

2.1

THE THINK TANK Prime Factorization and Factor Trees

Learning Goals

In this lesson, you will:

- ▶ Determine the prime factorization of a number.
- ▶ Understand the usefulness of prime factors.
- ▶ Recognize that each whole number has exactly one prime factorization.

Key Terms

- ▶ prime factorization
- ▶ Associative Property of Multiplication
- ▶ factor tree
- ▶ power
- ▶ base
- ▶ exponent
- ▶ Fundamental Theorem of Arithmetic

2

Did you know that some scientists devote their study to one species of animal? In fact, some teams of oceanographers in Texas and California have dedicated their study to octopi, squid, nautilus, and cuttlefish. Collectively, these animals are members of the class Cephalopoda, or commonly known as cephalopods (pronounced sef-e-le-pods). To study cephalopods, oceanographers use huge tanks to house the creatures in order to mimic the ocean environment.

Does your school have a fish tank? Do you or any of your classmates have a fish tank at home?

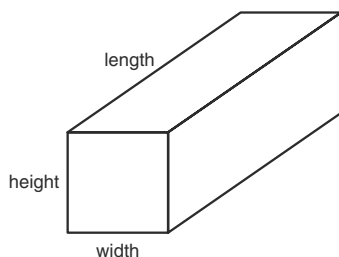
Problem 1 Dimensions of a Tank



Previously, you determined equal-sized groups to organize baseball cards. You also determined dimensions for display cases using factor pairs. Let's consider other ways to represent the factors of a number.

The Think Tank designs and creates customized tanks and aquariums for oceanographers. A team of oceanographers who study the characteristics of plankton requested a tank that has a volume of 240 cubic feet, but they didn't give the dimensions of the tank. You have been asked to help The Think Tank list possible tank dimensions using the information about the tank's volume.

Recall that volume is the amount of space occupied by an object. The only rule you need to follow is that none of the dimensions can be 1 foot. To calculate the volume of a tank, multiply the tank's length by its width and by its height.



Let l = length

Let w = width

Let h = height

$$V = lwh$$

2

1. Complete the table by listing three possible tank dimensions for the volume given.



Length (ft)	Width (ft)	Height (ft)	Volume (ft ³)
			240
			240
			240



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2. How did you determine the possible dimensions of the tank? How did you verify you were correct?



3. Compare your possible tank dimensions to your classmates' dimensions. What do you notice?

4. Write the volume as the product of prime numbers for each row in your table. What do you notice?



A natural number written as the product of primes is the *prime factorization* of that number. **Prime factorization** is the long string of factors that is made up of all prime numbers.

5. Does the order of factors make a difference? Explain your reasoning.

I can write any whole number—except for 1—as a product of prime numbers. So, 4 as a product of prime numbers would be 2×2 .



Recall that the Commutative Property of Multiplication states that changing the order of two or more factors in a multiplication statement does not change the product. For instance, look at the example shown.

$$\begin{aligned} 3 \times 5 &= 5 \times 3 \\ 15 &= 15 \end{aligned}$$

The **Associative Property of Multiplication** states that changing the grouping of the factors in a multiplication statement does not change the product. For any numbers a , b , and c , $(a \times b) \times c = a \times (b \times c)$.

For instance, look at the example of the Associative Property of Multiplication.

$$\begin{aligned}(2 \times 3) \times 3 &= 2 \times (3 \times 3) \\ 6 \times 3 &= 2 \times 9 \\ 18 &= 18\end{aligned}$$

2



6. What is the difference between the Commutative Property of Multiplication and the Associative Property of Multiplication?

7. Rewrite each set of factors using the Associative Property of Multiplication.

a. $(3 \times 4) \times 5 =$ _____

b. $6 \times (5 \times 3) =$ _____

c. $(2 \times 3) \times 5 =$ _____

8. Determine the product for each set of factors in Question 7. Which set of factors did you use, the set given or the set you rewrote using the Associative Property? Explain your choice.

The Properties can help you add and multiply numbers more efficiently because you can reorder and regroup.



