The Solar Flux

The flux from the Sun at distance D is simply the luminosity of the Sun spread over the surface of a sphere of radius D:

$$\text{Brightness or Flux} = \frac{\text{Luminosity}}{\text{Area}} = \frac{\text{Luminosity}}{4\pi D^2}$$

$$= 1400\left(\frac{\text{AU}}{\text{D}}\right)^2 \text{ Watts/m}^2$$

→ Planets further from the Sun than the Earth should be colder since the receive less solar heating

Solar Power At Earth

If the Earth intercepted all of the Sun's energy, the oceans would evaporate in 10 seconds.

The energy a planet receives (per second) is the incident flux (energy per unit area per second) times the projected area of the planet

Incident flux = (solar luminosity)/(area of sphere radius 1 AU)
$= L/4\pi D^2$ $D=1AU$

Projected area of the Earth = πR^2 R=radius of the earth

So the Earth receives
(incident flux)(projected area)$=LR^2/4D^2$
$= 2\times10^{17}$Watts of solar radiation

Energy Balance

That energy has to go somewhere – if it simply stayed on the Earth, the Earth would get steadily hotter with time

But the Earth's temperature is roughly constant, so the Earth must be losing the energy again

But how?

Key Ideas:

Temperature (Kelvin Scale)

- Measures internal energy content.

Blackbodies:

A hot, dense object produces a *continuous spectrum* (blackbody spectrum).

Stefan-Boltzmann law, Wien Law.

The Interaction of Light & Matter

Light & Matter can interact in a number of different ways:

- Matter can transmit light (glass, water).
- Matter can reflect light.
- Matter *gains* energy by *absorbing* light.
- Matter *loses* energy by *emitting* light.

The last two (absorption and emission) bear on the *internal energy* of the matter.

Temperature

Temperature is a measurement of the internal energy content of an object.

Solids:

- Higher temperature means higher average vibrational energy per atom or molecule.

Gases:

- Higher temperature means more average kinetic energy (faster speeds) per atom or molecule.


Cool Gas
Slow Average Speeds
Hot Gas
Faster Average Speeds

Kelvin Temperature Scale

An absolute temperature system:

- Developed by Lord Kelvin (19th century)
- Uses the Celsius temperature scale

<u>Absolute Kelvin Scale</u> (K):

- 0 K = Absolute Zero (all motion stops)
- 273 K = pure water freezes (0º Celsius)
- 373 K = pure water boils (100º C)

<u>Advantage</u>:

- The *total internal energy* is directly proportional to the temperature in Kelvins.

Black Body Radiation

A *Blackbody* is an object that *absorbs all light*.

- Absorbs at *all* wavelengths.
- As it absorbs light, it heats up.
- Characterized by its *Temperature.*

It is also *a perfect radiator*:

- Emits at *all* wavelengths (*continuous spectrum*)
- Energy emitted depends on Temperature.
- Peak wavelength depends on Temperature.

The Sun and the Planets are not perfect black bodies, but they are close enough….

Stefan-Boltzmann Law

Flux = energy emitted per second per area by a blackbody with *Temperature* (*T*):

$$F = \sigma T^4$$

σ is Boltzmann's constant (a number).

In Words:

"*<u>Hotter</u> objects are <u>Brighter</u> at All Wavelengths*"

Iron plate at temperature T radiates F.

Its total luminosity is L = F x Area.

