

The Polygon Angle-Sum Theorems

1. Plan

What You'll Learn

- To classify polygons
- To find the sums of the measures of the interior and exterior angles of polygons

... And Why

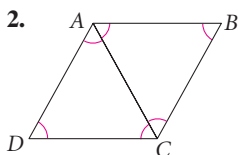
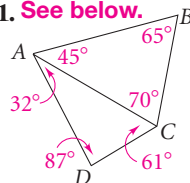
To find the measure of an angle of a triangle used in packaging, as in Example 5

Check Skills You'll Need

GO for Help Lessons 1-6 and 3-4

Find the measure of each angle of quadrilateral $ABCD$.

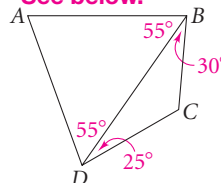
1. See below.



$$m\angle D = m\angle B = 60;$$

$$m\angle DAB = m\angle DCB = 120$$

3. See below.



New Vocabulary

- polygon
- convex polygon
- concave polygon
- equilateral polygon
- equiangular polygon
- regular polygon

$$1. m\angle DAB = 77; m\angle B = 65;$$

$$m\angle BCD = 131; m\angle D = 87$$

$$3. m\angle A = 70; m\angle ABC = 85;$$

$$m\angle C = 125; m\angle ADC = 80$$

1

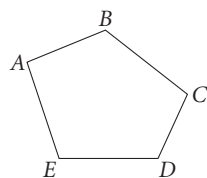
Classifying Polygons



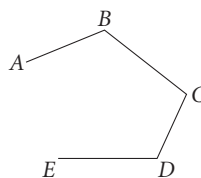
Real-World Connection

Polygons create striking designs on a soccer ball.

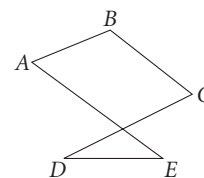
A **polygon** is a closed plane figure with at least three sides that are segments. The sides intersect only at their endpoints, and no adjacent sides are collinear.



A polygon



Not a polygon;
not a closed figure



Not a polygon;
two sides intersect
between endpoints.

To name a polygon, start at any vertex and list the vertices consecutively in a clockwise or counterclockwise direction.

1 EXAMPLE Naming Polygons

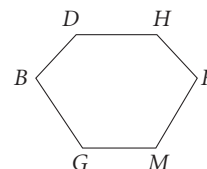
Name the polygon. Then identify its vertices, sides, and angles.

Two names for this polygon are $DHKMGB$ and $MKHDBG$.

vertices: $\overline{D, H, K, M, G, B}$

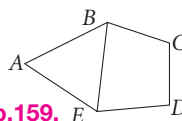
sides: $\overline{DH}, \overline{HK}, \overline{KM}, \overline{MG}, \overline{GB}, \overline{BD}$

angles: $\angle D, \angle H, \angle K, \angle M, \angle G, \angle B$



Quick Check

- 1 Three polygons are pictured at the right. Name each polygon, its sides, and its angles. See margin, p. 159.



Objectives

- To classify polygons
- To find the sums of the measures of the interior and exterior angles of polygons

Examples

- Naming Polygons
- Real-World Connection
- Finding a Polygon Angle Sum
- Using the Polygon Angle-Sum Theorem
- Real-World Connection



Math Background

Because each interior angle of a regular n -gon measures $\frac{180(n-2)}{n}$, one can readily find the set of all regular n -gons that tessellate a plane. Combinations of regular polygons that tessellate a plane can likewise be found with a bit more work and application of some straightforward number theory.

More Math Background: p. 124D

Lesson Planning and Resources

See p. 124E for a list of the resources that support this lesson.



Bell Ringer Practice

Check Skills You'll Need

For intervention, direct students to:

Using the Angle Addition Postulate

Lesson 1-6: Example 3
Extra Skills, Word Problems, Proof Practice, Ch. 1

Finding Angle Measures in Triangles

Lesson 3-4: Example 1
Extra Skills, Word Problems, Proof Practice, Ch. 3

Differentiated Instruction Solutions for All Learners

Special Needs L1

Have students draw a polygon with an exterior angle at each vertex. Students cut out their exterior angles and tape the vertices together. Students should recognize the sum of the exterior angles is 360° .

learning style: tactile

Below Level L2

Have students draw a polygon on paper and cut out the exterior angles. By placing the angles so that they are adjacent, students can verify Theorem 3-15.

learning style: tactile

2. Teach

Guided Instruction

1 EXAMPLE Math Tip

Remind students that there are also different ways to name sides and angles in this example. Ask: *What is another name for \overline{HK} ? $\angle M$? $\angle KMG$ or $\angle GMK$*

Connection to Science

The study of optics teaches that a convex lens causes rays of light to come together and that a concave lens causes rays of light to spread apart. Convex lenses are used in microscopes and telescopes. Eyeglasses may be either convex or concave.

