THE STRUT-AND-TIE MODEL

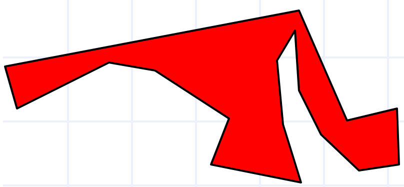
OF CONCRETE STRUCTURES

By

Dr. C. C. Fu, Ph.D., P.E,

The BEST Center

University of Maryland



Presented to

The Maryland State Highway Administration

August 21, 2001

Introduction

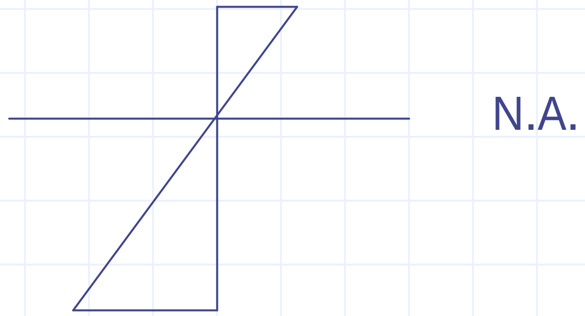
- The Strut-and-Tie is a unified approach that considers all load effects (*M, N, V, T*) simultaneously
- The Strut-and-Tie model approach evolves as one of the most useful design methods for shear critical structures and for other disturbed regions in concrete structures
- The model provides a rational approach by representing a complex structural member with an appropriate simplified truss models
- There is no single, unique STM for most design situations encountered. There are, however, some techniques and rules, which help the designer, develop an appropriate model

History and Specifications

- The subject was presented by Schlaich et al (1987) and also contained in the texts by Collins and Mitchell (1991) and MacGregor (1992)
- One form of the STM has been introduced in the new AASHTO LRFD Specifications (1994), which is its first appearance in a design specification in the US
- It will be included in ACI 318-02 Appendix A

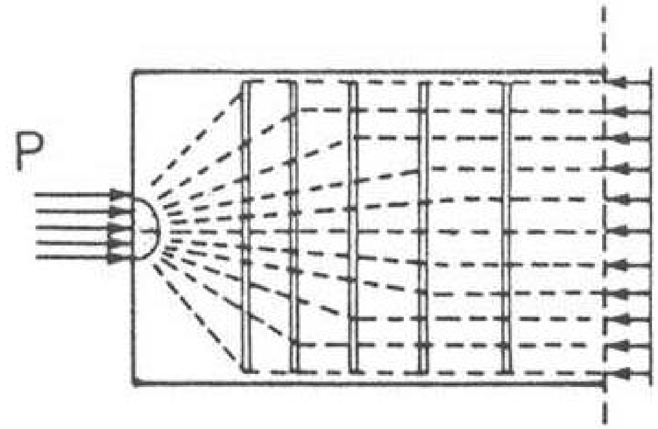
Bernoulli Hypothesis

- Bernoulli hypothesis states that: " Plane section remain plane after bending…"
- Bernoulli's hypothesis facilitates the flexural design of reinforced concrete structures by allowing a linear strain distribution for all loading stages, including ultimate flexural capacity

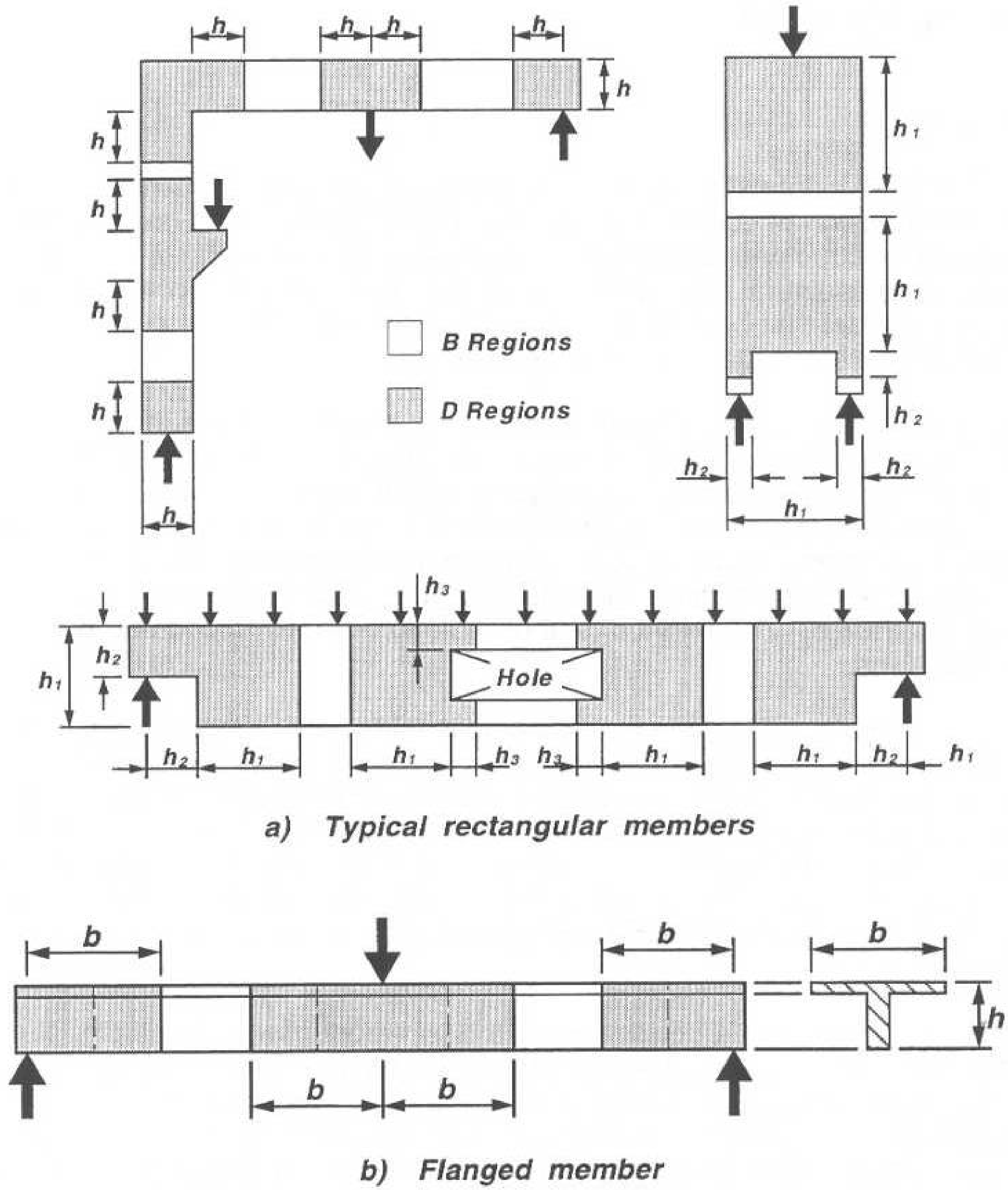


St. Venant's Principle

- St. Venant's Principle states that: " The localized effects caused by any load acting on the body will dissipate or smooth out within regions that are sufficiently away from the location of the load…"



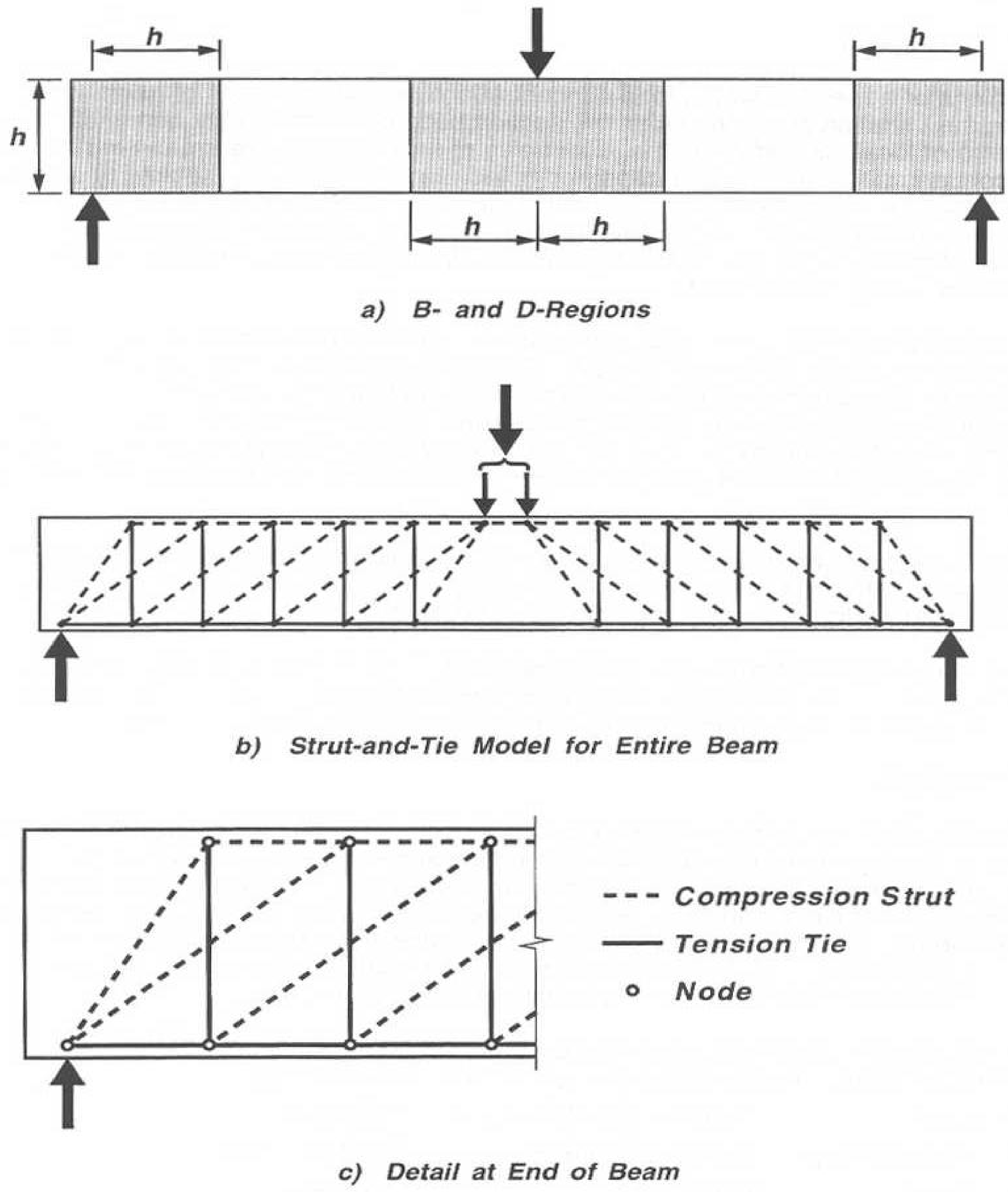
B- & D- Regions for Various Types of Members



Design of B & D Regions

- The design of B (Bernoulli or Beam) region is well understood and the entire flexural behavior can be predicted by simple calculation
- Even for the most recurrent cases of D (Disturbed or Discontinuity) regions (such as deep beams or corbels), engineers' ability to predict capacity is either poor (empirical) or requires substantial computation effort (finite element analysis) to reach an accurate estimation of capacity

STM for Simple Span Beam

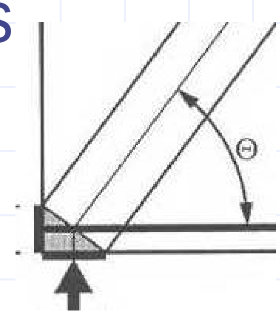


Feasible Inclined Angle q

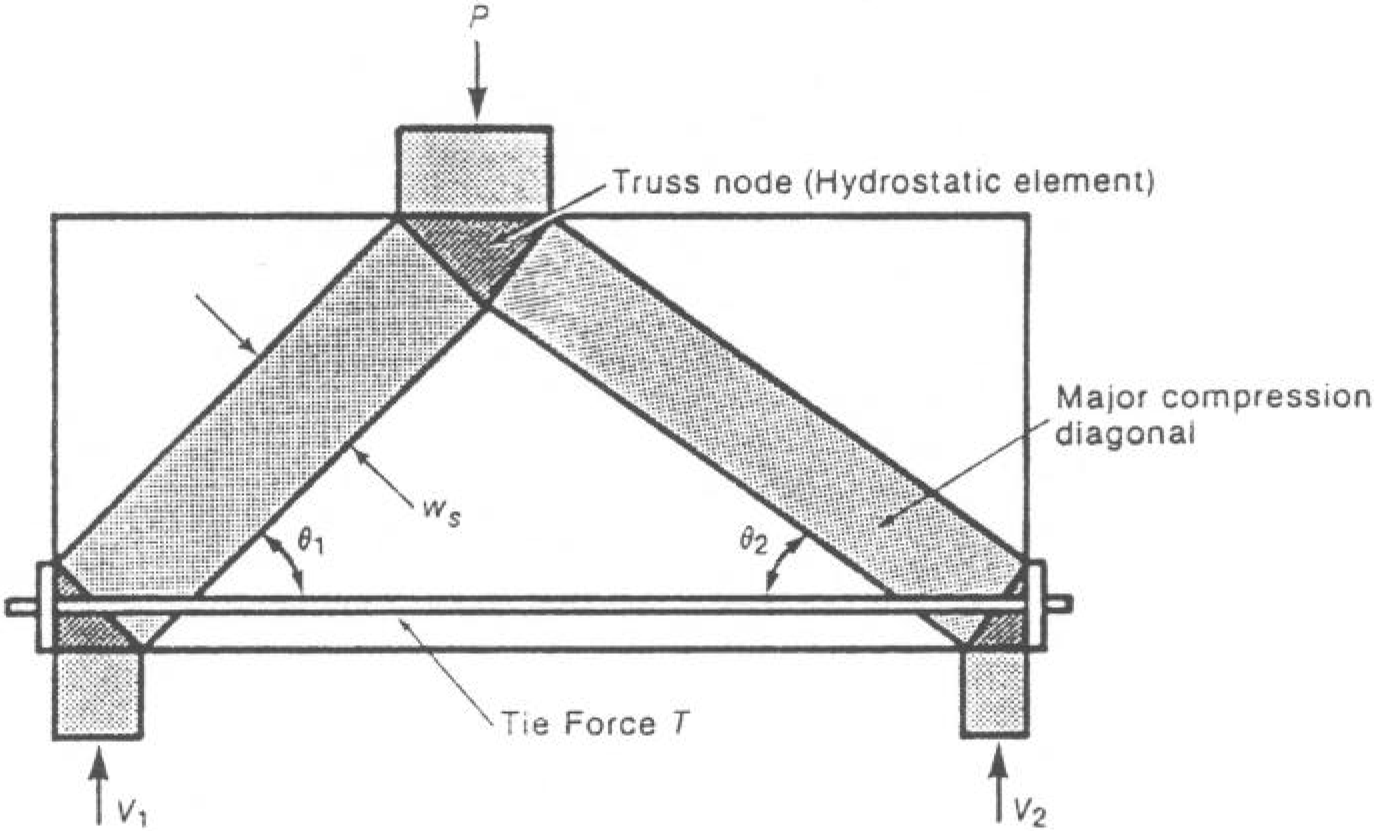
- Swiss Code: $0.5 \le \text{Cot}\ \theta \le 2.0$ ($\theta = 26°$ to $64°$)
- European Code: $3/5 \le \text{Cot}\ \theta \le 5/3$ ($\theta = 31°$ to $59°$)
- Collin's & Mitchells

  $\theta_{min} = 10 + 110(V_u/[\phi f_c' b_w jd])$ deg

  $\theta_{max} = 90 - \theta_{min}$ deg
- ACI 2002: $\theta_{min} = 25°$; ($25° \le \theta_{recom} \le 65°$ here)
- If small θ is assumed in the truss model, the compression strength of the inclined strut is decreased.