For a sphere: L = F x Area = F x 4 π R^2

Wien's Law

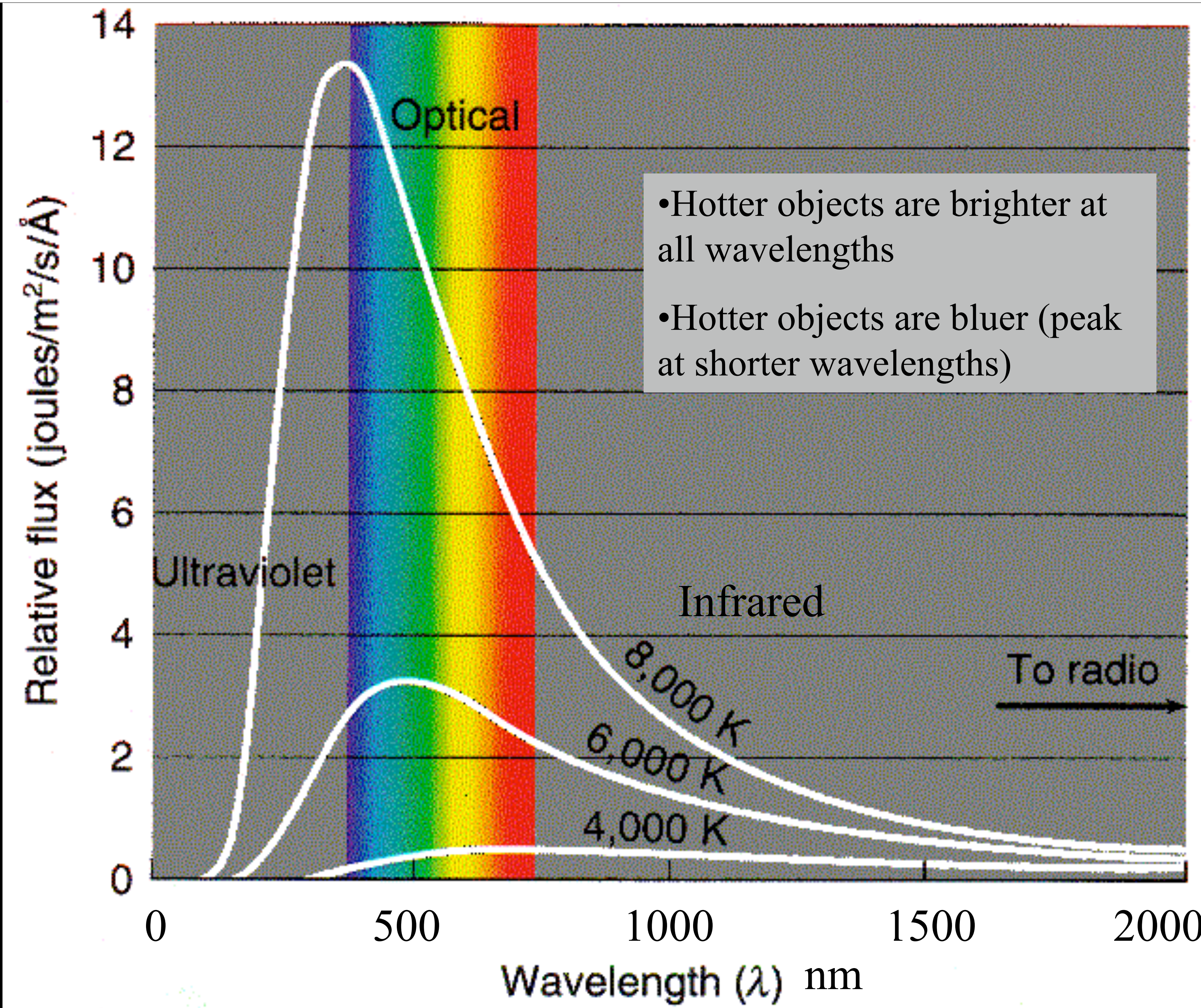
Relates *peak wavelength* and *Temperature*:

$$\lambda_{peak} = \frac{2{,}900{,}000\ \mathrm{nm}}{\mathrm{T}}$$

In Words:

"Hotter objects are BLUER" (have spectra that peak at shorter wavelength, higher frequency)

"Cooler objects are REDDER" (have spectra that peak at longer wavelength, lower frequency)



Relative flux (joules/m²/s/Å)
14
12
10
8
6
4
2
0
Optical
Ultraviolet
Infrared
•Hotter objects are brighter at all wavelengths
•Hotter objects are bluer (peak at shorter wavelengths)
8,000 K
6,000 K
4,000 K
To radio
0
500
1000
1500
2000
Wavelength (λ) nm

