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Introduction

This eBook was created as the perfect starter kit for any student planning to take the AP Calculus AB or BC exams. By beginning here, you'll have a better understanding of the test structure and key content, and you'll receive essential study advice to set yourself up for success.

This guide starts by explaining some of the foundational concepts in the AP Calculus AB-BC curriculum, then supplying practical advice for preparing for the test. With this eBook, you'll be able to confidently take action in creating your study plan and framing your goals.

This book features information from the <u>Albert Blog</u>, where new academic resources are published every day of the week. Be sure to regularly check the blog and subscribe to hear about our new posts.

E-mail us at hello@albert.io if you have any questions, suggestions, or comments!

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About Us

What is Albert?

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Zp, di Zp v.nds later. YO D

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Integration may be the most challenging concept in AP Calculus textbooks, but it is also arguably the most important! That being said, there is no shame in being nervous for the upcoming AP exam. In order to help quell your fears, I will walk you through the most important concepts of solving even the most challenging of integrals. Reading this AP Calculus review will outline all the tools you need, and hopefully calm the butterflies in your stomach.

Integrals can be split into two separate categories: definite and indefinite. A definite integral has bounds and yields a numerical answer, while an indefinite integral does not have bounds and yields an algebraic answer. Indefinite integrals will be addressed first, since the method for solving them is also used as a part of calculating definite integral solutions.

Indefinite integrals can be solved using two different methods, the anti-chain rule method and the substitution method. Solving an indefinite integral is the same thing as solving for the antiderivative, or undoing the derivative and solving for the original function.

We are now moving on to the fun part: seeing some examples. The anti-chain rule method is basically the reverse of the chain rule method implemented in the derivative section of your textbook. Instead of subtracting one from the exponent, you add one, and instead of multiplying the quantity by the new exponent, you divide it. Let's see an example:

Example 1: Integrate

So we set up the integral, keeping in mind the dx term:

$$\int x^3 + 6x^2 - 2x dx$$

Next, we integrate each term, adding one to the exponent and dividing the answer by the new exponent value:

$$\frac{1}{4}x^4 + 2x^3 - x^2 + c$$



The *c* term represents a constant value that is a result of integration and should never be forgotten, especially on multiple choice questions of the AP Calculus exam.

So far, so good, right? It gets worse. Let's now look at the substitution method, also known as u-substitution. What happens if you are presented with a function that can't be integrated with anti-chain rule? This occurs when multiple functions are multiplied together in ways that cannot be expanded, such as the function in Example 2 below. In this case, we must make a substitution of one of the quantities in order to solve the problem. This is where *dx* becomes much less like a decorative piece on the end of the integral and much more useful. Let's explain with an example:

Example 2: Integrate
$$9(x^2 + 3x + 5)^8 (2x + 3)$$

First, as always, set up the integral:

$$\int 9(x^2+3x+5)^8(2x+3)\,dx$$

As you can see, chain rule could not be easily applied without expanding the function, which would take a considerable amount of algebra. Let's see if the substitution rule will work. This requires choosing a smart quantity to substitute. I would choose the higher powered function first, but in some cases the choice might not be so clear.

Let
$$u = x^2 + 3x + 5$$



Since we are changing the function from being in terms of x to being in terms of u, we must solve for du and sub that into the integral function. Never fear! This is simpler than it seems.

If $u = x^2 + 3x + 5$, then du = 2x + 3dx, the derivative of u with respect to x.

We then make these substitutions into the above integral:

$$\int 9u^8(2x+3)\frac{du}{(2x+3)}$$

The (2x+3) term cancels out, and we are left with:

$$\int 9u^8 du$$

We can solve this using simple chain rule:

 $u^9 + c$

We then sub back in for *u* with our function in terms of *x*.

$$(x^2 + 3x + 5)^9 + c$$

Now we can clearly see the two main methods for solving indefinite integrals. That wasn't so bad, was it?

Let's take a quick side note to discuss how we treat trig functions in integrals. I'm sure you've tackled trig functions and their identities in your previous math classes. They are extremely important functions, but their integrals are relatively simple. The only method to solving these integrals is to memorize the solution. They were proved by famous mathematicians a long time ago, and in AP Calculus we don't need to worry about where they came from.

Function	Integral
$sin\left(x ight)$	$-cos\left(x ight)+c$
$cos\left(x ight)$	$sin\left(x ight) +c$
$sec(x)^2$	$tan\left(x ight)+c$
$sec\left(x ight)tan\left(x ight)$	sec(x) + c
$csc(x)^2$	$-cot\left(x ight)+c$
$csc\left(x ight)cot\left(x ight)$	$-csc\left(x ight)+c$

Table 1: The integrals of Trig Functions

Now that we are warmed up in the ways of solving indefinite integrals, we can move on to definite integrals. This discussion will involve some theory, but keep in mind that understanding some of the theory behind calculus will help aid your understanding.

Definite integrals are arguably the most important concept in calculus because they often yield real, hard numbers. From an engineering standpoint, this is ideal. Integral action is applied to many real-life problems such as finding velocity profiles of moving fluids in pipes. The best way to come to terms with definite integrals is to look at them from a graphical standpoint.

A definite integral represents the integral of some function from one point on the axis of a graph, *a*, to another point, *b*. We write this as:

$\int_{\mathbf{a}}^{\mathbf{b}} \mathbf{f}(\mathbf{x}) \, \mathbf{d}x$

This integral represents the area under a curve on the graph. This is a complicated concept to understand. The area under the curve could represent anything and normally depends on the concept behind the problem. For AP Calculus, the concept does not matter, and we just focus on the math; however, there may be concept questions asking what the integral represents, and what's the answer? That's right, the area under the curve. Let's look at an example to make this clear!



Example 3: Set up the integral for integrating the function $f(x) = x^2$ from x = -2 to x = 2 and graph it.

First, we write the integral in the standard notion:

$$\int_{-2}^{2} x^2 dx$$

We then graph this function and color in the section that we are interested in from -2 to 2.

As you can see, only the area that is literally between the function and the *x*-axis is colored in. When you compute a definite integral, you obtain the value of this area! Pretty neat, right? So how do we solve definite integrals on paper, without graphs? Let's find out!

When computing definite integrals in AP Calculus, we use the Fundamental Theorem of Calculus. If this name doesn't prove how important integration is in calculus, nothing will. The Fundamental Theorem states that if a function, f(x), is continuous from x=a to x=b, and F(x) is the antiderivative of f(x), then

$$\int_{a}^{b} f(x) \, dx = F(b) - F(a)$$

But what does continuous mean? In order for a function to be continuous, there must be no breaks, jumps, or vertical asymptotes in the function. So how do we solve these problems? The Fundamental Theorem, at first glance, does look like a lot of mumbo-gumbo.



Let's walk through an example.

Example 4: Solve this definite integral: $\int_{1}^{2} \sqrt{2x+1} dx$

First, we solve the problem as if it is an indefinite integral problem. The chain rule method would not easily apply to this situation so we will use the substitution method. We will let u = 2x + 1, and therefore, du = 2dx. Let's substitute these values in. The integral now becomes

$$\int_1^2 \sqrt{u} \frac{du}{2}$$

Solving the integral as if it were an indefinite integral, we obtain the solution

$$\frac{2}{3}u^{\frac{3}{2}} + c$$

We then substitute in the original function, and our answer becomes

$$\frac{2}{3}(2x+1)^{\frac{3}{2}}+c$$

The next part of the solution is solely based on the Fundamental Theorem. The constant value that is present in all indefinite integral solutions is a representation in the uncertainty of the answer. When we aren't give bounds or initial conditions, we cannot solve for the constant value. When we are solving definite integrals, however, we do have bounds, and a tangible solution can be obtained without the undefined constant.



In fact, the answer will have no algebraic parts at all! So in order to incorporate the Fundamental Theorem, we drop the constant term and we plug in to the Theorem equation:

$$\int_{1}^{2} \sqrt{2x+1} dx = F(2) - F(1)$$

In this case, $F(x) = \frac{2}{3}(2x+1)^{\frac{3}{2}}$, and so we calculate F(2) and F(1).

$$F(2) = rac{2}{3}(2(2)+1)^{rac{3}{2}} = 7.45$$

 $F(1) = rac{2}{3}(2(1)+1)^{rac{3}{2}} = 3.46$

Therefore,

$$\int_{1}^{2} \sqrt{2x+1} dx = 7.45 - 3.46 = 3.99$$

As you can see, the answer is purely numerical! Isn't that useful?

There are a few properties of definite integrals that are important to cover in any AP Calculus review. First, if a function is real and defined at a value of x = a, then:

$$\int_{a}^{a} f(x) \, dx = 0$$

This is apparent after you look at the Fundamental Theorem and see that F(a) - F(a) = 0 because they represent the same value.



Second, an integral can be reversed. What does this mean? This means that the bounds of the integral could be switched. The reverse integral would then just be the negative version. In calculus notation:

$$\int_{a}^{b}f\left(x
ight)dx=-\int_{b}^{a}f\left(x
ight)dx$$

Third, let's say that our function is continuous and defined at x = a, b, and c on a closed internal. Then, we could say that

$$\int_{a}^{c} f(x) dx = \int_{a}^{b} f(x) dx + \int_{b}^{c} f(x) dx$$

Definite integrals can also be incorporated into other theorems. For instance, the Mean Value Theorem can be adjusted to apply to integrals. Recall from your prior calculus knowledge that the Mean Value Theorem is normally defined as

$$f'\left(c
ight)=rac{f\left(b
ight)-f\left(a
ight)}{b-a}$$

We know that the Mean Value Theorem applies when f(x) is a continuous and defined function between x = a and b, then we know that there is at least one value of c such that the above equation holds true. When using definite integrals, the equation becomes:

$$f\left(c
ight)=rac{1}{b-a}\int_{a}^{b}f\left(x
ight)dx$$

This *c* value is also known as the average value, or the mean value, of a function and it can be calculated given the function and the interval of interest.

Another important theorem that uses definite integrals is the Second Fundamental Theorem of Calculus. Don't you think they could have come up with more creative names for these theorems? Mathematicians are a very blunt group. The Second Fundamental Theorem states that if a function is continuous on the desired interval from x = a to x = b, then the function $F(x)=\int_a^x f(t) dt$ has a derivative at every point x in the interval and therefore,

$$F'(x) = rac{d}{dx} \left(\int_a^x f(t) \, dt \right) = f(x)$$

Logical, right? I know these theorems are not the most exciting part of integration, but they are definitely as important as being able to solve an integral on the AP Calculus exam!

AP Calculus practice is one of the best ways to study for the big exam. College Board publishes the free response sections of the old exams online and I encourage you to work as many of the old exams as you can! In all types of math, practice really does make perfect. Here we will work an example free response question to get you started.

Example 5: 1999 AP Calculus AB Free Response, Question 1

A particle moves along the y-axis with velocity given by $v(t) = tsin(t^2)$ for 0.

- 1. In which direction (up or down) is the particle moving at time t = 1.5? Why?
- 2. Find the acceleration of the particle at time t = 1.5. Is the velocity of the particle increasing? Why or why not?
- 3. Given that y(t) is the position of the particle at time t and that y(0) = 3, find y(2).
- 4. Find the total distance traveled by the particle from t = 0 to t = 2.

First, let's take a step back and observe the framework of the problem. Like all AP Calculus free response problems, this example has multiple parts that ask for a variety of things. Parts (a) and (b) do not require integration, so we will skip those problems in this exercise.

Part (c) asks for the particles position at a specific time. We know that velocity is the derivative of position, and therefore we will need to integrate the given equation to solve for the velocity profile. Do we use a definite or an indefinite integral? We see that a definite integral would give us the total displacement of the particle. We do not want that. We also see that we are given an initial condition for the particle; therefore, we can use an indefinite integral and solve for the constant value!

Let's begin by setting up the integral:

$$\int {
m t} sin\left({t^2 }
ight) dt$$

We see that we need to make a substitution in order to solve this problem. Let's make $u = t^2$ and therefore, du = 2dt. Plugging in, we get:

$$\int tsin(u) \frac{du}{2t}$$

We can cancel the two *t*'s and solve the integral in terms of *u*:

$$\int rac{1}{2} sin\left(u
ight) du = -rac{1}{2} cos\left(u
ight) + c$$



We then plug our original substitution back in:

 $-rac{1}{2}cos\left(t^{2}
ight)+c$

Now we can solve for *c* with our initial condition:

$$-rac{1}{2}cos\left(0^2
ight)+c=3$$
 $0+c=3$ $c=3$

Therefore, our position equation becomes:

$$y\left(t
ight)=-rac{1}{2}cos\left(t^{2}
ight)+3$$

Solving for when t = 2, we get:

$$y\left(2
ight)=-rac{1}{2}cos\left(2^{2}
ight)+3=3.527$$

Part (d) is also an integration problem. The total distance traveled by the particle is another way to ask for the displacement of the particle from t = 0 to t = 2. We therefore know that we need to use a definite integral. We also know that the first step of solving an integral is solving for the indefinite integral equation, which we have already done in part (c).

Therefore, we can skip to using the Fundamental Theorem of Calculus:

 $\int_{0}^{2} t sin(t^{2}) dt = y(2) - y(0)$

We realize that we have already calculated both of these values! Therefore, we just plug in and solve:

Displacement=y(2) - y(0) = 3.527 - 3 = 0.527

Eureka! The AP Calculus exam is often filled with little shortcuts like this because the exam is timed. Be sure to watch out for them so that you can finish the exam!

As we have seen, integration is not so bad. You now have all the tools to master the AP Calculus exam! Keep in mind, however, that practice makes perfect! Do you think we have left something out? Let us know! Applications for integration are covered in other articles!



In this article, we're going to talk in depth about definite integrals and how to solve them. Studying for the AP Calculus exams may seem like a long and arduous task given the amount of information that you need to understand to ace the exams. But don't worry! Here at Albert.io, we have developed the best AP Calculus review articles possible to help you learn everything you need to know, from integration techniques to series. As a part of that, we have developed these Crash Course AP Calculus review articles to break every single topic that will be covered on the AP Calculus exams down into smaller, more understandable pieces. If you're here, you've taken your first step to developing a comprehensive understanding of AP Calculus.

What do you already know about integrals? Take a moment to write down everything you know about integrals and integration. If your list is long, that's awesome! This article will help you review and master some of the things already on that list, and set you up with the knowledge you need to understand the rest. If your list is short, don't worry! Using plenty of explanations and examples, we're going to walk through the process of evaluating definite integrals step-by-step until we've covered everything needed to make you a pro at solving them. To start, we're going to look at what an integral is, and what makes a definite integral different from an indefinite integral. Then, we'll look at some example problems, and go through the solutions step-by-step. Finally, we'll take a look at a question on definite integrals from a real AP Calculus exam!

Are you ready? Let's get started!



What is an Integral?



Image Source: Wikimedia Commons

Without delving into too much high-level mathematics, an integral is a mathematical tool used to calculate the amount of something between two bounds. Often, when speaking abstractly in terms of graphs, an integral is a tool to calculate the area under a curve. The bounds of an integral, or the bounds of the area calculation, depend on the field of interest. Integrals can be used to analyze how a function changes over time, or to look at how the area under a curve varies based on the region of interest. Within the realm of the AP Calculus exam, all you need to know about integrals from a theoretical point-of-view is that they are used to sum areas under curves produced by functions. What is more important to know for the AP Calculus exam is how to integrate, and the different integration techniques.



What is a Definite Integral and How Does it Differ From an Indefinite Integral?

Let's start with what an indefinite integral is. An indefinite integral integrates a function across the entire span of the variable in question. For example, the indefinite integral $\int f(x) dx$ integrates the function f(x) from $x = -\infty$ to $x = \infty$, or across the entire range of the x-axis. In doing so, the resultant function describes the area under the curve of the function f(x) at any point along the x-axis.

Indefinite integrals can be used to find this describing function (which we'll call F(x)) for any function using any of the variables that appear in the function. However, when evaluating indefinite integrals, you must include a constant (which we will refer to as C). This is because multiple different F(x) functions can be differentiated to produce the same f(x), even though the areas under each curve are not necessarily the same. This constant can be solved for using known conditions (such as intercepts) of F(x).

Definite integrals, on the other hand, always produce a finite number. Instead of integrating from $x = -\infty$ to $x = \infty$, definite integrals are bounded by real numbers^{*}. For example, $\int_0^{100} f(x)$ is a definite integral because it produces the value of the area under the curve f(x) from x = 0 to x = 100. Definite integrals also do not require the use of a constant C when integrating because the bounds of the integral function act as the "known condition" we talked about before.

*Note: Definite integrals can be bounded by other variables or other functions, especially when taking double $(\iint f(x, y) dydx)$ or triple $(\iint f(x, y, z) dxdydz)$ integrals. However, for the purposes of AP Calculus review, we will say that definite integrals are bounded by real numbers or variables representing real numbers (for example, $\int_a^b f(x) dx$ where a and b are real-valued numbers).



How do I Solve Definite Integrals?

If you know how to solve indefinite integrals, you know how to get most of the way through solving definite integrals. If you don't know how to solve indefinite integrals, check out <u>this article</u> on how to integrate before you continue onto integration of definite integrals. Assuming you do know how to solve indefinite integrals (or have finished reading the Crash Course article on how to solve integrals), we'll keep going!

I've found that explanations for solving definite integrals that do not involve some kind of example tend to leave students more confused than they were before. To avoid this, we're going to work through an example problem first, and I will explain some of the math in words along the way.

Example #1:

$$\int_0^{10} x^2 + 1 dx$$

This is a fairly basic definite integration problem and is of a difficulty comparable to an easier question on the AP Calculus exam.

