The Stress Visualization Coloring Book

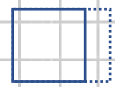
"A coloring book in college, really?!?" *Yes, and it's not totally novel. Anatomy and physiology courses have been using coloring books for decades to help nursing and biology students learn the complex systems of the human body, so it is a time-proven concept. Please give it a try before judging it. Besides, there is a lot more here than just coloring; there is something for everybody. If you find it too easy, childlike, busy-work, I apologize, but I doubt that will be the case. But even if it is, you may still make a few "stupid" mistakes – hopefully. Mistakes are awesome – they are the best teachers. I highly recommend making lots of them especially while you are in school – all kinds of them, smart ones, dumb ones, big ones, small ones, red ones and blue ones. If you find yourself having to think while coloring, then you are spending your time doing what you came to college to do – grow new brain cells! In any case, I hope you enjoy doing something somewhat creative and certainly different than "crunching numbers", and mostly, I hope your understanding of stress grows at least a little. And for sure, this book will provide lots of opportunities for you to ASK QUESTIONS!*

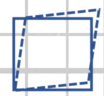
"But Professor, this is a waste of time. I already know how to calculate stresses in beams – we did that last year." *Design is performed in the human mind – not on paper or on a computer. There is a <u>fundamental</u> difference between knowing how to use equations to calculate stress and truly understanding stress well enough to design parts capable of withstanding demanding loads. Paper and computers are tools that a well-trained mind can use to develop world-changing creations – but it is the **mind** that does the creating! This book offers exercises for the mind helping you develop a deeper understanding of mechanical stress (and perhaps psychological stress as well?). Before you can understand stress in a complex part, you need to really understand it thoroughly in simple structures like beams. By actively coloring in the stresses in simple structures, you will develop a greater appreciation for what stress is, how it varies in a structure, and how it "flows" in a part. With this deeper understanding, you will be ready to design highly complex parts that can survive the most strenuous loading conditions yet to be imagined… …(was that a bit "over the top"?).*

-Dr. Ken Lulay

*-PS, it's okay if you don't stay completely within the lines, but an engineer's work does need to be neat and clear. So be sure to **use a straight edge** and **circle template** where appropriate.*

"What are stress and strain?" The answers can be found in complex mathematical descriptions, but complexity is not what this book is about. Simply put, strain is a change in length or shape due to forces (mechanical strain) and/or change in temperature (thermal strain):

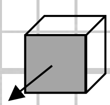
Normal strain is a change in length per length  $(\varepsilon_x = \partial u/\partial x)$

Shear strain is a change in angle  $(\gamma_{xy} = \partial v/\partial x)$:

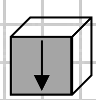
> *Rhetorical question (one that you should ponder, but do not need to answer here) – changes in temperature cause things to expand/contract. Can changes in temperature cause shear strain? We can discuss in class if you ask!*

Experimentally, engineers measure strain using strain gages. However, since most failure theories are based on stress, engineers are typically more interested in stress than strain. But stress can **not** be directly measured, it can only be calculated based on other measurements. Fortunately, most engineering materials behave linearly and therefore, Hooke's law can be used to determine stress from strain measurements. Stress is also determined analytically based on equilibrium. Although a deep understanding of strain is important, this workbook focuses on stress.

"So, what is stress?" When force is applied to a solid material, it "flows through" the material to maintain equilibrium. Stress is not force, but it does describe how the force is distributed. The more "concentrated" the force, the higher the stress. If 10 pounds is applied to an axial bar and the bar has one-square inch cross-section through which the load is carried, there will be 10 pounds per square inch stress (10psi). If 10 pounds is applied to a thinner bar, say one-half square inch, it will still be carrying 10 pounds, but the stress will be greater, 20psi in this example. When the force is acting normal (perpendicular) to an internal area, it is referred to as normal stress:

Normal Stress  **F_{normal}/dA (in the limit that A approaches zero)**

When the force is acting transverse to an internal area, it is referred to as shear stress:

Shear Stress  **$F_{transverse}/dA$ (in the limit that A approaches zero)**

Normal and shear stress are fundamentally different from each other – each causes material to react in differently.

Plusses and Minuses, Greater and Lesser?

Normal stresses come in two 'varieties' – tension and compression. Tension ("stretching") are by convention indicated as positive values while compression ("squishing") are indicated by negative values. Since algebraically, negative values are less than positive values, compression stresses are "less than" zero, and tension stresses are greater than zero. Therefore, even high magnitude compression stresses are considered to be "less than" low magnitude tension stresses – at least mathematically.

Shear stresses, on the other hand, do not have a positive and negative convention. For the duration of this workbook, consider all shear stresses to be positive. The only exception is to indicate that the direction of an arrow shown on a stress element is going in the opposite direction to reality – then use a negative sign.

"Professor, I get confused. Is stress a scalar quantity, you know – stress is 10kpsi in the bar, or is it a vector, because it's kind of like force… or what ….it all gets very confusing." Confusion is normal. Mathematically, stress is a completely new concept for students. It is related to force, but it is NOT force! **Stress is neither a scalar nor is it a vector**! Scalars (zeroth order tensors) have magnitude, vectors (first order tensors) are described by magnitude and direction, but stress and strain are second order tensors; therefore, three things are required to describe them: magnitude, direction and "type" (shear and normal stress and strain). Stress and strain are mathematically and conceptually more complex than force. This coloring book is meant to help you see that (literally).

Mohr's Circle

Strength of materials texts provide details and equations that may be used to create Mohr's circle. While equations are critical in engineering and extremely pervasive in school, they can obstruct the usefulness of Mohr's circle. Mohr's circle provides the engineer a way of visualizing stress at a point that equations cannot. You will be asked to draw many circles in this workbook, please create and use Mohr's circle graphically and avoid equations. And use your circle template – it is not called Mohr's egg! Seriously, if you are going to use the circle to determine principal stresses, etc., **it needs to be a circle drawn to scale.**

Plane Stress, Plane Strain

Plane stress and plane strain are two special conditions that are mutually exclusive. Plane strain condition exists when the strains in at least one direction are zero; hence all of the non-zero strains lay in a single plane. Plane strain is a very uncommon condition. However, plane stress is extremely common. The surface of any part, as long as it is not acted upon by an external force, will be in a plane stress condition. The direction normal to the surface will be stress free; therefore, all of the non-zero stresses lay in the plane of the surface. That makes Newton happy.

3D Mohr's Circle for Plane Stress

Stress and strain are always three-dimensional, even if in one or more directions they are zero. Creating 3D Mohr's circle is quite simple for plane stress conditions. Since stress is three-dimensional, there are always three principle stresses – be sure to show them on the Mohr's circles in this workbook. For plane stress, one of the principal stresses will be zero (otherwise, it is not plane stress). Remember, the principal stresses are always orthogonal to each other.

COLORING

In this book, you will apply the basic equations you learned in the strength of materials course, but you will take this a step further. You may be asked to calculate stresses in 3-dimensions (x, y, and z) at every point in a structure. You are also asked to determine the three principal stresses and the maximum normal stresses at every point in the structure. You will use color to represent stress magnitude. In many cases, this will be easier than it may appear – the stress is often zero and does not require an equation to be "calculated". And don't forget about linearity – it is your friend if you need to interpolate.

You will be asked to fill in the color scale using various colors. Please follow the typically convention used in finite element analysis of stress where red is used to indicate maximum stress, dark blue or black indicate minimum stress, and green is half-way between the two. Depending upon the colors you have, fill in between these with a reasonable color progression.

After creating the color scale, you will color the various structures. When coloring in the structure, select the color that fits within the color range you have created. Here are an examples of recommended color bars:

Five colors:

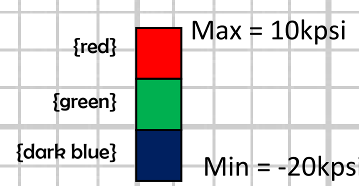
Seven colors:

{sorry, gray scale doesn't show colors}

Before creating the color scale, you will first need to determine the maximum and minimum stresses you wish to show. The minimum is often zero or compression (negative). The algorithm to determine the range of each color band is:

Range = (Max Stress – Min Stress)/(# of colors)

For a simple example, let's use three colors: red, green, dark blue:



The maximum and minimum stresses to show are 20ksi and -10ksi, respectively.

Range = [10kpsi – (– 20kpsi)] / 3colors = 10kpsi per color

Then each color represents a range of stresses as follows:

Dark blue: min+range = -20kpsi+10kpsi= -10kpsi (dark blue represents stress from -20kpsi to -10kpsi)

Green: previous+range = -10kpsi+10kpsi=0kpsi (green represents stress from -10kpsi to 0kpsi)

Red: previous+range = 0kpsi+10kpsi=10kpsi (red represents stress from 0kpsi to 10kpsi)

If you have a stress of -7kpsi that you want to represent with color, green would be the appropriate choice using the above color scale. If you have a stress that is greater than the maximum or less than the minimum, then use the color for the maximum or minimum, respectively. In our example, if stress is 35kpsi, then it should be colored red and a stress of -28kpsi should be colored dark blue.

I am sure you are anxiously awaiting the chance to start coloring, but there are several other details first. …and one last word about sketching and coloring: please **use a straight edge!**

Engineering Assumptions

"Professor, I've heard that when we assume we 'make an ass out of u and me' - isn't making assumptions a dangerous thing for engineers?" Assumptions are not the problem; the problem is thinking you are not making assumptions when in fact you are. We all make assumptions daily. We assume the car coming towards us will stay on their side of the road or will see us in the cross-walk and stop. If we don't realize that those are assumptions, then that is when something can potentially go very wrong. Almost all engineering analysis involves making assumptions. Engineers need to be aware of that and understand the implications of their assumptions. We need to consider how the "error" or lack of precision introduced by making simplifying assumptions affect our design. Stating assumptions that you are making is the first step in acknowledging the limits of your knowledge. The next step is to design parts accordingly.

Consider the simple engineering question "how long will it take an object to fall 100 meters." The appropriate equation is: time=(2*distance/acceleration)$^{1/2}$. Solve this problem and discuss two important assumptions you made, why you made them, and evaluate how they may have affected your answer (how would your answer have been different if you did not make each assumption). Also, discuss some conditions that would *invalidate* your assumptions.

Since almost all analysis involves some assumptions, ***judgement is <u>required</u>*** when engineers interpret the results/conclusions/answers they arrived at through analysis. The numerical result is only a part of the answer – the number must be interpreted. The following are several assumptions that are frequently made in stress analysis; discuss them as requested below.

Assumption: "***The material is linear elastic and remains so while loaded***." Almost all engineering alloys are quite linear in the elastic region (they obey Hooke's law well), but many polymers are not. Is "linearity" always an important assumption in stress analysis? In other words, are the results equally valid for non-linear materials such as rubber as they are for linear materials such as steel? Briefly explain why or why not for each of the fundamental loading conditions (the answers may vary, and equations provide clues to the answer):

a) uniaxial loads ($\sigma = F/A$)

b) bending loads ($\sigma = My/I$)

c) torsion loads ($\tau = Tr/J$)

Assumption: "***The material is isotropic and homogeneous***." No material is perfectly isotropic or homogeneous. However, the degree of anisotropy and inhomogeneity of most engineering materials will not significantly affect the results for most application. Therefore, these are usually reasonable assumptions. Composites are a significant exception. Composites such as graphite/epoxy and wood are highly anisotropic having *very* significantly different properties in the fiber direction compared to other directions. To "pretend" (aka "assume") they are isotropic and homogeneous could lead to highly erroneous answers. Cold rolled alloys are typically assumed to be isotropic but in reality, strength does vary by a few percent in different directions in cold rolled, cold worked, or work-hardened materials. If you need to know the strength precisely in a certain direction, then assuming isotropic behavior may be ill advised for such materials.

Nanotechnology is an evolving field in engineering (google it if you are unfamiliar with it). At the nano-scale, materials such as 2024-T3 aluminum do not behave as if they were isotropic and homogeneous. Briefly explain why that may be the case and briefly discuss the reasonableness or unreasonableness of using material properties such as yield strength and Young's modulus (obtained from ASTM E-8 testing) for designing nano-scaled machines.

Assumption: ***"Body forces, such as weight and magnetic forces, are negligible."*** For all terrestrial designs, gravity exists; therefore, body forces will be present. In stress analysis. what conditions allow engineers to neglect weight of the structure itself?

Assumption: "***Only small deflections occur when the load is applied (no large deflections occur)."*** This is generally a reasonable assumption, especially with engineering alloys. Since most alloys are very stiff (moduli of elasticity on the order of 10^6psi or hundreds of giga-Pascals), even with large loads, deflection is generally very small. However, polymers are much more compliant (much less stiff) and deflections with modest loads may be large. It is easy to stretch a rubber band to many times its original length without it breaking.

Consider this: a 100-pound weight is applied to the end of a 10-foot long rubber diving board (aka a cantilever beam). Sketch to scale (not full scale) the diving board before the load is applied (assume it is straight), and then with the load applied. Assume the end deflects 3 feet downward when loaded. If you were to analyze the bending stress in the diving board based on the "small deflection" assumption, how may the large deflection affect the actual stress the beam? In other words, how "good" is your analysis if it is based on small deflections?

UNIAXIAL LOADING

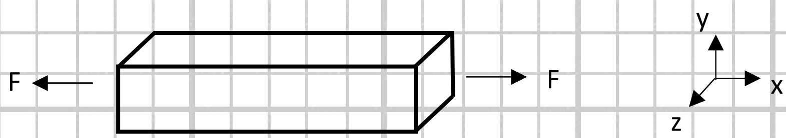
Uniaxial loading is the most basic and fundamental loading condition. ASTM E8 *Standard Test Methods for Tension Testing of Metallic Materials* utilizes uniaxial loading to create stress-strain diagrams which are used to determine several basic and critically important material properties including yield strength, tensile strength, ductility (elongation, %EL), and the modulus of elasticity. Uniaxial loading is also a commonly used loading condition for determining Poisson's ratio. Sketch a test specimen allowed per ASTM E8 (as was used in EGR270 Materials Laboratory) and indicate the direction force is applied:

Stress-Strain curve

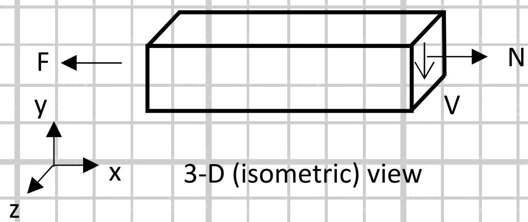
Neatly sketch a stress-strain diagram for 2024-T3 aluminum alloy. Use typical 2024-T3 properties: yield strength = 50kspi, tensile strength = 70kpsi, ductility of 20%EL, and Young's modulus of 10Mpsi.

STRESSES IN UNIAXIAL LOADING

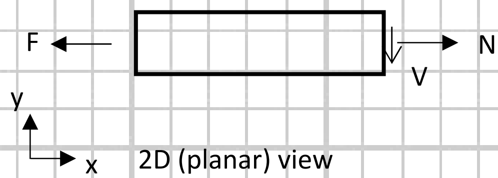
Given: a long bar with a 10mmX10mm cross-section. A uniform 1kN axial load (F) is applied at the ends in the x-direction. Assume stress is uniform. Uniform stress means that stress is equal at all points; however, it does NOT mean the stress has the same magnitude in all directions, nor does it mean shear stress is equal to normal stress.



a) Using equilibrium equations, determine the internal normal force, N, on a plane perpendicular to the loading axis. Also, determine the shear force, V, (even if it is zero) on this section. Then determine the normal stress and shear stress created by these internal forces. The force will depend upon orientation (N and V are not equal), and so will the stress depend upon direction.

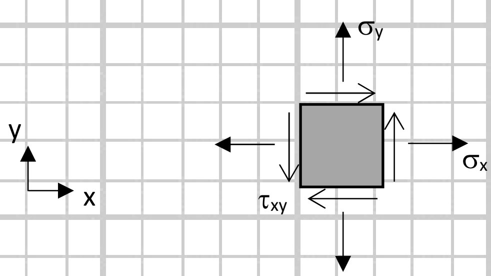


3-D (isometric) view



2D (planar) view

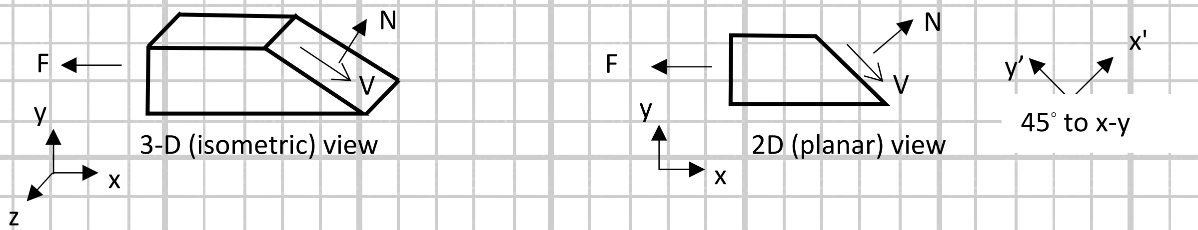
b) The stress element shown below corresponds to stress at any point in the bar (oriented with x-y axis). Using your work from part "a", determine the stresses on the stress element below (σ_x, σ_y, τ_{xy}). Where appropriate, identify the stresses as principal stresses or maximum shear stress at the point. Use negative values if arrows are drawn in the opposite direction to reality.



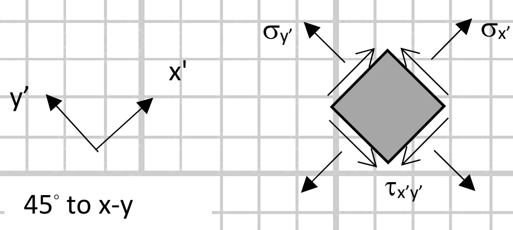
σ_x = σ_y = τ_{xy} =

and not shown in the element: σ_y = τ_{yz} = τ_{xz} =

c) Using equilibrium equations, determine the internal normal force, N, and shear force, V, on an internal plane inclined 45 degrees to the loading axis. N and V are in the x-y plane (oriented 45° to x-y axis). Then determine the normal stress and shear stress created by these internal forces. A reminder: the area of the internal plane that these forces are acting on is **not** 10mmX10mm.



d) The stress element shown below corresponds to stress at any point in the bar (oriented 45° to the x-y axis). Using your work from part "c", determine the stresses on the stress element below ($\sigma_{x'}$, $\sigma_{y'}$, $\tau_{x'y'}$) oriented 45 degrees from the x-axis. Where appropriate, identify the stresses as principal stresses or maximum shear stress at the point. Use negative values if arrows are drawn in the opposite direction to reality.



