NCTM 2019

Paper # 1

The Teaching of Worked Examples: Chinese Approaches in U.S. Classrooms

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CITATION:
Milewski Moskal, M., & Varano, A. (2019). *The teaching of worked examples: Chinese approaches in U.S. classrooms*. Paper presented at 2019 NCTM research conference as part of a research symposium, Ding, M (2019, organizer). *Enhancing elementary mathematics instruction: A U.S.-China collaboration*. San Diego, CA.

ACKNOWLEDGMENT:
This work is supported by the National Science Foundation CAREER award (No. DRL-1350068) to Dr. Meixia Ding at Temple University. Any opinions, findings, and conclusions in this study are those of the authors and do not necessarily reflect the views of the funding agency. We are grateful to the feedback provided by our discussant, Dr. Jinfa Cai, to the paper presented at the 2019 NCTM research conference.

For the first two years of our NSF supported U.S.-China Collaboration Study, each of the participating Grades 1-4 math teachers from both countries videotaped four early algebra lessons from their corresponding math curriculum. During year three of our study, teachers from both countries were given the opportunity to view, learn, compare, and comment on each others' translated video lessons. After viewing several videos, it was evident that the Chinese teachers used a teaching method that was unfamiliar to us, the teaching of worked examples. This method was used through representations of everyday life situations and skillful, purposeful, deep questioning that led to intentional talks between students and teachers. Our math curriculum in the U.S. does not parallel this method of teaching. Observing this approach to teaching math related facts inspired us, the authors of this action research, to explore ways that we could model using worked examples despite not having such images or dialogue in our textbooks and curriculum. We are particularly interested in exploring ways to design worked examples similar to the Chinese teachers in our own math classrooms to encourage deep thinking and collaborative discussions through real-life situations. The purpose of this study is to report our effort in improving our math instruction and lessons we learned from this endeavor.

Literature Review

Research on worked examples has a long tradition. From the cognitive load perspective, worked examples can help students establish relevant schema for solving new problems, which decreases cognitive load and enhances students' problem solving (Sweller & Cooper, 1985; Sweller, 2006). However, the worked example effect has not been well utilized in many US classrooms (Stigler & Hiebert, 1999). For instance, teachers in reformed classrooms often rushed with asking students to solve problems before even helping them learn an essential method through an example task. When students got lost in the process of problem solving, some teachers encouraged students to seek peer help (Ding, Li, Piccolo, & Kulm, 2007). This type of problem solving with minimum teacher guidance has been criticized by cognitive psychologist due to its low efficiency (Kirschner, Sweller, & Clark, 2006). Consequently, worked examples were recommended for teachers to enhance students' problem solving. However, existing studies in worked examples were mainly conducted in laboratory settings with secondary grade students (Cooper & Sweller, 1987; Renkle, Atkinson, Maier, & Staley, 2002) rather than elementary children in actual classrooms. In many laboratory studies, the solutions to worked examples were often directly presented to students through direct instruction. In these cases, students' learning appears to be more passive. In fact, other cognitive research on worked example suggested that asking students to self-explain their solutions (Chi, 2000) and gradually fade out solution steps (Renkl, Atkinson, & Grobe, 2004) will enhance the worked example effect. Our observation of the Chinese teaching approach to worked examples clearly illustrated and perhaps, further enriched the research assertion on effective use of worked examples. As to be explained in the Methods section, we noticed that Chinese worked examples were generally situated in real-life situations. Students were actively involved in discussion of the problem situation, which was then faded out into the targeted numerical representations.

The above Chinese approach to worked examples was supported by the literature. In an article, "Developing Number Sense through Real-Life Situations in School," Yang (2006) discussed a fourth grade math lesson that he observed in Taiwan. The teacher stepped out of the textbook structure and developed a real world problem for students to learn the worked example. Yang stated that "children's number sense can be developed through interaction with everyday objects in a realistic setting . . . through real classroom interactions." In this Taiwan lesson, the students actively participated in deep discussions, explained their thinking, and were able to apply what they learned to help them with realistic problems. This directly relates to important pedagogical implications that we, also, observed in classrooms in China. In contrast, though, the textbooks in China provide students with the opportunity to engage in real life situations. Teachers are given directly related real-life situations already implemented in their textbooks and curriculum. With the knowledge of worked examples from the literature, we are curious how the Chinese approach may be used in our US classrooms. Taking the perspective of classroom teachers, we explored this main research question as an action research team and documented the process of our experimenting as well as our success and challenges, which has some implications for current math education research and practices.

