THE USE OF CONTEXTUAL PROBLEMS TO SUPPORT MATHEMATICAL LEARNING

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Abstract

This paper examines the use of contextual problems to support mathematical learning based on current classroom practice. The use contextual problems offers some potentials to engage and motivate students in learning mathematics but it also presents some challenges for students in classrooms. Examples of the use of contextual problems from several primary classrooms in Indonesia will be discussed. Contextual problems do not lend themselves to a meaningful learning for students. Teachers need to engage students in interpreting the context in order to explore key mathematical ideas. It is critical to establish explicit links between the context and the mathematics ideas to support students' progression in their mathematical thinking.

Keyword: *Contextual Problems, Context, Mathematical Learning*

Abstrak

Makalah ini membahas penggunaan masalah kontekstual untuk mendukung pembelajaran matematika berdasarkan pada praktek pengajaran terkini. Masalah penggunaan kontekstual menawarkan beberapa potensi untuk terlibat dan memotivasi siswa dalam belajar matematika tetapi juga menyajikan beberapa tantangan bagi siswa di kelas. Contoh penggunaan masalah kontekstual dari kelas utama beberapa di Indonesia akan dibahas. Masalah kontekstual tidak meminjamkan diri untuk belajar bermakna bagi siswa. Guru perlu melibatkan para siswa dalam menafsirkan konteks untuk mengeksplorasi ide-ide matematika kunci. Hal ini penting untuk membangun hubungan eksplisit antara konteks dan ide-ide matematika untuk mendukung perkembangan siswa dalam berpikir matematika mereka.

Kata Kunci: *Masalah Kontekstual, Konteks, Pembelajaran Matematika*

The role of contexts in mathematics teaching and learning has gained much attention. Lee (2012) presents examples of contextual problems dated over 1500 years ago in China so clearly the use of context is not a novelty. In Realistic Mathematics Education theory, a context plays a significant role as a starting point of learning for students to explore mathematical notions in a situation that is 'experientially real' for them (Gravemeijer & Doorman, 1999). Gravemeijer and Doorman (1997) underlines that experientially real situation does not exclude pure mathematical problem and "experiential reality grows with the mathematical development of the student." (p. 127). Freudenthal (1991) is critical about the use of context to help students in exploring mathematics and progressing in their mathematical thinking. He underlines that a context is not "a mere garment clothing nude mathematics" (p. 75) One of the key characteristics of good contextual problems is its' capacity to bring out a variety of mathematical interpretations and solution strategies. These informal strategies serve as a basis for progression to a more formal and sophisticated mathematics. In brief, contextual problems should support students' mathematisation process.

In Indonesia, PMRI (Pendidikan Matematika Realistik Indonesia) has been advocating the use of contextual problems to engage students in learning mathematics for more than a decade (Sembiring, Hoogland, & Dolk, 2010). Classroom research involving PMRI schools have documented evidence of students worked together with contextual problems as a basis for exploring mathematical concepts (Sembiring, Hadi, & Dolk, 2008; Widjaja, Dolk, & Fauzan, 2010; Widjaja, 2012; Wijaya, 2008). A variety of contextual problems including problems adapted from other resources, traditional games, and mathematical modelling problems which involve some data collection at the beginning of activity have been implemented in Indonesian classrooms. Some examples from the use of contextual problems in classrooms will be reviewed. Issues including potentials and challenges in using contextual problems in our classrooms will be examined.

METHOD

There is now a widespread body of knowledge that document potentials and challenges in using contextual problems to advance children learning of mathematics (see e.g., Boaler, 1993; Carraher & Schliemann, 2002; Van den Heuvel-Panhuizen, 1999). The meaning attached to the contextual problems can make problems more accessible and more likely to engage children in learning. Van den Heuvel-Panhuizen (1999) argues that contextual problems allow students to start with informal strategies. Hence the contextual problems offer opportunities for students to solve the problems at different levels of formality. Studies that incorporate the use of contextual problems in Indonesian classroom confirm this assertion (Widjaja, Dolk, & Fauzan, 2010; Dolk, Widjaja, Zonneveld, & Fauzan, 2010). Contextual problems permit students to start with diagrammatic representations or less efficient strategies. It provides opportunities for teachers to discuss the progression of strategies with students rather than focusing only on the most efficient strategies associated with formal algorithm.