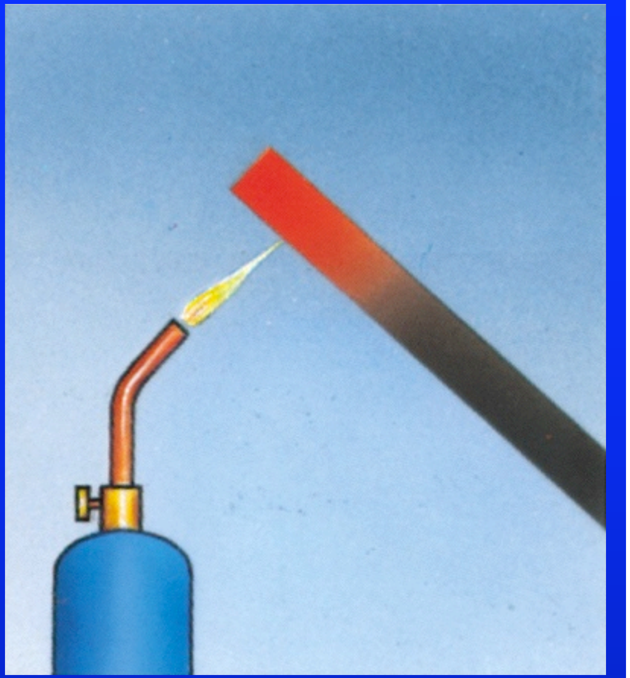
Examples:

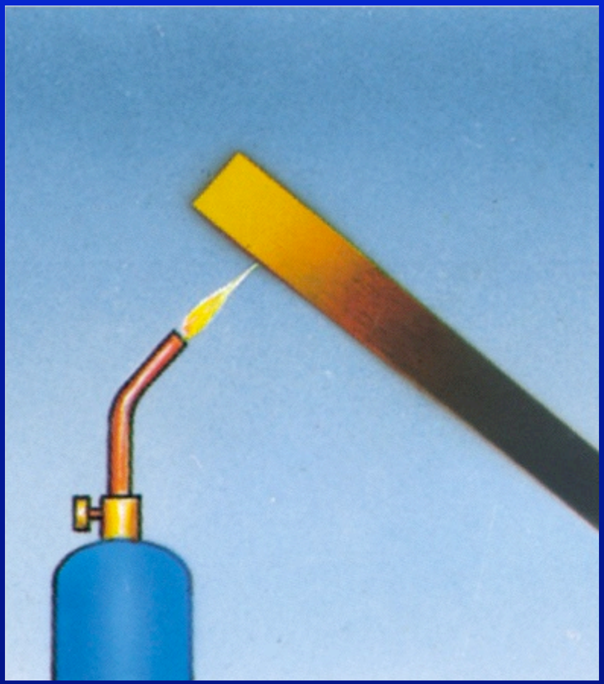
Heat an iron bar from 300 to 600K

- Temperature increases by 2×
- Brightness increases by $2^4 = 16\times$
- Peak wavelength shifts towards the blue by 2× from ~10μm in the mid-Infrared to ~5μm in the near-Infrared.

Result of heating any blackbody:

- Gets *brighter* at all wavelengths
- *Gets bluer* in color





Example – The Light Bulb

The filament in a light bulb is about 0.6m long and 0.000064m in diameter, giving it an area of

area $A=(\text{length})(\text{circumference})=\pi(\text{length})(\text{diameter})=10^{-4}\text{m}^2$

Using electricity we then heat the filament to about T=2200K, so it emits

flux $F=\sigma T^4=1.3\times10^6$ Watts/m^2

Giving a total luminosity of

L=(flux)(area)=FA=130 Watts

At a peak wavelength of

$\lambda_{max}=(2.9\times10^6/T)$nm=1300nm (it peaks in the infrared)

Incandescent bulbs produce more heat than light, which is why fluorescent bulbs (which are not black bodies) are more efficient