Methods

As the authors of this action research, we are two elementary teachers. We teach second grade and third grade at an urban elementary K-8 school in Philadelphia. We have participated in local university on-going professional developments broadening our understanding of how students learn mathematics. We have participated in training to learn how to actively monitor student assessments and target our instruction based on the data. This interest in mathematics led us to this project to analyze our own teaching methods and those of our colleagues in China. During this project we taught lessons in addition and subtraction inverse relations and multiplication and division relations.

The Chinese Approach to a Worked Example

Aligned with their textbooks, the Chinese teachers taught worked examples that were situated in real-world contexts. We noticed the picture image used (i.e. children at a pool with some in the pool and some by the pool) that allowed the teacher to lead a deep discussion about the targeted topic, inverse operations. Figure 1 illustrates the two worked example tasks in the first grade Chinese textbook.

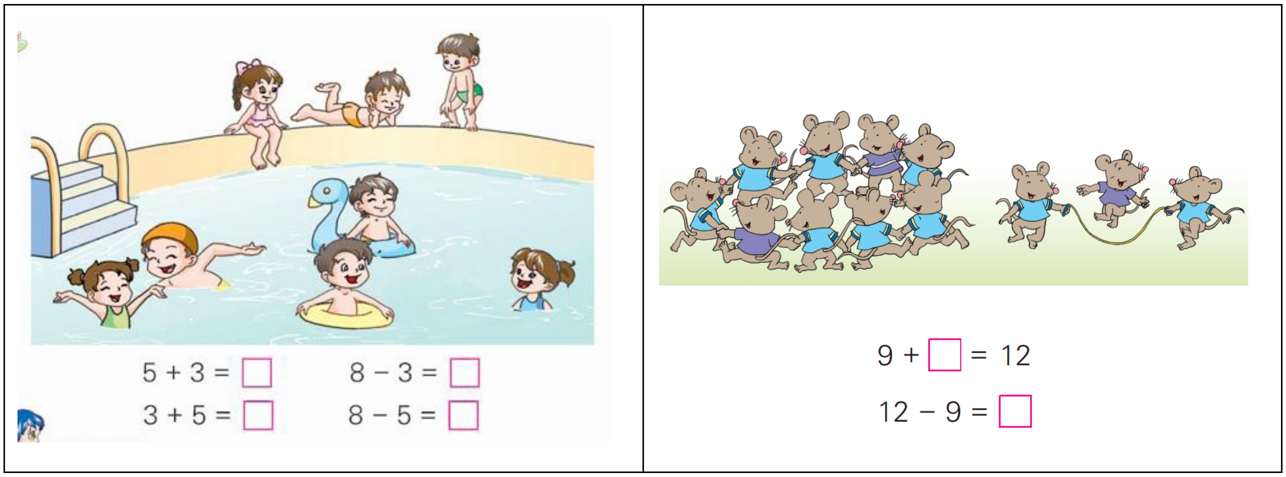


Figure 1. Worked examples in Chinese textbooks.

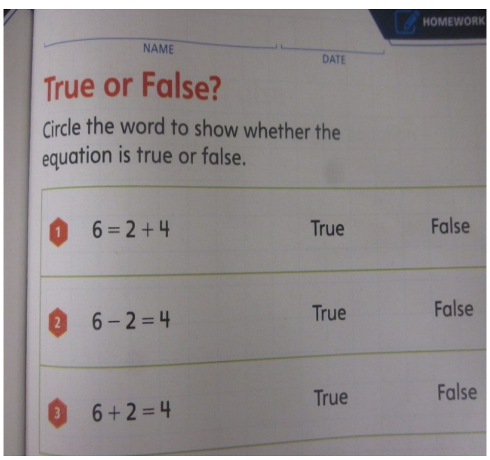
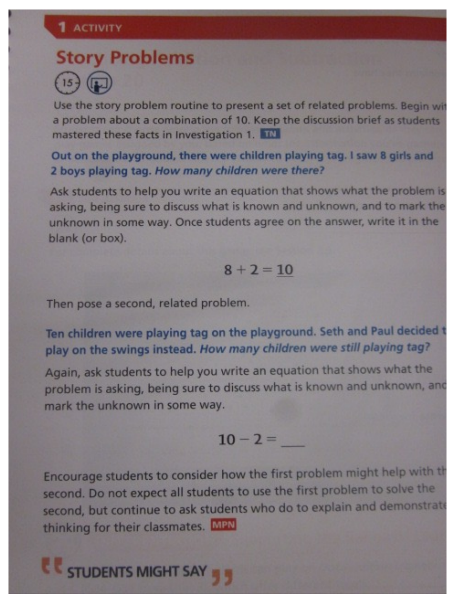
As part of the project, we observed the Chinese lessons on inverse relations based on the swimming pool example (see Figure 1 left), which was the same topic taught in our own video-taped lessons. We noticed that the Chinese teacher did not present the equations immediately. Rather, she only presented the image of the children at the pool. Based on this vivid real-world image, she asked a series of purposeful questions. The teacher began with a simple introduction: Do you like swimming? Her questioning led a discussion about this image for 15 minutes. The level of rich questioning and explanation between teacher and students lead the students to understand a new concept without direct demonstration and teacher modeling. Some of her questions are listed below:

1. Look at this picture carefully. What mathematical information have you obtained?
2. Can you pose a question that is solved by addition?
3. Can you write a number sentence to solve her question?
4. What do you observe from these number sentences?
5. Can you pose a problem that can be solved by subtraction?
6. Are there any connections between these four number sentences? Exchange your ideas with your desk mate.

Overall, the Chinese approach is supported by the literature on worked examples. Yet, its process of engaging students in unpacking the example adds to the literature because in the Chinese approach, students are active learners rather than passive receivers of the worked example solution.

Available Materials in our Own Teaching

In contrast to the Chinese curriculum and method of unpacking student understanding of inverse relations, our own curriculum necessitates students to use discussions requiring them to use abstract thinking. Below is an image of a first grade student workbook page practicing inverse relations for addition and subtraction for the first time (see Figure 2 left). There are no supporting pictures, which is quite different from the Chinese textbook for practicing related facts (see Figure 1).

US first grade student textbook	US first grade teacher guide
	

HOMEWORK

NAME DATE

True or False?

Circle the word to show whether the equation is true or false.

1. $6 = 2 + 4$ True False
2. $6 - 2 = 4$ True False
3. $6 + 2 = 4$ True False

1 ACTIVITY

Story Problems

15

Use the story problem routine to present a set of related problems. Begin wi a problem about a combination of 10. Keep the discussion brief as students mastered these facts in Investigation 1. TN

Out on the playground, there were children playing tag. I saw 8 girls and 2 boys playing tag. *How many children were there?*

Ask students to help you write an equation that shows what the problem is asking, being sure to discuss what is known and unknown, and to mark the unknown in some way. Once students agree on the answer, write it in the blank (or box).

$$8 + 2 = \underline{10}$$

Then pose a second, related problem.

Ten children were playing tag on the playground. Seth and Paul decided t play on the swings instead. *How many children were still playing tag?*

Again, ask students to help you write an equation that shows what the problem is asking, being sure to discuss what is known and unknown, and mark the unknown in some way.

$$10 - 2 = \_\_\_$$

Encourage students to consider how the first problem might help with th second. Do not expect all students to use the first problem to solve the second, but continue to ask students who do to explain and demonstrat thinking for their classmates. MPN

STUDENTS MIGHT SAY

Figure 2. Worked examples in US textbooks in comparison with the Chinese lessons.

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Likewise, the teacher guide (Figure 2, right) for how to teach inverse relations also did suggest using visual situations. Teachers are instructed to use the following questioning techniques:

1. Out on the playground, there were children playing tag. I saw 8 girls and 2 boys playing tag. How many children were there? (Ask students to help you write an equation.)
2. Then pose a second, related problem.
3. Ten children were playing on the playground. Seth and Paul decided to play on the swings instead. How many children were still playing tag? (Ask students to help you write an equation.)
4. Encourage students to consider how the first problem might help with the second problem.

Even though the above teacher guide did suggest two stories for classroom teaching, in comparison with the Chinese textbook example, these story problems were directly presented for students. This is different from Chinese textbook presentation where a concrete real-world situation was presented, which enabled students to pose story problems by themselves. The lack of visual pictures seems to make a difference in students learning opportunities, that is, "been given story problems" versus "posing story problems by students." Clearly, both the US student textbook and teacher guide expected students to immediately jump to abstract reasoning.