Teaching Tip

Students may want to know that a seven-sided polygon is called a *heptagon*.

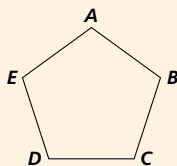
Visual Learners

To help students learn the names of polygons, have small groups make charts to be displayed in the classroom with the name of each polygon, its number of sides, and an appropriate figure.

PowerPoint

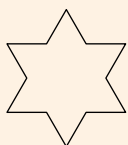
Additional Examples

1 Name the polygon. Then identify its vertices, sides, and angles.



\overline{ABCDE} ; vertices: A, B, C, D, E ;
sides: $\overline{AB}, \overline{BC}, \overline{CD}, \overline{DE}, \overline{EA}$;
angles: $\angle A, \angle B, \angle C, \angle D, \angle E$

2 Classify the polygon below by its sides. Identify it as convex or concave. **dodecagon; concave**



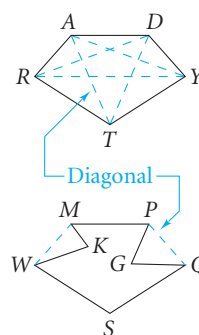
Vocabulary Tip

A **diagonal** of a polygon is a segment that connects two nonconsecutive vertices.

You can classify a polygon by the number of sides it has. The table at the right shows the names of some common polygons.

Sides	Name
3	triangle
4	quadrilateral
5	pentagon
6	hexagon
8	octagon
9	nonagon
10	decagon
12	dodecagon
n	n -gon

Polygons are classified as convex or concave.



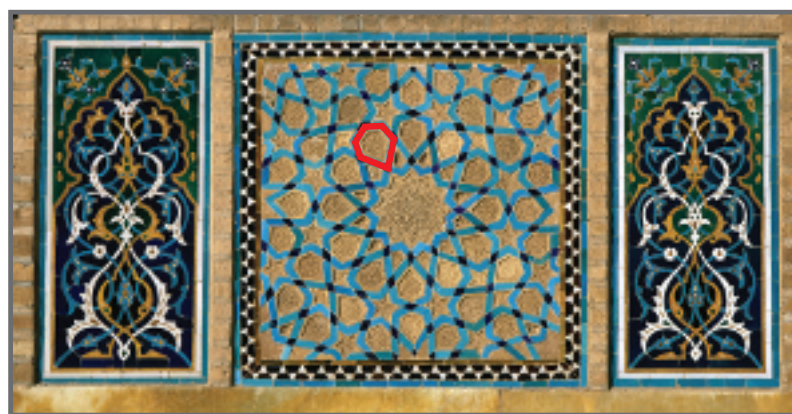
A **convex polygon** has no diagonal with points outside the polygon.

A **concave polygon** has at least one diagonal with points outside the polygon.

In this textbook, a polygon is convex unless stated otherwise.

2 EXAMPLE Real-World Connection

Tilework The tilework in the photo is a combination of different polygons that form a pleasing pattern. Classify the polygon outlined in red by using the table above. Then classify the polygon as convex or concave.



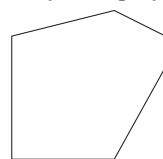
The polygon outlined in red has 6 sides. Therefore, it is a hexagon.

No diagonal of the hexagon contains points outside the hexagon. The hexagon is convex.



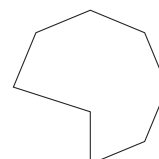
2 Classify each polygon by its sides. Identify each as convex or concave.

a.



hexagon; convex

b.



octagon; concave

c. the 12-pointed star at the center of the tilework pictured above **24-gon; concave**

Differentiated Instruction Solutions for All Learners

Advanced Learners L4





Have students do the Activity, discuss whether Theorem 3-14 applies to *concave* n -gons, and justify their reasoning.

English Language Learners ELL

Some students may not know the meaning of *adjacent sides* in the definition of a polygon. Show how adjacent sides share a vertex just as *adjacent angles* share a side

Activity: The Sum of Polygon Angle Measures

You can use triangles and the Triangle Angle-Sum Theorem to find the sum of the measures of the angles of a polygon. Record your data in a table like the one begun below.

