

CONTEMPORARY MATHEMATICS

The Selberg Trace Formula and Related Topics

**Proceedings of a Summer
Research Conference held
July 22–28, 1984**

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The Selberg Trace Formula and Related Topics

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Volume 53

The Selberg Trace Formula and Related Topics

**Proceedings of the AMS-IMS-SIAM
Joint Summer Research Conference
held July 22–28, 1984, with support
from the National Science Foundation**

**Dennis A. Hejhal, Peter Sarnak,
and Audrey Anne Terras, Editors**

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INTRODUCTION TO THE PROCEEDINGS OF THE CONFERENCE ON
SELBERG'S TRACE FORMULA AND RELATED TOPICS

Over a year has passed since the conference held at Bowdoin College in July of 1984. We have all forgotten how we ate too many lobsters and, at last, the papers have been collected and retyped. The volume includes a very diverse set of attitudes toward the trace formula: e. g., that of mathematical physics, classical analytic number theory, modern adelic number theory, modern group representations, classical harmonic analysis, and even that of those that would like to do without it. I hope that this very diversity will stimulate more interaction between the various camps and even lead to the creation of new ones. There are some missing papers and this is regrettable. But most of the speakers did send in their contributions and I thank them for doing so. I am sorry that the conference did not include more non-U.S. speakers. This was due to the rules of these conferences. In particular, I regret the absence of the many Soviet researchers on the trace formula, such as A. B. Venkov. Finally let me thank the NSF, the AMS, IMS, and SIAM for supporting these weekly summer conferences, in particular, I must thank the conference coordinator Carole Kohanski for a job well done. More finally I would like to thank my co-organizers Dennis Hejhal and Peter Sarnak for their invaluable efforts, and record my extreme gratitude to the typist Annetta Whiteman for a fantastic job. And even most finally let me add that this volume is dedicated to Atle Selberg, whose work on the trace formula is still the most important of all.

Audrey Terras
Encinitas, California
December 30, 1985