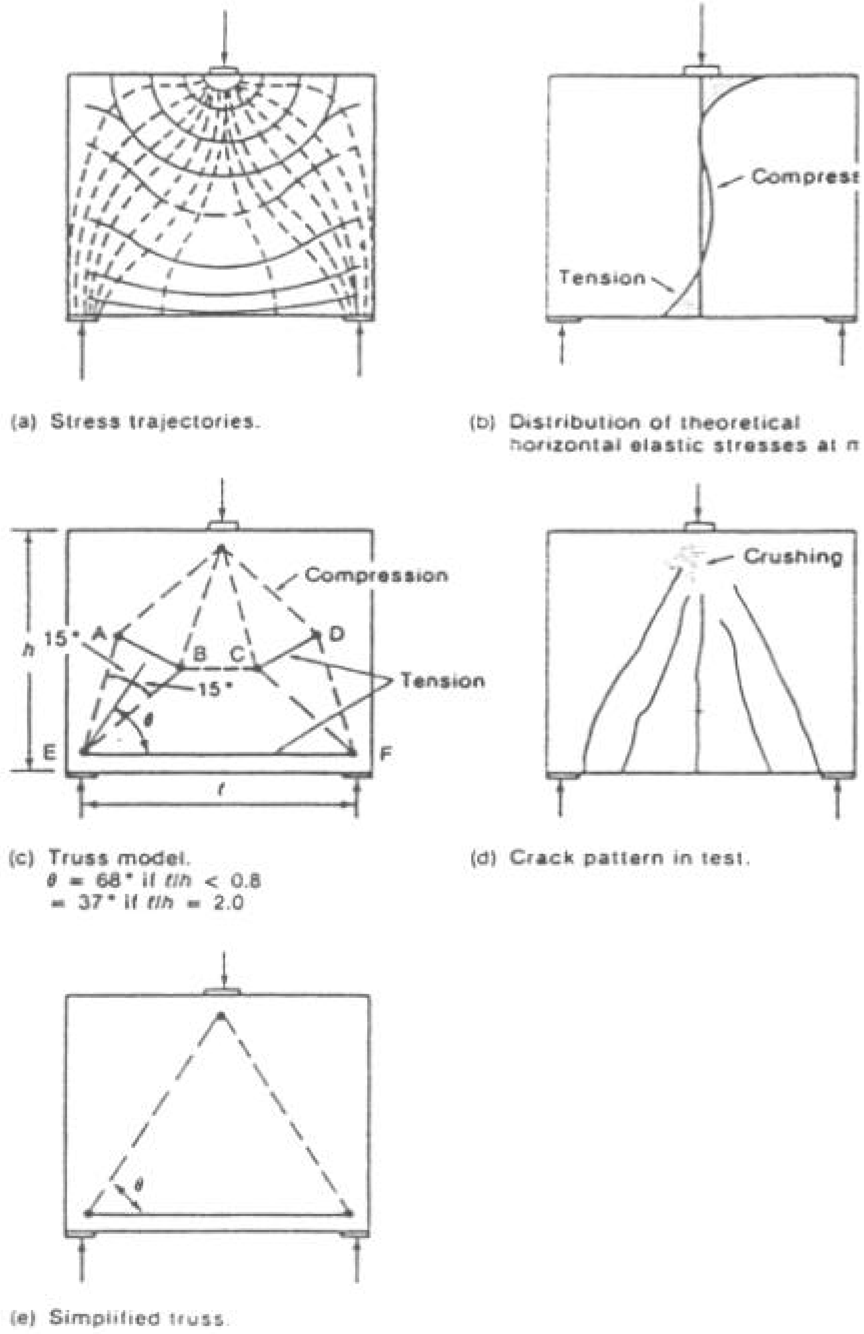


STM of a Deep Beam

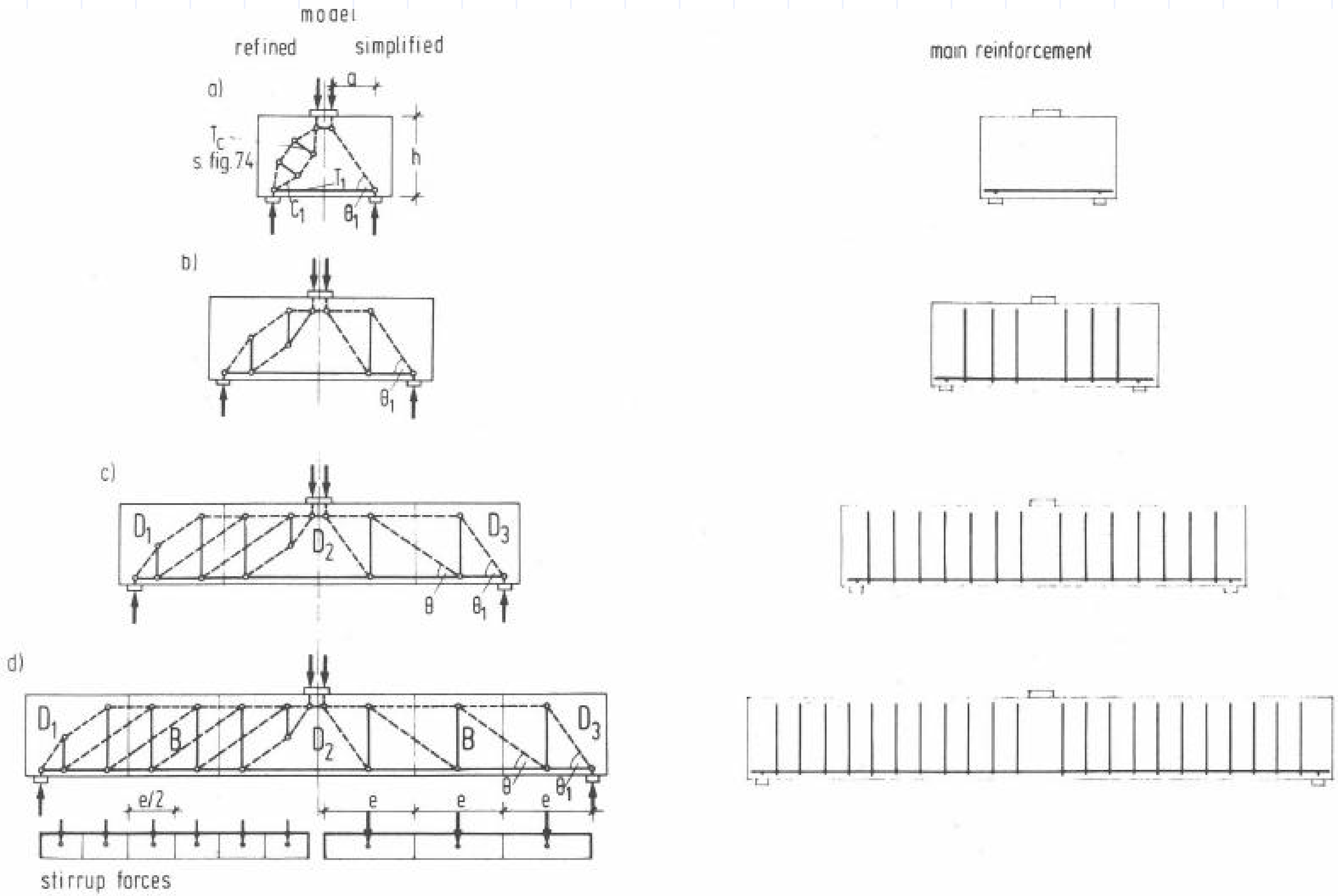


ACI Section 10.7.1 For Deep Beam:
L/d < 5/2 for continuous span; < 5/4 for simple span
ACI Section 11.8: L/d <5 (Shear requirement)

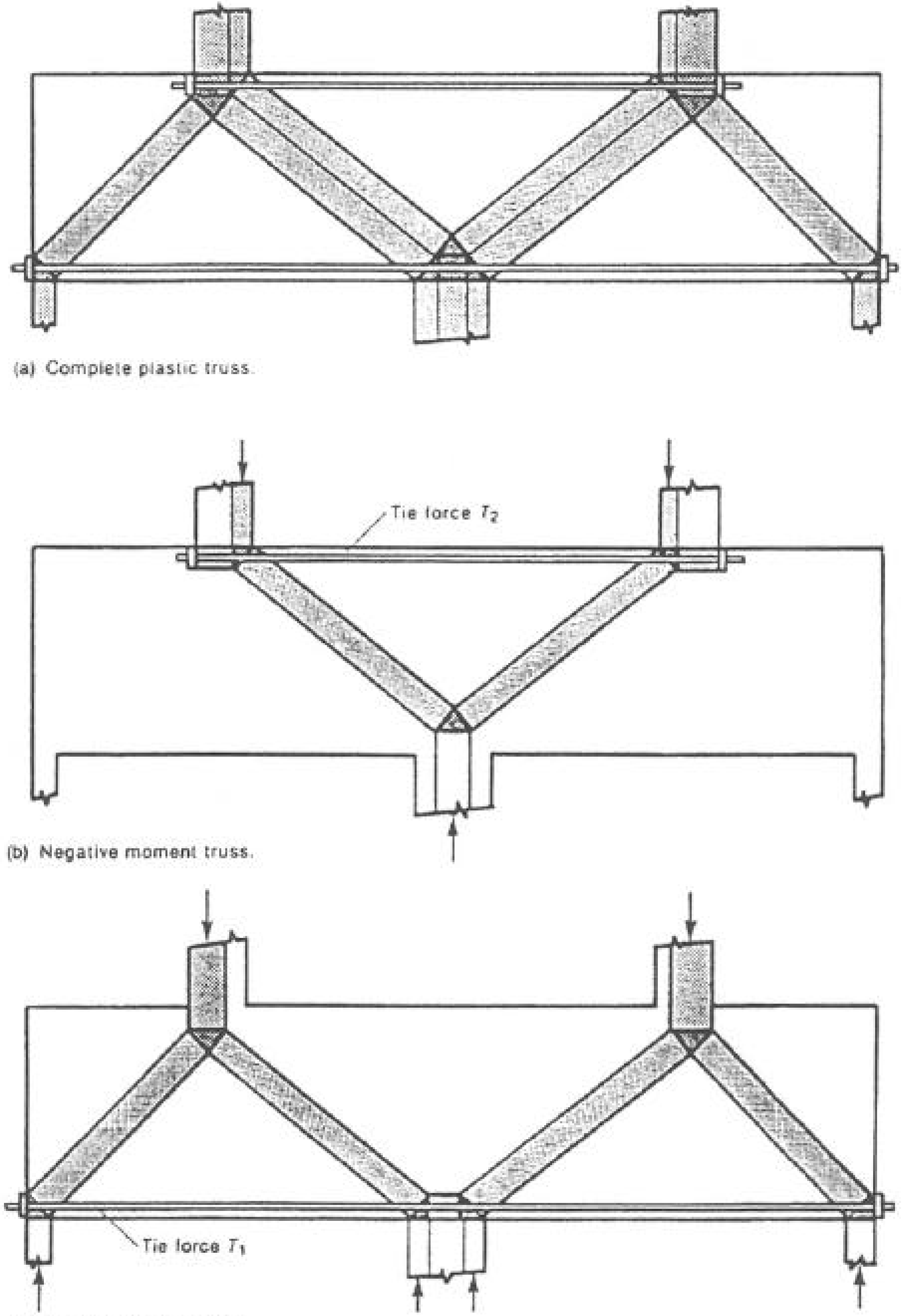
Deep Beam Stress and Its STM Model



Transition from Deep Beam to Beam



STM Model for a Two-span Continuous Beam



(a) Complete plastic truss.

(b) Negative moment truss.

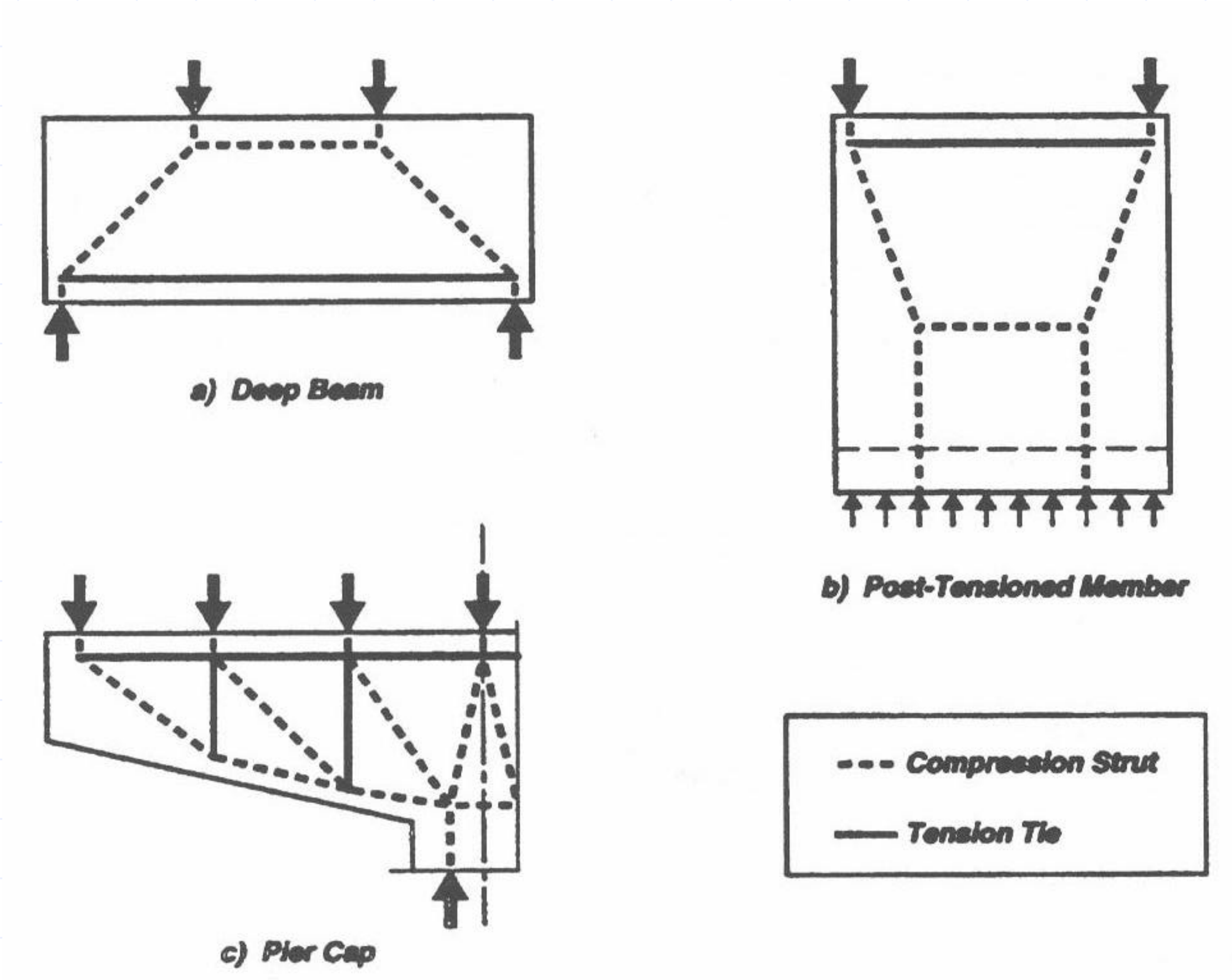
(c) Positive moment truss.

Basic Concepts

Strut-and-Tie Model: A conceptual framework where the stress distribution in a structure is idealized as a system of

Strut	Compression Member	Concrete
Tie or Stirrup	Tension Member	Reinforcement
Node	Connection	Concrete

Examples of STM Models



Strut Angle of STM Model

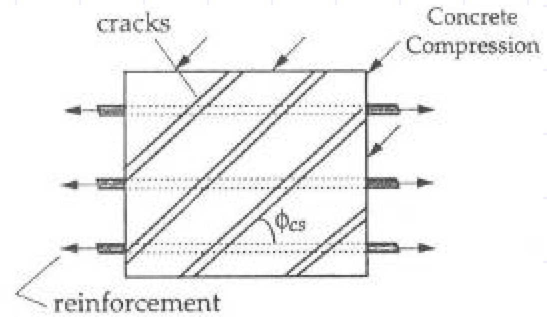
- A STM developed with struts parallel to the orientation of initial cracking will behave very well
- A truss formulated in this manner also will make the most efficient use of the concrete because the ultimate mechanism does not require reorientation of the struts

Lower Bound Theorem of Plasticity

- A stress field that satisfies equilibrium and does not violate yield criteria at any point provides a lower-bound estimate of capacity of elastic-perfectly plastic materials
- For this to be true, crushing of concrete (struts and nodes) does not occur prior to yielding of reinforcement (ties or stirrups)

Limitation of The Truss Analogy

- The theoretical basis of the truss analogy is the lower bound theorem of plasticity
- However, concrete has a limited capacity to sustain plastic deformation and is not an elastic-perfectly plastic material
- AASHTO LRFD Specifications adopted the compression theory to limit the compressive stress for struts with the consideration of the condition of the compressed concrete at ultimate



Prerequisites

- Equilibrium must be maintained
- Tension in concrete is neglected
- Forces in struts and ties are uni-axial
- External forces apply at nodes
- Prestressing is treated as a load
- Detailing for adequate anchorage

Problems in STM Applications

1. How to construct a Strut-and-Tie model?
2. If a truss can be formulated, is it adequate or is there a better one?
3. If there are two or more trusses for the same structure, which one is better?

Struts

A. Compression struts fulfill two functions in the STM:
   1. They serve as the compression chord of the truss mechanism which resists moment
   2. They serve as the diagonal struts which transfer shear to the supports

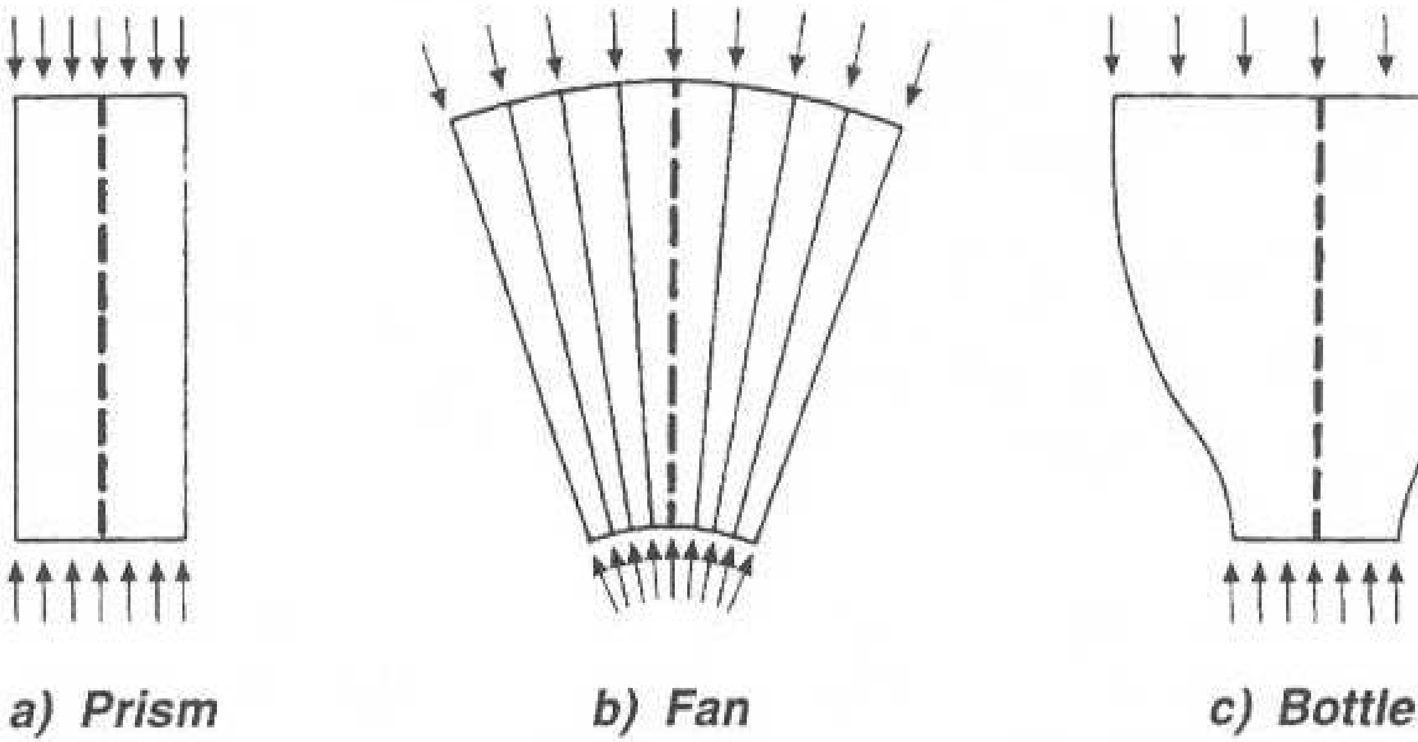
B. Diagonal struts are generally oriented parallel to the expected axis of cracking

Types of Struts

There are three types of struts that will be discussed:

