# THE ORIFICE PLATE DISCHARGE COEFFICIENT EQUATION THE EQUATION FOR ISO 5167-1 

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## SUMMARY

This report describes the final work undertaken to achieve the equation which is being balloted in ISO/TC 30/SC 2 for inclusion in ISO 5167-1. It is described as the Agreed Equation since it has the support of the ISO/TC 28 delegation. It includes a description of the work done to check that a change in the expansibility equation would not have a significant effect on discharge coefficient equations fitted to the database.

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## NOTATION

$A \quad$ Function of orifice Reynolds number (see equation (3))
$C \quad$ Orifice discharge coefficient
$C_{c} \quad$ Orifice discharge coefficient using comer tappings
$C_{s} \quad$ Dependence of $C_{c}$ on Reynolds number
$C_{\infty} \quad C_{c}$ for infinite Reynolds number for $D \geq 71.12 \mathrm{~mm}$ (2.8 inch)
$C_{\Sigma} \quad C_{\infty}+C_{s}+\Delta C_{D}$
$\Delta C_{D} \quad$ Small pipe diameter term
$\Delta C_{\text {down }} \quad$ Downstream tapping term
$\Delta C_{\text {up }} \quad$ Upstream tapping term
$D \quad$ Pipe diameter
d Orifice diameter
$L_{1} \quad$ Quotient of the distance of the upstream tapping from the upstream face of the plate and the pipe diameter

Quotient of the distance of the downstream tapping from the downstream face of the plate and the pipe diameter
$M_{2}{ }^{\prime} \quad$ Quotient of the distance of the downstream tapping from the downstream face of the plate and the dam height (as in equation (3))
$R e_{D} \quad$ Pipe Reynolds number
Standard deviation of the data in the database about an equation
$\beta \quad$ Diameter ratio
$\varepsilon_{1} \quad$ Expansibility referred to upstream conditions

Although the orifice plate is the recognized flowmeter for the measurement of natural gas and light hydrocarbon liquids, the orifice discharge coefficient equation in use in the international standard ISO 5167-1: $1991^{(1)}$ is based on data collected more than 50 years ago. Moreover, for many years the United States and Europe have used different equations, a discrepancy with serious consequences for the oil and gas industry since many companies are multinational. Europe uses the Stolz Equation in ISO 5167-1: 1991 which was previously in ISO 5167: 1980. The United States used the Buckingham Equation ${ }^{(2)}$ until the ReaderHarris/Gallagher (RG) Equation (see Reference 3 and below) was adopted in 1990.

The Buckingham equation is based on the data collected by Beitler in the early 1930s at the Ohio State University Engineering Experimental Station ${ }^{(4)}$. Stolz ${ }^{(5)}$ used 303 points from the Ohio State University data together with a table of flow coefficients from ISO/R $541^{(6)}$. The ISO/R 541 table was established by the German VDI and is based on original data of Witte which are no longer available.

To resolve discrepancies between equations data on orifice plate discharge coefficients were collected in Europe and the United States over more than ten years in order to provide a new database from which an improved discharge coefficient equation could be obtained which would receive international acceptance.

In November 1988 a joint meeting of API (American Petroleum Institute) and EEC flow measurement experts in New Orleans accepted an equation derived by NEL ${ }^{(7)}$. At that time the database contained 11346 points, collected in pipes whose diameters ranged from 50 to 250 mm ( 2 to 10 inch ); 600 mm ( 24 inch ) data were being collected but had not yet been included in the database. 600 mm data have since been collected in gas and in water and extend the database both in pipe diameter and in Reynolds number.

The data which were least well fitted by the equation presented at New Orleans were the 50 mm data, and following the meeting the American standard API 2530 was revised to include the Reader-Harris/Gallagher (RG) Equation based on the NEL equation accepted at New Orleans with an additional term proposed by Gallagher for pipes whose diameter lies below 71.12 mm ( 2.8 inch ). Since there was no physical explanation for the additional term for small pipe diameter, additional data were collected in 50 mm pipe in water and oil and included in the database. Measurements by NEL of the edge radius of the plates used in European tests showed that orifices whose diameter is less than 50 mm tend to have edge radii outside those permitted by ISO 5167-1. An equation was therefore derived which included an additional term for small orifice diameter rather than one for small pipe diameter, and it was put forward in a report to EC BCR numbered PR14 ${ }^{(8)}$. It is also described in papers at the North Sea Flow Measurement Workshop in $1992^{(9)}$ and in Flow Measurement and Instrumentation ${ }^{(10)}$.

However, neither this equation nor the attempt at compromise put forward at a meeting of ISO/TC 30 in Paris in 1993 were acceptable to the ISO/TC 28 delegation. It was considered by the ISO/TC 28 delegation that the PR14 Equation gave an insufficiently good fit to the data collected in 75 mm ( 3 -inch) pipes, and their need was for an equation which was of the same form as the RG Equation in API MPMS 14.3: $1990^{(3)}$ for $R e_{D} \geq 4000$; only the
constants and exponents might be changed to give the best fit to the final database. Moreover, measurements by SwRI of the orifice edge radius for the orifice plates used in 50 mm pipe were smaller than those made by NEL. Since one of the principal objectives of the last fourteen years of work was to obtain a common equation for use worldwide, an equation of the form of the RG Equation has been derived and is presented here. It is being ballotted in ISO/TC 30/SC 2 for inclusion in ISO 5167-1. It is described here as the Agreed Equation since it has the support of the ISO/TC 28 Delegation. Given the definition that $s$ is the standard deviation of the data in the database about an equation, the value of $s$ for the Agreed Equation is very similar to the value of $s$ for the PR14 Equation.

## 2 THE DATABASE

In addition to the database used for deriving the PR14 orifice plate discharge coefficient equation in 1992 an additional 146 points have been added: these are from SwRI, Texas ${ }^{(11)}$; their inclusion makes essentially no difference to the final fitted equation.

## 3 THE AGREED EQUATION

As stated in the Introduction the Agreed Equation had to be of the same form as that of the RG Equation for $R e_{D} \geq 4000$ with only the constants and exponents changed.

### 3.1 The Tapping Terms

The form of the upstream and downstream tapping terms, $\Delta C_{\mathrm{up}}$ and $\Delta C_{\text {down }}$, in the RG equation (and also in the NEL equation for New Orleans described in Reference 7) is as follows:

$$
\begin{equation*}
\Delta C_{\mathrm{up}}=\left(c_{1}+c_{2} e^{-f_{1} L_{1}}-\left(c_{1}+c_{2}\right) e^{-f_{2} L_{1}}\right)\left(1-c_{3} A\right) \frac{\beta^{4}}{1-\beta^{4}} \tag{1}
\end{equation*}
$$

and

$$
\begin{equation*}
\Delta C_{\text {down }}=c_{4}\left(M_{2}^{\prime}-c_{5} M_{2}^{f_{5}}\right)\left(1-c_{6} A\right) \beta^{f_{4}} \tag{2}
\end{equation*}
$$

where

$$
\begin{equation*}
A=\left(\frac{19000 \beta}{R e_{D}}\right)^{0.8} \text { and } M_{2}=\frac{2 \dot{L_{2}}}{1-\beta} . \tag{3}
\end{equation*}
$$