Step 1: Integrate! We've already gone over how to integrate, so I'm going to leave this one to you. Below, you can find the process of integrating our example problem. If you find the process confusing, or don't understand how some of the math works, make sure to review general integration problems again before you move on. For those who haven't seen this notation, the bar at the end of the integrated equation represents the bounds of the definite integral, and are the same bounds that we saw on the original problem.

$$\int_{0}^{10} x^2 + 1 dx
ightarrow [(rac{x^3}{3} + x)|_{0}^{10}]$$



Step 2: Substitute in your bounds! From here, you need to substitute the bounds of your equation into your newly integrated equation. This is done by taking the upper bound and substituting it for x in your integrated equation, taking the lower bound and substituting it for x in your integrated equation, and then subtracting the latter from the former.

 $[(rac{x^3}{3}+x)|_0^{10}] o [(rac{10^3}{3}+10)-(rac{0^3}{3}+0)]$

Step 3: Subtraction! We're almost finished! The last step is actually to subtract the two values that we found in Step 2. In this case, the lower bound causes part of the equation to be zero, but bear in mind that this is not the case for all definite integrals! Always solve out your definite integrals completely, as it is common for substitution of both bounds into your integrated equation to result in non-zero numbers.

$$[(\tfrac{10^3}{3} + 10) - (\tfrac{0^3}{3} + 0)] = \tfrac{1030}{3} - 0 = \tfrac{1030}{3}$$

And now we have our answer! This process can be applied to any the integration of any definite integral, no matter how complicated or simple the problem is.



Examples of AP Calculus Exam Questions on Definite Integrals

So now you know how to solve definite integrals, but you're probably still wondering how you might be asked to apply this knowledge on the AP Calculus exam. For multiple choice questions that require the use of definite integrals, you will likely be asked to solve a definite integral based on a situation, such as <u>question 15 on page 16</u> of the CollegeBoard AP Calculus review guide.

- 15. A rain barrel collects water off the roof of a house during three hours of heavy rainfall. The height of the water in the barrel increases at the rate of $r(t) = 4t^3e^{-1.5t}$ feet per hour, where t is the time in hours since the rain began. At time t = 1 hour, the height of the water is 0.75 foot. What is the height of the water in the barrel at time t = 2 hours?
 - (A) 1.361 ft
 - (B) 1.500 ft
 - (C) 1.672 ft
 - (D) 2.111 ft

Questions like these can be solved by evaluating the definite integral based on the bounds provided (in this case t = 1 to t = 2) using integration by parts, and then add the resultant value to the initial condition provided by the problem.

For free response questions, the process of solving the definite integral may become a bit more complicated.



Another question from the <u>CollegeBoard AP Calculus review guide</u> (page 18, problem 1, part b) requires you to take the definite integral of a given function to determine when the minimum area under that function's curve is achieved.

Free Response: Section II, Part A

A graphing calculator is required for problems on this part of the exam.



1. The height of the water in a conical storage tank, shown above, is modeled by a differentiable function h, where h(t) is measured in meters and t is measured in hours. At time t = 0, the height of the water in the tank is 25 meters. The height is changing at the rate

$$h'(t) = 2 - \frac{24e^{-0.025t}}{t+4}$$
 meters per hour for $0 \le t \le 24$.

- (a) When the height of the water in the tank is *h* meters, the volume of water is $V = \frac{1}{3}\pi h^3$. At what rate is the volume of water changing at time t = 0? Indicate units of measure.
- (b) What is the minimum height of the water during the time period 0 ≤ t ≤ 24? Justify your answer.
- (c) The line tangent to the graph of h at t = 16 is used to approximate the height of the water in the tank. Using the tangent line approximation, at what time t does the height of the water return to 25 meters?

In doing this problem, you are asked to find h(t), find the value of t where h(t) is minimized, and find the value of h(t) at that minimum.

We will walk through this problem using the skills we've already covered below:

Step 1: Integrate! In the case of this problem, the first thing you want to do is determine a function for h(t). Since $h(t) = h(a) + \iint_a t h'(t) dt$ is the general equation for a definite integral with a known initial condition. we can use the initial condition (h(0) = 25) and first derivative $h'(t) = 2 - \frac{24e^{-0.025t}}{t+4}$) given in the problem to find h(t). Maxima and minima only occur at critical points or at the endpoints of the region we're looking at, so we need to evaluate h(t) at each of those points. Our endpoints are given as t = 0 and t = 24, but to find our critical points (points where h'(t) = 0 or is undefined), we set the first derivative of the h(t) equal to zero, and solve for t. There is only one solution for h'(t) = 0 at t = 6.261. Also, the first derivative is defined everywhere in our interval, so t = 6.261 is our only critical point.

From there, you can integrate the definite integral below to determine the value of the height at our endpoints and critical point.

$$h(t) = 25 + \int_0^t h'(t) dt$$

Step 2: Substitute in your bounds!

$$25 + \int_0^t h^{'}(t) dt = 25 + \left[(h(6.261) - h(0)) \right]$$

Step 3: Subtraction!

$$25 + [(h(6.261) - h(0)] = 16.339$$



If we repeat these steps for our endpoints, we find h(0) = 25 and h(24) = 34.562. We can now confirm that the value at our critical point h(6.261) = 16.339 is the minimum height in this interval.

And there you have it! Using this method, you can easily solve any problem that you come across on the AP Calculus exams. Congratulations, you made it through another Crash Course article! With the information that we've covered today, you now have the tools to approach any problem on the AP Calculus exam involving definite integrals and ace it. Make sure to practice your integration techniques, and review definite integrals, as the exam approaches, so that you don't forget the skills you've learned today. And, as always, we love feedback! If you have a AP Calculus review book that you can't get enough of, or you want us to write a guide for another AP Calculus topic, or even if you just want to say hi, don't hesitate to reach out!

Happy studying!





Image Source: Wikimedia Commons

One of the most challenging aspects of calculus is optimization. Many AP Calculus students struggle with optimization problems because they require a bit more critical thinking than a normal problem. Reading this article will give you all the tools you need to solve optimization problems, including some examples that I will walk you through. Together, we will beat all of your fears and confusion. Let's get started.



First, what is optimization? Optimization is when we are looking for the extrema of a function. Extrema are the maximum or minimum values. We can have absolute extrema and local extrema. Local extrema are the peaks and troughs in an equation. Absolute extrema are the overall maximum values or the overall minimum values. Absolute extrema can be within the function or they can be at the ends of the interval we are searching for the extrema on.

There are many different types of optimization problems. We could be optimizing volume, area, distance, length, and many other quantities. These are just some common, simple examples.

The types of optimization problems that we will be covering in this article involve something called a constraint. A constraint can be an equation, and a constraint is always true in the concept of the problem. These problems become difficult in AP Calculus because students can become confused about which equation we are trying to optimize and which equation represents the constraint. The best way to prevent this confusion is to read the problem very carefully, draw picture representations of whatever you are trying to optimize, and label your equation and your constraint. Always do this first before solving any problem.

Let's work through several examples of optimization problems in order to gain a better understanding of the concept. In each example, pay attention to the precise wording of the problem. Reading as many examples as you can and becoming more acquainted with the structure of these problems will help you get better at interpreting them.

Example 1

We want to create a box with an open top and square base with a surface area of 300 square inches. What height will produce a box with the maximum possible volume?



Step 1: Identify the equation we want to maximize.

Reading the problem, we see that we want to maximize the volume, but solve for the height of the box. The equation for the volume of a cube is:

$$V = x^2 h$$

In this equation, the x represents the two side measurements of the box and h represents the height of the box.

Step 2: Identify the constraint equation.

When working these optimization problems, it is important to remember that we always need two equations. Why? The reason will become clear later. Reading the problem, we see that the surface area of the box is a constant 300 square inches. This is the constraint. We can turn this into an equation of the surface area:

$$SA = x^2 + 4xh = 300$$

The x^2 term represents the square base of the box and the 4xh term represents the four sides of the box. We couldn't employ the normal surface area equation for a cube to this example because the top of the box is open.

Step 3: Eliminate a variable.

Looking at both of our equations, we see that each is composed of two variables. We know that in order to solve this equation, we must somehow eliminate one variable.



Because we have two equations, (I told you I'd explain the importance) we can solve the constraint equation to be in terms of one of the variables and plug that equation into our volume equation – the equation we are trying to maximize. Our first instinct would be to solve the constraint equation for in order to eliminate it from the volume equations (since our goal is to find the optimal height) but we see that solving for x would be relatively more difficult than solving for h. So, for our own ease, we will solve for h and eliminate it for now. Don't worry, we will solve for it at the end of the problem.

$$h=rac{300-x^2}{4x}$$

We can then plug this equation into the volume equation in place of height.

$$V = rac{x(300-x^2)}{4}$$

Step 4: Determine the bounds of the problem.

There are two extremes to this problem:

- 1. The box is all height and no side length (x=0).
- 2. The box is all side length and there is no height, (solve the height equation for when h=0).

$$0 = rac{300 - x^2}{4x}$$
 $x = \sqrt{300} = 17.32$



Therefore, the bounds that we are looking for the maximum on are from x = 0 to x = 17.32. This is useful information because we cannot go outside the scope of our constraint with these bounds and possibly solve for the wrong maximum.

Step 5: Find the critical numbers.

Critical numbers are the values of *x* where the function has a slope of zero. These critical numbers will tell us where it is possible for extrema to occur. In this problem we are trying to find the maximum volume. We can find these critical numbers by taking the derivative of the volume equation and setting it equal to zero.

$$rac{dV}{dx}=75-rac{3}{4}x^2=0$$
 $x=10$

At *x* = 10, either a maximum or a minimum occurs.

Step 6: Find the maximum volume.

We now test each of our bounds and our critical numbers in the volume equation to see which one has the maximum value.

For
$$x = 0, V = 0$$
.
For $x = 10, V = 500$.
For $x = 17.32, V = 0.0044$



We see from this example that the maximum value of volume is 500 cubic inches.

Step 7: Solve for height.

Because we know the side length value that corresponds to the maximum volume, we can solve for the corresponding height using the constraint equation we rearranged in Step 3.

$$h = rac{300 - 10^2}{4(10)}$$

From this example, we see the basic algorithm for solving these problems. We identify the equations, solve for a variable, use derivatives to find the critical numbers, and then test to find the maximum or minimum value. The most difficult part of any optimization problem is interpreting the problem statement. Let's see another example.

Example 2

Find the point on the curve $y = x^2$ that is closest to the point (1,5).

At the onset of this problem we realize that we want to minimize the distance between the given curve and a specific point on our coordinate system.

Step 1: Identify the equation we want to minimize.

You would think that we need to minimize the curve equation as a first guess, but this guess is not correct. We want to minimize the distance between a point on the curve and the given point.


Therefore, we want to minimize the distance equation:

$$d = \sqrt{(x-5)^2 - (y-1)^2}$$

Step 2: Identify the constraint equation.

The only equation that we know is constantly true in this scenario is the equation of the curve; therefore, it becomes our constraint equation.

$$y = x^2$$

Step 3: Eliminate a variable.

This step is easier in this example than the last one. We see that the distance equation has both *x* and *y*. Our constraint equation is already solved for *y*! Let's just plug it in:

$$d = \sqrt{(x-5)^2 - (x^2-1)^2}$$

Step 4: Determine the bounds of the problem.

In this specific example there are no specific bounds from the constraint equation that we need to test for. Therefore, we define the bounds at negative infinity and positive infinity.



Step 5: Find the critical numbers.

Remember that critical numbers are found by setting the derivative of our equation equal to zero!

$$rac{dd}{dx} = rac{-(2x^3 - 3x + 5)}{\sqrt{-x^4 + 3x^2 - 10x + 24}} = 0$$
 $x = -1.718853$

Step 6: Find the minimum distance.

Plug the bounds and the critical values into the distance formula we derived and see which number has the minimum distance. We won't try our bounds in this case because they aren't finite numbers.

For x = -1.718853, d = 6.4283

Since we only have one critical number, we only have one distance. This is the minimum distance then, by default.

Step 7: Solve for the y value

We just plug the *x* value into the constraint equation.

$$y = (-1.718853)^2$$

 $y = 2.954455$



Now we are starting to really understand these problems. Let's look at one more example.

Example 3

A high school student wants to draw a rectangle into a semicircle of radius 5. If one side must be on the semicircle's diameter, what is the area of the largest rectangle the student can draw?

First, we notice that we are trying to maximize the area of a rectangle that is inside a circle.

Step 1: Identify the equation we want to minimize.

As we said before, we want to minimize the area of a rectangle. The area of a square is given by the equation:

$$A = 2xy$$

In this equation, *x* is half the width of the base of the rectangle.

Step 2: Identify the constraint equation.

The area of the rectangle is constrained by the bounds of the semicircle. So the equation of the semicircle is the constraint.

$$y=\sqrt{5^2-x^2}$$



Step 3: Eliminate a variable.

We see that our area equation has two variables, *x* and *y*. We will eliminate the *y* variable by plugging the semicircle equation in for *y*:

$$A = 2x\sqrt{5^2 - x^2}$$

Step 4: Determine the bounds of the problem.

We realize that the base of the semicircle can only be as long as the diameter of the semicircle since one side is resting on the diameter. Therefore, can only be as long as the radius of the semicircle, which is 5. The bounds, then, are from x = 0 to x = 5.

Step 5: Find the critical numbers.

We find the critical numbers of the area equation by taking the derivative of the area function. We then set the derivative equal to zero and solve for the corresponding *x* values.

$$egin{aligned} rac{dA}{dx} &= 2\sqrt{25-x^2} - rac{2x^2}{\sqrt{25-x^2}} = 0 \ x &= rac{-5\sqrt{2}}{2} \ x &= rac{5\sqrt{2}}{2} \end{aligned}$$

We see that the negative critical number is not within the bounds we set in step 4. Therefore, we throw it out.



Step 6: Find the maximum area.

We will plug each of the bounds and valid critical numbers into the area equation we derived to figure out which value yields the maximum rectangle area.

For x=0, A=0.For $x=rac{5\sqrt{2}}{2}, A=25$ For x=5, A=0

We see from this analysis that the maximum area is achieved at our critical number. We do not need to proceed to step 7 since we have already calculated the maximum area of the rectangle, 25.

When working these problems, it is important to always remember these steps that I have demonstrated in these example problems. Even though it is not explained in this article, it is also important to be able to take derivatives of functions. Some functions are more tricky, so be sure to review derivatives when studying for the AP Calculus test.

Another important thing to note is the ability to craft geometric equations. I bet you never thought your geometry class would ever become useful, but things change. It is very important to be able to recall the area, surface area, and volume of all the common shapes, including squares, rectangles, circles, triangles, spheres, cubes, prisms, and pyramids. It is also important to know where these equations came from in cases like the first example, where a shape may be missing a side. You may end up deriving your own equations!



The most important way to prepare for optimization problems on the AP Calculus exam is to practice. I know I've already mentioned that in this article, but practice is extremely important. Go back and work the homework problems your teacher gave you. Work these examples without looking at their solutions. Look up additional problems online. There are plenty of resources out there for you to harness.

Optimization is one of the most challenging parts of AP Calculus. If you can conquer these problems, you can do anything. I wish you the best of luck with your AP Calculus review. Remember to practice.

Do you know something that we haven't covered in this article? Let us know!



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Start Practicing





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Welcome to one of Albert.io's AP Exam Crash Course Reviews! These articles are an awesome way to get ahead on studying material from any of the available AP exams, or to catch up on topics that you've fallen behind in. These guides take small topics that are often found on AP exams and break them down into even smaller pieces, so that when you encounter them on the exam, you know exactly what to do!



In this review guide, we're going to take a look at an AP Calculus review topic: calculating derivatives of inverse function! Calculating the derivative of an inverse function requires you to apply derivation skills you've already learned to a specific type of function, inverse functions, which don't always behave in a clear-cut manner. Have you worked with inverse functions before? If so, you'll learn how to combine your knowledge of inverse functions with your knowledge of derivatives. If you haven't, don't worry! We will review what an inverse function is first, and then talk about how to take the derivative of one. In order to make sure you have the best possible understanding of how to calculate the derivative on an inverse function, we're going to start with a step-by-step explanation containing plenty of examples, and then do a practice problem from an actual AP Calculus exam! By the end of this AP Calculus review article, you should be a pro at calculating the derivative of inverse functions. Are you ready? Let's get started!

What is an Inverse Function?

To start, let's define an inverse function. If we have an equation f(x) = x + 1, then the corresponding inverse function to this function is represented by $f^{-1}(x)$. Anytime you see a function with a $^{-1}$ in the superscript, you can assume that the function is an inverse function. In order to calculate an inverse function, you should set f(x) equal to x, and replace every instance of x within the formula with y. From there, you should solve the equation for y. Once you've solved the equation, replace y with $f^{-1}(x)$, and you have your inverse function! In the case of the example function we started with f(x) = x + 1, finding the inverse function would be done as follows:

Step 1: Replace every instance of x with y, and every instance of y with x.

To begin, take all of the x's in your equation, and replace them with y's, and vice versa. If your function begins with f(x) instead of a y, replace the f(x) with a y first, then change your variables.

$$f\left(x\right)=x+1\rightarrow y=x+1\rightarrow x=y+1$$



Step 2: Solve the equation for y.

Next, solve the equation for *y*, as you normally would.

x=y+1
ightarrow x-1=y
ightarrow y=x-1

Step 3: Replace y with $f^{-1}(x)$

Last step! Lastly, replace the y that you just solved for with $f^{-1}(x)$.

$$f^{-1}\left(x\right)=y=x-1$$

And now you're done! This same process can be used for any function that has an inverse.

In words, the inverse of a function $f^{-1}(x)$ is the line that results from reflecting the function f(x) about the line y = x. If you want to make sure your inverse function has been calculated correctly, do so in two ways. First, you can add $f^{-1}(x)$ and f(x) together. When added together, functions and their inverse functions always equal 1. Second, you can graph both lines to see whether they are reflections of each other.

How Do I Take the Derivative of an Inverse Function?

Now that we know what an inverse function is, we can learn to take the derivative of it! There are two main ways to approach differentiating an inverse function, and we will look at both.