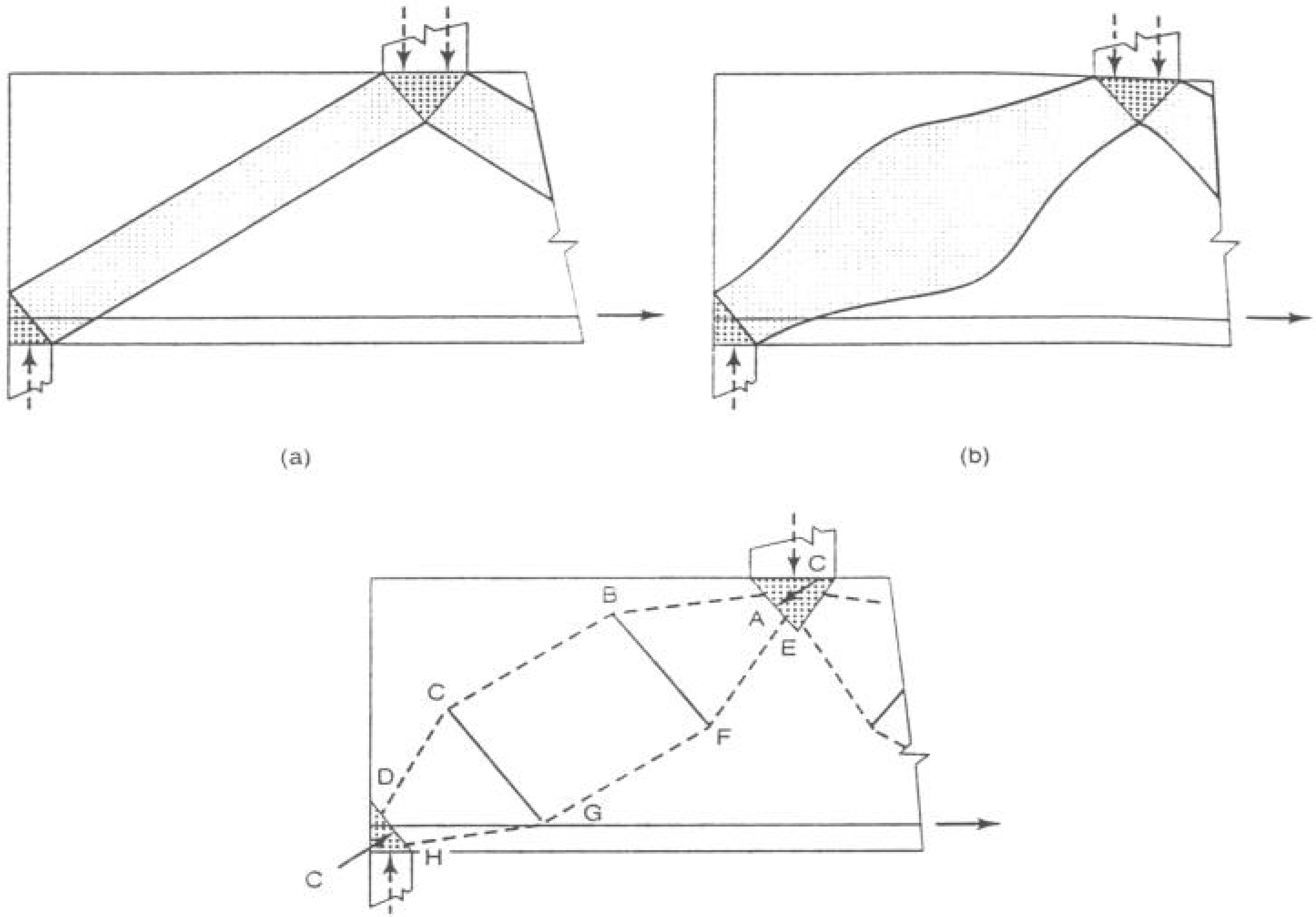
1. The simplest type is the “prism” which has a constant width
2. The second form is the “bottle” in which the strut expands or contracts along its length
3. The final type is the “fan” where an array of struts with varying inclination meet at or radiate from a single node

Three Types of Struts



Three Types of Struts (Adapted from Schlaich et al 1987)

Compression Struts



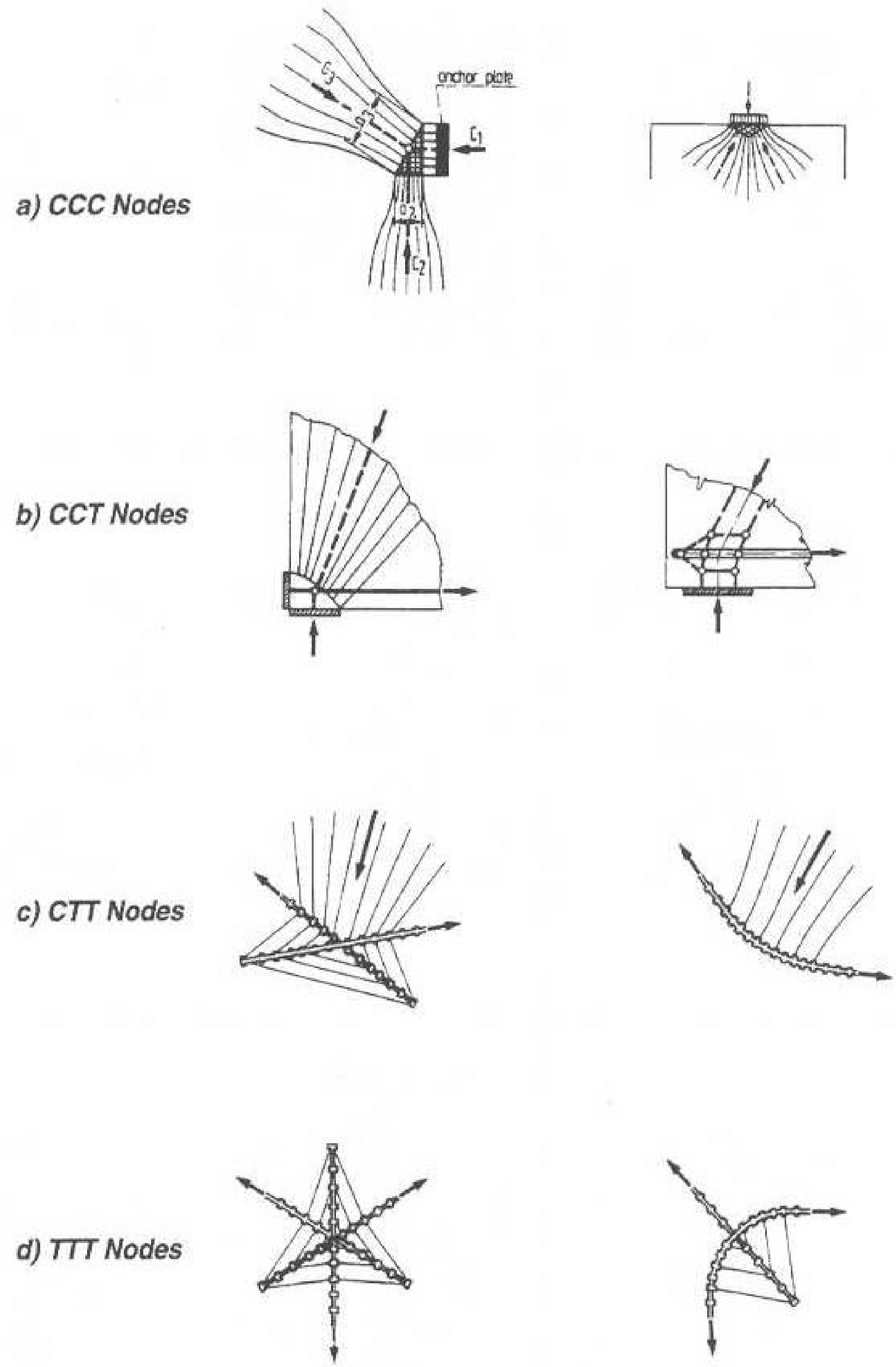
Ties

- Tensions ties include stirrups, longitudinal (tension chord) reinforcement, and any special detail reinforcement
- A critical consideration in the detailing of the STM is the provision of adequate anchorage for the reinforcement
- If adequate development is not provided, a brittle anchorage failure would be likely at a load below the anticipated ultimate capacity

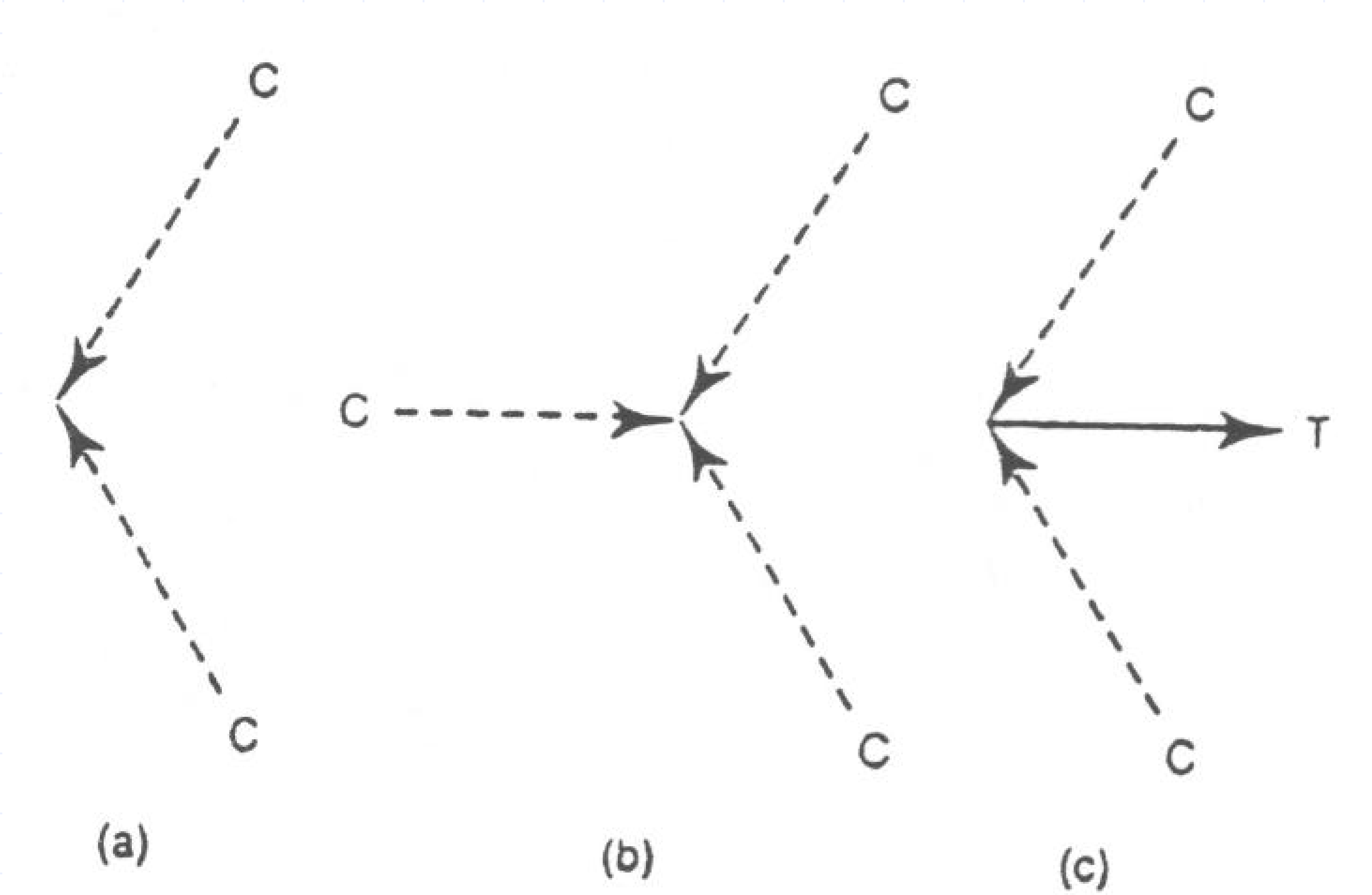
Nodes

- Nodes are the connections of the STM, i.e., the locations at which struts and ties converge
- Another way of describing a node is the location at which forces are redirected within a STM

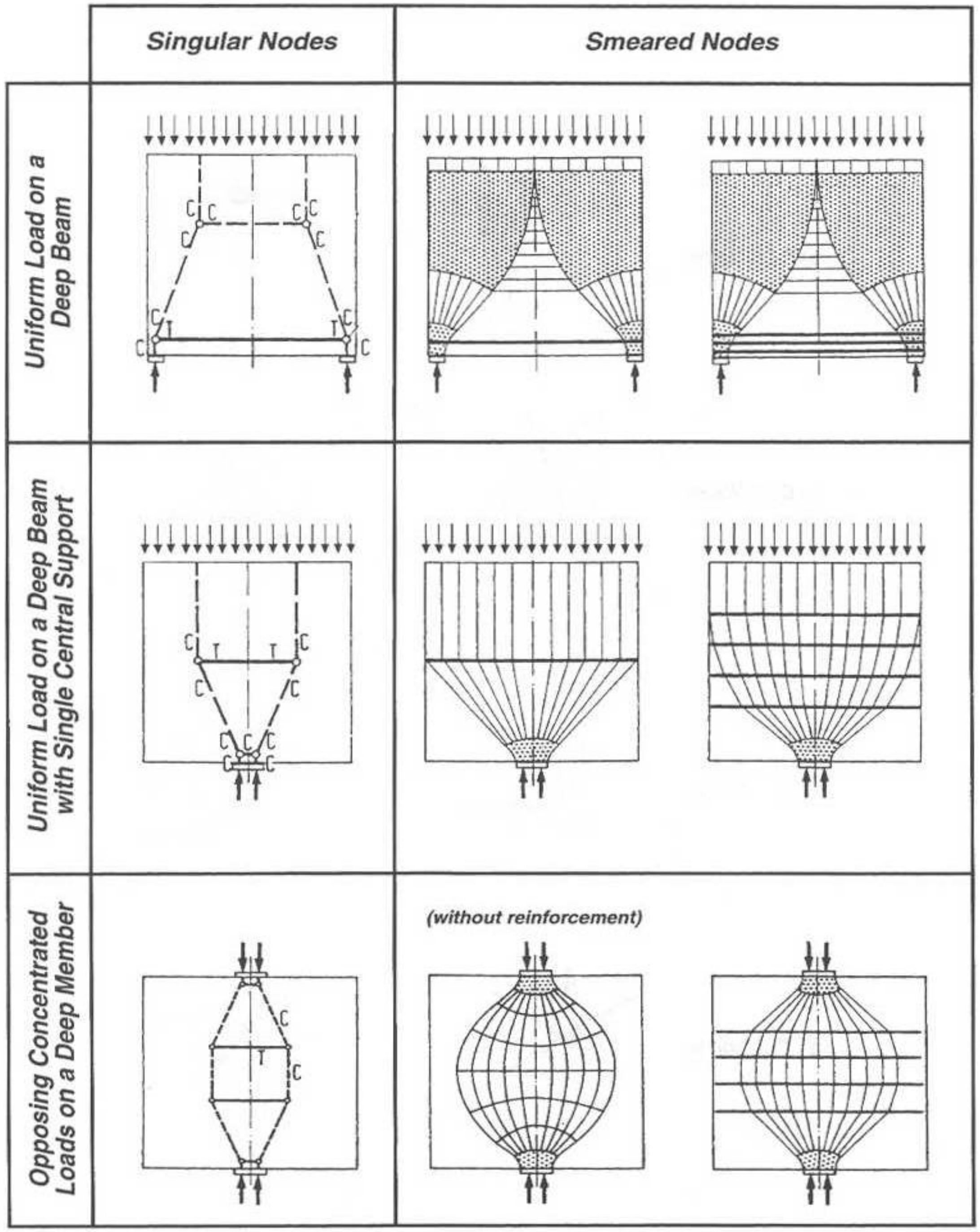
Type of Singular Nodes (Schlaich et al 1987)



Idealized Forces at Nodal Zones



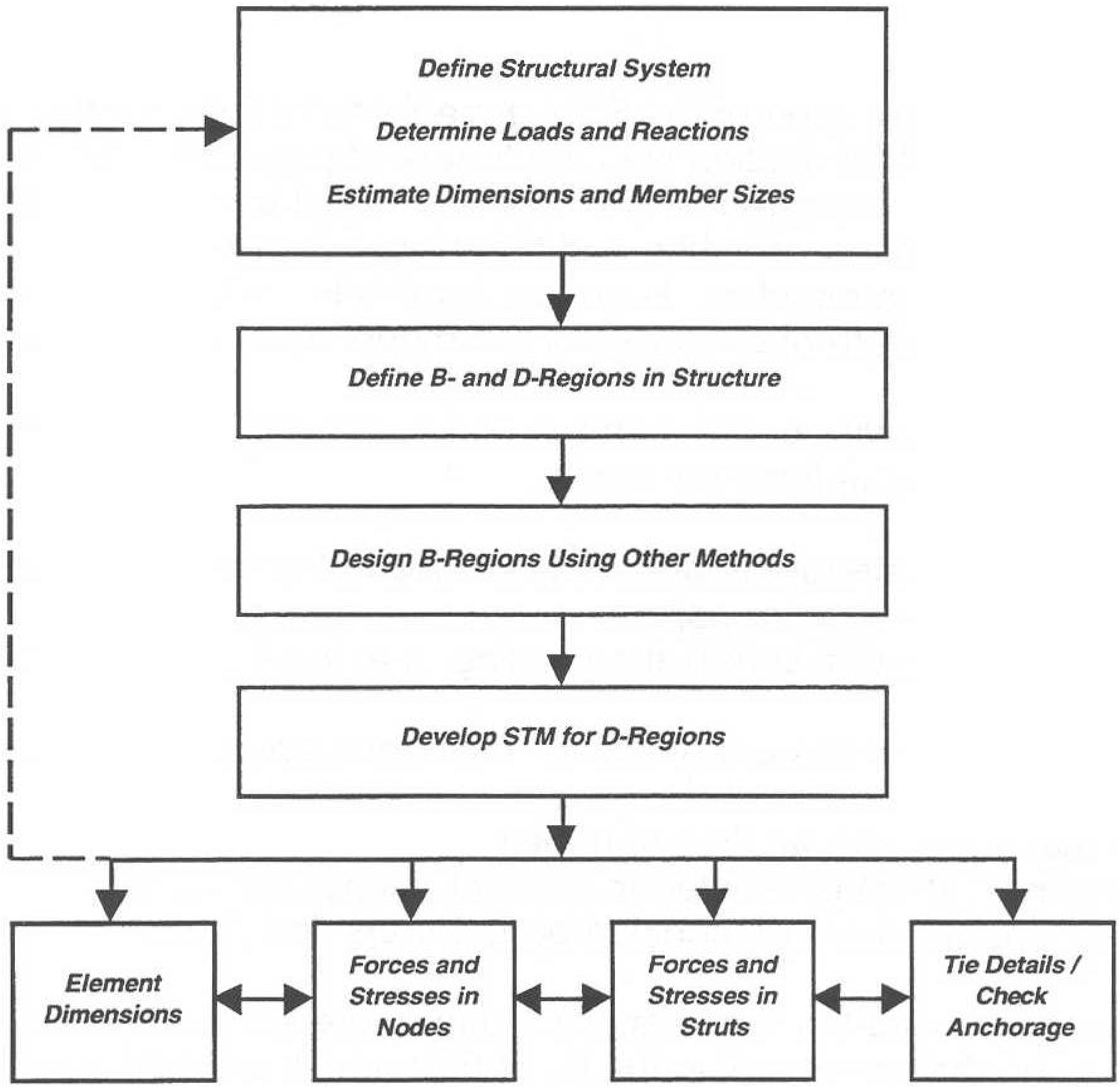
Singular and Smeared Nodes



STM Model Design Concept

- The successful use of the STM requires an understanding of basic member behavior and informed engineering judgment
- In reality, there is almost an art to the appropriate use of this technique
- The STM is definitely a design tool for thinking engineers, not a cookbook analysis procedure
- The process of developing an STM for a member is basically an iterative, graphical procedure