For high Reynolds numbers the tapping terms are identical in form to those obtained in deriving the PR14 Equation based on the enlarged database. Therefore, the coefficients $c_{1}, c_{2}$, $c_{4}$, and $c_{5}$ and the exponents $f_{i}(i=1,4)$, in the tapping terms for high Reynolds number are taken to be the same as those in the PR14 Equation ${ }^{(8 \cdot 10)}$, except that it was found that the overall fit to the database was improved by making $c_{2}=0.08$ instead of 0.09 . (The upstream tapping term for high Reynolds number is plotted in Figure 1 on the same basis as in References 8,9 and 10.) Moreover, when the tapping terms for lower Reynolds numbers for the PR14 Equation were derived it was found that the dependence of the sum of the tapping
terms on $R e_{D}$ for $R e_{D} \geq 4000$ could be expressed by making $\Delta C_{\text {up }}$ alone a function of $R e_{D}$. When determining the values of $c_{3}$ and $c_{6}$ to give the best fit of the form of the complete Agreed Equation to the database it was found that to permit a non-zero value of $c_{6}$ gives a negligible reduction in the standard deviation of the data about the equation. With $c_{6}=0$ the best-fit value of $c_{3}$ was 0.11 . So that a good fit for the complete database (including data for $R e_{D}<4000$ ) should still be obtained it is necessary to include an additional term in $\Delta C_{\text {down }}$ for $R e_{D} \leq 3700$ identical to that in the PR14 Equation. Therefore, the final tapping terms are given by

$$
\begin{equation*}
\Delta C_{\mathrm{up}}=\left(0: 043+0.080 e^{-10 L_{1}}-0.123 e^{-7 L_{1}}\right)(1-0.11 A) \frac{\beta^{4}}{1-\beta^{4}} \tag{4}
\end{equation*}
$$

and

$$
\begin{equation*}
\Delta C_{\text {down }}=-0.031\left(M_{2}^{\prime}-0.8 M_{2}^{1.1}\right)\left\{1+8 \max \left(\log _{10}\left(3700 / R e_{D}\right), 0.0\right)\right\} \beta^{1.3} \tag{5}
\end{equation*}
$$

### 3.2 The Small Pipe Diameter Term

Instead of a small orifice diameter term the RG Equation uses a small pipe diameter term, $\Delta C_{D}$, of the following form:

$$
\begin{equation*}
\Delta C_{D}=h_{1}\left(h_{2}-\beta\right) \max \left(h_{3}-D / 25.4,0.0\right) . \quad(D: \mathrm{mm}) \tag{6}
\end{equation*}
$$

The form of this term has no known physical basis (although the requirement for the term may be due to the edge radius of the orifice plates used in the 50 mm pipes), but the term gives a good fit to the database: $C$ was larger in 50 mm pipes than in larger pipes even with corner and $D$ and $D / 2$ tappings. Since there are essentially only discrete values of $D$ in the database $h_{3}$ cannot be determined: following the RG Equation the value of $h_{3}$ was taken to be 2.8 inch ( 71.12 mm ), 0.1 inch ( 2.54 mm ) smaller than the internal diameter for 3-inch schedule 80 pipe. The quality of fit to the complete database improves as $h_{2}$ reduces to 0.75 , the smallest value which ensures that this term does not become negative over the permissible range of use. If $\left(h_{2}-\beta\right)$ were replaced by $\max \left(h_{2}-\beta, 0\right)$ then the best fit is obtained with $h_{2}$ equal to 0.7 , but the improvement in fit is very small and the equation significantly different from the RG Equation and also more complicated. With $h_{2}=0.75 h_{1}$ was determined by fitting the complete database and is equal to 0.011 . Therefore, the small pipe diameter term is given by

$$
\begin{equation*}
\Delta C_{D}=0.011(0.75-\beta) \max (2.8-D / 25.4,0.0) . \quad(D: \mathrm{mm}) \tag{7}
\end{equation*}
$$

### 3.3 The $C_{\text {a }}$ and Slope Terms

The $C_{\infty}$ and slope terms are of exactly the same form in the RG and in the PR14 Equations and so in the Agreed Equation are of the following form:

$$
\begin{align*}
& C_{\infty}+C_{s}=a_{1}+a_{2} \beta^{m_{1}}+a_{3} \beta^{m_{2}}+b_{1}\left(10^{6} \beta / R e_{D}\right)^{n_{1}}  \tag{8}\\
& +\left(b_{2}+b_{3} A\right) \beta^{\prime} \max \left\{\left(10^{6} / R e_{D}\right)^{n_{2}}, g_{1}-g_{2}\left(R e_{D} / 10^{6}\right)\right\}
\end{align*}
$$

The exponents used for the PR14 Equation were used for the final equation with the exception that $m_{1}$ was taken to be 2 since this value is used in the RG Equation and gives a better fit to the complete database. With $m_{1}, m_{2}, l, n_{1}$ and $n_{2}$ given by $2,8,3.5,0.7$ and 0.3 , respectively, the tapping terms given in equations (4) and (5) and the small pipe diameter term given in equation (7) the optimum values of the constants in equation (8) were determined and the following equation obtained:

$$
\begin{align*}
& C_{\infty}+C_{s}=0.5961+0.0261 \beta^{2}-0.216 \beta^{8} \\
& +0.000521\left(10^{6} \beta / R e_{D}\right)^{0.7}  \tag{9}\\
& +(0.0188+0.0063 A) \beta^{3.5} \max \left\{\left(10^{6} / R e_{D}\right)^{0.3}, 22.7-4700\left(R e_{D} / 10^{6}\right)\right\}
\end{align*}
$$

### 3.4 The Complete Equation

Therefore, the complete orifice plate discharge coefficient equation is as follows:

$$
\begin{align*}
& C=0.5961+0.0261 \beta^{2}-0.216 \beta^{8} \\
& +0.000521\left(10^{6} \beta / R e_{D}\right)^{0.7} \\
& +(0.0188+0.0063 A) \beta^{3.5} \max \left\{\left(10^{6} / R e_{D}\right)^{0.3}, 22.7-4700\left(R e_{D} / 10^{6}\right)\right\} \\
& +\left(0.043+0.080 e^{-10 L_{1}}-0.123 e^{-7 L_{1}}\right)(1-0.11 A) \frac{\beta^{4}}{1-\beta^{4}}  \tag{10}\\
& -0.031\left(M_{2}^{\prime}-0.8 M_{2}^{\prime 1.1}\right)\left\{1+8 \max \left(\log _{10}\left(3700 / R e_{D}\right), 0.0\right)\right\} \beta^{1.3} \\
& +0.011(0.75-\beta) \max (2.8-D / 25.4,0.0) . \quad(D: \mathrm{mm})
\end{align*}
$$

For $R e_{D} \geq 4000$ this equation can be written as follows:
For $D \geq 71.12 \mathrm{~mm}$ ( 2.8 inch)

$$
\begin{align*}
& C=0.5961+0.0261 \beta^{2}-0.216 \beta^{8} \\
& +0.000521\left(10^{6} \beta / R e_{D}\right)^{0.7}+(0.0188+0.0063 A) \beta^{3.5}\left(10^{6} / R e_{D}\right)^{0.3} \\
& +\left(0.043+0.080 e^{-10 L_{1}}-0.123 e^{-7 L_{1}}\right)(1-0.11 A) \frac{\beta^{4}}{1-\beta^{4}}  \tag{11a}\\
& -0.031\left(M_{2}^{\prime}-0.8 M_{2}^{1.1}\right) \beta^{1.3} .
\end{align*}
$$

Where $D<71.12 \mathrm{~mm}$ (2.8 inch) the following term should be added to equation (11a):

$$
\begin{equation*}
+0.011(0.75-\beta)\left(2.8-\frac{D}{25.4}\right) . \quad(D: \mathrm{mm}) \tag{11b}
\end{equation*}
$$

Equation (11), comprising equation (11a) with the additional term (11b), is the Agreed Equation.