Polygon	Number of Sides	Number of Triangles Formed	Sum of the Interior Angle Measures
	4		 $\cdot 180 =$ 

- Sketch polygons with 4, 5, 6, 7, and 8 sides.
 - Divide each polygon into triangles by drawing all diagonals that are possible from one vertex.
 - Multiply the number of triangles by 180 to find the sum of the measures of the angles of each polygon.
1. Look for patterns in the table. Describe any that you find. *See back of book.*
2. **Inductive Reasoning** Write a rule for the sum of the measures of the angles of an n -gon. **The sum of the measures of the angles of an n -gon is $(n - 2) \cdot 180$.**

By dividing a polygon with n sides into $n - 2$ triangles, you can show that the sum of the measures of the angles of any polygon is a multiple of 180.

Guided Instruction**Activity**

Have students work in pairs to draw polygons and display the sum of their angles using geometry software. Have students manipulate the polygons to see that the sum remains constant.

Alternative Method

Students can use inductive reasoning to write the sum of the measures of the angles of an n -gon as $180n - 360$. Have them draw a hexagon and segments from an interior point to each vertex. Ask: *How many triangles are there?* **6** *What is the sum of the angle measures of all the triangles?* **$180 \cdot 6$, or 1080** *What is the sum of the angle measures of the triangles drawn from an interior point to the vertices of a polygon with n sides?* **$180n$** Point out that the sum of the angle measures around the interior point is 360. Discuss why 360 must then be subtracted from $180n$.

Vocabulary Tip

An n -gon is a polygon with n sides, where n can be 3, 4, 5, 6, ...

Key Concepts**Theorem 3-14 Polygon Angle-Sum Theorem**

The sum of the measures of the angles of an n -gon is $(n - 2)180$.

3 EXAMPLE Error Prevention

Some students may think the answer should be $15 \cdot 180$. Review the activity to correct them.

PowerPoint

Additional Examples

- 3** Find the sum of the measures of the angles of a decagon. **1440**

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3 EXAMPLE Finding a Polygon Angle Sum

Find the sum of the measures of the angles of a 15-gon.

For a 15-gon, $n = 15$.

$$\begin{aligned} \text{Sum} &= (n - 2)180 && \text{Polygon Angle-Sum Theorem} \\ &= (15 - 2)180 && \text{Substitute.} \\ &= 13 \cdot 180 && \text{Simplify.} \\ &= 2340 \end{aligned}$$

- The sum of the measures of the angles of a 15-gon is 2340.

Quick Check

- 3 a.** Find the sum of the measures of the angles of a 13-gon. **1980**
b. Critical Thinking The sum of the measures of the angles of a given polygon is 720. How can you use $\text{Sum} = (n - 2)180$ to find the number of sides in the polygon?
You can solve the equation $(n - 2)180 = 720$.
You will sometimes use algebra with the Polygon Angle-Sum Theorem to find measures of polygon angles.

Quick Check

1. ABE ; sides: \overline{AB} , \overline{BE} , \overline{EA} ;
angles: $\angle A$, $\angle ABE$, $\angle BEA$

$BCDE$; sides: \overline{BC} , \overline{CD} ,
 \overline{DE} , \overline{EB} ; \triangle : $\angle EBC$, $\angle C$,
 $\angle D$, $\angle DEB$

$ABCDE$; sides: \overline{AB} , \overline{BC} ,
 \overline{CD} , \overline{DE} , \overline{EA} ;
 \triangle : $\angle A$, $\angle ABC$, $\angle C$, $\angle D$,
 $\angle AED$

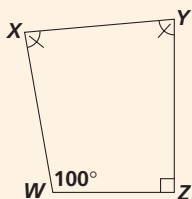
5 EXAMPLE

Point out that the angles of a polygon can be called *interior* angles. Also point out that the exterior angles of a regular polygon are congruent.

PowerPoint

Additional Examples

4 Find $m\angle X$ in quadrilateral $XYZW$.



85

5 Explain how you know that all the angles labeled $\angle 1$ in Example 5 have equal measures. **Sample:** Because the hexagon is regular, all its angles are congruent. An exterior angle is the supplement of a polygon's angles, and supplements of congruent angles all have the same measure.