$\sigma_{x'}$ =

$\sigma_{y'}$ =

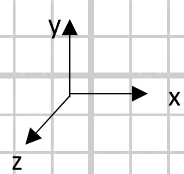
$\tau_{x'y'}$ =

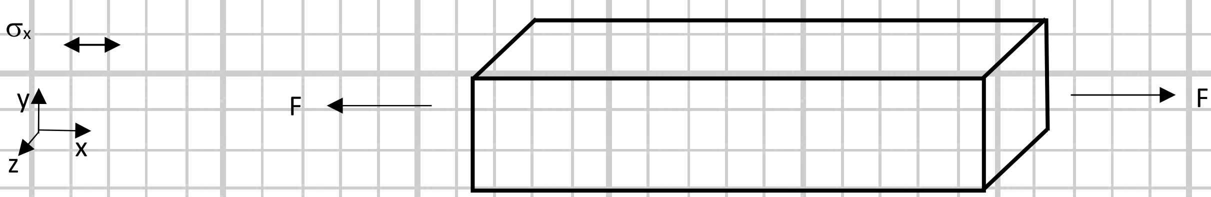
e) Draw Mohr's circle for any point in the bar (if the stress is uniform, all points are equal). One point on the circle corresponds to the orientation in part "b" – label this on Mohr's circle with an "X". One point corresponds to the orientation in parts "d" – label this with an "O".

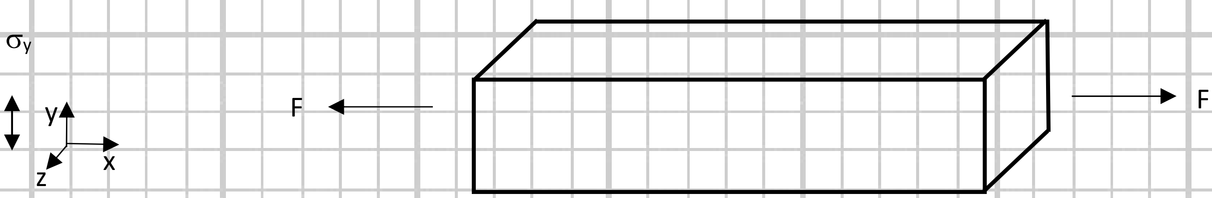
f) Using eight (8) colors, create a color-scale bar using colors recommended. If you don't have those colors, select colors similar, but choose wisely, *Grasshopper*, as you will be using the same color scale throughout this book (although specific values for the maximum and minimums will change, and you will be using fewer colors on some problems).

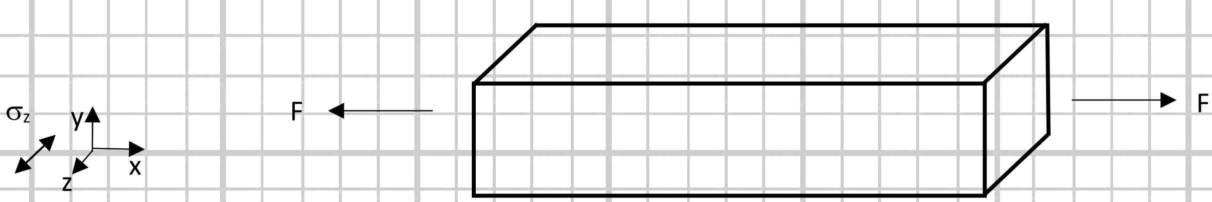
{red range: 8.75MPa to 10MPa, or higher}
{orange range: 7.5MPa to 8.75MPa)
{yellow range: 6.25MPa to 7.5MPa}
{green range: 5.0MPa to 6.25MPa}
{light blue range: 3.75MPa to 5.0MPa}
(blue range: 2.50MPa to 3.75MPa}
{dark blue range: 1.25MPa to 2.50MPa}
{black range: 0.0MPa (or less) to 1.25MPa}

Using the color-scale bar, sketch the stresses as requested in the following.

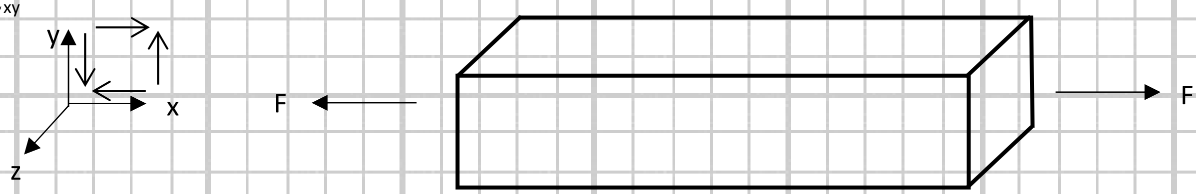




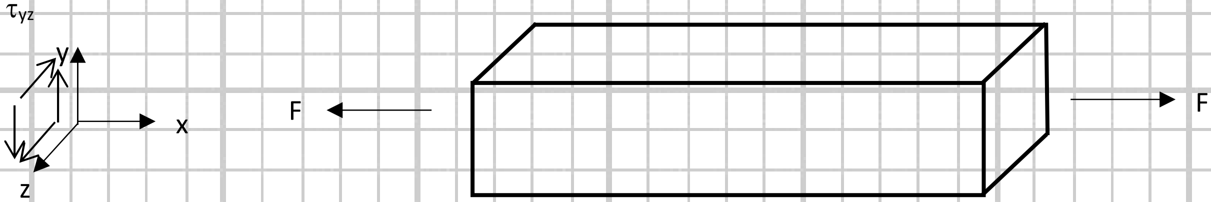




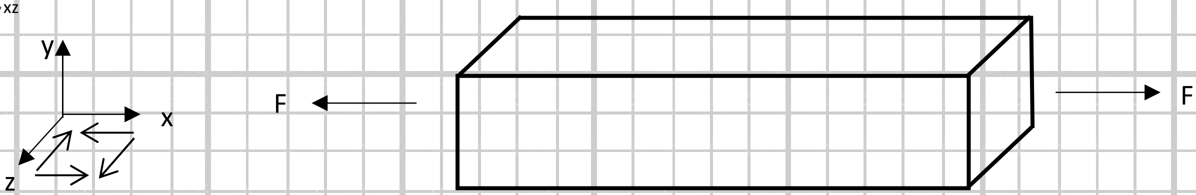
τ_{xy}



τ_{yz}

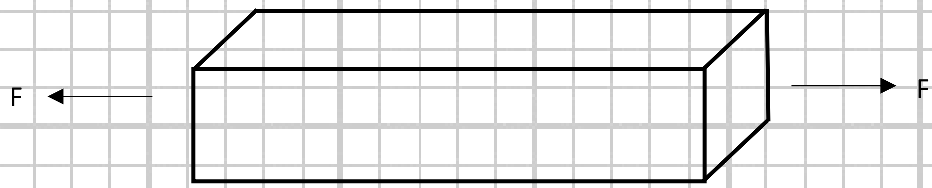


τ_{xz}

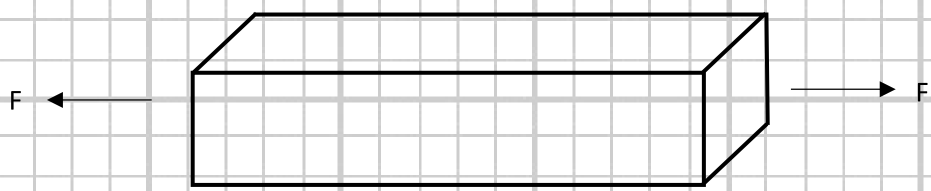


In addition to coloring, indicate the direction of the following stresses using arrows similar to what was done above.

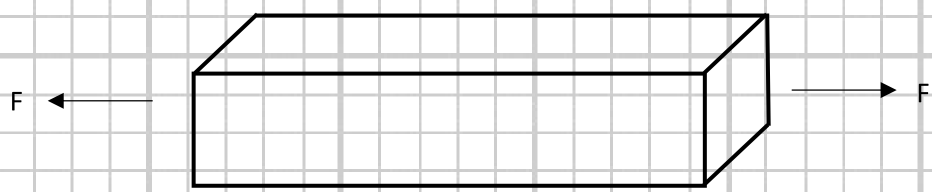
σ_1



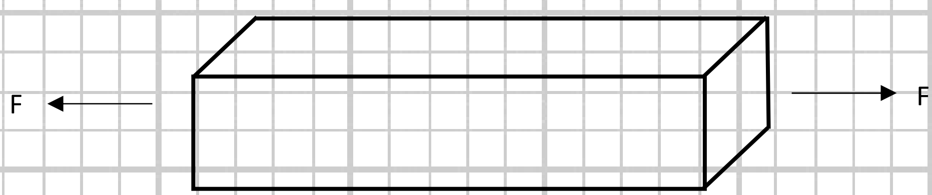
σ_2



σ_3



τ_{max}



Does uniaxial loading result in uniform stress (stress being equal at all points)?

Does uniaxial loading produce a plane stress condition at all points, some points, no points?

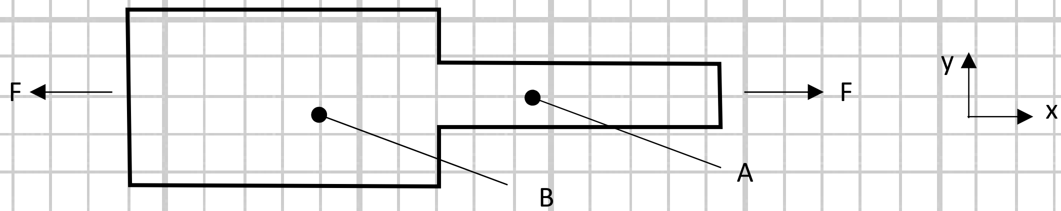
Does uniaxial loading produce a plane strain condition at all points, some points, no points?

What is the orientation of the maximum shear stress (τ_{max}) with respect to the loading axis (x-axis)?

What is the relative magnitude between maximum shear stress and the axial stress for uniaxial loading?

What is the importance of the bar being straight? Will the actual stress in the bar be what you calculated if the bar is somewhat curved?

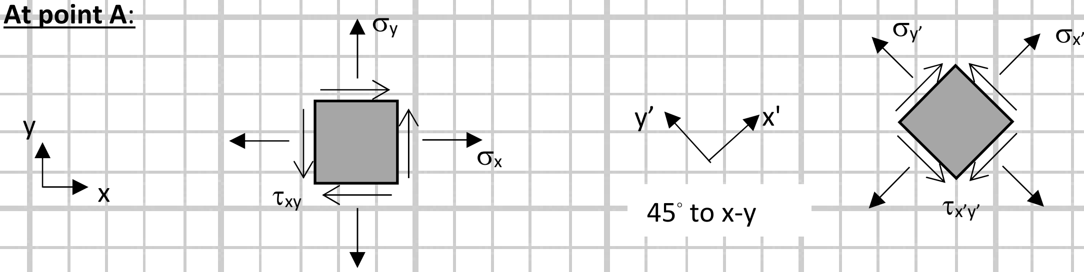
Given: a 1000mm long plate 10mm thick. One end is 30mm wide, the other end is 10mm wide. A uniform 1kN axial load (F) is applied at the ends in the x-direction. The following sketch is not to scale.



a) **Without using a calculator***, calculate stresses necessary to draw Mohr's circle for stresses at point A and point B and draw the two 3D Mohr's circles. State any additional important assumptions that may be necessary due to the geometry of the plate. **But Professor, why NO CALCULATOR? I will always have a calculator with me whenever I need to do some math!* ANSWER*: it can be embarrassing to need a calculator to do simple math – <u>YOU</u> DO NOT NEED a calculator for the following – you play as you practice. If you have to think a little when doing this math, then your big strong brain will thank you some day for giving it good exercise today! No pain, no gain.*

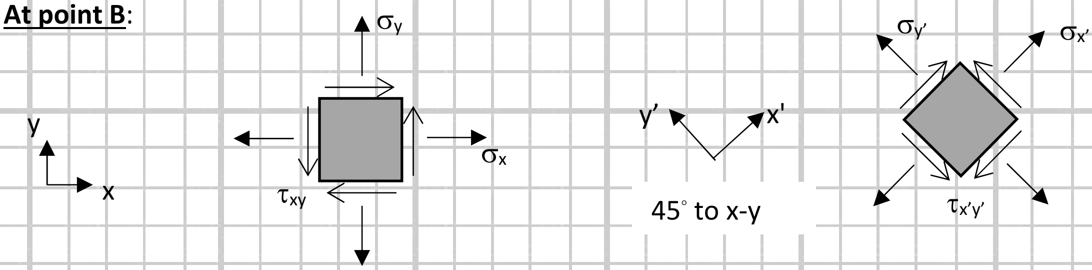
b) The four stress elements shown below corresponds to stress at point A and point B. Using the two Mohr's circles you've created, determine the stresses in the stress elements. Where appropriate, identify the stresses as principal stresses and maximum shear stress. Use negative values if arrows are drawn in the opposite direction to reality.

<u>**At point A**</u>:



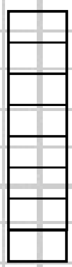
$\sigma_x =$ $\sigma_y =$ $\tau_{xy} =$ $\sigma_{x'} =$ $\sigma_{y'} =$ $\tau_{x'y'} =$

At point B:

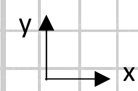


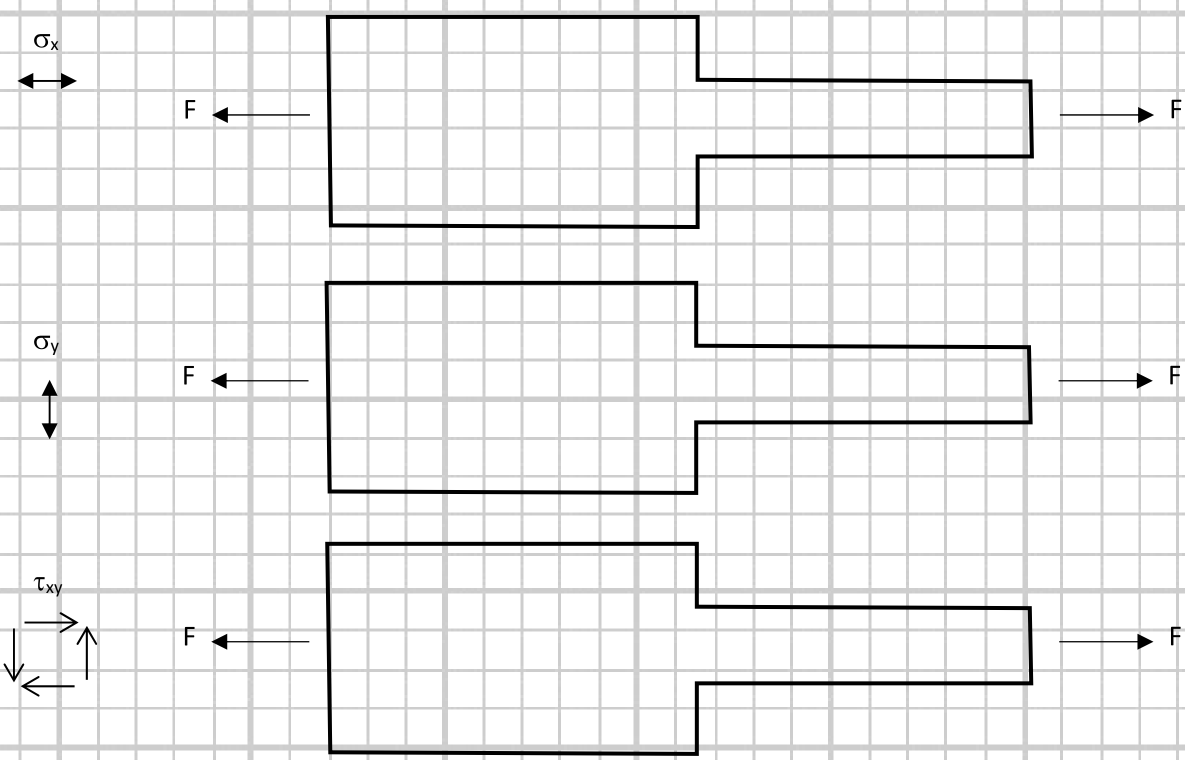
$\sigma_x =$ $\sigma_y =$ $\tau_{xy} =$ $\sigma_{x'} =$ $\sigma_{y'} =$ $\tau_{x'y'} =$

c) Using eight (8) colors, create a color-scale bar same as previous. Using the color-scale bar, sketch the stresses as requested in the following.

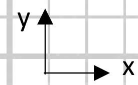


{red range: 8.75MPa to 10MPa, or higher}
{orange range: 7.5MPa to 8.75MPa)
{yellow range: 6.25MPa to 7.5MPa}
{green range: 5.0MPa to 6.25MPa}
{light blue range: 3.75MPa to 5.0MPa}
(blue range: 2.50MPa to 3.75MPa}
{dark blue range: 1.25MPa to 2.50MPa}
{black range: 0.0MPa (or less) to 1.25MPa}

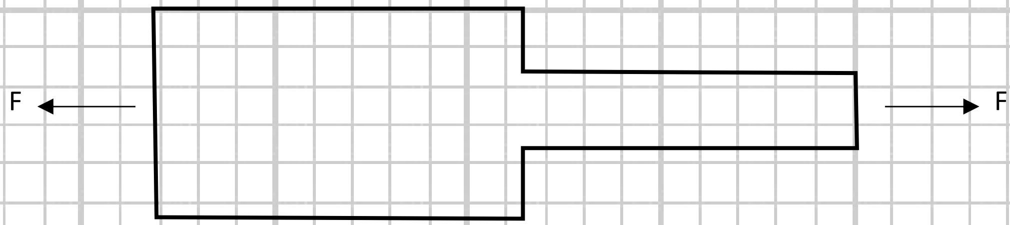




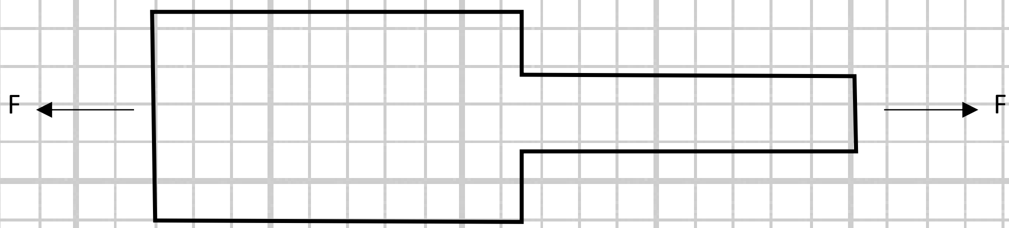
In addition to coloring, add arrows to indicate the direction of the stresses as you did previously.