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Derivatives of Inverse Functions: AP Calculus Crash Course Cont.

The first way to approach differentiating an inverse function is to differentiate the inverse function directly. Continuing to use the example of f(x) = x + 1, you would differentiate the inverse function by taking the derivative of it as follows:

$$f^{\prime}\left(f^{-1}\left(x
ight)
ight)=_{dx}^{d}f^{-1}\left(x
ight)+_{dx}^{d}\left(x-1
ight)=1$$

The second way to approach taking the derivative of an inverse function is to create a formula that allows you to find the value of the derivative of the inverse function at any point using the original function that the inverse function is based on. Assuming that the function f(x) is differentiable (that is, it is a real-valued continuous function with no corners or cusps), the derivative of the inverse of f(x) can determined using the following formula:

$$(f^{-1})'(x) = rac{1}{f'(f^{-1}(x))}$$

Now, this formula looks pretty complicated, so let's look at an example of how to use it using our equation f(x) = x + 1:

Step 1: Find the inverse of f(x) using the process outlines above

$$f\left(x
ight)=x+1
ightarrow f^{-1}\left(x
ight)=x-1$$

Step 2: Find the first derivative of *f*(*x*)

$$_{dx}^{d}f(x) = _{dx}^{d}(x+1) = 1$$



Step 3: Substitute the inverse of *f*(*x*) for all instances of *x* in the first derivative

 $f'(f^{-1}\left(x\right))=f'(x-1)=0(x+1)+1=1$

Step 4: Find the reciprocal of Step 3

$$rac{1}{f'(f^{-1}(x))} = rac{1}{1} = 1 = rac{d}{dx} \; (f^{-1})'(x)$$

And then you're done! Although the equation may seem complicated, if you solve out the problem step by step, you'll have no problem finding the derivative of an inverse function. Now, let's look at some examples from the AP Calculus exam!

AP Calculus Exam Examples

On the 2007 AP Calculus AP exam free response section, <u>one of the</u> <u>questions</u> required you to take the derivative of an inverse function in order to complete it. Specifically, the question provided a table of values for a function g(x) and its first derivative g'(x), and said that assuming $g^{-1}(x)$ is the inverse of g(x), write the equation of the line tangent to the graph of $g^{-1}(x)$ at x = 2. This problem also does not contain defined functions, so you must rely on your conceptual knowledge of inverse functions in order to solve it (in other words, the method of differentiation that we discussed earlier will not be useful for a problem like this). Let's go over how this problem would be solved, step-by-step, using our knowledge of derivatives of inverse functions.

Step 1: Find the first derivative of g(x).

These values are given in the table provided, so we can come back to this once we know the inverse of g(x).



Step 2: Find the inverse of g(x).

This value is slightly trickier to find. We know that the value of g(x) at x = 1 is 2, and we know that the function $g^{-1}(x)$ is the function g(x) reflected about the line y = x. The easiest way to determine the value of $g^{-1}(x)$ then, is to swap the x and y values of g(1) = 2 to see that $g^{-1}(2) = 1$. This method can be validated by plotting g(1) and y = x on a graph and determining what point mirrors g(1) = 2 on the other side of the line y = x. In this case, that value is 1 at x = 2, so $g^{-1}(2) = 1$.

Step 3: Substitute the inverse of f(x) for all instances of x in the first derivative.

Knowing that $g^{-1}(2) = 1$, we can now substitute $g^{-1}(x)$ into g'(x), which gives us the formula g'(1). To determine the value of g'(1), we can look at the table provided, which will show us that g'(1) = 5.

Step 4: Find the reciprocal of Step 3

The last step to solving this portion of the problem is to take the reciprocal of the value determined in step three. This will give us the derivative of the inverse function $g^{-1}(x)$ at x = 2, which, in this case, is equal to one-fifth.

From there, you can use this value as the slope of the tangent line for your equation, and use the point-slope formula to determine the equation of the tangent line to $g^{-1}(x)$ at x = 2.

And there you have it! You've just solved part of a free response question that appeared on an actual AP Calculus AB exam! Taking the derivative of an inverse function is a skill that is extremely important to understand, as it often appears in both the multiple choice and free response sections of the AP Calculus exams.



This AP Calculus review is your first step to totally understanding how to approach questions involving derivatives of inverse functions on the AP Calculus exam, but as you study, you should make a point of practicing questions that involve taking the derivative of inverse functions to make sure that you are prepared for any question that may appear on the exam.

Congratulations, you now have the tools to tackle any AP Calculus exam question about derivatives of inverse functions! If you've just started studying for the AP Calculus exam, check out <u>some of our other review guides</u>, so that you can set up the best possible plan to ace the AP Calculus exam. Alternatively, if you're interested in more AP Calculus review articles that tackle smaller and more specific areas of the exam, check out some of our other <u>Crash Course</u> articles. We love feedback, so if you want to reach out and tell us about how you're studying for your AP exams, or which books you're using, feel free to reach out! Otherwise, happy studying!



The Best AP Calculus Review Books of 2015



Here at Albert.io, we've reviewed the best AP Calculus review books available in 2015 and compiled a list of some of our favorites to study from for you to reference as the AP Calculus exam rounds the corner. If you prefer reading review books in print rather than online, grabbing a copy of an AP Calculus prep book can be a great way to prepare for your AP Calculus exam.



Barron's AP Calculus, 13th Edition by David Bock and Shirley Hockett



Armed with four practice tests for AP Calculus AB and an additional four for Calculus BC, the Barron's review book is jam packed with practice. Unsurprisingly, the majority of student feedback is that these tests either similarly reflect the exam, or are slightly harder than the exam. But that's classic Barron's living up to its reputation of being a rigorous studying option when approaching AP exam prep.

Even though the practice tests are slightly harder than the real thing, the best part about the Barron's book is that it can really help you pinpoint your strengths and weaknesses. Naturally, if you're not one to take on a challenge, you may want to look into trying a more comparable review book that has multiple choice questions that are slightly easier like the Princeton Review.



However, if you're like us and love a good challenge, the Barron's book is great because sometimes you'll face questions that you have no idea how to solve the question, but once you walk through the solution, you'll have a stronger grasp of the core concept.

Be careful and don't be afraid to get feedback from your teacher for certain questions you're not sure about though; like many of these massive AP review books, there are a few errors/typos. Some of the explanations could use little more detail as well. In terms of review material, the Princeton Review does a slightly better job of providing overviews of concepts you need to know. Overall though if you have a good grasp of Calculus, these practice tests are some of the best available in print to really test your understanding of core intuitions. Check out this book if you're interested in just getting your Calc reps in; look elsewhere if you're looking to learn Calculus from square one.



Cracking the AP Calculus AB Exam 2015 Edition by Princeton Review



After many years of having a consolidated version of the book, the Princeton Review finally decided to make the split and divide its AP Calculus Books into two different separate books. The best part about this Princeton Review book is that it does a comprehensive but concise job at summarizing the key principles of AP Calculus AB. That means unlike how the Barron's books often go into too much of detail, the Princeton Review gets it just about right.

Heed with caution though. Something that didn't sit well with us was how the book encouraged us to memorize formulas. We understand where they are coming from in the sense that the student picking up this book is probably cramming, but we would have liked if the book had encouraged students to instead fundamentally understand the intuition behind how to derive certain functions and equations.



Calculus is similar to studying Economics in that if you understand the core intuitions behind it, you can save yourself a lot of time from memorizing tons of formulas if you can derive them yourself.

Aside from this, the book provided actionable strategies for students to prepare for the AP Calculus AB exam. The part we especially liked was how it asked openended questions to encourage students like you to think about how you actually approach certain types of questions. The examples in the book were clearly laid out, though some parts of the book could have used more textual explanations than just working out the steps in a mathematical fashion. It's no surprise previous students have found some of these examples confusing a lot of the times since the Princeton Review can skip a few steps here and there.

Overall, this book is a good start, but the focus on memorization of problem formats and solution patterns was a bit disconcerting. While you are probably taking an AP Calculus class to get a great AP score and to have an opportunity to gain college credit, you should strive to understand the fundamental concepts that are being taught in the class.

The book offers three full-length practice exams, so make sure you time them carefully if you choose this book. We would recommend taking one before you begin actively studying, then another halfway through the book, and then the final one at the end of your AP exam prep.



Cracking the AP Calculus BC Exam 2015 Edition by Princeton Review



The Princeton Review's Calc BC book is eerily similar to its Calc AB book in terms of the strategies that are taught to students. Similar to our feedback for its Calc AB review book, we thought while the concept overviews were helpful and for the most part easy to understand, we couldn't help but feel a bit concerned by the fact that the book had such an emphasis on the rote memorization of solution techniques and formulas.

The best part about the Princeton Review's AP Calculus BC review book is that it provides helpful shortcuts for students to solve certain questions faster. We're a big fan of studying smart, not hard, so this sat well with us.



Aside from the shortcuts, the book itself features two full length practice tests. In the past, the Princeton Review has been known to have fairly representative practice questions to the actual AP exam. Overall, the rigor is good; however, the depth of the detailed explanations was a bit lacking. This note hit us hard since the book orients so much of its pedagogy to memorizing formulas and solution techniques that we fear that students could potentially miss the whole point of why a particular question is being asked in the practice exam and then they are left with a situation in which they don't understand the detailed explanation either.

There are known errs in explanations for the calculator inactive section of the FRQ section. The answer explanations refer to using a calculator to solve the problem, when students aren't supposed to be using calculators to solve it. Thanks for pointing this out, Scott Hopper!



5 Steps to a 5 AP Calculus AB, 2014-2015 Edition by William Ma



When it comes to AP Calculus review books, 5 Steps to a 5 tends to be a little forgotten when compared to Barron's and the Princeton Review, but this edition of McGraw Hill's 5 Steps to a 5 is a good overall review book for reference. The book itself has two full length practice exams, similar to the Princeton Review.

What we personally liked about the book was how it started by detailing and helping you form a study plan. Unlike the Princeton Review or Barron's books, it spent a bit more time walking through the three different plans it offers for how to use the book. This was super helpful because something we've come to understand from doing <u>online AP prep</u> is that there are tons of different types of students when it comes to preparing for AP exams, so you have to try to cater to as many different student demographics as possible.



This book outlines itself around three core strategies: if you're a full-year prep student, a one-semester prep student (for those who take a Calculus class that teaches AB in Fall and the BC starting in January), and a six-week prep student.

The main gripe we had about the book is its explanations. While it does a good job framing different study plans for students, it really fails to deliver when it comes to the same depth of explanations we've seen from other review books. The concept review was good quality, but the explanations just really varied in terms of overall quality.



5 Steps to a 5 AP Calculus BC, 2014-2015 Edition by William Ma



Most of feedback for the AB version of 5 Steps to a 5 AP Calculus AB carries over here. The thing is back in the day, these books were a consolidated book like the Barron's book, so the core content in terms of the way the author structures this BC version is very much the same.

Again, like its AB counterpart, the 5 Steps book struggles when it comes to provide clear explanations to students. The consistency of the detailed answer choices is not the best, so we'd recommend checking out Barron's or Princeton Review if you need more of an explanation for practice questions. The book features three practice tests though, so if you need additional practice, it's a good place to start.



AP Calculus AB/BC All Access by Stu Schwartz & REA



This review book doesn't get a lot of attention, but it's quite good. The nice part about REA's AP Calc review book is that it integrates three different ways of studying for you: via book, web, and mobile. However, the alternative forms do have additional fees associated with them. The strategies included are mainly your classic exam prep tips; the review content itself is clear and straightforward. One of the best parts about the book is the fact the author writes to reflect the latest changes to the AP Calc test so while many of the other books still have old content inside, most of the older content has been removed for this book.

The explanations could have used more detail, as you can probably tell by now this is usually the most difficult thing to find when it comes to print review books. We would recommend this as a potential supplement to a classroom since there are quite a few practice questions to work through but we wouldn't solely rely on this to prepare when you're starting your AP Calculus review.



There you have it. The best AP Calculus review books of 2015. In our opinion, if you prefer to prepare in writing, we'd recommend going with either Barron's or the Princeton Review. While the other books are good books, they just do not compete in terms of the overall consistency with the actual AP Calculus tests.

Have a great review book that's not on our list? Let us know!



How to Study for AP Calculus



Congratulations on signing up for the AP Calculus AB or AP Calculus BC exams! Both AP Calculus exams cover a fair amount of material, and you may not be sure where to begin studying, or what you should be studying in the first place. Don't worry! You've come to the right place. Here at Albert.io, we've written up several articles specifically designed to help you ace the AP Calculus exams.

In this article, we're going to go over how exactly you should be studying for the AP Calculus exams, and what topics you need to know to ace the exam. This article will not be going over any practice problems (those can be found in the <u>Albert.io AP Calculus AB-BC section</u>), and instead will focus mostly on creating an AP Calculus study plan and tips for acing the AP Calculus exams. If you're looking for practice problems or review articles on topics that will be on the AP Calculus exams, feel free to check out the links scattered throughout the rest of this article, or surf around <u>Albert.io's AP Calculus</u> blog articles.



An Overview of the AP Calculus Exams

To start, let's talk about the format of the AP Calculus exams. The AP Calculus AB and BC exams are formatted in the same way but cover different material. AP Calculus AB covers a subset of the information covered on the AP Calculus BC exam. Both exams are divided into a multiple choice section and a free response section, and you will be given three hours and fifteen minutes in total to complete each exam. There are two multiple-choice sections and two free response sections. You will be given 60 minutes to complete the first multiple choice section, which contains 30 questions, and you will not be allowed to use a calculator. You will then be given 45 minutes to complete the second multiple choice section, which contains 15 questions, and you will be allowed to use a calculator. From there, you will move on to the first free response section, which you will be given 30 minutes to complete and which contains two questions.

Finally, you will complete the second free response section, which you will be given 60 minutes to complete and which contains four questions. Unlike the multiple choice section, you can use a calculator on the first free response section, but cannot on the second.

Feeling a bit exhausted already? Don't worry! The exam format is nice to know so that you know what to expect when you take the exam, but you will become very familiar with it as you study for the AP Calculus exams through practice tests and problems.

Big Ideas

So, knowing the format of the exam should be helpful for you, but that still doesn't tell you what exactly is going to be on the exam. The <u>CollegeBoard AP</u> <u>Calculus AB/BC Course and Exam Description</u> breaks down the material that is covered in both AP Calculus exams into four Big Ideas. We will go through each Big Idea individually to highlight what they encompass.



1. Limits (AB/BC)

The first Big Idea – Limits – can be considered the foundation that all of the other Big Ideas are based on. This section requires you to be able to compute several different types of limits and know how to apply limits at specific points of a function.

2. Derivatives (AB/BC)

The second Big Idea – Derivatives – applies the properties of limits to looking at the rate of change of a function or variable over time. This section covers several different definitions of the derivative, analysis of graphs of functions, and properties of derivatives.

3. Integrals and the Fundamental Theorem of Calculus (AB/BC)

The third Big Idea – Integrals – builds upon the first two. This section covers the topics of the Fundamental Theorem of Calculus, indefinite and definite integrals, Riemann sums, and applications of integrals (largely based in physics).

4. Series (BC only)

The fourth Big Idea – Series – is only addressed on the AP Calculus BC exam, and can, therefore, be disregarded if you plan to take the AP Calculus AB exam. This section covers the topics of series of numbers, power series, and the convergence or divergence of a series. Problems from this section may involve writing a power series based on a given function, using Taylor series, or estimating the sum of a series.



Learning Objectives

If the format of the AP Calculus exams is thought of as a birds-eye view of the exam, then the Learning Objectives can be thought of as the problems on the exams themselves. The Learning Objectives of the AP Calculus exams are the skills that CollegeBoard expects students to have gained over the course of the year and are the skills that you will be tested on during the exam. The Learning Objectives can be rather specific, and there can be up to fifteen Learning Objectives for each Big Idea, so we won't be going into detail on the specifics of each learning objective. If you are interested in learning more about what the specific Learning Objectives are, you can find them in the <u>CollegeBoard AP</u> Calculus AB/BC Course and Exam Description.

Resources to Gather Before Studying

AP Calculus CollegeBoard Materials

- <u>AP Calculus Exam and Course Description</u>
 - contains the Learning Objectives and Big Ideas of AP Calculus in more detail and sample problems.
- <u>AP Calculus Sample Questions</u>
 - contains more sample questions for the AP Calculus AB and BC exams.

<u>AP Calculus AB Overview</u>

- contains an overview of the topics covered on the AP Calculus AB exam and in the course.
- AP Calculus AB Released Exam (1988)
 - ▶ contains a full AP Calculus AB exam from 1988.
- <u>AP Calculus AB Past FRQs</u>
 - contains the released free response questions from AP exams administered in recent years.



<u>AP Calculus BC Overview</u>

 contains an overview of the topics covered on the AP Calculus BC exam and in the course.

• AP Calculus BC Released Exam (1988)

▶ contains a full AP Calculus BC exam from 1988.

• AP Calculus BC Past FRQs

 contains the released free response questions from AP exams administered in recent years.

AP Calculus Videos

• <u>AP Calculus – YouTube</u>

 contains a compilation of AP Calculus videos from a range of different YouTube channels.

<u>Khan Academy</u>

 contains instructional videos that cover topics on the AP Calculus AB and AP Calculus BC exams.

AP Calculus Albert.io Posts

In addition to this article, Albert.io has tens of other articles to help you prepare for the AP Calculus exams. These include our review articles, which can help you study for the overall exam, our Crash Course articles, which break down smaller topics on the exam into understandable pieces, and our tips articles, which give advice on how best to approach certain problems on the exams. Below are some examples of what Albert.io has to offer!

- The Ultimate List of AP Calculus Tips
 - ▶ contains a list of 104 (!!!) tips for acing the AP Calculus exams.

