartment?);
- collect relevant information related to the alternatives to be considered in making a decision;



- summarize the advantages and disadvantages of the alternatives to a decision, using lists and organization charts;
- compare alternatives by rating and ranking information and by applying mathematical calculations and analysis, as appropriate (e.g., calculating loan payments or interest rates; constructing graphs or tables), using technology;
- explain the process used in making a decision and justify the conclusions reached;
- identify the advantages and disadvantages to the purchaser of various types of selling (e.g., retail store, catalogue, telemarketing, multilevel marketing, Internet) and techniques of selling (the use of loss leaders, the use of incentives such as coupons or Air Miles);
- compare the value of the Canadian dollar with the values of foreign currencies over a period of time and identify possible effects on purchasing and travel decisions.

### ***Investigating Career Opportunities***

By the end of this course, students will:

- identify the advantages and disadvantages of a variety of occupations of personal interest;
- compare the expected income for a variety of occupations with the costs of the education or training required;
- analyse employment trends to identify some occupations that are in high demand, and identify the skills required and the education paths recommended in order to qualify for these occupations.

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## **Mathematics for Everyday Life, Grade 11, Workplace Preparation**

(MEL3E)

This course enables students to broaden their understanding of mathematics as it is applied in important areas of day-to-day living. Students will solve problems associated with earning money, paying taxes, and making purchases; apply calculations of simple and compound interest in saving, investing, and borrowing; and calculate the costs of transportation and travel in a variety of situations.

**Prerequisite:** Mathematics, Grade 9, Academic or Applied

## Earning, Paying Taxes, and Purchasing

### Overall Expectations

By the end of this course, students will:

- solve problems involving different types of remuneration;
- describe various forms of taxation;
- solve problems involving the purchasing of items.

### Specific Expectations

#### *Earning Money*

By the end of this course, students will:

- solve problems involving various ways that an employee can be paid (e.g., salary, hourly rate, overtime, commission), using calculators or appropriate software;
- explain the differences between gross pay and net pay, and describe possible payroll deductions (e.g., for a pension plan, a savings plan, employment insurance, union dues);
- calculate gross pay and net pay for given situations;
- describe the effects on personal spending habits of the frequency of pay period (e.g., weekly, biweekly, monthly);
- determine the remuneration for chosen occupations, including salary and benefits, and evaluate it in terms of purchasing power and living standards.

#### *Describing Forms of Taxation*

By the end of this course, students will:

- solve problems involving the estimation and calculation of provincial and federal sales taxes;
- identify the information and documents required for filing a personal income tax return, and explain why they are required;
- identify agencies in the community that will complete or help to complete a personal income tax return;
- identify other forms of taxation (e.g., taxes included in the prices of gasoline and tobacco).

#### *Purchasing Items*

By the end of this course, students will:

- provide the correct change for an amount offered (e.g., provide the correct change for a charge of \$13.87 when the amount offered is \$20.00; provide the correct change for a charge of \$13.87 when the amount offered is \$15.12);
- select compatible amounts to offer for a given charge to reduce the number of coins received in the change (e.g., what payments might a person offer for a charge of \$46.36 in order to reduce the number of coins received in the change?);
- estimate and calculate the unit prices of comparable items to determine the best buy;
- estimate and calculate discounts, sale prices, and after-tax costs;
- identify various incentives to make purchases (e.g., Air Miles, coupons, stamps, interest-free loans), and explain their characteristics;
- estimate and calculate the price in Canadian funds of items bought in or ordered from another country;
- make a decision regarding the purchase of a costly item by identifying and ranking criteria for the comparison of possible choices;
- identify, calculate, and compare the interest costs involved in making purchases under various plans (e.g., instalment, layaway, credit card, credit line), using technology (e.g., spreadsheets, money-management software).

## Saving, Investing, and Borrowing

### Overall Expectations

By the end of this course, students will:

- calculate simple and compound interest;
- solve problems involving savings and investment alternatives;
- solve problems involving different ways of borrowing.

### Specific Expectations

#### *Calculating Simple and Compound Interest*

By the end of this course, students will:

- calculate interest earned and total amount in applications involving simple interest;
- describe the differences between simple interest and compound interest;
- calculate compound interest by using the simple-interest formula and a given spreadsheet template;
- solve problems involving the amount ( $A$ ) resulting from compound-interest calculations, using the formula  $A = P(1 + i)^n$ ;
- identify, through the use of technology, the effects of different compounding periods on the amount of a loan or an investment;
- construct graphs to represent the growth in the value of an investment over time, using spreadsheets or graphing technology.

#### *Understanding Saving and Investing*

By the end of this course, students will:

- explain the features of various savings alternatives commonly available from financial institutions (e.g., savings accounts, GICs, mutual funds);
- identify the types of transactions available through automated teller machines and online banking, and the fee(s) related to each type;
- interpret and check the accuracy of transaction codes and entries on a monthly banking or financial statement or in a passbook;

- determine the effect of compound interest on deposits made into savings accounts, using a given spreadsheet template for repeated calculations (e.g., the effect of saving a small amount on a regular basis);
- identify the characteristics of different types of investments (e.g., mutual funds, bonds, stocks);
- demonstrate, through the use of technology, the advantages of early deposits to long-term savings plans (e.g., compare the results of making a deposit of \$1000 to an RRSP, beginning at age 20, with the results of making a deposit of \$3000, beginning at age 50);
- monitor the value of investments (e.g., mutual funds, stocks) over a period of time, using technology (e.g., a spreadsheet, the Internet);
- demonstrate an understanding of risk tolerance and how it changes during different life stages.

#### *Understanding Borrowing*

By the end of this course, students will:

- describe the features of various credit cards and debit cards;
- demonstrate, through the use of technology, the effects of delayed payment on a credit card balance, using current rates;
- describe the features and conditions of various short-term loans (e.g., car loans, loans to consolidate debt, lines of credit);

- generate an amortization table for a personal loan whose features are described, using a given spreadsheet template;
- calculate the total amount of interest paid over the life of a personal loan, using a given spreadsheet template, and compare this amount with the original principal of the loan;
- compare the effects of various payment periods on the length of time needed to pay off loans, using a given spreadsheet template;
- explain the advantages and disadvantages of borrowing.

## Transportation and Travel

### Overall Expectations

By the end of this course, students will:

- demonstrate an understanding of the costs involved in owning and operating an automobile;
- demonstrate an understanding of the costs involved in travelling by automobile;
- compare the costs of making a trip by automobile, by train, by bus, or by airplane.

### Specific Expectations

#### *Understanding the Costs of Owning and Operating a Vehicle*

By the end of this course, students will:

- describe the procedures and costs involved in obtaining a driver's licence;
- compare the procedures, costs, advantages, and disadvantages involved in buying a new versus a used vehicle;
- compare the costs involved in buying versus leasing the same new vehicle;
- identify the factors and costs involved in insuring a vehicle;
- identify the costs of failing to operate a vehicle responsibly (e.g., fines, legal costs);
- calculate the fixed and variable costs involved in owning and operating a vehicle;
- complete a project involving the purchase or lease of a new vehicle or the purchase of a used vehicle, including the costs of insurance;
- compare the costs of owning or leasing and maintaining a vehicle with the costs of using public transportation.

#### *Understanding the Costs of Travelling by Automobile*

By the end of this course, students will:

- plan a travel route, by considering a variety of factors (e.g., the estimated distances involved, the purpose of the trip, the time of year, probable road conditions, personal interest);
- estimate the costs involved in a trip by automobile (e.g., gasoline, accommodation, food, entertainment), using real data acquired from authentic sources (e.g., automobile association travel books, travel guides, the Internet);
- explain the cost estimate for a trip by automobile in a clear, detailed presentation.

#### *Comparing Travel Costs*

By the end of this course, students will:

- identify sources of information for routes, schedules, and fares for travel by airplane, train, or bus;
- interpret airline, train, or bus schedules;
- compare the costs of travelling to a given destination by airplane, train, or bus;
- describe the advantages and disadvantages of travelling to a given destination by airplane, train, and bus.

