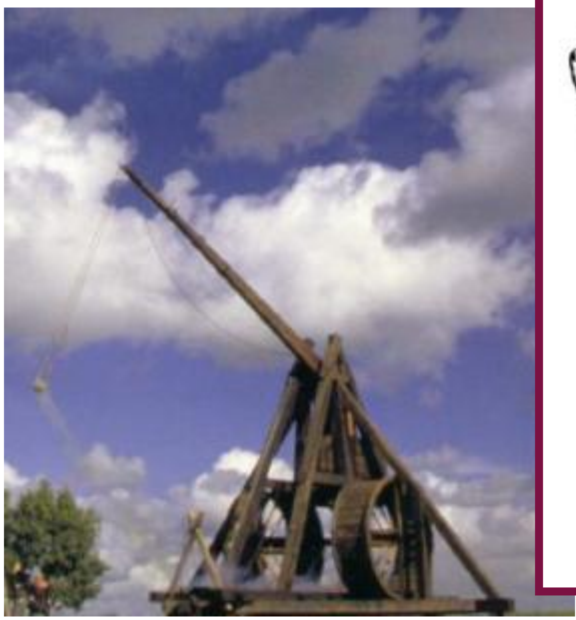
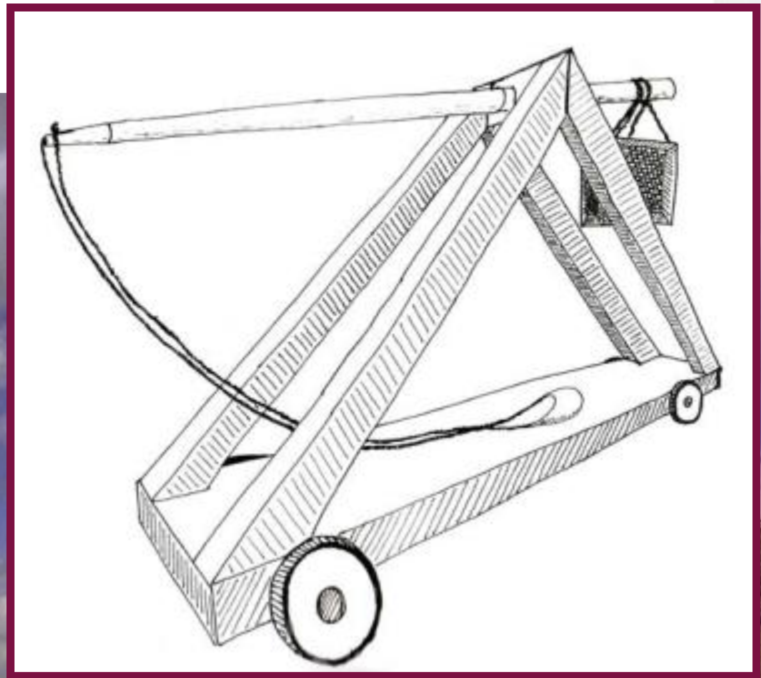
The Trebuchet

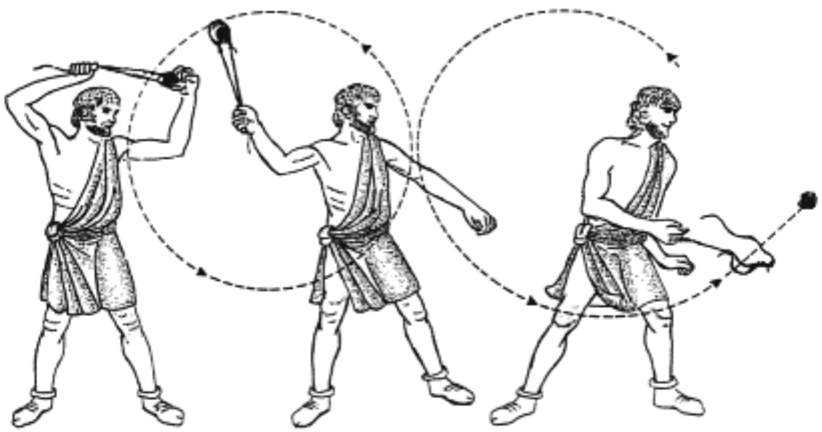
History and Physics of Mechanical War Engines

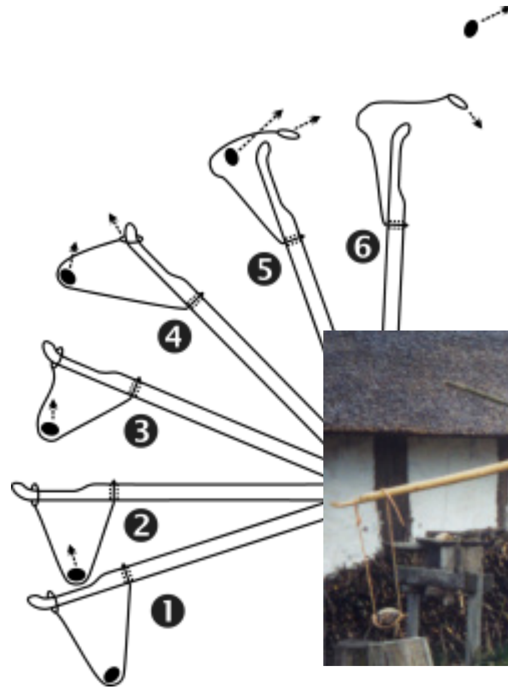

Sling

The *shepherd's sling* is one of the oldest projectile weapons. Sling effectively extends the length of the throwing arm by about 50 cm. Before the composite bow, slingers were as effective as archers.

The *stave sling* (or staff sling) is a sling on the end of a pole. Stave slings are more powerful because the stave can be made as long as two meters; Roman sources give their range as about one tenth of a mile.



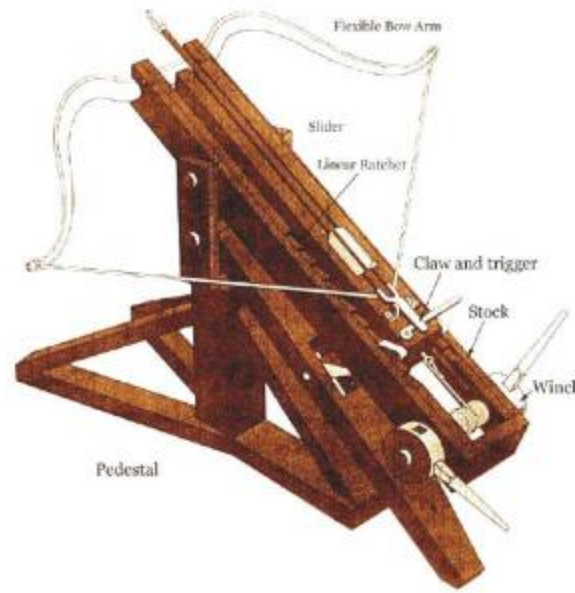
Shepherd's sling is thrown over or under handed, usually in a single swing.



Gastraphetes

The *gastraphetes* (`belly-bow´) was invented about 400 BC and is considered the first mechanical weapon. It was cocked by resting the stomach on the stock and pressing down.

The military effect of this weapon during the siege of Motya (Sicily) 397 BC encouraged the Greek engineers to develop a larger gastraphetes, mounted on a carriage with a windlass to cock it.