The notation of ISO 5167-1 has been used with the following additions:

$$
A=\left(\frac{19000 \beta}{R e_{D}}\right)^{0.8} \text { and } M_{2}^{\prime}=\frac{2 \dot{L}_{2}^{\prime}}{1-\beta} .
$$

## 4 QUALITY OF FIT

The quality of the fit of equation (10) to the database is very good: the overall standard deviation of the data for $R e_{D} \geq 4000$ about the equation is 0.259 per cent; the mean deviations of the data about the equation as a function of $\beta, D, \mathbb{R e}_{D}$ and pair of tappings used and of pairs of these independent variables are both small and well-balanced.

The quality of fit is quantified in Tables 1 to 8 . Table 1 gives a description of the meaning of the different lines in Tables 2 to 8. These tables give the deviations of the data in the database about the equation as a function of $\beta, D, R e_{D}$ and pair of tappings used and certain combinations of these. The range of values of $\beta$ corresponding to each nominal value of $\beta$ is given in Table 2. The tappings described as Corner (GU) are tappings in the corners which were designed by Gasunie and are simpler to make than those in ISO 5167-1. They are described in Reference 12. The database used is the complete EEC/API database as described except that data with $R e_{D}<4000$ were excluded in Tables 5 to 8.

## 5 COMPARISON BETWEEN EQUATIONS ON THE BASIS OF DEVIATIONS

A direct comparison between equation (11) and the RG and PR14 Equations for $R e_{D} \geq 4000$ is given here. To do this the data in Table 5 (and similar tables of deviations on the basis of $D$ and $\beta$ for the other equations) were analysed: the number of boxes (ranges of $D$ and $\beta$ ) over which an equation gave a mean deviation greater than 0.1 per cent was counted and is given in Table 9; the number of boxes for which the mean deviation was greater than 0.2 per cent was also counted and is given in Table 9. A similar count was undertaken for Tables 6-8 and the results are also given in Table 9. These figures provide a measure of possible bias in an equation. The standard deviation of the data about each equation is also given as a measure of the quality of fit. The quality of fit for the Agreed Equation is similar to that of the PR14 Equation. They are both better than the RG Equation.

Since the Stolz Equation in ISO 5167-1: 1991 is applicable over a more limited range of values of $R e_{D}$ and $\beta$ than the three equations previously considered, the standard deviation and the number of boxes with mean deviations greater than 0.1 or 0.2 per cent are given in Table 10 for all four equations over the range of applicability of the Stolz ISO 5167-1: 1991 Equation. The need for an improved equation even over the limited range of $R e_{D}$ and $\beta$ is obvious.

## 6 THE EFFECT OF THE EXPANSIBILITY EQUATION

Since doubt has been expressed regarding the accuracy of the expansibility equation used for orifice plates in ISO $5167-1^{1}$ (see Kinghorn ${ }^{(13)}$ and Seidl ${ }^{(14)}$ ) it has been suggested that the new discharge coefficient equation given in equation (10) and others based on the same database may have been significantly affected by errors in the discharge coefficient data
caused by errors in the expansibility, $\varepsilon_{1}$, since the value of $\varepsilon_{1}$ given in Section 8.3.2.2 of ISO 5167-1: 1991 was used in the computation of discharge coefficient.

In order to test this theory, for each point of the database the value of discharge coefficient which would have been obtained if an alternative equation for $\varepsilon_{1}$ had been used was calculated and the equation refitted. If the value of discharge coefficient given in the database is termed $C_{I}$, based on the expansibility given by the equation in Section 8.3.2.2 of ISO 5167-1: 1991, $\varepsilon_{1, l}$, then $C_{N}$, the value of discharge coefficient based on an alternative value of expansibility, $\varepsilon_{1, N}$, is given by

$$
\begin{equation*}
C_{N} \varepsilon_{l, N}=C_{I} \varepsilon_{l, I}, \tag{12}
\end{equation*}
$$

where

$$
\begin{equation*}
\varepsilon_{1, I}=1-\left(0.41+0.35 \beta^{4}\right) \frac{\Delta p}{\kappa p_{1}} . \tag{13}
\end{equation*}
$$

Where the value of $\varepsilon_{1, I}$ is given in the database it is possible to calculate $\kappa$ using the values of $\beta, \Delta p$ and $p_{1}$ and then to calculate $\varepsilon_{1, N}$. Where $\varepsilon_{1, I}$ is not given in the database it is necessary, in the first instance, to estimate what value of $\kappa$ might have been used on the basis of other data; if it were shown to be the case that the discharge coefficient equation fitted to $C_{N}$ differed significantly from that fitted to $C_{I}$ it would be necessary to obtain better values for $\kappa$.

The only sets of gas data for which $\varepsilon_{1, I}$ was not provided in the database were those from SwRI and Ruhrgas. For SwRI the downstream expansion factor, $Y_{2}$, (see Reference 3) was provided but not $\varepsilon_{1, l}$. So for SwRI and Ruhrgas values for $\kappa$ of 1.41 and 1.32, respectively, were used: 1.41 is appropriate for nitrogen; 1.32 is a typical value for natural gas.

Three alternative equations for $\varepsilon_{1, N}$ were used: they were as follows:

$$
\begin{align*}
& \varepsilon_{1, N, 1}=1-\left(0.35+0.38 \beta^{4}\right) \frac{\Delta p}{\kappa p_{1}},  \tag{14}\\
& \varepsilon_{1, N, 2}=1-\left(0.352+0.433 \beta^{4}\right) \frac{\Delta p}{\kappa p_{1}},  \tag{15}\\
& \varepsilon_{1, N, 3}=1-\left(0.357+0.557 \beta^{4}\right) \frac{\Delta p}{\kappa p_{1}} . \tag{16}
\end{align*}
$$

and
$\varepsilon_{1, N, 1}$ and $\varepsilon_{1, N, 2}$ were taken from equations (9) (rounded as in the conclusions of the paper) and (10) of Kinghorn ${ }^{(13)}$ and $\varepsilon_{1, N, 3}$ was taken from equation (8) (the recommended equation) of Seidl ${ }^{(14)}$.

Calculating $C_{N, i}$ on the basis of $\varepsilon_{1, N, i}$ for $i=1,3$ and using the tapping terms given in equations (4) and (5), the sum of the $C_{\infty}, C_{s}$ and $\Delta C_{D}$ terms, $C_{\Sigma}$, was refitted as follows:

$$
\begin{align*}
& C_{\Sigma}=C_{\infty}+C_{s}+\Delta C_{D}=a_{1}+a_{2} \beta^{2}+a_{3} \beta^{8} \\
& +b_{1}\left(10^{6} \beta / R e_{D}\right)^{0.7}+\left(b_{2}+b_{3} A\right) \beta^{3.5} \max \left\{\left(10^{6} / R e_{D}\right)^{0.3}, 22.7-4700\left(R e_{D} / 10^{6}\right)\right\}  \tag{17}\\
& +h(0.75-\beta) \max (2.8-D / 25.4,0.0) . \quad(D: \mathrm{mm})
\end{align*}
$$

In each case the standard deviation of the data in the database about the equation, $s$, the number of points which are shifted by more than 0.2 per cent, $N_{s}$, and the largest magnitude of shift, $S_{M}$, were calculated. The results were as foltows:

$$
\begin{align*}
& C_{\Sigma, N, 1}=0.59590+0.02638 \beta^{2}-0.21794 \beta^{8}+0.0005288\left(10^{6} \beta / R e_{D}\right)^{0.7} \\
& +(0.01904+0.005864 A) \beta^{3.5} \max \left\{\left(10^{6} / R e_{D}\right)^{0.3}, 22.7-4700\left(R e_{D} / 10^{6}\right)\right\} \\
& +0.01135(0.75-\beta) \max (2.8-D / 25.4,0.0) . \quad(D: \mathrm{mm}) \tag{18}
\end{align*}
$$