• 7 Ways to Approach AP Calculus Review

 contains thoughts on seven different ways to approach reviewing for the AP Calculus exams.



- The Best AP Calculus Review Books of 2015
 - contains a ranking of the five best AP Calculus review books from 2015.

Setting A Baseline: The Pre-Studying Practice Test

This may not be the first thing you think of as you plan out your study schedule, but the best way to begin preparing for the AP Calculus exams is to take a practice exam! Taking a practice exam before you begin studying helps you to understand what topics you understand fairly well, and therefore do not need to spend a ton of time studying, and what topics you do not understand, and therefore need to spend more of your time studying. You should take your practice exam under the same conditions as you would take the actual AP Calculus exam. Now, we'll talk about specific ways to take an AP Calculus practice exam, to best simulate taking the actual exam in May.

Conditions

In general, you should try to match the conditions that you will be taking the AP Calculus exam in May as much as possible when taking your practice exam. Do not try to take different sections on different days, or with long breaks between the sections, as that allows you more time to recover mentally from the exam than you will be given on the exam day. Take your practice exam alone, during a time when you will not be expected to interact with other people. If possible, ask a parent or friend to proctor the exam for you, to create the most accurate testing environment. Make sure you use a calculator that is allowed on the AP Calculus exams, as well as the appropriate writing utensils.

However, in the case of your first practice exam, you do not need to take the exam under the exact testing conditions. Be sure to record how long it takes you to finish each section, but don't worry if you don't finish the section in the time allotted. You will have plenty of time to become a faster and more accurate test taker as you continue to study for the exam.



Accuracy

Once you finish taking the practice exam, you should first look to see what problems you got wrong. See whether these problems fit into a certain type, such as problems that focus on Riemann Sums, or problems that use tables of values instead of formulas. This approach will give you a better idea of what types of problems you excel at, and what types of problems you need to focus on in your studies. Also, take note of which problems you answered correctly because you knew the answer, and which problems you answered correctly because you guessed. If you don't know the answer to a question on the actual AP Calculus exam, guessing is recommended (as you will not be penalized for incorrect answers), but knowing how to solve the problem is always a better option than guessing.

Timing

The next aspect of your practice exam results that you should look at is the amount of time it takes you to finish each section of the exam. If you're doing the first multiple-choice section of the exam, and it takes you 75 minutes to complete, that translates to you taking two and a half minutes to finish each question. On the actual exam, you will be given 60 minutes to complete the first section, which translates to two minutes to finish each question. This gives you a goal to work towards – shaving thirty seconds off of the speed at which you finish each multiple-choice section.

If, during the exam, you notice that you're stalling on one particular question or set of questions, take note of that! It may not be that you're taking two and a half minutes to complete each question, but that you're taking one and a half minutes to finish 28 of the questions and eighteen minutes to finish the last two. This is important, as it shows you that this is an area of AP Calculus that you need to focus on when you create your study plan.



Baseline Analysis

So, you've taken your first practice exam, and you're going through the exam to break down all the considerations that we've talked about in the past couple of sections. Here's a step-by-step method of going through each exam section that encompasses everything we've talked about:

1. Determine why you got a question wrong

As you go through each question, mark the questions that you got wrong. Then, go back over the questions that you got wrong, and determine exactly why you got those questions wrong. In some cases, you may not have known how to compute the math necessary to complete the problem. In other cases, you may not have understood the wording of the question. Each of these issues requires different approaches in your study plan. For questions where you understood the question, but could not do the math, reviewing the topics that the questions were based on may prove to be helpful to getting them right the next time. For questions where you did not understand the question, reviewing questions with similar wording and their answers may give you some insight into the true meaning of the question. Include questions that you guessed on, regardless of whether you got them right, in this analysis.

2. Determine your strengths and your weaknesses

After going through your practice exam to dissect the questions you got wrong, go through the questions that you got right (not including the ones that you guessed on). Make a list of the topics that those questions are based on. Those topics are your "Strengths." This is not to say that you do not have to study these topics, but you do not have to focus on them as much as you might on other topics. Next, take the questions that you got wrong or guessed on, and make a list of the topics that those questions are based on. These topics are your "Weaknesses." It is totally okay if one of these lists of longer than the other, as well as if there is overlap in topics between the two lists.



3. Make a study plan!

Using the Strengths and Weaknesses lists that you just made, we're going to create a study plan! The main key to creating a study plan is to focus on the topics listed in the Weaknesses list by practicing them daily while making sure to reinforce the topics on your Strengths list by practicing them every couple of days. Additionally, you should schedule in regular full AP Calculus practice exams, to evaluate your studying progress. We will talk more in detail about making study plans for different time frames in the next section.

4. Repeat!

Each time you take a new AP Calculus practice exam, repeat the three steps above to re-evaluate your Strengths and Weaknesses lists. As topics move from your Weaknesses list to your Strengths list, you will be able to focus less on those topics and more on the topics still on your Weaknesses list. The goal is to get to the point where your Weaknesses list is empty!

Study Plans

One of the main things that we've talked about in this AP Calculus review article is creating a study plan! Study plans are incredibly helpful for keeping you on track with your studying as the AP Calculus exams approach, whether they be three months or three weeks away. We're going to look at how to create study plans for three different time periods, so that, no matter when you decide to start studying for the AP Calculus exams, you will be as prepared as possible for the exams come testing day.

Long Term (more than five months)

If you're starting to study for the AP Calculus exams five months or more away from the exam date, congratulations! You are way ahead of the game, and your head start gives you more time to learn any material that you find difficult. It also allows you to have to study fewer days per week, which means less stress about the exam overall!



In preparing your study plan, block out a couple of hours on three or four days of the week that you plan to devote to studying. Take one or two Strengths, and assign it to one of your study days. On this day, you will only review that topic. Assign each of your other study days one or two Weaknesses. Every two weeks, take another practice exam and update your strengths and weaknesses accordingly. You do not have to review every topic on both lists within the two week period, as long as you make sure all of your Weaknesses become Strengths about two weeks before the exam date. During the two weeks before your exam, do daily practice tests (one section per day) to make sure that you are answering questions both accurately and quickly. Pay attention to the questions you get wrong during this period, and briefly review those topics after taking the practice exam.

Medium Term (2-3 months)

Most people begin studying for the AP Calculus exams two or three months before the exam date, so if you're starting here, you're in good company!

This study plan happens to be similar to the Long Term Plan, in that it approaches studying using your Strengths and Weaknesses (as opposed to breaking the material down by topic, which is a method we will use for the short term study plan). Instead of picking three or four days to study, however, you'll be studying six out of the seven days of the week. You should be studying for about two hours on each of these days. For this example, we'll say that Sunday is your day off. Two of the six days – Wednesday and Saturday – will focus on one or two topics in your Strengths list. The other four days – Monday, Tuesday, Thursday, and Friday – will each focus on one or two topics from your Weaknesses list. Again, you do not need to review every topic on each list each week, so long as all of your Weaknesses become Strengths by a week before the exam date. You will also be taking weekly practice exams (this is easiest done at the end of the week), and will be re-evaluating your Strengths and Weaknesses according to the steps we went over earlier. A week before the exam, switch to only taking practice exams on each of your six study days. You should still note the questions that you get wrong and why, and try to review those topics after finishing the practice exam.


How to Study for AP Calculus Cont.

Short Term (less than a month)

So, you have four weeks (or less) until the AP Calculus exam, and you're trying to figure out where to start with your study plan. The easiest way to break down the material on the AP Calculus is by the four Big Idea – Limits, Derivatives, Integrals, and Series. Assign each of the four weeks (or divide the number of days you have left by four, if you have less than four weeks) to one of those topics, and focus on mastering each of those Big Ideas by being able to accurate answer practice questions on those topics. Take practice exams every week, and continue to review material from previous Big Ideas at the end of each week. For a more detailed, day-by-day short term guide to studying for the AP Calculus exam in a month or less, check out our <u>30 Day Guide to Studying for AP Calculus</u>!

Strategies to Ace The AP Calculus Exams

By taking a practice exam to establish your baseline knowledge, and by applying that baseline knowledge to your study plan, you will be as prepared as possible come exam day. But what about exam day itself? Sometimes, even though you study according to your plan, you get nervous or stressed before the exam and don't do as well as you could have. With the knowledge that this may happen, we're going to talk about some strategies to ace the AP Calculus exams that are specifically aimed at the day of the exam.

1. Don't study on the day of the exam!

You might think that reviewing some of the material before the start of the exam will help you to remember the material in a few hours on the exam, but this often causes students to become stressed about the material they may not completely understand. If there is a topic that you don't understand, it is unlikely that you will be able to pick it up in a couple of minutes before the exam starts (Note: the AP Calculus exams are typically in the morning). Instead, have a relaxed, AP Calculus-free morning before your exam, and arrive at the test well-rested and ready to take the exam!



How to Study for AP Calculus Cont.

2. Eat breakfast (or don't!)

There are a lot of mixed messages about whether you need to eat breakfast in the first place, let alone whether you need to do so before an exam. If you are someone who can't get through their first class without having had something to eat that morning, you should make sure to have a delicious, hearty breakfast before you take the AP Calculus exam. Try to stay away from sugary cereals, as they may make your energy crash during the exam. If you're comfortable going the morning without eating anything (like me!), you can choose to skip breakfast in favor of some hot coffee or tea in the morning. Again, avoid sugary drinks like hot chocolate, as they may cause your energy to crash midway through the exam.

3. Take breaks between sections!

You will be given breaks between each section on the exam, so make the most of them! Try not to think about questions you may have answered wrong in the previous section, or worry about the questions on the next section. Instead, take a trip to the bathroom, or take a walk down the hallway, and think about anything that isn't AP Calculus. This gives your brain a break from the intense focus required during the testing periods of the AP Calculus exams, and will leave you feeling more refreshed once you start the next section.

4. Skip questions you don't know, and guess!

If you come across a question that takes you more than a minute or two to answer, or that you don't know the answer to at all, skip it and come back to it later! For the multiple choice sections, you will likely be able to answer more questions this way, and then can come back to the questions that you skipped near the end of the time allowed for the section and make an educated guess on the answer. For the free response sections, this means skipping a section of a problem to do the next sections, or to set up the math necessary for the next sections if they are dependent on the ones that you are stuck on. Always put an answer down for a question, even if it is a guess! You will not be penalized for guessing.



How to Study for AP Calculus Cont.

In this article, we've outlined some AP Calculus study plans, gone over some AP Calculus tips, talked about the format of the AP Calculus exams, and reviewed how to interpret the results of a practice exam. With the use of this information, you will have every resource and advantage you can to do well on the AP Calculus exams! Here at Albert.io, we love to get feedback on our articles from the students who use them. If you have an AP Calculus review book that you think is fantastic, or you have an article that you think we should write, don't hesitate to reach out to us! Otherwise, happy studying!



Ready to Score Higher?

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Start Practicing





Math can be the bane of many students' existences. Even mentioning the words 'AP Calculus exam' can send a shock of terror through any student's heart. But don't worry, we've got the tools that you'll need to succeed. We have developed a 30-day study plan that will help you solve problems, experiment, interpret results, and support your conclusions. We're here to sharpen your math skills and make you an AP Calculus AB/BC pro in no time. We can't make the AP Calculus exam questions any easier, but we can help sharpen your math skills. This AP study guide will give you everything you need to review, learn, and maintain for the AP Calculus AB/BC Exam. If you stick to the daily regimen we have laid out for you here, we will sharpen your calculus skills like never before and get you that much closer to that 5 on the AP exam.



Key Things to Remember While Using this AP Study Guide

• From The College Board: "AP Calculus AB is roughly equivalent to a firstsemester college calculus course devoted to topics in differential and integral calculus...AP Calculus BC is roughly equivalent to both first and second-semester college calculus courses and extends the content learned in AB to different types of equations and introduces the topic of sequences and series. The AP course covers topics in differential and integral calculus, including concepts and skills of limits, derivatives, definite integrals, the Fundamental Theorem of Calculus, and series. The course teaches students to approach calculus concepts and problems when they are represented graphically, numerically, analytically, and verbally, and to make connections amongst these representations. Students learn how to use technology to help solve problems, experiment, interpret results, and support conclusion.."

• Both the AB and BC exams consist of 45 multiple choice questions, for which you have 105 minutes to complete. This will make up 50 percent of your exam score. There will also be a single long response question and two shorter response questions that will make up the Free Response section of the exam. These three questions must be completed within 1 hour and 30 minutes, respectively. They account for 50 percent of your overall score.

• This is a guide, you don't have to follow every step if it's not working for you. Everybody learns differently and it's ok if some steps are not quite working for you. Change things up and mix our recommendations around. Just find whatever will work best for you.

• This may seem like an impossible statement, but make studying fun! Try not to get too stressed out when studying for the exam. True, math and calculus can seem a bit daunting, but don't get too caught up in you work. You will be doing a lot of work over the next 30 days, but stress and freaking out never helped anyone ace a test. Nothing is so important that it should make you miserable, so enjoy and feel pride in each of your accomplishments as we move forward. We're rooting for you too!



• On that note, stay healthy! Maintain a healthy eating and sleeping schedule. You can study all you want, but if you don't take care of your body, you will not be doing yourself any favors on test day. A healthy mind remembers things better. Also, if you ever feel yourself getting tired following this study plan, get up and do a couple of quick stretches, or go for a short walk. Your brain will appreciate the extra blood flow.

What You Will Need

• Access to Albert.io's <u>AP Calculus AB/BC homepage</u>. The best way to study for the AP Calculus exam is to practice, practice, and practice. And Albert.io has the questions you need. You will be getting very familiar with these practice questions over the next month.

• A flashcard site such as <u>Quizlet</u>. Or you just use regular notecards. These are going to help you get all of those math terms down, so make sure that you keep on top of your daily flashcards.

• Note taking materials. These can be done on your computer, tablet, etc. but you're also going to need plenty of paper and countless sharpened pencils. Just like the flashcards, you're going to be taking a lot of notes and practicing those equations on a daily basis, so make sure that you're comfortable using whichever version you choose.

• The College Board homepage for <u>Calculus AB</u> and/or <u>BC</u>. Make sure you thoroughly read the AP Calculus information provided by the College Board's website. Just like any other exam, it is necessary to understand the expectations of the people testing you. Read these materials and memorize them if you can.

• Your own Calculus Textbook or an online source of that will provide you with the same amount of detail. When it comes to studying, the more the better, so make sure you have at least one source for reference. Need some help deciding? Take a look at our examination of the best (and worst) <u>AP Calculus Review Books</u> on the market now.



Optional (but helpful) Stuff:

• Any AP-style workbooks or study guides your teacher provides, or any supplemental material you find helps your study of the main materials. As we've stated before, the more the merrier when it comes to studying for the AP exams.

• <u>Albert.io</u> has some excellent recommendations on the how to approach the AP Calculus AB/BC exams and other great recommendations on readings, study tips, etc. Take a look at everything the site has to offer and make your own decisions on what will work best for you.

• A dictionary, which can be in print or <u>online</u>. Some math concepts get a little tricky, so this might help in the long run.

• You are also going to want a graphing calculator for certain questions. Look at our **<u>FRQs</u>** for help on which ones to choose.

How to Use the Study Plan

We have designed this AP study guide to revolve around the specific expectation laid out by the College Board for the topic of Calculus. The College Board has laid out four key themes that you will need to understand before taking the exam:

- 1. Functions, Graphs, and Limits
- 2. Derivatives
- 3. Integrals
- 4. Polynomial Approximations and Series

NOTE: If you are planning on taking the Calculus AB exam, you do not need to study for key theme number IV. This 30-day study plan covers everything that you will need to know for both exams. For those who do not want to take that extra step into Calculus BC territory, we still recommend sticking with our study guide to the end since it will help you better understand the topics covered on the AB exam.



That said, the phrase "practice makes perfect" is going to become your motto over the next several weeks. Probably more than any other AP exam, studying for Calculus AB/BC is going to require doing math problems until you start dreaming in algorithms.

You should be spending a minimum of one to two hours studying each day. All AP exams are known for being difficult. You will need to dedicate a serious couple of hours each day if you want that 5.

But we can't have you overload, either. It is important that you stay happy and healthy. Never overexert yourself. Keep on top of your sleep, healthy eating, and physical fitness. This will ensure that the information you are learning will be retained.

We have also worked in a study break every once in a while. This will help to ensure that your brain doesn't get too cluttered when sorting out all of this information in such a short period of time.



THEME 1

Functions, Graphs, and Limits

Day 1

First thing's first, we are going to start off by taking a look at the AP Calculus course Overview for both the <u>Calculus AB</u> and <u>Calculus BC</u> Exams. Read this thoroughly and get to know what is expected of you. One of the most important keys to successfully passing exams is to understand the expectations of those giving the exam.

Read through pages 1 through 3 of the College Board's <u>AP Calculus AB/BC Course</u> and Exam Description, paying particular attention to the ways that the College Board has set up the AP course and exam process. Familiarize yourself with <u>Albert.io's AP Study Guide</u> pages, getting a feel for how our website works and where all of the information is located.

Now, let's get into the routine of this study plan:

For each and every day you are reading material or watching a video, you are going to want to make flashcards on all of the buzzwords you come across (either with note cards, a folded piece of paper, or online on a site such as Quizlet). Choose a term of importance, writing it down on one side of the card, while putting the definition and a helpful hint down on the other side.

For Example, Right off the bat, you may not know what a vector function is. So, make a flashcard that looks like this:

(Side A) Vector Function

(Side B) a mathematical function of one or more variables whose range is a set of multidimensional vectors or infinite-dimensional vectors. The input of a vector-valued function could be a scalar or a vector. The dimension of the domain is not defined by the dimension of the range.



Helpful Hint: Here are some equations that relate to the term Vector Function,

r(t) = f(t) i + g(t) jr(t) = f(t) i + g(t) j + h(t) k

Sometimes it helps to leave some room in the significance section, so you can add more info as you learn.