Resources

- Daily Notetaking Guide 3-5 **L3**
- Daily Notetaking Guide 3-5—Adapted Instruction **L1**

Closure

If the sum of the interior angles of a polygon equals the sum of the exterior angles, what is the name of the polygon? **quadrilateral** If each exterior angle of a regular polygon measures 30, how many sides does the polygon have? **12**



For: Regular Polygon Activity
Use: Interactive Textbook, 3-5

4 EXAMPLE Using the Polygon Angle-Sum Theorem

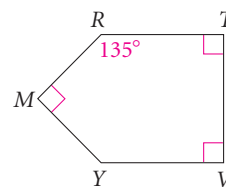
Algebra Find $m\angle Y$ in pentagon $TVYMR$ at the right. Use the Polygon Angle-Sum Theorem for $n = 5$.

$$m\angle T + m\angle V + m\angle Y + m\angle M + m\angle R = (5 - 2)180$$

$$90 + 90 + m\angle Y + 90 + 135 = 540 \quad \text{Substitute.}$$

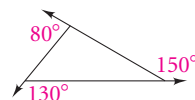
$$m\angle Y + 405 = 540 \quad \text{Simplify.}$$

$$m\angle Y = 135 \quad \text{Subtract 405 from each side.}$$

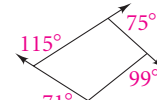


Quick Check **4** Pentagon $ABCDE$ has 5 congruent angles. Find the measure of each angle. **108**

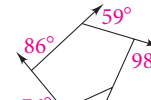
You can draw exterior angles at any vertex of a polygon. The figures below show that the sum of the measures of the exterior angles, one at each vertex, is 360. This can be proved as a theorem in a way suggested in Exercise 46.



$$80 + 150 + 130 = 360$$



$$115 + 75 + 99 + 71 = 360$$



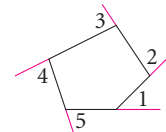
$$86 + 59 + 98 + 41 + 76 = 360$$

Key Concepts

Theorem 3-15 Polygon Exterior Angle-Sum Theorem

The sum of the measures of the exterior angles of a polygon, one at each vertex, is 360.

For the pentagon,
 $m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 + m\angle 5 = 360$.



An **equilateral polygon** has all sides congruent. An **equiangular polygon** has all angles congruent. A **regular polygon** is both equilateral and equiangular.

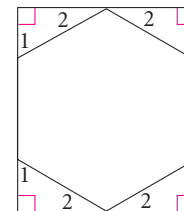
5 EXAMPLE Real-World Connection

Packaging The game board at the right has the shape of a regular hexagon. It is packaged in a rectangular box outlined beneath it. The box uses four right triangles made of foam in its four corners. Find $m\angle 1$ in each foam triangle.



Find the measure of an angle of the hexagon first.

- A regular hexagon has 6 sides and 6 congruent angles. The sum of the measures of the interior angles = $(6 - 2)180$, or **720**.
- The measure of one interior angle is $\frac{720}{6}$, or 120.
- The measure of its adjacent exterior angle, $\angle 1$, is $180 - 120$, or 60.



Quick Check **5** a. Find $m\angle 1$ by using the Polygon Exterior Angle-Sum Theorem. **60**
 b. Find $m\angle 2$. Is $\angle 2$ an exterior angle? Explain. **30; no, it is not formed by extending one side of the polygon.**

EXERCISES

For more exercises, see *Extra Skill, Word Problem, and Proof Practice*.

Practice and Problem Solving

A Practice by Example

Example 1
(page 157)



5. *MWBFX*; sides: \overline{MW} , \overline{WB} , \overline{BF} , \overline{FX} , \overline{XM} ; \triangle : $\angle M$, $\angle W$, $\angle B$, $\angle F$, $\angle X$

7. *HEPTAGN*; sides: \overline{HE} , \overline{EP} , \overline{PT} , \overline{TA} , \overline{AG} , \overline{GN} , \overline{NH} ; \triangle : $\angle H$, $\angle E$, $\angle P$, $\angle T$, $\angle A$, $\angle G$, $\angle N$

Example 2
(page 158)

Example 3
(page 159)

Example 4
(page 160)

Example 5
(page 160)

Is the figure a polygon? If not, tell why.

1. **yes**

2. **No; it has no sides.**

3. **No; it is not a plane figure.**

4. **No; two sides intersect between endpoints.**

Name each polygon by its vertices. Then identify its sides and angles.

5. **$KCLP$; sides: \overline{KC} , \overline{CL} , \overline{LP} , \overline{PK} ; \triangle : $\angle K$, $\angle C$, $\angle L$, $\angle P$**

6. **$KCLP$; sides: \overline{KC} , \overline{CL} , \overline{LP} , \overline{PK} ; \triangle : $\angle K$, $\angle C$, $\angle L$, $\angle P$**

7. **$KCLP$; sides: \overline{KC} , \overline{CL} , \overline{LP} , \overline{PK} ; \triangle : $\angle K$, $\angle C$, $\angle L$, $\angle P$**

Find a polygon in each photograph. Classify the polygon by its number of sides. Tell whether the polygon is convex or concave.