$s_{N, 1}=0.0016775 ; \quad N_{s, 1}=191 ; \quad S_{M, 1}=0.98$ per cent.
$C_{\Sigma, N, 2}=0.59591+0.02645 \beta^{2}-0.21778 \beta^{8}+0.0005286\left(10^{6} \beta / R e_{D}\right)^{0.7}$
$+(0.01895+0.005894 A) \beta^{3.5} \max \left\{\left(10^{6} / R e_{D}\right)^{0.3}, 22.7-4700\left(R e_{D} / 10^{6}\right)\right\}$
$+0.01133(0.75-\beta) \max (2.8-D / 25.4,0.0)$. ( $D: \mathrm{mm}$ )
$s_{N, 2}=0.0016768 ; \quad N_{s, 2}=177 ; \quad S_{M, 2}=0.85$ per cent.

$$
\begin{align*}
& C_{\Sigma, N .3}=0.59592+0.02662 \beta^{2}-0.21740 \beta^{8}+0.0005281\left(10^{6} \beta / R e_{D}\right)^{0.7} \\
& +(0.01876+0.005965 A) \beta^{35} \max \left\{\left(10^{6} / R e_{D}\right)^{0.3}, 22.7-4700\left(R e_{D} / 10^{6}\right)\right\} \\
& +0.01129(0.75-\beta) \max (2.8-D / 25.4,0.0) . \quad(D: \mathrm{mm}) \tag{20}
\end{align*}
$$

$s_{N, 3}=0.0016784 ; \quad N_{s, 3}=139 ; \quad S_{M, 3}=0.76$ per cent.
Since small differences between equations were being investigated the constants for $C_{\Sigma}$ with the same number of decimal places as for $C_{\Sigma, N, i}$ are also required where $\varepsilon_{1, I}$ was used:

$$
\begin{align*}
& C_{\Sigma, I}=0.59615+0.02609 \beta^{2}-0.21675 \beta^{8}+0.0005216\left(10^{6} \beta / R e_{D}\right)^{0.7} \\
& +(0.01874+0.006071 A) \beta^{3.5} \max \left\{\left(10^{6} / R e_{D}\right)^{0.3}, 22.7-4700\left(R e_{D} / 10^{6}\right)\right\} \\
& +0.01101(0.75-\beta) \max (2.8-D / 25.4,0.0) . \quad(D: \mathrm{mm}) \tag{21}
\end{align*}
$$

$$
s_{t}=0.0016747
$$

The constants in equations (7) and (9) have been rounded and then rebalanced to ensure that there is no mean deviation between equation and database.

It can be seen that the differences in $s$ and thus in overall quality of fit are very small but that $\varepsilon_{1, l}$ gives the best result. Moreover the coefficients in equations (18) - (21) are very similar. The largest value of $S_{M}$ for equations (18) - (20) occurs for equation (18); however, even in this case the largest magnitude of difference between the equation and equation (21) is 0.04 per cent for any values of $\beta, D$ and $R e_{D}$ except at the very lowest end of the Reynolds number range (below 4000). Therefore the choice of the expansibility equation has very little effect on the discharge coefficient equation and there is no problem in putting equation (11) in ISO 5167-1. However, the choice of expansibility equation has a significant effect both on some individual data points in the database and when it is used in the field, and it is important that the best equation is obtained.

## 7 CONCLUSIONS

The derivation of the Agreed discharge coefficient equation which is being ballotted in ISO/TC 30/SC 2 for inclusion in ISO 5167-1 has been described; it has been shown that possible changes to the expansibility equation would have only a small effect on the discharge coefficient equation. Deviations of the data in the database from the Agreed equation have been tabulated and a comparison made with deviations from the PR14, RG and Stolz ISO 5167-1: 1991 equations.

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## TABLE_1

## GENERAL INFORMATION ABOUT THE ANALYSIS OF DEVIATIONS IN TABLES 2 TO 8

For each cell, line 1 - Mean per cent error
line 2 - Per cent standard deviation
line 4 - Number of observations
line 5 - Per cent standard deviation about equation.

For the $\mathrm{i}^{\text {th }}$ point in a cell Per cent error, $P_{i}=\frac{\left(C_{i m}-C_{i e}\right)}{C_{i m}} \times 100$,
where $C_{i m}$ is the measured discharge coefficient of the ith point, and
$C_{i e}$ is the corresponding discharge coefficient from the equation.
Mean per cent error, $m=\frac{\sum_{i=1}^{N} P_{i}}{N}$,
where $N$ is the number of points in the cell.
Per cent standard deviation $=\sqrt{\frac{\sum_{i=1}^{N}\left(P_{i}-m\right)^{2}}{N-1}}$.
Per cent standard deviation about equation $=\sqrt{\frac{\sum_{i=1}^{N} P_{i}^{2}}{N}}$.
Statistics for the entire population appear in the bottom right hand cell.

TABLE 2
RESIDUALS FROM EQUATION (10) AS AFUNCTION OF $\beta$ and $D$

| $\begin{gathered} \hline D(\mathrm{~mm}) \\ \beta \\ \hline \end{gathered}$ | 50 | 75 | 100 | 150 | 250 | 600 | Summary <br> by <br> $\beta$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{gathered} 0.100 \\ (0.0991 \text { to } \\ 0.1028) \end{gathered}$ | 0.000 | 0.000 | 0.000 | 0.157 | -0.004 | 0.000 | 0.078 |
|  | 0.000 | 0.000 | 0.000 | 0.238 | 0.269 | 0.000 | 0.265 |
|  | - | - | - | - | $\overline{7}$ | - | - |
|  | 0 | 0 | 0 | 81 | 79 | 0 | 160 |
|  | 0.000 | 0.000 | 0.000 | 0.284 | 0.268 | 0.000 | 0.276 |
| $\begin{gathered} 0.200 \\ (0.1982 \text { to } \\ 0.2418) \end{gathered}$ | 0.012 | -0.047 | 0.079 | 0.088 | -0.170 | 0.028 | -0.019 |
|  | 0.467 | 0.124 | 0.228 | 0.107 | 0.187 | 0.239 | 0.300 |
|  | - | - | - | - | - | - | - |
|  | 507 | 57 | 652 | 111 | 714 | 394 | 2435 |
|  | 0.467 | 0.131 | 0.241 | 0.138 | 0.253 | 0.240 | 0.300 |
| $\begin{gathered} 0.375 \\ (0.3620 \text { to } \\ 0.3748) \end{gathered}$ | 0.073 | -0.003 | 0.088 | 0.138 | -0.106 | 0.037 | 0.031 |
|  | 0.296 | 0.097 | 0.255 | 0.261 | 0.159 | 0.113 | 0.225 |
|  | - | - | - | - | - | - | - |
|  | 444 | 106 | 469 | 133 | 439 | 591 | 2182 |
|  | 0.304 | 0.097 | 0.269 | 0.295 | 0.191 | 0.119 | 0.227 |
| $\begin{gathered} 0.500 \\ (0.4825 \text { to } \\ 0.5003) \end{gathered}$ | -0.030 | 0.138 | 0.135 | 0.098 | 0.033 | -0.063 | 0.016 |
|  | 0.296 | 0.054 | 0.190 | 0.103 | 0.164 | 0.118 | 0.205 |
|  | - | . | - | - | - | - | - |
|  | 398 | 69 | 300 | 109 | 392 | 526 | 1794 |
|  | 0.297 | 0.148 | 0.233 | 0.142 | 0.168 | 0.134 | 0.205 |
| $\begin{gathered} 0.570 \\ (0.5427 \text { to } \\ 0.5770) \end{gathered}$ | -0.109 | 0.095 | 0.028 | 0.009 | 0.074 | -0.090 | 0.009 |
|  | 0.393 | 0.076 | 0.233 | 0.143 | 0.260 | 0.117 | 0.255 |
|  | - | - | - | - | - | - | - |
|  | 348 | 72 | 1008 | 136 | 1123 | 567 | 3254 |
|  | 0.408 | 0.121 | 0.235 | 0.143 | 0.270 | 0.148 | 0.255 |
| $\begin{array}{\|c\|} 0.660 \\ (0.6481 \text { to } \\ 0.6646) \end{array}$ | -0.088 | 0.101 | -0.018 | -0.097 | 0.050 | -0.136 | -0.038 |
|  | 0.305 | 0.100 | 0.245 | 0.204 | 0.197 | 0.143 | 0.233 |
|  | - | - | - | - | - | - | - |
|  | 498 | 64 | 642 | 92 | 823 | 643 | 2762 |
|  | 0.318 | 0.142 | 0.246 | 0.225 | 0.204 | 0.197 | 0.236 |
| $\begin{gathered} 0.750 \\ (0.7239 \text { to } \\ 0.7509) \end{gathered}$ | -0.051 | 0.092 | 0.120 | 0.107 | -0.024 | -0.251 | -0.004 |
|  | 0.342 | 0.106 | 0.322 | 0.333 | 0.321 | 0.338 | 0.340 |
|  | - | - | - | - | - | - | - |
|  | 866 | 101 | 1024 | 130 | 1478 | 336 | 3935 |
|  | 0.345 | 0.140 | 0.344 | 0.349 | 0.322 | 0.421 | 0.340 |
| $\begin{gathered} \text { Summary } \\ \text { by } \\ D \end{gathered}$ | -0.032 | 0.062 | 0.067 | 0.073 | -0.013 | -0.073 | -0.002 |
|  | 0.359 | 0.113 | 0.264 | 0.229 | 0.262 | 0.199 | 0.274 |
|  |  |  | - | - | - | - | - |
|  | 3061 | 469 | 4095 | 792 | 5048 | 3057 | 16522 |
|  | 0.361 | 0.129 | 0.272 | 0.240 | 0.262 | 0.212 | 0.274 |