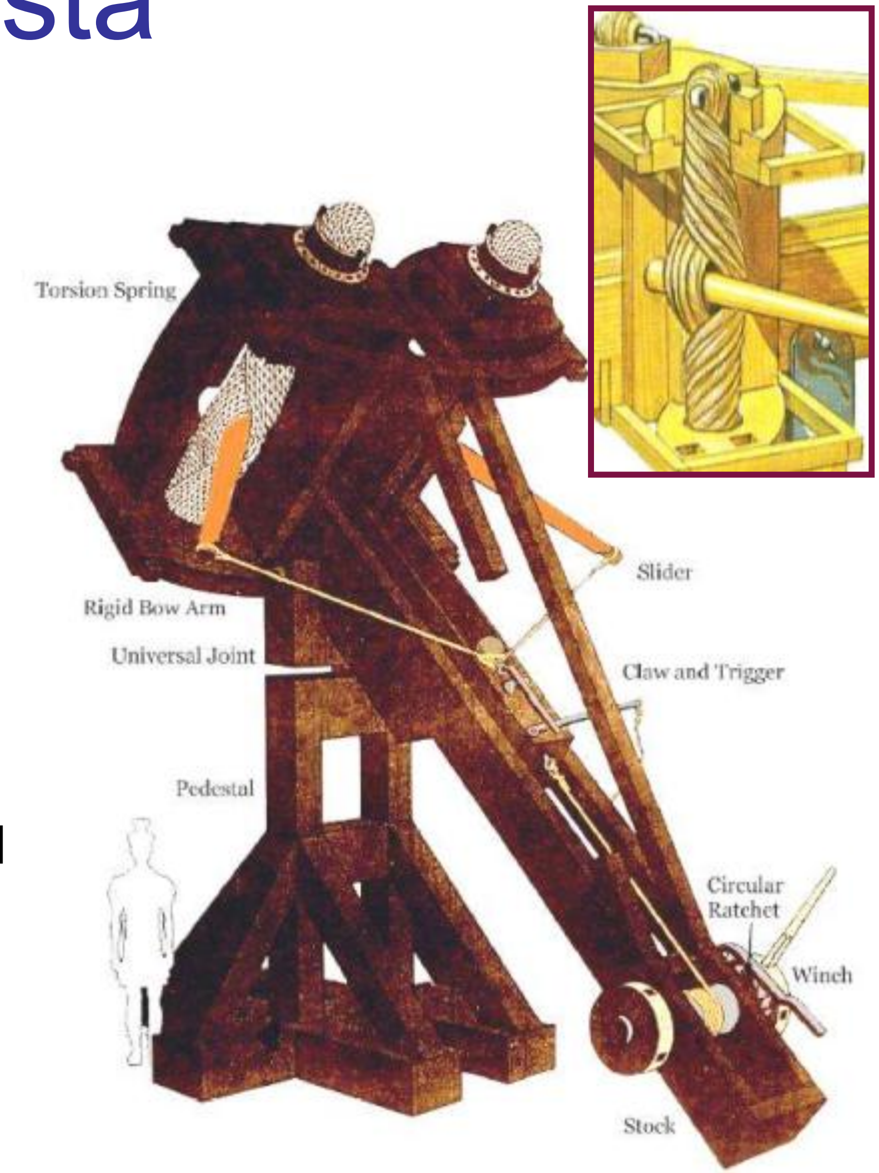
Ballista

Physical limits prevented further enlargement of the composite bow. In the mid-fourth century BC torsion springs, made from horse-hair or sinew, replaced the bow. Such a spring could be enlarged indefinitely.

Inscriptions on the Acropolis of Athens first mention torsion spring catapults there about 330 BC and Alexander the Great employed them on his campaigns.

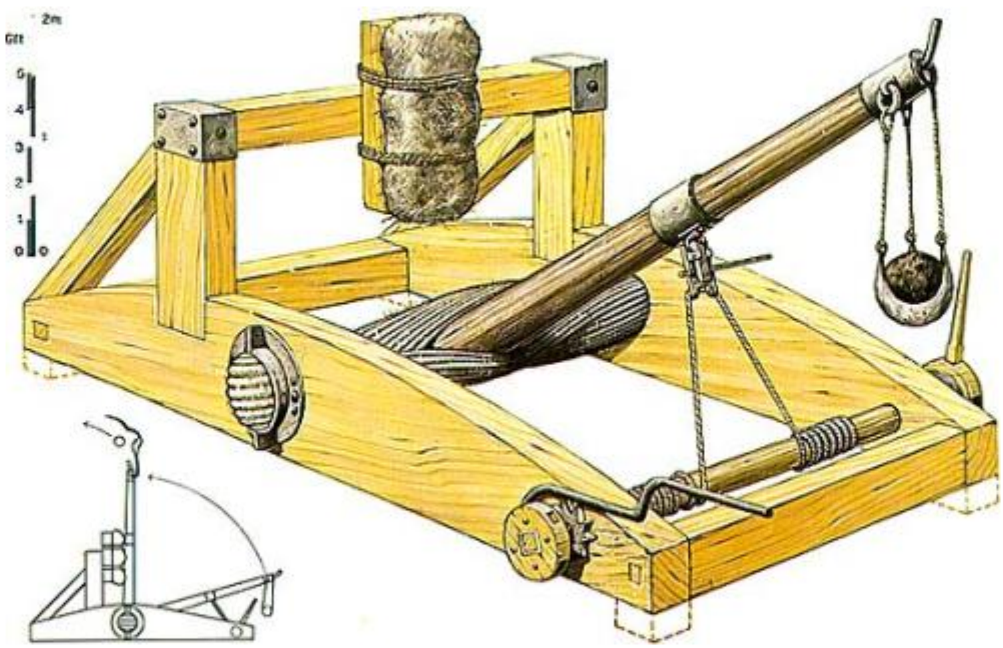
The Greeks used two types: the *euthytonon* for shooting arrows and the *palintonon* for throwing stone balls.

These war engines are better known today by their Roman name: the *ballista*.



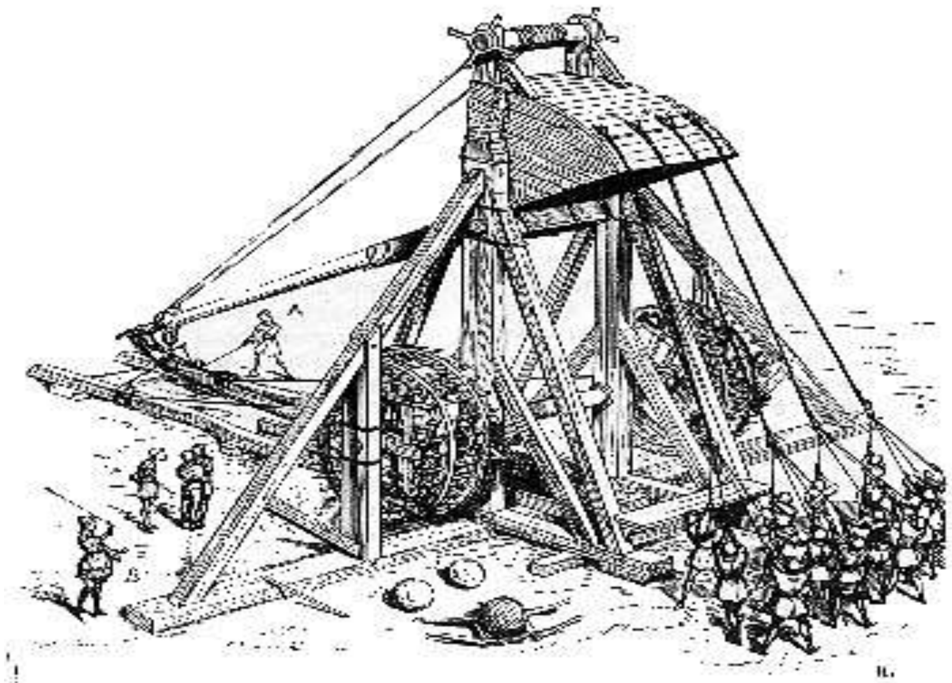
Onager

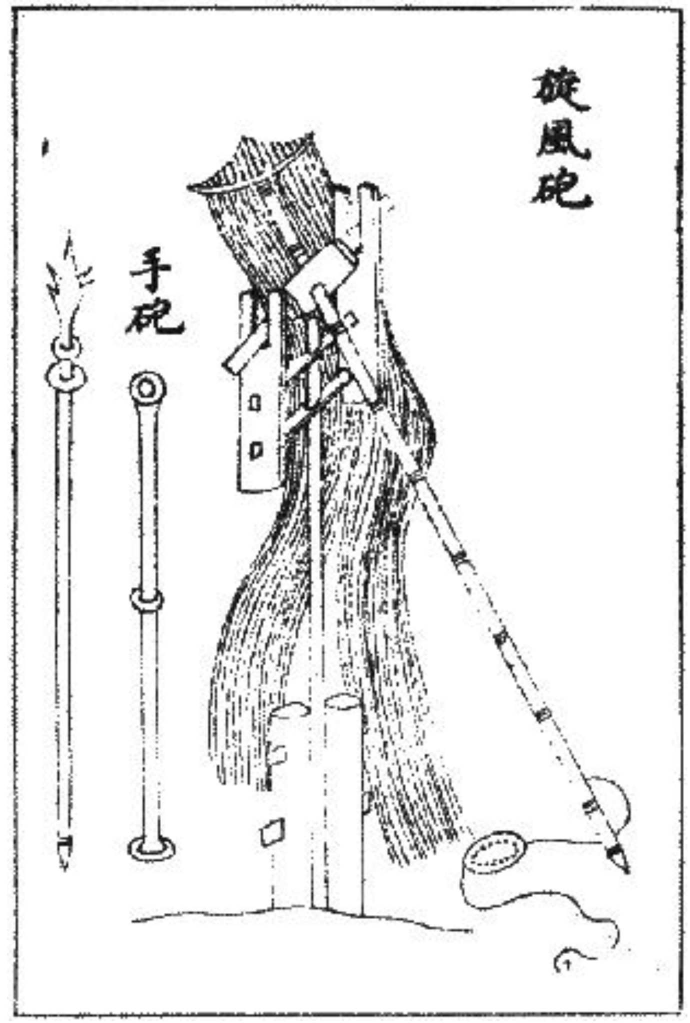
The onager (wild donkey), was a torsion engine similar to the ballista but only one arm. This catapult was the largest weapon used by the Roman army, almost exclusively for siege warfare.



Traction Trebuchet

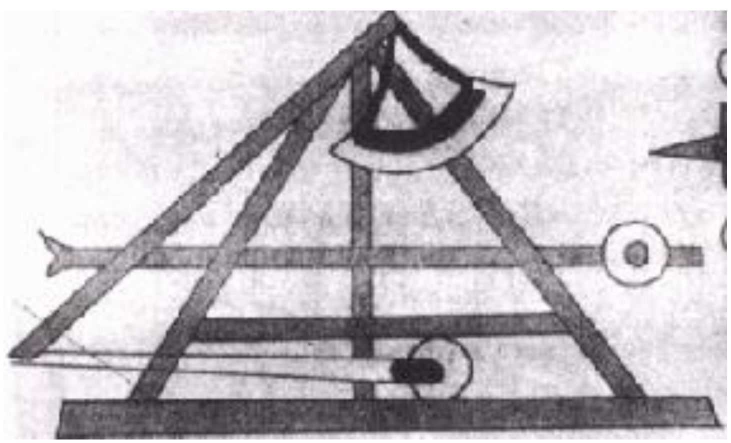
In the 6th century AD, the Chinese developed a large-scale stave sling with a crew that pulled the pole. Known today as a traction trebuchet.