8. **pentagon; convex**

9. **decagon; concave**

10. **pentagon; concave**

Find the sum of the measures of the angles of each polygon.

11. **1080**

12. dodecagon **1800**

13. decagon **1440**

14. 20-gon **3240**

15. 1002-gon **180,000**

Example 4 x^2 Algebra Find the missing angle measures.

16. **102**

17. **103**

18. **145**

19. **37**

20. **60, 60, 120, 120**

21. **113, 119**

Find the measures of an interior angle and an exterior angle of each regular polygon.

22. pentagon **108; 72**

23. dodecagon **150; 30**

24. 18-gon **160; 20**

25. 100-gon **176.4; 3.6**

Lesson 3-5 The Polygon Angle-Sum Theorems **161**

3. Practice

Assignment Guide

1 A B 1-10, 50-53

2 A B 11-49, 54-56

C Challenge 57-63

Test Prep 64-70

Mixed Review 71-86

Homework Quick Check

To check students' understanding of key skills and concepts, go over Exercises 8, 22, 32, 37, 55.

Exercises 8–10 Point out that it is usually easier to count vertices than sides of a polygon. Because the number of vertices and sides are equal, either method is acceptable.

Connection to Language Arts

Exercises 12, 13 Point out that the two polygons differ only in the prefix *do-*, which means "two." By adding it and the prefix *deca-*, which means "ten," you get $2 + 10 = 12$.

Differentiated Instruction Resources

GPS Guided Problem Solving **L3**

Enrichment **L4**

Reteaching **L2**

Adapted Practice **L1**

Practice **L3**

Practice 3-5 Lines in the Coordinate Plane

Write an equation of the line with the given slope that contains the given point.

1. $F(3, -6)$, slope $\frac{1}{2}$ 2. $Q(5, 2)$, slope -2 3. $A(3, 3)$, slope 7 4. $R(-4, -1)$, slope $-\frac{1}{2}$
5. $L(-1, -2)$, slope $\frac{3}{2}$ 6. $R(15, 10)$, slope $\frac{2}{3}$ 7. $D(1, -9)$, slope -4 8. $W(0, 6)$, slope -1

Graph each line using slope-intercept form.

9. $2y = 8x - 2$ 10. $2y = \frac{1}{2}x - 10$ 11. $3x + 9y = 18$ 12. $-x + y = -1$
13. $y + 7 = 2x$ 14. $4x - 2y = 6$ 15. $5 - y = \frac{1}{2}x$ 16. $\frac{1}{2}x = \frac{1}{2}y - 1$

Graph each line.

17. $y = 5x + 4$ 18. $y = \frac{1}{2}x - 3$ 19. $x = -2$ 20. $y = -2x$
21. $y = -5$ 22. $y = x$ 23. $y = \frac{2}{3}x + 2$ 24. $x = 2.5$

Write an equation of the line containing the given points.

25. $A(2, 7)$, $B(3, 4)$ 26. $F(-1, 3)$, $G(0, 4)$ 27. $S(0, 2)$, $T(2, -2)$ 28. $D(7, -4)$, $E(-4, 2)$
29. $G(-2, 0)$, $H(3, 10)$ 30. $B(3, 5)$, $C(-6, 2)$ 31. $X(-1, -1)$, $Y(4, -2)$ 32. $M(0, -3)$, $N(7, 3)$

Write equations for (a) the horizontal line and (b) the vertical line that contain the given point.

33. $P(2, -11)$ 34. $D(0, 2)$ 35. $R(-4, -4)$ 36. $F(-1, 8)$

Graph each line using intercepts.

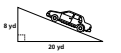
37. $3x - y = 12$ 38. $2x + 4y = -4$ 39. $\frac{1}{2}x + \frac{1}{3}y = 3$ 40. $12x - 3y = -6$
41. $2x - 2y = 8$ 42. $\frac{1}{2}x + 2y = 2$ 43. $-6x + 15y = 18$ 44. $0.2x + 0.3y = 1.8$

45. **Hourly Wages** The equation $P = \$3.90 + \$0.10h$ represents the hourly pay (P) a worker receives for loading a number of boxes onto a truck.

- a. What is the slope of the line represented by the given equation?
b. What does the slope represent in this situation?
c. What is the y -intercept of the line?

d. What does the y -intercept represent in this situation?