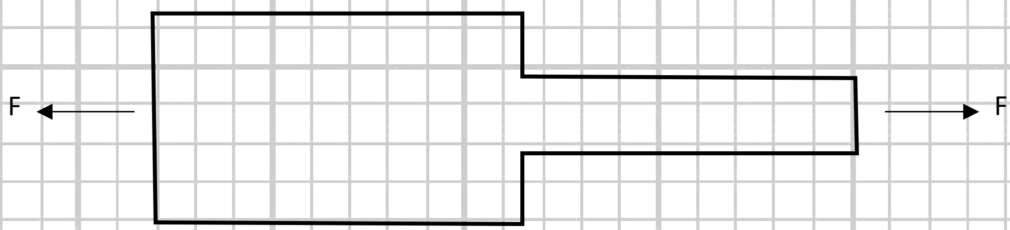
σ_1



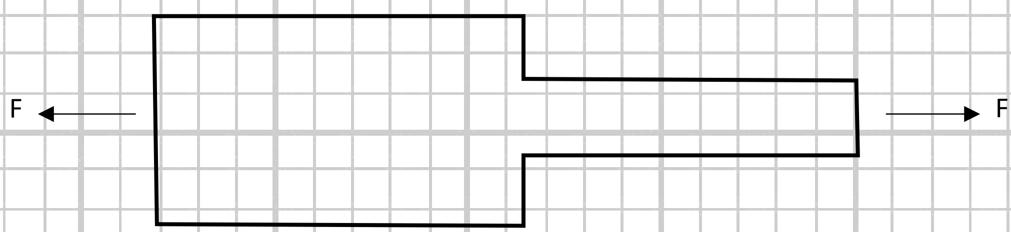
σ_2



σ_3 (there are <u>always</u> three principal stresses – even with plane stress conditions)



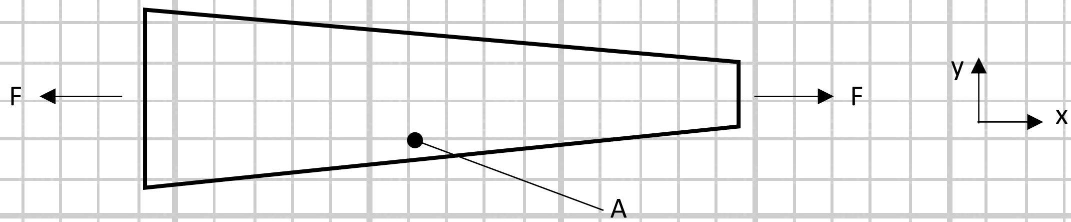
τ_{max}



The axial stress is greater in the 10mm wide section than in the 30mm wide section. Is the stress uniform (the same at every point) in the wider (30mm) section – even in the corners?

Speculate…what do you think happens to the stress at the transition from the 10mm wide to the 30mm wide section? Is there a step-function change from higher stress to lower stress, or is there some sort of more gradual transition in the stress levels?

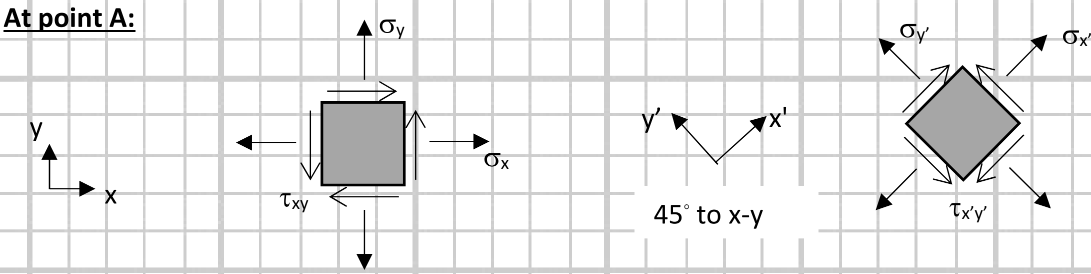
Given: a 1000mm long plate 10mm thick. One end is 30mm wide, the other end is 10mm wide. It is uniformly tapered between the ends. A uniform 1kN axial load (F) is applied at the ends in the x-direction.



a) Point A lies mid-plate (500mm from either end). **Without using a calculator**, calculate stresses necessary to draw Mohr's circle for stress at point A and then draw Mohr's circle (3D). State any additional important assumptions that may be necessary due to the geometry of the plate.

b) The stress elements shown below corresponds to stress at point A. Using the Mohr's circle you've created, determine the stresses in the stress elements. Where appropriate, identify the stresses as principal stresses and maximum shear stress. Use negative values if arrows are drawn in the opposite direction to reality.

<u>At point A:</u>



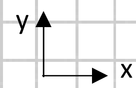
$\sigma_x =$ $\sigma_y =$ $\tau_{xy} =$ $\sigma_{x'} =$ $\sigma_{y'} =$ $\tau_{x'y'} =$

And not shown on the elements, but do exist in reality (although they may be zero):

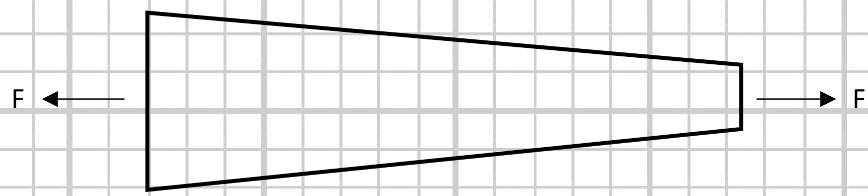
$\sigma_z =$ $\tau_{yz} =$ $\tau_{xz} =$ $\sigma_{z'} =$ $\tau_{y'z'} =$ $\tau_{x'z'} =$

c) Using eight (8) colors, create a color-scale the same as in the previous problem. Calculate stresses as required to complete the sketches below. Sketch the stresses using the color-scale bar.

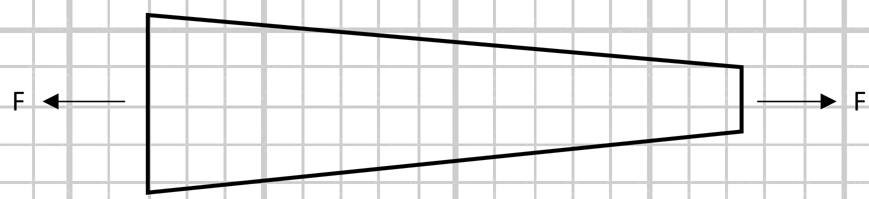
{red range: 8.75MPa to 10MPa, or higher}
{orange range: 7.5MPa to 8.75MPa)
{yellow range: 6.25MPa to 7.5MPa}
{green range: 5.0MPa to 6.25MPa}
{light blue range: 3.75MPa to 5.0MPa}
(blue range: 2.50MPa to 3.75MPa}
{dark blue range: 1.25MPa to 2.50MPa}
{black range: 0.0MPa (or less) to 1.25MPa}



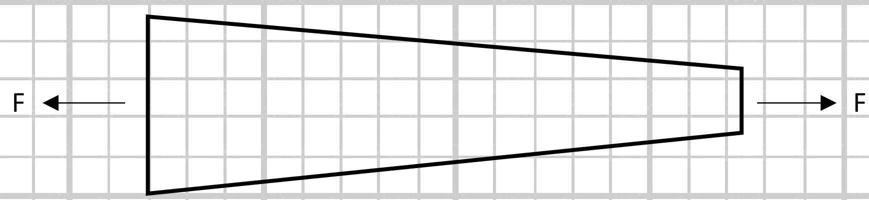
σ_x



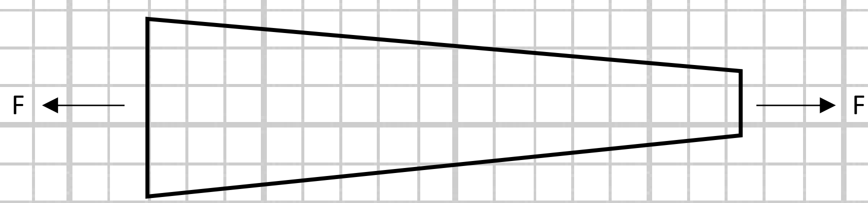
σ_y

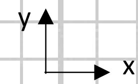


σ_z

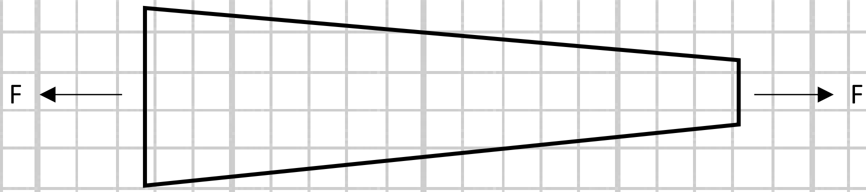


τ_{xy}



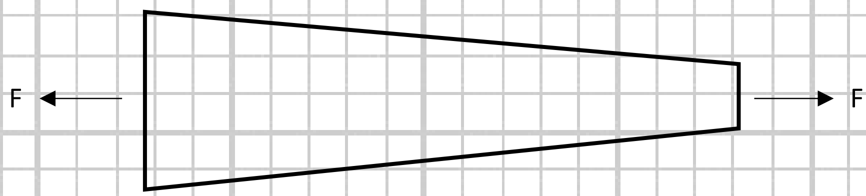


σ_1

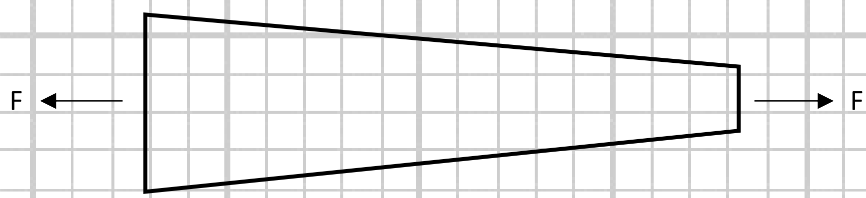


σ_2

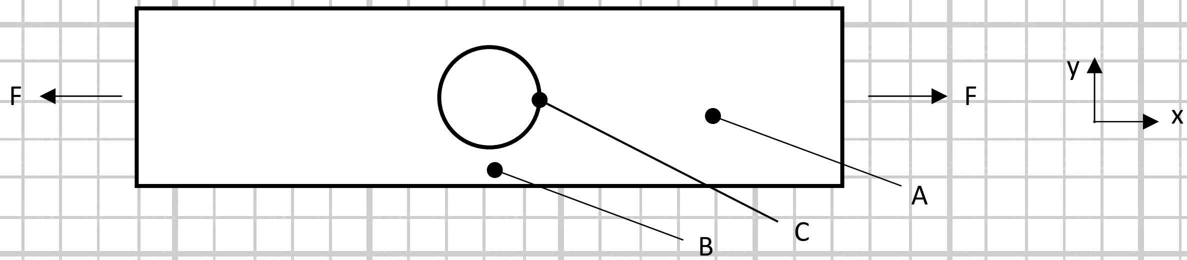
σ_3



τ_{max}

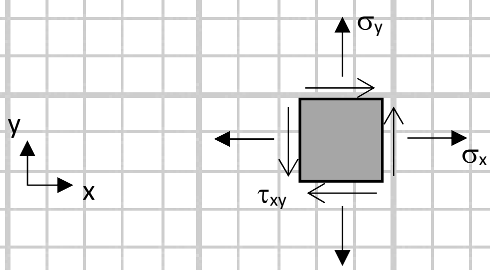


Given: a 1000mm long plate 10mm thick, 30mm wide. There is a 20mm diameter hole in the middle. A uniform 1kN axial load (F) is applied at the ends in the x-direction.



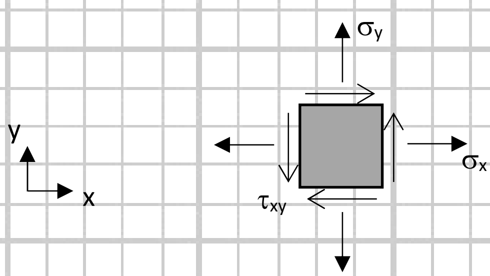
Without using a calculator, determine the stresses at Points A, B, C. Then draw Mohr's circles for them.

<u>At Point A:</u>



$\sigma_x =$ $\sigma_y =$ $\tau_{xy} =$

<u>At Point B:</u>

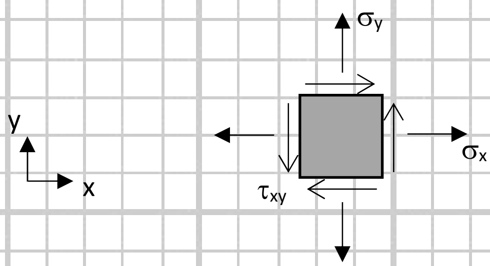


$\sigma_x =$ $\sigma_y =$ $\tau_{xy} =$

At Point C

(point C is located at the hole; therefore, the left side of the stress element is the surface of the hole).

How would Newton answer this? (That’s a hint, not a question).

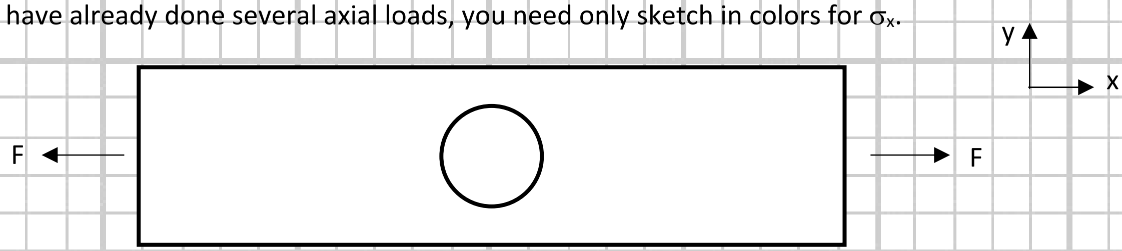


$\sigma_x =$ $\sigma_y =$ $\tau_{xy} =$

- {red range: 8.75MPa to 10MPa, or higher}
- {orange range: 7.5MPa to 8.75MPa)
- {yellow range: 6.25MPa to 7.5MPa}
- {green range: 5.0MPa to 6.25MPa}
- {light blue range: 3.75MPa to 5.0MPa}
- (blue range: 2.50MPa to 3.75MPa}
- {dark blue range: 1.25MPa to 2.50MPa}
- {black range: 0.0MPa (or less) to 1.25MPa}

Since you have already done several axial loads, you need only sketch in colors for σ_x.

σ_x



Discuss the effect of the hole on the stress in the plate.

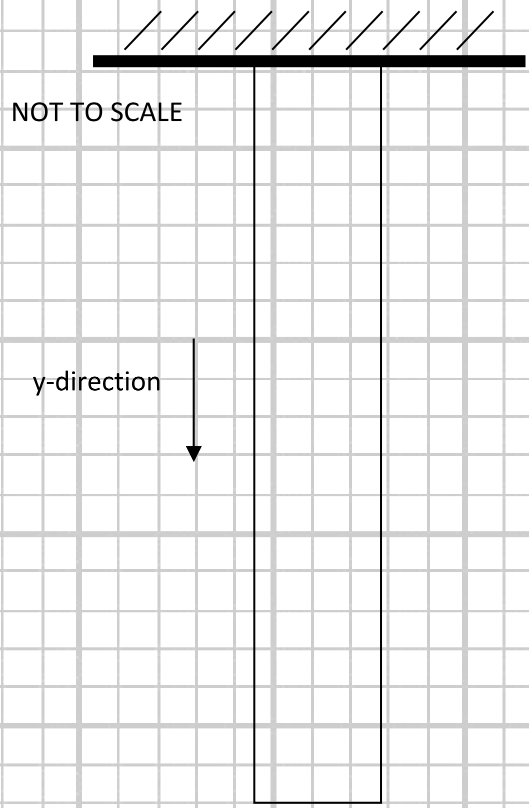
Given: a 20 meter long steel bar with a 10mmX10mm cross-section. It is suspended from the top and hangs freely. For most applications, we choose to neglect weight of the structure itself; however, for this problem, the weight of the bar is the only load. Determine the appropriate scale and color the stress in the y-direction due to the weight of the bar itself.

Change to 5 colors

Color scale: Max = __________ MPa

Min = __________ MPa

σ_y



BENDING LOADS (BEAMS)

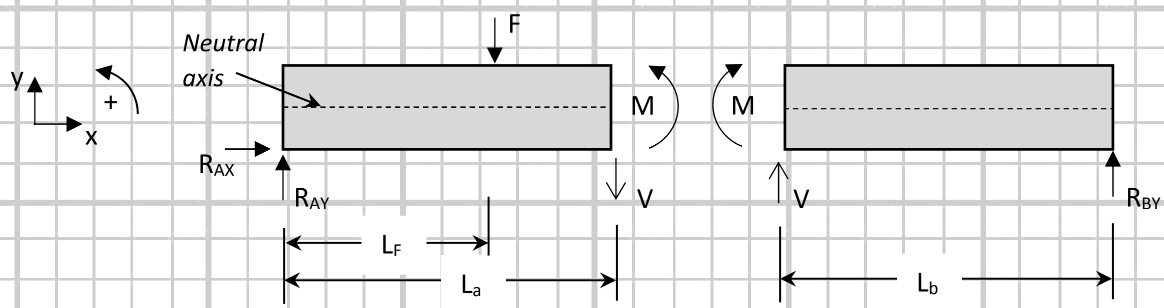
Wikipedia offers a good description of a beam: "*A beam is a structural element that primarily resists loads applied laterally to the beam's axis. Its mode of deflection is primarily by bending. The loads applied to the beam result in reaction forces at the beam's support points. The total effect of all the forces acting on the beam is to produce shear forces and bending moments within the beam, that in turn induce internal stresses, strains and deflections of the beam. Beams are characterized by their manner of support, profile (shape of cross-section), length, and their material.*"

Bending loads are one of the three fundamental loading conditions (axial and torsion are the other two). The following investigates the implications of the bending stress equation and shear stresses in beams. Most of the questions asks you to create:

- Free body diagram (FBD) – **include dimensions and coordinate system, show and label all forces**.
- Determine reactions (using the FBD)
- Shear diagram
- Bending moment diagram
- Deflection diagram (**sketch the "exaggerated" deflection shape without doing any analysis**). If you are not sure how the beam would deflect, get a stick and bend it – observe. Also, identify the approximate location of the maximum deflection.