Now is going to be a good time to read through the <u>How to Study for AP Calculus</u> <u>Exam Tips</u> page that Albert.io has set up. This article contains some very helpful tips on how to become an excellent test-taker, especially for the AP exams.

Now, read through <u>Chapter 1</u> from the MIT Calculus page. It asks, "why study calculus?" As a thought experiment, answer this. What do you think the point of calculus is? How much do you know about it?

After you've finished reading these, take out your notebook and rearrange these recommendations to fit your learning style. Which is most important to you as someone learning Calculus for the first time? The point of doing this is to get you to think about where you are going to focus your efforts for the next several weeks. Make it work for you!

OK, now get some rest and we will begin again tomorrow!

Day 2

It is going to be absolutely important that you take notes on the information you are gathering right not only today, but in the weeks that follow. Your notepad should become your best friend and really should never leave your side as you study with us. Make sure that you're taking notes on key concepts, ideas that you might find confusing and need to come back to later, and tips on how to become a better studier.



Take a look at this <u>"What is a limit?"</u> video from the site calculus-help.com. Don't forget to take notes on all of the key concepts that are being covered here. Use these notes to create flashcards. Also make sure to write down what you find.

It's ok if you're a little lost. This is Day 2 after all! If you are confused about a certain topic or idea, just make a note of it, so you can come back later and go back over the details.

Now let's take a look at pages 4 through 6 of the College Board's <u>AP Calculus</u> <u>AB/BC Course and Exam Description</u>.

This is going to be a good time to get used to how the <u>Albert.io</u> site is set up. We just want you to select 20 questions from any category, at random. You don't need to care about the topic or even if you get the answers correct. The point here is to just familiarize yourself with how these are phrased and the types of questions that will be asked. Don't worry if you can't answer them correctly yet, that time will come.

One of the most important things to remember is that Albert.io will provide you with helpful tips and places where you can do better. Take note of your weak spots and write them down in your notebook. These are the places you will have to spend more time on as you progress forward in the days ahead.

Day 3

Today we begin the serious business of our routine. The more we do these steps day by day, the more familiar you should become with this study guide. And remember, if anything just isn't feeling right, feel free to make your own tweaks.

This also means that we are going full speed ahead. Keep up and be proud of all of the progress that you'll be making.

We are going to start today off with these videos: <u>"When does a limit</u> exist?" and <u>"How do you Evaluate Limits?"</u>



After you've watched these, go ahead and read through <u>Chapters 2 and 3</u> from MIT's Calculus page.

Keep on top of your flashcard terms and your note taking.

Next, you're going to want to take your first real look at the Free Response Questions. Just like we did yesterday, with the example multiple choice questions, select 5 FRQs from the <u>College Board website</u>, read through them, and see if you can answer any of them. Again, this is just to get to know the questions; you shouldn't know how to do these yet. If you do, you're awesome, but don't sweat it if you don't. Take notes on how these are phrased and possible ways to respond. We'll take a look at these again later...

Let get into your first real set of practice questions. Go ahead and start answering the first 20 questions in the <u>Functions, Graphs, and Limits</u> Try not to look at any notes and treat this as if you were taking the real test itself. But it's also early on in the study guide, so don't sweat it if you need to look at some notes at this point.

Try not to get frustrated if you're having a hard time at this point. We are only a few days in. The more you stick to this 30-day study plan, the easier these questions will get.

Great job thus far. Hopefully, you're getting into the swing of things!

Day 4

Let's go ahead and starts our day with some more videos. Take a look at the videos that cover <u>Lessons 3 and 4</u>. Remember, don't watch these without your textbook(s) open and your notebook ready to be written in.

On the topic of note taking, take a look at <u>Chapter 4</u> from MIT's Calculus for Beginners site and keep jotting down what you think is important or what you might be struggling with. .



Now's that time again—let's do some practice questions. Today we are going to do those titled <u>Limit of Functions and Continuity and Differentiability</u>. Keep your eye on the recommendations that Albert.io is providing. These are going to help you grow into a better mathematician.

Now would be a good time to dive right into the FRQs as well. You're going to want to sit down and do those that were given out in the year 2002 for both the <u>Calculus AB</u> and the <u>Calculus BC</u> AP exam. It is quite all right if you cannot answer these. But try to do so at the best of your ability. But pay particular attention to the sample answers that are provided. Take note on what has worked in the past and what has not.

Day 5

We are going to be finishing up the first theme for Calculus AB/BC today. If things feel like they're flying by, don't worry. We will have plenty of time to go back over everything that we covered in the Review section of this study guide.

With this in mind, go ahead and take a moment to review anything that might be confusing you to this point.

Once you feel like you've caught back up, quiz yourself on all of the flashcards that you have created. Again, as a friendly reminder, keep on top of these every time a new idea or term pops up in this study guide.

Now, you're going to want to finish off the remaining videos from the Calculus-Help page. These would be <u>Lessons 5 and 6</u>

We are going to end today's session with a couple of Free Response Questions. Choose the 2003 FRQs from the College Board's <u>Calculus AB</u> and the <u>Calculus</u> <u>BC</u> Again, a little peek or two at your notes is ok, but now is a good time to start trying not to rely on them too heavily.



Let's go ahead and finish up the <u>Functions, Graphs, and Limits</u> Complete the remaining questions that you have not finished yet for this section. Now that we are finishing up this course theme, you shouldn't really be relying on notes anymore. You should start treating these practice tests more like the real deal.

Before we finish up with the first theme of this course, review everything that we covered up to this point. Is there anything that you are unclear about? If so, go back over your notes or reread some of the material that we covered, so you can get the ideas down.

Day 6

Ok, you need a little break. Take a long walk, hopefully for at least an hour.

On the walk go over everything you covered this week. Let the ideas flow. Try to remember what you can in your own words.

When you return from your walk, look up the material you couldn't remember on your walk. Spend the next hour skimming all of your notes—nothing too serious.

If you can, try your best to get a study-buddy, friend, or family member in on the action. Explain everything you've learned this week in your own terms. Ask them if your explanations were clear. Also have them ask you questions of their own. This will help you understand the concepts in your own way.

This last part is particularly important. It may sound weird, trying to explain math problems out loud like this. But the more you practice explaining the concepts, the better you will understand them. Try to think about how you would teach the concepts to a room full of students learning calculus for the first time.

Start your AP Calculus Prep today



THEME 2

Derivatives

Day 7

Before we jump full force into the Derivatives theme, write out an outline that covers the main themes and concepts that have been covered up to this point. Make sure that you are connecting your ideas together, showing that you have a solid understanding of how functions, graphs, and limits all relate to one another and the roles that they play in the world of calculus.

Read through pages 6-17 of the College Board's <u>AP Calculus AB/BC Course and</u> <u>Exam Description</u>.

Once you've read through this, let's get going with the Theme #2 videos and readings. Take a look at the first lesson of the Derivatives section from <u>Calculus-Help</u>.

Once you've got these concepts down, go back to MIT's calculus for beginners page and read through <u>Chapters 4 through 6</u>, taking note of all the new material that is being covered.

Back to practicing the multiple choice questions. Get working on the first 50 questions from the <u>Derivatives section</u> that's been supplied by Albert.io. If you get any of these wrong, look back over the questions once more. Ask yourself what the best way to answer this type of question would be. Even though you already know the answers, retake these questions. The point is to get the formulas down so when similar questions pop up in the future, you'll know the routine.

Make sure that you are taking note of where you are struggling, so you can come back to it later. Continue writing key concepts on your flashcards, but also, keep track of what you find confusing in your notebook. These will prove useful for when it comes time to review.



Let's go ahead and finish off the day with another FRQ. Select a question from the year 2003 for today's practice. It doesn't really matter whether you choose from the <u>Calculus AB</u> or the <u>Calculus BC</u> exam right now. What matters is that you need to stop using your notes at this point and start treating these practices are like the real deal.

Day 8

Review all of your flashcards up to this point. Give yourself a little test by randomly selecting 15 cards and see if you can get all of the definitions correct.

Go ahead and click on the <u>"The Power Rule"</u> video link and watch the video.

Once you've finished taking your notes on this video, go ahead and read through <u>Chapters 6-7</u> from MIT's page. After you've done this, take out your notebook and reflect on which of these learning styles fit you better. Are the Calculus-Help videos more of your style? If so, keep taking detailed notes on these. If you're struggling with MIT's Chapters, you may have to reread them in order to get the information out. Remember, not everyone learns the same way.

And now we begin with the multiple choice questions. Go to our <u>AP Calculus</u> <u>page</u> and complete the next two sections worth of questions (47 questions in all). Keep taking notes on where you are struggling. Make sure that you focus on these problem areas as we move forward in the weeks.

Now on to another FRQ example. Work through the other <u>Calculus AB</u> or the <u>Calculus BC</u> FRQ from 2003 (whichever you did not choose from the day before. Make sure that you are no longer relying on your notes for these. Take your time and work through these on your own.

Day 9

Today we are going to do something a little different. Today is going to be a practice day (remember, practice makes perfect when it comes to math and calculus). We will be working on practice multiple choice questions and FRQs like we usually do, but with a twist.



As you work through the following problems, write out an explanation for every step you are taking to solve the equations. For example, you may come across a problem that is asking you to find a limit of a function as X approaches a constant. In solving this problem, you may want to write out that you are going to "circumvent the indeterminate form by factoring both the numerator and denominator" right next to the numerical representation of your solution.

You are going to do this with every problem you work on today, so let's get started!

GO through this process with all of the questions in the **<u>Differential Equations</u>**, **<u>Second Derivatives</u>**, and Parametric and Polar Differentiation example questions.

And go ahead and do the same thing with both of the <u>Calculus AB</u> or the <u>Calculus</u> <u>BC</u> FRQs from the year 2004.

Once you have read through all of the answers to these questions, go ahead and end today's study session.

Day 10

Before we get straight into the readings and videos for this week, how did you like the explanatory part of the problem solving from yesterday? If you found it useful, go ahead and do this every once and a while when you are answering your practice questions. Sure, it takes more time, but it will help you understand the concepts in more detail.

Now, back to the <u>Calculus-Help</u> Take a look at "The Product Rule" video.

Now is going to be a good time to go through <u>Chapters 8-9</u> from MIT's page. After you've done this, write out a couple of sentences that explain what you've learned today. Think about this as if you were going to explain them to someone else who has never taken calculus before.



Back to practice question time. Go ahead and answer the <u>Trig, Exponential, and</u> <u>Log Derivatives</u> section. Now will be an excellent time to start timing yourself. If you do not have a timer or stopwatch app on your phone/computer, you can access one here: <u>online-stopwach.com</u>. Try to treat this like a real exam, paying close attention to the time you have used up and how much time you have remaining.

Select and work on the FRQs from both the <u>Calculus AB</u> and <u>Calculus BC</u> FRQs from the year 2005. You are going to want to have someone else take a look at what you've written. If you have a family member or friend who is familiar with this kind of math, have them look through them and double-check your work. Otherwise, get your teachers in on the action.

Day 11

Today we are going to finish up with the Derivatives theme of this course. What you are going to start today with is the creation of an outline. Think about everything that we've covered up to this point. If you were to teach these themes to a new student, how would you do this? Probably, most importantly how to you explain the connections between all of the topics that we have covered?

Watch the remaining video lessons from <u>Calculus-Help</u>. Don't start slacking on your notes and flashcards. Make sure that you are keeping up with the new ideas and anything that you might be finding confusing.

Let's finish up this theme with a review session, <u>Chapters 10.</u> After reading through this, how does it compare to the outline that you created earlier? Is there anything you need to add to your own notes on what we have covered?

It's time to finish up with the <u>Derivatives</u> section of practice questions. Complete any that remains. Take notes on which questions you are struggling with and make sure to review these topics from the videos and other readings we have been going over.



Now go ahead and work on the <u>Calculus AB</u> and <u>Calculus BC</u> FRQs from the year 2005. Remember, a calculator is not allowed on several of these questions, so pay attention to the requirements. You need to be practicing these as if they were the real test.

If you haven't had the chance to finish all of your multiple choice questions or FRQs, go ahead and catch up now. Also, make sure that you are paying attention to the recommendations that Albert.io provides.

Day 12

Congratulations, you've made it to another day of rest.

Take your walk, clear your head. Do the same as last time: think about all of the concepts you've gone over up to this point.

Keep thinking about how you would teach this topic to other people. What are the best concepts to focus on? What types of problems are repeatedly coming up on the exams?

When you get back from your walk. Go over your flashcards once more; keep mental and/or physical notes on where you need the most work.

Start your AP Calculus Prep today



THEME 3

Integrals

Day 13

Since we are getting close to the halfway point of our 30-day study guide, now would be a good time to take a look at <u>Albert.io's checklist for the AP Calculus</u> <u>AP/BC exam</u>. Look through those sections that we have covered and check of only those you can answer with confidence. Anything that does not get checked off should be reviewed once more.

After this, take a thorough look at pages 17-34 of the College Board's <u>AP Calculus</u> <u>AB/BC Course and Exam Description</u>. After finishing this, is there anything you would add to the course outline that you began on day one? Are there any concepts that you are missing or would choose to focus more time on?

Calculus-Help is still updating its video selection, so we are going to have to switch gears here and take a look at a series of Calculus videos from a YouTube playlist. For now, we are going to watch the <u>videos 41 through 47</u>. Make sure you take good notes on these as well.

NOTE: If you've got the time, you should watch all of these videos. They're also very good and will act as a perfect source for review.

So, luckily MIT's Calculus for Beginners page covers all of the material that we need, so we don't have to change anything there. Since you know the deal, go ahead and read through <u>Chapters 11 and 12</u>.

Complete all of the questions from <u>Interpretations and Properties</u>. Review your notes and textbook on any areas you got wrong, and try to figure out why you answered the question wrong. Are you missing an important piece of knowledge about the course material, or did you just get tripped up on the wording of the question?



Let's go ahead and work on some more <u>Calculus AB</u> and <u>Calculus BC</u> FRQs from the College Board website. Do those from the year 2006. Keep working on explaining the steps that you are taking. After you've done this in writing, ask a friend or family member if you can explain the process to them. Go over how you've answered each question, explaining the process in detail. Encourage them to ask questions.

Day 14

Review all of your flashcards up to this point. Give yourself a little test by randomly selecting 30 cards and see if you can get all of the definitions correct.

Go back to that YouTube and watch videos 48 through 54.

Now go ahead and read through <u>Chapters 13 and 14</u>. Take some time to think about the ways that Integrals and Derivatives are fundamental parts of Calculus. Use these readings and the videos to explain the ways they are connected and how they function.

And now we begin with the multiple choice questions. Go to our <u>AP Calculus</u> <u>AB/BC page</u> and work on the following to sections of multiple choice questions (41 questions in all).

Once you have done that, it's time to move on to the FRQs. Complete the questions from the year2007 for both the <u>Calculus AB</u> and <u>Calculus BC</u>.

Day 15

Congratulations, you've made it to the halfway mark of this study guide! You should take a moment to feel proud of the work you've done. Keep up with the good work and let's finish off like a champ.

Watch the videos 55 through 62. How do you like these compared to the others we've watched? Which is better? Why? How would you create an informative video like these?



Read through MIT's <u>Chapter 15</u>. Don't slack on your flashcards and note taking now. You've come so far!

Complete all of the questions from the **<u>Techniques of Antidifferentiation</u>**.

Work on the 2008 FRQs from both Calculus AB and Calculus BC.

Day 16

Before we begin with any new material today, go ahead and look through all of your notes and take a quick assessment of all of the videos that we have watched up to this point. Are there any topics that you have not quite gotten a full grasp of? Go back and re-watch any videos that cover topics that you're struggling with.

Do the same with all of the reading material to this point, including all of the MIT stuff and any textbook material that you've been using.

Now spend the next 30 or so minutes coming up with your own Calculus problems. Imagine that you're creating a test for the class that you've been creating outlines for. Come up with three original math problems for each topic that we have covered (Integrals, Derivatives, and Functions, Graphs, and Limits). Along with these questions, provide detailed explanations of how to answer the questions.

Now do the same with example FRQs. Come up with 2 of your own questions and write out detailed explanations for how the problems should be solved.

It's always fun to get a friend or family member involved, so ask another studymate (you could even ask them to create their own questions and answer them too) or your parents to solve the problems that you've just created. When they're done, explain to them out loud the best solutions, using everything you have learned about Calculus up to this point.

Keep on top of your flashcard terms and your note taking. It may be a good time to review those that you have made up to this point.



Day 17

Now, let's get back on schedule with the readings and videos. Click on the links for <u>videos 63 through 69</u>. Make sure that you're still taking detailed notes.

Back o MIT's Calculus for Beginners page. Go ahead and read through Chapter 16.

Finish off the remaining Integral multiple choice questions. You should be caught up with all of the questions we have covered. If not, spend the next 20 minutes going back over those you may have missed and catch up with the end of this theme.

Since we have already completed a number of example FRQs, now would be a good time to review those you have already completed. Read over all of the answers that you have provided thus far. How would you answer them differently? If you were grading the exam as a member of the College Board, what grade would you give yourself? Have a teacher, friend or parent read some of these as well and ask them if your thought process is clear. If they were to provide you with a grade, ask them what they would give you.

Day 18

The end of another major theme!

Spend today reflecting on everything you've learned in the last couple of weeks. What are you struggling with the most? Are you still stuck on certain ideas? If so, write these down and we'' come back to them during the review section of the 30-day study guide.

Stay on top of you physical fitness, sleep, and diet.

Start your AP Calculus Prep today



THEME 4

Polynomial Approximations and Series

Day 19

This is probably the most important period of this 30-day study guide. You may be tempted to start slacking; but you are so close to the end. Three words: Don't stop now.

Even if you are taking the AP Calculus AB exam, stick through the rest of the study plan. We've designed the entire 30-day study routine to help both Calculus AB and BC students. You will only be cutting yourself short if you quit now.