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## Advanced Functions and Introductory Calculus, Grade 12, University Preparation

(MCB4U)

This course builds on students' experience with functions and introduces the basic concepts and skills of calculus. Students will investigate and apply the properties of polynomial, exponential, and logarithmic functions; broaden their understanding of the mathematics associated with rates of change; and develop facility with the concepts and skills of differential calculus as applied to polynomial, rational, exponential, and logarithmic functions. Students will apply these skills to problem solving in a range of applications.

**Prerequisite:** Functions and Relations, Grade 11, University Preparation, or Functions, Grade 11, University/College Preparation

## Advanced Functions

### Overall Expectations

By the end of this course, students will:

- determine, through investigation, the characteristics of the graphs of polynomial functions of various degrees;
- demonstrate facility in the algebraic manipulation of polynomials;
- demonstrate an understanding of the nature of exponential growth and decay;
- define and apply logarithmic functions;
- demonstrate an understanding of the operation of the composition of functions.

### Specific Expectations

#### *Investigating the Graphs of Polynomial Functions*

By the end of this course, students will:

- determine, through investigation, using graphing calculators or graphing software, various properties of the graphs of polynomial functions (e.g., determine the effect of the degree of a polynomial function on the shape of its graph; the effect of varying the coefficients in the polynomial function; the type and the number of  $x$ -intercepts; the behaviour near the  $x$ -intercepts; the end behaviours; the existence of symmetry);
- describe the nature of change in polynomial functions of degree greater than two, using finite differences in tables of values;
- compare the nature of change observed in polynomial functions of higher degree with that observed in linear and quadratic functions;
- sketch the graph of a polynomial function whose equation is given in factored form;
- determine an equation to represent a given graph of a polynomial function, using methods appropriate to the situation (e.g., using the zeros of the function; using a trial-and-error process on a graphing calculator or graphing software; using finite differences).

#### *Manipulating Algebraic Expressions*

By the end of this course, students will:

- demonstrate an understanding of the remainder theorem and the factor theorem;
- factor polynomial expressions of degree greater than two, using the factor theorem;
- determine, by factoring, the real or complex roots of polynomial equations of degree greater than two;
- determine the real roots of non-factorable polynomial equations by interpreting the graphs of the corresponding functions, using graphing calculators or graphing software;
- write the equation of a family of polynomial functions, given the real or complex zeros [e.g., a polynomial function having non-repeated zeros 5,  $-3$ , and  $-2$  will be defined by the equation  $f(x) = k(x - 5)(x + 3)(x + 2)$ , for  $k \in \mathbb{R}$ ];
- describe intervals and distances, using absolute-value notation;
- solve factorable polynomial inequalities;
- solve non-factorable polynomial inequalities by graphing the corresponding functions, using graphing calculators or graphing software and identifying intervals above and below the  $x$ -axis;



- solve problems involving the abstract extensions of algorithms (e.g., a problem involving the nature of the roots of polynomial equations: If  $h$  and  $k$  are the roots of the equation  $3x^2 + 28x - 20 = 0$ , find the equation whose roots are  $h + k$  and  $hk$ ; a problem involving the factor theorem: For what values of  $k$  does the function  $f(x) = x^3 + 6x^2 + kx - 4$  give the same remainder when divided by either  $x - 1$  or  $x + 2$ ?).

#### ***Understanding the Nature of Exponential Growth and Decay***

By the end of this course, students will:

- identify, through investigations, using graphing calculators or graphing software, the key properties of exponential functions of the form  $a^x$  ( $a > 0$ ,  $a \neq 1$ ) and their graphs (e.g., the domain is the set of the real numbers; the range is the set of the positive real numbers; the function either increases or decreases throughout its domain; the graph has the  $x$ -axis as an asymptote and has  $y$ -intercept = 1);
- describe the graphical implications of changes in the parameters  $a$ ,  $b$ , and  $c$  in the equation  $y = ca^x + b$ ;
- compare the rates of change of the graphs of exponential and non-exponential functions (e.g., those with equations  $y = 2x$ ,  $y = x^2$ ,  $y = x^{\frac{1}{2}}$  and  $y = 2^x$ );
- describe the significance of exponential growth or decay within the context of applications represented by various mathematical models (e.g., tables of values, graphs);
- pose and solve problems related to models of exponential functions drawn from a variety of applications, and communicate the solutions with clarity and justification.

#### ***Defining and Applying Logarithmic Functions***

By the end of this course, students will:

- define the logarithmic function  $\log_a x$  ( $a > 1$ ) as the inverse of the exponential function  $a^x$ , and compare the properties of the two functions;
- express logarithmic equations in exponential form, and vice versa;
- simplify and evaluate expressions containing logarithms;
- solve exponential and logarithmic equations, using the laws of logarithms;
- solve simple problems involving logarithmic scales (e.g., the Richter scale, the pH scale, the decibel scale).

#### ***Understanding the Composition of Functions***

By the end of this course, student will:

- identify composition as an operation in which two functions are applied in succession;
- demonstrate an understanding that the composition of two functions exists only when the range of the first function overlaps the domain of the second;
- determine the composition of two functions expressed in function notation;
- decompose a given composite function into its constituent parts;
- describe the effect of the composition of inverse functions [i.e.,  $f(f^{-1}(x)) = x$ ].

## Underlying Concepts of Calculus

### Overall Expectations

By the end of this course, students will:

- determine and interpret the rates of change of functions drawn from the natural and social sciences;
- demonstrate an understanding of the graphical definition of the derivative of a function;
- demonstrate an understanding of the relationship between the derivative of a function and the key features of its graph.

### Specific Expectations

#### *Understanding Rates of Change*

By the end of this course, students will:

- pose problems and formulate hypotheses regarding rates of change within applications drawn from the natural and social sciences;
- calculate and interpret average rates of change from various models (e.g. equations, tables of values, graphs) of functions drawn from the natural and social sciences;
- estimate and interpret instantaneous rates of change from various models (e.g. equations, tables of values, graphs) of functions drawn from the natural and social sciences;
- explain the difference between average and instantaneous rates of change within applications and in general;
- make inferences from models of applications and compare the inferences with the original hypotheses regarding rates of change.

#### *Understanding the Graphical Definition of the Derivative*

By the end of this course, students will:

- demonstrate an understanding that the slope of a secant on a curve represents the average rate of change of the function over an interval, and that the slope of the tangent to a curve at a point represents the instantaneous rate of change of the function at that point;

- demonstrate an understanding that the slope of the tangent to a curve at a point is the limiting value of the slopes of a sequence of secants;
- demonstrate an understanding that the instantaneous rate of change of a function at a point is the limiting value of a sequence of average rates of change;
- demonstrate an understanding that the derivative of a function at a point is the instantaneous rate of change or the slope of the tangent to the graph of the function at that point.

#### *Connecting Derivatives and Graphs*

By the end of this course, students will:

- describe the key features of a given graph of a function, including intervals of increase and decrease, critical points, points of inflection, and intervals of concavity;
- identify the nature of the rate of change of a given function, and the rate of change of the rate of change, as they relate to the key features of the graph of that function;
- sketch, by hand, the graph of the derivative of a given graph.

## Derivatives and Applications

### Overall Expectations

By the end of this course, students will:

- demonstrate an understanding of the first-principles definition of the derivative;
- determine the derivatives of given functions, using manipulative procedures;
- determine the derivatives of exponential and logarithmic functions;
- solve a variety of problems, using the techniques of differential calculus;
- sketch the graphs of polynomial, rational, and exponential functions;
- analyse functions, using differential calculus.

### Specific Expectations

#### *Understanding the First-Principles Definition of the Derivative*

By the end of this course, students will:

- determine the limit of a polynomial, a rational, or an exponential function;
- demonstrate an understanding that limits can give information about some behaviours of graphs of functions [e.g.,  $\lim_{x \rightarrow 5} \frac{x^2 - 25}{x - 5}$  predicts a hole at (5, 10)];
- identify examples of discontinuous functions and the types of discontinuities they illustrate;
- determine the derivatives of polynomial and simple rational functions from first principles, using the definitions of the derivative function,

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \text{ and}$$

$$f'(a) = \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a};$$

- identify examples of functions that are not differentiable.