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9. Write the prime factorization of 240. List the factors in order from least to greatest.

Problem 2 Factor Trees



A **factor tree** is a way to organize and help you determine the prime factorization of a number. Factor trees use branches to show how a number is broken down into prime numbers.

You can use the X or \cdot symbols to represent multiplication. Make sure you use one symbol throughout each multiplication sentence.



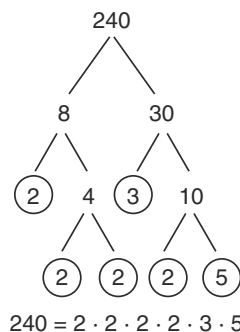
An example of a factor tree for 240 is shown.

Begin by writing the number 240.

Choose any factor pair, but do not use 1 as a factor. Draw a branch from 240 to each factor.

If one of the factors is a prime number, circle the number to indicate the branch is complete.

If one of the factors is a composite number, continue to choose factor pairs (but do not use 1 as a factor) until all the factors are prime numbers.



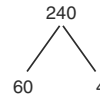


1. Complete each factor tree. Then, write the prime factorization as a mathematical statement.

a. 240



b. 240



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c. 720



d. 720



e. 360



f. 360



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Did you notice that 2 and 3 were repeated factors of the prime factorizations in the factor trees you created? The prime factorizations were long, and you had to write a lot of numbers. Because 2 and 3 are repeated factors, you can write their repeated multiplication using a shorthand method. This method is called a *power*. A **power** has two elements: the base and the exponent, as shown.

$$2 \times 2 \times 2 \times 2 = 2^4$$

The **base** of a power is the factor that is multiplied repeatedly in the power, and the **exponent** of the power is the number of times the base is used as a factor of repeated multiplication.

You can read a power in the following ways:

“2 to the fourth power”

“2 raised to the fourth power”

“the fourth power of 2”



2. Identify the base and exponent in each power. Then, write each power in words.

a. 7^2

b. 4^3



3. Write the prime factorizations using powers for the factor trees you completed in Question 1.



The **Fundamental Theorem of Arithmetic** states that every natural number is either prime or can be written as a unique product of primes.



4. Determine the prime factorization of each number using factor trees. Then, write the prime factorization using powers.

a. $180 =$

b. $81 =$

c. $48 =$

2

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Essential Ideas

- Prime factorization is the expression of a composite number as a product of prime numbers.
- The Associative Property of Multiplication states that changing the grouping of the factors in a multiplication statement does not change the product.
- A factor tree is a way to organize and help you determine the prime factorization of a number. It uses branches to show how a number is broken down into prime numbers.
- Powers are used to express a repeated factor. The base of a power is the factor and the exponent of the power is the number of times the factor is repeated.
- The Fundamental Theorem of Arithmetic states that every natural number is either prime or can be uniquely written as a product of primes.

Common Core State Standards for Mathematics

6.NS The Number System

Compute fluently with multi-digit numbers and find common factors and multiples.

4. Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor.

6.EE Expressions and Equations

Apply and extend previous understandings of arithmetic to algebraic expressions.

1. Write and evaluate numerical expressions involving whole-number exponents.

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WORKSHOP RESOURCES • COURSE 1 • COURSE 2 • COURSE 3

2

Overview

The volume of a rectangular solid is used as a context. Students write possible sets of dimensions of the rectangular solid and express the sets of dimensions as the product of prime numbers. Prime factorization is introduced and students will use the Associative Property of Multiplication to understand the grouping of prime factors does not affect the value of the product.

Students use factor trees to organize the prime factorization of a number. Power notation is used as a shortcut to write repeated multiplication of prime factors. The Fundamental Theorem of Arithmetic will help students understand that every natural number is either prime or can be uniquely written as a product of primes.

Warm Up

Write two different factor pairs for each number.