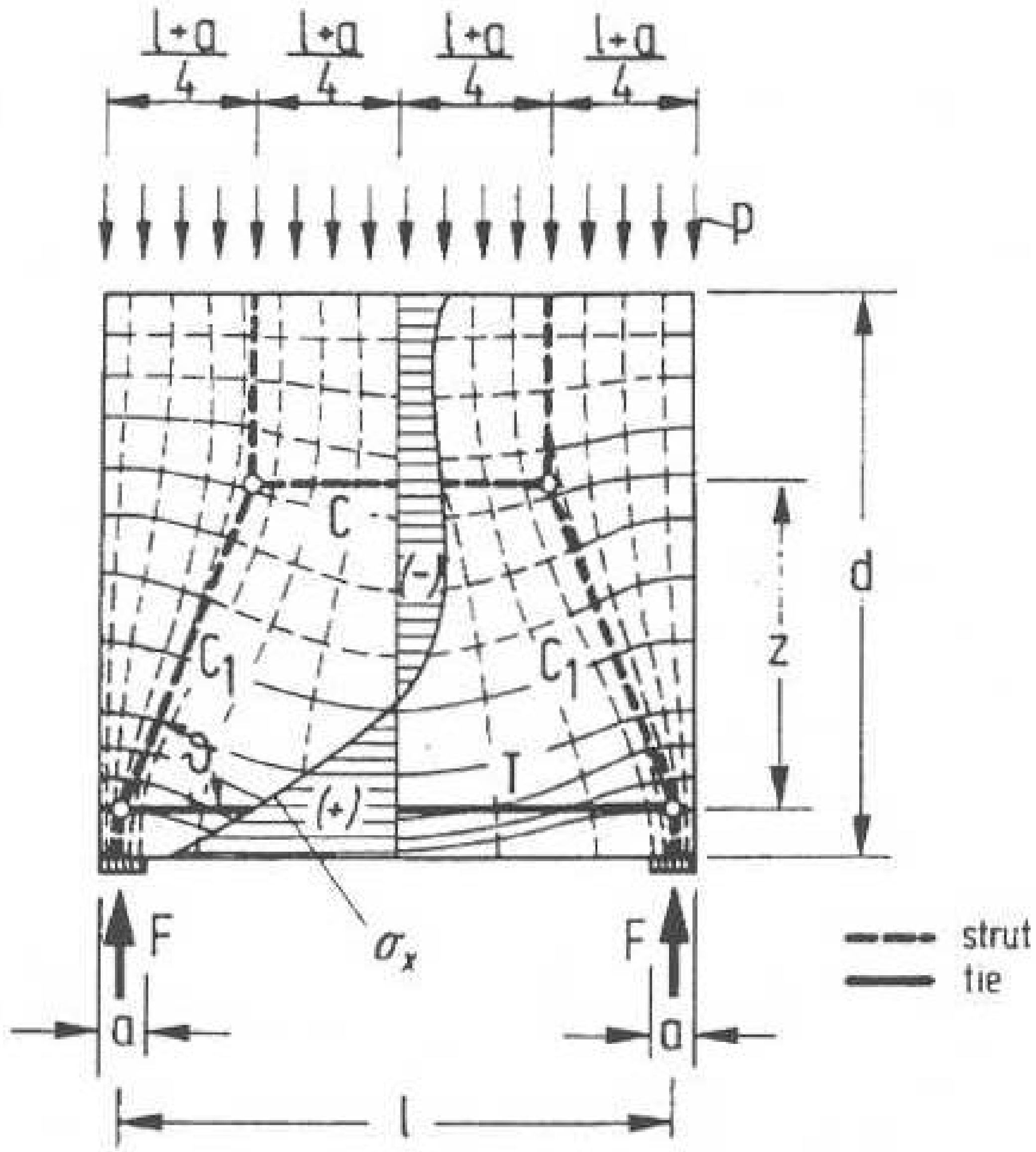
STM Model Design Flow Chart



Methods for Formulating STM Model

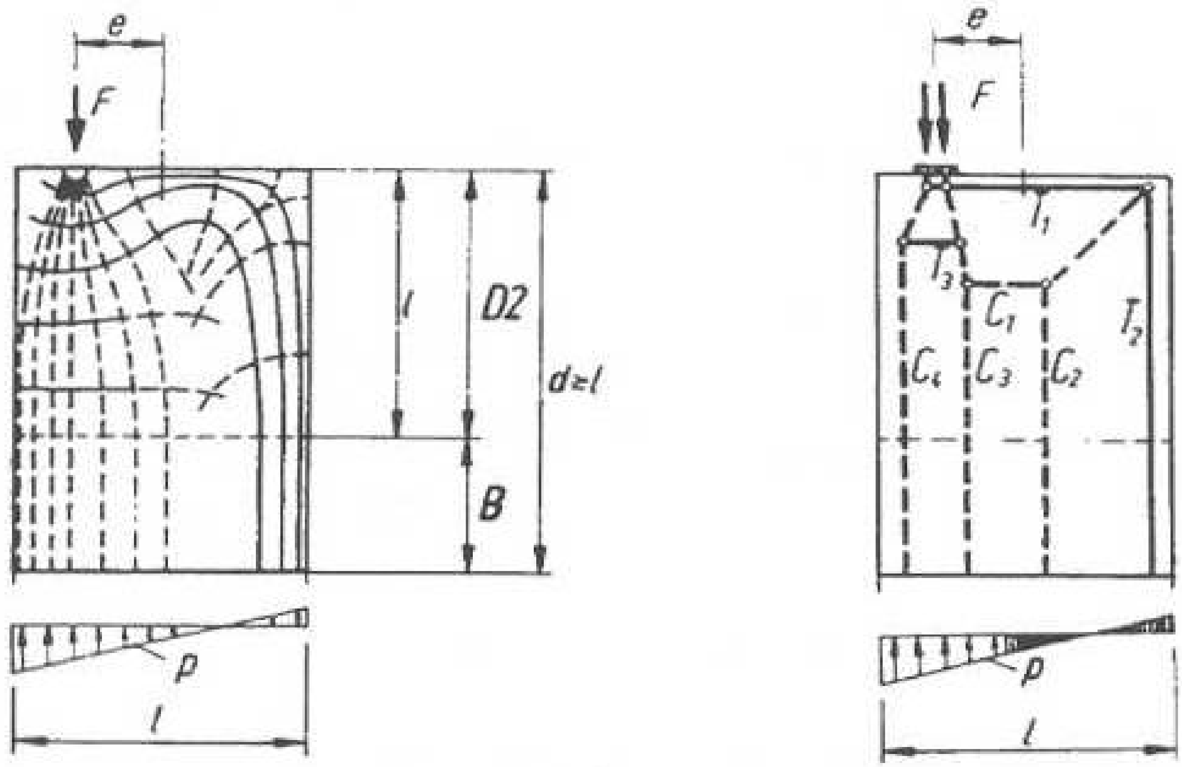
- Elastic Analysis based on Stress Trajectories
- Load Path Approach
- Standard Model

Elastic Analysis for the STM Model A



a) Uniform Load on a Deep Beam

Elastic Analysis for the STM Models B & C

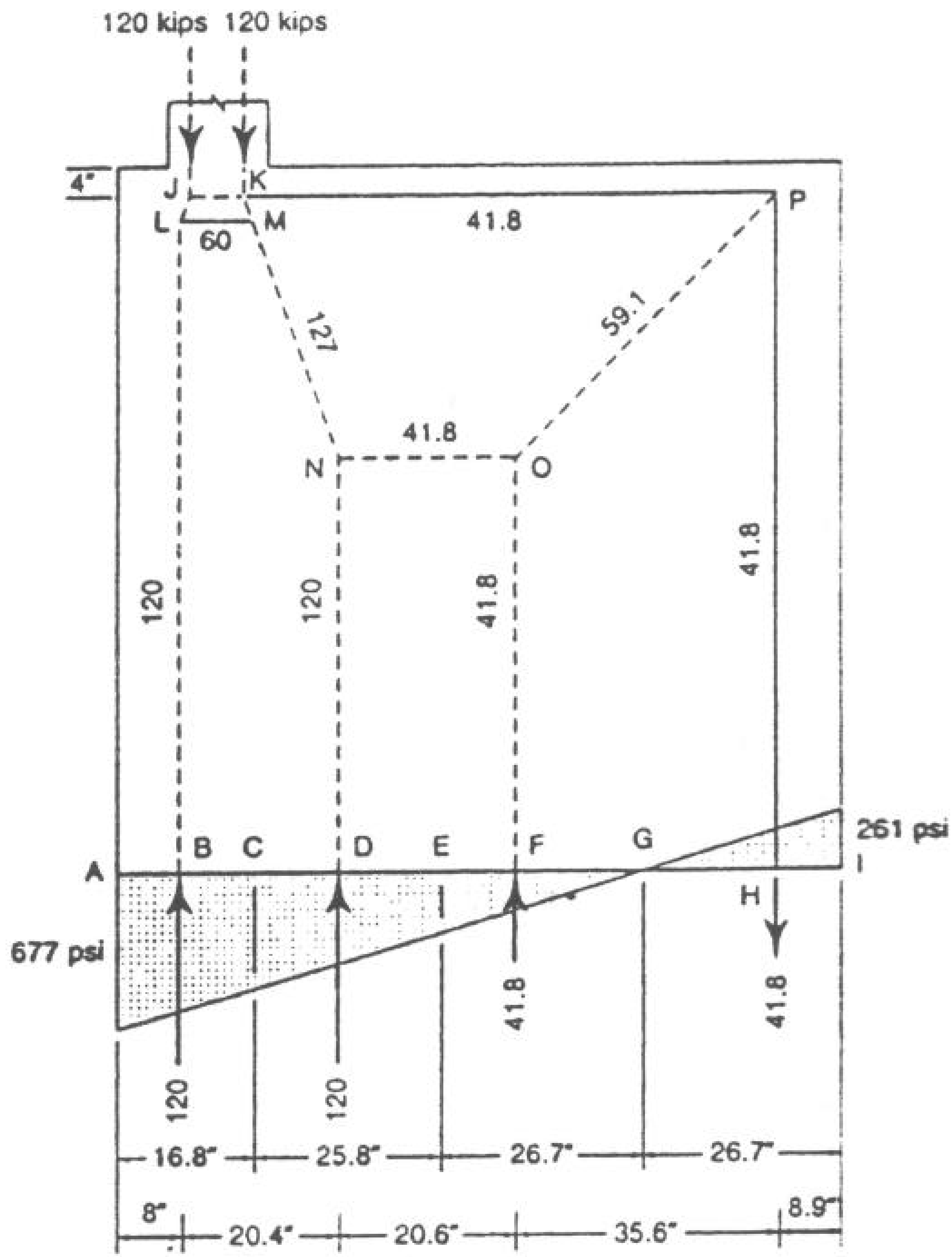


b) Eccentric Concentrated Load on Long Member

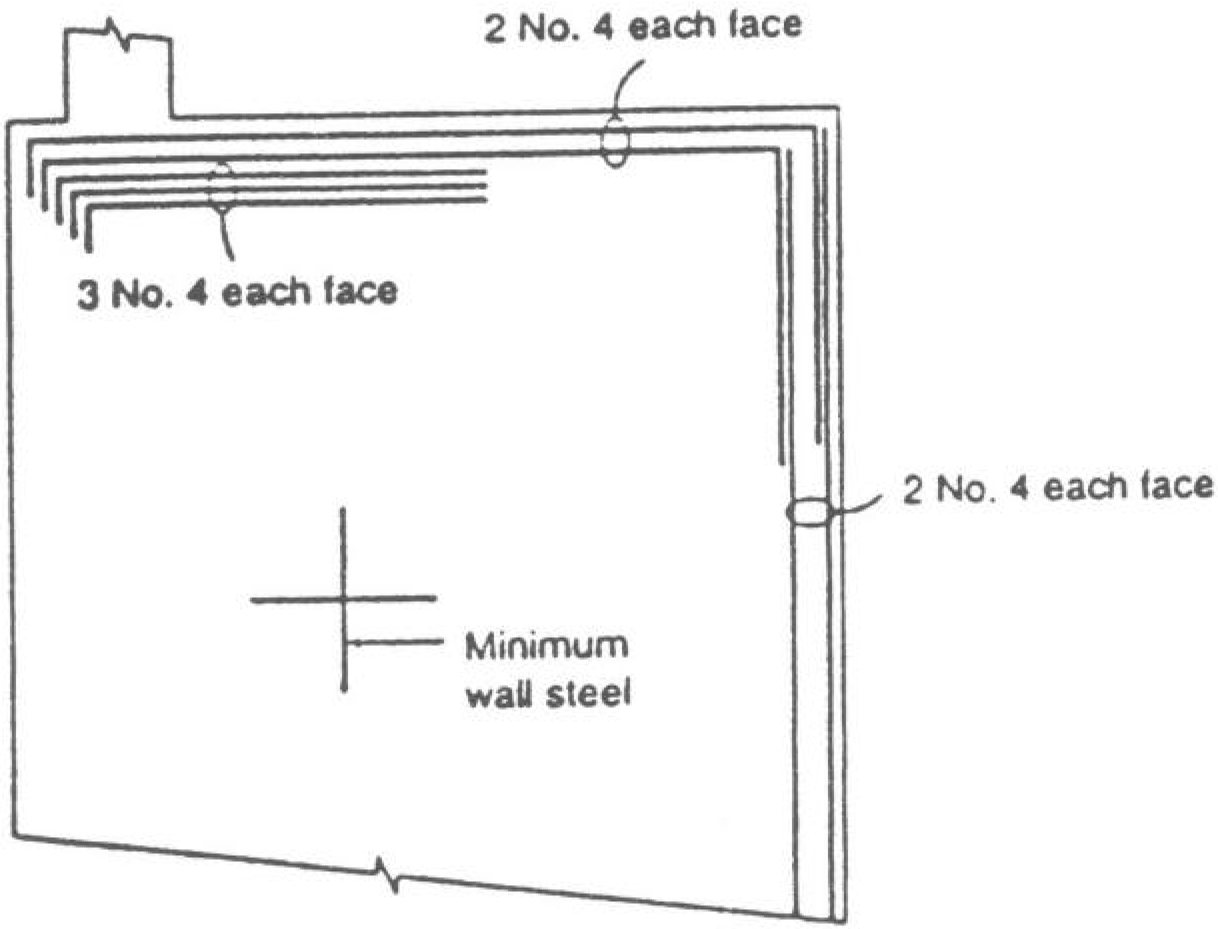
Elastic Analysis Approach Procedures

1. Isolate D-regions
2. Complete the internal stresses on the boundaries of the element
3. Subdivide the boundary and compute the force resultants on each sub-length
4. Draw a truss to transmit the forces from boundary to boundary of the D-region
5. Check the stresses in the individual members in the truss

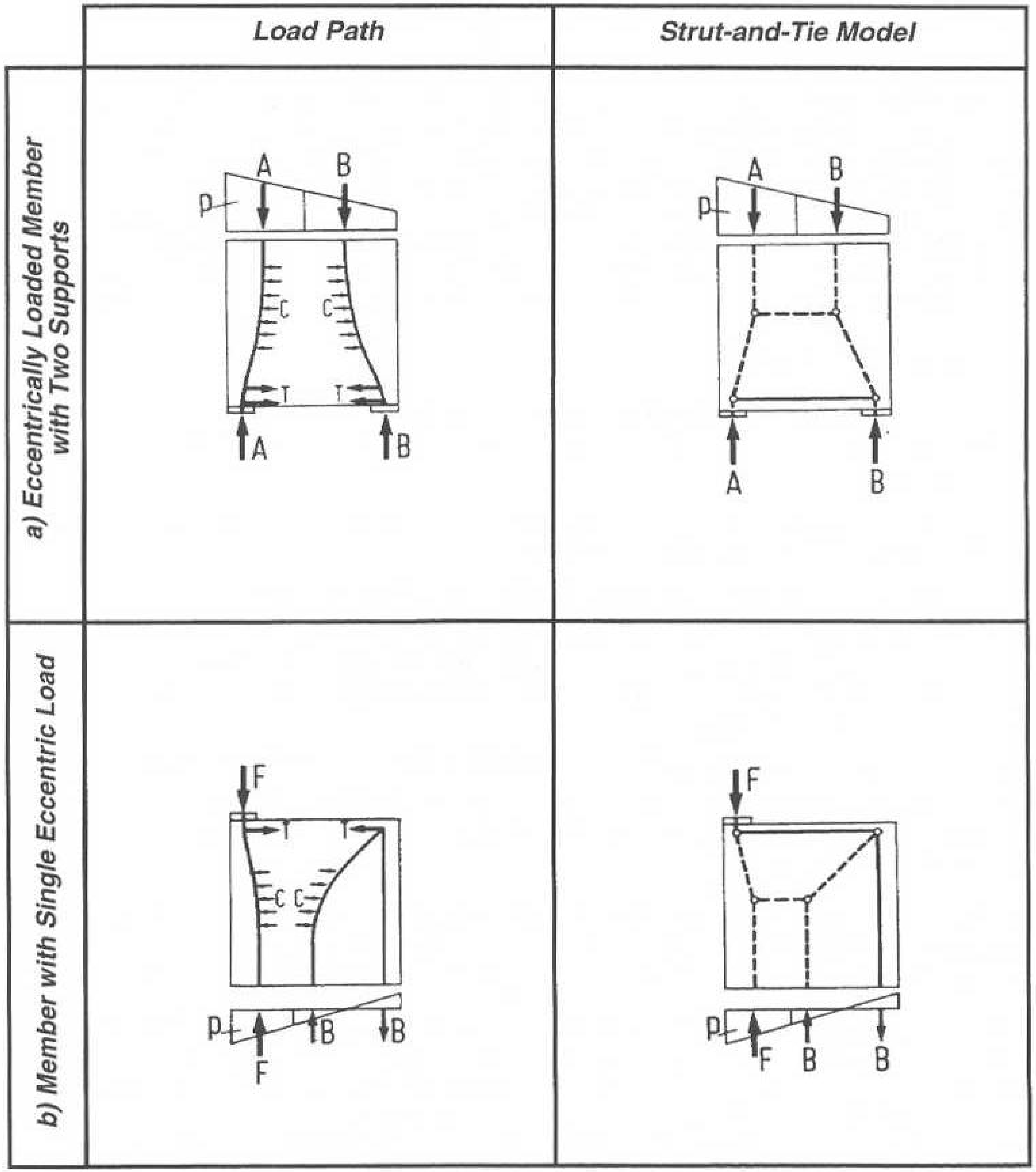
STM Model C Example using Elastic Analysis



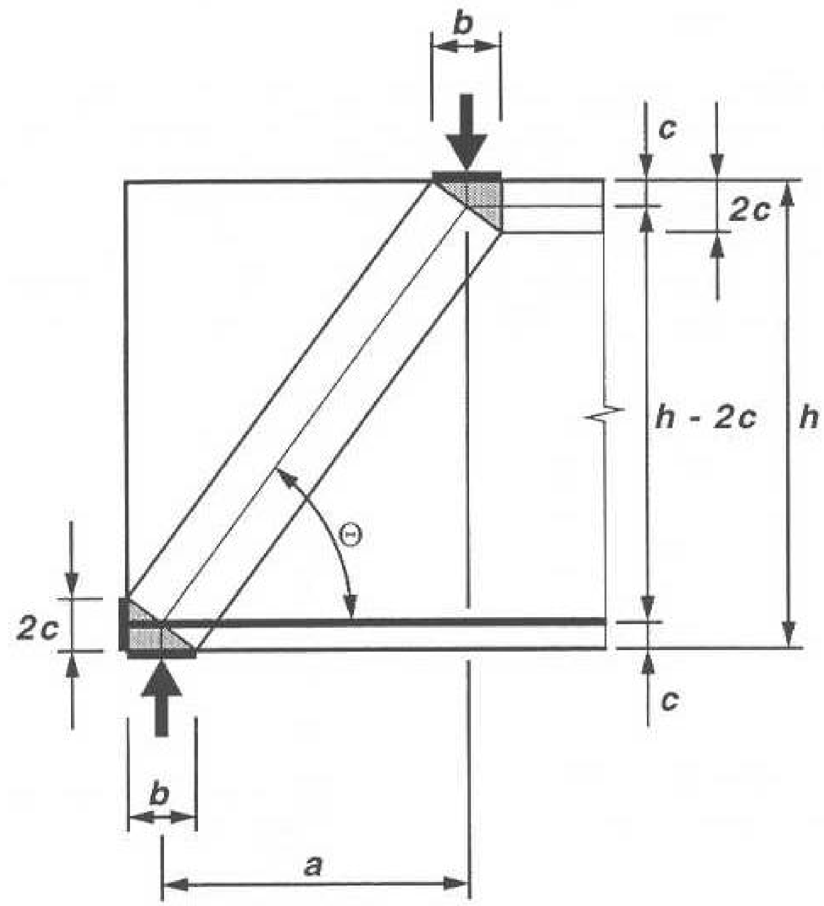
STM Model C Example Reinforcement



Load Path Approach (Schlaich et al. 1987)