A number of studies report challenges that students face in solving contextual problems. Some contextual problems have little in common with those faced in life which led to students' decision to consider them as school problems covered with 'real-world' associations. Students often fail to interpret the context as intended or ignore the context and proceed to solve it as a bare mathematical problem. Gravemeijer (1994) reports a case when students disengage with the context of sharing bottles of Coca-cola drink evenly among students because they do not like Cola-cola. In this case, students consider their personal experience as a factor to decide if the context is relevant for them or not. When the problem involves students as part of the context, it is critical to ensure that the story reflects the reality of students in the classroom. Similarly, Dolk, Widjaja, Zonneveld, & Fauzan (2010) observe grade four students struggle to make sense of the context of four groups of children sharing 5 bars of chocolate because there are five groups in the class. Carraher and Schliemann (2002) identify that students' interpretations of the context are most likely different than adults so it could be a challenge to assist children in unpacking the contextual problems in a meaningful way.

RESULT

The following examples of contextual problems were implemented in Indonesian classrooms. Due to a limited space, this paper will focus on issues in students' interpretation of the context and ways to support students' progression in their mathematical thinking as observed in some classroom practice. It should be noted that the intention of presenting these examples is not to compare the different experiences but to learn from these experiences of implementing contextual problems in classrooms.

The first problem is an adaptation of a sandwich problem developed by the Math in Context curriculum (Galen & Wijers, 1997). The problem was introduced to one Grade 4 class consisting of 25 students in Padang (see Dolk, Widjaja, Zonneveld, & Fauzan, 2010 for more detail) as part of Design research workshop activities involving the classroom teacher. The students worked in 5 small groups while working on the problem over 2 days.

1. Four groups of kids are going on a trip and the teacher gives them some chocolate bars to share. In the first group, there are 4 kids and they get 3 chocolate bars together. In group 2 there are 5 kids, and they get 4 chocolate bars. Group 3 contains 8 kids, and they get 7 chocolate bars. Whereas group 4 has 5 kids who get 3 chocolate bars. Did the teacher distribute the chocolate fairly?

There were a few learning points from this classroom experience. The context failed to engage students because students noticed that it did not represent the number of groups in their class. The teacher tried to bring this problem to a concrete level by introducing a diagram to represent the chocolate bars (Figure 1) and showing real chocolate bars. This helped students to make a connection between the context of sharing chocolate bars in different groups and fractions. However, the notion of a whole in fractions was not yet fully grasped by students and some students focused only on the numerators when comparing two fractions. These situations proved to be challenging for both the teacher and students. In retrospect, students would benefit from a whole class discussion at the beginning to help them make the connection between the act of sharing chocolate and its representations using fraction. It is critical to help them realise the relationship between the whole and the parts in fraction because during the partitioning process often lead students to treat the parts as a new whole. Students also need to be encouraged to negotiate their interpretations of the context and discuss their mathematical strategies without relying too much on teacher's help.

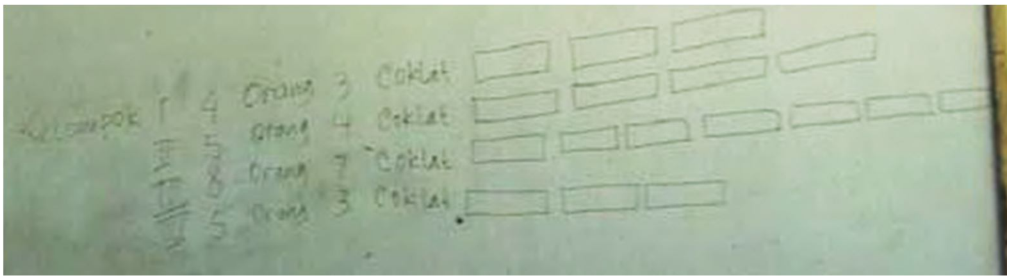

Figure 1. Rephrasing the problem

The second problem was designed and implemented in 2 Grade 5 classes during the Design research workshops in Yogyakarta (Widjaja, Dolk, & Fauzan, 2010). Similar to the first problem, this problem presents a situation related to fractions. In the classroom, both teachers decided to tell the story as their own problem and asked their students to help the teacher to solve this problem.

> 2. A family buys 25 kilograms of rice and eats ¾ of a kilo each day. How many days can 25 kilograms of rice lasts for?