Enjoy your rest day today.

Keep up the good work!

Day 20

Again, before we start this new theme, let's take another look at the <u>checklist</u> you've been working on. Your confidence level should be going up as you check these off. You've done some great work, so feel good about it. Reread and review everything that you are still struggling with.

After this, read the rest of the College Board's <u>AP Calculus AB/BC Course and</u> <u>Exam Description</u>. At this point, you should have a good idea of how the exam will be structured and what will be expected from you on exam day.

Now, go ahead and skim over everything that you've already read from the College Board's website, including the exam description. Take your notebook out and write down everything those topics that you think you need to study more. Combine this with any other topics you need to review from the checklist that you worked on earlier.



Keep these notes handy. We're going to use them later on in this study guide. Also, as we continue on, keep adding your own notes on where you think you need to improve the most.

Great, now that the review stuff for today is out of the way, let's get into the math. Take a look at videos 70 through 78.

Now, click on MIT's Calculus for Beginners page and read through <u>Chapter 17</u>. Keep taking notes on where and how to improve.

After that, work on the Series Basics and Concepts.

Work on the FRQ from the year 2009 from both <u>Calculus AB</u> and <u>Calculus BC</u>. If you haven't done so yet, get a teacher to take a look at the ways you've been answering these. Ask for some test-taking tips.

Day 21

Let's begin today's study session by watching <u>videos 79 through 89.</u> While you are watching these, make 5 flashcards for every video you watch. At this point, your flashcard collection should be getting pretty big and scary looking.

After you've made your flashcards, read through Chapters 18 through 19.

Now, try your hand at the **Advanced Series multiple choice questions**.

We are going to finish off the day with the FRQs from the year 2010 from both <u>Calculus AB</u> and <u>Calculus BC</u>. At this point, you should be feeling comfortable with the timing of these FRQs. If you still find yourself rushing to finish these up within the allotted amount of time, spend today's practice paying special attention to the clock and ways to manage your time better.



Day 22

Today we are going to actually finish up with all of our reading and video watching. So, before we officially begin this material today, take a quick review of your notes to make sure that you're up to speed on all of the material. Pay special attention to those places you are struggling.

Go ahead and finish off the remaining videos from the <u>YouTube channel on</u> <u>Calculus</u>.

After that, finish reading the rest of MIT's Calculus for Beginners.

Doesn't that feel great to finally be done with these? But make sure that you're not slacking off just because this is the last chunk of new material.

After you've got these down, review all of your flashcards up to this point. Give yourself a little test by randomly selecting 30 cards and see if you can get all of the definitions correct.

Finish off the rest of the **multiple choice questions**.

Let's go ahead and finish off the day with another round of FRQs. Select and answer the FRQs from the year 2011. Do both <u>Calculus AB</u> and <u>Calculus BC</u>.

Day 23

We are going to spend the majority of today preparing for the review section coming up after out theme break. Make sure that you take good notes today since we want you to focus your energy reviewing topics that will help you out the most.

First, let's take another look at the <u>checklist</u>. Make sure that you think long and hard on this. It's ok to be struggling on certain topics, everyone does. SO, be honest with yourself when checking those boxes. Take notes on where you are struggling the most and where you think you should focus your review.



Do the same thing with all of the notes you've created. Go back over those comments to yourself on where you need to improve. Keep adding these to your growing list.

Now, read through the blog article on <u>The Ultimate List of AP Calculus Tips</u>. What part of this blog post is most useful to you? Why?

After you've finished reading this, take a look at our List of 7 Ways to Approach <u>AP Calculus Review</u>.

After you've read these, ask yourself: Have I covered all of these steps? Which ones have you been slacking on the most? Which were the most helpful tips? Write down all of your answers in your notes.

We are going to make two lists. For the first, make a Top Ten list of topics that you need to review. Rank them in order of those ideas that are most confusing being first, and so on. Tape this to your study area. By the end of next week you'll have them all off that list.

For the second list, number the best study tips that you've read or heard yet. Which ones work best for you? The point here is to figure out what kind of learner you are, so you can focus on your strengths during our final review week.

Lastly, for today, go ahead and finish up with any FRQs or practice multiple choice questions you may not have completed yet.

Day 24

Another day of rest! Great job, you should be feeling fantastic about your progress.

This is probably the most critical period. At this point, you might be feeling a little burned out. But stay with us, we'll get you through this.



You may be tempted to take it easy on the following review section, but don't. We've designed this 30-day study guide to guide you through a successful AP exam. The next several days are essential in helping you retain everything you've worked so hard to learn.

Get some rest, and onward!

Start your AP Calculus Prep today



REVIEW

Day 25

We are going to start off today by looking at those lists you've created. Your plan for today is to focus your review on what you need to learn before the exam. Your goal will be to cross these off the list by the end of our session. Emphasize on numbers 1 through 3, in particular.

Next, select 10 to 20 questions that apply to each concept that you've written on this list. Use the <u>Albert.io questions</u> that we've been working through this entire study guide. All of these should be repeats, since we've finished them, but you need to work on your must-learn list, so a little repetition makes sense, right?

For example, if you are having problems figuring out derivatives, work on those questions from the "Derivative at a Point" and Second Derivatives" sections.

Next, look through the <u>Checklist</u> once again. Go over this and check everything that applies. If there's a topic that you haven't quite got down yet, keep studying until you can say, with confidence, that you can give it a check.

Next, take out your notes, outlines, and example tests for the AP Calculus course writing exercise we've been doing. Would you make any changes to the ways you've described the AP Calculus course? What would you add? Why? What skills have you learned about studying for this exam that you would share with others?

After working on this thought exercise for ten to twenty minutes, do a full review of your flashcards. Make sure that you've got all of these terms down.

Now, go ahead and work on the <u>Calculus AB</u> and <u>Calculus BC</u> FRQs from the year 2012.

Make sure that you are all caught up on your FRQs and example questions. Now is the time to complete everything you have missed. Also, make sure that you are caught up on all of your reading, online videos, and flashcards.



Day 26

Before we get into any more practice questions for today, go back through all of your notes and review those topics that you are still struggling with.

After reviewing your notes, take out your notebook. Let's make sure that you've got a solid understanding of the central themes that make up the AP Calculus course. Write out, in your own words, the main points of each of the themes that we have covered. Each explanation should take up a page, showing all of the key concepts, connections with the other themes, and hints on how the math works.

Now is the time to retake all of the example questions that you may have gotten wrong over the last several weeks. Keep taking them until you get the right answer, reading through Albert.io's recommendations after each attempt. Do this again with each of the FRQs.

After you have done this, select another **<u>30 example questions</u>** at random.

Next, do the Calculus AB and Calculus BC FRQs from the year 2013.

OK, so for the next two days, we are going to do a couple of practice exams and try to recreate the exam itself as best we can. So, spend the rest of the day setting up the perfect practice exam space for the next few days. Make sure that you are going to be in a quiet space, where you can focus for two or three hours and nobody will disturb you. Set up some pencils, paper, water, snacks, your calculator, stopwatch, etc. and everything else you will need.

Day 27

Today, we take our practicing seriously. Get ready to start this as if it were the real deal.

Have your stopwatch or clock ready, so you make sure that you are staying in line with the time limits required by the College Board. For today and tomorrow, absolutely no notes.



Reminder, you have 105 minutes to complete the multiple choice section (45 questions) and 90 minutes to complete the FRQs section (3 questions).

And now we begin with the multiple choice questions. Go to the multiple choice questions for the year <u>1998</u> and complete them within the time limit. These are actual questions from past exams, so take note on how they are phrased, the best solutions, etc.

Once you have done that, it's time to move on to the FRQs. Complete the questions from the year <u>2014</u>.

You did it! Great job. You should be feeling really positive about your work and all of the effort you've put in.

Last, you want to check all of your answers and think about ways to improve.

Day 28

OK, today is going to go the same as yesterday. Good luck!

Go to the multiple choice questions for the year <u>1996</u> and complete them within the time limit.

Next, complete the FRQs from the year 2015.

Check all of your answers once more and take any notes on where you may want to spend your review time tomorrow.

Great job for all of your hard work!



Day 29

Today is going to be your final day of review. You should be feeling very confident about your future exam. Now's the time to get those final problem areas off of your list.

First off, take a look over all of your notes, including all of the outlines you have created, the outlines that you have made, and take a skim-through of the videos and readings we've worked on. Read your notes thoroughly and double-check your familiarity with all of the concepts and ideas.

If there are still some problem areas, re-watch any videos on the topic. It's ok to not know every piece of information, but you do want to have a good feeling about all topics.

Next, you are going to want to review all of your flashcards. DO one last quiz on yourself. Or better yet, get a friend to help you study. Power in numbers, right? (Like the math pun?)

Review all of your example question scores from the <u>AP Calculus AB/BC site by</u> <u>Albert.io</u>. If you still have some questions that were answered incorrectly, go back and retake these until you have reached 100 percent correct.

Review all of the FRQ you have answered. Ask yourself: Which ones were the strongest? Weakest? Can you see yourself getting progressively better? Take notes in your notebook on where you need to do better. Be as specific as possible.

Make sure that you are paying attention to all of the comments that Albert.io and the other sources are providing you, so you can figure out the best way to approach the solutions to the problems.,

Next, take one last look at the list of test-taking tips that you made. Look this over once more and remember the tips that have worked best for you. Use your strengths to get that 5.



Day 30

You did it! You made it to the last day. Awesome job!

Today is a day that you just need to prepare for the exam. Don't stress out. Feel good about your work. And be proud that you made it through our 300-day study guide.

You're going to want to focus on being prepared for the exam itself. Get your stuff ready. Just like the preparation for your last couple of practice exams, do you have your pencils? Water? Snacks? Calculator? Get all that together the day before the exam, so you're spending the day thinking only about Calculus.

Again, stay healthy! Drink plenty of water to stay hydrated and eat healthy foods.

Probably most important, get your sleep! Don't throw the last 30 days away by being tired and brain-fogged during the exam.

You've worked really hard to get to this point, so congratulations on staying on top of this 30-day study plan—you'll do great!

If you have kept up with this daily study guide, you will have:

Notes, key terms, and flashcards on the four central themes of AP Calculus.

Gained the skills to graphically, numerically, analytically, and verbally explain how calculus works in both theory and in applied mathematical equations.

Observed dozens of visual examples of economic processes work through expansive informational videos and explanatory texts.

Completed a broad range of example Free Response Questions and practiced with hundreds multiple choice questions!

Applied tried and true test-taking tips to your AP Calculus study regiment.



If you have any extra time between the end of this 30-day study guide and your actual test, stay on top of our work. You are going to want to go over all of the material provided by <u>Albert.io</u> as often as possible.

Finally, maintain that feeling of confidence. Desire and discipline pave the road to success. The fact that you have shown your dedication and resilience in completing this 30-day study guide shows that you are a strong learner. Work with your strengths and always feel proud!

For information on other AP exams and the other study guides we offer, head to <u>Albert.io</u> or read more of our other blog posts.

<u>Let us know</u> what has worked for you. What did you like best about this one month study guide? Do you have recommendations of your own on how to study for the AP Calculus exam?

Start your AP Calculus Prep today



7 Ways to Approach AP Calculus Review

WIND Log X Sin tg 0,01x Sin cos 0,1x

Image Source: Flickr

The AP Calculus exam can be one of the most daunting exams that a student takes during his or her high school career. Fortunately, by following these tips for AP Calculus review, you can be confident in your ability to receive a high score on the exam when the time comes.


7 Ways to Approach AP Calculus Review Cont.

1. Write Out What Will Be On The Exam

This first step may seem unusual, but before beginning to review for the AP Calculus exams, it is important to make sure you understand exactly what topics will be covered. The material for the two AP Calculus exams differ, but comprehensive lists of all the topics you will need to know are <u>available on the</u> <u>CollegeBoard website</u> in the course description files for each course. There are a breadth of topics that you should be well versed in before taking the exam. You do not need to know the details of each topic, but you should have a working knowledge of the relevant formulas and basic concepts for each topic. Given this, making a list of the topics that will be on the exam is the appropriate way to begin reviewing, as most of the following steps to review depend on your familiarity with the material on the exam.

2. Take a Practice Test

Taking old AP exams (exams for AB and BC can be found <u>here</u> and <u>here</u>, respectively) from previous years is one of the best ways to prepare for the AP Calculus exams, because these practice exams cover the same material in the same manner as the actual exam. You will want to take as many practice exams as possible before the actual exam to make sure you are comfortable with the format of the exam, but at this step in your AP Calculus review process, you should use the practice test to assess what you need to review, and the forms in which they appear (multiple choice vs. free response), before the exam.

3. Divide Questions by Understanding and Review Wrong Answers

Once you've taken your first practice exam, it is important that you review your answers. Understanding why you are getting questions wrong is important, because it will help you not to make the same mistakes on the actual exam. If you got a question wrong because you did not understand the topic, you should review that topic or ask for help.

7 Ways to Approach AP Calculus Review Cont.

If you got a question wrong because you did not understand the question, you should look at similar questions and their answers to get an idea of what the question is asking for. You should also review the questions with a teacher or a student who has previously done well on the exam, as they should be able to tell you whether you are applying the material correctly. Reviewing your wrong answers also helps you to determine how much focus you should put on reviewing each topic between subsequent practice exams, allowing you to maximize your study time.

4. Take a Timed Full AP Calculus Practice Exam

While understanding the material that will be covered on the exam is probably the most important part of preparing for the AP Calculus, be aware that one of the most challenging parts of the exam has nothing to do with the questions, and everything to do with the time you have to answer them. This is also the part that most students do not prepare for. You will be given 105 minutes to answer 45 multiple choice questions and 90 minutes to answer six free response questions, and although this may seem like a long period of time, you will be surprised how quickly time flies during the exam. By taking a timed practice exam before the actual exam, you can get an idea of how much faster or slower you should be working.

5. Review Everything

Following the theme of time, the time that you have to review before taking the AP Calculus exam is precious, and should not be wasted. At some point before the exam, you should briefly review all of the material that will be covered on the exam, including the material that you have been getting right on the practice exams. As this is a review, you do not have time to relearn all of the material that will be covered on the exam, and trying to do so will likely result in you not being able to properly cover or understand the majority of the material that will be on the exam.



7 Ways to Approach AP Calculus Review Cont.

Your review should consist of reading over a summary of each topic and the completion of a couple practice problems from each topic. By doing this, you may answer any questions you had on the practice exam, as well as reinforce topics you already understand.

6. Figure Out How You Test

The perfect time to figure out how you should prepare to take the exam is when you are reviewing. This may sound redundant, but there is more to review than just taking the actual test. Try to get an idea of the conditions under which you test best. You cannot take the test at a time other than when it is given, but you can eat before the exam and get a good night's sleep. Before taking practice exams, try different combinations of eating and sleeping (such as eating more carbs versus eating more protein before the exam) so that when you take the actual exam, you are prepared in every way possible.

7. Know Your Limits

Reviewing the material is one of the easiest ways to score high on the AP Calculus exams, but it is also one of the easiest ways to score low. While review is important, some students review to the point of creating more stress and anxiety for themselves within the days before the exam, causing them to panic on the exam and to perform worse than they have the potential to perform. Review everything you need to know for the exam, but be sure to take breaks periodically to do something relaxing, and to not over-tax yourself. There are various techniques that you can use to break up your study sessions. Most involve alternating periods of studying and rest, such as taking an hour to study and then taking a fifteen minute break. Whatever you do, make sure it leads to both effective AP Calculus review and a positive state of mind as the exam approaches.





Learn the language and vocabulary, because half of the battle is knowing what a question is asking and/or telling you! Give yourself the benefit of analyzing the information presented in a question, because then you can execute on the knowledge and skills that you have practiced!

In order for you to score a 4 or 5 on the AP Calculus exam (AB or BC), it is important for you to follow the tips outlined below. So far in 2015, only 38.4% of students who took the AP Calculus AB exam received a grade of 4 or 5. However, 61.5% of students that took the AP Calculus BC exam in 2015 received a 4 or 5 score. The AP Calculus BC exam covers all of the topics in the AP Calculus AB exam plus some additional ones as well. Take the time to review the following tips and you'll be well on your way to earning the highest possible score on your AP Calculus exam. Relax, read and absorb the tips as you go! Good luck!



How to Study for AP Calculus Exam Tips

1. Know the exam's content: The AP Calculus exam will cover a number of specific concepts besides just differentiation (Derivatives) and integration (Integrals). See the following for a comprehensive list of topics and concepts covered in the exam:

List of Topics and Concepts on the AP Calculus Exam	
> Analysis and behavior of graphs	> Integrals• Interpretations• Properties
> Limits of functions (one and two sided)	 Applications Techniques
> Asymptotic and unbounded behavior	Numerical approximationsDefinite
> Continuity	 Indefinite Areas under curve Euler's Method
 > Derivatives • Concept • At a point • As a function • Applications 	 Integration by parts
 Higher Order derivatives Techniques Max/Min problems 	> Implicit differentiation
> The Fundamental Theorem of Calculus	> L'Hôpital's rule
> Antidifferentiation	> Asymptotes



As Calculus is a systematic assembly of concepts, take the time to familiarize yourself with the terminology, and, more importantly, understand the actual concepts. Calculus is a highly conceptualized subject matter and it is extremely important that you have a firm grasp on all of the major concepts. Once you have mastered the individual concepts you are ready for the next step (Tip 2).

2. Practice makes perfect: In order to score a 4 or 5 on your AP Calculus exam you will need to practice lots of problems. The more problems you do, the more adept you will be at deciphering the way in which each type of problem is presented. There are numerous textbooks on calculus that are suggested study guides for students. Get one or two of these textbooks and start doing the problems in each chapter. You may also use online study guides for the same purpose. One such valuable resource is available at the <u>Albert.io</u> website. Do yourself a big favor and search for additional online resources as time permits. There are plenty of them! If you have extra time, we highly recommend visiting College Board's AP Calculus course homepage.