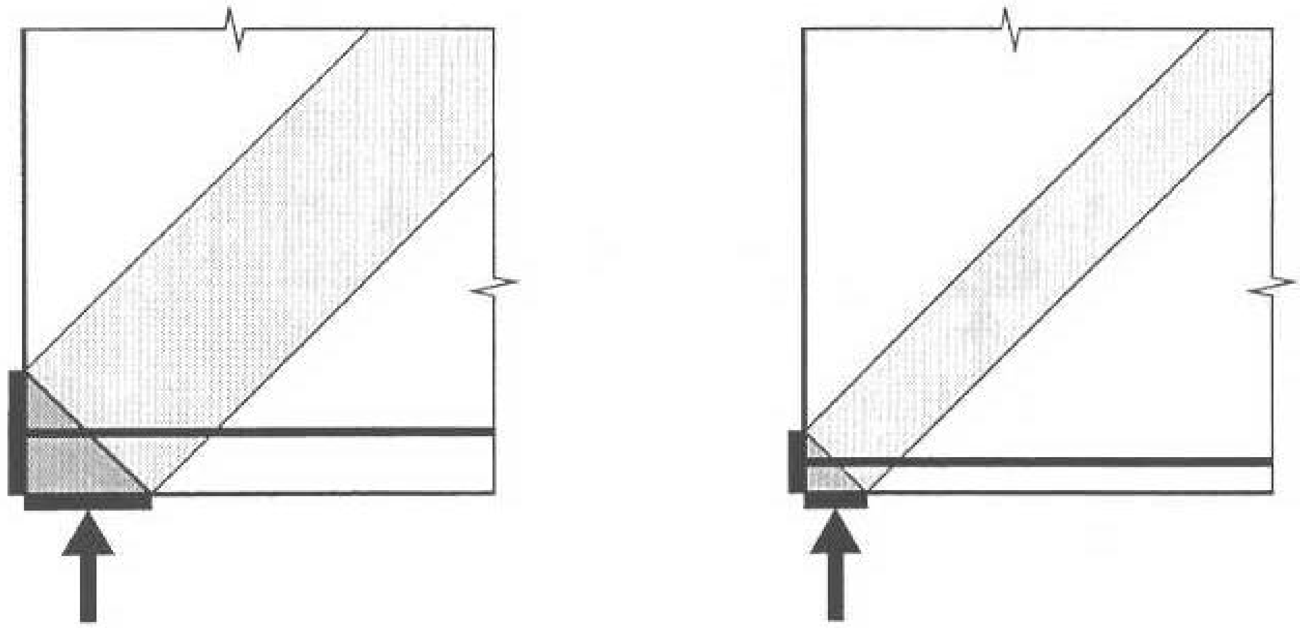


Example of Determining STM Model Geometry



By similar triangles: $\tan\theta = \frac{h-2c}{a} = \frac{b}{2c}$

Factors Affecting Size of Compression Strut



- Location and distribution of reinforcement (tie) and its anchorage
- Size and location of bearing

Nodal Zones

- These dimensions are determined for each element using

  (1) the geometry of the member and the STM,

  (2) the size of bearings,

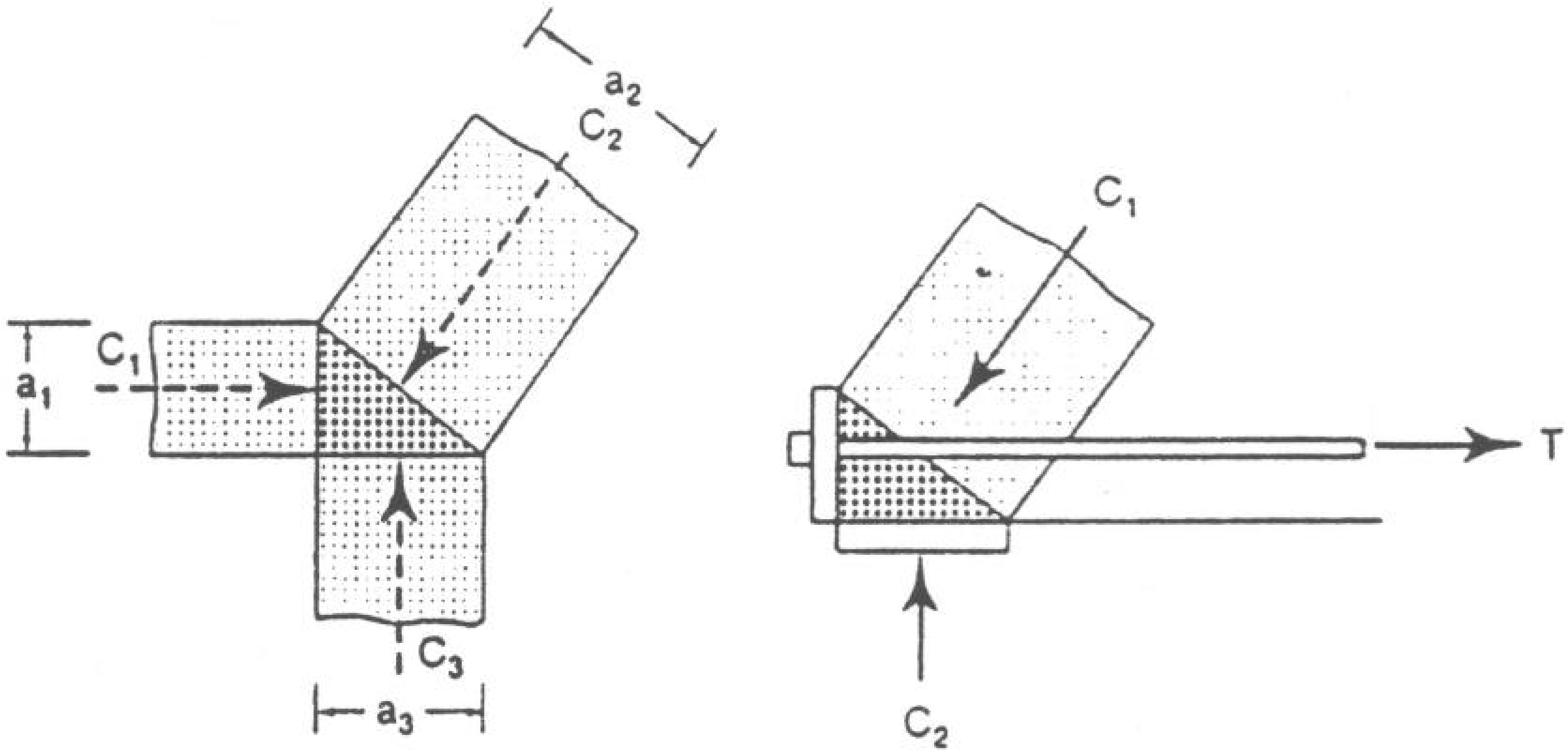
  (3) the size of loaded areas,

  (4) the location and distribution of reinforcement, and

  (5) the size of tendon anchorages, if any

- Struts and ties should be dimensioned so that the stresses within nodes are hydrostatic, i.e., the stress on each face of the node should be the same

Hydrostatic Nodal Zones



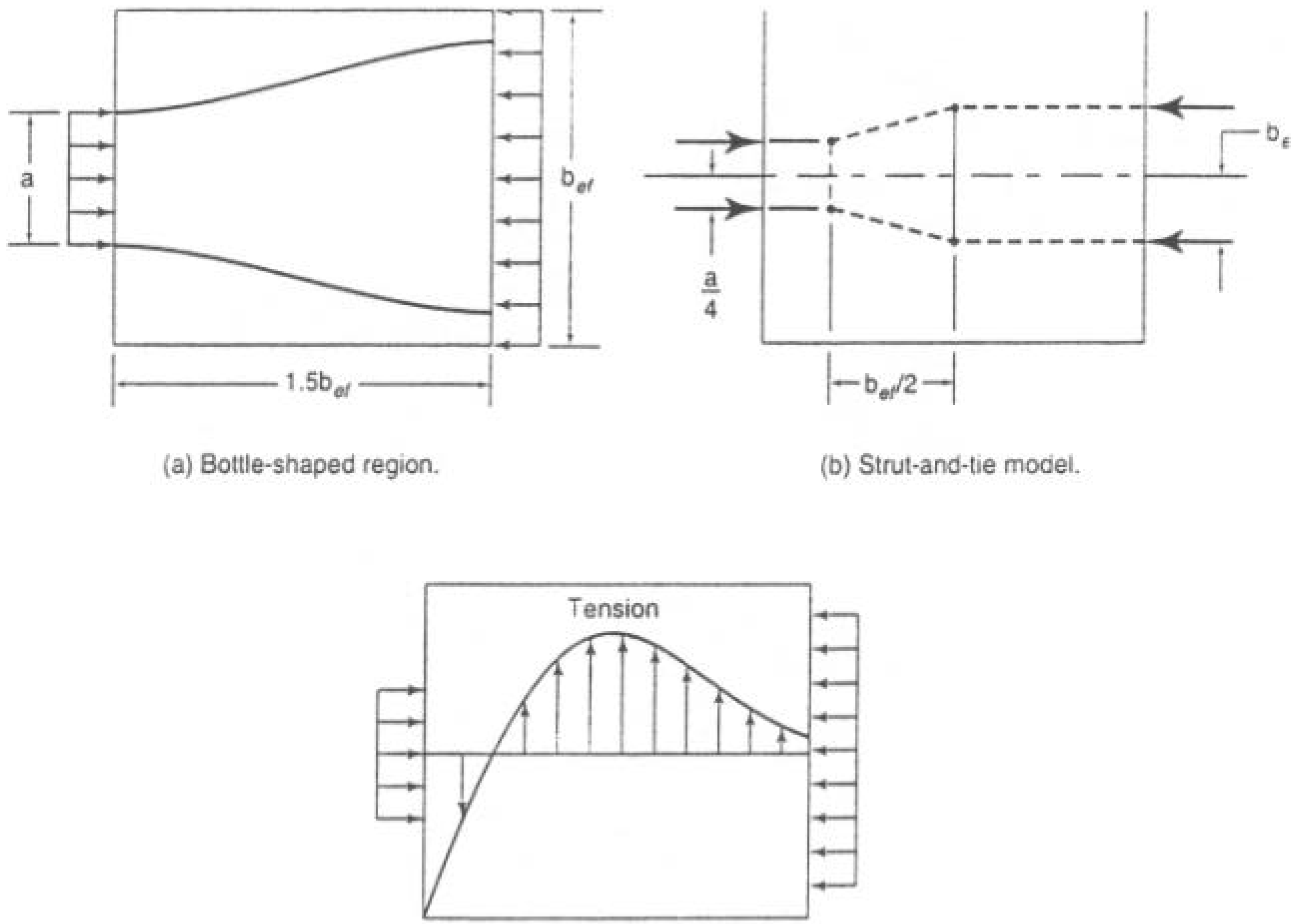
(a) CCC Node

(b) CCT Node

Cracking of Compression Strut

$b_{ef} = a + l/6$

$T = C(1 - a/b_{ef})/4$

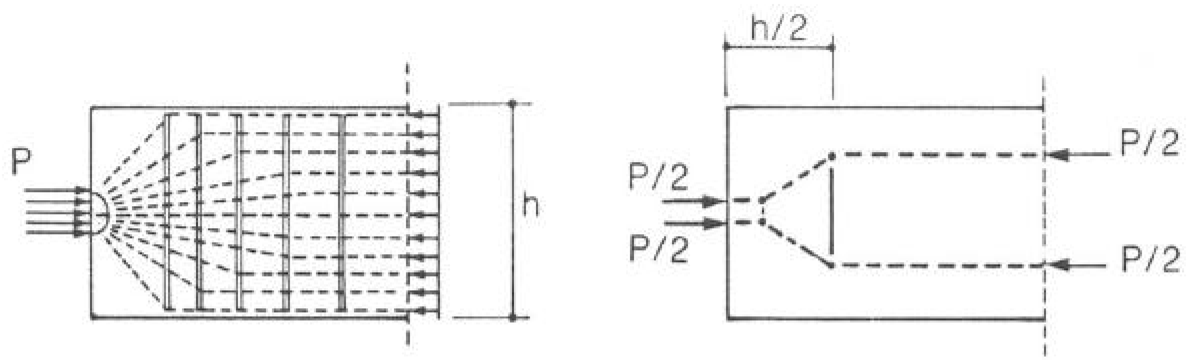


(a) Bottle-shaped region.

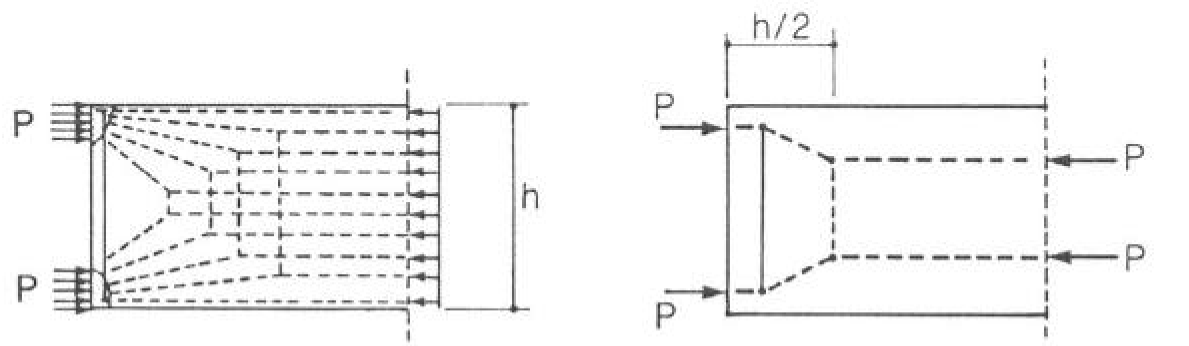
(b) Strut-and-tie model.

(c) Transverse tensions and compressions.