You may also be asked to create free body diagrams at various sections. They should look something like the following example (filling in appropriate values for forces). In all cases, show all of your work. Attach extra paper if needed. **Use a straight edge!**

FBD at Section A-A:



$L = 160in$ $L_F = 80in$ $L_a = 100in$ $L_b = 60in$ $F = 100lb$

$R_{AX} = 0lb$ $R_{AY} = 50lb$ $R_{BY} = 50lb$

(*note to students: reaction forces must be determined first from the FBD of the non-sectioned structure*)

Determine V and M: using FBD on right side,

$\Sigma F_Y = -V + R_{BY} = 0;\ V = R_{BY} = 50lb$ V=50lb

$\Sigma M_{(at\ section)} = R_{BY}(L_b) - M = 0;\ M = R_{BY}(L_b) = (50lb)(60in) = 300lb\text{-}in$ M=300lb-in

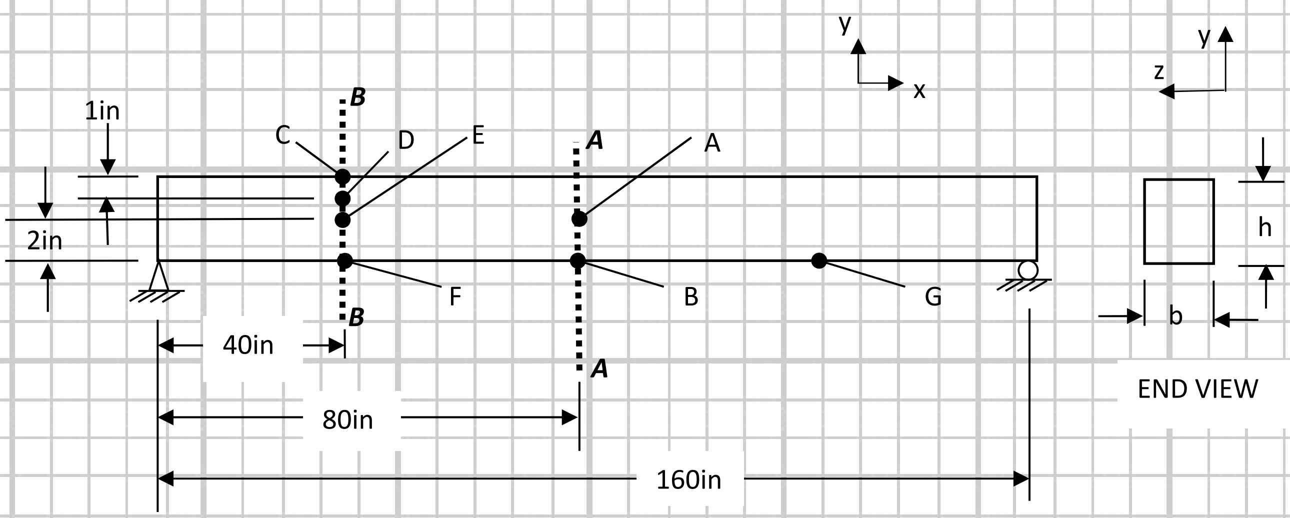
NOTE: "V" is the transverse shear force (the direction perpendicular to the beam's axis). The bending moment (M) at the section is indicated by the curved arrows, but the curved arrows do not represent the actual load distribution within the beam created by the bending moment.

Reminder: The transverse shear force (V) is ***not*** uniformly distributed across the transverse section. It is maximum at the neutral axis and approaches zero at the top and bottom surfaces of the beam ($\tau_{xy} = 0$ at the surface as mandated by equilibrium – *why does equilibrium require this to be true?*). Therefore, the transverse shear stress is **not** equal to V/A. Refer to a strength of materials textbook for details or search the web for a reliable source.

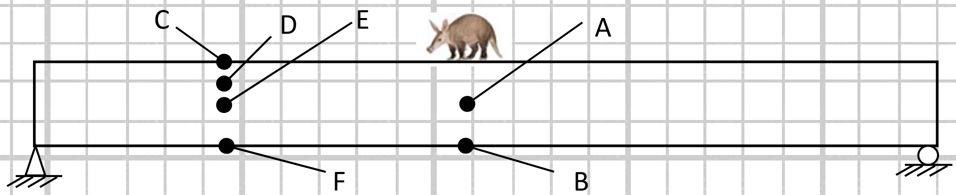
Rhetorical question (something meant to actively engage your mind, so try to answer it for yourself – then **we can discuss in class if you ask**): *Will a non-linear material (such as steel exceeding its yield strength) have a linearly varying stress distribution from the neutral axis? In other words, would the load still vary linearly from the neutral axis, as suggested by the equation $\sigma = My/I$, if the beam is being plasticly deformed?*

Unless stated otherwise, for all beam problems use the following:

The applied load is 1000 pounds regardless of whether it is an aardvark or bunny rabbit or camel (this is called "artistic license"). The drawings are not to scale, the total beam length (L) is 160 inches, beam height (h) is 4 inches, beam width (b) (z-axis direction) is 3 inches (therefore, $I = bh^3/12 = (3in)(4in)^3/12 = 16in^4$). For most of the beams, you are asked to determine stresses at several locations: two at ***Section A-A*** and four at ***Section B-B***. ***Section A-A*** is at mid-span and ***Section B-B*** is at quarter length (40" from end). Point A is in the middle of the beam and point B is on the bottom of the beam in ***Section A-A***. At ***Section B-B***, point C is at the top, D is a quarter down, E is mid-way between top and bottom, and F is at the bottom. Point G is on the bottom of the beam, 40 inches from the right end.

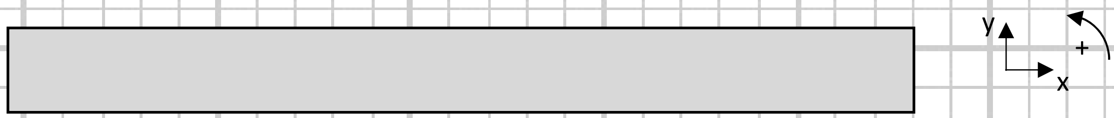


<u>Problem "Aardvark"</u> The aardvark is standing on the middle of the simply supported beam. **All dimensions and loads are given above. b**eam and aardvark are not to scale. Create the appropriate diagrams, etc. as requested. NO CALCULATOR IS NEEDED FOR ANY PART OF THIS PROBLEM! Thinking causes brain cells to grow – and they come in handy when least expected!



Use symbols only for the following, add actual values later on.

FBD:



Shear:

Moment

When asked for the deflection curve, sketch the shape the beam will take (if you have difficulty envisioning this, get a stick and try it). Mark the approximate location of the maximum deflection.

Deflection

Determine reaction forces for the FBD above. Calculate using brain – the ultimate portable calculator.

For ***Sections A-A*** and ***B-B***, draw the FBD and determine internal forces (V) and moments (M). Assume the aardvark is standing ever-so-slightly to the right of center.

Section A-A:

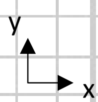
Section B-B:

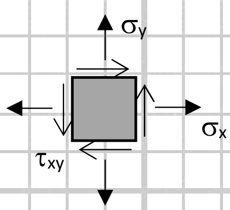
Next, you are asked to determine stresses at various points in the beam. First, determine σ_x, σ_y, and τ_{xy} for the requested point. Then create Mohr's circle and then using the Mohr's circle, determine the stresses oriented 45 degrees ($\sigma_{x'}$, $\sigma_{y'}$, $\tau_{x'y'}$). Use negative values if arrows are drawn in the opposite direction to reality.

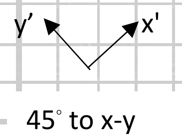
At point A:

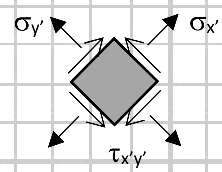
Stress calculations (determine σ_x, σ_y, and τ_{xy})

Mohr's circle:







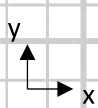


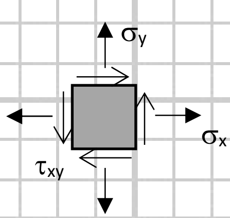
σ_x = σ_y = τ_{xy} = $\sigma_{x'}$ = $\sigma_{y'}$ = $\tau_{x'y'}$ =

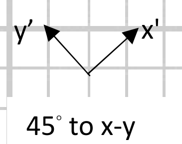
At point B:

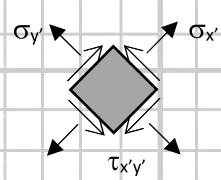
Stress calculations (determine σ_x, σ_y, and τ_{xy})

Mohr's circle:







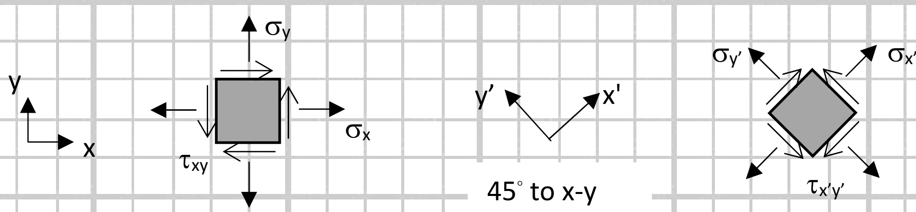


σ_x = σ_y = τ_{xy} = $\sigma_{x'}$ = $\sigma_{y'}$ = $\tau_{x'y'}$ =

At point C:

Stress calculations (determine σ_x, σ_y, and τ_{xy})

Mohr's circle:

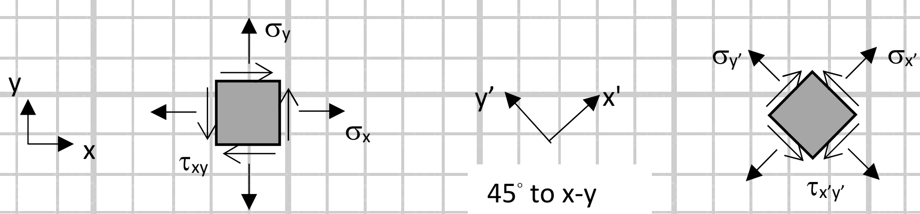


σ_x = σ_y = τ_{xy} = $\sigma_{x'}$ = $\sigma_{y'}$ = $\tau_{x'y'}$ =

At point D: (you might need a calculator for this one)

Stress calculations (determine σ_x, σ_y, and τ_{xy})

Mohr's circle:

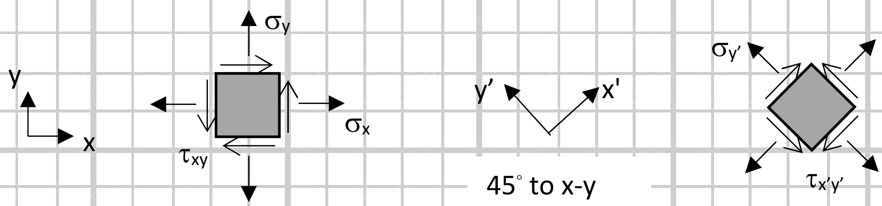


σ_x = σ_y = τ_{xy} = $\sigma_{x'}$ = $\sigma_{y'}$ = $\tau_{x'y'}$ =

At point E:

Stress calculations (determine σ_x, σ_y, and τ_{xy})

Mohr's circle:

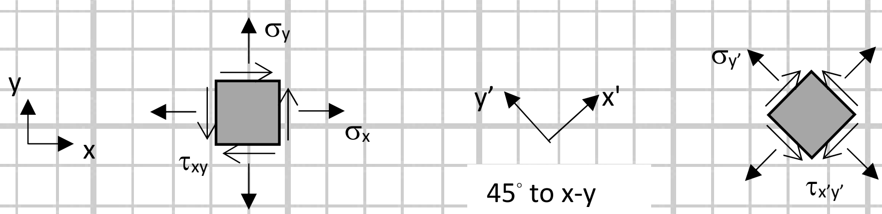


σ_x = σ_y = τ_{xy} = $\sigma_{x'}$ = $\sigma_{y'}$ = $\tau_{x'y'}$ =

At point F:

Stress calculations (determine σ_x, σ_y, and τ_{xy})

Mohr's circle:



σ_x = σ_y = τ_{xy} = $\sigma_{x'}$ = $\sigma_{y'}$ = $\tau_{x'y'}$ =

Before beginning the coloring exercises for the “Aardvark beam”, briefly answer the following questions.

What are the principal stresses and maximum shear stress at the points analyzed above? Circle the greatest magnitude of σ_1, σ_3, and τ_{max}.

Point A:	$\sigma_1 =$	$\sigma_2 =$	$\sigma_3 =$	$\tau_{max} =$
Point B:	$\sigma_1 =$	$\sigma_2 =$	$\sigma_3 =$	$\tau_{max} =$
Point C:	$\sigma_1 =$	$\sigma_2 =$	$\sigma_3 =$	$\tau_{max} =$
Point D:	$\sigma_1 =$	$\sigma_2 =$	$\sigma_3 =$	$\tau_{max} =$
Point E:	$\sigma_1 =$	$\sigma_2 =$	$\sigma_3 =$	$\tau_{max} =$
Point F:	$\sigma_1 =$	$\sigma_2 =$	$\sigma_3 =$	$\tau_{max} =$

Is the bending stress (σ=My/I) the same thing as the normal stress in the x-direction (σ_x) for all points in the beam?

At what locations in the beam are the bending stresses a principal stress?

In locations that the bending stress is not a principal stress, is it approximately equal to the principal stress?

Does the bending stress vary linearly from end to end? Are there conditions that must be valid that potentially are not valid, but were assumed to be?

Does bending stress vary linearly from top to bottom? Briefly explain or discuss appropriate equation. Are there conditions that must be valid that potentially are not valid, but were assumed to be?

Does the transvers shear stress (τ_{xy}) vary linearly from top to bottom? Briefly explain or discuss appropriate equation.

For Point A, compare and discuss the magnitude and direction of the transverse shear stress (τ_{xy}) with the maximum shear stress (τ_{max}).

For Point B, compare and discuss the magnitude and direction of the transverse shear stress (τ_{xy}) with the maximum shear stress (τ_{max}).

Assume the beam was made from laminated timber (feel free to google "laminated timber"). If the glue line has low shear strength, where would be the worst place to locate the bond? The glue-line lays in the x-z plane along the length of the beam (each piece of lumber would be 160 inches long).

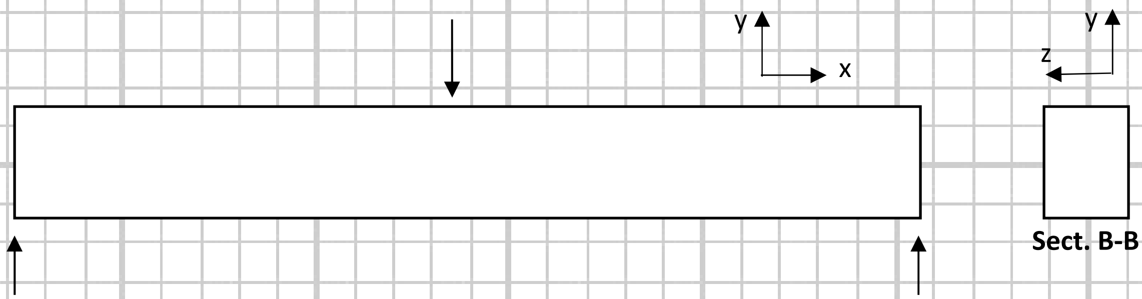
Often in beam analysis, we focus on the bending stress and neglect analyzing the transverse shear stress. This is because the bending stress dominates the transverse shear stress. However, if the beam is short enough, that is not the case. Consider a different simply supported beam. All of the loads and dimensions are the same as the "Aardvark beam" except the length, which is shorter (less than 160 inches). The 1000 pound load is applied mid-span. What length would be required such that the transverse shear stress (τ_{xy}) at the center of the mid-length of beam equals the maximum shear stress (τ_{max}) at the bottom of the mid-length of the beam?

Using the knowledge you have created for the "Aardvark beam," show how the stress is distributed throughout the beam by coloring the following. Using five (5) colors, create a color-scale similar to axial loaded sketches. Sketch the stresses using the color-scale bar.

{red: 3000psi to 5000psi, and higher}
{orange: 1000psi to 3000psi}
{green: -1000psi to 1000psi}
{light blue: -2000psi to -3000psi}
{dark blue: -3000psi to -5000psi (or lower)}

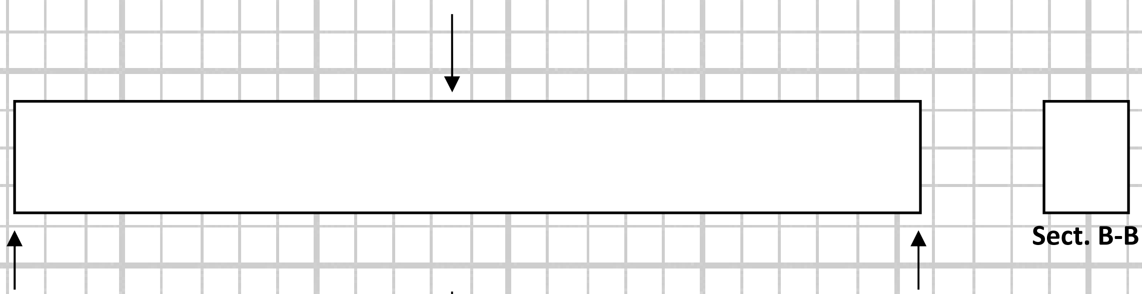
HINT for drawing colors: identify locations that separate the colors with a small "x" (-3000psi, -1000psi, 1000psi, 3000psi). Connect the points and fill in the appropriate colors.

σ_x

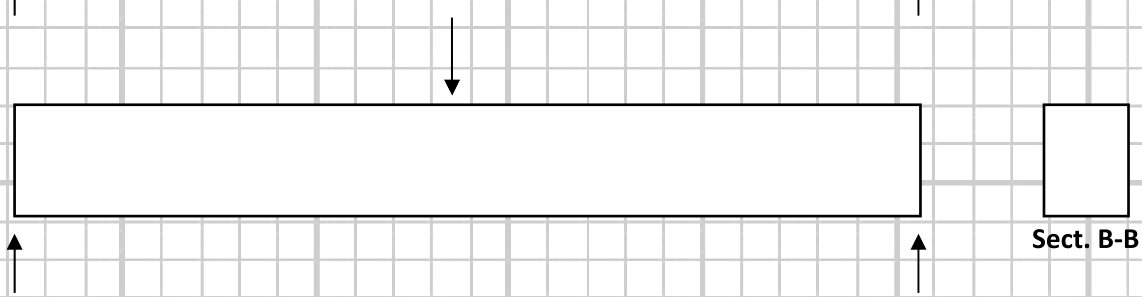


When coloring in the principal stresses, use the convention $\sigma_1 \geq \sigma_2 \geq \sigma_3$. Hint, for beams, σ_1 is tension and σ_3 is compression. And knowing that the transverse shear stress is zero or near zero at all locations should help you understand the relation between bending stress and the principal stresses.

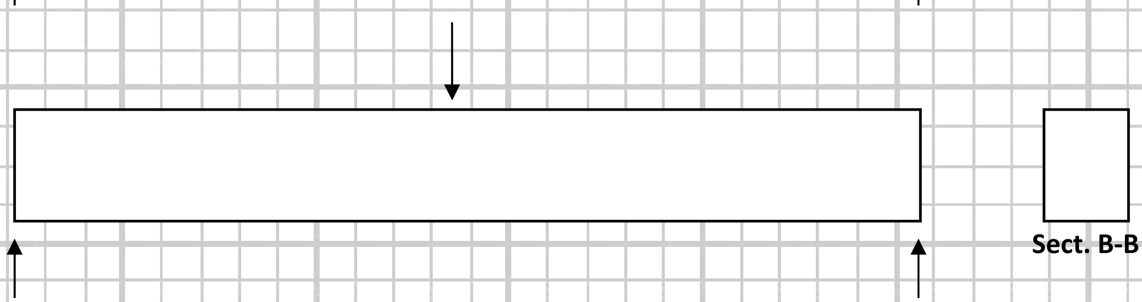
σ_1



σ_2



σ_3

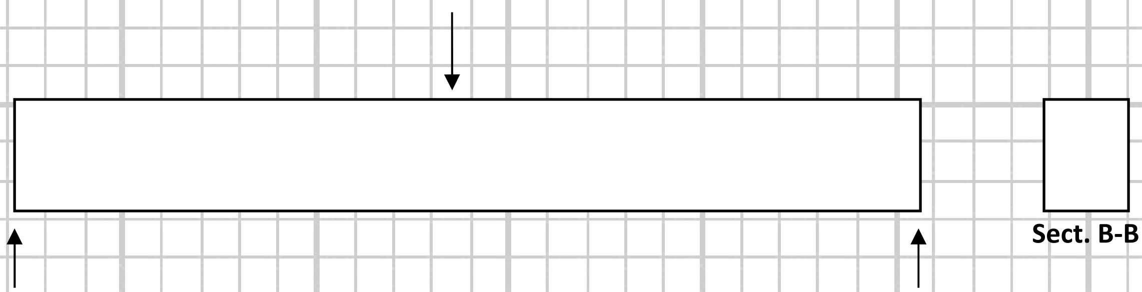


For the shear stresses, use the scale provided below. Does it make sense that the shear stresses scale is half the magnitude of the bending stress scale? Briefly explain. Refer to the Mohr's circles you've created.