3. Assess the exam and gauge your time appropriately: There are two multiplechoice and two Free-Response Question (FRQ) sections of the exam. Both the multiple-choice and FRQ sections are broken up into four separate time segments. You will have two segments where you will be allowed to use a calculator and two where you will not be allowed to use a calculator. For each FRQ (there are a total of six) your time allotment should be 15 minutes. Try your best to pace your time on these questions. Since there are a total of 45 multiplechoice questions and a total time of 105 minutes, you should pace yourself at two minutes per multiple-choice question. By skimming the exam before you start, you can quickly tackle the problems you feel the most comfortable with. Then, you can tackle the more difficult questions.

4. Pay attention to detail: Especially when working with the AP Calculus FRQs, it is important that you are detail oriented in solving the more complex problem of an FRQ. In other words, make sure that you detail every step in the process. For example, if they have two or three equations that require differentiation or integration at some point, then show this work in detail.



You should draw a box around any important interim equations that you will need for figuring out the final answer. The picture below is an excellent example illustrating the use of this tip:



Image Source: askmrcalculus.com



Also, make sure that your final answer to a FRQ is clearly identified as such. If you are having problems with a particular multiple-choice question use the process of elimination. Try your best to narrow down your answer to one or two of the multiple-choice selections. For example, if you know that the answer needs to be in the derivative form then you can eliminate any and all options containing the integral form.

5. Review and double-check your final answers: We cannot emphasize this tip enough. As time permits, always perform a quick review of an answer. OK, if you are so totally confident of a particular answer then don't waste the time on rechecking its correctness. Move onto the next question and so on. For problems that you are not able to answer during your first skim, place a check mark or circle them so that it will be easy for you to find them later on during the exam. This will certainly be time well spent.

6. Review important trigonometric identities: Make sure that you know the common trigonometric identities you learned in earlier advanced math courses. You will most definitely need them for many of the problems on the AP Calculus exam. Here is a list of the trig identities you should know by memory:



TRIGONOMETRIC IDENTITIES

Image Source: Blair's Blog



7. Show all of your work: Make sure that when performing solutions to either the multiple-choice or FRQ problems that you show all your work. This is especially important for the FRQs as partial credit will apply. Multiple-choice questions do not have partial credit. They are correct or wrong, so there is no in-between.

8. Write down the equations: If you are using your calculator to solve an equation, be sure to write down the equation first. An answer without an equation may not get full credit even if it is correct. Also, if you use your calculator to find the value of a definite integral or derivative, write down the integral or derivative equation first.

9. Use your calculator only for the following: Graphing functions, computing numerical values for derivatives and definite integrals, and for solving complex equations. In particular, do not use your calculator to determine max/min points, concavity, inflection points, increasing/decreasing, domain, and range. (You can explore all these with your calculator, but your solution must stand alone in order to receive credit).

10. Know both the product and quotient rules for derivatives: These are some of the most frequently used rules in all of Calculus. You must know them well for the AP exam. We have always used the following to remember the product rule: The derivative of the product of two functions of the same variable is equal to the first function times the derivative of the second function plus the second function times the derivative of the first. Symbolically the product rule is:

$$rac{d}{dx}\left(g\left(x
ight)h\left(x
ight)
ight)=g\left(x
ight)rac{d}{dx}h\left(x
ight)+h\left(x
ight)rac{d}{dx}g\left(x
ight)$$

The quotient rule is more complex and can be memorized as follows:

If,
$$f\left(x
ight)=rac{g(x)}{h(x)}$$

Then, $f'(x)=rac{g'(x)h(x)-g(x)h'(x)}{[h(x)]^2}$



In words you can memorize the quotient rule as follows: The derivative of a quotient of two functions of the same variable is equal to the derivative of the top function times the bottom function MINUS the top function times the derivative of the bottom function ALL over the bottom function squared.

11. Indefinite Integrals: If you are ever asked to integrate a specific function always remember to add + C as part of the answer after the equal sign. Leaving it out will cost you points. For example, what is the integral of the function?

$$Y = f\left(x\right) = x^3 + x^2$$

The correct answer is:

$$\int Y dx = rac{x^4}{4} + rac{x^3}{3} + C$$

Start your AP Calculus Prep today



AP Calculus Multiple-Choice Review Tips

In this section we review some specific tips regarding the multiple-choice portions of the AP Calculus exam. Although some of these may also be extended to the FRQ portion, these tips are more tailored to the multiple-choice questions on the test.

1. Understand what the question is asking: Make sure that you read the question carefully and understand what they are asking for. Usually this is easier with mathematics type exams since math is more symbolic in nature. So it should be quite easy and apparent when you are trying to understand an AP Calculus question.

For example: What is the value of f'(x) for the function $f(x) = 3x^3 + x^2 + 4$ at the point x = 4?

In this example they are asking for the numerical value of the first derivative of the function at a particular point. So you will first need to find the first derivative of the function $f(x) = 3x^3 + x^2 + 4$ which is $f'(x) = 9x^2 + 2x$ and then plug in the value 4 for x into the first derivative equation. Therefore, the answer is

$$f'(x) = 152$$
 at $x = 4$.

2. The Process of Elimination Technique (POE): If you really know your stuff you will normally not need to use this technique. However, there are almost always a few problems that you may be having trouble with on the exam and this technique would definitely come in very handy. If there are answers that are obviously wrong it would be a good idea to place a mark on those answers telling you that they are incorrect. Since there are five possible answers to each multiple-choice question, eliminating one answer increases your odds by 20%. Not too shabby! Eliminating two answers increases your odds of a correct response by 40% and so on. Again, use this technique only if necessary. Otherwise, solve the problem and choose the correct answer.



3. Learn to spot distracters: The individuals who "craft" or "create" new multiplechoice questions and answers use what are commonly referred to as distracters in the answers section. The multiple-choice item consists of a problem, also known as the stem, and a list of suggested solutions known as "alternatives". Contained in the list of the alternatives is the correct or best alternative, which is the answer, and incorrect or inferior alternatives, known as distracters. Your job is to select the correct answer/alternative. Oftentimes on math exams distracters can be made to be very close in nature to the actual answer. One example of a common distracter is to use the correct numerical answer but to put a minus sign in front of it. Or if the answer is supposed to be a negative number they will place a distracter with a positive sign in front of it.

4. Use an <u>AP Calculus study guide</u> or <u>textbook</u> to prepare: Using a study guide or a good textbook can be a tremendous help in getting a high score on your AP Calculus exam. The reason for this is that many of the types of problems that occur on the AP Calculus exam come from these study guides. You can find many of them on the internet by doing a simple internet search. The most important thing to remember is that the more problems you do at home or with a study partner, the better your chances will be for getting a high score. Using study guides and textbooks will also give you exposure to the types of problems that will appear on the AP Calculus exam. This leads us to the next tip.

5. Know the different types of calculus problems: There are many different types of Calculus problems. One of the most common is the min/max problem. These problems require you to find the derivative of a specified function and set it equal to zero. You would then solve the resulting equation (the derivative) to find its roots and apply these roots to the original function to determine the min or max. Another common type of problem is that of finding limits of functions. Starting in 2016, the Calculus AP exam will contain problems relating to L'Hôpital's rule. This is a great way of determining the limit of a function divided by another function. You can visit this page to understand how to use L'Hôpital's rule when finding the limit of a quotient of two functions. The other types of problems found on the AP exam are the following: continuity of functions, asymptotic functions, antidifferentiation and the Fundamental Theorem of Calculus.



6. Understand and master the Chain Rule: It is very important that you know how to use the Chain Rule. This rule is used for the computation of derivatives of the composition of two or more functions. See this <u>page</u> for more information on how to use the Chain Rule. Knowing this rule will allow you to easily calculate the derivative of a multi-functioned problem containing at least two or more functions and their respective variables.

7. Identify your weaknesses: If there are certain types of AP Calculus problems you generally have issues with, practice doing more of those before taking the AP Calculus exam. Focus on the underlying concept of these (or any other) problems. Mastering and understanding the underlying and fundamental concepts of AP Calculus will greatly help you improve your score.

8. Review AP Calculus practice multiple choice questions online: One of the best ways to prepare for the multiple choice portion of the AP Calculus exam is to use online practice sites. These sites either have actual past AP Calculus multiple choice questions or they are modeled to be strikingly similar to past exams. Here is one such site with the 2006 AP Calculus practice exams along with the key to the answers. Going through each and every problem on this one website is like the equivalent of taking the AP Calculus test a total of five times! You get a real flavor of what to expect on the actual AP Calculus exam if you go through these problems one by one. In this manner, you will also be keen as to what types of problems you can expect to find when you take your own exam.

9. Study the multiple-choice problems in the 2012 College Board AP Calculus course description: The <u>course description manual</u> published by College Board AP covers both AP Calculus AB and AP Calculus BC exams. It contains just about all of what you'll need in order to get the best possible idea of taking the actual exam, the philosophy and principles behind it and a number of multiple choice and FRQ examples. We strongly recommend that you read and understand the entire document and then do the sample questions. Better still, we also recommend that you time yourself. Part A of the multiple-choice section contains 28 questions (55 minutes) and Part B contains 17 questions in (50 minutes).

Start your AP Calculus Prep today



AP Calculus Free Response Question Tips

The following tips are specifically designed to help you master the FRQ section of the exam. An FRQ is like a series of multiple-choice questions with the caveat that the FRQ is fashioned in the form of a chain of reasoning exercise. In essence, the FRQ is digging deeper in attempting to assess your more global understanding of Calculus.

1. Underline key equations: FRQs for the AP Calculus exam are essentially word problems that come in multiple parts. They are usually between 3-5 parts per FRQ. Visit AP Central <u>here</u> for examples of the AP Calculus Free Response Questions from 1999-2015. The first thing you should do is underline any key equations that are given. FRQs are designed to test your ability for an "extended chain of reasoning".

2. Show all your work: Since partial credit is given for FRQ's it is especially important to show all of your work. For example, you may be given a function f(x) and will need its derivative f'(x). Make sure that you actually write down the first derivative f(x) and underline or box it in since it will be an important equation you will need for the other parts of the problem. Here is an actual example from the College Board website. Your work should be clearly written in the space provided and your answers also should be provided in the proper space. Sometimes you will be asked to justify your answer. In this case briefly describe how you arrived at the answer. Indicate what concepts or equations you used to get to the correct answer. We recommend placing the words "Final Answer" or the letters "FA" right next to the boxed in final answers. This will make it very clear to the graders of your exam that this is your final answer.

3. Budget your time: You will have a total of six FRQs. Part A will contain two FRQs and is 30 minutes long (calculator permitted). Part B will contain four FRQs (calculator not permitted) and is one hour long. Important note! If you still have time when you finish Part B, you are allowed to go back and finish Part A if you need to. Try to spend no more than 10 minutes per FRQ. This will allow you some time to re-visit a particular question.



4. Read the data and question parts slowly and carefully: The AP Calculus Free Response Question has essentially two parts. The first part is the data or information needed to solve the individual problems. The second part is the individual questions. You will need to understand both parts in order to correctly answer and provide solutions to all of the questions.

5. Be specific and brief in your justification answers: If you are asked to justify your answer, don't write a book about it! Be brief and to the point. Make sure you include all pertinent aspects of how you arrived at the solution. For example, if you are asked to provide justification on how you determined that a certain polynomial function has a max or a min at a certain point you must show the individual steps you took in order to arrive at the solution. To do this, <u>make a</u> <u>table showing the original function</u>, the function's <u>first derivative</u> and <u>its</u> <u>roots</u> and <u>all of the critical points</u>. Then finally show how you determined that the point was a min or max. See the example below:

Find the absolute maxima and minima of the function,

$$f(x) = x^3 - 6x^2 + 9x + 2, \quad 0 \le x \le 4$$

Take the first derivative,

$$f'(x) = 3x^2 - 12x + 9$$

Find the roots which are:



These are the only critical points of **f**. We consider the following table of the endpoints and the critical points of **f**:

X	<i>f(x)</i>
0	2
4	6
1	6
3	2

The absolute maximum occurs at both x = 1 and x = 4 and is 6 and the absolute minimum occurs at both x = 0 and x = 3 and is 2.

6. Pay attention to details: Don't forget to give units if required. For example, if they ask you how many cubic feet of water are flowing through a pipe at a certain time, make sure that your final answer includes both the number and the units. In this case the units would be ft³. Another example would be to include the constant C whenever you integrate a given function. Leaving the C out will cost you points. Also, and this is especially important, make sure that your standard Calculus notations are correct and complete. For example,

$$rac{d^{2}}{dx^{2}}f\left(x
ight)=rac{d}{dx}f^{'}\left(x
ight)=f^{''}\left(x
ight)$$

7. Rounding of numerical answers: Make sure that any numerical answers are given to the nearest thousandth (3 places after the decimal point). Also, store any interim values in your calculator and use those numbers to calculate the final answer. You will lose points if your answers are not properly rounded.



8. Be neat when working on FRQs: Try to be as neat as possible when showing your work or answers on Free Response Questions. This is good for two important reasons. Firstly, if the graders cannot read your work they will not give you partial credit. Secondly, it is good for you because if you have made a mistake on the early part of a question, say section (a), then your answers for sections (b), (c) and (d) will be wrong as well since the FRQs are designed to challenge your chain of reasoning.

9. Make up your own problems and then solve them: This may sound silly but it will actually test your true aptitude in Calculus. Start with making up your own polynomial equation and then find its first derivative. Then, in order to prove to yourself that you understand the Fundamental Theorem of Calculus, take the integral of the derivative you just found. You should, of course, come up with the original polynomial. Take this a step further and make the original function more complex. Take a polynomial and multiply or divide it by a trigonometric function. Write it down and try taking its derivative. Once you have gotten an answer, plug in some numerical values and then check it with your calculator. Just think of how many possibilities you could come up with. Stick with reasonable expressions and don't overdo it. Write all of your work down on a piece of paper and keep it as your own personal study guide.

10. Never forget +C. When integrating a function, remember that you need to account for an arbitrary constant C. It may seem frivolous after you've done more complex calculations, but it's not. Check through your answers at the end of the section to make sure you have included a constant when necessary. I often forgot this during the school year, so I made sure to commit it to memory while studying. When the test arrived, I was incredibly glad I had. C is a crucial part of the equation and you will be marked down if you forget it!

11. Don't doodle! Many students feel discouraged and think it funny to draw instead of answering the question, try not to do this. Doodling is the equivalent of leaving a multiple choice question blank — that is, you will get zero points no matter what. Even if you're stumped, try rewriting what you know and brainstorming some ways you might be able to solve it. You might get a couple of points anyway. Better yet, you might have an insight that helps you work through the problem.



12. Memorize the key derivatives and integrals of common trigonometric and other functions: This tip is very important because it will save you time by not having to explicitly derive already known expressions. They will almost certainly appear on the AP Calculus exam so it is best that you know them by heart much like you learned multiplication tables in grade school using flash cards. You can also memorize these key derivatives and integrals by simply writing them down in your personal notes. Whenever you study for the AP Calculus exam, take the time to memorize, but more importantly, to understand the following:

Differentials	Integrals
$d(\text{constant}) = 0$ $d(u^n) = nu^{n-1} du$	$\int u^n du = \frac{u^{n+1}}{n+1} \qquad (n \neq -1)$
$d(e^u)=e^udu$	$\int e^u du = e^u$
$d(\ln u) = \frac{1}{u} du$	$\int \frac{1}{u} du = \ln u $
$d(\sin u) = \cos u du$	$\int \cos u du = \sin u$
$d(\cos u)=-\sin udu$	$\int \sin u du = -\cos u$
$d(\tan u) = \sec^2 u du$	$\int \sec^2 u du = \tan u$
$d(\cot u) = -\csc^2 u du$	$\int \csc^2 u du = -\cot u$
$d(\sec u) = \sec u \tan u du$	$\int \sec u du = \ln \sec u + \tan u $
$d(\csc u) = -\csc u \cot u du$	$\int \csc u du = -\ln \csc u + \cot u $
$d(\arcsin u) = \frac{1}{\sqrt{1-u^2}} du$	$\int \frac{1}{\sqrt{1-u^2}} du = \arcsin u$
$d(\arctan u) = \frac{1}{1+u^2}du$	$\int \frac{1}{1+u^2} du = \arctan u$

Source: Cliffsnotes



Tips by AP Calculus Teachers

The following are a collection of tips specifically designed for the student to optimize his/her performance on the AP Calculus exam. Most all of the tips are from the writer of this article. I first learned Calculus at the age of 16 and was self-taught in the beginning. I also searched the internet extensively as it related to the AP examination process in order to get more authoritative sources such as College Board AP Central.

1. Relax and Enjoy! I know it sounds somewhat counterintuitive to enjoy taking a test but it's actually very possible to do. Remember, you are taking this exam because you are smart and because you may even enjoy Calculus. Think of taking the exam as an opportunity and not a chore or commitment. You should put yourself in the frame of mind that it is an honor for you to be taking the exam in the first place and you now have that great opportunity to demonstrate your advanced skills. Stay positive in your thinking and simply focus on each individual question as if it were like any other test you have taken in school.

2. Prepare yourself both mentally and physically: A few days before you actually take the exam, try not to think of it constantly. Doing so will only increase your anxiety level. Make sure that you relax your mind and get a good night's sleep the night before the exam. Remember that taking the exam is an opportunity and not a necessity. While taking the exam keep this theme in mind and focus only on getting the correct answers. That should be your only goal.

3. Do as many problems as possible beforehand: Six to eight weeks before the exam start your preparation by doing a few Calculus problems each day. Make a habit of it. It will only take an hour a day and you will be ready and ultimately rewarded with a high score on the AP exam. You must do your due diligence and get in the groove of doing the problems each day during this time period. Don't worry, be patient, practice daily and you'll be well on your way to success. The score of 4 or 5 on the Calculus AP exam will happen by itself if you take this advice and persevere.