STM Models A & B for Anchorage Zones

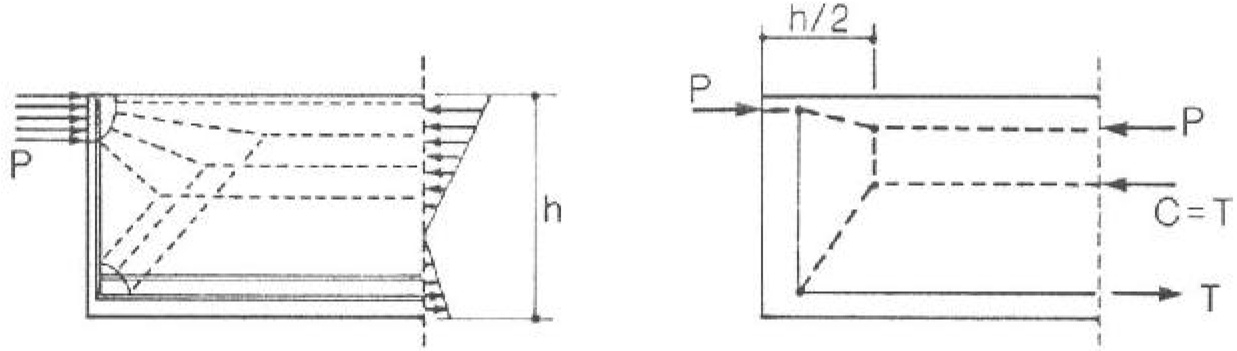


(a) Centrally located bearing plate

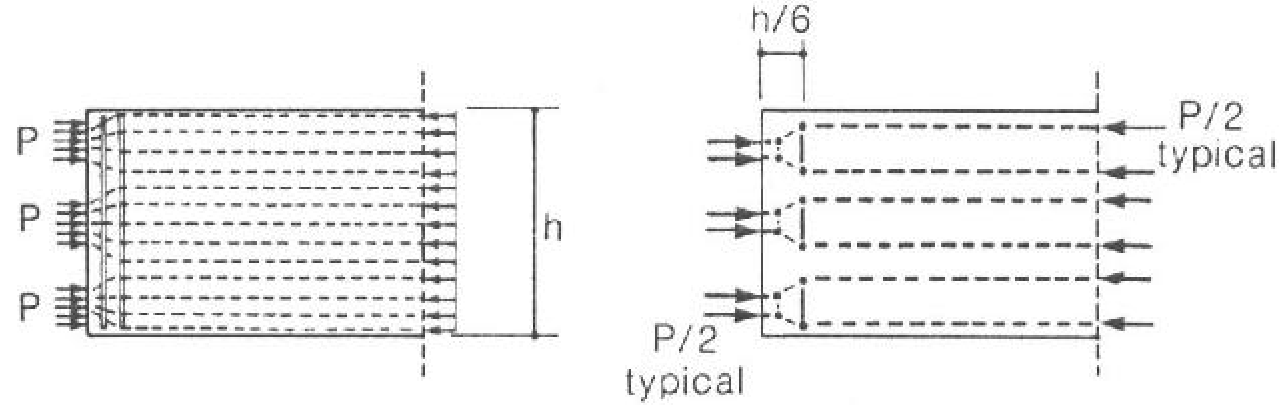


(b) Bearing plates at top and bottom

STM Models C & D for Anchorage Zones

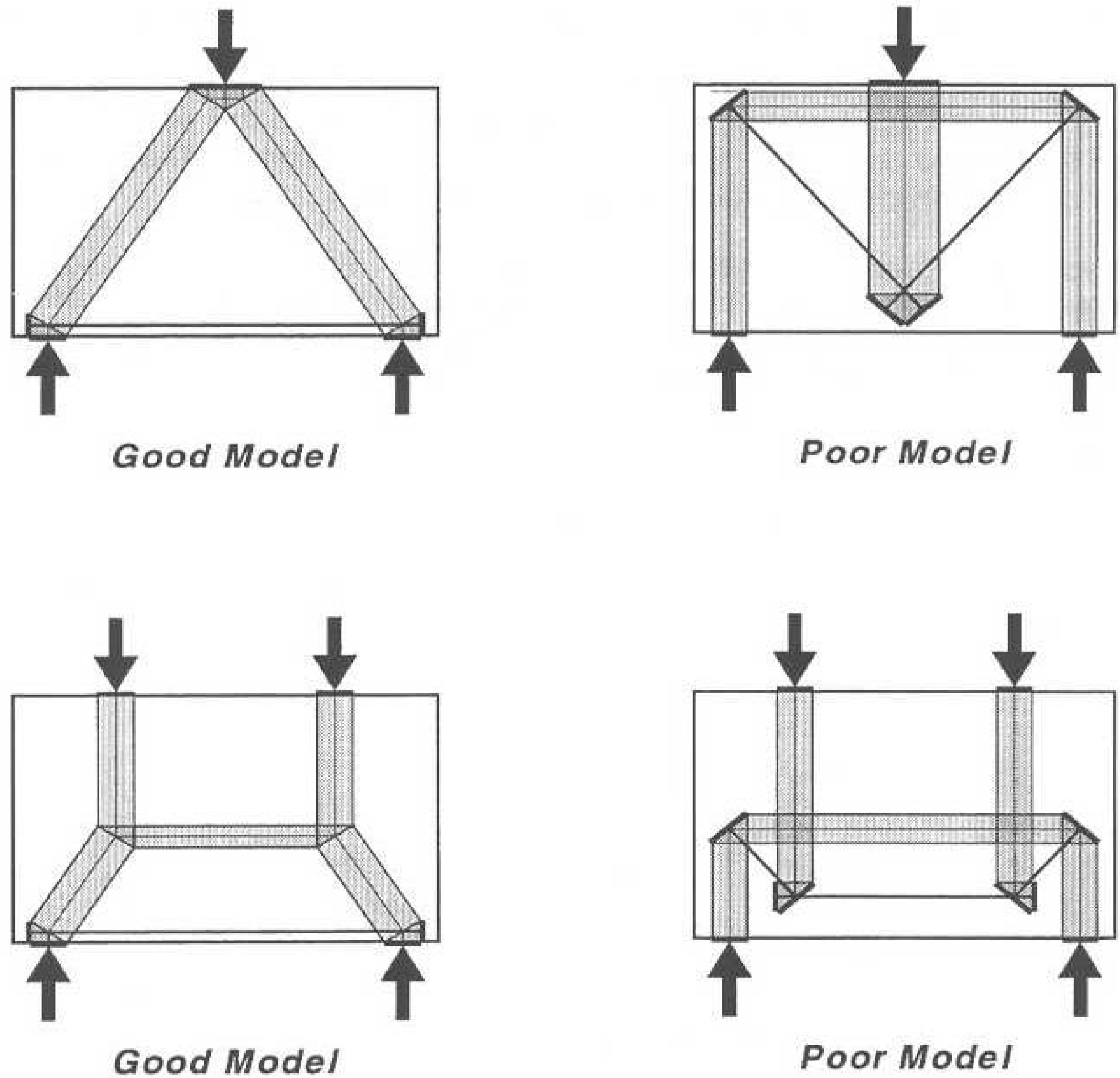


(c) Bearing plate at top



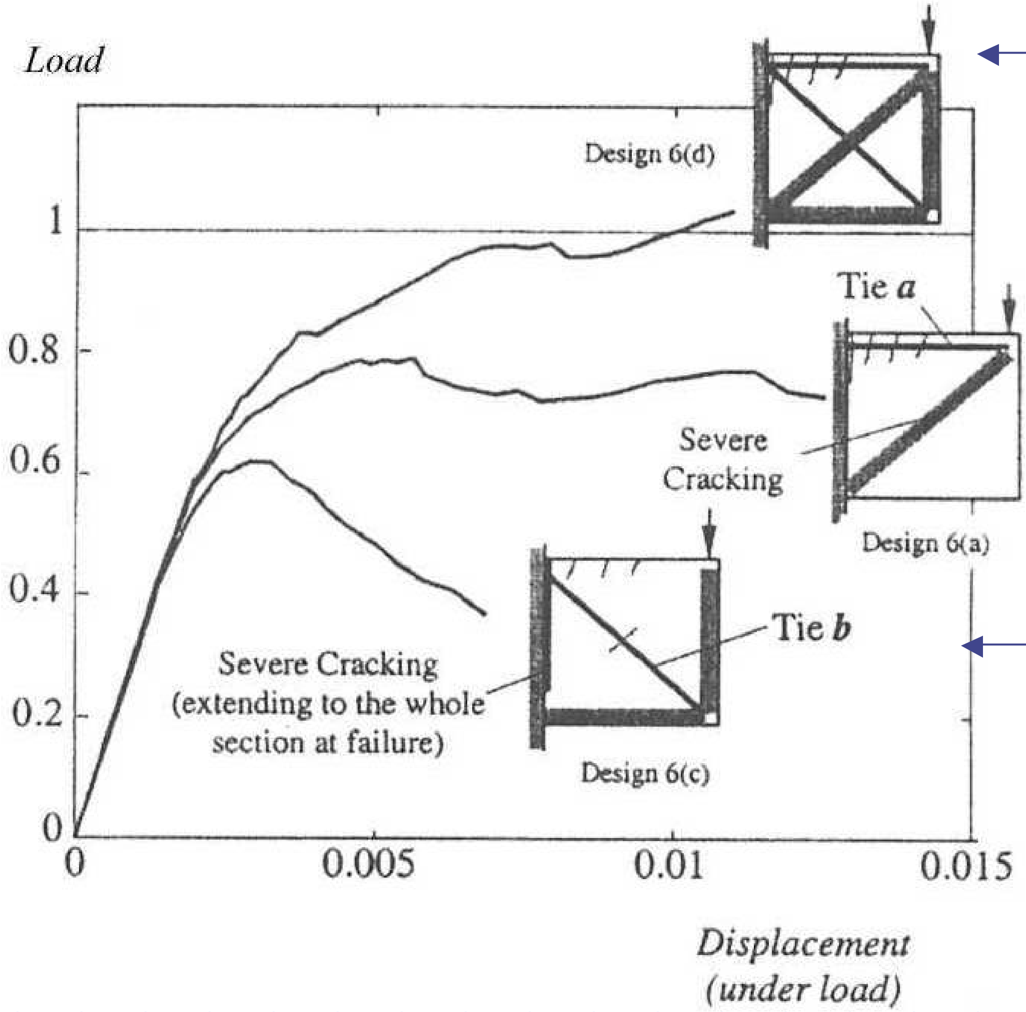
(d) Three symmetrically located bearings

Examples of Good and Poor STM Models



- Good Model is more closely approaches to the elastic stress trajectories
- Poor model requires large deformation before the tie can yield; violate the rule that concrete has a limited capacity to sustain plastic deformation

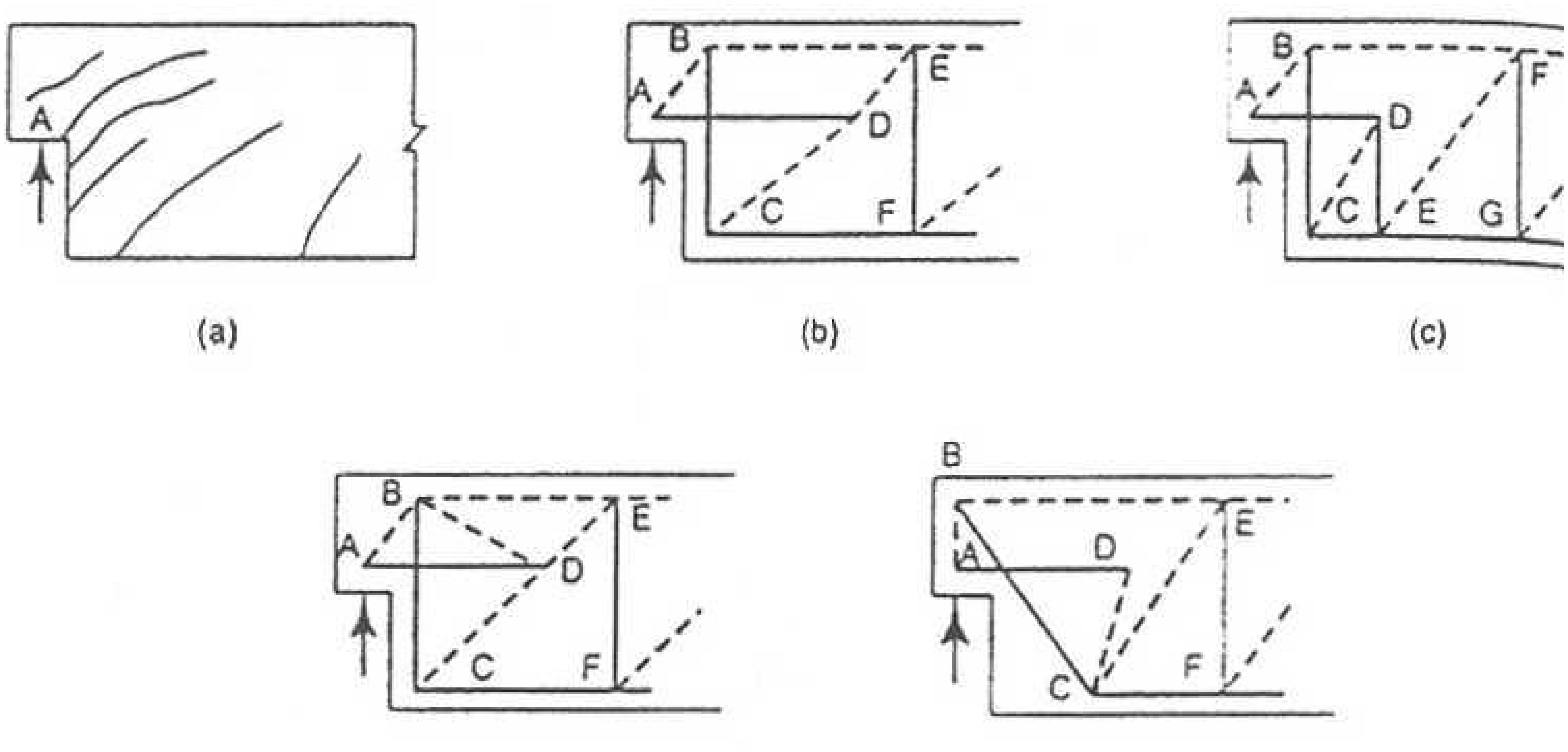
Nonlinear finite element comparison of three possible models of a short cantilever



(d) behaves almost elastically until anticipated failure load

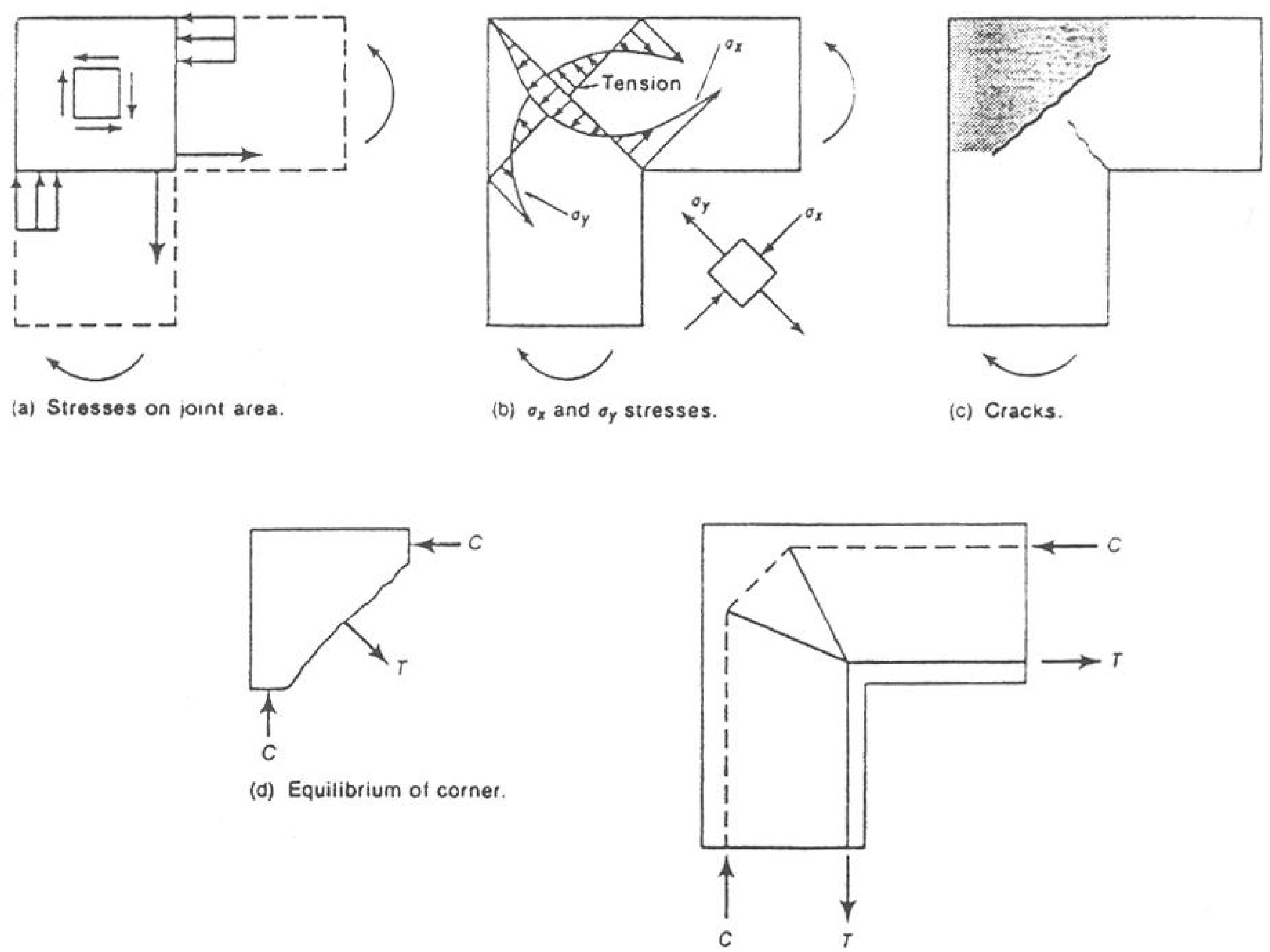
(c) requires the largest amount of plastic deformation; thus it is more likely to collapse before reaching the failure load level

STM Model for a Ledged End



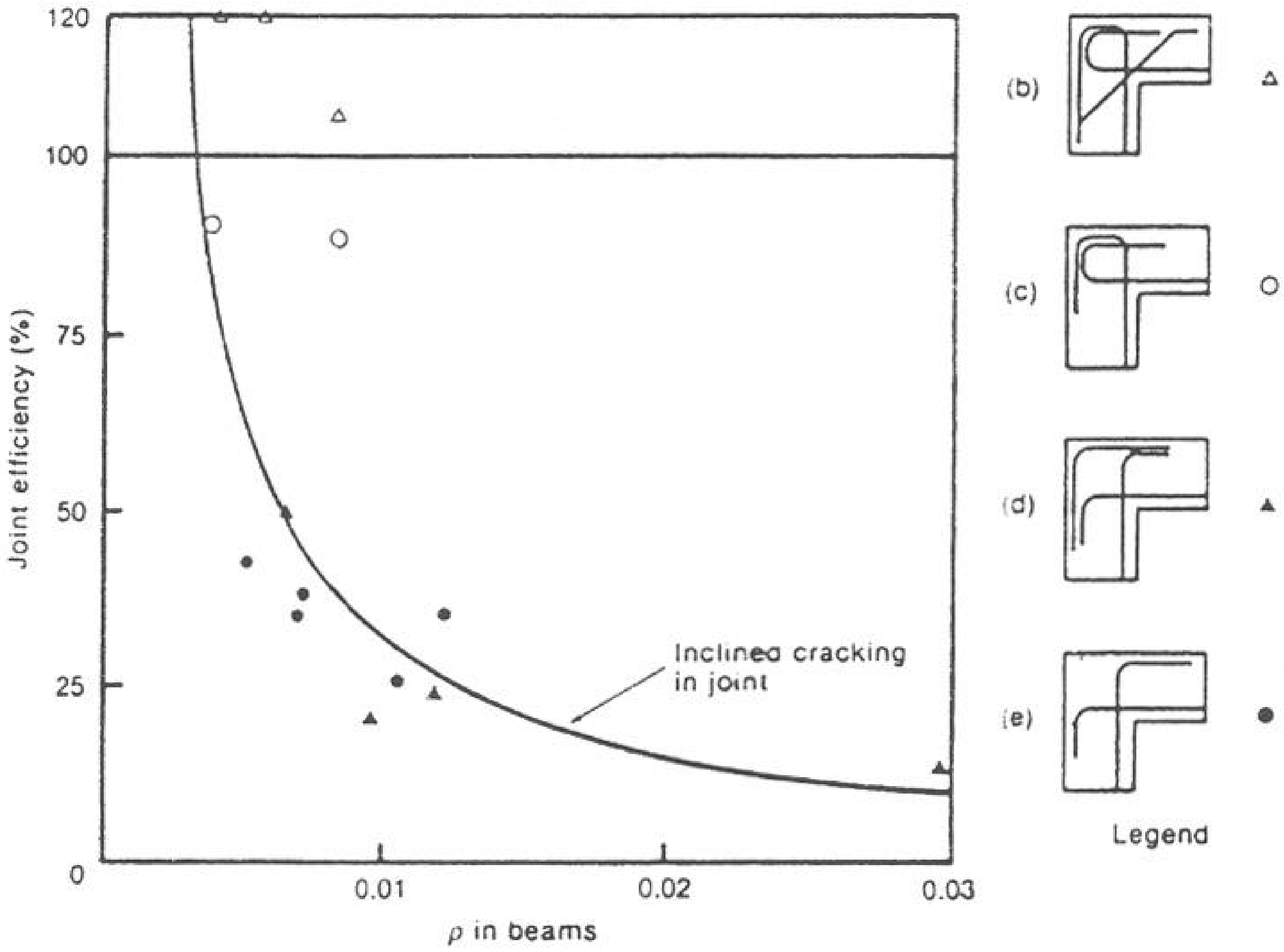
(a) (b) (c) (d) (e)

Beam-Column Opening Joints



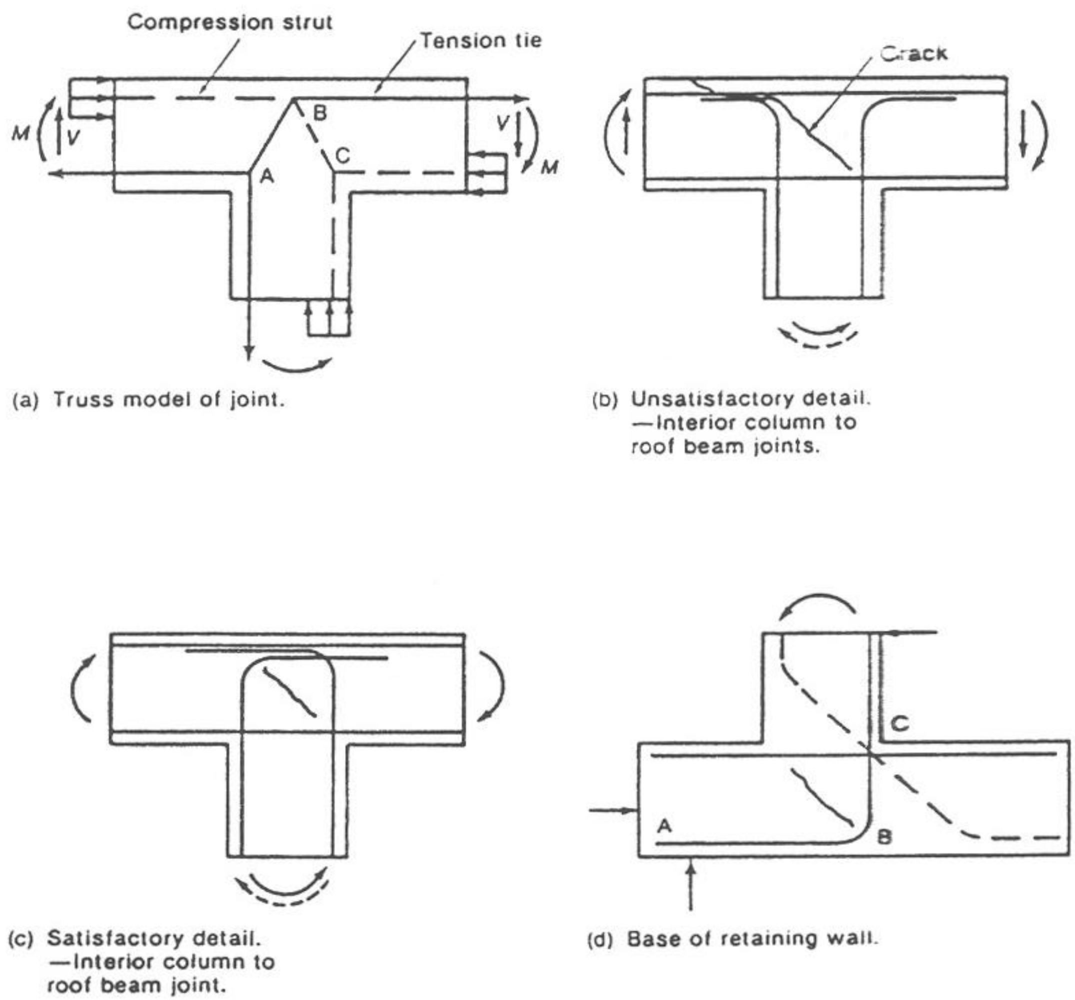
(a) Stresses on joint area.
(b) σ_x and σ_y stresses.
(c) Cracks.
(d) Equilibrium of corner.
(e) Truss model for joint.