#### *Determining Derivatives*

By the end of this course, students will:

- justify the constant, power, sum-and-difference, product, quotient, and chain rules for determining derivatives;
- determine the derivatives of polynomial and rational functions, using the constant, power, sum-and-difference, product, quotient, and chain rules for determining derivatives;
- determine second derivatives;
- determine derivatives, using implicit differentiation in simple cases (e.g.,  $4x^2 + 9y^2 = 36$ ).

#### *Determining the Derivatives of Exponential and Logarithmic Functions*

By the end of this course, students will:

- identify  $e$  as  $\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n$  and approximate the limit, using informal methods;
- define  $\ln x$  as the inverse function of  $e^x$ ;
- determine the derivatives of the exponential functions  $a^x$  and  $e^x$  and the logarithmic functions  $\log_a x$  and  $\ln x$ ;
- determine the derivatives of combinations of the basic polynomial, rational, exponential, and logarithmic functions, using the rules for sums, differences, products, quotients, and compositions of functions.

**Using Differential Calculus to Solve Problems**

By the end of this course, students will:

- determine the equation of the tangent to the graph of a polynomial, a rational, an exponential, or a logarithmic function, or of a conic;
- solve problems of rates of change drawn from a variety of applications (including distance, velocity, and acceleration) involving polynomial, rational, exponential, or logarithmic functions;
- solve optimization problems involving polynomial and rational functions;
- solve related-rates problems involving polynomial and rational functions.

**Sketching the Graphs of Polynomial, Rational, and Exponential Functions**

By the end of this course, students will:

- determine, from the equation of a rational function, the intercepts and the positions of the vertical and the horizontal or oblique asymptotes to the graph of the function;
- determine, from the equation of a polynomial, a rational, or an exponential function, the key features of the graph of the function (i.e., intervals of increase and decrease, critical points, points of inflection, and intervals of concavity), using the techniques of differential calculus, and sketch the graph by hand;
- determine, from the equation of a simple combination of polynomial, rational, or exponential functions (e.g.,  $f(x) = \frac{e^x}{x}$ ), the key features of the graph of the combination of functions, using the techniques of differential calculus, and sketch the graph by hand;

- sketch the graphs of the first and second derivative functions, given the graph of the original function;
- sketch the graph of a function, given the graph of its derivative function.

**Using Calculus Techniques to Analyse Models of Functions**

By the end of this course, students will:

- determine the key features of a mathematical model of an application drawn from the natural or social sciences, using the techniques of differential calculus;
- compare the key features of a mathematical model with the features of the application it represents;
- predict future behaviour within an application by extrapolating from a mathematical model of a function;
- pose questions related to an application and answer them by analysing mathematical models, using the techniques of differential calculus;
- communicate findings clearly and concisely, using an effective integration of essay and mathematical forms.

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## **Geometry and Discrete Mathematics, Grade 12, University Preparation**

(MGA4U)

This course enables students to broaden mathematical knowledge and skills related to abstract mathematical topics and to the solving of complex problems. Students will solve problems involving geometric and Cartesian vectors, and intersections of lines and planes in three-space. They will also develop an understanding of proof, using deductive, algebraic, vector, and indirect methods. Students will solve problems involving counting techniques and prove results using mathematical induction.

**Prerequisite:** Functions and Relations, Grade 11, University Preparation

# Geometry

## Overall Expectations

By the end of this course, students will:

- perform operations with geometric and Cartesian vectors;
- determine intersections of lines and planes in three-space.

## Specific Expectations

### *Operating with Geometric and Cartesian Vectors*

By the end of this course, students will:

- represent vectors as directed line segments;
- perform the operations of addition, subtraction, and scalar multiplication on geometric vectors;
- determine the components of a geometric vector and the projection of a geometric vector;
- model and solve problems involving velocity and force;
- determine and interpret the dot product and cross product of geometric vectors;
- represent Cartesian vectors in two-space and in three-space as ordered pairs or ordered triples;
- perform the operations of addition, subtraction, scalar multiplication, dot product, and cross product on Cartesian vectors.

### *Determining Intersections of Lines and Planes in Three-Space*

By the end of this course, students will:

- determine the vector and parametric equations of lines in two-space and the vector, parametric, and symmetric equations of lines in three-space;
- determine the intersections of lines in three-space;
- determine the vector, parametric, and scalar equations of planes;

- determine the intersection of a line and a plane in three-space;
- solve systems of linear equations involving up to three unknowns, using row reduction of matrices, with and without the aid of technology;
- interpret row reduction of matrices as the creation of a new linear system equivalent to the original;
- determine the intersection of two or three planes by setting up and solving a system of linear equations in three unknowns;
- interpret a system of two linear equations in two unknowns and a system of three linear equations in three unknowns geometrically, and relate the geometrical properties to the type of solution set the system of equations possesses;
- solve problems involving the intersections of lines and planes, and present the solutions with clarity and justification.

## Proof and Problem Solving

### Overall Expectations

By the end of this course, students will:

- prove properties of plane figures by deductive, algebraic, and vector methods;
- solve problems, using a variety of strategies;
- complete significant problem-solving tasks independently.

### Specific Expectations

#### *Proving Properties of Plane Figures by Deductive, Algebraic, and Vector Methods*

By the end of this course, students will:

- demonstrate an understanding of the principles of deductive proof (e.g., the role of axioms; the use of “if . . . then” statements; the use of “if and only if” statements and the necessity to prove them in both directions; the fact that the converse of a proposition differs from the proposition) and of the relationship of deductive proof to inductive reasoning;
- prove some properties of plane figures (e.g., circles, parallel lines, congruent triangles, right triangles), using deduction;
- prove some properties of plane figures (e.g., the midpoints of the sides of a quadrilateral are the vertices of a parallelogram; the line segment joining the midpoints of two sides of a triangle is parallel to the third side) algebraically, using analytic geometry;
- prove some properties of plane figures, using vector methods;
- prove some properties of plane figures, using indirect methods;
- demonstrate an understanding of the relationship between formal proof and the illustration of properties that is carried out by using dynamic geometry software.

#### *Using a Variety of Strategies to Solve Problems*

By the end of this course, students will:

- solve problems by effectively combining a variety of problem-solving strategies (e.g., brainstorming, considering cases, choosing algebraic/geometric/vector or direct/indirect approaches, working backwards, visualizing by using concrete materials or diagrams or software, iterating, varying parameters, creating a model, introducing a coordinate system);
- generate multiple solutions to the same problem;
- use technology effectively in making and testing conjectures;
- solve complex problems and present the solutions with clarity and justification.

#### *Completing Significant Problem-Solving Tasks Independently*

By the end of this course, students will:

- solve problems of significance, working independently, as individuals and in small groups;
- solve problems requiring effort over extended periods of time;
- demonstrate significant learning and the effective use of skills in tasks such as solving challenging problems, researching problems, applying mathematics, creating proofs, using technology effectively, and presenting course topics or extensions of course topics.

## Discrete Mathematics

### Overall Expectations

By the end of this course, students will:

- solve problems, using counting techniques;
- prove results, using mathematical induction.

### Specific Expectations

#### *Using Counting Techniques*

By the end of this course, students will:

- solve problems, using the additive and multiplicative counting principles;
- express the answers to permutation and combination problems, using standard combinatorial symbols  
[e.g.,  $\binom{n}{r}$ ,  $P(n, r)$ ];
- evaluate expressions involving factorial notation, using appropriate methods (e.g., evaluate mentally, by hand, by using a calculator);
- solve problems involving permutations and combinations, including problems that require the consideration of cases;
- explain solutions to counting problems with clarity and precision;
- describe the connections between Pascal's triangle, values of  $\binom{n}{r}$ , and values for the binomial coefficients;
- solve problems, using the binomial theorem to determine terms in the expansion of a binomial.

#### *Using Mathematical Induction to Prove Results*

By the end of this course, students will:

- demonstrate an understanding of the principle of mathematical induction;
- use sigma notation to represent a series or the sum of a series;
- prove the formulas for the sums of series, using mathematical induction;
- prove the binomial theorem, using mathematical induction;
- prove relationships between the coefficients in Pascal's triangle, by mathematical induction and directly.



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## Mathematics of Data Management, Grade 12, University Preparation

(MDM4U)

This course broadens students' understanding of mathematics as it relates to managing information. Students will apply methods for organizing large amounts of information; apply counting techniques, probability, and statistics in modelling and solving problems; and carry out a culminating project that integrates the expectations of the course and encourages perseverance and independence. Students planning to pursue university programs in business, the social sciences, or the humanities will find this course of particular interest.