TABLE 3
RESIDUALS FROM EQUATION (10)AS A FUNCTION OF D AND PAIR OF TAPPINGS

| Tappings $D(\mathrm{~mm})$ | Corner (ISO) | Flange | $D \& D / 2$ | Corner (GU) | Summary by $D$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 50 | 0.008 | -0.037 | -0.063 | 0.000 | -0.032 |
|  | 0.411 | 0.314 | 0.393 | 0.000 | 0.359 |
|  | . | - | - | - | - |
|  | 728 | 1605 | 728 | 0 | 3061 |
|  | 0.411 | 0.316 | 0.398 | 0.000 | 0.361 |
| 75 | 0.000 | 0.062 | 0.000 | 0.000 | 0.062 |
|  | 0.000 | 0.113 | 0.000 | 0.000 | 0.113 |
|  | - | - | - | - | . |
|  | 0 | 469 | 0 | 0 | 469 |
|  | 0.000 | 0.129 | 0.000 | 0.000 | 0.129 |
| 100 | 0.040 | 0.064 | 0.104 | 0.000 | 0.067 |
|  | 0.224 | 0.257 | 0.313 | 0.000 | 0.264 |
|  | - | - | - | - | - |
|  | 1084 | 2078 | 933 | 0 | 4095 |
|  | 0.228 | 0.265 | 0.330 | 0.000 | 0.272 |
| 150 | 0.000 | 0.073 | 0.000 | 0.000 | 0.073 |
|  | 0.000 | 0.229 | 0.000 | 0.000 | 0.229 |
|  | - | - | - | - | - |
|  | 0 | 792 | 0 | 0 | 792 |
|  | 0.000 | 0.240 | 0.000 | 0.000 | 0.240 |
| 250 | 0.026 | -0.068 | -0.009 | 0.044 | -0.013 |
|  | 0.249 | 0.212 | 0.280 | 0.320 | 0.262 |
|  | - | - | - | - | - |
|  | 1155 | 1841 | 1167 | 885 | 5048 |
|  | 0.250 | 0.223 | 0.280 | 0.322 | 0.262 |
| 600 | -0.156 | 0.003 | -0.066 | -0.097 | -0.073 |
|  | 0.199 | 0.160 | 0.200 | 0.209 | 0.199 |
|  | - | - | - | - | - |
|  | 828 | 876 | 1130 | 223 | 3057 |
|  | 0.253 | 0.160 | 0.211 | 0.230 | 0.212 |
| Summary by Tappings | -0.013 | 0.005 | -0.009 | 0.016 | -0.002 |
|  | 0.283 | 0.248 | 0.301 | 0.306 | 0.274 |
|  | - | - | $\stackrel{-}{-7}$ | - | ${ }^{-}$ |
|  | 3795 | 7661 | 3958 | 1108 | 16522 |
|  | 0.283 | 0.249 | 0.301 | 0.306 | 0.274 |

TABLE 4
RESIDUALS EROM EQUATION (10) AS A FUNCTION OF $\beta$ AND Re ${ }_{D}$

| $R e_{D}$ $\beta$ | $\begin{gathered} 10 \\ \text { to } \\ 4000 \end{gathered}$ | $\begin{gathered} 4000 \\ \text { to } \\ 10^{4} \end{gathered}$ | $\begin{gathered} 10^{4} \\ \text { to } \\ 10^{5} \end{gathered}$ | $\begin{gathered} 10^{5} \\ \text { to } \\ 10^{6} \\ \hline \end{gathered}$ | $\begin{gathered} 10^{6} \\ \text { to } \\ 10^{7} \\ \hline \end{gathered}$ | $\begin{gathered} 10^{\prime} \\ \text { to } \\ 10^{8} \\ \hline \end{gathered}$ | $\begin{gathered} \hline \text { Summary } \\ \text { by } \\ \beta \\ \hline \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.100 | 0.095 | 0.108 | 0.037 | 0.000 | 0.000 | 0.000 | 0.078 |
|  | 0.189 | 0.282 | 0.305 | 0.000 | 0.000 | 0.000 | 0.265 |
|  | - | - | - | - | - | - | - |
|  | 52 | 49 | 59 | 0 | 0 | 0 | 160 |
|  | 0.211 | 0.299 | 0.305 | 0.000 | 0.000 | 0.000 | 0.276 |
| 0.200 | -0.027 | 0.047 | -0.050 | -0.051 | 0.102 | 0.000 | -0.019 |
|  | 0.556 | 0.354 | 0.249 | 0.240 | 0.170 | 0.000 | 0.300 |
|  | - | - | - | - | - | - | - |
|  | 237 | 238 | 1190 | 454 | 316 | 0 | 2435 |
|  | 0.556 | 0.357 | 0.254 | 0.245 | 0.198 | 0.000 | 0.300 |
| 0.375 | 0.376 | 0.125 | 0.008 | -0.041 | 0.049 | 0.082 | 0.031 |
|  | 0.509 | 0.322 | 0.175 | 0.157 | 0.084 | 0.083 | 0.225 |
|  | - | - | - | - | - | - | - |
|  | 125 | 133 | 748 | 711 | 325 | 140 | 2182 |
|  | 0.631 | 0.344 | 0.175 | 0.162 | 0.097 | 0.116 | 0.227 |
| 0.500 | -0.057 | -0.093 | 0.034 | 0.071 | -0.090 | -0.057 | 0.016 |
|  | 0.760 | 0.281 | 0.195 | 0.163 | 0.134 | 0.085 | 0.205 |
|  | - | - | - | - | - | - | - |
|  | 33 | 83 | 436 | 788 | 205 | 249 | 1794 |
|  | 0.750 | 0.294 | 0.198 | 0.177 | 0.162 | 0.102 | 0.205 |
| 0.570 | -0.546 | -0.334 | -0.040 | 0.002 | 0.083 | 0.028 | 0.009 |
|  | 0.968 | 0.459 | 0.241 | 0.195 | 0.273 | 0.224 | 0.255 |
|  | - | - | - | - | - | - | - |
|  | 18 | 59 | 502 | 1430 | 782 | 463 | 3254 |
|  | 1.087 | 0.564 | 0.244 | 0.195 | 0.285 | 0.226 | 0.255 |
| 0.660 | -0.087 | -0.325 | -0.110 | 0.012 | 0.025 | -0.100 | -0.038 |
|  | 0.116 | 0.522 | 0.307 | 0.203 | 0.193 | 0.177 | 0.233 |
|  | - | - | - | - | - | - | - |
|  | 5 | 35 | 466 | 1121 | 475 | 660 | 2762 |
|  | 0.135 | 0.609 | 0.326 | 0.204 | 0.194 | 0.203 | 0.236 |
| 0.750 | 0.000 | 0.242 | -0.073 | 0.038 | -0.029 | -0.053 | -0.004 |
|  | 0.000 | 0.514 | 0.404 | 0.303 | 0.315 | 0.355 | 0.340 |
|  | . | - | - | - | - | - | - |
|  | 0 | 78 | 615 | 1704 | 1062 | 476 | 3935 |
|  | 0.000 | 0.565 | 0.410 | 0.305 | 0.316 | 0.359 | 0.340 |
| Summary by $R e_{D}$ | 0.071 | 0.019 | -0.038 | 0.013 | 0.024 | -0.041 | -0.002 |
|  | 0.589 | 0.411 | 0.273 | 0.230 | 0.255 | 0.239 | 0.274 |
|  | - | - | - | - | - | - | - |
|  | 470 | 675 | 4016 | 6208 | 3165 | 1988 | 16522 |
|  | 0.593 | 0.411 | 0.276 | 0.231 | 0.257 | 0.242 | 0.274 |