1. 18

$$2 \times 9, 3 \times 6$$

2. 24

$$3 \times 8, 4 \times 6$$

3. 30

$$2 \times 15, 3 \times 10$$

4. 36

$$6 \times 6, 4 \times 9$$

5. 75

$$3 \times 25, 5 \times 15$$

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Does your school have a fish tank? Do you or any of your classmates have a fish tank at home?

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Problem 1

Volume is defined and the formula for the volume of a rectangular solid is given. Students use a specified volume for a rectangular fish tank and list possible sets of dimensions. They will express each set of dimensions as the product of prime numbers, and conclude that each product has the same factors, but the factors are written in different orders. Prime factorization is introduced and the Associative Property of Multiplication will show students that the order in which the prime factors are grouped does not change the product of the factors. Students then rewrite sets of factors using the Associative Property of Multiplication and perform prime factorization.

Grouping

- Ask a student to read the introduction before Question 1 aloud. Discuss the context as a class.
- Have students complete Questions 1 through 4 with a partner. Then share the responses as a class.

Discuss Phase, Introduction

- What is an oceanographer?
- What do oceanographers do?
- What is a think tank?

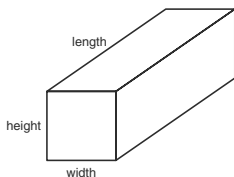
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Recall that volume is the amount of space occupied by an object. The only rule you need to follow is that none of the dimensions can be 1 foot. To calculate the volume of a tank, multiply the tank's length by its width and by its height.



Let l = length

Let w = width

Let h = height

$$V = lwh$$



1. Complete the table by listing three possible tank dimensions for the volume given.

Length (ft)	Width (ft)	Height (ft)	Volume (ft ³)
4	6	10	240
12	5	4	240
15	2	8	240



- How does the volume of a rectangular fish tank limit the possible dimensions of the tank?
- How do you distinguish between the length and the width?

Share Phase, Questions 1 through 4

- Are there other possible dimensions for the tank besides the ones you included in the table?
- Why is the measurement of volume a cubic measurement?
- If the orders in which the prime factors of a product are written are different, does that change the value of the product?

Grouping

- Ask a student to read the information about prime factorization aloud. Complete Question 5 as a class.
- Ask a different student to read the information about the Commutative Property of Multiplication and the Associative Property of Multiplication aloud. Then discuss the examples as a class.

2. How did you determine the possible dimensions of the tank? How did you verify you were correct?

I started with a factor pair of $240 = 4 \times 60$, and then determined another factor pair of 60 to be 6×10 . I checked my answer by multiplying all three factors together to make sure the product was 240.



3. Compare your possible tank dimensions to your classmates' dimensions. What do you notice?

There are other combinations of three factors which result in a product of 240. My classmates had the same factors, but in a different order.

4. Write the volume as the product of prime numbers for each row in your table. What do you notice?

$$2 \times 2 \times 2 \times 3 \times 2 \times 5 = 240$$

$$3 \times 2 \times 2 \times 5 \times 2 \times 2 = 240$$

$$3 \times 5 \times 2 \times 2 \times 2 \times 2 = 240$$

Each product has the same factors, but the factors are in different orders.

I can write any whole number—except for 1—as a product of prime numbers. So, 4 as a product of prime numbers would be 2×2 .



A natural number written as the product of primes is the *prime factorization* of that number. **Prime factorization** is the long string of factors that is made up of all prime numbers.

5. Does the order of factors make a difference? Explain your reasoning.
Order does not make a difference because when I multiply all the factors together, I get the original number.

Recall that the Commutative Property of Multiplication states that changing the order of two or more factors in a multiplication statement does not change the product. For instance, look at the example shown.

$$3 \times 5 = 5 \times 3$$

$$15 = 15$$

2

Discuss Phase, Definitions

- What is a prime factor?
- How is the Associative Property of Multiplication used?
- How does this meaning of the word associate reflect the Associative Property of Multiplication?

Grouping

Have the students to complete Questions 6 through 9 with a partner. Then share the responses as a class.