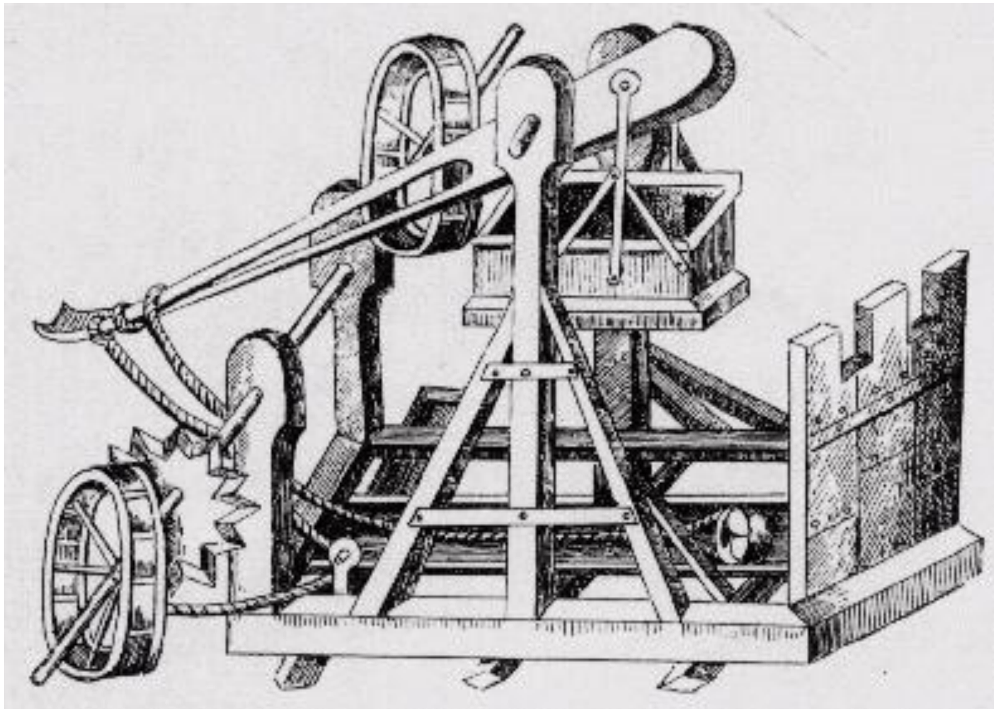
Trebuchet

- Trebuchet was the ultimate achievement in mechanical siege engines.
- Developed in the Far East and brought west in the 10th century by the Mongol and Muslim armies.



Trebuchet in Europe

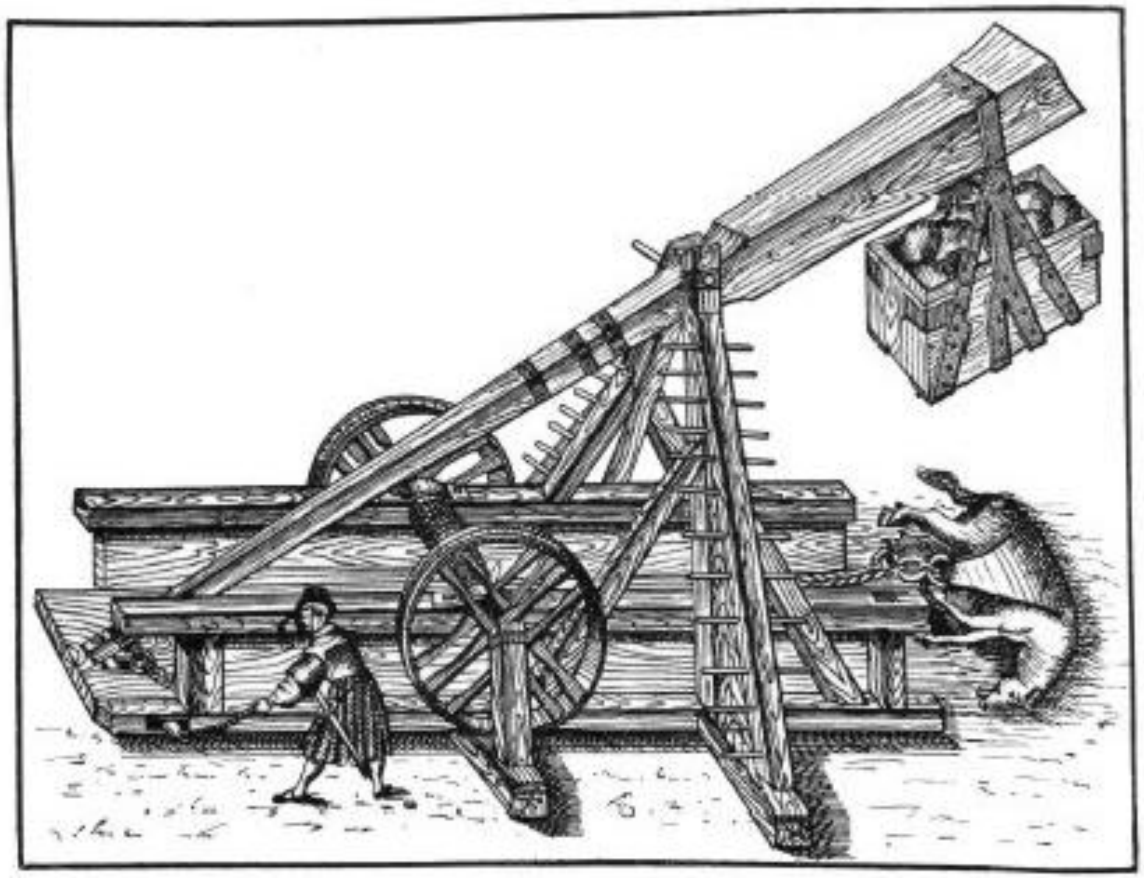
European armies encounter and adopt the trebuchet during the Crusades of the 12th century.



Biological Warfare

During medieval times, siege machines were the first weapons used for biological warfare.

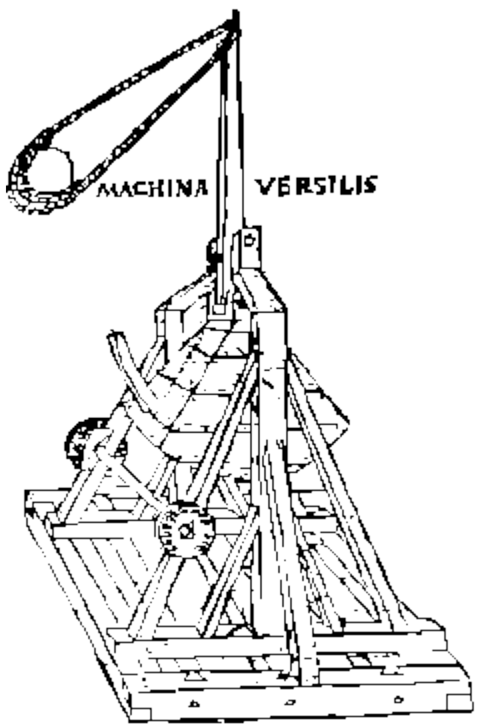
The carcasses of diseased animals and those who had perished from the Black Death were thrown over castle walls to infect those barricaded inside.

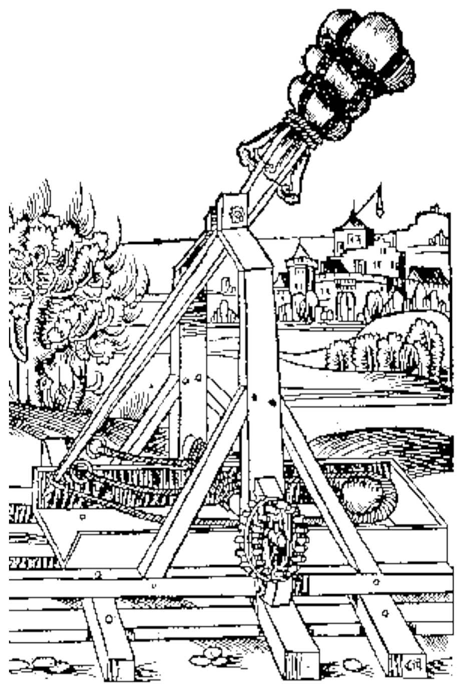


Casting a dead horse into a besieged town
(from *Il Codice Atlantico*, Leonardo da Vinci)