**Prerequisite:** Functions and Relations, Grade 11, University Preparation, or Functions, Grade 11, University/College Preparation

## Organization of Data for Analysis

### Overall Expectations

By the end of this course, students will:

- organize data to facilitate manipulation and retrieval;
- solve problems involving complex relationships, with the aid of diagrams;
- model situations and solve problems involving large amounts of information, using matrices.

### Specific Expectations

#### *Organizing Data*

By the end of this course, students will:

- locate data to answer questions of significance or personal interest, by searching well-organized databases;
- use the Internet effectively as a source for databases;
- create database or spreadsheet templates that facilitate the manipulation and retrieval of data from large bodies of information that have a variety of characteristics (e.g., a compact disc collection classified by artist, by date, by type of music).

#### *Using Diagrams to Solve Problems*

By the end of this course, students will:

- represent simple iterative processes (e.g., the water cycle, a person's daily routine, the creation of a fractal design), using diagrams that involve branches and loops;
- represent complex tasks (e.g., searching a list by using algorithms; classifying organisms; calculating dependent or independent outcomes in probability) or issues (e.g., the origin of global warming), using diagrams (e.g., tree diagrams, network diagrams, cause-and-effect diagrams, time lines);
- solve network problems (e.g., scheduling problems, optimum-path problems, critical-path problems), using introductory graph theory.

#### *Using Matrices to Model and Solve Problems*

By the end of this course, students will:

- represent numerical data, using matrices, and demonstrate an understanding of terminology and notation related to matrices;
- demonstrate proficiency in matrix operations, including addition, scalar multiplication, matrix multiplication, the calculation of row sums, and the calculation of column sums, as necessary to solve problems, with and without the aid of technology;
- solve problems drawn from a variety of applications (e.g., inventory control, production costs, codes), using matrix methods.

## Counting and Probability

### Overall Expectations

By the end of this course, students will:

- solve counting problems and clearly communicate the results;
- determine and interpret theoretical probabilities, using combinatorial techniques;
- design and carry out simulations to estimate probabilities.

### Specific Expectations

#### *Solving Counting Problems*

By the end of this course, students will:

- use Venn diagrams as a tool for organizing information in counting problems;
- solve introductory counting problems involving the additive and multiplicative counting principles;
- express the answers to permutation and combination problems, using standard combinatorial symbols, [e.g.,  $\binom{n}{r}$ ,  $P(n, r)$ ];
- evaluate expressions involving factorial notation, using appropriate methods (e.g., evaluating mentally, by hand, by using a calculator);
- solve problems, using techniques for counting permutations where some objects may be alike;
- solve problems, using techniques for counting combinations;
- identify patterns in Pascal’s triangle and relate the terms of Pascal’s triangle to values of  $\binom{n}{r}$ , to the expansion of a binomial, and to the solution of related problems (*Sample problem:* A girl’s school is 5 blocks west and 3 blocks south of her home. Assuming that she leaves home and walks only west or south, how many different routes can she take to school?);
- communicate clearly, coherently, and precisely the solutions to counting problems.

#### *Determining and Interpreting Theoretical Probabilities*

By the end of this course, students will:

- solve probability problems involving combinations of simple events, using counting techniques [i.e.,  $P(A \text{ or } B)$ ,  $P(A \text{ and } B)$ , and  $P(\sim A)$ ];
- identify examples of discrete random variables (e.g., the sums that are possible when two dice are rolled);
- construct a discrete probability distribution function by calculating the probabilities of a discrete random variable;
- calculate expected values and interpret them within applications (e.g., lottery prizes, tests of the fairness of games, estimates of wildlife populations) as averages over a large number of trials;
- determine probabilities, using the binomial distribution (*Sample problem:* A light-bulb manufacturer estimates that 0.5% of the bulbs manufactured are defective. If a client places an order for 100 bulbs, what is the probability that at least one bulb is defective?);
- interpret probability statements, including statements about odds, from a variety of sources.

***Simulating and Predicting***

By the end of this course, students will:

- identify the advantages of using simulations in contexts;
- design and carry out simulations to estimate probabilities in situations for which the calculation of the theoretical probabilities is difficult or impossible (*Sample problem*: A set of 6 baseball cards can be collected from cereal boxes. If the different cards are evenly distributed throughout the boxes, carry out a simulation to determine the probability of collecting one complete set in a purchase of 14 boxes);
- assess the validity of some simulation results by comparing them with the theoretical probabilities, using the probability concepts developed in the course (*Sample problem*: A light-bulb manufacturer estimates that 0.5% of the bulbs manufactured are defective. Carry out a simulation to estimate the probability that at least one bulb is defective in an order of 100 bulbs).

## Statistics

### Overall Expectations

By the end of this course, students will:

- demonstrate an understanding of standard techniques for collecting data;
- analyse data involving one variable, using a variety of techniques;
- solve problems involving the normal distribution;
- describe the relationship between two variables by interpreting the correlation coefficient;
- evaluate the validity of statistics drawn from a variety of sources.

### Specific Expectations

#### *Collecting Data*

By the end of this course, students will:

- demonstrate an understanding of the purpose and the use of a variety of sampling techniques (e.g., a simple random sample, a systematic sample, a stratified sample);
- describe different types of bias that may arise in surveys (e.g., response bias, measurement bias, non-response bias, sampling bias);
- illustrate sampling bias and variability by comparing the characteristics of a known population with the characteristics of samples taken repeatedly from that population, using different sampling techniques;
- organize and summarize data from secondary sources (e.g., the Internet, computer databases), using technology (e.g., spreadsheets, graphing calculators).

#### *Analysing Data Involving One Variable*

By the end of this course, students will:

- compute, using technology, measures of one-variable statistics (i.e., the mean, median, mode, range, interquartile range, variance, and standard deviation), and demonstrate an understanding of the appropriate use of each measure;
- interpret one-variable statistics to describe characteristics of a data set;

- describe the position of individual observations within a data set, using z-scores and percentiles.

#### *Solving Problems Involving the Normal Distribution*

By the end of this course, students will:

- identify situations that give rise to common distributions (e.g., bimodal, U-shaped, exponential, skewed, normal);
- demonstrate an understanding of the properties of the normal distribution (e.g., the mean, median, and mode are equal; the curve is symmetric about the mean; 68% of the population are within one standard deviation of the mean) and use these properties to solve problems;
- make probability statements about normal distributions, on the basis of information drawn from a variety of applications.

#### *Describing the Relationship Between Two Variables*

By the end of this course, students will:

- define the correlation coefficient as a measure of the fit of a scatter graph to a linear model;
- calculate the correlation coefficient for a set of data, using graphing calculators or statistical software;

- demonstrate an understanding of the distinction between cause-effect relationships and the mathematical correlation between variables;
- describe possible misuses of regression (e.g., use with too small a sample, use without considering the effect of outliers, inappropriate extrapolation).

***Evaluating Validity***

By the end of this course, students will:

- explain examples of the use and misuse of statistics in the media;
- assess the validity of conclusions made on the basis of statistical studies, by analysing possible sources of bias in the studies (e.g., sampling bias) and by calculating and interpreting additional statistics, where possible (e.g., measures of central tendency, the standard deviation);
- explain the meaning and the use in the media of indices based on surveys (e.g., the consumer price index, the cost of living index).

## Integration of the Techniques of Data Management

### Overall Expectations

By the end of this course, students will:

- carry out a culminating project on a topic or issue of significance that requires the integration and application of the expectations of the course;
- present a project to an audience and critique the projects of others.

### Specific Expectations

#### *Carrying Out a Culminating Project*

By the end of this course, students will:

- pose a significant problem whose solution would require the organization and analysis of a large amount of data;
- select and apply the tools of the course (e.g., methods for organizing data, methods for calculating and interpreting measures of probability and statistics, methods for data collection) to design and carry out a study of the problem;
- compile a clear, well-organized, and fully justified report of the investigation and its findings.

#### *Presenting and Critiquing Projects*

By the end of this course, students will:

- create a summary of a project to present within a restricted length of time, using communications technology effectively;
- answer questions about a project, fully justifying mathematical reasoning;
- critique the mathematical work of others in a constructive fashion.