Share Phase, Questions 6 through 9

- If a product has 3 factors, how many different ways can they be associated?
- If a product has 4 factors, how many different ways can they be associated?
- If a product has 5 factors, how many different ways can they be associated?
- If a product has 6 factors, how many different ways can they be associated?

The **Associative Property of Multiplication** states that changing the grouping of the factors in a multiplication statement does not change the product. For any numbers a , b , and c , $(a \times b) \times c = a \times (b \times c)$.

For instance, look at the example of the Associative Property of Multiplication.

$$\begin{aligned} (2 \times 3) \times 3 &= 2 \times (3 \times 3) \\ 6 \times 3 &= 2 \times 9 \\ 18 &= 18 \end{aligned}$$

 6. What is the difference between the Commutative Property of Multiplication and the Associative Property of Multiplication?

The difference between the two properties is that the Commutative Property of Multiplication states that changing the order of the factors does not change the product, while the Associative Property of Multiplication states that changing the grouping of the factors does not change the product.

7. Rewrite each set of factors using the Associative Property of Multiplication.

a. $(3 \times 4) \times 5 = \underline{\hspace{2cm}} 3 \times (4 \times 5) \underline{\hspace{2cm}}$

b. $6 \times (5 \times 3) = \underline{\hspace{2cm}} (6 \times 5) \times 3 \underline{\hspace{2cm}}$


c. $(2 \times 3) \times 5 = \underline{\hspace{2cm}} 2 \times (3 \times 5) \underline{\hspace{2cm}}$

8. Determine the product for each set of factors in Question 7. Which set of factors did you use, the set given or the set you rewrote using the Associative Property? Explain your choice.

a. 60
 I used the set of factors I rewrote. It was easier to multiply 4×5 first, then multiply by 3.

b. 90
 I used the set of factors I rewrote. It was easier to multiply 6×5 first, then multiply by 3.

c. 30
 I used the set of factors given. It was easier to multiply 2×3 first, then multiply by 5.



The Properties can help you add and multiply numbers more efficiently because you can reorder and regroup.



9. Write the prime factorization of 240. List the factors in order from least to greatest.

$$2 \times 2 \times 2 \times 2 \times 3 \times 5$$

Problem 2

Students use factor trees to organize and determine the prime factorization of a number. They will complete factor trees for several numbers and write the prime factorization as a mathematical statement.

Grouping

Ask a student to read the introduction aloud. Discuss the definition and the worked example of a factor tree as a class.

Discuss Phase, Worked Example

- Do you think a tree is a good name for this diagram? Why or why not?
- Do you think it makes any difference which whole number factors you pick to start the factor tree?
- How are factor trees used to perform prime factorization?
- What would you equate with the roots of the tree?
- Can there be more than one factor tree for a number?
- Why does the factor tree work?

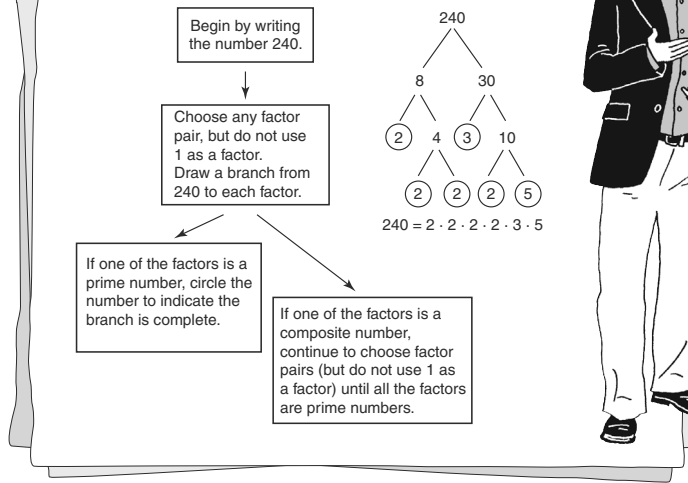
Problem 2 Factor Trees



A **factor tree** is a way to organize and help you determine the prime factorization of a number. Factor trees use branches to show how a number is broken down into prime numbers.