Mangonel

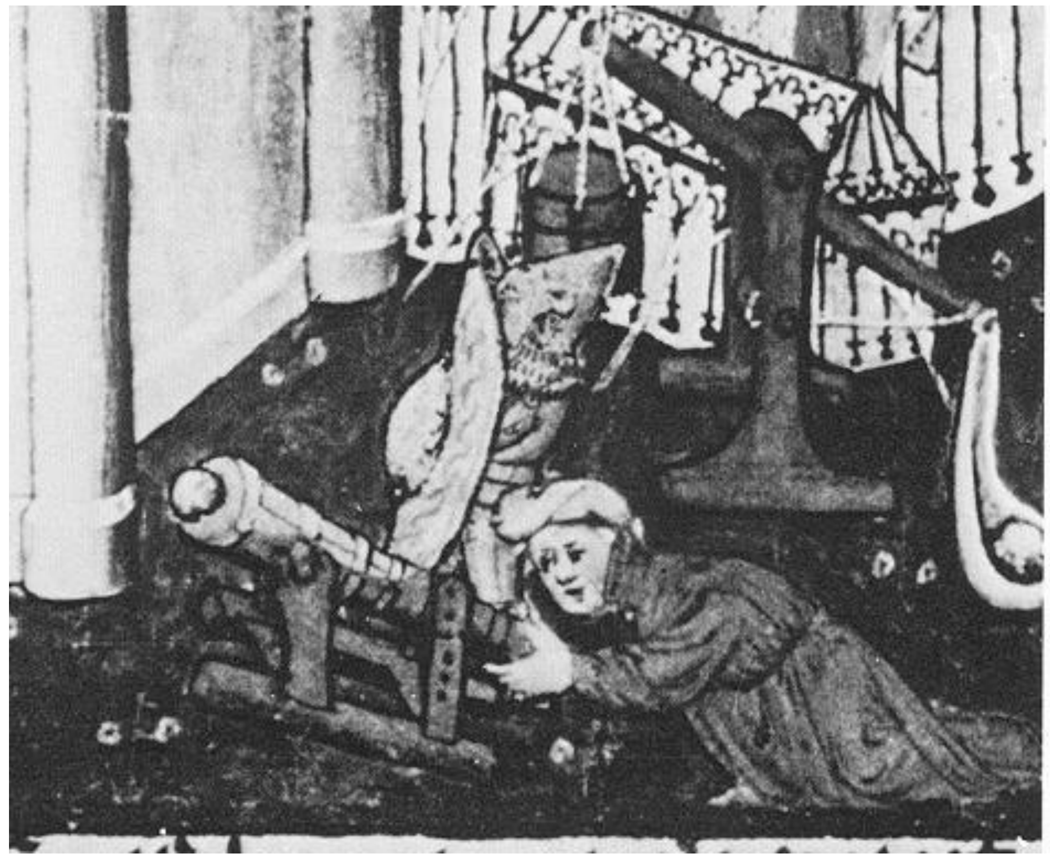
The mangonel is a simplified trebuchet using a fixed counter-weight.





Decline of the Trebuchet

In this siege scene, taken from a 14th century document, a trebuchet stands behind the weapon that would ultimately replace it, the cannon.

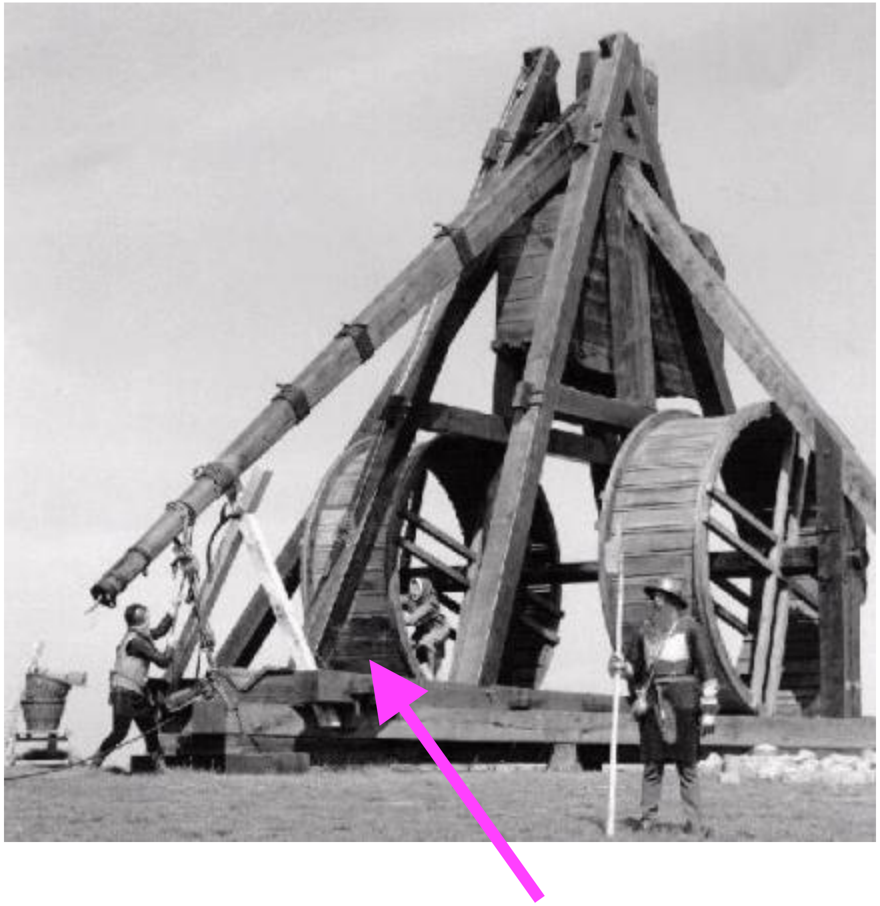


Return of the Trebuchet

Many modern reproductions of trebuchets exist today.

The trebuchet in Denmark's Medieval Center is built of oak and medieval craft techniques were used in its construction.

Has a ballast of 15 tons and is designed to throw projectiles of up to 300 kg.



Counterweight lifted by human powered "hamster wheels"

Punkin Chunkin

Yearly pumpkin throwing contest in Delaware

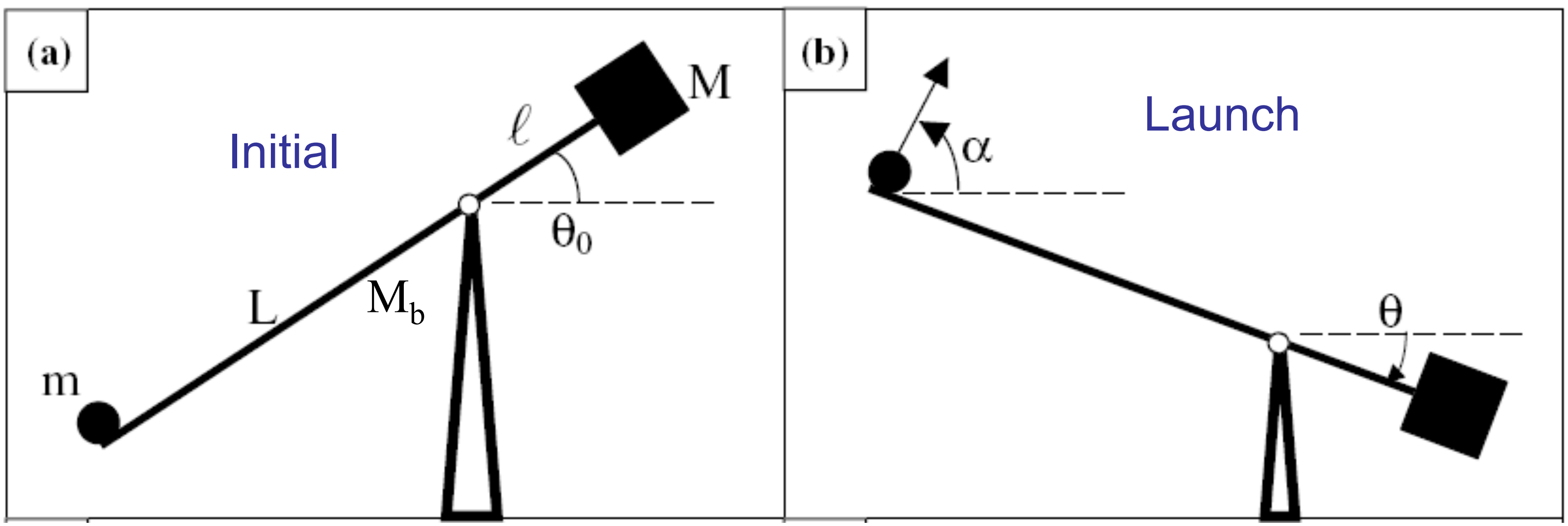
Ballista

Onager

Pumpkin-Throwing Trebuchet

Lever Arm Engine

Simplest case is a mangonel without a sling, which is called the lever arm engine. (see video with Holbrook jr.)