TABLE 5
RESIDUALS FROM EQUATION (10)AS A FUNCTION OF $\beta$ and $D\left(R_{e_{D}} \geq 4000\right)$

| $\begin{gathered} \hline D(\mathrm{~mm}) \\ \beta \end{gathered}$ | 50 | 75 | 100 | 150 | 250 | 600 | Summary by $\beta$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & 0.100 \\ & (0.0991 \text { to } \\ & 0.1028) \end{aligned}$ | 0.000 | 0.000 | 0.000 | 0.267 | -0.004 | 0.000 | 0.069 |
|  | 0.000 | 0.000 | 0.000 | 0.277 | 0.269 | 0.000 | 0.296 |
|  | - | - | - | - | - | - | - |
|  | 0 | 0 | 0 | 29 | 79 | 0 | 108 |
|  | 0.000 | 0.000 | 0.000 | 0.381 | 0.268 | 0.000 | 0.302 |
| $\begin{gathered} 0.200 \\ (0.1982 \text { to } \\ 0.2418) \end{gathered}$ | 0.052 | -0.047 | 0.079 | 0.095 | -0.170 | 0.028 | -0.018 |
|  | 0.341 | 0.124 | 0.232 | 0.079 | 0.187 | 0.239 | 0.258 |
|  | - | - | - | - | - | - | - |
|  | 324 | 57 | 626 | 83 | 714 | 394 | 2198 |
|  | 0.345 | 0.131 | 0.244 | 0.123 | 0.253 | 0.240 | 0.258 |
| $\begin{aligned} & 0.375 \\ & (0.3620 \text { to } \\ & 0.3748) \end{aligned}$ | 0.022 | -0.003 | 0.061 | 0.081 | -0.106 | 0.037 | 0.010 |
|  | 0.190 | 0.097 | 0.200 | 0.175 | 0.159 | 0.113 | 0.174 |
|  | - | - | - | - | - | - | - |
|  | 344 | 106 | 455 | 122 | 439 | 591 | 2057 |
|  | 0.191 | 0.097 | 0.209 | 0.192 | 0.191 | 0.119 | 0.174 |
| $\begin{gathered} 0.500 \\ (0.4825 \text { to } \\ 0.5003) \end{gathered}$ | -0.028 | 0.138 | 0.135 | 0.098 | 0.033 | -0.063 | 0.017 |
|  | 0.211 | 0.054 | 0.190 | 0.103 | 0.164 | 0.118 | 0.179 |
|  | - | - | - | - | - | - | - |
|  | 365 | 69 | 300 | 109 | 392 | 526 | 1761 |
|  | 0.213 | 0.148 | 0.233 | 0.142 | 0.168 | 0.134 | 0.180 |
| $\begin{gathered} 0.570 \\ (0.5427 \text { to } \\ 0.5770) \end{gathered}$ | -0.086 | 0.095 | 0.028 | 0.009 | 0.074 | -0.090 | 0.012 |
|  | 0.322 | 0.076 | 0.233 | 0.143 | 0.260 | 0.117 | 0.242 |
|  | - | - | - | - | - | - | - |
|  | 330 | 72 | 1008 | 136 | 1123 | 567 | 3236 |
|  | 0.333 | 0.121 | 0.235 | 0.143 | 0.270 | 0.148 | 0.243 |
| $\begin{array}{\|c\|} \hline 0.660 \\ (0.6481 \text { to } \\ 0.6646) \end{array}$ | -0.088 | 0.101 | -0.018 | -0.097 | 0.050 | -0.136 | -0.038 |
|  | 0.307 | 0.100 | 0.245 | 0.204 | 0.197 | 0.143 | 0.233 |
|  | - | - | - | - | - | - | - |
|  | 493 | 64 | 642 | 92 | 823 | 643 | 2757 |
|  | 0.319 | 0.142 | 0.246 | 0.225 | 0.204 | 0.197 | 0.236 |
| $\begin{gathered} 0.750 \\ (0.7239 \text { to } \\ 0.7509) \end{gathered}$ | -0.051 | 0.092 | 0.120 | 0.107 | -0.024 | -0.251 | -0.004 |
|  | 0.342 | 0.106 | 0.322 | 0.333 | 0.321 | 0.338 | 0.340 |
|  | - | - | - | - | - | - | - |
|  | 866 | 101 | 1024 | 130 | 1478 | 336 | 3935 |
|  | 0.345 | 0.140 | 0.344 | 0.349 | 0.322 | 0.421 | 0.340 |
| Summaryby$D$ | -0.037 | 0.062 | 0.064 | 0.061 | -0.013 | -0.073 | -0.004 |
|  | 0.305 | 0.113 | 0.259 | 0.217 | 0.262 | 0.199 | 0.259 |
|  | . | - | - | - | - | - | - |
|  | 2722 | 469 | 4055 | 701 | 5048 | 3057 | 16052 |
|  | 0.308 | 0.129 | 0.267 | 0.225 | 0.262 | 0.212 | 0.259 |