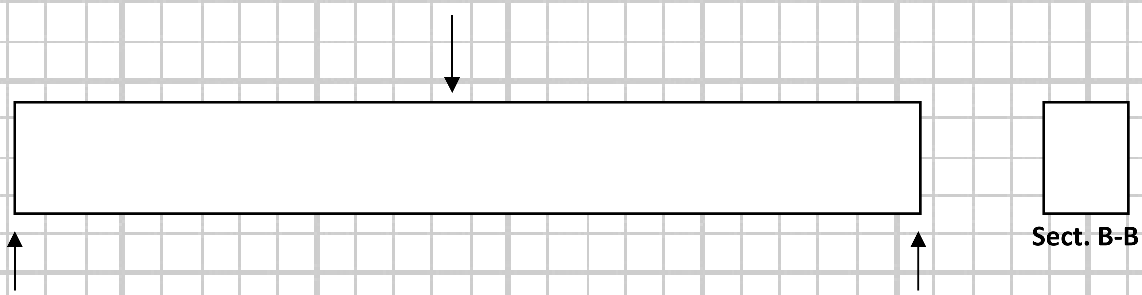
- {red: 1500psi to 2500psi, and higher}
- {orange: 500psi to 1500psi}
- {green: -500psi to 500psi}
- {light blue: -500psi to -1500psi}
- {dark blue: -1500psi to -2500psi (or lower)}

The shear stresses do not vary linearly, so use engineering judgement when coloring in – how "good" is "good enough"? (rhetorical question)

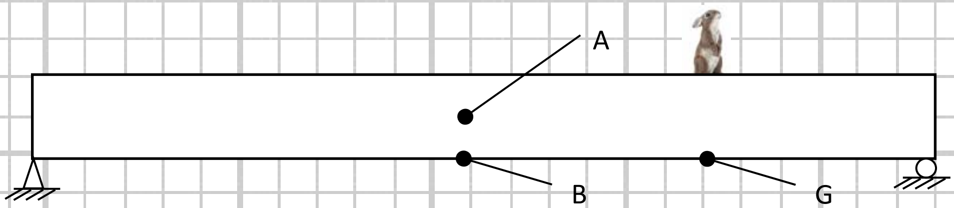
τ_{xy}



τ_{max}

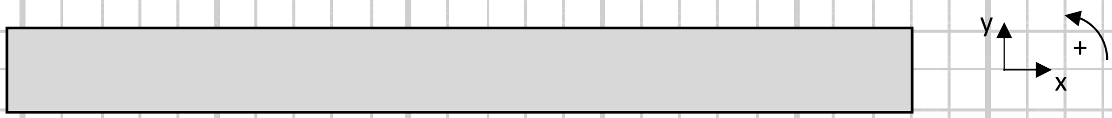


<u>Problem "Bunny Rabbit"</u> The 1000 pound bunny rabbit(!) is standing 40 inches from the right side of the simply supported beam (one quarter the length of the beam's length). **All dimensions and loads are given above** (beam and bunny rabbit are not to scale). Create the appropriate diagrams, etc. as requested. NO CALCULATOR IS NEEDED FOR ANY PART OF THIS PROBLEM! So eat a carrot and exercise your mind.



Use symbols only in the following. When asked to determine specific values, that is the time to use quantities.

FBD:



Shear:

Moment

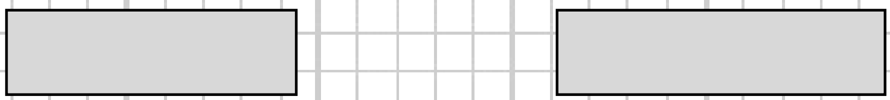
Deflection:

(Mark approximate location of maximum deflection)

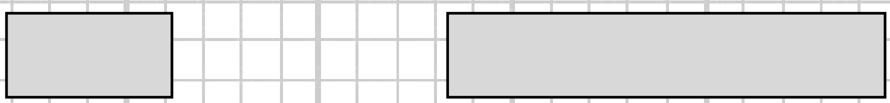
Determine reaction forces for the FBD above:

For ***Sections A-A*** and ***B-B***, draw the FBD and determine internal forces (V) and moments (M)

Section A-A:
(mid-beam)



Section B-B:
(quarter-length)

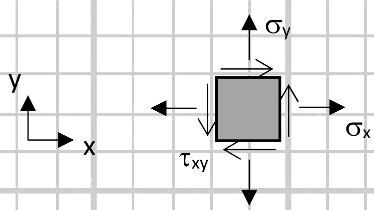


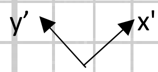
As with the Aardvark beam, determine σ_x, σ_y, and τ_{xy} for the requested points. Using theses, create Mohr's circle and then using the Mohr's circle, determine the stresses oriented 45 degrees ($\sigma_{x'}$, $\sigma_{y'}$, $\tau_{x'y'}$). Use negative values if arrows are drawn in the opposite direction to reality.

At point A:

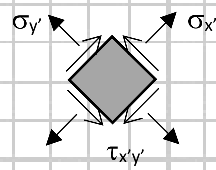
Stress calculations (determine σ_x, σ_y, and τ_{xy})

Mohr's circle:





45° to x-y

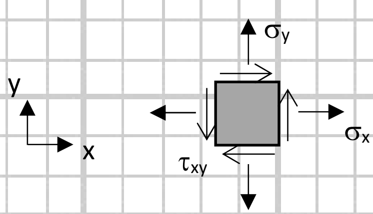


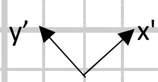
$\sigma_x =$ $\sigma_y =$ $\tau_{xy} =$ $\sigma_{x'} =$ $\sigma_{y'} =$ $\tau_{x'y'} =$

At point B:

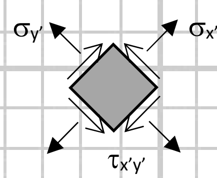
Stress calculations (determine σ_x, σ_y, and τ_{xy})

Mohr's circle:





45° to x-y

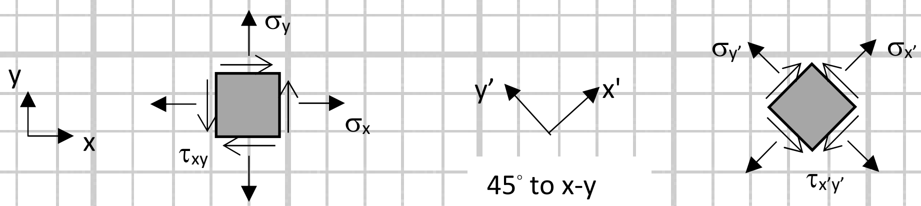


$\sigma_x =$ $\sigma_y =$ $\tau_{xy} =$ $\sigma_{x'} =$ $\sigma_{y'} =$ $\tau_{x'y'} =$

At point G:

Stress calculations (determine σ_x, σ_y, and τ_{xy})

Mohr's circle:



$\sigma_x =$ $\sigma_y =$ $\tau_{xy} =$ $\sigma_{x'} =$ $\sigma_{y'} =$ $\tau_{x'y'} =$

From the Mohr's circles you have created, what are the principal stresses and maximum shear stress at the following points? Circle the greatest magnitudes of σ_1, σ_3, and τ_{max}.

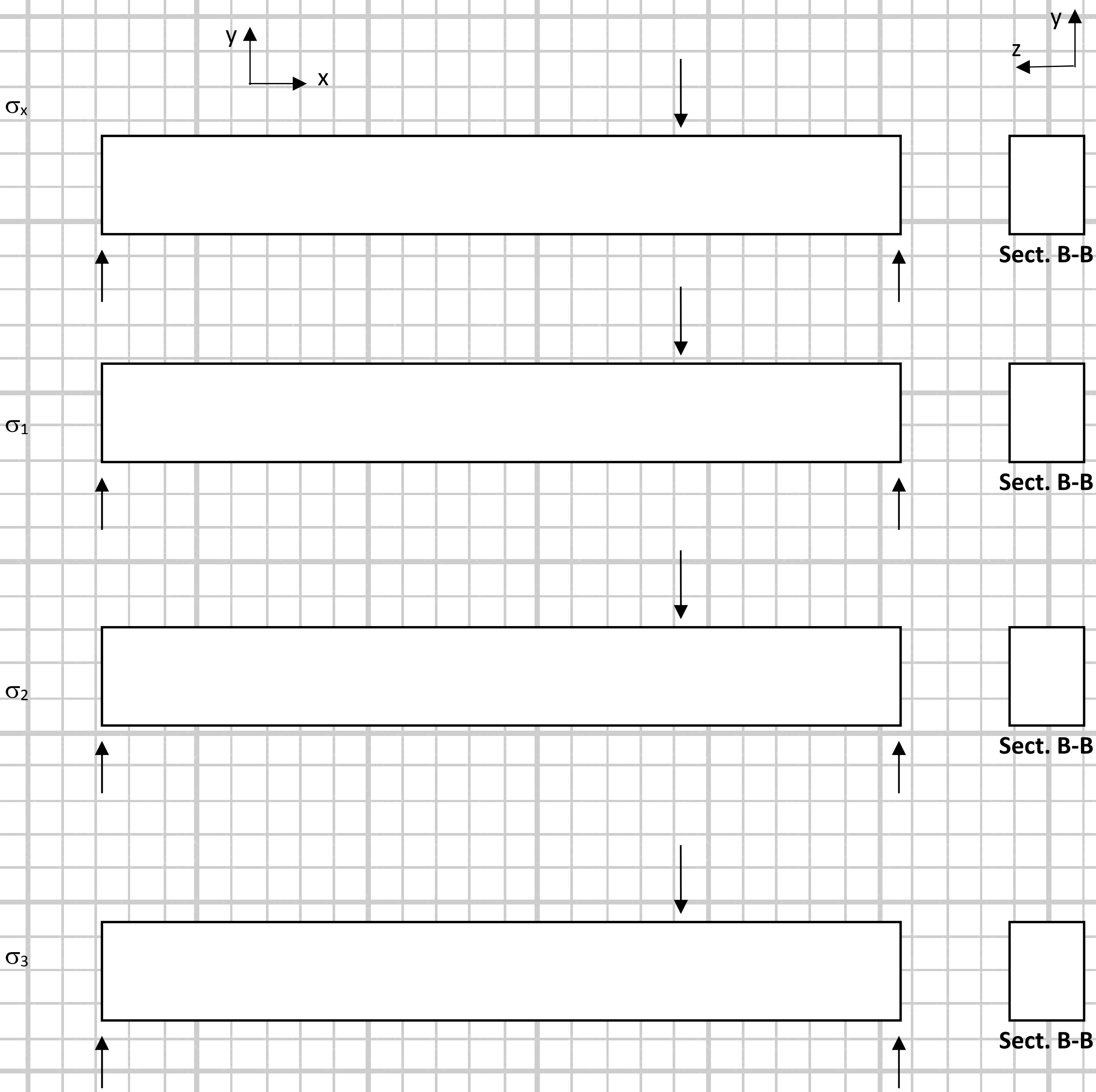
Point A: $\sigma_1 =$ $\sigma_2 =$ $\sigma_3 =$ $\tau_{max} =$

Point B: $\sigma_1 =$ $\sigma_2 =$ $\sigma_3 =$ $\tau_{max} =$

Point G: $\sigma_1 =$ $\sigma_2 =$ $\sigma_3 =$ $\tau_{max} =$

Using the knowledge you have created for the “bunny rabbit beam,” show how the stress is distributed throughout the beam by coloring the following. Sketch the stresses using the same color-scale bar even though stresses are less in this problem.

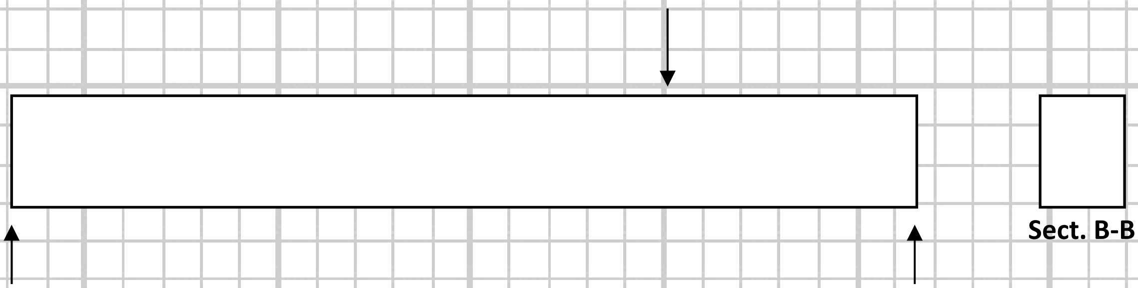
{red: 3000psi to 5000psi, and higher}
{orange: 1000psi to 3000psi}
{green: -1000psi to 1000psi}
{light blue: -1000psi to -3000psi}
{dark blue: -3000psi to -5000psi (or lower)}



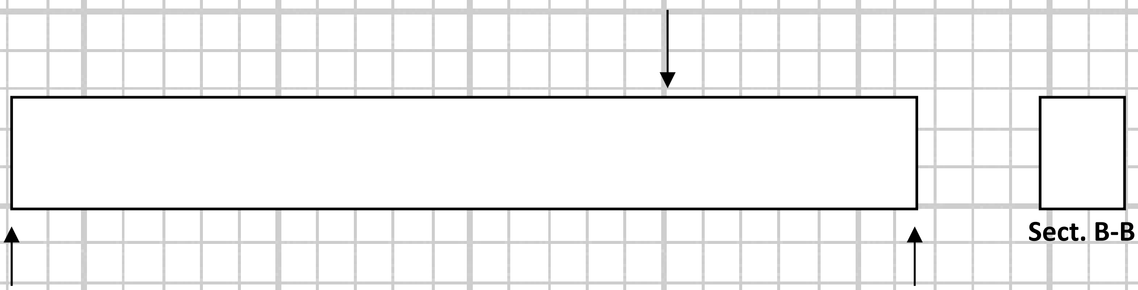
For the shear stresses, use the following scale (which is half the magnitude of the normal stress scale).

- {red: 1500psi to 2500psi, and higher}
- {orange: 500psi to 1500psi}
- {green: -500psi to 500psi}
- {light blue: -500psi to -1500psi}
- {dark blue: -1500psi to -2500psi (or lower) }

τ_{xy}



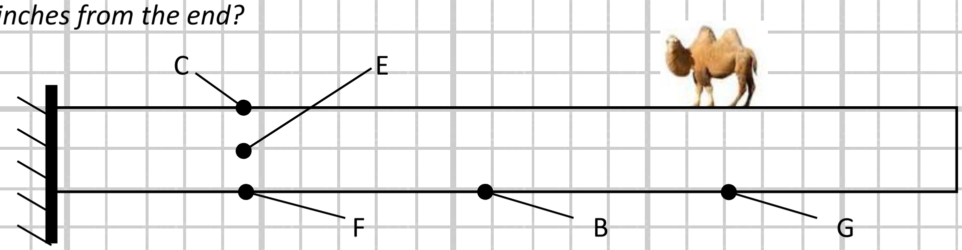
τ_{max}



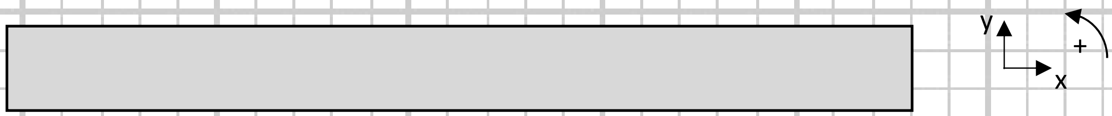
Comparing Aardvark and Bunny Rabbit loading conditions, comment on the magnitude of the reaction forces. Would the differences allow/require an engineer to design the two beams differently?

Comparing Aardvark and Bunny Rabbit loading conditions, comment on the magnitude of the principal stresses and maximum shear stresses. Would the difference allow/require an engineer to design the two beams differently?

<u>Problem "Camel".</u> The 1000 pound Camel (finally, a "right weight" animal) is standing 40 inches from the right side of the cantilever beam (one quarter the distance of the beam's length). The beam is supported on the left side. **All dimensions and loads are given above** (beam and camel are not to scale). Create the appropriate diagrams, etc. as requested. All points are the same as in the Bunny Rabbit beam. NO CALCULATOR IS NEEDED FOR ANY PART OF THIS PROBLEM! So drink lots of water and exercise your mind. *Rhetorical question*: *does it make sense to claim that a 1000 pound camel is standing 40 inches from the end?*



FBD:



Shear:

Moment

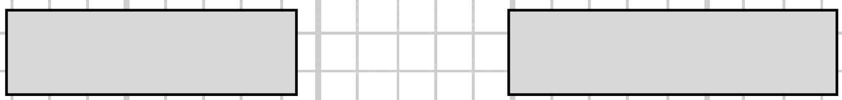
Deflection:

(Mark approximate location of maximum deflection)

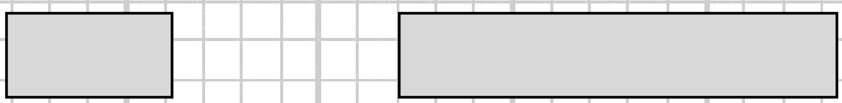
Determine reaction forces for the FBD above:

For ***Sections A-A*** and ***B-B***, draw the FBD and determine internal forces (V) and moments (M)

Section A-A:
(mid-beam)



Section B-B:
(quarter-length)

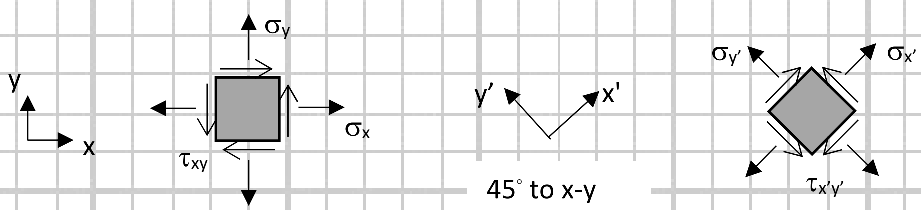


You know what to do next...