Procedures, Data Collection, and Analysis

After viewing the Chinese lessons in year 3 of the project, we realized that our own videotaped lessons in year 1 did not utilize real life situations that would involve the students in a rich discussion. In the first year videotaped lesson on additive reasoning, the second grade teacher author of this study used seven connecting cubes to model related number facts in addition and subtraction. Although cubes are concrete materials, this type of representational tool did not indicate a real-life situation and thus did not offer rich opportunities for students' authentic discussion like what the swimming pool situation could afford.

For our year four video filming, motivated by the Chinese approaches to worked examples, we conducted an Action Research on our own teaching. This included a process of planning, teaching, and reflection. First, we as a action research team discussed the possibilities of using similar "worked examples" that would reflect real life situations. After much discussion, we decided to try using a similar technique in our classrooms. These images, as simple as they may seem, were difficult to duplicate, since these types of images were not available in our current textbooks or curriculum. So, we needed to create our own images of "worked examples" for each of our lessons to introduce the inverse relationship between addition/subtraction and multiplication/division. After planning the lessons and the images, we taught the lessons, which were videotaped by the project researchers. We created our own images using Notebook for Smartboard. After each lesson, we were interviewed during which we were prompted to reflect upon the successes and challenges teaching worked examples.

At the end of year four we were given the amazing opportunity to observe math lessons in China across several schools in two different cities. We observed that the teachers were using real life situations with deep questioning and discussions throughout many of their lessons. This further inspired us to continue to hone our practices during year five of our study.

Even though our lessons in year four were more reflective of our Chinese counterparts, we still felt that we could reach our students by using better images to spark students' knowledge of inverse relations. As such, in year five we created our own images, to improve our students understanding of number relations.

The video data in years 1 and 4 as well as our self-collected data in year 5 were reviewed and compared by the two authors of this study. We also re-listened to our interviews shared by the project. All these data were qualitatively analyzed by writing memos and seeking patterns so to answer the research questions on our successes and challenges. Below we report our findings.

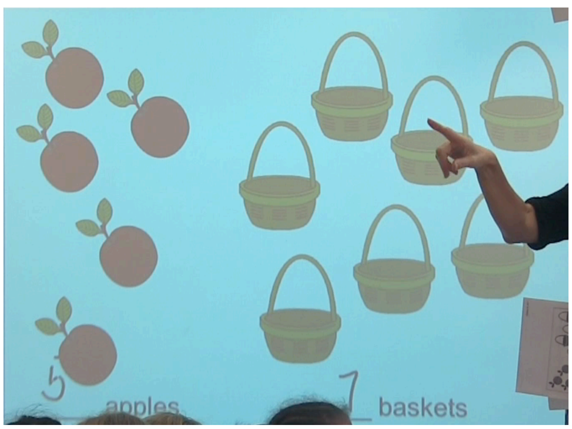
Results

Being exposed to the method of introducing math concepts through "worked examples" with familiar situational images, we were able to explore a new way of engaging children in rich mathematical discussions. The success lied in the aspect that the teaching of worked examples was situated in a real-world situation. This allowed students to think mathematically and be able to verbalize their thoughts through concrete images. We have discovered a powerful new tool to introduce new concepts. However, this success resulted from a process of exploration due to the challenges we faced. This challenge was evident in our year 4 videotaped lessons. They were not as successful as what we had hoped, mainly related to the primitive images that we had created using Notebook for the Smart Board. The images that we created on Notebook for the Smartboard were more illustrative than situational. Nevertheless, the creating of images was already a challenging and time-consuming task for us with novice technological expertise. We realized that we would need to create and match a visual situational image to provoke a rich discussion to introduce the addition/subtraction and multiplication/division inverse relationships. Based on continuous discussions and reflections, we decided to improve our classroom design by adaptively using the Chinse approaches to worked examples. Below we report stories in our second and third grade classrooms.

Second Grade Classroom Story

As mentioned above, during Y4 of classroom teaching, the second grade teacher created worked example images that were illustrative. Figure 4 shows two typical examples. Lesson 1 (left picture) was a created image to illustrate a lesson on number comparison. The textbook lesson asks students to imagine a classroom situation to find out if there will be enough pencils for the class and if so, will there be extra, if not how many more will you need. This is a difficult concept without having a concrete image, so the apples with baskets was created. The created lesson asked, "Will every apple have a basket?" The smaller number of apples and baskets with images gave students an entry point to understand the concept. Although, this was a concrete example, it still lacked the real life situation that was observed in the Chinese videos. As indicated by the interview, the second grade teacher was unsatisfied by these images as they did not elicit rich conversations as expected.