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## College and Apprenticeship Mathematics, Grade 12, College Preparation

(MAP4C)

This course equips students with the mathematical knowledge and skills they will need in many college programs. Students will use statistical methods to analyse problems; solve problems involving the application of principles of geometry and measurement to the design and construction of physical models; solve problems involving trigonometry in triangles; and consolidate their skills in analysing and interpreting mathematical models.

**Prerequisite:** Mathematics of Personal Finance, Grade 11, College Preparation, or Functions, Grade 11, University/College Preparation (or Functions and Relations, Grade 11, University Preparation)



## Applications of Statistics

### Overall Expectations

By the end of this course, students will:

- collect, analyse, and evaluate data involving one variable;
- collect, analyse, and evaluate data involving two variables;
- analyse significant problems or issues, using statistics;
- evaluate the validity of the use of statistics in the media.

### Specific Expectations

#### *Collecting, Analysing, and Evaluating Data Involving One Variable*

By the end of this course, students will:

- determine appropriate methods for collecting, storing, and retrieving, from primary or secondary sources, data involving one variable;
- design questionnaires for gathering data through surveys, giving consideration to possible sources of bias;
- demonstrate an understanding of the distinction between the terms *population* and *sample*;
- choose from and apply a variety of sampling techniques (e.g., random, stratified);
- represent data in appropriate graphical forms (e.g., histograms, bar graphs), using technology;
- identify and describe properties of common distributions of data (e.g., normal, bimodal, exponential, skewed);
- calculate the mean, median, mode, range, variance, and standard deviation of a data set, using standard statistical notation and technology;
- describe the significance of results drawn from analysed data (e.g., the shape of the distribution, the mean, the standard deviation);
- make and justify statements about a population on the basis of sample data.

#### *Collecting, Analysing, and Evaluating Data Involving Two Variables*

By the end of this course, students will:

- determine appropriate methods for collecting, storing, and retrieving, from primary or secondary sources, data involving two variables;
- construct a scatter plot to represent data, using technology;
- determine an equation of a line of best fit, using the regression capabilities of graphing technology;
- calculate and interpret the correlation coefficient, using appropriate technology;
- describe possible misuses of regression (e.g., use with too small a sample, use without considering the effect of outliers, inappropriate extrapolation);
- describe the relationship between two variables suggested by a scatter plot (e.g., no relationship, a positive correlation, a negative correlation);
- make and justify statements about a population on the basis of sample data.

#### *Analysing Problems*

By the end of this course, students will:

- collect, organize, and analyse data to address problems or issues, and calculate relevant statistical measures;
- formulate a summary conclusion to a problem or an issue, by synthesizing interpretations of individual statistical measures;

- formulate extending questions related to the conclusion reached in the investigation of a problem or an issue;
- communicate the process used and the conclusions reached in the investigation of a problem or an issue, using appropriate mathematical forms (e.g., oral and written explanations, tables, graphs, formulas).

***Evaluating Validity***

By the end of this course, students will:

- explain the use and misuse in the media of graphs and commonly used statistical terms (e.g., *percentile*), and expressions (e.g., *19 times out of 20*);
- assess the validity of conclusions made on the basis of statistical studies, by analysing possible sources of bias in the studies (e.g., sampling bias);
- explain the meaning, and the use in the media, of indices based on surveys (e.g., the consumer price index).

## Applications of Geometry, Measurement, and Trigonometry

### Overall Expectations

By the end of this course, students will:

- demonstrate an understanding of the relationship between three-dimensional objects and their two-dimensional representations;
- solve problems involving measurement;
- solve problems involving trigonometry in triangles.

### Specific Expectations

#### *Understanding Two-Dimensional and Three-Dimensional Shapes*

By the end of this course, students will:

- identify, through observation and measurement, the uses of geometric shapes and the reasons for those uses, in a variety of applications (e.g., product design, architecture, fashion);
- represent three-dimensional objects in a variety of ways (e.g., front, side, and top views; perspective drawings; scale models), using concrete materials and design or drawing software;
- create nets, plans, and patterns from physical models related to a variety of applications (e.g., fashion design, interior decorating, building construction), using design or drawing software;
- design and construct physical models of things (e.g., structures, equipment, furniture), satisfying given constraints and using concrete materials, design software, or drawing software.

#### *Solving Problems Involving Measurement*

By the end of this course, students will:

- solve problems related to the perimeter and area of plane figures, and the surface area and volume of prisms, pyramids, cylinders, spheres, and cones, including problems involving combinations of these objects;

- demonstrate accuracy and precision in working with metric measures;
- demonstrate an understanding of the use of the imperial system in a variety of applications (e.g., bolt and screw sizes; tool sizes; quantities of soil, water, or cement);
- demonstrate a working knowledge of the measurement of length and area in the imperial system, in relation to applications (e.g., design, construction);
- perform required conversions between the imperial system and the metric system, as necessary within projects and applications;
- use calculators effectively in solving problems involving measurement, and judge the reasonableness of the answers produced.

#### *Solving Problems Involving Trigonometry in Triangles*

By the end of this course, students will:

- solve problems involving trigonometry in right triangles;
- demonstrate an understanding of the signs of the sine, cosine, and tangent of obtuse angles;
- determine side lengths and angle measures in oblique triangles, using the cosine law and the sine law, and solve related problems;
- identify applications of trigonometry in occupations and in postsecondary programs related to the occupations.

## Analysis of Mathematical Models

### Overall Expectations

By the end of this course, students will:

- interpret and analyse given graphical models;
- interpret and analyse given formulaic models;
- interpret and analyse data given in a variety of forms.

### Specific Expectations

#### *Interpreting and Analysing*

##### *Given Graphical Models*

By the end of this course, students will:

- interpret a given linear, quadratic, or exponential graph to answer questions, using language and units appropriate to the context from which the graph was drawn;
- interpret the rate of change and initial conditions (i.e., the slope and  $y$ -intercept) of a linear model given within a context;
- make and justify a decision or prediction and discuss trends based on a given graph;
- describe the effect on a given graph of new information about the circumstances represented by the graph (e.g., describe the effect of a significant change in population on a graph representing the size of the population over time);
- communicate the results of an analysis orally, in a written report, and graphically.

#### *Interpreting and Analysing*

##### *Given Formulaic Models*

By the end of this course, students will:

- evaluate any variable in a given formula drawn from an application by substituting into the formula and using the appropriate order of operations on a scientific calculator;
- construct (e.g., combine or modify) formulas to solve multi-step problems in particular situations (e.g., determine the amount of paint required to paint two coats on a large cylindrical water tank);

- rearrange a formula to isolate any variable in it (e.g., to determine the values of a variable in a formula, using a spreadsheet);
- judge the reasonableness of answers to problems;
- demonstrate mastery of key algebraic skills, including the ability to solve linear equations, to solve systems of linear equations, to graph a linear function from its equation, and to determine the slope and intercepts of a linear function from its equation;
- factor expressions of the form  $ax^2 + bx + c$ ;
- solve quadratic equations by factoring.

#### *Interpreting and Analysing Data*

##### *Given in a Variety of Forms*

By the end of this course, students will:

- retrieve information from various sources (e.g., graphs, charts, spreadsheets, schedules);
- identify options that meet certain criteria, using more than one chart, spreadsheet, or schedule (e.g., the schedules of connecting flights; the spreadsheets of mortgage-payment plans);
- make informed decisions, using data provided in chart, spreadsheet, or schedule format and taking into account personal needs and preferences;
- enter data or a formula into a graphing calculator and retrieve other forms of the model (e.g., enter data and retrieve a scatter graph or a table of values; enter a formula and retrieve a table of values or the graph of a function).

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## Mathematics for College Technology, Grade 12, College Preparation

(MCT4C)

This course equips students with the mathematical knowledge and skills needed for entry into college technology programs. Students will investigate and apply properties of polynomial, exponential, and logarithmic functions; solve problems involving inverse proportionality; and explore the properties of reciprocal functions. They will also analyse models of a variety of functions, solve problems involving piecewise-defined functions, solve linear-quadratic systems, and consolidate key manipulation and communication skills.