At point B:

Stress calculations (determine σ_x, σ_y, and τ_{xy})

Mohr's circle:

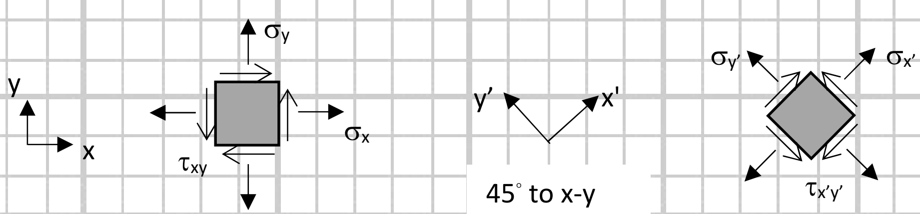


σ_x = σ_y = τ_{xy} = $\sigma_{x'}$ = $\sigma_{y'}$ = $\tau_{x'y'}$ =

At point C:

Stress calculations (determine σ_x, σ_y, and τ_{xy})

Mohr's circle:

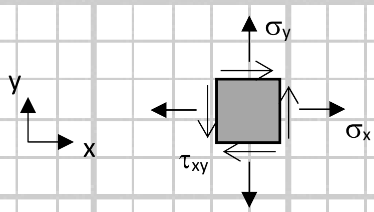


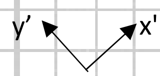
σ_x = σ_y = τ_{xy} = $\sigma_{x'}$ = $\sigma_{y'}$ = $\tau_{x'y'}$ =

At point E:

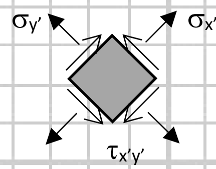
Stress calculations (determine σ_x, σ_y, and τ_{xy})

Mohr's circle:





45° to x-y

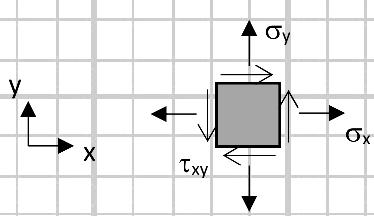


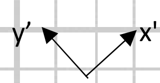
σ_x = σ_y = τ_{xy} = $\sigma_{x'}$ = $\sigma_{y'}$ = $\tau_{x'y'}$ =

At point F:

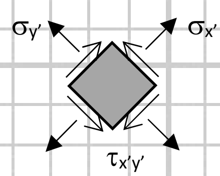
Stress calculations (determine σ_x, σ_y, and τ_{xy})

Mohr's circle:





45° to x-y

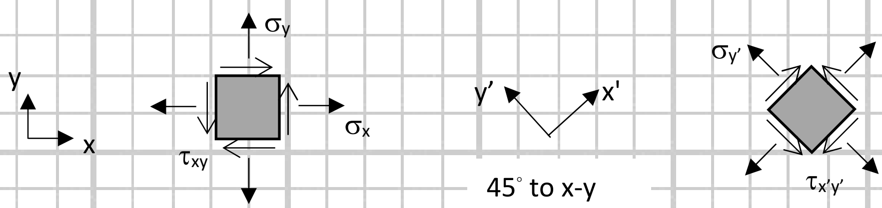


σ_x = σ_y = τ_{xy} = $\sigma_{x'}$ = $\sigma_{y'}$ = $\tau_{x'y'}$ =

At point G:

Stress calculations (determine σ_x, σ_y, and τ_{xy})

Mohr's circle:



σ_x = σ_y = τ_{xy} = $\sigma_{x'}$ = $\sigma_{y'}$ = $\tau_{x'y'}$ =

From the Mohr's circles you have created, what are the principal stresses and maximum shear stress at the following points? Circle the greatest magnitudes of σ_1, σ_3, and τ_{max}.

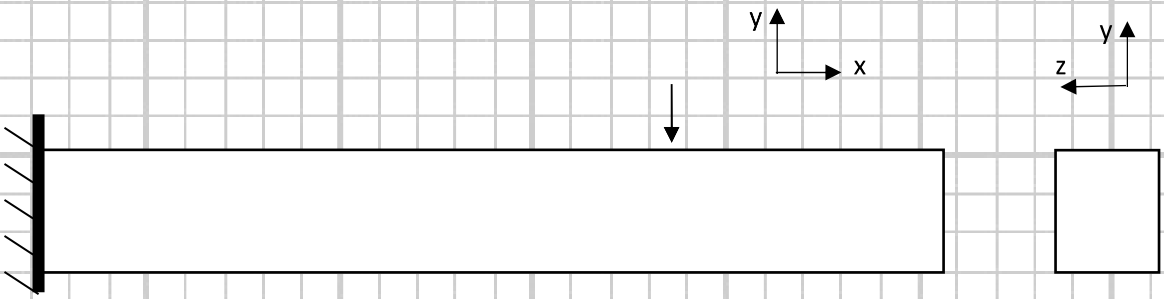
Point B:	σ_1 =	σ_2 =	σ_3 =	τ_{max} =
Point C:	σ_1 =	σ_2 =	σ_3 =	τ_{max} =
Point E:	σ_1 =	σ_2 =	σ_3 =	τ_{max} =
Point F:	σ_1 =	σ_2 =	σ_3 =	τ_{max} =
Point G:	σ_1 =	σ_2 =	σ_3 =	τ_{max} =

Although you have analyzed several points on the beam, none of the locations requested (Points B, C, etc.) are at the point of highest bending stress. Where is the highest bending stress in the "Camel beam" and what is its magnitude?

Using the knowledge you have created for the “Camel beam,” show how the stress is distributed throughout the beam by coloring the following. Using five (5) colors, create a color-scale same as you did previously – however, now you to determine appropriate values for the maximum and minimum stresses for the Camel beam, and determine the range for each color. Sketch the stresses using the color-scale bar.

	{red:	to	}
	{orange:	to	}
	{green:	to	}
	{light blue:	to	}
	{dark blue:	to	}

σ_x

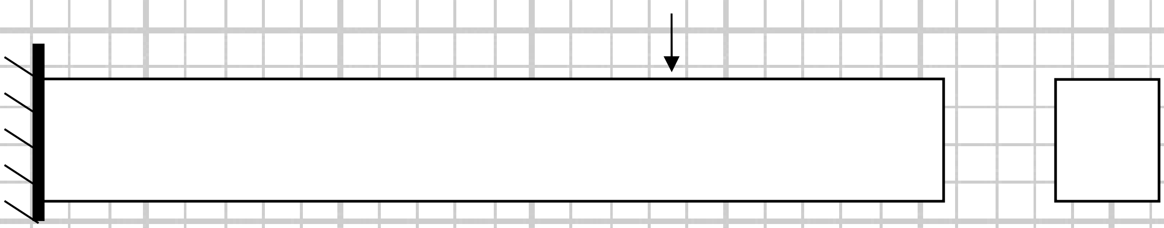


Sect. B-B

Create a new scale for the shear stress with half the magnitude of the normal stress scale.

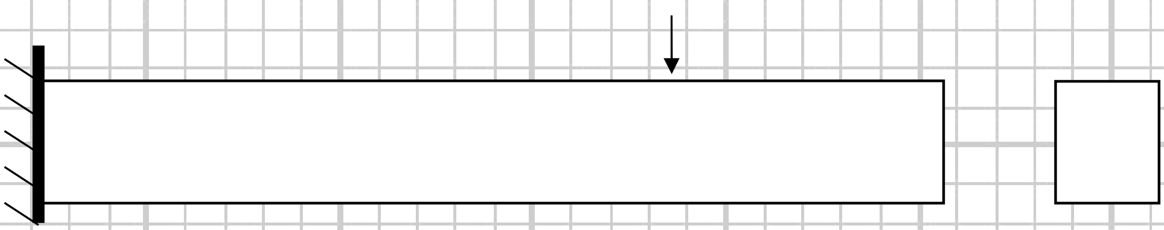
	{red:	to	}
	{orange:	to	}
	{green:	to	}
	{light blue:	to	}
	{dark blue:	to	}

τ_{xy}



Sect. B-B

τ_{max}



Sect. B-B

It is easy to draw many things on paper (or on a computer) but easy-of-drawing does not equate to easy-of-production. Comment on the relative complexity of creating a cantilever beam compared to a simply supported beam; in particular, is attaching a cantilever beam to the "wall" as *easy* as supporting a simply supported beam?

The three beams above (Aardvark, Bunny Rabbit, and Camel) all have the same dimensions and load. They differ in where the load is applied and how the beam is supported. Discuss the stresses in these beams – does one have significantly higher principal stress than the others, does one have lower stresses, or are they all nearly the same. How do the maximum shear stresses compare?

If you are designing a new beam to carry a load, comment on what could potentially be done to minimize stress. Things to consider: overall length, location of the load, supports (cantilever vs. simply supported), etc.

All of the loads in this book so far have been in multiples of 10 (10kN, 1000lb). Applying a "unit load" (load of 1 pound, 1kN, etc.) or multiples of 10 is common practice in stress analysis when designing a structure because we often do not know what the actual load will be. Using unit loads makes it trivial to then scale the stresses for differing loads. For the three beams above (aardvark, bunny rabbit, camel), what would the first principal stress (σ_1) and maximum shear stress (τ_{max}) become at Point B if the loads were doubled to one ton (2000 pounds)? Fill in the table.

Beam	Load: 1000lb		Load: 2000lb	
	σ_1	τ_{max}	σ_1	τ_{max}
Aardvark				
Bunny rabbit				
Camel				

Now for each of the three beams, determine the maximum applied load (weight of the animal) allowed based on the following criteria:

Maximum allowable shear stress in the beam is 50kpsi

Aardvark beam (maximum allowable aardvark weight):

Bunny rabbit beam (maximum allowable weight of the bunny rabbit)

Camel beam (maximum allowable weight of the camel)

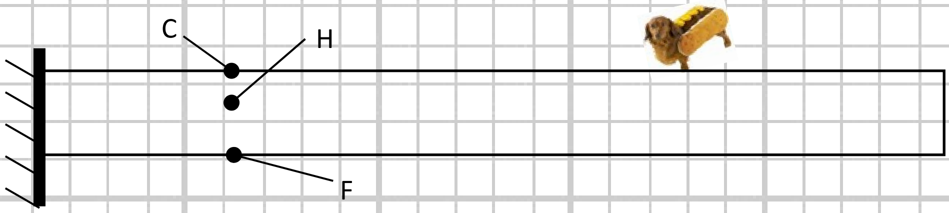
Maximum allowable principal stress in the beam is 50kpsi

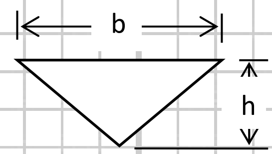
Aardvark beam (maximum allowable aardvark weight):

Bunny rabbit beam (maximum allowable weight of the bunny rabbit)

Camel beam (maximum allowable weight of the camel)

Problem "Dachshund". The 1000 pound Dachshund is doing a "paw-stand" with her paw 40 inches from the right side of the cantilever beam (one quarter the distance of the beam's length). The beam is supported on the left side. This beam is identical to the Camel beam, except it has a triangular cross-section rather than rectangular. The purpose of this exercise is to study the effect of a non-symmetric cross-section. You will be asked to determine stresses at only three locations, all being 40 inches from the attached end. Points C and F are on the top and bottom (as they were in the previous beams) and point H is at the neutral axis. The neutral axis is **not** halfway between the top and bottom. No calculator needed, except perhaps for Point H. You should be getting pretty good at this by now.





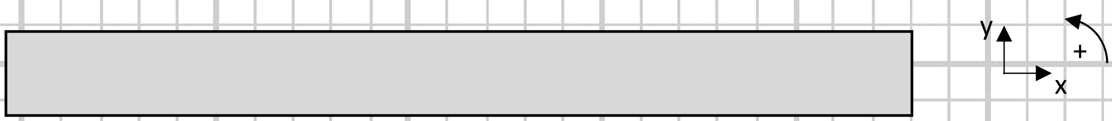
END VIEW
(NOT TO SCALE)

The cross-section dimensions are: b=9 inches, h = 4 inches. What is the area moment of inertia and how does it compare with the previous beams?

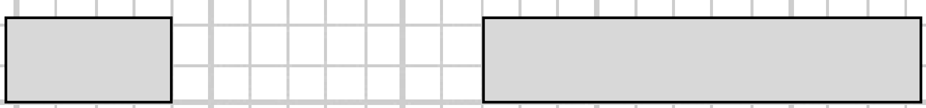
Where is the neutral axis located?

Create the FBD for the beam, and for the ***Section B-B*** (which contains Points C, F, and H)

FBD:



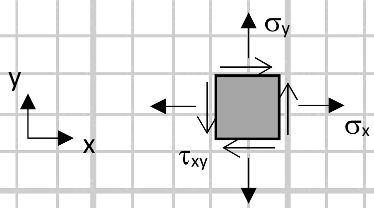
Section B-B:

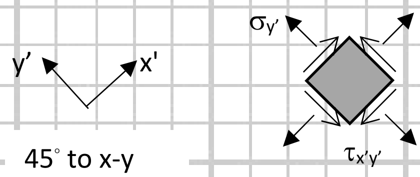


At point C:

Stress calculations (determine σ_x, σ_y, and τ_{xy})

Mohr's circle:





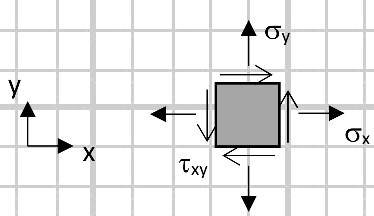
45° to x-y

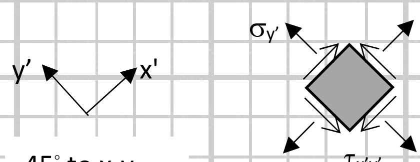
σ_x = σ_y = τ_{xy} = $\sigma_{x'}$ = $\sigma_{y'}$ = $\tau_{x'y'}$ =

At point F:

Stress calculations (determine σ_x, σ_y, and τ_{xy})

Mohr's circle:





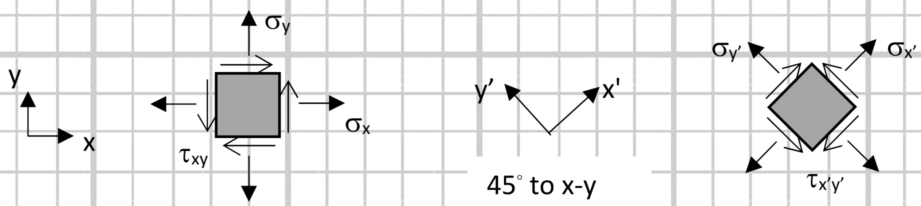
45° to x-y

σ_x = σ_y = τ_{xy} = $\sigma_{x'}$ = $\sigma_{y'}$ = $\tau_{x'y'}$ =

At point H:

Stress calculations (determine σ_x, σ_y, and τ_{xy})

Mohr's circle:



σ_x = σ_y = τ_{xy} = $\sigma_{x'}$ = $\sigma_{y'}$ = $\tau_{x'y'}$ =

From the Mohr's circles you have created, what are the principal stresses and maximum shear stress at:

Point C:	σ_1 =	σ_2 =	σ_3 =	τ_{max} =
Point F:	σ_1 =	σ_2 =	σ_3 =	τ_{max} =
Point H:	σ_1 =	σ_2 =	σ_3 =	τ_{max} =

If the material is linear, does the bending stress vary linearly from top to bottom?

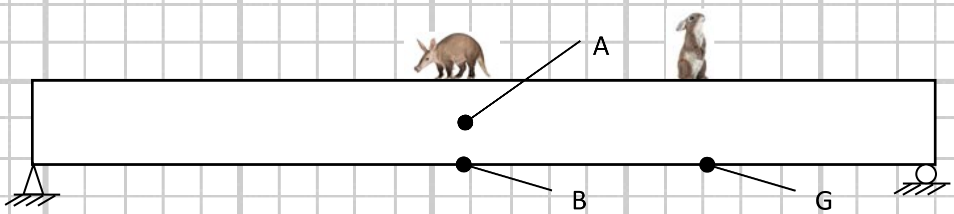
Does the strain vary linearly from the top to the bottom?

Why is the neutral axis **not** located halfway between the top and the bottom?

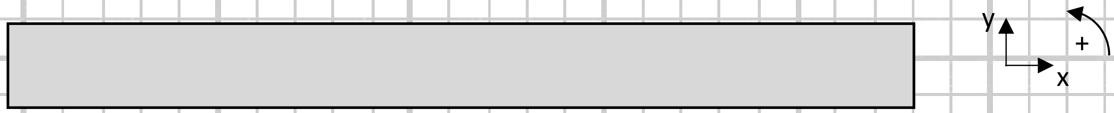
Problem "Superposition." The 1000 pound aardvark and a 1000 pound bunny rabbit are standing on the beam as they did before – but now they both are on the same beam simultaneously. **All dimensions and loads are given above**.

Engineering Wiki describes: *The Principle of Superposition is a method used to solve complex problems with multiple loads and/or reactions acting on the member. Superposition helps us solve these problems by breaking the member down as many times as necessary for each force acting on it. Once all the stresses or deflections for the point of interest are found, they can then be added all together to get a final answer.*

In other words, you have already solved this problem. You did so in two separate steps, now combine those steps. If (and only if) the beam remains linear, you can use superposition to determine the stresses in this problem by algebraically adding the stresses from the aardvark and bunny rabbit beam. You can do this for the stresses laying in the same directions (you can only add "apples" to "apples": $\sigma_{x\text{-combined}} = \sigma_{x\text{-aardvark}} + \sigma_{x\text{-bunny}}$. But can you do so with the principal stresses or maximum shear stress? That's another one of those rhetorical questions, for now.



FBD:



Shear:

Moment

Deflection:

(Mark approximate location of maximum deflection).

Determine reaction forces for the FBD above:

For ***Sections A-A***, draw the FBD and determine internal forces (V) and moments (M)

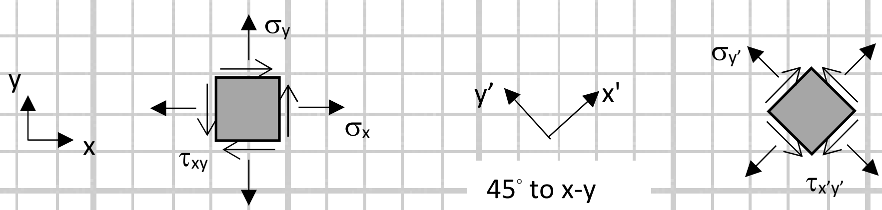
Section A-A:
(mid-beam)

Before determining stresses at the Points A, B and G in the beam, answer this question: where do you expect the bending stress to be maximum? This is an important question to be able to answer – otherwise, as an engineer, you'll end up analyzing stresses at numerous points needlessly.