Lesson 1



Lesson 3

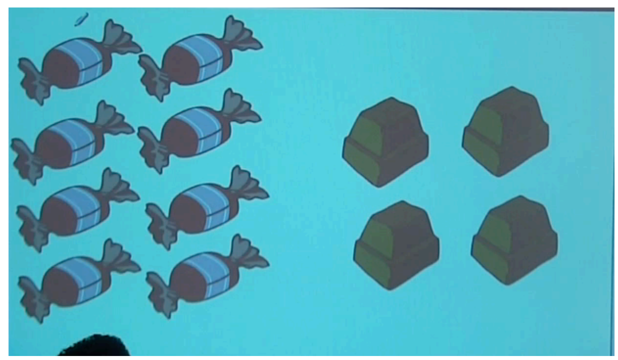
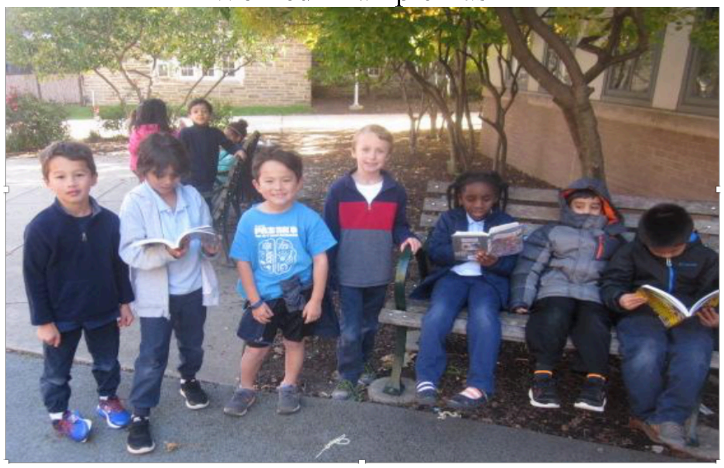


Figure 3. Worked example images created on Notebook using a Smartboard in year 4.

The opportunity to observe Chinese lessons both through video observations and onsite classroom visits inspired us to revise the images we had designed to be more realistic. In this study, the second grade teacher created a real life situation by photographing her students on the playground and in the classroom. She used the Chinese model for deep questioning and engaged her second grade students to discuss possible story problems that could be connected to the pictures. Below is one such picture (see Figure 4, left) with an example of a student notebook showing the picture representation of related facts (see Figure 4, right). These related facts were derived from a rich classroom discussion about the playground picture. Students were then asked to write a story problem based on the picture and one of the equations. They were able to write both addition and subtraction story problems.

Worked Example Task



Student Response

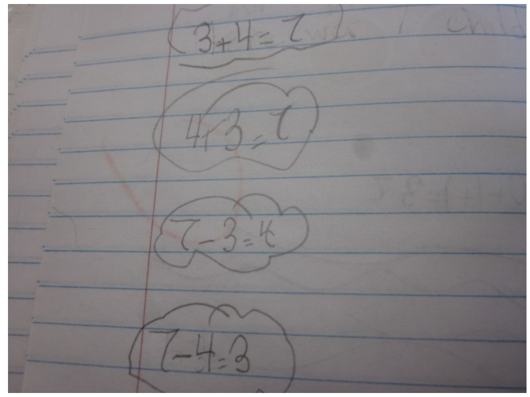


Figure 4. Creating worked example tasks by photographing on the playground in year 5.

The use of images has supported student learning concepts that are taught abstractly through textbooks. Incorporating real life images has benefited students in understanding inverse relations and more difficult concepts, such as, unknown change in additive reasoning. Below is another image from the second grade classroom that facilitated the discussion of unknown change in additive reasoning using real life images.

Worked example task

Student responses

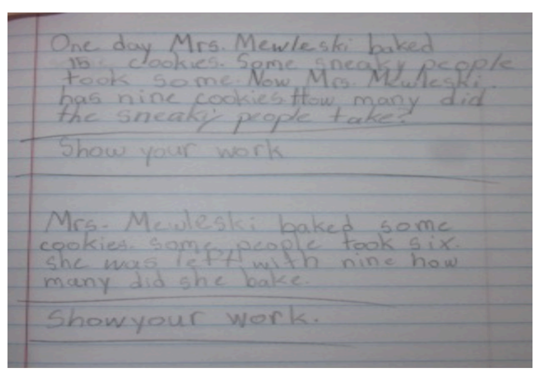


Figure 5. Creating worked example tasks by photographing in the classroom in year 5.