**Prerequisite:** Functions, Grade 11, University/College Preparation (or Functions and Relations, Grade 11, University Preparation)

# Polynomial Functions and Inverse Proportionality

## Overall Expectations

By the end of this course, students will:

- determine, through investigation, the characteristics of the graphs of polynomial functions of various degrees;
- demonstrate facility in the algebraic manipulation of polynomials;
- demonstrate an understanding of inverse proportionality;
- determine, through investigation, the key properties of reciprocal functions.

## Specific Expectations

### *Investigating the Graphs of Polynomial Functions*

By the end of this course, students will:

- determine, through investigation, using graphing calculators or graphing software, various properties of the graphs of polynomial functions (e.g., determine the effect of the degree of a polynomial function on the shape of its graph; the effect of varying the coefficients in the polynomial function; the type and the number of  $x$ -intercepts; the behaviour near the  $x$ -intercepts; the end behaviours; the existence of symmetry);
- describe the nature of change in polynomial functions of degree greater than two, using finite differences in tables of values;
- compare the nature of change observed in polynomial functions of higher degree with that observed in linear and quadratic functions;
- sketch the graph of a polynomial function whose equation is given in factored form;
- determine an equation to represent a given graph of a polynomial function, using methods appropriate to the situation (e.g., using the zeros of the function; using a trial-and-error process on a graphing calculator or graphing software; using finite differences).

### *Manipulating Algebraic Expressions*

By the end of this course, students will:

- demonstrate an understanding of the remainder theorem and the factor theorem;
- factor polynomial expressions of degree greater than two, using the factor theorem;
- determine, by factoring, the real or complex roots of polynomial equations of degree greater than two;
- determine the real roots of non-factorable polynomial equations by interpreting the graphs of the corresponding functions, using graphing calculators or graphing software;
- write the equation of a family of polynomial functions, given the real or complex zeros [e.g., a polynomial function having non-repeated zeros 5,  $-3$ , and  $-2$  will be defined by the equation  $f(x) = k(x - 5)(x + 3)(x + 2)$ , for  $k \in \mathbb{R}$ ];
- describe intervals and distances, using absolute-value notation;
- solve factorable polynomial inequalities;
- solve non-factorable polynomial inequalities by graphing the corresponding functions, using graphing calculators or graphing software and identifying intervals above and below the  $x$ -axis.

***Understanding Inverse Proportionality***

By the end of this course, students will:

- construct tables of values, graphs, and formulas to represent functions of inverse proportionality derived from descriptions of realistic situations (e.g., the time taken to complete a job varies inversely as the number of workers; the intensity of light radiating equally in all directions from a source varies inversely as the square of the distance between the source and the observer);
- solve problems involving relationships of inverse proportionality.

***Determining the Key Properties of Reciprocal Functions***

By the end of this course, students will:

- sketch the graph of the reciprocal of a given linear or quadratic function by considering the implications of the key features of the original function as predicted from its equation (e.g., such features as the domain, the range, the intervals where the function is positive or negative, the intervals where the function is increasing or decreasing, the zeros of the function);
- describe the behaviour of a graph near a vertical asymptote;
- identify the horizontal asymptote of the graph of a reciprocal function by examining the patterns in the values of the given function.

## Exponential and Logarithmic Functions

### Overall Expectations

By the end of this course, students will:

- demonstrate an understanding of the nature of exponential growth and decay;
- define and apply logarithmic functions.

### Specific Expectations

#### *Understanding the Nature of Exponential Growth and Decay*

By the end of this course, students will:

- identify, through investigations, using graphing calculators or graphing software, the key properties of exponential functions of the form  $a^x$  ( $a > 0$ ,  $a \neq 1$ ) and their graphs (e.g., the domain is the set of the real numbers; the range is the set of the positive real numbers; the function either increases or decreases throughout its domain; the graph has the  $x$ -axis as an asymptote and has  $y$ -intercept = 1);
- describe the graphical implications of changes in the parameters  $a$ ,  $b$ , and  $c$  in the equation  $y = ca^x + b$ ;
- compare the rates of change of the graphs of exponential and non-exponential functions (e.g., those with equations  $y = 2x$ ,  $y = x^2$ ,  $y = x^{\frac{1}{2}}$ , and  $y = 2^x$ );
- describe the significance of exponential growth or decay within the context of applications represented by various mathematical models (e.g., tables of values, graphs, equations);

- pose and solve problems related to models of exponential functions drawn from a variety of applications, and communicate the solutions with clarity and justification.

#### *Defining and Applying Logarithmic Functions*

By the end of this course, students will:

- define the logarithmic function  $\log_a x$  ( $a > 1$ ) as the inverse of the exponential function  $a^x$ , and compare the properties of the two functions;
- express logarithmic equations in exponential form, and vice versa;
- simplify and evaluate expressions containing logarithms, using the laws of logarithms;
- solve simple problems involving logarithmic scales (e.g., the Richter scale, the pH scale, the decibel scale).



## Applications and Consolidation

### Overall Expectations

By the end of this course, students will:

- analyse models of linear, quadratic, polynomial, exponential, or trigonometric functions drawn from a variety of applications;
- analyse and interpret models of piecewise-defined functions drawn from a variety of applications;
- solve linear-quadratic systems and interpret their solutions within the contexts of applications;
- demonstrate facility in carrying out and applying key manipulation skills.

### Specific Expectations

#### *Analysing Models of Functions*

By the end of this course, students will:

- determine the key features of a mathematical model (e.g., an equation, a table of values, a graph) of a function drawn from an application;
- compare the key features of a mathematical model with the features of the application it represents;
- predict future behaviour within an application by extrapolating from a given model of a function;
- pose questions related to an application and use a given function model to answer them.

#### *Analysing and Interpreting Models of Piecewise-Defined Functions*

By the end of this course, students will:

- demonstrate an understanding that some naturally occurring functions cannot be represented by a single formula (e.g., the temperature at a particular location as a function of time);
- graph a piecewise-defined function, by hand and by using graphing calculators or graphing software;

- analyse and interpret a given mathematical model of a piecewise-defined function, and relate the key features of the model to the characteristics of the application it represents;
- make predictions and answer questions about an application represented by a graph or formula of a piecewise-defined function;
- determine the effects on the graph and formula of a piecewise-defined function of changing the conditions in the situation that the function represents.

#### *Solving Linear-Quadratic Systems*

By the end of this course, students will:

- determine the key properties of a linear function or a quadratic function, given the equation of the function, and interpret the properties within the context of an application;
- solve linear-quadratic systems arising from the intersections of the graphs of linear and quadratic functions;
- interpret the solution(s) to a linear-quadratic system within the context of an application.

***Consolidating Key Skills***

By the end of this course, students will:

- perform numerical computations effectively, using mental mathematics and estimation;
- solve problems involving ratio, rate, and percent drawn from a variety of applications;
- solve problems involving trigonometric ratios in right triangles and the sine and cosine laws in oblique triangles;
- demonstrate facility in using manipulation skills related to solving linear, quadratic, and polynomial equations, simplifying rational expressions, and operating with exponents.

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## **Mathematics for Everyday Life, Grade 12, Workplace Preparation**

(MEL4E)

This course enables students to broaden their understanding of mathematics as it is applied in important areas of day-to-day living. Students will use statistics in investigating questions of interest and apply principles of probability in familiar situations. They will also investigate accommodation costs and create household budgets; solve problems involving estimation and measurement; and apply concepts of geometry in the creation of designs.

**Prerequisite:** Mathematics for Everyday Life, Grade 11, Workplace Preparation

## Statistics and Probability

### Overall Expectations

By the end of this course, students will:

- construct and interpret graphs;
- formulate questions, and collect and organize data related to the questions;
- apply principles of probability to familiar situations;
- interpret statements about statistics and probability arising from familiar situations and the media.

### Specific Expectations

#### *Constructing and Interpreting Graphs*

By the end of this course, students will:

- represent given data in a variety of graphical forms, using spreadsheets or other suitable graphing technology;
- select an effective graphical form for a given set of data and explain reasons for the choice;
- interpret graphs by identifying trends and describing the meaning of the trends within the context of the data.