TABLE 6
RESIDUALS FROMEQUATION (10)AS A FUNCTIONOF $\beta$ AND $R e_{D}\left(R e_{D} \geq 4000\right)$

| $\begin{gathered} R e_{D} \\ \beta \\ \hline \end{gathered}$ | $\begin{gathered} 10 \\ \text { to } \\ 4000 \\ \hline \end{gathered}$ | $\begin{gathered} 4000 \\ \text { to } \\ 10^{4} \\ \hline \end{gathered}$ | $\begin{gathered} 10^{4} \\ \text { to } \\ 10^{5} \\ \hline \end{gathered}$ | $\begin{gathered} 10^{5} \\ \text { to } \\ 10^{6} \\ \hline \end{gathered}$ | $\begin{gathered} 10^{0} \\ \text { to } \\ 10^{7} \\ \hline \end{gathered}$ | $\begin{gathered} 10^{7} \\ \text { to } \\ 10^{8} \\ \hline \end{gathered}$ | $\begin{gathered} \hline \text { Summary } \\ \text { by } \\ \beta \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.100 | 0.000 | 0.108 | 0.037 | 0.000 | 0.000 | 0.000 | 0.069 |
|  | 0.000 | 0.282 | 0.305 | 0.000 | 0.000 | 0.000 | 0.296 |
|  | - | - | - | - | - | - | - |
|  | 0 | 49 | 59 | 0 | 0 | 0 | 108 |
|  | 0.000 | 0.299 | $0.30{ }^{\circ}$ | 0.000 | 0.000 | 0.000 | 0.302 |
| 0.200 | 0.000 | 0.047 | -0.050 | -0.051 | 0.102 | 0.000 | -0.018 |
|  | 0.000 | 0.354 | 0.249 | 0.240 | 0.170 | 0.000 | 0.258 |
|  | - | - | - | - | - | - | - |
|  | 0 | 238 | 1190 | 454 | 316 | 0 | 2198 |
|  | 0.000 | 0.357 | 0.254 | 0.245 | 0.198 | 0.000 | 0.258 |
| 0.375 | 0.000 | 0.125 | 0.008 | -0.041 | 0.049 | 0.082 | 0.010 |
|  | 0.000 | 0.322 | 0.175 | 0.157 | 0.084 | 0.083 | 0.174 |
|  | - | - | - | - | - | - | - |
|  | 0 | 133 | 748 | 711 | 325 | 140 | 2057 |
|  | 0.000 | 0.344 | 0.175 | 0.162 | 0.097 | 0.116 | 0.174 |
| 0.500 | 0.000 | -0.093 | 0.034 | 0.071 | -0.090 | -0.057 | 0.017 |
|  | 0.000 | 0.281 | 0.195 | 0.163 | 0.134 | 0.085 | 0.179 |
|  | - | - | - | - | - | - | - |
|  | 0 | 83 | 436 | 788 | 205 | 249 | 1761 |
|  | 0.000 | 0.294 | 0.198 | 0.177 | 0.162 | 0.102 | 0.180 |
| 0.570 | 0.000 | -0.334 | -0.040 | 0.002 | 0.083 | 0.028 | 0.012 |
|  | 0.000 | 0.459 | 0.241 | 0.195 | 0.273 | 0.224 | 0.242 |
|  | . | - | - | - | - | - | - |
|  | 0 | 59 | 502 | 1430 | 782 | 463 | 3236 |
|  | 0.000 | 0.564 | 0.244 | 0.195 | 0.285 | 0.226 | 0.243 |
| 0.660 | 0.000 | -0.325 | -0.110 | 0.012 | 0.025 | -0.100 | -0.038 |
|  | 0.000 | 0.522 | 0.307 | 0.203 | 0.193 | 0.177 | 0.233 |
|  | - | - | - | - | - | - | - |
|  | 0 | 35 | 466 | 1121 | 475 | 660 | 2757 |
|  | 0.000 | 0.609 | 0.326 | 0.204 | 0.194 | 0.203 | 0.236 |
| 0.750 | 0.000 | 0.242 | -0.073 | 0.038 | -0.029 | -0.053 | -0.004 |
|  | 0.000 | 0.514 | 0.404 | 0.303 | 0.315 | 0.355 | 0.340 |
|  | - | - | - | - | - | - | - 393 |
|  | 0 | 78 | 615 | 1704 | 1062 | 476 | 3935 |
|  | 0.000 | 0.565 | 0.410 | 0.305 | 0.316 | 0.359 | 0.340 |
| $\begin{gathered} \text { Summary } \\ \text { by } \\ R e_{D} \end{gathered}$ | 0.000 | 0.019 | -0.038 | 0.013 | 0.024 | -0.041 | -0.004 |
|  | 0.000 | 0.411 | 0.273 | 0.230 | 0.255 | 0.239 | 0.259 |
|  | - | - | - | - | - | - | - |
|  | 0 | 675 | 4016 | 6208 | 3165 | 1988 | 16052 |
|  | 0.000 | 0.411 | 0.276 | 0.231 | 0.257 | 0.242 | 0.259 |

## TABLE7

DEVIATIONS FROM EQUATION (10) AS A FUNCTION OF D AND $R e_{D}\left(R e_{D} \geq 4000\right)$

|  | $\begin{gathered} 10 \\ \text { to } \\ 4000 \\ \hline \end{gathered}$ | $\begin{gathered} 4000 \\ \text { to } \\ 10^{4} \\ \hline \end{gathered}$ | $\begin{gathered} 10^{4} \\ \text { to } \\ 10^{5} \\ \hline \end{gathered}$ | $\begin{gathered} 10^{5} \\ \text { to } \\ 10^{6} \\ \hline \end{gathered}$ | $\begin{gathered} 10^{\circ} \\ \text { to } \\ 10^{7} \\ \hline \end{gathered}$ | $\begin{gathered} 10^{\prime} \\ \text { to } \\ 10^{8} \\ \hline \end{gathered}$ | Summary by D |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 50 | 0.000 | -0.031 | -0.082 | 0.095 | 0.000 | 0.000 | -0.037 |
|  | 0.000 | 0.467 | 0.278 | 0.178 | 0.000 | 0.000 | 0.305 |
|  | - | - | - | - | - | - | - |
|  | 0 | 403 | 1749 | 570 | 0 | 0 | 2722 |
|  | 0.000 | 0.467 | 0.289 | 0.202 | 0.000 | 0.000 | 0.308 |
| 75 | 0.000 | -0.047 | 0.046 | 0.086 | 0.000 | 0.000 | 0.062 |
|  | 0.000 | 0.137 | 0.119 | 0.097 | 0.000 | 0.000 | 0.113 |
|  | . | - | - | - | - | - | - |
|  | 0 | 22 | 209 | 238 | 0 | 0 | 469 |
|  | 0.000 | 0.141 | 0.127 | 0.129 | 0.000 | 0.000 | 0.129 |
| 100 | 0.000 | 0.149 | 0.041 | 0.057 | 0.114 | 0.000 | 0.064 |
|  | 0.000 | 0.252 | 0.271 | 0.259 | 0.228 | 0.000 | 0.259 |
|  | - | - | - | - | - | - | - |
|  | 0 | 134 | 1111 | 2276 | 534 | 0 | 4055 |
|  | 0.000 | 0.292 | 0.274 | 0.265 | 0.255 | 0.000 | 0.267 |
| 150 | 0.000 | 0.225 | 0.149 | -0.031 | -0.119 | 0.000 | 0.061 |
|  | 0.000 | 0.226 | 0.244 | 0.125 | 0.095 | 0.000 | 0.217 |
|  | - | - | - | - | - | - | - |
|  | 0 | 68 | 275 | 328 | 30 | 0 | 701 |
|  | 0.000 | 0.317 | 0.286 | 0.129 | 0.151 | 0.000 | 0.225 |
| 250 | 0.000 | -0.175 | -0.153 | -0.032 | 0.011 | 0.194 | -0.013 |
|  | 0.000 | 0.360 | 0.212 | 0.222 | 0.296 | 0.231 | 0.262 |
|  | - | - | - | - | - | - | - |
|  | 0 | 48 | 646 | 2348 | 1490 | 516 | 5048 |
|  | 0.000 | 0.397 | 0.261 | 0.225 | 0.296 | 0.301 | 0.262 |
| 600 | 0.000 | 0.000 | -0.323 | -0.078 | 0.002 | -0.123 | -0.073 |
|  | 0.000 | 0.000 | 0.336 | 0.194 | 0.196 | 0.179 | 0.199 |
|  | - | - | - | - | - | - | - |
|  | 0 | 0 | 26 | 448 | 1111 | 1472 | 3057 |
|  | 0.000 | 0.000 | 0.461 | 0.209 | 0.196 | 0.218 | 0.212 |
| $\begin{gathered} \text { Summary } \\ \text { by } \\ R e_{D} \end{gathered}$ | 0.000 | 0.019 | -0.038 | 0.013 | 0.024 | -0.041 | -0.004 |
|  | 0.000 | 0.411 | 0.273 | 0.230 | 0.255 | 0.239 | 0.259 |
|  | . | - | - | - | - | - | - |
|  | 0 | 675 | 4016 | 6208 | 3165 | 1988 | 16052 |
|  | 0.000 | 0.411 | 0.276 | 0.231 | 0.257 | 0.242 | 0.259 |