As seen in Figure 5 (left), this photograph contained two sub-pictures: Students initially looking at 15 cookies and then walking away with cookies now missing from the tray. After showing the image, the students were asked what they think the picture portrays. They related many different "stories" of what was going on in the picture. After a lengthy discussion, the second grade teacher asked them to write a story problem that could go along with the images. A student example above says, "One day Ms. Milewski baked 15 cookies. Some sneaky people took some. Now Ms. Milewski has nine cookies. How many did the sneaky people take? Another story problem read, "Ms. Milewski baked some cookies. Some people took six. She was left with nine. How many did she bake?" This image and subsequent discussion gave students the opportunity to use their mathematical thinking.

Third Grade Classroom Story

Students in the third grade classroom encountered similar instructional approach to worked examples as what was reported with the second grade classroom. After learning how the Chinese teachers taught inverse relationships through many visual images, the third grade teacher created examples on the smart board to visualize what the images represented, which has enabled students to create story problems, and promote classroom math discourse. Figure 6 illustrated a few images.

Figure 6. Images created by the third grade classroom teacher.

More encouragingly, the third grade teacher provided the pre- and post-tests (designed by the NSF project) before and after students were taught the unit. Student learning gains provide strong evidence for the success of our classroom experiences with the Chinese approach. In the pre-test only 10 % could convert the worked examples to number expressions to form a fact family that matched the images. After the unit was tweaked and taught, students were able to answer the visual image to the multiplicative related fact family with a success rate of 88%. Figure 7 illustrated student typical work.

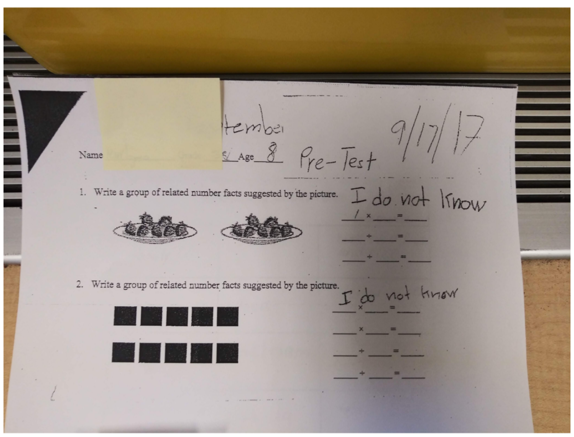
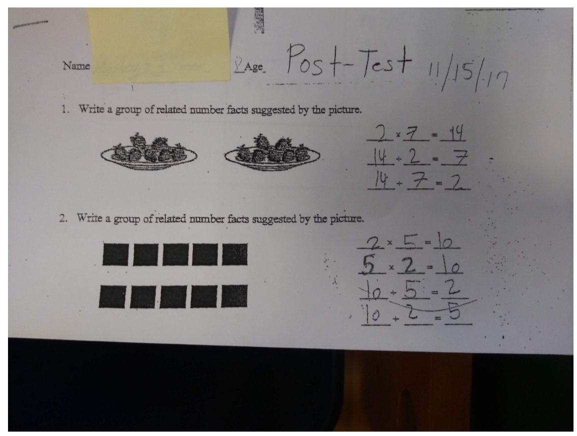
Student Pretest	Student posttest
	

Figure 7. Typical student work in the pre- and post-tests.

Discussion

Our successes and challenges in this action research call for better support for teachers if we are to enhance our classroom instruction. As indicated by our NSF project, the Chinese textbooks often start each lesson with a well-designed worked example with vivid images that leads to a deep discussion of mathematical concepts. In the United States textbooks use pictures that are not related to or supportive of students' thinking. After viewing Chinese primary grade math lessons using realistic situations, which enabled accountable talk, we realize the mathematical importance for children. These types of images and rich questioning is not available to mathematics teachers. Instead we are made to prepare and create our own worked examples. This type of resource is available to teachers in China, which has enabled Chinese teachers to spend time on instructional design instead of creating images. As such, an urgent support for teachers is to create a curriculum that allows teachers to have a bank of images and leading questions to use “worked examples” as a method to introduce math concepts such as inverse relations. Classroom teachers have an enormous amount of daily work to prepare in their teaching day. Being able to have a resource such as worked example images and questions for collaborative talks corresponding to math standards, would enable teachers to enrich student math lessons. Such a resource, would allow all students across the United States to participate in rich mathematical discussions and greater understanding of mathematical concepts.

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