You can use the X or · symbols to represent multiplication. Make sure you use one symbol throughout each multiplication sentence.

An example of a factor tree for 240 is shown.



- What is the role of multiples in the factor tree?
- How can you tell if all of the factors are prime factors?

Grouping

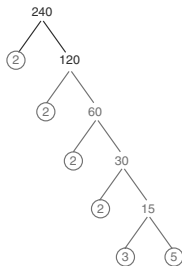
Have students complete Question 1 with a partner. Then share the responses as a class.

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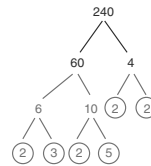
1. Complete each factor tree. Then, write the prime factorization as a mathematical statement.

a. 240



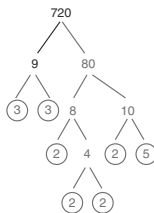
$240 = 2 \times 2 \times 2 \times 2 \times 3 \times 5$

b. 240



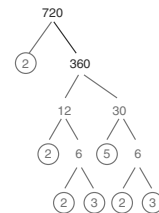
$240 = 2 \times 2 \times 2 \times 2 \times 3 \times 5$

c. 720



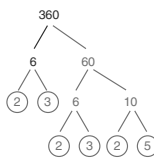
$720 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5$

d. 720



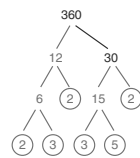
$720 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5$

e. 360



$360 = 2 \times 2 \times 2 \times 3 \times 3 \times 5$

f. 360



$360 = 2 \times 2 \times 2 \times 3 \times 3 \times 5$

Grouping

- Ask a student to read the information about powers, bases, and exponents aloud. Discuss the definitions and example as a class.
- Have students complete Questions 2 and 3 with a partner. Then share responses as a class.

Share Phase, Questions 2 and 3

- What is the purpose of the power notation? Why use it?
- What is the base with respect to power notation?
- How do you know which number is the base?
- What is the exponent with respect to power notation?
- How do you know which number is the exponent?
- How many powers are there?
- What else can be expressed using powers besides prime factorization?



Did you notice that 2 and 3 were repeated factors of the prime factorizations in the factor trees you created? The prime factorizations were long, and you had to write a lot of numbers. Because 2 and 3 are repeated factors, you can write their repeated multiplication using a shorthand method. This method is called a **power**. A **power** has two elements: the base and the exponent, as shown.

$$2 \times 2 \times 2 \times 2 = 2^4$$

The **base** of a power is the factor that is multiplied repeatedly in the power, and the **exponent** of the power is the number of times the base is used as a factor of repeated multiplication.

You can read a power in the following ways:

“2 to the fourth power”
 “2 raised to the fourth power”
 “the fourth power of 2”



2. Identify the base and exponent in each power. Then, write each power in words.

a. 7^2

The base is 7, and the exponent is 2.

“Seven squared.”

“Seven raised to the second power.”

b. 4^3

The base is 4, and the exponent is 3.

“Four cubed.”

“Four raised to the third power.”



3. Write the prime factorizations using powers for the factor trees you completed in Question 1.

a. $2^4 \times 3 \times 5 = 240$

b. $2^4 \times 3 \times 5 = 240$

c. $2^4 \times 3^2 \times 5 = 720$

d. $2^4 \times 3^2 \times 5 = 720$

e. $2^3 \times 3^2 \times 5 = 360$

f. $2^3 \times 3^2 \times 5 = 360$

2

Grouping

Ask a student to read the Fundamental Theorem of Arithmetic aloud. Then discuss the theorem as a class.

Discuss Phase, Definition

- What is the big idea behind the Fundamental Theorem of Arithmetic?
- What does “uniquely written as a product of primes” mean?
- Can there be more than one set of prime factors for a product?

Grouping

Have the students complete Questions 4 through 8 with a partner. Then share the responses as a class.

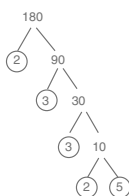


The **Fundamental Theorem of Arithmetic** states that every natural number is either prime or can be written as a unique product of primes.