M = 10,000 kg, m = 100 kg, M_b = 2000 kg,

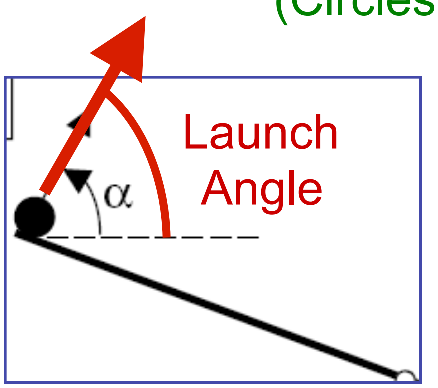
L + l = 12 m, 3 < L/l < 5, θ_0 = 60º

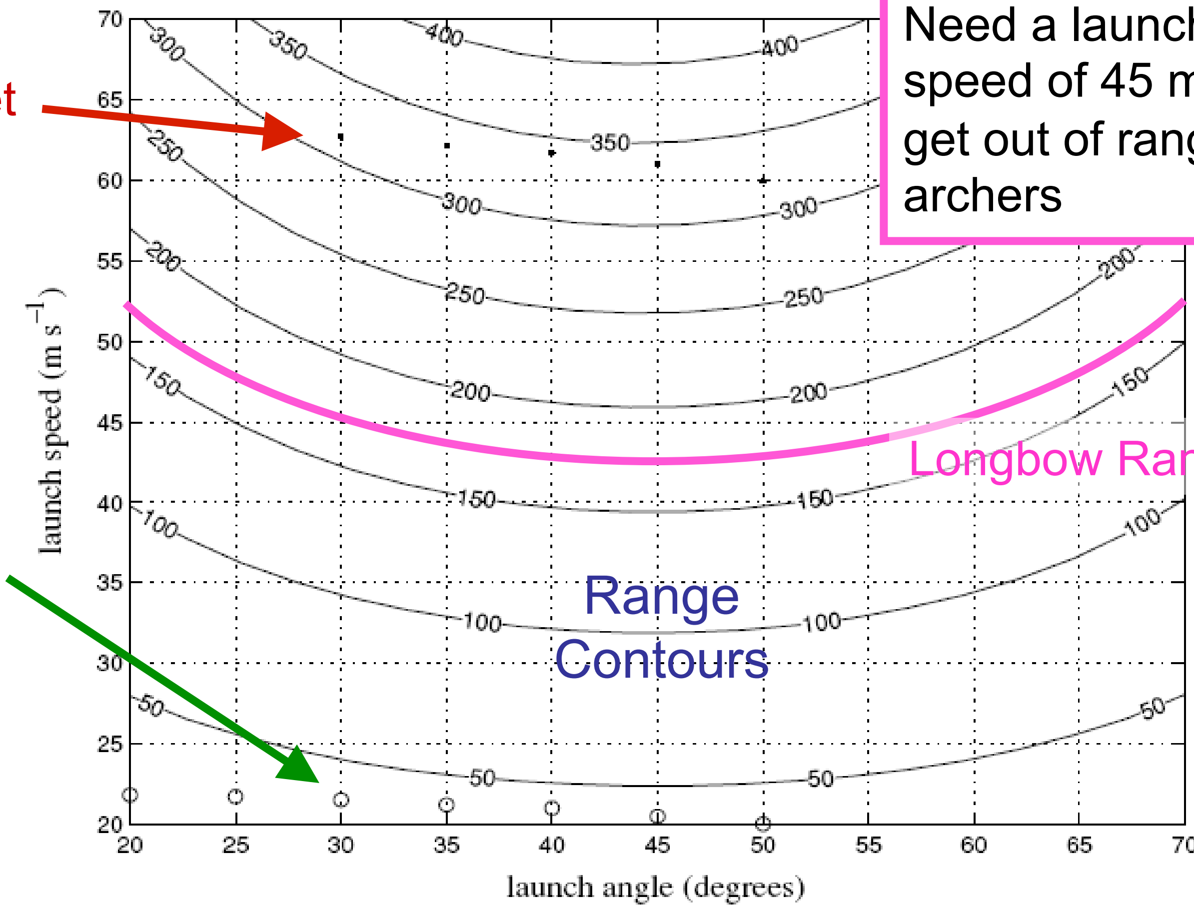
Launch Angle

Trebuchet (Dots)

Optimum angle, including effect of drag, is between 40º and 45º.

Lever Arm Engine (Circles)





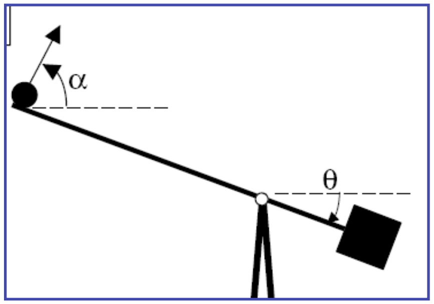
Need a launch speed of 45 m/s to get out of range of archers

Figure 2. Launch speed u versus launch angle α, with range contours. $u(\alpha)$ is shown for the simplest 'lever' engine (open circles, below the 50 m range contour) and for a fully-developed trebuchet (dots, between 300 m and 350 m). For both cases, maximum range is obtained for $40° < \alpha < 45°$.

Analysis of Lever Arm Engine

Calculate launch speed from energy conservation.

$$u(\alpha) = L\sqrt{2\frac{V}{I}(\sin(\theta_0) + \cos(\alpha))}.$$



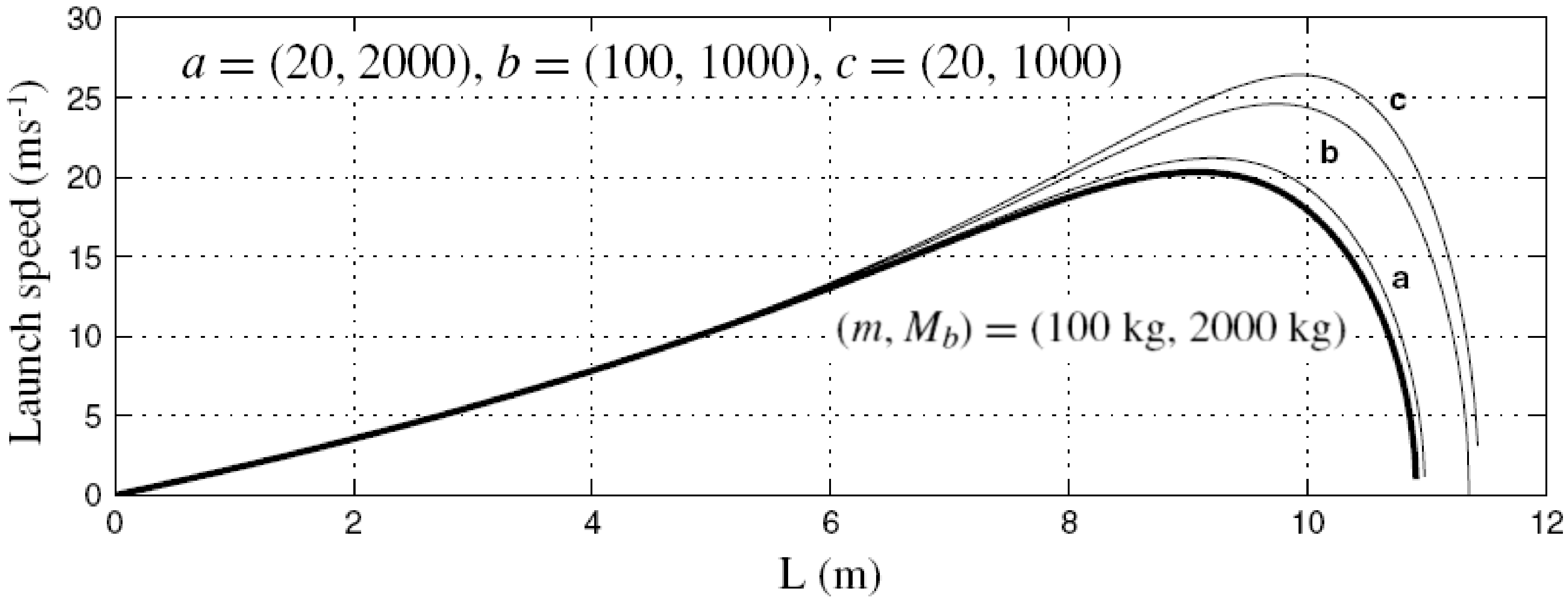
where

$$V \equiv Mg\ell - mgL - \frac{1}{2}M_b g(L - \ell)$$

$$I \equiv M\ell^2 + mL^2 + I_b \qquad I_b = \frac{1}{3}M_b\left(\frac{L^3 + \ell^3}{L + \ell}\right)$$

Note that if $M >> m, M_b$ then $u \propto L/\sqrt{l}$, *independent* of M.