46. **Rolling** The Blackberry[®] driveway is difficult to get up the winter ice and snow because of its slope. What is the equation of the line that represents the Blackberry[®] driveway?



Error Prevention!

Exercise 36 Ask: How many sides does this polygon have? **5** What is the sum of the interior angles for this figure? **540°**

Exercise 37 Have students investigate the shapes of sports stadiums and report how polygons are used in designing them.

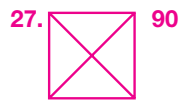
Exercise 38 If necessary, review how to solve equations with a variable in the denominator. Remind students that this is a regular polygon.

Technology Tip

Exercise 57 This would be a nice classroom exercise using the table feature of a graphing calculator with overhead display.

Connection to Algebra

Exercise 58 Part b introduces the idea of the asymptote of a function, which students will encounter in their next algebra course.



B Apply Your Skills

Packaging The nut container at the right has the shape of a regular octagon. It fits in a square box. A cheese wedge fills each corner of the box.

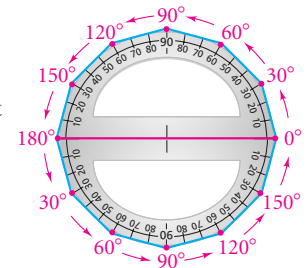


26. Find the measure of each angle of a cheese wedge. **45, 45, 90**
 27. **Critical Thinking** Show how to rearrange the four pieces of cheese to make a regular polygon. What is the measure of each angle of the polygon? **See left.**

Use a protractor. Sketch each type of regular polygon.

Sample: dodecagon

Use the protractor to equally space 12 points around a circle. ($360^\circ \div 12 = 30^\circ$, so mark a point every 30° .) Connect these points to form a regular dodecagon. **28–31. See margin.**

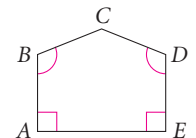


28. triangle 29. quadrilateral
 30. hexagon 31. octagon

The sum of the measures of the angles of a polygon with n sides is given. Find n .

32. 180 **3** 33. 1080 **8** 34. 1980 **13** 35. 2880 **18**

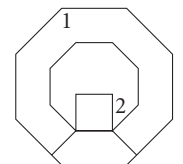
36. **Multiple Choice** In the figure at right, $\angle B \cong \angle D$ and $m\angle C = 130$. What is $m\angle B$? **C**
 (A) 25 (B) 50
 (C) 115 (D) 160



37. **Stage Design** The diagram at the right shows platforms constructed for a theater-in-the-round stage. Describe the largest platform by the type of regular polygon it suggests. Find the measure of each numbered angle.

- octagon; $m\angle 1 = 135$; $m\angle 2 = 45$**
 38. **Error Analysis** Miles said that he measured an angle of a regular polygon to be 130° . Explain why this result is impossible. **If you solve $\frac{(n-2)180}{n} = 130$, you get $n = 7.2$. This number is not an integer.**

39. **Critical Thinking** A triangle has two congruent angles and an exterior angle with measure 100. Find two possible sets of measures for the angles of the triangle. **20-80-80; 50-50-80**



The measure of an exterior angle of a regular polygon is given. Find the measure of an interior angle, and find the number of sides.

40. 72 **108; 5** 41. 36 **144; 10** 42. 18 **162; 20** 43. 30 **150; 12** 44. x **$180 - x$; $\frac{360}{x}$**

45. **Probability** Find the probability that the measure of an angle of a regular n -gon is a positive integer if n is an integer and $3 \leq n \leq 12$. **$\frac{4}{5}$**



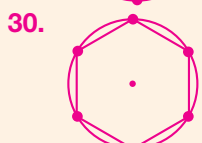
46. **Algebra** A polygon has n sides. An interior angle of the polygon and an adjacent exterior angle form a straight angle.

- a. What is the sum of the measures of the n straight angles? **$n \cdot 180$**
 b. What is the sum of the measures of the n interior angles? **$(n - 2)180$**
 c. Using your answers above, what is the sum of the measures of the n exterior angles? **$180n - 180(n - 2) = 360$**
 d. What theorem do the steps above lead to? **Polygon Ext. \angle -Sum Thm.**

Real-World Connection

At a theater-in-the-round, seats are arranged so the audience surrounds the stage.

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 Visit: PHSchool.com
 Web Code: aue-0305



57. a. (20, 162), (40, 171), (60, 174), (80, 175.5), (100, 176.4), (120, 177), (140, 177.4), (160, 177.8), (180, 178), (200, 178.2)

- c. It is very close to 180.
 d. No, two sides cannot be collinear.