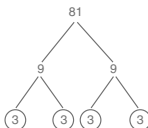


4. Determine the prime factorization of each number using factor trees. Then, write the prime factorization using powers.

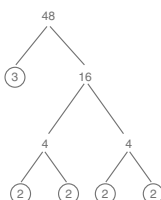
a. $180 = 2^2 \times 3^2 \times 5$



b. $81 = 3^4$



c. $48 = 2^4 \times 3$



Share Phase, Questions 4 through 8

- How did you decide which two factors to use to start your factor tree?
- Are these factors prime or composite numbers?
- What do you need to do if they are composite numbers?
- How did you know when you were finished factoring?
- What is the relationship between the number of prime factors and the value of the product?

5. Circle the prime factorization for the number 36. How do you know?

$2 \times 2 \times 3 \times 3$

2×18

3×12

I know the prime factorization I chose for the number 36 from the selections is correct because all the numbers are prime numbers, and the product is 36.

6. How do you know that the answer you selected is the only prime factorization for 36?

The Fundamental Theorem of Arithmetic tells me that there is a unique prime factorization for every number. For the selection, there was only one prime factorization choice that had a product of 36.

7. Use exponents to make each prime factorization true.

a. $24 \stackrel{\underline{\quad}}{=} 2 \times 3$
 $24 = 2^3 \times 3$

b. $45 \stackrel{\underline{\quad}}{=} 3 \times 5$
 $45 = 3^2 \times 5$

c. $54 \stackrel{\underline{\quad}}{=} 2 \times 3$
 $54 = 2 \times 3^3$

d. $72 \stackrel{\underline{\quad}}{=} 2 \times 3$
 $72 = 2^3 \times 3^2$



8. Khalil and Reyna are comparing their favorite numbers. Khalil says that the prime factorization of his number has three factors. Reyna says that the prime factorization of her number has four factors. Khalil says that his number must be less than Reyna's number since it has fewer factors in its prime factorization. Reyna thinks that Khalil is incorrect. Who do you think is correct? Use an example to support your answer.
- Reyna is correct. The number of factors doesn't matter, but the value does. Khalil's factors may all be higher values than Reyna's. For instance, if Khalil chose 30, his prime factors are 2, 3, and 5. If Reyna chose 24, her factors are three 2s and one 3. Although Reyna has more factors, they are 2s and 3s. Khalil has a 5 in his prime factorization.



Be prepared to share your solutions and methods.

Follow Up

Assignment

Use the Assignment for Lesson 2.1 in the Student Assignments book. See the Teacher's Resources and Assessments book for answers.

Skills Practice

Refer to the Skills Practice worksheet for Lesson 2.1 in the Student Assignments book for additional resources. See the Teacher's Resources and Assessments book for answers.

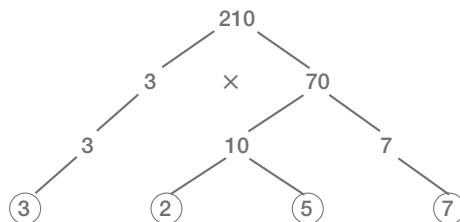
Assessment

See the Assessments provided in the Teacher's Resources and Assessments book for Chapter 2.

Check for Students' Understanding

Your school plans to make and sell homemade ice cream. They will be having a contest for the container design. One of the design requirements states that the length, width, and height must be whole numbers greater than 1, and the container must be designed to hold about 1 gallon of ice cream. One gallon of ice cream is approximately 210 cubic inches. Determine the possible dimensions of the container.

- Construct a factor tree for the number 210.



- Write the prime factorization of 210.

$$2 \times 3 \times 5 \times 7$$

- Use the prime factorization to list all of the possible dimensions of the container in the table.

Length (inches)	Width (inches)	Height (inches)	Volume (cubic inches)
10	7	3	210
21	5	2	210
7	5	6	210
15	7	2	210
35	3	2	210
5	3	14	210