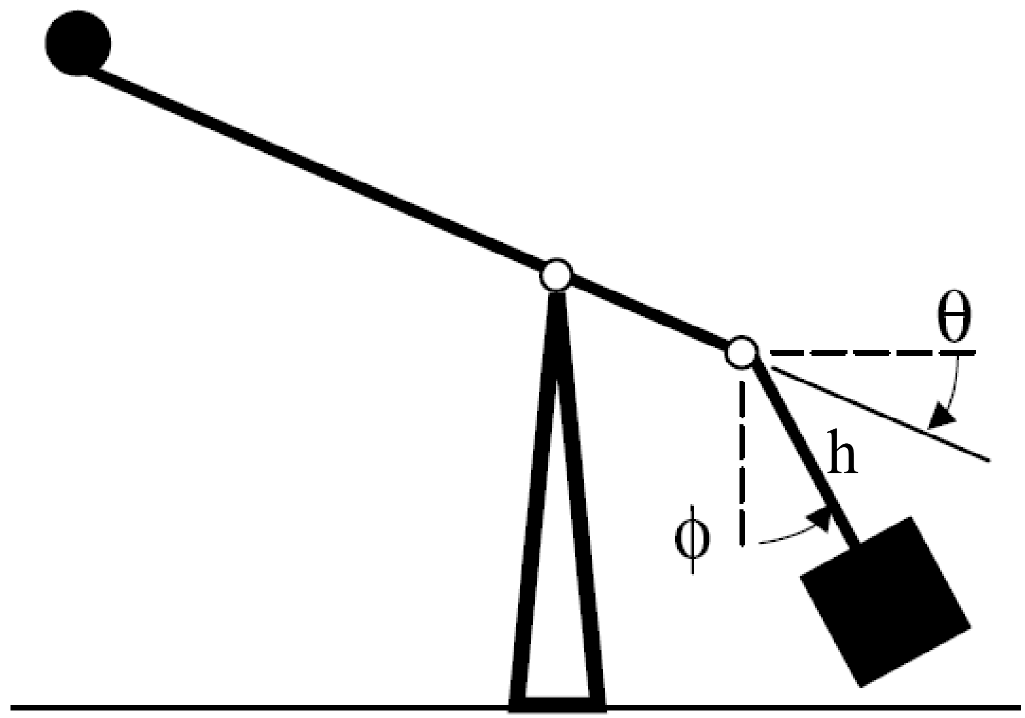
Results for Lever Arm Engine



For total arm length of 12 m, optimal throwing arm length about 9 m. Using lighter projectile or lighter beam increases launch speed yet range is still unacceptably short.

Sling-less Trebuchet

- Ballast attached to lever arm at second pivot.
- Pendulum has length h.



Sling-less Trebuchet

Lagrangian is

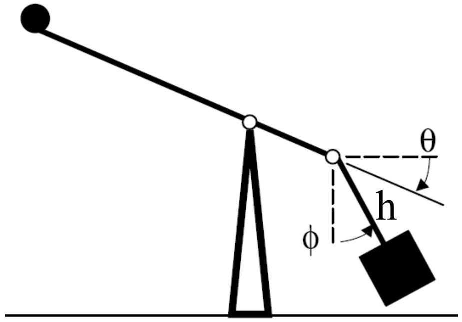
$$L_{\mathrm{II}} = \tfrac{1}{2} I \dot{\theta}^2 + \tfrac{1}{2} M h^2 \dot{\phi}^2 - M \ell h \dot{\theta} \dot{\phi} \sin(\theta + \phi) + V \sin(\theta) + M g h \cos(\phi)$$

Equations of motion are

$$I \ddot{\theta} = M \ell h \ddot{\phi} \sin(\theta + \phi) + M \ell h \dot{\phi}^2 \cos(\theta + \phi) + V \cos(\theta)$$

$$h \ddot{\phi} = \ell \ddot{\theta} \sin(\theta + \phi) + \ell \dot{\theta}^2 \cos(\theta + \phi) - g \sin(\phi).$$

Must solve numerically.



Sling-less Trebuchet Results

Rapid increase in projectile speed just before release due to surge in angular speed

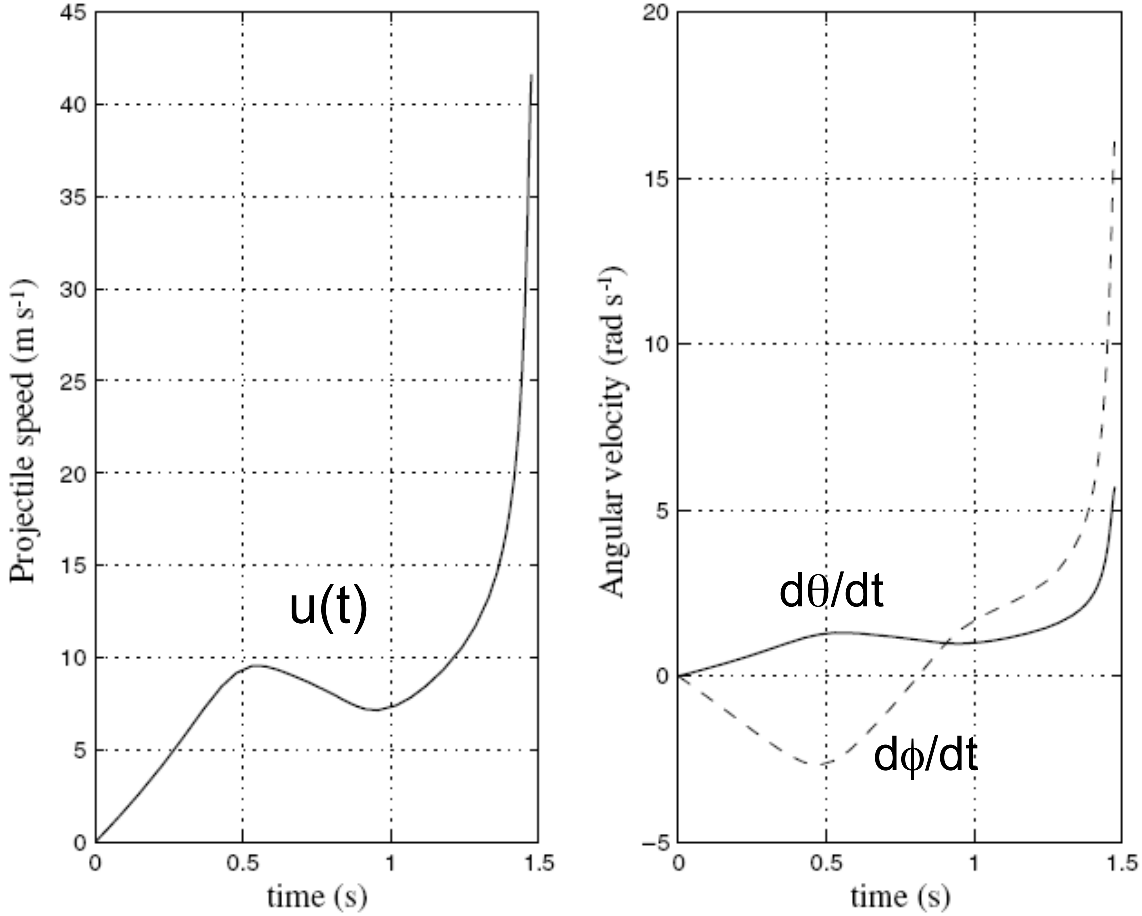
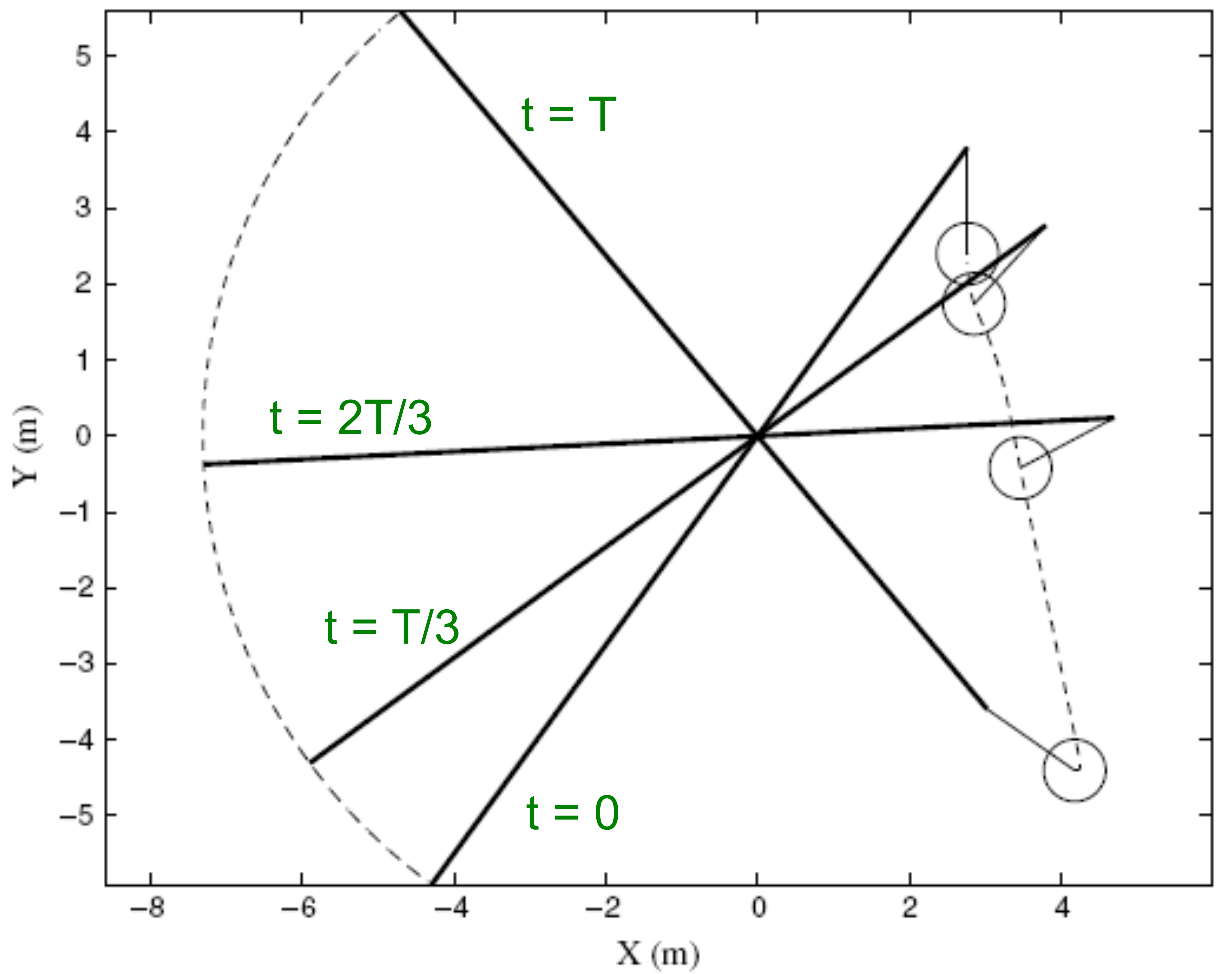


Figure 5. Case II. *Left*, projectile speed versus time; *right*, $\dot{\theta}$ (solid line) and $\dot{\phi}$ (broken line) versus time, obtained by numerical integration. The parameter values are $\theta_0 = 55°$, $L = 7.3$ m, $h = 1.4$ m, and the masses of equation (1). These parameters yield the maximum launch speed.