Start with Point B and determine stress in two different ways. First, calculate it based on the bending moment at Point B created by the two loads. Then calculate it based on superposition – that is to say, add σ_x, σ_y, and τ_{xy} from the Aardvark beam and the Bunny Rabbit beam. Compare the results from the two methods. For Points A and G, use superposition to determine the stresses.

At point B: (based on bending stress created by the two loads).

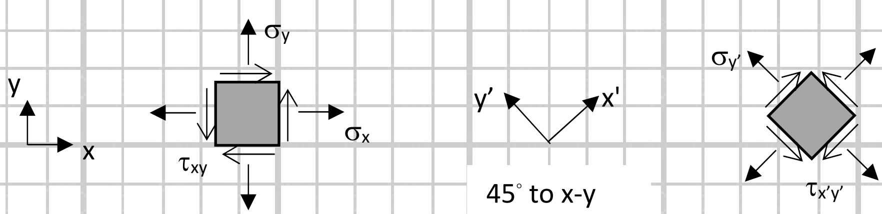
Stress calculations (determine σ_x, σ_y, and τ_{xy})   Mohr's circle:



σ_x = σ_y = τ_{xy} = $\sigma_{x'}$ = $\sigma_{y'}$ = $\tau_{x'y'}$ =

At point B: (Based on superposition adding your results from Aardvark to Bunny Rabbit)

Stress calculations (determine σ_x, σ_y, and τ_{xy} and also $\sigma_{x'}$, $\sigma_{y'}$, and $\tau_{x'y'}$)   Mohr's Circle:



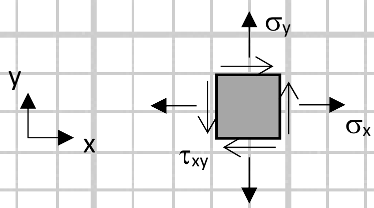
σ_x = σ_y = τ_{xy} = $\sigma_{x'}$ = $\sigma_{y'}$ = $\tau_{x'y'}$ =

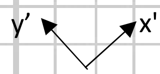
Discuss the results of the two methods used for determining stress (x, y direction and x', y' direction) at Point G. Are Mohr's circles the same?

At point A: (Based on superposition)

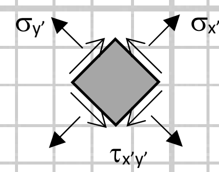
Stress calculations (determine σ_x, σ_y, and τ_{xy})

Mohr's circle:





45° to x-y

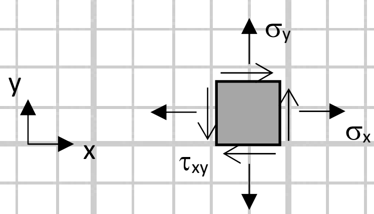


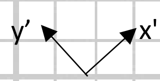
σ_x = σ_y = τ_{xy} = $\sigma_{x'}$ = $\sigma_{y'}$ = $\tau_{x'y'}$ =

At point G: (Based on superposition)

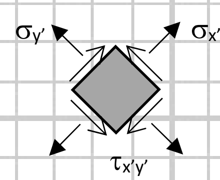
Stress calculations (determine σ_x, σ_y, and τ_{xy})

Mohr's circle:





45° to x-y



σ_x = σ_y = τ_{xy} = $\sigma_{x'}$ = $\sigma_{y'}$ = $\tau_{x'y'}$ =

For the Aardvark beam, the principal stresses and maximum shear stress were (recopy them here):

Point A: σ_1 = σ_2 = σ_3 = τ_{max} =

Point B: σ_1 = σ_2 = σ_3 = τ_{max} =

Point G: σ_1 = σ_2 = σ_3 = τ_{max} =

For the Bunny Rabbit beam, the principal stresses and maximum shear stress were (recopy them here):

Point A: σ_1 = σ_2 = σ_3 = τ_{max} =

Point B: σ_1 = σ_2 = σ_3 = τ_{max} =

Point G: σ_1 = σ_2 = σ_3 = τ_{max} =

Now add these together to help answer the question: *does superposition work for principal stresses and maximum shear stress*?

Point A: σ_1 = σ_2 = σ_3 = τ_{max} =

Point B: σ_1 = σ_2 = σ_3 = τ_{max} =

Point G: σ_1 = σ_2 = σ_3 = τ_{max} =

From the **Mohr's circles** you have created for the Superposition Beam what are the principal stresses and maximum shear stress at:

Point A: σ_1 = σ_2 = σ_3 = τ_{max} =

Point B: σ_1 = σ_2 = σ_3 = τ_{max} =

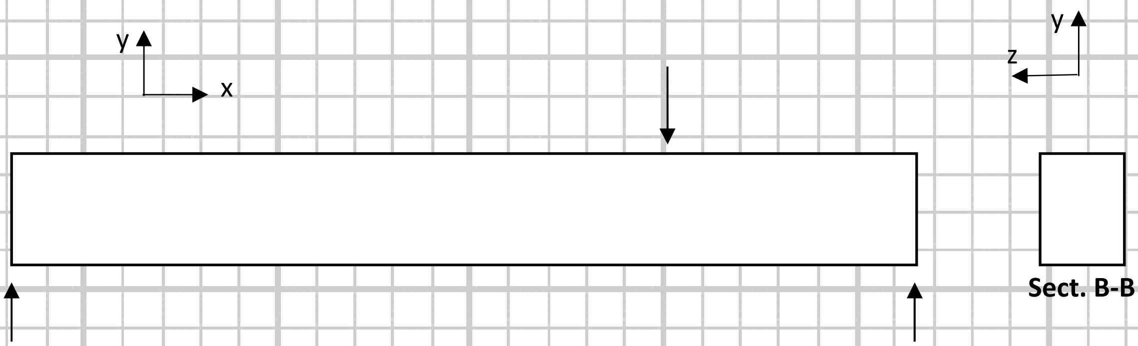
Point G: σ_1 = σ_2 = σ_3 = τ_{max} =

Explain why superposition worked for calculating certain stresses such as σ_x and τ_{xy}, but not the principal stresses or maximum shear stress.

Using the knowledge you have created for the “superposition beam,” show how the stress is distributed throughout the beam by coloring the following. Sketch the stresses using the color-scale bar:

{red: 6000psi to 10000psi, and higher}
{orange: 2000psi to 6000psi}
{green: -2000psi to 2000psi}
{light blue: -2000psi to -6000psi}
{dark blue: -6000psi to -10000psi, or lower}

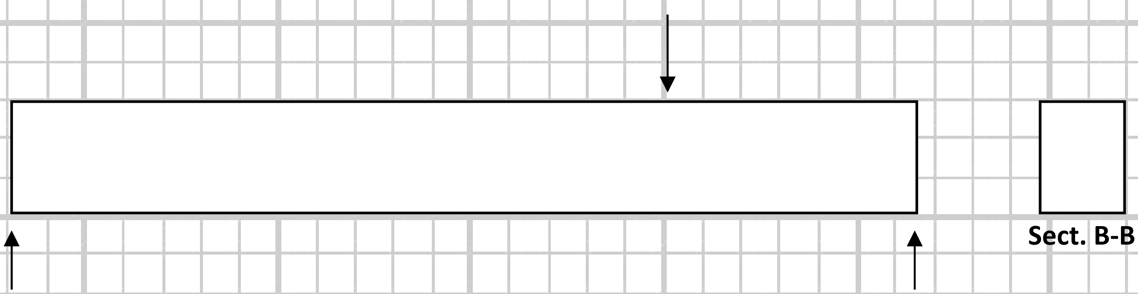
σ_x



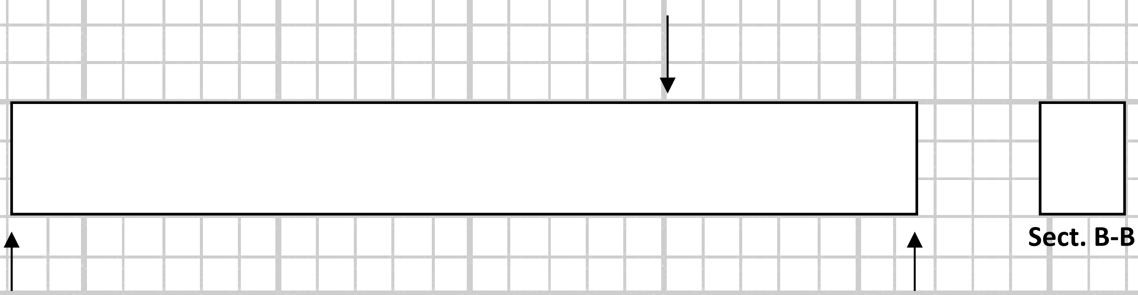
For the shear stresses, use the following scale.

{red: 3000psi to 5000psi, and higher}
{orange: 1000psi to 3000psi}
{green: -1000psi to 1000psi}
{light blue: -1000psi to -3000psi}
{dark blue: -3000psi to -5000psi, or lower}

τ_{xy}



τ_{max}



TORSION LOADS (SHAFTS)

Along with axial and bending loads, torsion is one of the three basic loading conditions. Shafts are the mechanical element that carry torsional loads. According to Wikipedia, "*A shaft is a rotating machine element, usually circular in cross section, which is used to transmit power from one part to another, or from a machine which produces power to a machine which absorbs power. The various members such as pulleys and gears are mounted on it.*" The power is transmitted through the shaft as the shaft carries a torque load and rotates. Similar to the bending stress problems you have completed, you will be asked to determine stress at various locations in shafts, and you will be asked to color in the stress levels.

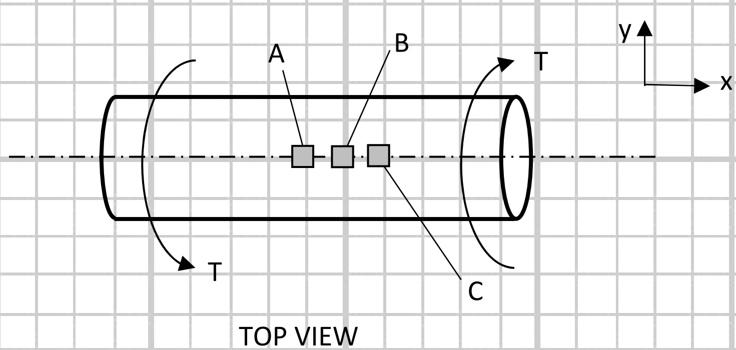
Stress in shafts is sometimes referred to as "pure shear loading". This can be a bit misleading. While there is no normal stress produced in the direction of the shaft axis, there are normal stresses in the shaft. Mohr's circle should make that apparent. After completing these exercises, you should, without even thinking, recognize that Mohr's circle for "pure shear loading" is centered on the $\sigma-\tau$ axis.

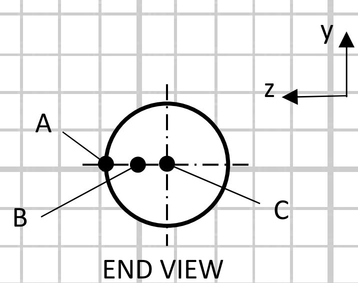
NOTE: due to axial symmetry, the use of Cartesian coordinates may be inappropriate. However, since we use Cartesian for axial and bending loads, we will continue to do so. As long fix our attention on one area of the bar and relate those points to the x-y-z axis, our results will be legitimate. But recognize that Point A in the first problem, for example, represents the stress at any point on the surface; regardless of its orientation with respect to the coordinate system we use.

Problem "Solid Circular Shaft". Consider the shaft with the axis laying in the x-direction. On the left side of the shaft is a gear driven by a motor (not shown in the diagram). The gear is 10 inches in diameter and a force of 2000 pounds is applied to the gear teeth. The power is transmitted through the shaft, and on the right side is another gear (not shown) that delivers the power out of the shaft. Determine stresses as requested. Diameter is 2.125 inches. Points A, B and C are points in the shaft; A is on the surface, B is half the radius from the center, and C is in the middle.

Shear stress, $\tau = Tr/J$ where J is the polar moment of inertia. For a solid circular bar, $J = \pi d^4/32$. If d=2.125" then $J=2.0in^4$ (that's nice, now you don't need a calculator).

NOTE: semi-isometric view is shown here to show the shaft is circular.



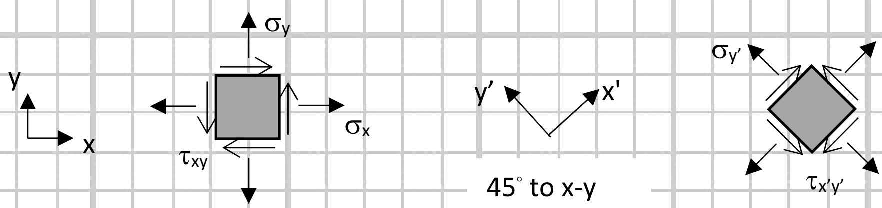


T =____________________________

At point A:

Stress calculations (determine σ_x, σ_y, and τ_{xy})

Mohr's circle:

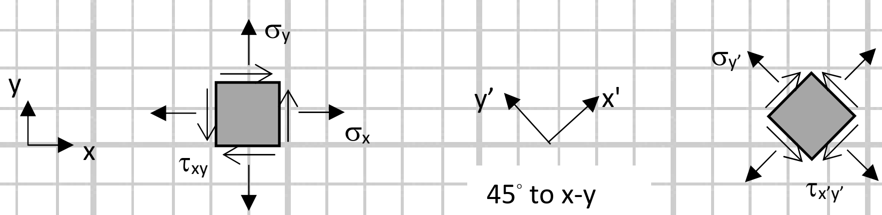


$\sigma_x =$ $\sigma_y =$ $\tau_{xy} =$ $\sigma_{x'} =$ $\sigma_{y'} =$ $\tau_{x'y'} =$

At point B:

Stress calculations (determine σ_x, σ_y, and τ_{xy})

Mohr's circle:

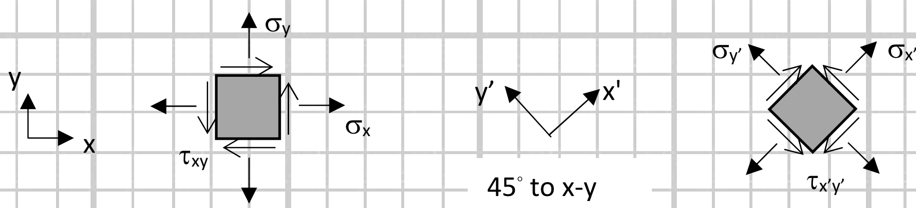


$\sigma_x =$ $\sigma_y =$ $\tau_{xy} =$ $\sigma_{x'} =$ $\sigma_{y'} =$ $\tau_{x'y'} =$

At point C:

Stress calculations (determine σ_x, σ_y, and τ_{xy})

Mohr's circle:



$\sigma_x =$ $\sigma_y =$ $\tau_{xy} =$ $\sigma_{x'} =$ $\sigma_{y'} =$ $\tau_{x'y'} =$

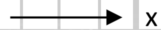
Does a shaft have to have a circular cross section in order for the equation τ=Tr/J to be valid? Explain.

Does the material need to be linear for the equation τ=Tr/J to be valid? Explain.

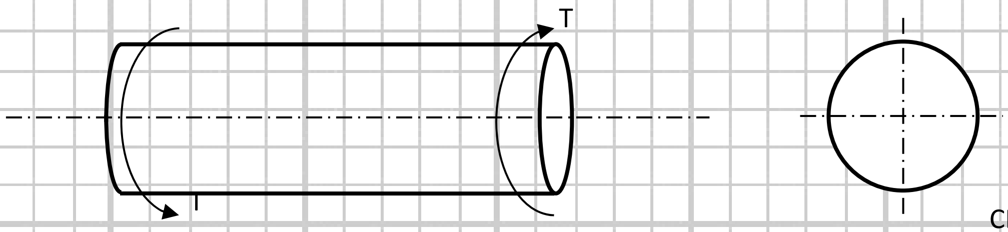
NOTE ON SYMMETRY: you will now color in the stresses. Recognize that "y-direction stress" (Cartesian coordinates) as done above, actually refers to the stress perpendicular to the axis (x-direction) and a direction tangent to the surface. For example, the stresses you calculate for Point A represent the stresses on the surface regardless of the angular position ("top" of the shaft, "side" of the shaft, etc.). Consider the y-direction to rotate along the surface as a local coordinate. Use that knowledge as you color in the shaft stresses. Stresses are symmetric around the shaft.

Color in the stresses:

- {red: 3000psi to 5000psi, and higher}
- {orange: 1000psi to 3000psi}
- {green: -1000psi to 1000psi}
- {light blue: -1000psi to -3000psi}
- {dark blue: -3000psi to -5000psi, or lower}

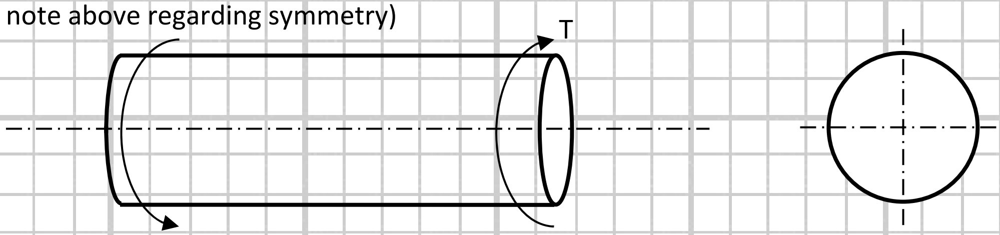


σ_x

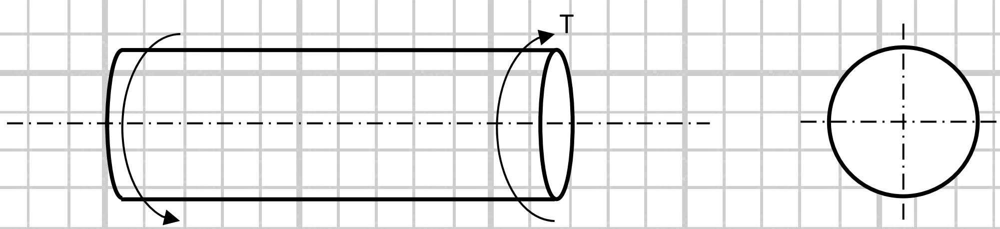


CENTER
SECTION

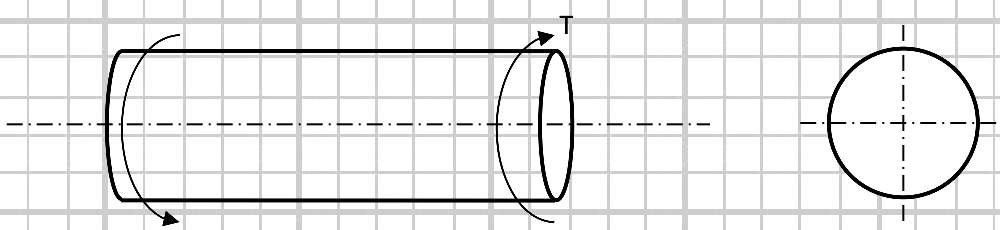
"σ_y" (see note above regarding symmetry)



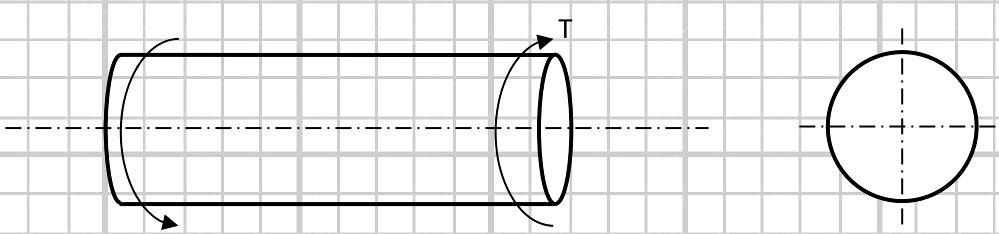
σ_1



σ_2



σ_3



Using the same scale as above, color in the shear stresses.