58. b. As n gets larger, the size of the angles get closer to 180. The more sides it has, the closer the polygon is to a circle.

48. $w = 72, x = 59,$
 $y = 49, z = 121; \triangle$

Problem Solving Hint

In Exercises 51–53, sketch a figure to meet the first condition. Then adjust it to meet the second condition.

54. **Yes; the sum of the measures of \triangle at the int. point is 360. The sum of the measures of all the \triangle is $180n$. $180n - 360 = (n - 2)180$**



55. **Writing** Tell what you know about the figure at the right. **See above right.**

56. The measure of an interior angle of a regular polygon is three times the measure of an exterior angle of the same polygon. What is the name of the polygon? **octagon**

Challenge



57. **a. Graphing Calculator** Find the measure of an angle of a regular n -gon for $n = 20, 40, 60, 80, \dots, 200$. Record your results to the nearest tenth as ordered pairs in the form $(n, \text{measure of each angle})$. **See margin p. 162.**

b. Plot the ordered pairs using a window like the one shown at the right. **See back of book.**

c. Data Analysis Based on the graph from part (b), make a statement about the measure of an angle of a regular 1000-gon. **c–d. See margin p. 162.**

d. Is there a regular n -gon with an angle of 180° ? Explain.

58. **a.** Explain why the measure of an angle of a regular n -gon is given by the formulas $\frac{180(n-2)}{n}$ and $180 - \frac{360}{n}$. **$[180(n-2)] \div n = \frac{180n-360}{n} = 180 - \frac{360}{n}$.**

b. Use the second formula to explain what happens to the measures in the angles of regular n -gons as n becomes a large number. Explain also what happens to the polygons. **See margin p. 162.**

59. Two rays bisect two consecutive angles of a regular decagon and intersect in the decagon's interior. Find the measure of the acute angles formed by the intersecting rays. **36**

Draw, if possible, the concave quadrilateral described. If not possible, explain. 60–63. See margin.

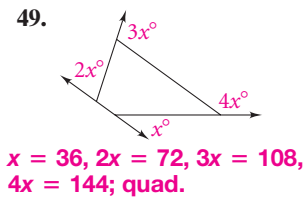
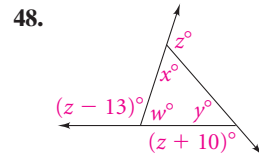
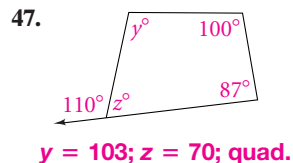
60. with two pairs of congruent adjacent sides

61. with two pairs of congruent opposite sides

62. with three congruent sides

63. with four congruent sides

x² Algebra Find each missing angle measure. Then name the polygon.



Open-Ended Sketch each figure described in Exercises 50–53.

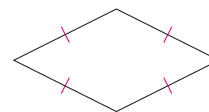
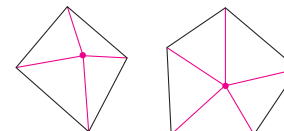
50. a quadrilateral that is not equiangular **50–53. See back of book.**

51. an equiangular quadrilateral that is not regular

52. an equilateral polygon that is not equiangular

53. an equiangular polygon that is not equilateral

54. **Critical Thinking** Ellen says she has another way to find the sum of the measures of the angles of a polygon. She picks a point inside the polygon, draws a segment to each vertex, counts the number of triangles, multiplies by 180, and then subtracts 360. Does her method work? Explain. **See left.**



Xmin = 0 Ymin = 160
Xmax = 200 Ymax = 184
Xscl = 20 Yscl = 4

57. **a. Graphing Calculator** Find the measure of an angle of a regular n -gon for $n = 20, 40, 60, 80, \dots, 200$. Record your results to the nearest tenth as ordered pairs in the form $(n, \text{measure of each angle})$. **See margin p. 162.**

b. Plot the ordered pairs using a window like the one shown at the right. **See back of book.**

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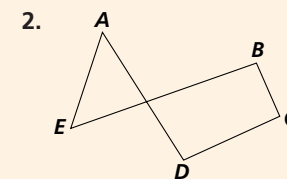
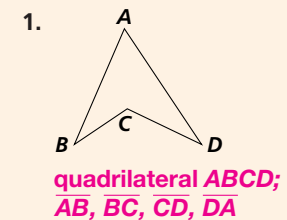
60. with two pairs of congruent adjacent sides

61. with two pairs of congruent opposite sides

62. with three congruent sides

63. with four congruent sides

For Exercises 1 and 2, if the figure is a polygon, name it by its vertices and identify its sides. If the figure is not a polygon, explain why not.



not a polygon because two sides intersect at a point other than endpoints

3. Find the sum of the measures of the angles in an octagon. **1080**

4. A pentagon has two right angles, a 100° angle and a 120° angle. What is the measure of its fifth angle? **140**

ABCDEFGHIJ is a regular decagon.