Students' initial attempts also suggested that some students ignored the context despite the teacher's effort to make the context engaging for the students. Formal algorithms such as the division algorithm, repeated subtraction and the multiplication of fractions including some misconceptions about fractions were observed. Realising students' difficulties, the teachers introduced a diagram to represent the whole and asked some students to represent three quarters of the whole (Figure 2). The whole class discussion on students' interpretation of ¾ with respect to the diagram served as a good starting point to support students in establishing the link between the context and the mathematics. Students were able to build on this representation and explore more strategies (Figure 3).Teacher's probing questions which highlight the context in the problem was critical step in helping students to comprehend the problem in a more meaningful way as contrast to their initial solutions. The fact that students were comfortable in sharing their strategies and expressing their confusions enabled the whole class to make a good progress from this learning experience. Clearly the contextual problem did not lend itself to a meaningful learning for students; it requires both the teacher and students to work together in making this context becomes meaningful for students.

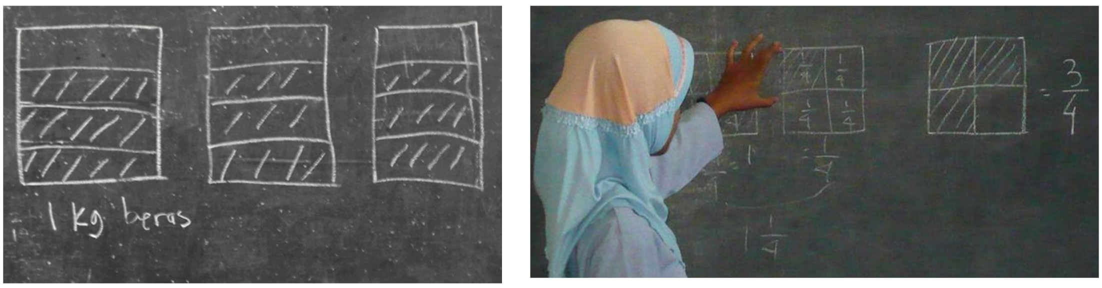

Figure 2. Using a diagram to represent ¾

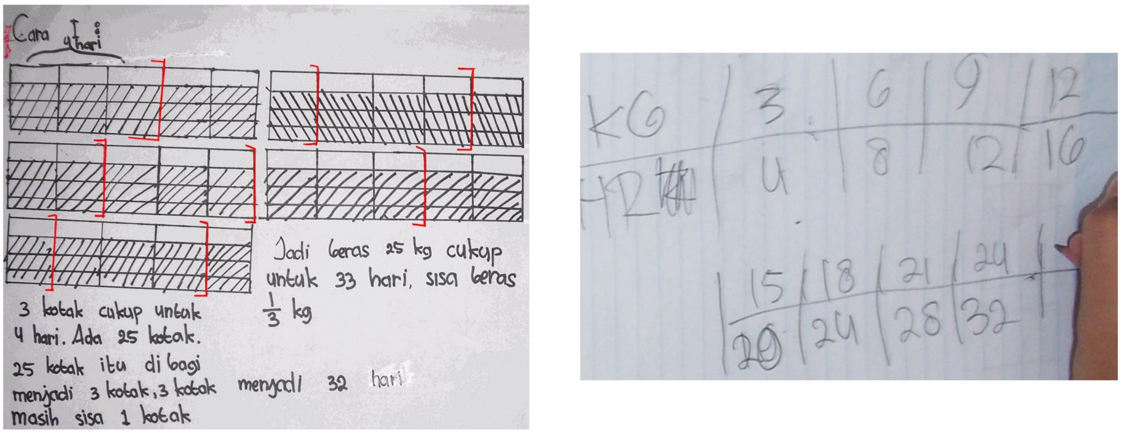

Figure 3. Samples of students' strategies

The third problem was designed and implemented in two grade 6 classrooms in Yogyakarta as part of a learning sequence designed for a PMRI research project (Widjaja, Julie, Prasetyo, 2009). Students were required to collect data at the beginning of the lesson. The problem was set as a starting point for students to explore the rate of the water flow (known as "debit" in Indonesian).

3. You are going to collect data of water flowing through two plastic bottles with different size of holes in 10 seconds. For each bottle, you should collect the data three times. Your task as a group is to represent the data so that others could understand them. Please think of as many different ways as you can to represent the data.