Efficiency of Opening Joints



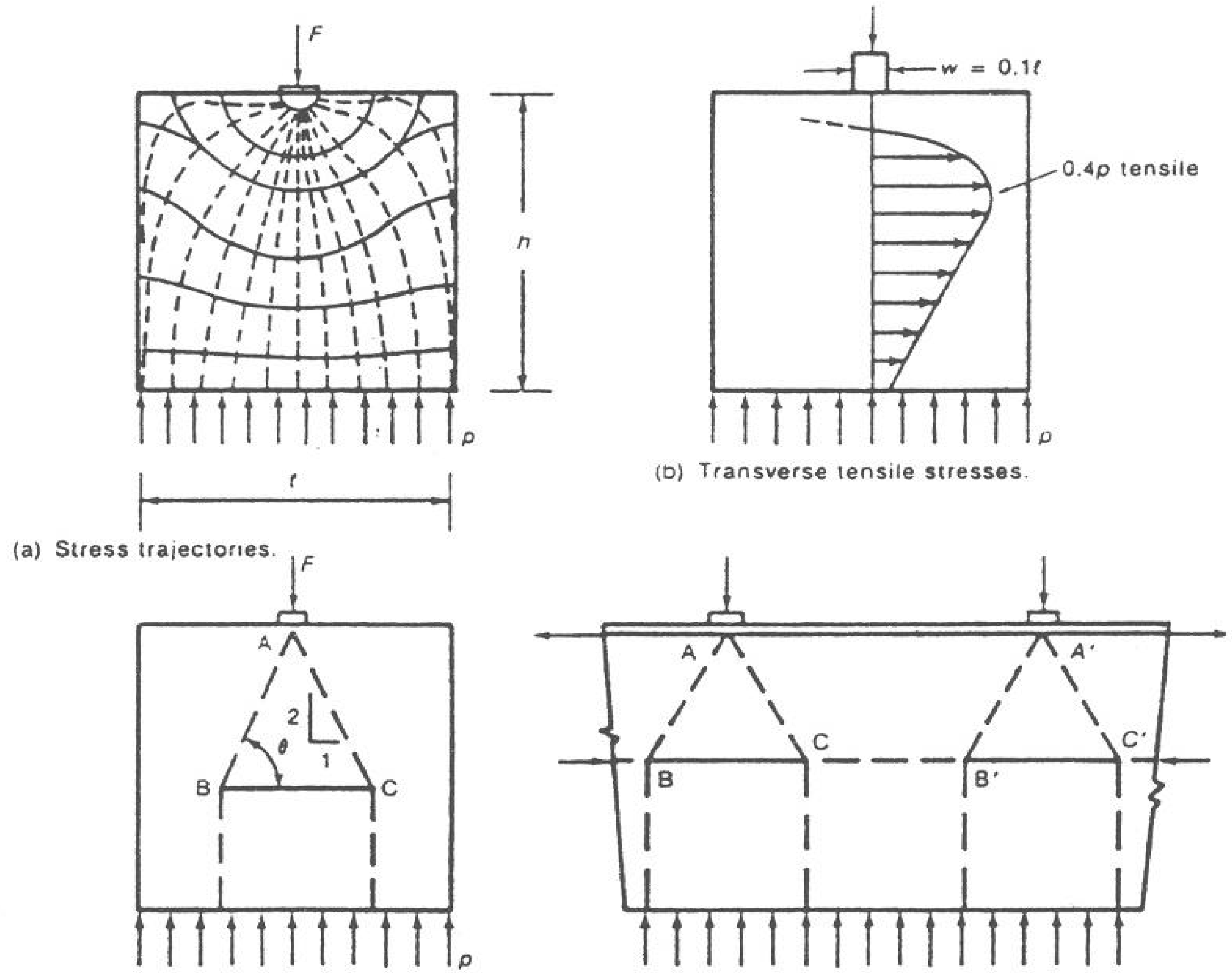
(a) Test data.

T-Joints



Compression strut
Tension tie
M
V
B
C
A
V
M
(a) Truss model of joint.
Crack
(b) Unsatisfactory detail.
—Interior column to
roof beam joints.
(c) Satisfactory detail.
—Interior column to
roof beam joint.
C
A
B
(d) Base of retaining wall.

Concentrated Load on a Bearing Wall



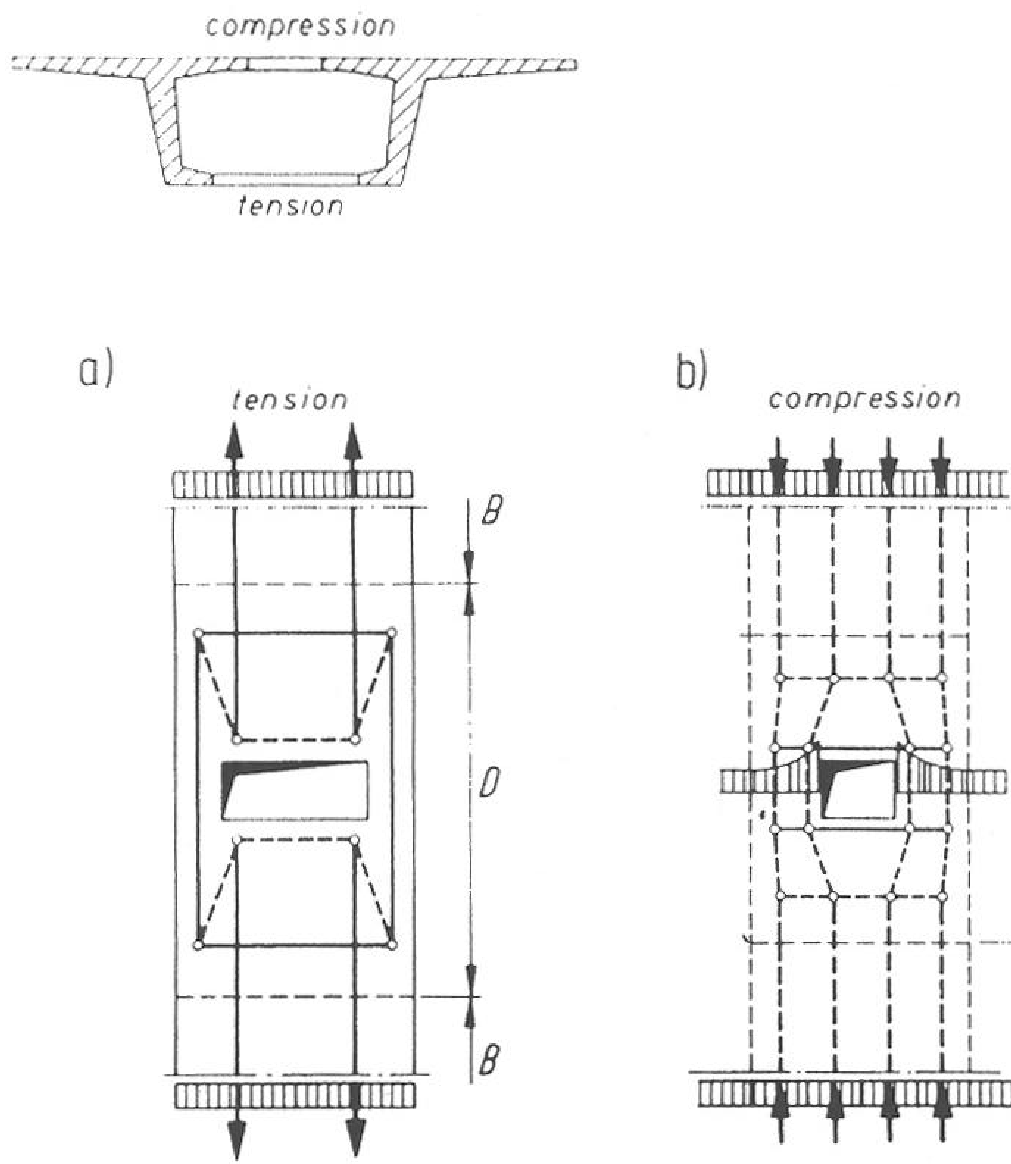
(a) Stress trajectories.

(b) Transverse tensile stresses.

(c) Truss model.

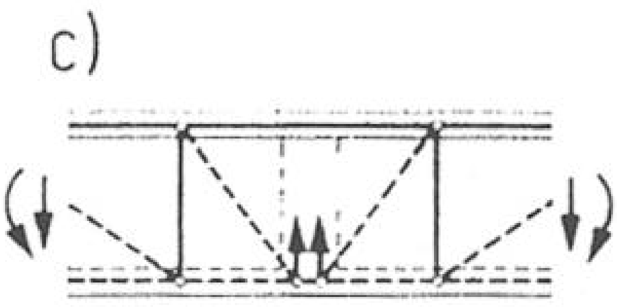
(d) Truss model for continuous wall.

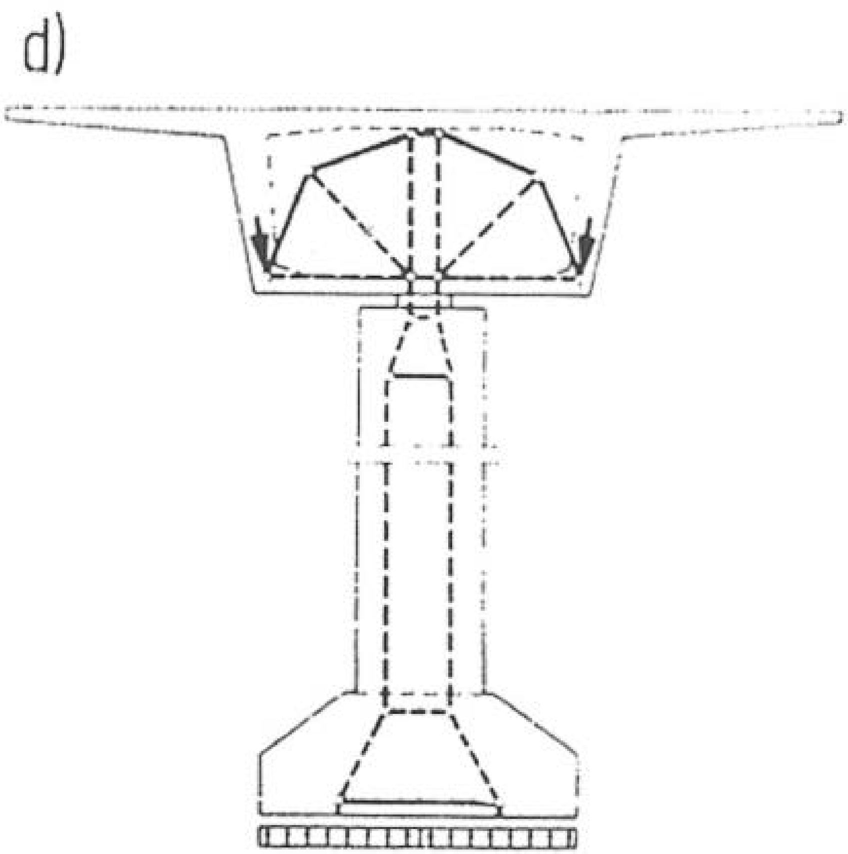
STM Models (a) Tensile Flange w/Opening (b) Compression Flange w/Opening