Example - You

You have a surface area of about
area $A=2\times(2.0m\times0.5m)=2m^2$

Your body temperature is about T=310K, so it emits
flux $F=\sigma T^4=500$ Watts/m^2

Giving a total luminosity of
L=(flux)(area)=FA=1000 Watts

At a peak wavelength of
$\lambda_{max}=(2.9\times10^6 /T)$nm=10000nm (in the infrared)

But – everything around you is about the same temperature, so the NET energy loss (or gain) is much smaller

The Sun

The surface temperature of the Sun is T=5800 K, so the flux emerging from the surface of the Sun is

$$F=\sigma T^4=6.4\times10^7 \text{ Watts/m}^2 = 64 \text{ megawatts/m}^2$$

So a patch 4m×4m puts out the power of a 1 gigawatt electrical plant!

The total solar luminosity is the flux times the surface area of the sun

$$L_\odot=4\pi R_\odot{}^2F=3.9\times10^{26} \text{ Watts}$$

The peak wavelength is

$$\lambda_{max}=(2.9\times10^6 /T)\text{nm}=500\text{nm (it peaks in the green)}$$

Your eyes work where the Sun puts out most of its light. It doesn't look green because of the details of how the eye determines color and the shape of the black body spectrum – we see black bodies in the sequence red, yellow, blue-white, white as they get hotter (just like the light bulb experiment).

The Earth

The mean surface temperature of the Earth is $T=293$ K (20^o C). Assuming it is a black body, the emitted flux is

$$F=\sigma T^4=420 \text{Watts/m}^2$$

The total luminosity is the flux times the surface area of the Earth

$$L=4\pi R^2 F=2.1\times10^{17} \text{ Watts}$$

At a peak wavelength of

$$\lambda_{max}=(2.9\times10^6/T)\text{nm}=10000\text{nm (the infrared)}$$

The Earth

This emitted luminosity is almost exactly equal to the amount coming from the Sun – this is not a coincidence

Heating by the Sun has to be exactly balanced by cooling if the Earth’s temperature is to be (roughly) constant

The only way the Earth can cool is to emit radiation into space.

Thus, the average temperature of a planet is a competition between the radiation coming from the sun and the radiation emitted by the planet with the temperature of the planet is determined by where the two balance.

The Expected Temperature of a Planet

For a planet of radius R at distance D, the solar heating rate (Watts) is

$$L = (\text{solar flux at the planet}) \times (\text{projected area of planet})$$
$$= (L_{sun}/4\pi D^2) \times (\pi R^2_{planet})$$
$$= L_{sun} R^2_{planet} / 4D^2$$

For a planet of radius R and temperature T, the cooling rate (Watts) is

$$L = (\text{black body flux}) \times (\text{surface area of planet})$$
$$= (\sigma T^4_{planet}) \times (4\pi R^2_{planet}) = 4\pi R^2 \sigma T^4$$

The Expected Temperature of a Planet

In equilibrium, these two must be equal (or the temperature would either rise or fall to compensate)

$$\sigma T^4_{planet} = L_{sun}/16\pi D^2$$

Note that the radius of the planet cancels and the equilibrium temperature depends only on the luminosity of the sun and the distance of the planet from the sun

$$T = 279\ (AU/D)^{1/2}\ \ K$$

For the Earth, this is close to freezing (273K) and colder than the actual mean temperature (293K)

Does $T=279(AU/D)^{1/2}$ K work?

Planet	D/AU	actual T	predicted T	
Mercury	0.39	623K	450K	rotation~orbit
Venus	0.72	750K	328K	huge green house effect!
Earth	1.00	293K	279K	
Mars	1.52	220K	226K	
Jupiter	5.20	163K	122K	radiates more than it gets
Saturn	9.57	93K	90K	
Uranus	19.19	57K	64K	
Neptune	30.07	57K	51K	
Pluto	39.54	50K	44K	

Sorta, kinda, right, but we missed some details – planets reflect some sunlight, and clouds modify things (greenhouse effect).