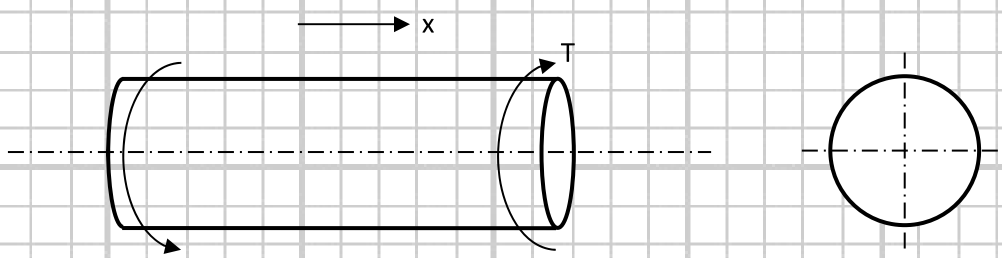
Why is it appropriate to use the same scale for shear stress as for normal stress in this torsion problem, but for the beam loads, the shear stress scale was half that of the normal stress? Refer to Mohr's circles.

Color in the stresses:

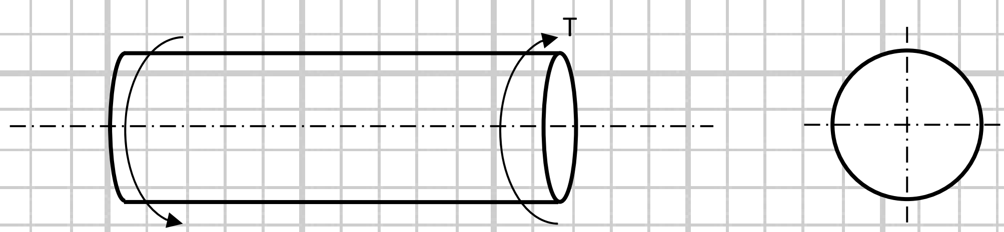
{red: 3000psi to 5000psi, and higher}
{orange: 1000psi to 3000psi}
{green: -1000psi to 1000psi}
{light blue: -1000psi to -3000psi}
{dark blue: -3000psi to -5000psi, or lower}

Regarding "τ_{xy}" and "τ_{yz}", see note above about symmetry.

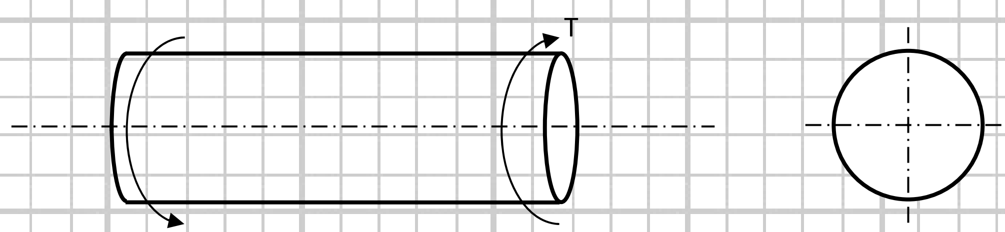
"τ_{xy}"



"τ_{yz}"

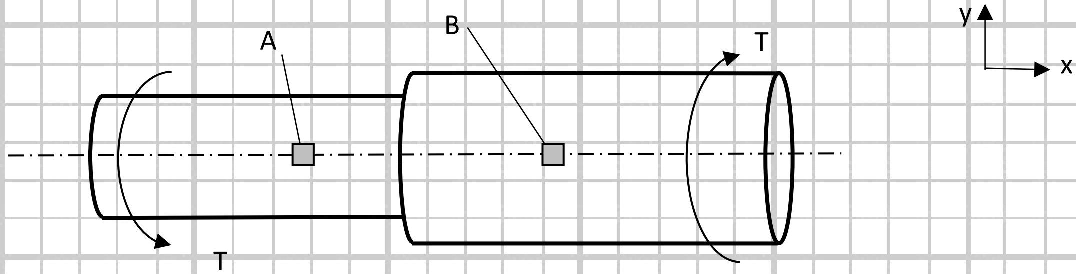


τ_{max}



Problem "Solid Circular Shaft – Two Diameters". Consider the shaft with the axis laying in the x-direction. An input torque of 100in-lb is applied to the left side. Determine stresses as requested. The shaft has two different diameters: 2.125 inches and 3.177 inches. Point A is on the surface of the 2.125 inch diameter section and Point B is on the surface of the larger diameter section. You may use a calculator.

Regarding Cartesian coordinates, see note above about symmetry

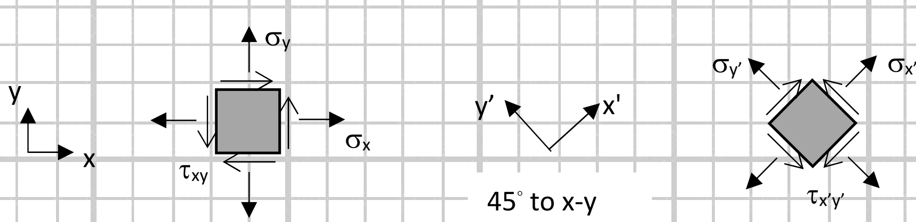


Assumptions:

At point A (on the surface of the 2.125 inch diameter section).

Stress calculations (determine σ_x, σ_y, and τ_{xy})

Mohr's circle:

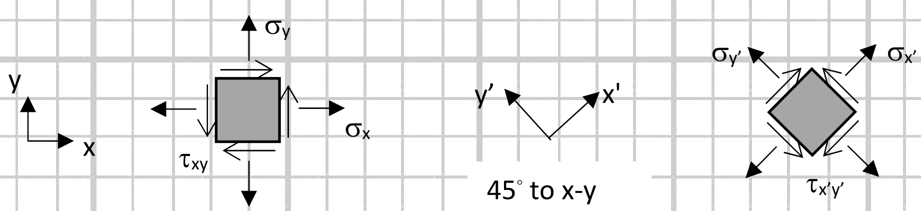


$\sigma_x =$ $\sigma_y =$ $\tau_{xy} =$ $\sigma_{x'} =$ $\sigma_{y'} =$ $\tau_{x'y'} =$

At point B (on the surface of the 3.177 inch diameter section)

Stress calculations (determine σ_x, σ_y, and τ_{xy})

Mohr's circle:



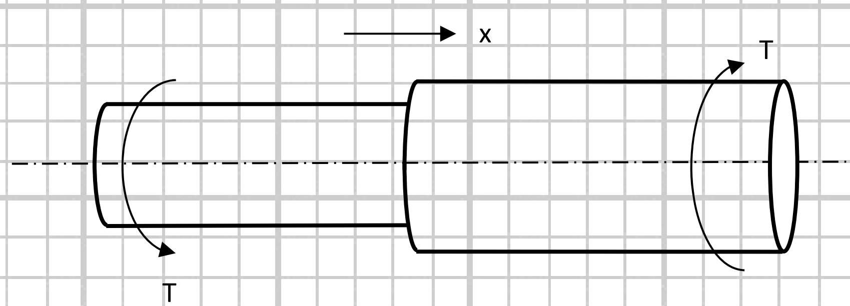
$\sigma_x =$ $\sigma_y =$ $\tau_{xy} =$ $\sigma_{x'} =$ $\sigma_{y'} =$ $\tau_{x'y'} =$

What is τ_{xy} at a point 1.063 inches from the center of the shaft, just below point B?

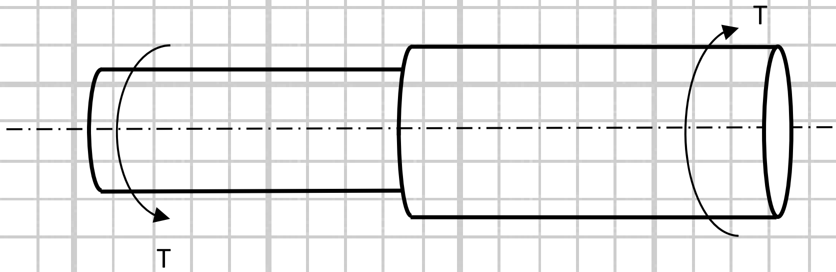
Color in the stresses: (see note above regarding symmetry)

- {red: 3000psi to 5000psi, and higher}
- {orange: 1000psi to 3000psi}
- {green: -1000psi to 1000psi}
- {light blue: -1000psi to -3000psi}
- {dark blue: -3000psi to -5000psi, or less}

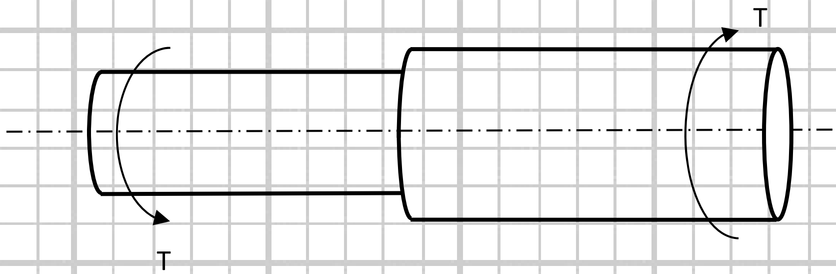
σ_x



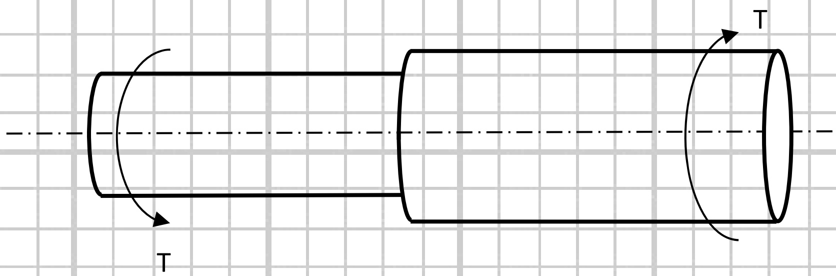
σ_1



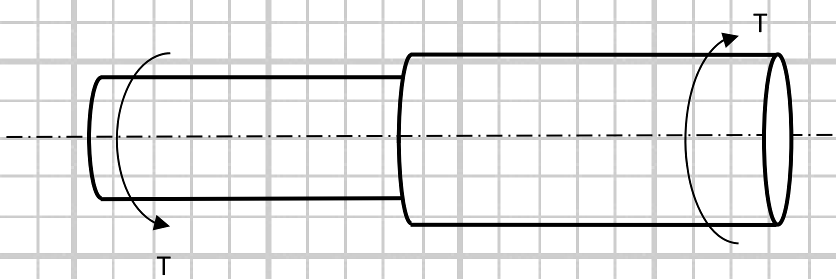
σ_2



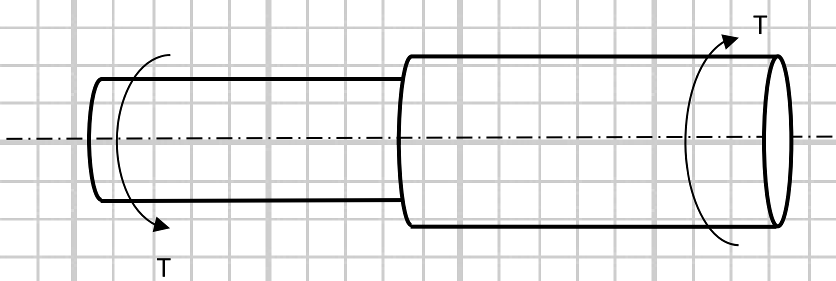
σ_3



"τ_{xy}"



τ_{max}



Comment on the effect of a relatively small change in diameter (2.125", 3.177") on stresses.

To save weight, many shafts are hollow. Determine the thickness required for following criteria:

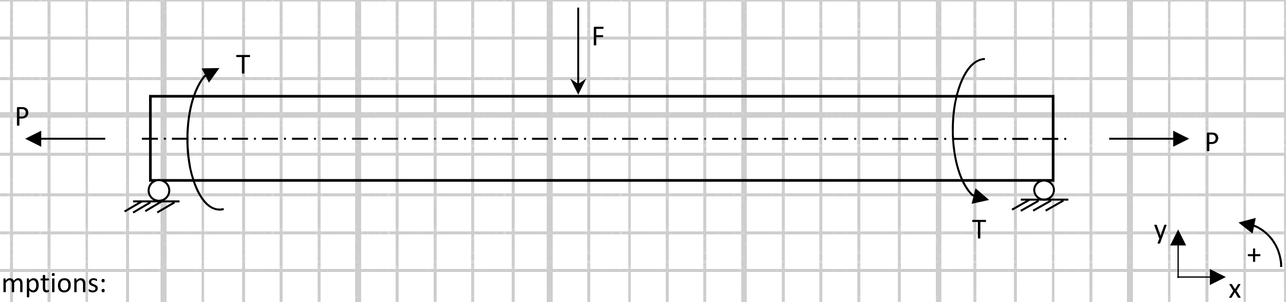
1) Same stresses on the surface as are present in the 2.125" diameter solid shaft, with 100in-lb torque.
2) Outside diameter is 3.177 inches.
3) Applied torque: 100in-lb

After determining the wall-thickness, determine how much weight saving (percentage) would be achieved by using the hollow shaft compared to the solid 2.125" shaft.

Assumptions:

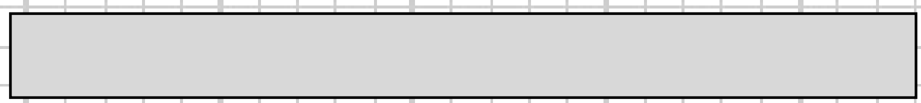
Solution:

Problem "Super-duper Super Position" Now let's put this all together. Consider the following complex loaded simple structure: a solid round bar with an axial load, bending load, and torsional load. This is a stationary structure (unlike rotating shafts). The bending load (F) is 100 pounds applied mid-span, the torque (T) is 100in-lb, and the axial load (P) is 1000 pounds. The structure is 30 inches long and 2.0 inches in diameter. A calculator may be required; although your math acumen by now may be so good, you won't need one!



Assumptions:

FBD:



Shear:

Moment

Determine reaction forces for the FBD above:

Identify the location of the greatest principal stress (greatest σ_1) and greatest shear stress (greatest τ_{max}). To determine this location, use your understanding of how stress varies in a structure such as this due to each of the three different loads:

How does stress vary in a uniaxial loaded part? Are there general areas or a specific location where you expect stress to be greatest?

How does stress vary in beam due to bending? Are there general areas or a specific location where you expect stress to be greatest?

How does stress vary in shafts due to torsion? Are there general areas or a specific location where you expect stress to be greatest?

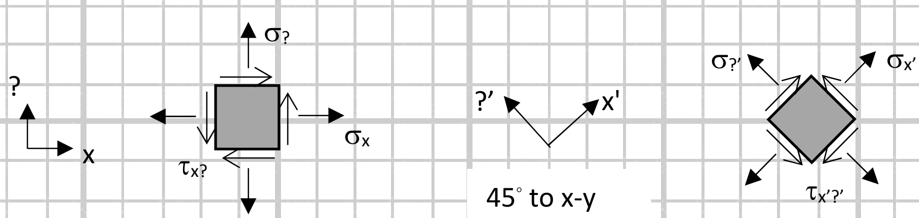
Where is the stress greatest in this part with all three loads combined?

Next, for the point you expect to have the highest stress, determine the stresses produced by the axial load, bending load, and torsion load. Then using superposition, determine the stresses at that point produced by the loads combined. Create Mohr's circles for the axial load, bending load, torsion load, and all loads combined (four circles). No coloring required.

Axial load:

Stress calculations

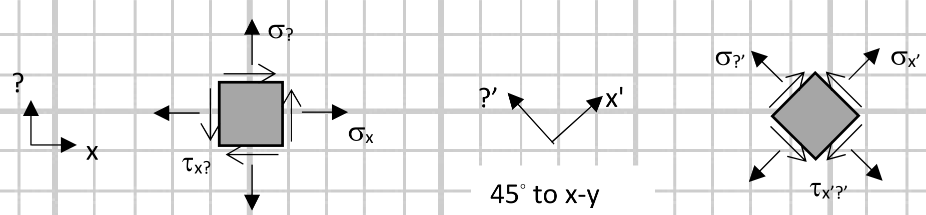
Mohr's circle:



$\sigma_x =$ $\sigma_? =$ $\tau_{x?} =$ $\sigma_{x'} =$ $\sigma_{?'} =$ $\tau_{x'?'} =$

Bending load:

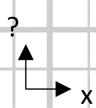
Stress calculations Mohr's circle:

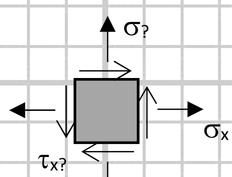


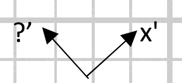
$\sigma_x =$ $\sigma_y =$ $\tau_{xy} =$ $\sigma_{x'} =$ $\sigma_{y'} =$ $\tau_{x'y'} =$

Torsion load:

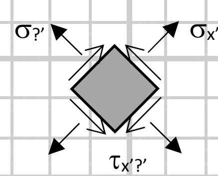
Stress calculations Mohr's circle:







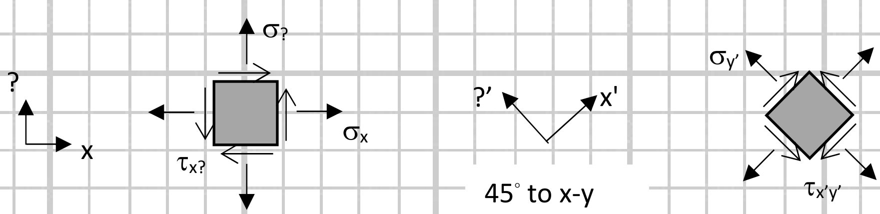
45° to x-y



$\sigma_x =$ $\sigma_y =$ $\tau_{xy} =$ $\sigma_{x'} =$ $\sigma_{y'} =$ $\tau_{x'y'} =$

All loads – determine the greatest stresses in this part.

Stress calculations (superposition) Mohr's circle:



σ_x = σ_y = τ_{xy} = $\sigma_{x'}$ = $\sigma_{y'}$ = $\tau_{x'y'}$ =

The End!