Figure 4. Data collection process

Various representations were offered by students and the classroom norms encouraged students to understand other group's choice of representations and to raise questions if they did not understand. Students were able to make appropriate links between the context and various forms of representations. Various strategies that were observed showed a range of data representations that they have learnt before such as tables, pie charts, picture graphs and bar graphs. There was an exception for one group who misinterpreted 'different' representations as any representation that "look different" without sound mathematical basis (Widjaja, 2012). This misunderstanding was resolved during the whole class discussion by having the student to explain his thinking using a diagram. The teacher orchestrated the discussion by inviting other students to share their thoughts in helping their classmate to realise that the representation had to be mathematically sound.

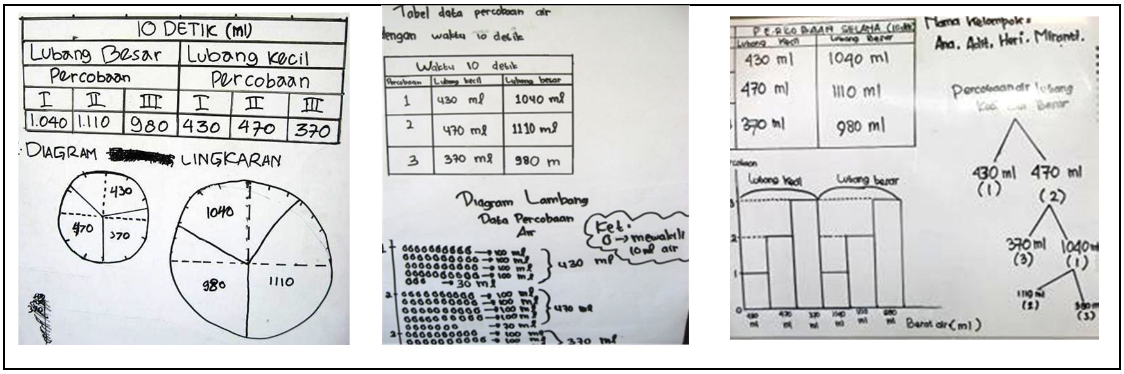

Figure 5. Samples of posters

The last problem is a model eliciting problem with two versions designed and implemented in Singapore and Indonesia (Chan, Widjaja, Ng, 2011). The Indonesian version problem was implemented in one Grade 4 class in Yogyakarta over the course of three days.

Menentukan jalur bus paling efisien

Bu Mustari mendapat kunjungan tamu guru dari Singapore. Beliau ingin mengantar rombongan guru ini menggunakan bus dari terminal Jombor ke tempat kerajinan perak di Kota Gede. Ada 3 alternatif jalur bus yang dapat dipilih seperti tercantum pada peta. Ketiga alternatif jalur bus ditandai dengan warna biru, kuning dan merah muda pada peta. Pemberhentian bus di sepanjang jalur-jalur ini ditandai dengan stiker dengan warna yang sama dengan jalur bus.

Tugas kalian adalah membantu bu Mustari untuk memberi rekomendasi dengan menjawab pertanyaan berikut:

1. Bagaimana kelompok kalian menentukan jalur bus yang "paling efisien"?
2. Asumsi apa yang harus kalian buat untuk dapat membantu bu Mustari?
3. Bagaimana kalian menggunakan matematika untuk membantu Bu Mustari memilih jalur bus yang paling efisien?
4. Bagaimana kelompok kalian meyakinkan kelompok lain lewat penjelasan matematika bahwa jalur bus yang kalian pilih adalah yang paling efisien?
5. Rekomendasi akhir jalur bus yang paling efisien untuk bu Mustari.

   Untuk membantu kelompok lain memahami penjelasan kelompok kalian, lampirkan dalam poster kalian:

(a) Peta yang memuat jalur bus yang kalian pilih
(b) Informasi penting yang kalian gunakan untuk menyelesaikan masalah