Trebuchet "Pump" Effect

Massive ballast falls in a nearly vertical motion.

Swing is like the first cycle of a driven oscillator, such as when "pumping" a playground swing.



Trebuchet Movie

Watch initial fall of the ballast and acceleration of the beam and sling just before release.

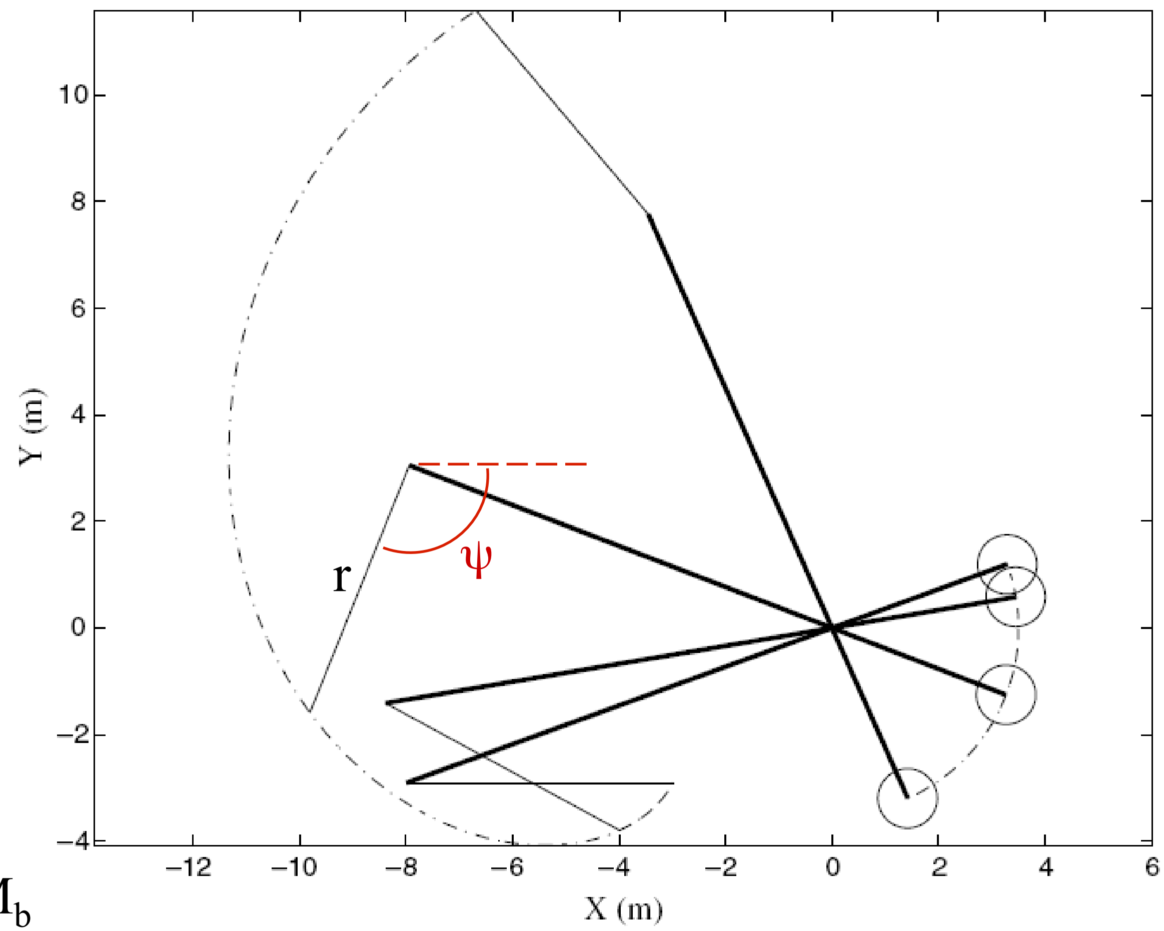
https://www.youtube.com/watch?v=EIphLNUpw2Y

Slinged Mangonel (Simplified)

Results almost the same as for the sling-less mangonel (i.e., lever-arm machine) but with a longer arm.

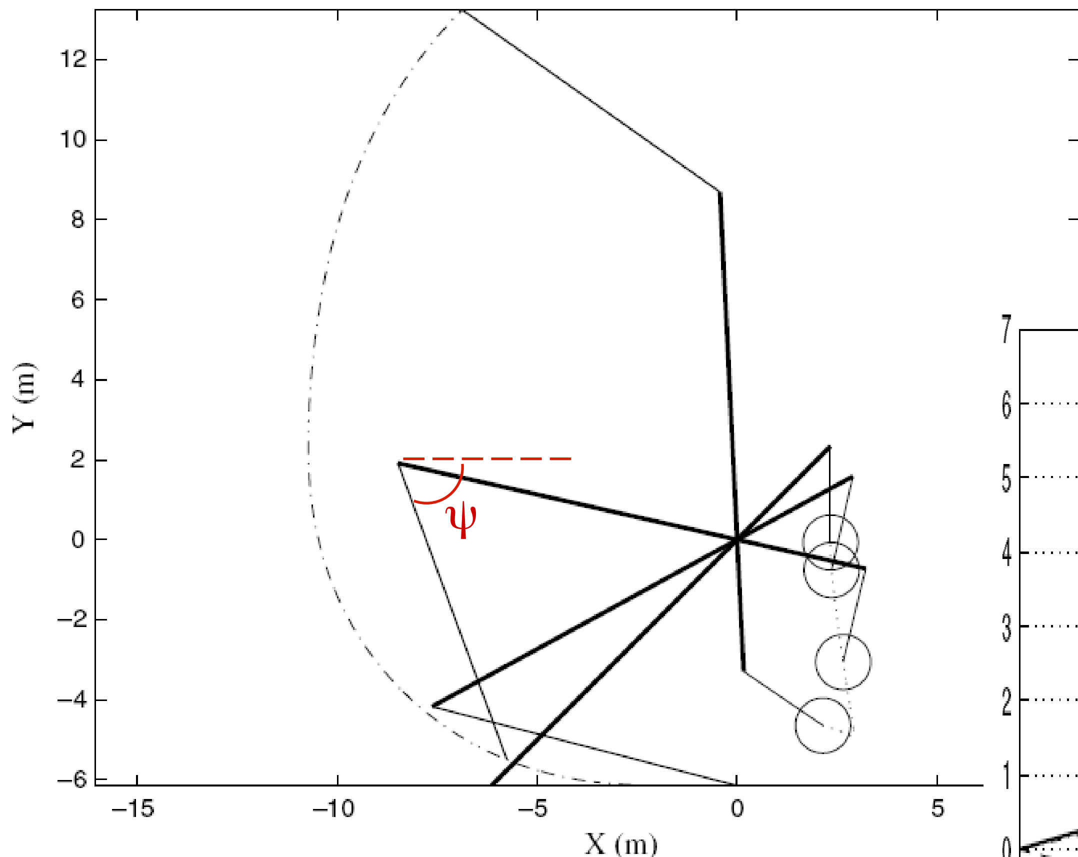
Sling has the advantage of added mechanical advantage without added weight.

Recall that if $M >> m, M_b$ then $u \propto L/\sqrt{l}$



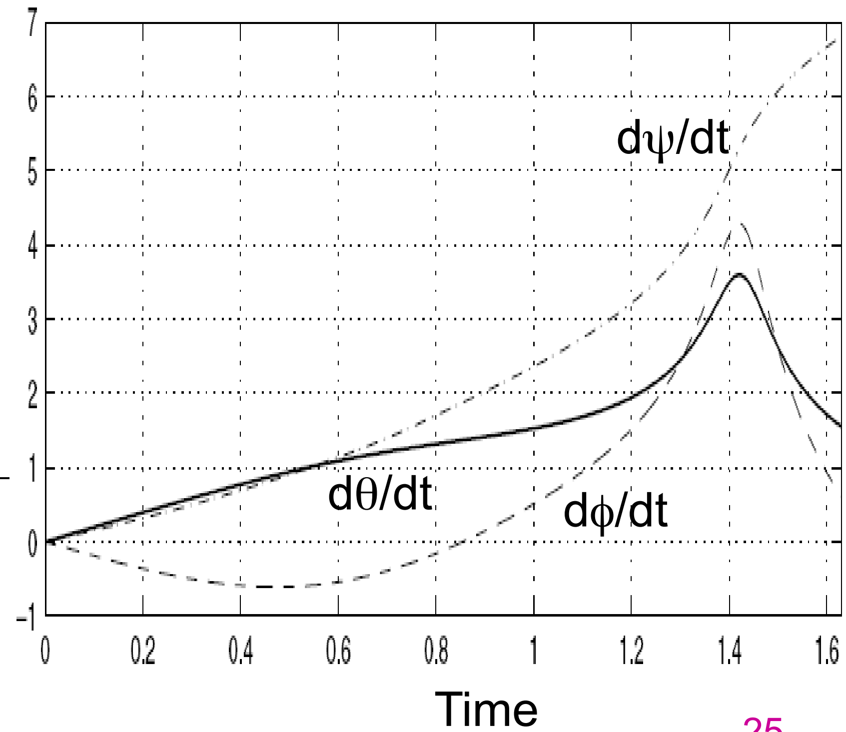
Simplification: Sling doesn't touch the ground

Slinged Trebuchet



Sling moves along the ground until lifted into the air.