TABLE 8
DEVIATIONS FROM EQUATION (10) AS A FUNCTION OF $\beta$ AND PAIR OF TAPPINGS $\left(R e_{D} \geq 4000\right)$

| Tappings <br> B | $\begin{aligned} & \hline \text { Corner } \\ & \text { (ISO) } \end{aligned}$ | Flange | D\&D/2 | Corner <br> (GU) | Summary by $\beta$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | 0.000 | 0.069 | 0.000 | 0.000 | 0.069 |
|  | 0.000 | 0.296 | 0.000 | 0.000 | 0.296 |
| 0.100 | - | - | - | - | - |
|  | 0 | 108 | 0 | 0 | 108 |
|  | 0.000 | 0.302 | 0.000 | 0.000 | 0.302 |
|  | 0.018 | -0.034 | -0.002 | -0.098 | -0.018 |
|  | 0.274 | 0.233 | 0.272 | 0.259 | 0.258 |
| 0.200 | - | - | - | - | - |
|  | 537 | 925 | 582 | 154 | 2198 |
|  | 0.275 | 0.235 | 0.272 | 0.276 | 0.258 |
|  | -0.006 | 0.026 | 0.006 | -0.072 | 0.010 |
|  | 0.175 | 0.178 | 0.155 | 0.170 | 0.174 |
| 0.375 | - | - | - | - | - |
|  | 382 | 1101 | 466 | 108 | 2057 |
|  | 0.175 | 0.180 | 0.155 | 0.184 | 0.174 |
|  | 0.012 | 0.020 | 0.029 | -0.037 | 0.017 |
|  | 0.242 | 0.154 | 0.160 | 0.178 | 0.179 |
| 0.500 | - | - | - | - | - |
|  | 369 | 884 | 403 | 105 | 1761 |
|  | 0.241 | 0.155 | 0.163 | 0.181 | 0.180 |
|  | -0.002 | -0.035 | 0.084 | 0.080 | 0.012 |
|  | 0.231 | 0.197 | 0.274 | 0.314 | 0.242 |
| 0.570 | - | - | - | - | - |
|  | 794 | 1366 | 845 | 231 | 3236 |
|  | 0.231 | 0.200 | 0.286 | 0.323 | 0.243 |
|  | -0.068 | -0.045 | 0.006 | -0.040 | -0.038 |
|  | 0.231 | 0.215 | 0.256 | 0.244 | -0.233 |
| 0.660 | - | - | - | - | - |
|  | 671 | 1200 | 690 | 196 | 2757 |
|  | 0.241 | 0.220 | 0.256 | 0.246 | 0.236 |
| 0.750 | -0.026 | 0.060 | -0.147 | 0.106 | -0.004 |
|  | 0.324 | 0.290 | 0.381 | 0.381 | 0.340 |
|  | - | - | - | - | - |
|  | 963 | 1765 | 893 | 314 | 3935 |
|  | 0.325 | 0.296 | 0.408 | 0.395 | 0.340 |
| Summary by Tappings | -0.017 | 0.004 | -0.011 | 0.016 | -0.004 |
|  | 0.262 | 0.229 | 0.291 | 0.306 | 0.259 |
|  |  | - | - | - | - |
|  | 3716 | 7349 | 3879 | 1108 | 16052 |
|  | 0.263 | 0.229 | 0.291 | 0.306 | 0.259 |

## TABLE 9

ANALYSIS OF DEVIATIONS FOR THE DATABASE (Re $e_{D} \geq 4000$ )

|  | $\begin{gathered} \text { AGREED } \\ \text { EQUATION (11) } \end{gathered}$ |  | $\begin{gathered} \text { PR14 } \\ \text { EQUATION } \end{gathered}$ |  | $\begin{gathered} \text { RG } \\ \text { EQUATION } \end{gathered}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | ${ }^{\prime}$ | $\mathrm{N}_{2}$ | $N_{1}$ | $\mathrm{N}_{2}$ | $N_{1}$ | $\mathrm{N}_{2}$ |
| $\beta \vee D$ | 10 | 2 | 14 | 2 | 18 | 6 |
| $\beta \vee R e_{D}$ | 8 | 3 | 6 | 5 | 14 | 6 |
| $D \vee R e_{D}$ | 10 | 2 | 7 | 2 | 12 | 2 |
| $\beta \vee$ Tappings | 2 | 0 | 2 | 0 | 6 | 1 |
| TOTAL | 30 | 7 | 29 | 9 | 50 | 15 |
| $s$ (per cent) | 0.259 |  | 0.254 |  | 0.292 |  |

## TABLE 10

## ANALYSIS OF DEVIATIONS FOR THE DATABASE OVER THE RANGE OF THE STOLZ ISO 5167-1:1991 EOUATION ( $R e_{D} \geq 4000$ )

|  | $\begin{array}{r} \mathrm{A} \\ \mathrm{EQU} \end{array}$ | $\begin{aligned} & \overline{\mathrm{ED}} \\ & \mathrm{~N}(11) \end{aligned}$ | PR14 | TION |  |  | ISO <br> EQ | :1991 <br> ON |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $N_{1}$ | $\mathrm{N}_{2}$ | $N_{1}$ | $\mathrm{N}_{2}$ | $N_{1}$ | $\mathrm{N}_{2}$ | $N_{1}$ | $\mathrm{N}_{2}$ |
| $\beta \vee D$ | 8 | 1 | 15 | 2 | 17 | 6 | 26 | 16 |
| $\beta \vee R e_{D}$ | 4 | 0 | 3 | 1 | 10 | 4 | 16 | 12 |
| $D \vee R e_{D}$ | 7 | 2 | 10 | 3 | 10 | 3 | 16 | 10 |
| $\beta \vee$ Tappings | 2 | 0 | 2 | 0 | 5 | 0 | 13 | 7 |
| TOTAL | 21 | 3 | 30 | 6 | 42 | 13 | 71 | 45 |
| $s$ (per cent) | 0.245 |  | 0.247 |  | 0.277 |  | 0.390 |  |

## DEFINITIONS FOR TABLES 9 AND 10:

$N_{1}$ is the number of boxes with the mean deviation greater than 0.1 per cent. $N_{2}$ is the number of boxes with the mean deviation greater than 0.2 per cent. $s$ is the standard deviation of the data in the database about the equation.


FIG 1 Upstream tapping term as a function of $L_{1}$