STM Models

(c) Web supported by Diaphragm
(d) Pier and Diaphragm w/Single Support

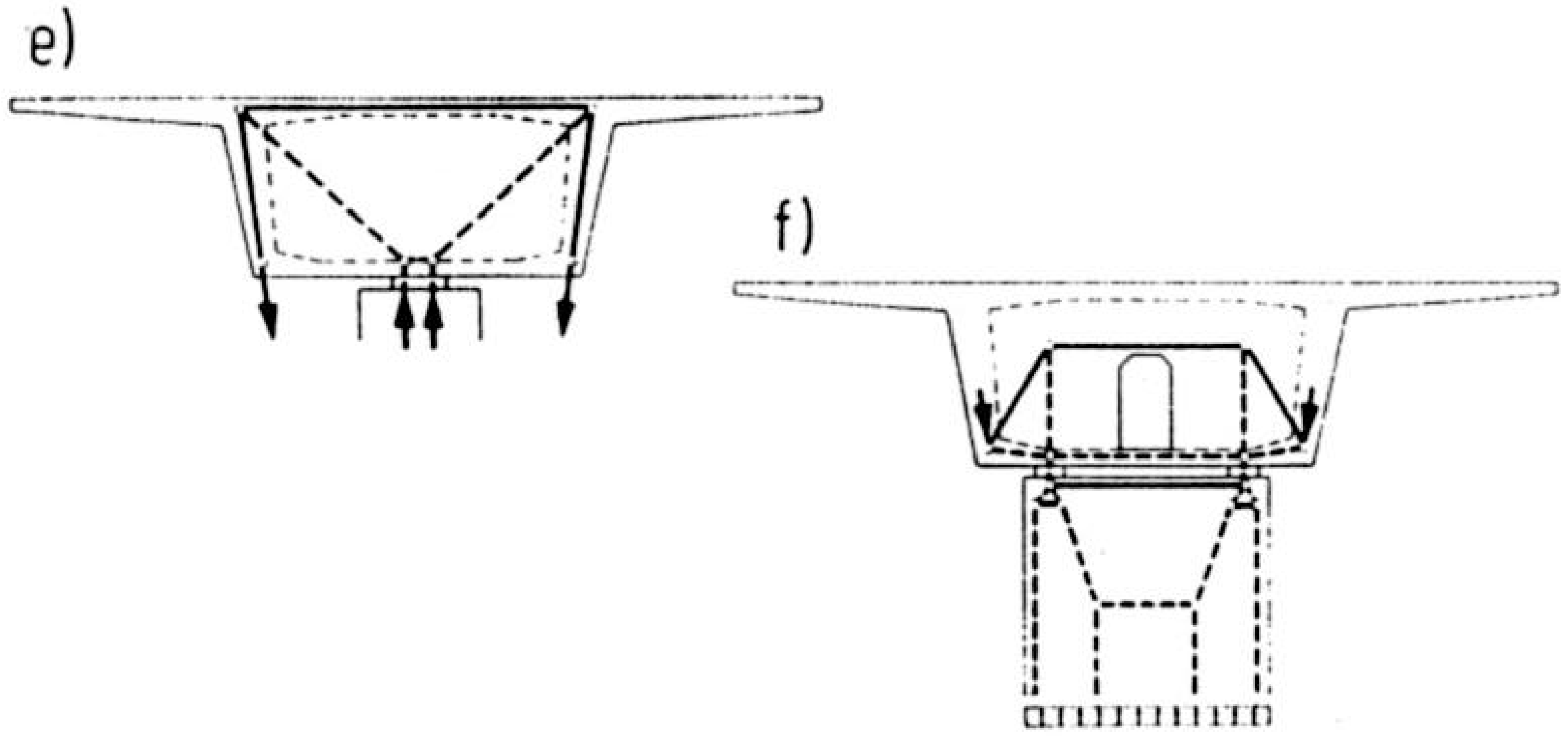




STM Models

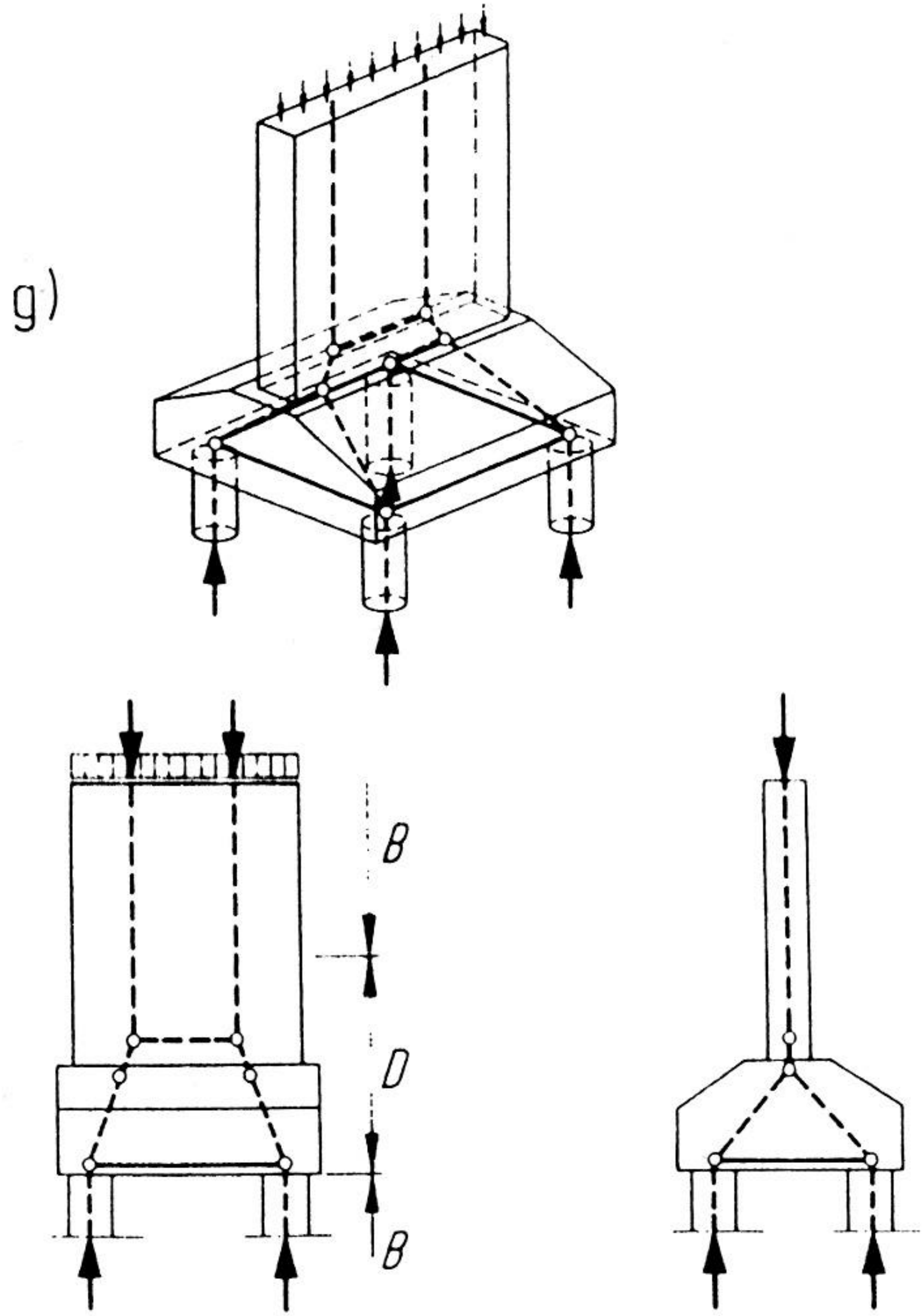
(e) Other Model for Diaphragm

(f) Pier and Diaphragm w/Two Supports

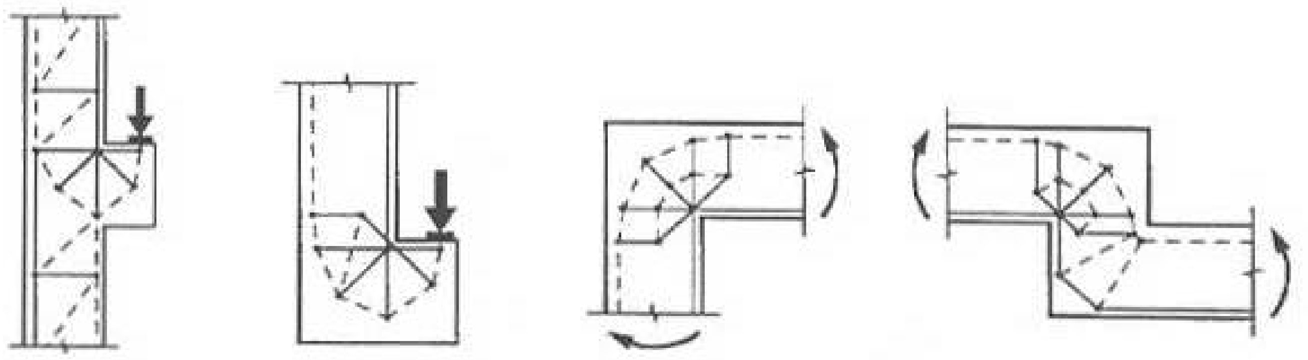


STM Models

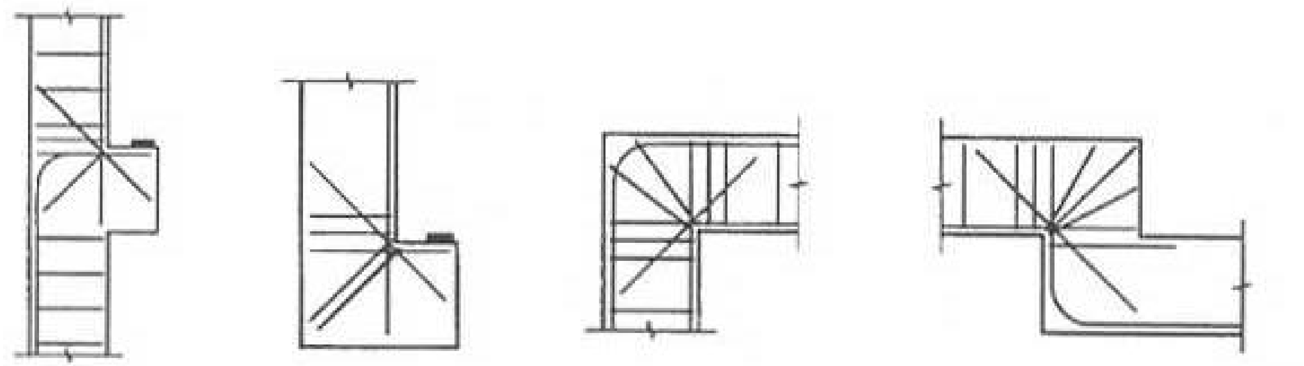
(g) Piers on a Pile Cap



Examples of STM Models & Reinforcement (Schlaich et al 1987)

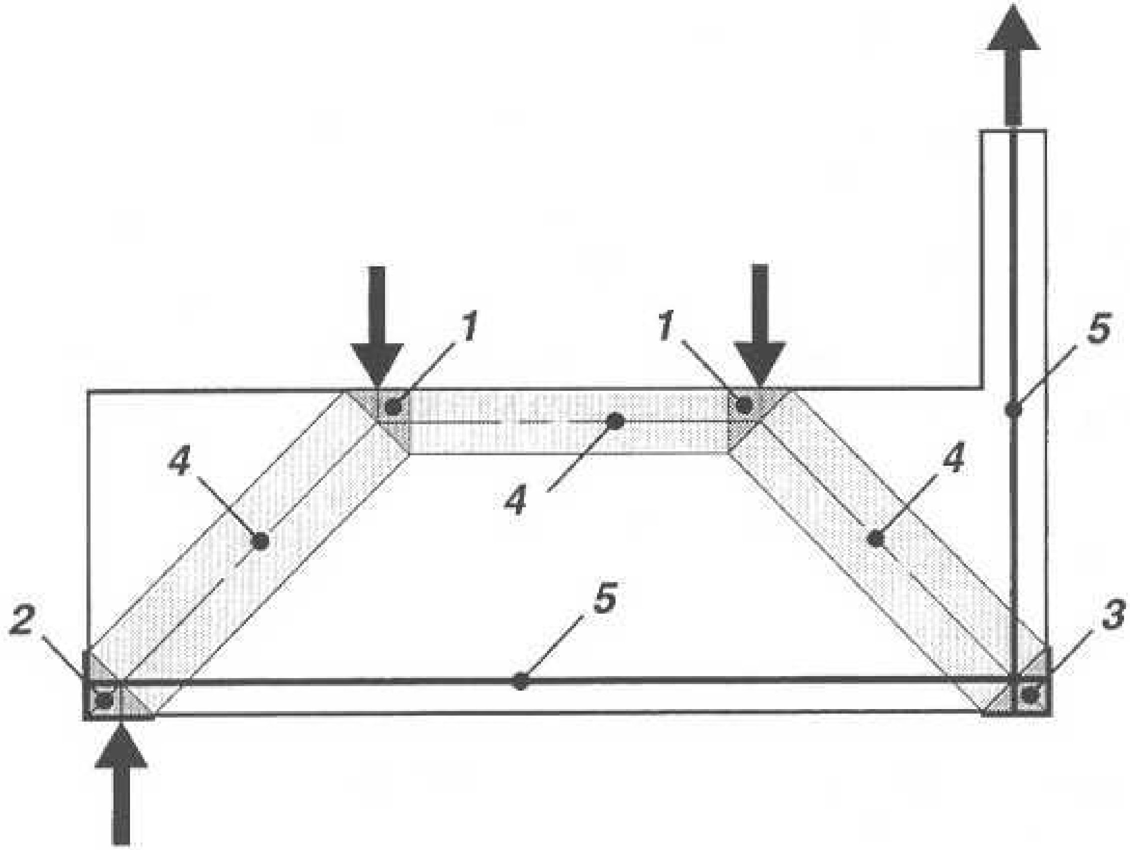


a) *Strut-and-Tie Model*



b) *Reinforcement*

Limiting Stresses for Truss Elements



Element	Limiting Stress
1 - CCC Node	$0.85\ \phi\ f'_c$
2 - CCT Node	$0.75\ \phi\ f'_c$
3 - CTT or TTT Node	$0.65\ \phi\ f'_c$
4 - Strut	f_{cu}
5 - Tie	f_y or $(f_{pe}+f_y)$

Limiting Compressive Stress in Strut AASHTO LRFD 5.6.3.3.3

$$f_{cu} = \frac{f_c^{'}}{0.8 + 170 e_1} \leq 0.85 f_c^{'}$$

where:

e_1 = $(e_s + 0.002) \cot^2 a_s$

f_{cu} = the limiting compressive stress

a_s = the smallest angle between the compressive strut and adjoining tension ties (DEG)

e_s = the tensile strain in the concrete in the direction of the tension tie (IN/IN)

Simplified Values for Limiting Compressive Stress in Strut, f_{cu} (Schlaich et al. 1987)

For an undisturbed and uniaxial state of compressive stress:

$f_{cu} = 1.0\ (0.85\ f_c^{?}) = 0.85\ f_c^{?}$

If tensile strains in the cross direction or transverse tensile reinforcement may cause cracking parallel to the strut with normal crack width:

$f_{cu} = 0.8\ (0.85\ f_c^{?}) = 0.68\ f_c^{?}$

As above for skew cracking or skew reinforcement:

$f_{cu} = 0.6\ (0.85\ f_c^{?}) = 0.51\ f_c^{?}$

For skew cracks with extraordinary crack width – such cracks must be expected if modeling of the struts departs significantly from the theory of elasticity's flow of internal forces:

$f_{cu} = 0.4\ (0.85\ f_c^{?}) = 0.34\ f_c^{?}$

Strength of Compressive Strut AASHTO LRFD 5.6.3.3.3

$P_r = F\ P_n$ (LRFD 5.6.3.2-1)

$P_n = f_{cu}\ A_{cs}$ (LRFD 5.6.3.3.1-1)

where:

F = 0.70 for compression in strut-and-tie models (LRFD 5.5.4.2.1)

A_{cs} = effective cross-sectional area of strut (LRFD 5.6.3.3.2)

ACI 2002 STM Model

Design of struts, ties, and nodal zones shall be based on:

$$fF_n \geq F_u$$

The nominal compressive strength of a strut without longitudinal reinforcement:

$$F_{ns} = f_{cu} A_c$$

The effective compressive strength of the concrete in a strut is:

$$f_{cu} = 0.85 \boldsymbol{b}_s f_c^{'}$$

ACI 2002 STM Model

The strength of a longitudinally reinforced strut is:

$$F_{ns} = f_{cu} A_c + A_s^{'} f_s^{'}$$

The nominal strength of a tie shall be taken as:

$$F_{nt} = A_{st} f_y + A_{ps} \left(f_{se} + \Delta f_p \right)$$

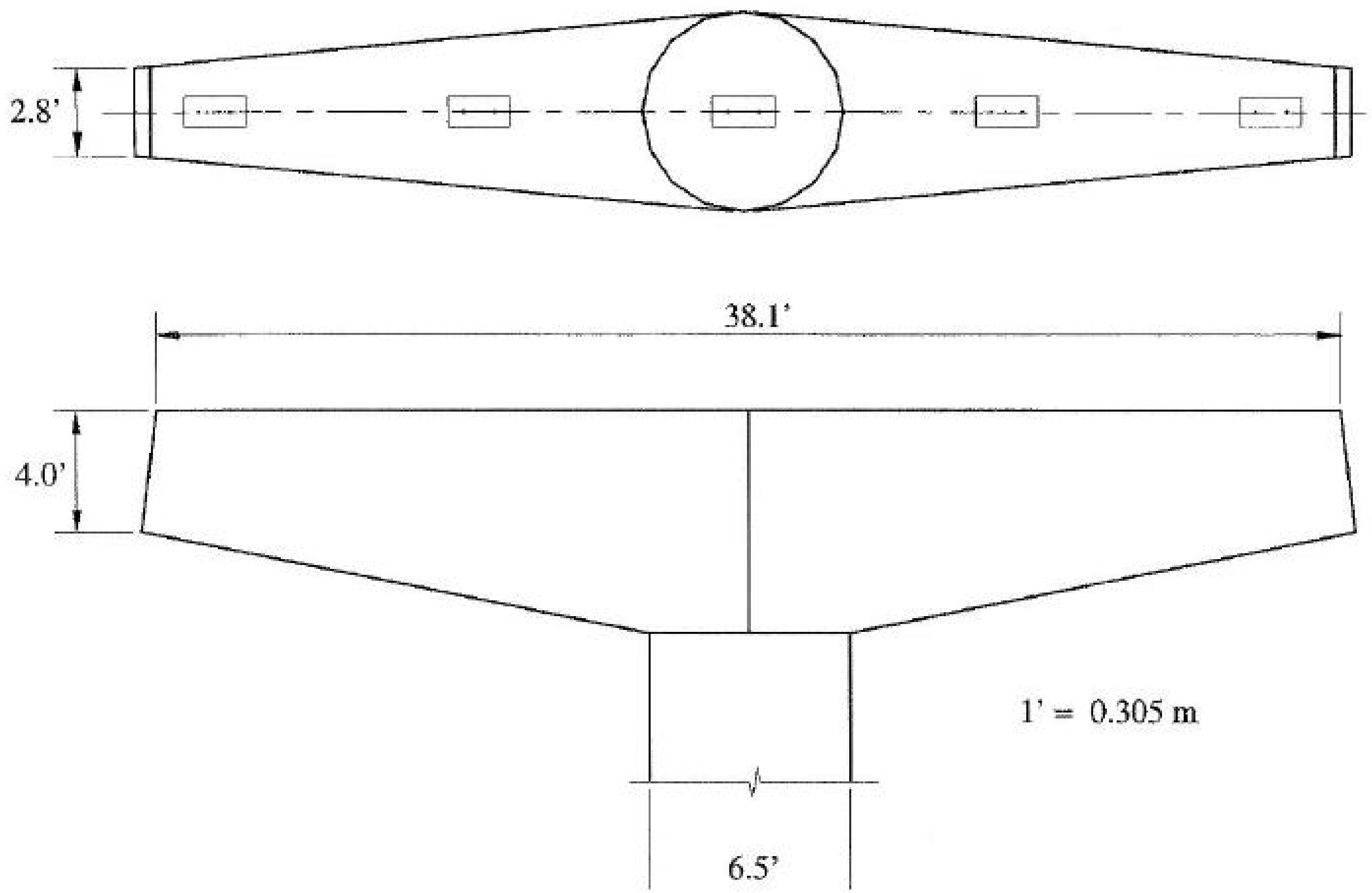
The nominal compression strength of a nodal zone shall be:

$$F_{nn} = f_{cu} A_n$$

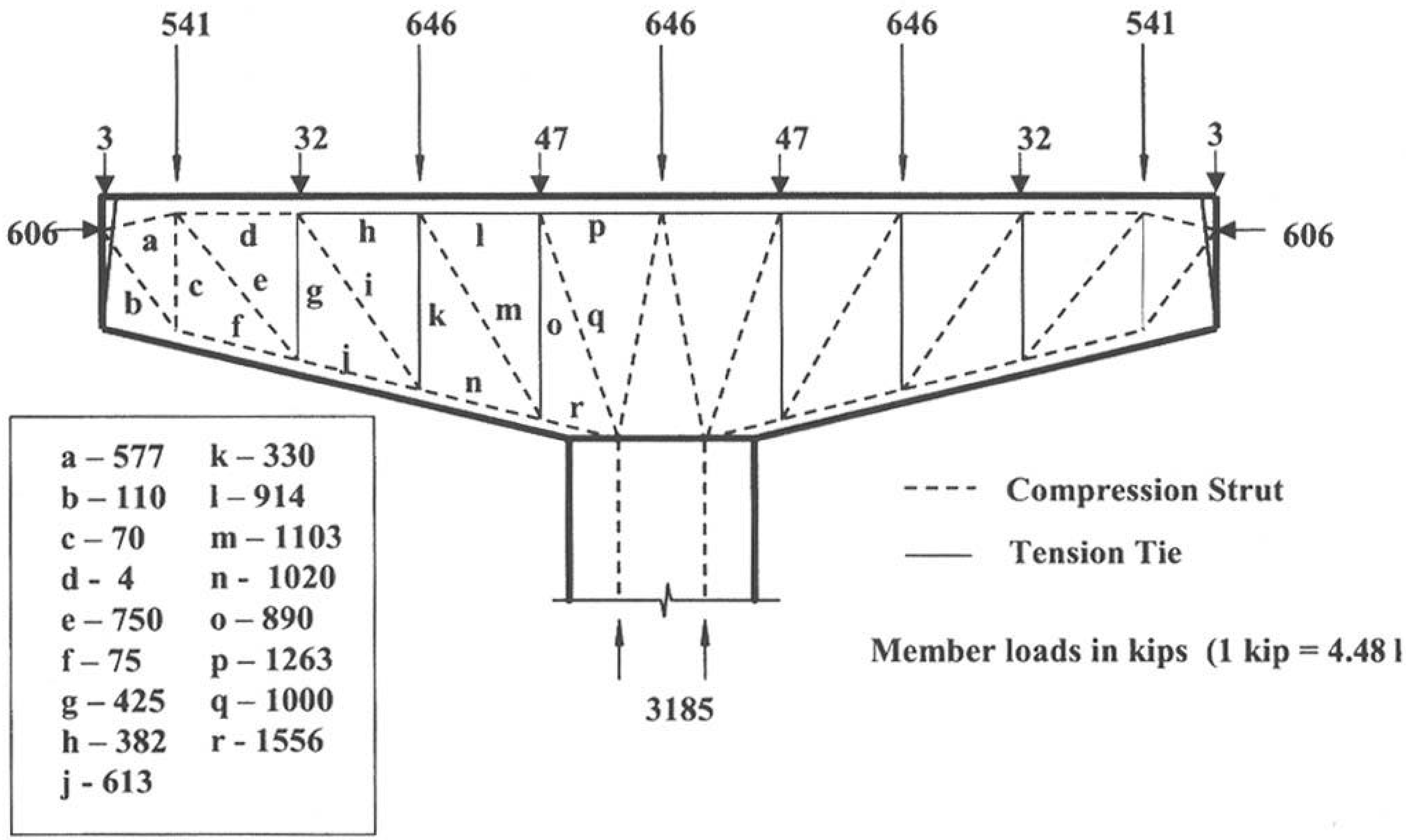
Findings of STM Model

- The STM formulation that requires the least volume of steel will be the solution that best models the behavior of a concrete member
- This approach holds great promise for DOTs and design offices which could develop or obtain standard STMs for certain commonly encountered situations
- Standard reinforcement details based on an STM could be developed for common situations
- The STM then could be reviewed and revised if any parameters change

Hammerhead Pier Example



Hammerhead Pier STM Model



Spreadsheet Calculation of STM Model Examples

- Abutment on Pile Model Example
- Walled Pier Model Example