Sling angle, ψ, pumped just prior to release



Analysis of Slinged Trebuchet

Lagrangian is

$$L_{\mathrm{IV}} = \tfrac{1}{2} I \dot{\theta}^2 + \tfrac{1}{2} M h^2 \dot{\phi}^2 + \tfrac{1}{2} m r^2 \dot{\psi}^2 - M \ell h \dot{\theta} \dot{\phi} \sin(\theta + \phi) - m L r \dot{\theta} \dot{\psi} \cos(\psi - \theta)$$
$$+ V \sin(\theta) + M g h \cos(\phi) + m g r \sin(\psi) - \underbrace{\lambda [L \sin(\theta) - r \sin(\psi) + L \sin(\theta_0)]}_{\text{Lagrange multiplier}}.$$

Equations of motion are

$$I \ddot{\theta} = M \ell h \ddot{\phi} \sin(\theta + \phi) + m L r \ddot{\psi} \cos(\psi - \theta) + M \ell h \dot{\phi}^2 \cos(\theta + \phi)$$
$$- m L r \dot{\psi}^2 \sin(\psi - \theta) + (V - \lambda L) \cos(\theta)$$
$$h \ddot{\phi} = \ell \ddot{\theta} \sin(\theta + \phi) + \ell \dot{\theta}^2 \cos(\theta + \phi) - g \sin(\phi)$$
$$r \ddot{\psi} = L \ddot{\theta} \cos(\psi - \theta) + L \dot{\theta}^2 \sin(\psi - \theta) + \left(g + \frac{\lambda}{m}\right) \cos(\psi)$$
$$\lambda \sin(\psi) = L(\sin(\theta) + \sin(\theta_0)).$$

where normal force, λ, is initially –mg; once $\lambda = 0$ it remains zero (sling leaves the ground).

Human Mangonel

Rich guy and his trebuchet
https://youtu.be/-wVADKznOhY

Computer Games

Trebuchet & Hollywood

Trebuchets appear in the battle scenes of two recent movies.

References

Main reference

Siege engine dynamics, M. Denny, *European Journal of Physics* **26** 561 (2005)

Supplementary reference

The Trebuchet, P. Chevedden and L. Eigenbrod, *Scientific American* **273** #1 (1995)

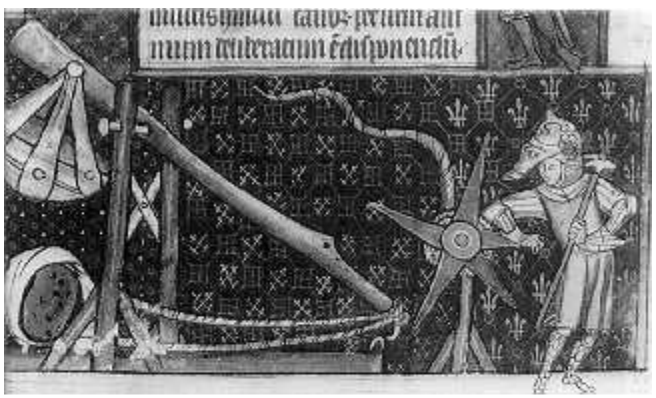
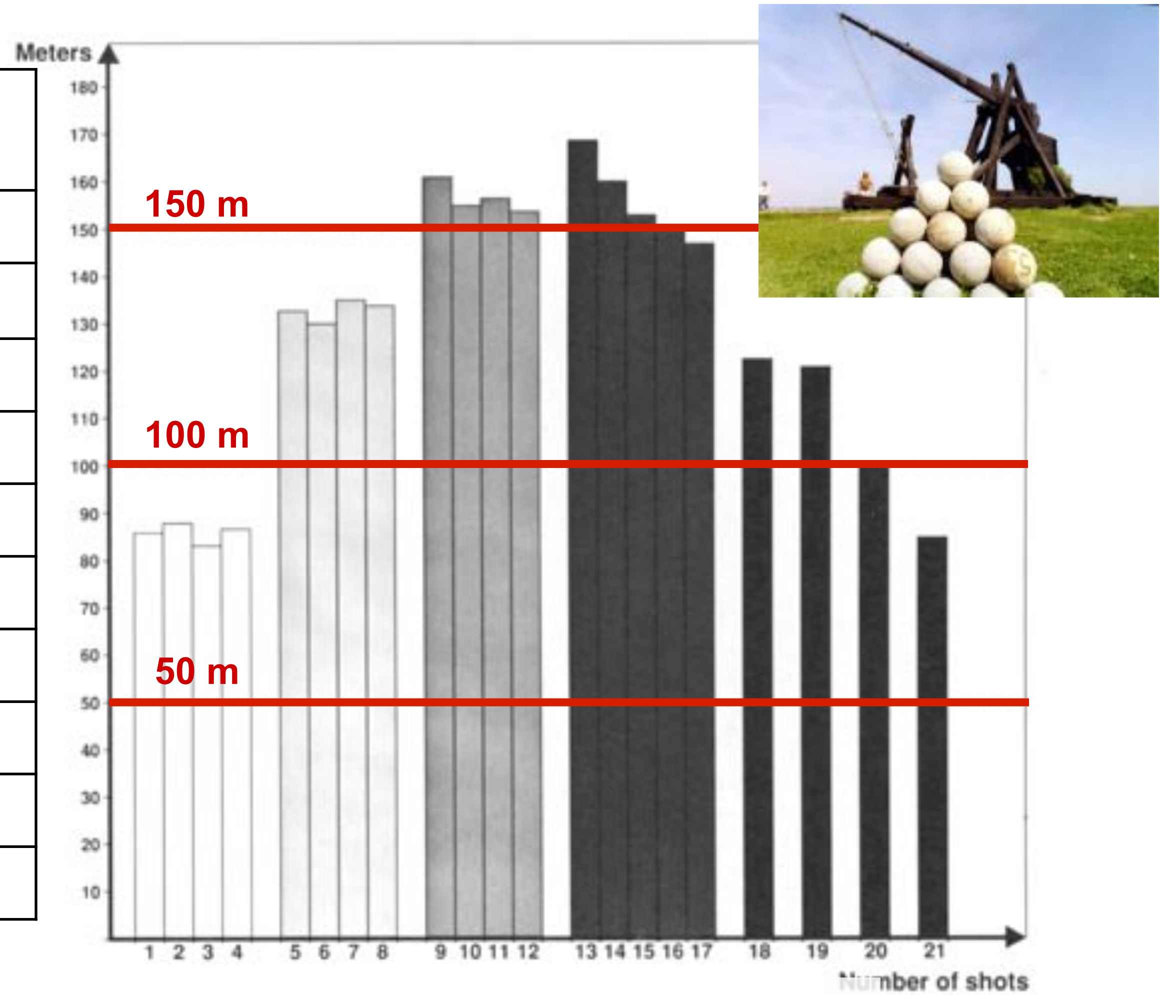


Fig. 16. Trick shot of the trébuchet firing.
Note the curved trajectory of the ball.
Photo: A. Knudsen.

Experimental Data

Shots	Ballast	Sling	Projectile
1-4:	1000 kg	5 m	15 kg.
5-8:	1500 kg	5 m	15 kg
9-13:	2000 kg	5 m	15 kg
14:	1000 kg	5 m	15 kg
15:	2000 kg	5 m	15 kg
16:	2000 kg	5 m	15 kg
17:	2000 kg	5 m	15 kg
18:	2000 kg	4 m	15 kg
19:	2000 kg	5 m	20 kg
20:	2000 kg	5 m	25 kg
21:	2000 kg	5 m	47kg



Experimental Reconstruction of a Medieval Trébuchet
by Dr. Peter Vemming Hansen, Nyköbing Falster, Denmark
Acta Archaeologica vol. 63, 1992, pp. 189

A simulation of trebuchets in action can be seen in the 2003 movie *The Lord of the Rings: The Return of the King*. The defenders of Minas Tirith fired their trebuchets from the top of the city's battlements. Although this appears very effective, it was never historically used, as castle walls were not big enough to hold a good trebuchet, and the forces exerted by such a siege engine on the walls would destroy them.

I made four catapults, the trebuchet arms of which would swing to 56 feet and would flip a hundred pound ball about 400 meters.

Modern Trebuchet in Action