#### *Collecting and Organizing Data*

By the end of this course, students will:

- identify issues or questions of interest and collect related data, using an appropriate sampling technique;
- construct tables and graphs to represent collected data, using spreadsheets or other suitable graphing technology;
- draw appropriate conclusions about questions or issues on the basis of the interpretation of graphs;
- explain conclusions clearly.

#### *Applying Principles of Probability*

By the end of this course, students will:

- express probabilities of simple events as the number of favourable outcomes divided by the total number of outcomes;
- express probabilities as fractions, decimals, and percents, and interpret probabilities expressed in each of these forms;

- describe the results obtained in carrying out probability experiments related to familiar situations involving chance (e.g., rolling dice, spinning spinners, flipping coins);
- compare predicted and experimental results for familiar situations involving chance, using technology to extend the number of experimental trials (e.g., using a random number generator on a spreadsheet or on a graphing calculator);
- simulate familiar situations involving chance and explain the choice of simulation (e.g., simulate the gender of children in a family by the repeated flipping of a coin and explain why coin flipping was used).

#### *Interpreting Statements About Statistics and Probability*

By the end of this course, students will:

- interpret information about probabilities to assist in making informed decisions in a variety of situations (e.g., evaluating risk versus reward in the purchase of lottery tickets);
- interpret and assess statistical and probabilistic information used in the media and in common conversation (e.g., vague statements such as “four out of five dentists recommend”; statements about odds; scales on graphs).

## Everyday Financing

### Overall Expectations

By the end of this course, students will:

- determine the costs involved in renting an apartment;
- determine the costs involved in buying a house;
- design household budgets for given circumstances.

### Specific Expectations

#### *Determining the Costs of Renting an Apartment*

By the end of this course, students will:

- determine, through investigation, the costs of apartment rentals in the surrounding community;
- describe the alternatives available (e.g., leasing, renting month to month) and the procedures involved (e.g., paying a deposit) in renting an apartment;
- describe the rights and responsibilities of an apartment tenant and an apartment landlord;
- calculate the monthly costs involved in maintaining an apartment.

#### *Determining the Costs of Buying a House*

By the end of this course, students will:

- determine, through investigation, patterns in the cost of housing in the surrounding community (e.g., what kind of house can be purchased for \$75 000? \$140 000? \$250 000? \$400 000?);
- describe the procedures and costs involved in purchasing a house;
- identify the costs involved in maintaining a house;
- calculate the monthly costs involved in maintaining a given house.

#### *Designing Budgets*

By the end of this course, students will:

- identify typical components (e.g., accommodation, food, savings) and their dispersion in a household budget;
- determine the type of housing affordable in the surrounding community by a person with a given income and family responsibilities;
- design an appropriate monthly budget for a person living in the surrounding community who has a given income, family responsibilities, and long-term savings goals;
- present a budget in a clear fashion, using appropriate mathematical forms (e.g., written or oral explanations, charts, tables, graphs, calculations);
- investigate the effect on an overall budget of changing one component, using a given spreadsheet template or budgeting software.

## Applications of Measurement and Geometry

### Overall Expectations

By the end of this course, students will:

- use measurement and strategies of estimation in a variety of applications;
- solve problems involving measurement and design;
- apply transformation geometry in creating effective designs.

### Specific Expectations

#### *Measuring and Estimating*

By the end of this course, students will:

- demonstrate a working knowledge of the metric system;
- measure lengths accurately, using the metric system and the imperial system;
- estimate distances in metric units and in imperial units by applying personal referents (e.g., the width of a finger is approximately 1 cm; the length of a piece of standard loose-leaf paper is about 1 foot);
- estimate capacities in metric units by applying personal referents (e.g., a can of pop is about 350 mL);
- estimate, with reasonable accuracy, large numbers that are illustrated visually (e.g., books on a wall in a library, pictures of crowds, populations of high-rise buildings), and explain the strategies used.
- demonstrate an understanding of the effect on the area of familiar objects (e.g., a photograph, a television screen, a road map) of multiplying each dimension by the same factor;
- make a two-dimensional scale drawing of a room, using design or drawing software effectively;
- create a three-dimensional drawing of the interior of a room, using design or drawing software effectively;
- construct, with reasonable accuracy, a scale model of an environment of personal interest (e.g., a building, a garden, a bridge);
- estimate and calculate the surface area and volume of objects and containers in the surrounding environment that approximate the shape of rectangular prisms and cylinders;

#### *Solving Problems Involving*

#### *Measurement and Design*

By the end of this course, students will:

- demonstrate an understanding of the Pythagorean theorem, by constructing on a floor a rectangular region having accurate right-angled corners;
- determine the perimeter and area of regular and irregular figures from given diagrams;
- estimate, with reasonable accuracy, perimeters and areas of large regions (e.g., a playing field), and explain the strategies used;

- investigate the making of a household improvement (e.g., landscaping a property, decorating a room), design the improvement, and estimate and calculate the cost, using technology (e.g., spreadsheets, design or drawing software).

#### *Applying Transformation Geometry*

By the end of this course, students will:

- describe the use of translations, reflections, rotations, and dilatations as they relate to symmetry and design in logos, with the aid of technology (e.g., dynamic geometry software, design or drawing software);

- analyse the geometric aspects of interesting and appealing applications (e.g, logos found in advertising, designs found in fabric or wallpaper);
- create a personal logo, using the mathematics of symmetry, translations, reflections, rotations, or dilatations, with the aid of technology (e.g., dynamic geometry software, design or drawing software);
- determine, through investigations, using concrete materials and technology, the characteristics of shapes that will tile the plane;
- create designs involving tiling patterns (e.g., Escher-type designs, wallpaper or fabric designs), using technology (e.g., dynamic geometry software, design or drawing software).

## Some Considerations for Program Planning in Mathematics

Teachers who are planning a program in mathematics must take into account considerations in a number of important areas. Essential information that pertains to all disciplines is provided in *The Ontario Curriculum, Grades 9 to 12: Program Planning and Assessment, 2000*. The areas of concern to all teachers that are outlined there include the following:

- types of secondary school courses
- education for exceptional students
- the role of technology in the curriculum
- English as a second language (ESL) and English literacy development (ELD)
- career education
- cooperative education and other workplace experiences
- health and safety

Additional considerations that have particular relevance for program planning in mathematics are noted here.

***Education for Exceptional Students.*** The Education Act and regulations made under the act require school boards to provide exceptional students with special education programs and services that are appropriate for their needs.

An Individual Education Plan (IEP) must be developed and maintained for each student who is identified as exceptional by an Identification, Placement, and Review Committee (IPRC). The IEP must outline, as appropriate, any modified or alternative curriculum expectations and any accommodations (i.e., the specialized support and services) that are required to meet the student's needs. The IEP must also identify the methods by which the student's progress will be reviewed. For exceptional students who are fourteen years of age or older and who are not identified solely as gifted, the IEP must contain a plan to help them make the transition to postsecondary education, apprenticeship programs, or the workplace, and to help them live as independently as possible in the community.

An IEP may be prepared for a student with special needs who is receiving special education programs and/or services but who has not been identified as exceptional by an IPRC.

Exceptional students may require program modifications that specifically address their strengths and needs in learning mathematics. Some students will benefit from additional time to internalize important mathematical concepts. In an effort to help students understand concepts, teachers may include more concrete experiences, vary the pace of the learning, use additional examples, or insert cumulative reviews. The appropriate use of technologies (e.g., concrete materials, dynamic geometry software) will support the teacher in meeting the needs of exceptional students as set out in their Individual Education Plan.



***The Role of Technology in the Curriculum.*** Technology helps to make students more powerful learners by giving them the means to explore mathematical concepts more effectively. In the time gained by using technology, students can study fundamental ideas in greater depth, develop higher skill levels, and explore more applications.

Various forms of technology have application in many different areas of mathematics learning. Calculators save students time in performing complex arithmetic calculations. Graphing utilities enable students to explore properties of the graphs of functions. Statistical software allows students to collect and analyse data effectively and quickly. Dynamic geometry software allows students to visualize spatial relationships, to test hypotheses, and to investigate the properties of loci. The use of technology in learning and doing mathematics also gives students excellent opportunities to develop their abilities in algorithmic thinking, for example, by creating templates in spreadsheets or by writing sequences of instructions in application programs as part of a problem-solving process.