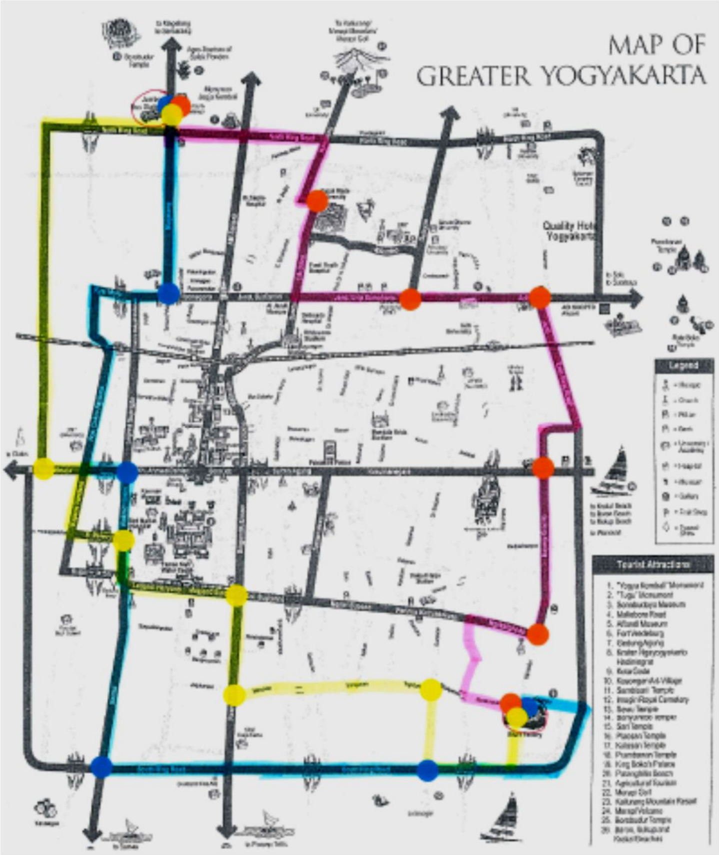

Figure 6. The problem of finding the shortest bus route (Chan, Widjaja, Ng, 2010, p. 129)

The task offered opportunities for children to engage in a rich discussion on their interpretations on the meaning of efficiency and assumption before exploring relationships between relevant variables such as distance and time (Figure 7). The discussion at the beginning to negotiate these interpretations was critical to assist children in understanding the context based on information provided on the map. Students' knowledge of length and its relationship to distance, time and units of time, and speed were evident in the various strategies observed during the whole class discussion. Children are encouraged to ask questions and to justify their thinking in public. It was evident that this classroom norm has been established and practiced by the teacher and students which supported a rich whole class discussion.

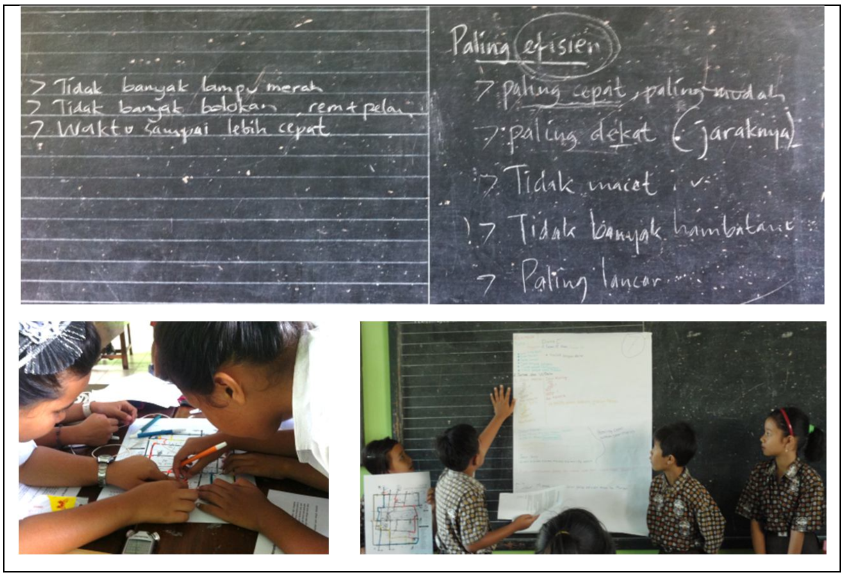

Figure 7. Working together in finding the shortest bus route

CONCLUDING REMARKS

As pointed out by many researchers, contextual problems *do not directly* make mathematics easier and motivating for students (Boaler, 1993; Carraher & Schliemann, 2002). Students bring to classrooms different learning experiences which will affect their interpretations of the context. Studies show that students often ignore the context altogether. The openness of contextual problems allows rooms for diverse interpretations including misconceptions or misunderstandings. Hence opportunities to negotiate their interpretations of the context are critical to establish an appropriate link between the context and mathematical ideas. Introducing a diagram or a representation to create a link to the mathematics from the context is helpful. Teachers facilitate discussions with questions that support students to progress from the context to more formal mathematics. Our experiences show that context can lead to a meaningful learning when students take an active role in the discussion, by asking questions for clarifications, explaining, and justifying their reasoning. In conclusion, I would like to reiterate Lee's (2012) points to make *an easy entry* to contextual problems but *end with a higher mathematics*.

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