4. Be extremely confident in yourself: In order to get a good grade on any important standardized exam, you need to be extremely confident in yourself. Think positively by saying to yourself, "I know I can get a 4 or 5 on this AP Calculus test because I have prepared so well and for so long to get this far". Also, say to yourself, "I've done so well on all the other exams I've taken so why not just treat this exam like those?" Speak with your friends and tell them how much you are looking forward to taking the exam. Their responses will not only reinforce your confidence levels but also will make you feel better about taking the exam with the expectation of achieving a 4 or 5.

5. Focus on learning and understanding fundamental concepts: This is the most important tip I give to all my students. Calculus is all about solid, valid and proven concepts. Know that the derivative of a function at a certain point is the slope of the line that is tangent to that curve at that point. Know that integration is the inverse of differentiation and vice versa. Also know that the net area bounded by the curve between any two points on the curve is equal to the definite integral between those two points. See <u>Wikipedia</u>.

6. Review past math concepts: It is also a great idea to make sure that you are still fluent with the other math courses and concepts you have learned in the past. Surely you will want to be familiar with advanced algebra and graphing polynomials. You will also want to make sure that your trigonometry is up to par. Many Calculus problems will use trig functions so it would be best for you to review trigonometric identities and common trigonometric formulas. Thanks for the tip from the folks at the College Board.

7. Study with friends or other students: If any of your fellow friends or students will also be taking the AP Calculus exam, it would be a great idea for you to get together with them every so often and do some problems together. Remember the old saying that "Two heads are better than one". Studying with other people with the same goals is great because it adds an element of personality to your academic understanding of a particular subject matter. Most serious students do this during High School or College and it also helps take the stress out of the anticipation of taking such a high-level exam such as AP Calculus. Thanks for the tip from the folks at the College Board.



8. Use outside resources: You should always use outside sources for preparing for the AP Calculus exam. Relying on only one textbook would give you a narrower understanding of the types of problems that might occur on the AP test. Remember that the AP exam is prepared by a committee that likely uses multiple resources to create the problems. You should also diversify your portfolio, so to speak, and use the internet or other good textbooks on Calculus. There are scores of websites devoted to sample Calculus problems. I would visit as many as possible without over doing it of course. Thanks for the tip from the folks at the College Board.

9. Ask for help: If you are having difficulty with certain types of problems then it would be a good idea to seek help. You might want to schedule some time with your teacher at school or perhaps look into hiring a good Calculus tutor. Thanks for the tip from the folks at the College Board.

10. Help other students: Besides just getting together with other students or friends for a Calculus studying session, it would be good for you to help other students that may be having problems with understanding Calculus. If you are very comfortable with having an above average understanding and knowledge of Calculus, it would be great for you to help someone else out. This would be a winwin situation for both of you since teaching a subject matter always reinforces the concepts with yourself. You will gain additional confidence and understanding of the basic principles of Calculus by teaching someone else. Thanks for the tip from the folks at the College Board.

11. Learn from students that have taken the test before: You may already know some people that have taken the AP Calculus exam before. If so, speak with them about their experience in taking the exam. They may have some tips for you as well. Ask them questions like, "What types of problems were on the test?" or "Did you have enough time to finish all of the problems?". You may also ask them if they thought the test was easy or difficult. Try to get as much information from them as you can. It certainly can't hurt. Thanks for the tip from the folks at the College Board.



12. Don't procrastinate: Some students are better than others at not procrastinating. Make sure that you are serious about preparing for the exam way in advance and start doing problems on a daily or otherwise frequent basis. It is important that you stick to a plan and execute it properly. Don't say to yourself, "Oh, I already know this stuff so I will put it off until the week before the exam". If you do that, it will definitely ensure that you will not be fully prepared for the exam. Always remember that practice makes perfect and just make sure that you do a lot of practicing. Again, like I said before, the more you practice, the better are your chances of getting a 4 or 5 on the AP Calculus exam. Good luck!

13. Answer all questions: When you are finished with the exam, make sure that all questions have an answer. If you leave a question blank you will not get any points for that question. This could make the difference between a 4 or a 5 score if you leave too many questions unanswered. Thanks for the tip from the folks at the College Board.

14. Wear a watch: Although there may be a clock on the wall in the room, there is no guarantee. Even if they do have a clock in the room it would be a lot better if you were to simply look down at your wrist rather than looking up to the wall clock all the time. Thanks for the tip from the folks at the College Board.

15. If you feel overwhelmed or your hand cramps, take a short break. Taking thirty seconds to shake out your wrist after all that writing and look up from your paper will not be harmful. It's a timed test, but it's not a time trial. Taking short breaks when you feel confused allow you to clear your head and relax.

16. Remember, you don't need to score 100% to pass. The AP Calculus test is supposed to be difficult. While you might have done very well on unit tests in your Calculus class, most people do not score nearly as well on the AP test. This is how it is designed. To receive a 5 in recent years, examinees have only needed to answer 63% of material correctly. This is not a test to ensure you know everything, but to measure what you do.



17. When it's done, relax and celebrate! Once you have turned in your exam, there is nothing more you can do for a couple of months. Do not worry too much about your results until they come out. You have worked hard all year to earn college credit, and now your studying and practice has paid off!

18. Work AP problems ALL YEAR LONG! Don't wait until the end of the year for the review. We review by chapter, the day before or after each test. Thanks for the tip from Ms. Gisella C. from Boca Raton Community High.

19. When things get difficult, focus on what you DO know as a starting point—not what you don't. Pull together the ideas of what you DO know and good things will happen. Thanks for the tip from Aaron P. from Crystal Lake Central High.

20. Be sure to label all work correctly. For instance, it you taking the derivative be sure to label f prime (x). Setting up an integral use the integral symbol etc. As a reader it is amazing how many students work a problem with no labeling or worse mix f and f prime. Thanks for the tip from Mr. Waddell.

21. Review all formulas the night before the exam and then go to sleep. No need to try to cram...you just can't for this test. It's what I always tell them. Well, that and to learn every little detail about every little Calculus theorem there is. Thanks for the tip from Pamela L. from Ralph L. Fike High.

22. In doing derivatives, I always stress the difference between $y = x^n$, $y = a^x$, $y = a^n$, and $y = x^x$. All of them look similar, but different rules apply: Power Rule, Exponential Rule, Constant Rule, and Logarithmic Differentiation. I usually do this by presenting 4 specific problems on the board, such as the ones with n = e, a = pi. I ask them which rules apply. They always try to use the power rule on all 4. However, with practice at the board, they get the difference. Thanks for the tip from Ron T.

23. If a student boxed in an answer on the FRQ's, that was the only answer we could consider. Too many students had the correct answer elsewhere and I could read that answer, unless they had a boxed in answer. Thanks for the tip from Laura S. from Chatham High.



24. Learn the language and vocabulary, because half of the battle is knowing what a question is asking and/or telling you! Give yourself the benefit of analyzing the information presented in a question, because then you can execute on the knowledge and skills that you have practiced! Thanks for the tip from Brittany A. at YesPrep Public Schools.

25. The best thing I can think is for students to practice "AP level" problems as much as possible. Students should find past AP tests, the AP course description or review books and do as many problems as they can. Thanks for the tip from Richard S.

26. There may be stuff on the test that you do not know. That is okay. Just don't panic and do well on the stuff you do know how to do. Thanks for the tip from Ned E. from Lebanon High.

27. Relax and don't be afraid of making mistakes...you are sharper than you think. Thanks for the tip from Ms. L.

28. Have the unit circle memorized so that you can fill in a blank one in 5 minutes or less. It will save you precious time on the AP exam. Thanks for the tip from Liesa K. at Art College Prep, Jill P., and Michelle J.

29. One tip I give them to be successful on the AP exam is to mentally get after it. They are a little confused at what I mean by that at first so I relate it to sports. If you are in a wrestling match, you 'get after it' physically. On the exam you get after it mentally. There are no times where you lose focus. Your mind is working it's tail off. If you get tired or start to lose focus, you have to pep yourself up and get after it even harder. When the test is over, you will feel drained but proud of your efforts. Thanks for the tip from Brad S.

30. When you have worked the problem, do not write the answer until you go back and read the question. This applies to both parts of the test. Thanks for the tip from Mr. B.



31. The best tip I could give for the multiple choice sections is called "Triage." On the first pass through the multiple choice problems, do only those you immediately and absolutely know how to do. Circle the ones that you cannot do without some effort. Make a second pass through, this time skipping those you feel you have no idea how to do. Go through the problems one last time and attempt those problems you believe to be most difficult. This makes the best use of your time and keeps you from expending mental energy early in the test on problems that are difficult. Thanks for the tip from Mr. G.

32. Do not simplify a numerical answer ! If you get 1(1) + 2(3) + 5cos
45.....stop!!! You are finished. I took away over 100 points when I graded the exam because students told me 1(1) = 2. Thanks for the tip from KC H. at Freedom Area High.

33. Don't let AP readers guess at what you mean. Write the responses to your Free Response questions completely, leaving nothing to the readers imagination. Thanks for the tip from Ryan H. at Southern Lehigh High.

34. Don't simplify your answers for the FRQs. Thanks for the tip from David O. at St. Johns Country Day School.

35. Questions have to do with one of the following areas: limits, derivatives, or integrals (definite or indefinite). If you get stuck on a problem, just ask yourself which of these the question is asking you to find. Thanks for the tip from Dan M. at Broad Run High.

36. AP stands for Always Practicing. You can not cram prior to the exam to be successful on any AP test. You must put the time and work in all year long. This includes weekends, holidays, and even snow days. If you have consistently done this, your scores will reflect your efforts. Thanks for the tip from Dan M. at Broad Run High.

37. The value of a limit of a function as x approaches "a" is NEVER the value of the y coordinate of an isolated "dot". Thanks for the tip from James M.



38. EVERY point on the free response is its own little test. Write down all that you know and don't get freaked out about stuff in an earlier part of a question you may NOT have known. STAY IN THE GAME! Thanks for the tip from Bo G.

39. THINK GRAPHICALLY. A picture IS worth a thousand words, and your ability to picture what is happening in the context of the problem will help you understand if the derivative should be negative or positive, if the value should be big or small, if the second derivative (acceleration in many cases) is positive or negative, if your integral value should be positive or negative, etc. To reason with a visual supports your algebra in many ways. Thanks for the tip from Chris L.

40. Sit on your ego! Know that you will be challenged and that it is not a reflexion on you! Thanks for the tip from Damien J.

41. The more you practice a variety of problems from different resources the better you begin to recognize and understand types of problems. Thanks for the tip from Kristy H.

42. Bring extra batteries for your calculator! Thanks for the tip from David P. at Torrey Pines High.

43. You must know the why and how, not just the what. Thanks for the tip from John S.

44. Don't forget your calculator on exam day and be sure to put fresh batteries in it. Thanks for the tip from Dawn D. at FCHS.

45. Make sure your calculator is in radians (check the settings in general). Thanks for the tip from Dawn D. at FCHS.

46. Use your calculator when there are shortcuts to solving a problem rather than performing the calculus by hand. Thanks for the tip from Dawn D. at FCHS.

47. Answer only the question asked, don't waste time doing something that isn't needed or required to a problem. Thanks for the tip from Dawn D. at FCHS.



48. If a graph is given, be sure to identify which function it represents....is it the function or a derivative? Thanks for the tip from Dawn D. at FCHS.

49. Don't oversimplify. The test doesn't require it, and oversimplifying can use valuable time! Thanks for the tip from Misty P. at Virtual Virginia.

50. I would tell students to know their major theorems, both hypothesis and conclusion, cold. To me these are IVT, EVT, Rolle's, MVT (derivatives), FTC I (including its use for Net Change on a rate), FTC II, and MVT (integrals). After that, practice justifying numerical and graphical problems on previous FRQs using the information given (Ex: Given a graph of f(x), do not simply say f'(x) is positive, say f'(x) is positive as the given graph, f(x), is increasing, thereby connecting your justification to the information given). Finish by checking your work with the sample 9/9 FRQ student response provided by the College Board. Thanks for the tip from Shaun B. at James Clemens High.

51. Use 3 decimal places. Thanks for the tip from Keith L. at SHHS.

52. Make sure students know that a critical number must be in the domain of the function. So in f(x) = (x + 5)/(x - 3), 3 cannot be a critical number. Thanks for the tip from Jane W.

53. On application questions with lots of information, highlight the units on the given functions (equation, graph, or table) in the problem and ask yourself what needs to be done (integrate or differentiate) in order to arrive at the correct units in your answer. Thanks for the tip from Joseph S.

54. Learn to do mental math. You save time on the Multiple Choice, no calculator part. Thanks for the tip from Leena G.

55. My tip is to never leave a free response question blank and if you are unsure what to do, set the given equation equal to zero and solve it and set the derivative of the given equation equal to zero and solve it. Thanks for the tip from Craig G.

56. Do Saturday work sessions by topic and give mock exams. Thanks for the tip from Valerie P.



57. On free response questions especially, practice understanding in words what you are doing mathematically. Like knowing by the words if they want a average rate of change or the average of the integral. Knowing how those are asked and what the answers mean, make it easier not only to answer the question but to justify the answer as well. Thanks for the tip from Natalie B. from Memorial Early College High.

58. Time can be your enemy. When taking practice exams, set your timer to give yourself 5 fewer minutes than you would when taking the actual exam. You'll get used to working at a quicker pace and you'll have extra time to check your work before time is called. Thanks for the tip from Mary B.

59. Realize this is not a UIL competition test. They are looking for a reason to give you credit for Calculus – not trying to make a test no one can make a 100 on. Practice being thorough in your answers so they can see you understand the concept. Thanks for the tip from Michael B.

60. Starting In January do one free response question a night. <u>This</u> <u>attachment</u> has the problems by topic, year, and number. Thanks for the tip from Eric H.

61. Continue to ask yourself WHY? as you progress through the topics. If you are doing that, and able to answer that, then you will continue to understand the concepts deeply, and the unique problems from the AP will be easily tackled. Thanks for the tip from Eric S.

62. Answer every part of a FRQ. Label each part a), b), c), etc. If there are multiple answers, roots, solutions, points to a single question, make sure to sum up your answers in one Box. Thanks for the tip from Mary S. from Grafton High.

63. The best tip I have for my students is to keep up with the current homework, homework is where most of your learning takes place. If you have questions make sure to ask them. Work with a buddy or buddies if you can and discuss your ideas. Catching up is very difficult. Thanks for the tip from Carolanne F.



64. Read the question carefully and answer it completely. – Many FR questions require the student to complete multiple tasks. Many students lose points because they do not answer the questions appropriately. Thanks for the tip from Jessica S. from Cypress Bay High.

65. Know the big concepts, but pay attention to the details. Show all work and do not skip steps. Thanks for the tip from Beth P. from Martin Luther King, Jr. Magnet School.

66. Here's a tip I use to try to get kids to take a breath before starting a free response question: Basically, Calculus is derivatives and integrals, so ask yourself before doing any work, "Is this a derivative or an integral question?" Then, at least, they have somewhere to start. Thanks for the tip from Steve S. from Kents High.

67. The mean value theorem and its converse must be understood at all levels and one must be able to provide a simplistic physics example to explain it. Thanks for the tip from Jonathan H.

68. Limits must be understood to bind the whole of calculus together and must not be treated as the red headed step child, and this includes graphing them by hand using precalculus and algebra 2 rules and not being reliant on a calculator. Thanks for the tip from Jonathan H.

69. There are 3 phases of calculus: position, velocity, and acceleration. You must know which phase the problem is in currently and which phase you need to transform it to. This will help you decide if you are going to integrate or differentiate. Thanks for the tip from Rachael A.

70. Understanding graphs: keep in mind that a function can decrease while its derivative increases. Although tangent lines are getting less steep, their slopes are becoming less negative. Thanks for the tip from Mr. S.



71. Practice, practice, practice, and more practice. There are so many nonsecure, practice multiple-choice and free-response questions available that there is no reason not to take full advantage of them in preparation for the exam. Exposure to the question formats, common notation/terminology, etc. is key to applying content knowledge for applicable scenarios and, ultimately, being successful on the actual exam. Thanks for the tip from Dominic B.

72. Even if you have the correct answer, linkage errors and presentation errors can make students lose points in the grading process. Go back and check for these before time is up. Linkage error example: 10 * 5 = 50 / 2 = 25 is a linkage error because 10 * 5 does not equal 50 / 2. You must start a new line rather than link them together; so 10 * 5 = 50; 50 / 2 = 25 Presentation error example: When doing limit problems, not putting the limit as h approaches 0 in front of each expression along the way to the answer results in a presentation error. Thanks for the tip from Doreen V.

Are you a teacher or student? Do you have an awesome tip? Let us know!



Frequently Tested Concepts for the AP Calculus Exam

Differential Calculus

Properties of limits, Properties of derivatives, Domain and range, One-sided limits, Limits at infinity, Continuity, 3 Types of Discontinuities, Product rule, Quotient rule, Power rule, Chain rule, Even functions, Odd functions, Periodic functions, Trig derivatives and inverse trig derivatives, Implicit differentiation, Higher order derivatives, Mean Value Theorem, L'Hopital's Rule, Tangent lines, Extreme Value Theorem, Newton's method of approximation.

Integral Calculus

Indefinite integrals, U-Substitution Integration by parts, Exponential growth and decay, Definite integrals, Riemann sums, Trapezoidal method, Fundamental Theorem of Calculus #1, Fundamental Theorem of Calculus #2, Average Value, Disk method, Washer method, Shell method.

This Ultimate List of AP Calculus Tips was written for the express purpose of giving the AP Calculus student a strong competitive advantage when taking the exam. The academic world is especially competitive these days and it is critical that even the best of students read and understand these tips. Make sure to bookmark this page so that you can refer to it throughout the school year as you navigate your way through AP Calculus AB/BC. Best of luck!

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