5. Find $m\angle ABC$. **144**

6. $\angle XBC$ is an exterior angle at vertex B . Find $m\angle XBC$. **36**

Alternative Assessment

Have students draw a polygon with 5 sides and a polygon with 8 sides. For each polygon, students should show the exterior angles and find the sum of the interior angles and the sum of the exterior angles. Then have them write a paragraph explaining how the sums illustrate the Polygon Angle-Sum Theorem and the Exterior Angle-Sum Theorem.



To graph the ordered pairs, use **STAT** and **STAT PLOT** on your graphing calculator.

Go Online
PHSchool.com

For: Graphing calculator procedures

Web Code: aue-2120

60–63. Answers may vary. Samples are given.

61. Not possible; opp. sides would overlap.

63. Not possible; opp. and adj. sides would overlap.



Test Prep

A sheet of blank grids is available in the Test-Taking Strategies with Transparencies booklet. Give this sheet to students for practice with filling in the grids.

Resources

For additional practice with a variety of test item formats:

- Standardized Test Prep, p. 198
- Test-Taking Strategies, p. 188
- Test-Taking Strategies with Transparencies



Test Prep

Gridded Response

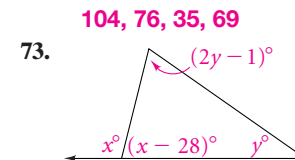
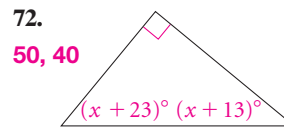
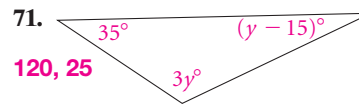
For Exercises 64–70, you may need the formula $(n - 2)180$ for the sum of the angle measures in a polygon with n sides.

64. What is the sum of the measures of the angles of a 25-gon? **4140**
65. A company is manufacturing a gear that has the shape of a regular polygon. The measure of each angle of the gear is 162. How many sides does the gear have? **20**
66. The car at each vertex of a Ferris wheel holds a maximum of 5 people. The sum of the measures of the angles of the Ferris wheel is 7740. What is the maximum number of people that the Ferris wheel can hold? **225**
67. What is the sum of the measures of the exterior angles, one at each vertex, of an octagon? **360**
68. Exactly three angles of a pentagon are congruent. The other two angles are complementary. What is the measure of one of the three congruent angles? **150**
69. The sum of the measures of the angles of a regular polygon is 4500. How many sides does the polygon have? **27**
70. What is the measure of an exterior angle of a regular polygon with 36 sides? **10**

Mixed Review



Lesson 3-4 Find each missing angle measure.



Lesson 2-4

Name the property that justifies each statement.

74. $4(2a - 3) = 8a - 12$
Distr. Prop.

76. $\overline{RS} \cong \overline{RS}$

78. If $2r = 18$, then $r = 9$.
Reflexive Prop. of \cong
Div. Prop.

Subst. Prop.

75. If $b + c = 7$ and $b = 2$, then $2 + c = 7$.

77. If $\angle 1 \cong \angle 4$, then $\angle 4 \cong \angle 1$.

79. If $AB = BC$ and $BC = 1$, then $AB = 1$.
Symm. Prop. of \cong
Trans. Prop.

Lessons 1-4, 1-6

Identify the following in the diagram.

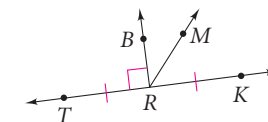
80. a pair of opposite rays $\overrightarrow{RT}, \overrightarrow{RK}$

81. two right angles $\angle BRT, \angle BRK$

82. two segments

83. an acute angle

85. a straight angle $\angle TRK$



84. an obtuse angle $\angle TRM$

86. a midpoint R

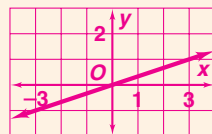
82. Answers may vary.
Sample: \overline{BR} and \overline{TK}

83. Answers may vary.
Sample: $\angle BRM$

164 Chapter 3 Parallel and Perpendicular Lines

page 165 Algebra 1 Review

10. The line is not as steep as line ℓ , but has the same tilt.



11. The line is steeper than line r , but has the same tilt.