The presence of technology as part of learning mathematics makes many new things possible, but it also places increasing importance on the ability of students to make mental judgements about expected results. For example, the student who uses a calculator to perform an arithmetic calculation should have the habit of using estimation to judge the reasonableness of the answer produced. Similarly, the student who produces a graph using technology should be capable of creating a mental approximation of the graph as a verification of the image on the screen.

This curriculum incorporates the use of technology in the learning expectations. While the use of technology makes many things possible in mathematics, acquiring the technology and gaining skills in using it effectively take time. Full implementation of the curriculum will occur as schools acquire the technology and teachers and students develop skills in using it.

***English As a Second Language and English Literacy Development (ESL/ELD).*** There are several key considerations for teachers of mathematics in planning programs for ESL/ELD students. Teachers must recognize the mathematical skills and knowledge that all students bring to the classroom, and should value the students' knowledge and build on their strengths. Teachers should approach with sensitivity the increased emphasis on communication in mathematics, especially in cooperative learning settings, so that difficulties with language do not inhibit the participation of ESL/ELD students and hinder their success. Students should be encouraged to communicate their thoughts and understandings about mathematics in oral and written form, using the language of mathematics. ESL/ELD students should be encouraged to discuss problems, justify reasons for answers, and compare ideas and strategies within the classroom. Teachers must ensure that reading levels are appropriate to students' abilities and must strive for clarity in the use of mathematical terminology. Where possible, teachers should use visual and interactive methods, including technology, to facilitate the learning of mathematics, and should make appropriate accommodations and modifications for assessment.

***Career Education.*** Teachers should promote students' understanding of the role of mathematics in daily life and its relation to career opportunities by exploring applications of concepts, providing opportunities for career-related project work, and promoting independent investigations. Such activities allow students the opportunity to investigate mathematics-related careers compatible with their interests, aspirations, and abilities.

***Cooperative Education and Other Workplace Experiences.*** For students in mathematics, the hands-on experience of work and of cooperative education can reveal career options previously unknown to them and can provide valuable lessons on the application of mathematics in work and society. Cooperative placements in many occupations (e.g., engineering, accounting, banking, dentistry, the skilled trades, teaching, landscape design, computer programming, medicine, and fashion design) can give students greater insight into the value of learning mathematics.

***Mathematics Anxiety.*** Mathematics anxiety is a state of mind relating to a student's perception of his or her ability to do mathematics. It is neither grade specific nor exclusively gender related. If left unchecked, it often leads to mathematics avoidance. To alleviate this anxiety in classrooms, teachers should:

- be accepting, patient, and understanding;
- defuse tense situations if they arise;
- make mathematics relevant by connecting the context with the student's life experience;
- provide many opportunities for students to be successful;
- set up programs for peer tutoring;
- use a variety of assessment techniques (journals, interviews, portfolios, projects);
- comment positively on material that is assessed;
- be aware of cultural biases.

Much mathematics anxiety can be avoided by using good teaching techniques. Being aware of situations that can cause tension and working with students in a caring, understanding, and encouraging way can do much to alleviate anxiety.



# The Achievement Chart for Mathematics

The achievement chart that follows identifies four categories of knowledge and skills in mathematics – Knowledge/Understanding, Thinking/Inquiry/Problem Solving, Communication, and Application. These categories encompass all the curriculum expectations in courses in the discipline. For each of the category statements in the left-hand column, the levels of student achievement are described. (Detailed information on the achievement levels and on assessment, evaluation, and reporting policy and its implementation is provided in *The Ontario Curriculum, Grades 9 to 12: Program Planning and Assessment, 2000*.)

The achievement chart is meant to guide teachers in:

- planning instruction and learning activities that will lead to the achievement of the curriculum expectations in a course;
- planning assessment strategies that will accurately assess students' achievement of the curriculum expectations;
- selecting samples of student work that provide evidence of achievement at particular levels;
- providing descriptive feedback to students on their current achievement and suggesting strategies for improvement;
- determining, towards the end of a course, the student's most consistent level of achievement of the curriculum expectations as reflected in his or her course work;
- devising a method of final evaluation;
- assigning a final grade.

The achievement chart can guide students in:

- assessing their own learning;
- planning strategies for improvement, with the help of their teachers.

The achievement chart provides a standard province-wide method for teachers to use in assessing and evaluating their students' achievement. A variety of materials is being made available to assist teachers in improving their assessment methods and strategies and, hence, their assessment of student achievement.

The ministry is providing the following materials to school boards for distribution to teachers:

- a standard provincial report card, with an accompanying guide
- instructional planning materials
- assessment videos
- training materials
- an electronic curriculum planner

When planning courses and assessment, teachers should review the required curriculum expectations and link them to the categories to which they relate. They should ensure that all the expectations are accounted for in instruction, and that achievement of the expectations is assessed within the appropriate categories. The descriptions of the levels of achievement given in the chart should be used to identify the level at which the student has achieved the expectations. Students should be given numerous and varied opportunities to demonstrate their achievement of the expectations across the four categories. Teachers may find it useful to provide students with examples of work at the different levels of achievement.

The descriptions of achievement at level 3 reflect the provincial standard for student achievement. A complete picture of overall achievement at level 3 in a course in mathematics can be constructed by reading from top to bottom in the column of the achievement chart headed “70–79% (Level 3)”.

**Achievement Chart – Grades 11 and 12, Mathematics**

<b>Categories</b>	<b>50–59% (Level 1)</b>	<b>60–69% (Level 2)</b>	<b>70–79% (Level 3)</b>	<b>80–100% (Level 4)</b>
<b>Knowledge/ Understanding</b>	<b>The student:</b>			
– understanding concepts	– demonstrates limited understanding of concepts	– demonstrates some understanding of concepts	– demonstrates considerable understanding of concepts	– demonstrates thorough understanding of concepts
– performing algorithms	– performs only simple algorithms accurately by hand and by using technology	– performs algorithms with inconsistent accuracy by hand, mentally, and by using technology	– performs algorithms accurately by hand, mentally, and by using technology	– selects the most efficient algorithm and performs it accurately by hand, mentally, and by using technology
<b>Thinking/Inquiry/ Problem Solving</b>	<b>The student:</b>			
– reasoning	– follows simple mathematical arguments	– follows arguments of moderate complexity and makes simple arguments	– follows arguments of considerable complexity, judges the validity of arguments, and makes arguments of some complexity	– follows complex arguments, judges the validity of arguments, and makes complex arguments
– applying the steps of an inquiry/problem-solving process (e.g., formulating questions; selecting strategies, resources, technology, and tools; representing in mathematical form; interpreting information and forming conclusions; reflecting on the reasonableness of results)	– applies the steps of an inquiry/problem-solving process with limited effectiveness	– applies the steps of an inquiry/problem-solving process with moderate effectiveness	– applies the steps of an inquiry/problem-solving process with considerable effectiveness	– applies the steps of an inquiry/problem-solving process with a high degree of effectiveness and poses extending questions
<b>Communication</b>	<b>The student:</b>			
– communicating reasoning orally, in writing, and graphically	– communicates with limited clarity and limited justification of reasoning	– communicates with some clarity and some justification of reasoning	– communicates with considerable clarity and considerable justification of reasoning	– communicates concisely with a high degree of clarity and full justification of reasoning
– using mathematical language, symbols, visuals, and conventions	– infrequently uses mathematical language, symbols, visuals, and conventions correctly	– uses mathematical language, symbols, visuals, and conventions correctly some of the time	– uses mathematical language, symbols, visuals, and conventions correctly most of the time	– routinely uses mathematical language, symbols, visuals, and conventions correctly and efficiently
<b>Application</b>	<b>The student:</b>			
– applying concepts and procedures relating to familiar and unfamiliar settings	– applies concepts and procedures to solve simple problems relating to familiar settings	– applies concepts and procedures to solve problems of some complexity relating to familiar settings	– applies concepts and procedures to solve complex problems relating to familiar settings; recognizes major mathematical concepts and procedures relating to applications in unfamiliar settings	– applies concepts and procedures to solve complex problems relating to familiar and unfamiliar settings

*Note:* A student whose achievement is below 50% at the end of a course will not obtain a credit for the course.

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