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Given: $\overline{\mathrm{AB}} \cong \overline{\mathrm{AC}}$
Prove: $\angle B \cong \angle C$


Proof Statement

Reason

Theorem 20: If two sides of a triangle are congruent, the angles opposite the sides are congruent. ( If $A$, then $\Delta$.)

Given: $\overline{\mathrm{AB}} \cong \overline{\mathrm{AC}}$
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Proof
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1. $\overline{\mathrm{AB}} \cong \overline{\mathrm{AC}}$
2. Given

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1. $\overline{\mathrm{AB}} \cong \overline{\mathrm{AC}}$
2. Given
3. Ref.

Theorem 20: If two sides of a triangle are congruent, the angles opposite the sides are congruent. ( If $A$, then $\Delta$.)

Given: $\overline{\mathrm{AB}} \cong \overline{\mathrm{AC}}$
Prove: $\angle B \cong \angle C$


Proof
Statement
Reason

1. $\overline{\mathrm{AB}} \cong \overline{\mathrm{AC}}$
2. $\overline{\mathrm{BC}} \cong \overline{\mathrm{BC}}$
3. $\triangle \mathrm{ABC} \cong \triangle \mathrm{ACB}$
4. Given
5. Ref.
6. SSS

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Reason

1. Given
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Theorem 21: If two angles of a triangle are congruent, the sides opposite the angles are congruent. ( If $\Delta$, then $\star$.)

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Given: $\angle B \cong \angle C$
Prove: $\overline{\mathrm{AB}} \cong \overline{\mathrm{AC}}$


Proof Statement

Theorem 21: If two angles of a triangle are congruent, the sides opposite the angles are congruent. (If $\Delta$, then $\Delta$.)

Given: $\angle B \cong \angle C$
Prove: $\overline{\mathrm{AB}} \cong \overline{\mathrm{AC}}$


Proof
Statement

1. $\angle \mathrm{B} \cong \angle \mathrm{C}$
2. $\overline{\mathrm{BC}} \cong \overline{\mathrm{BC}}$
3. $\triangle \mathrm{ABC} \cong \triangle \mathrm{ACB}$ 4. $\overline{\mathrm{AB}} \cong \overline{\mathrm{AC}}$

Reason

1. Given
2. Ref.
3. ASA
4. СРСТС

## The two theorems tell us:

If at least two sides of a triangle are congruent, the triangle is isosceles.

If at least two angles of a triangle are congruent, the triangle is isosceles.

> Theorem 20: If two sides of a triangle are congruent, the angles opposite the sides are congruent. ( If $A$, then $\Delta$. )

The inverse of Theorem 20 is true: If two sides of a triangle are not congruent, then the angles opposite them are not congruent, and the larger angle is opposite the longer side. (If $\notin$, then $\Delta$.)

# Theorem 21: If two angles of a triangle are congruent, the sides opposite the angles are congruent. (If $\Delta$, then $A$.) 

The inverse of Theorem 21 is true: If two angles of a triangle are not congruent, then the sides opposite them are not congruent, and the longer side is opposite the larger angle. ( If $\Delta$, then $\#$. )

## The median to the base of an isosceles triangle bisects the vertex angle.

The median to the base of an isosceles triangle bisects the vertex angle.
Given: Isosceles $\triangle \mathrm{ABC}$ with vertex $\angle A$ and median $\overline{\mathrm{AD}}$

Prove: $\angle \mathrm{BAD} \cong \angle \mathrm{CAD}$
Proof
Statement


Reason

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1. Isosceles $\triangle \mathrm{ABC}$ with vertex $\angle A$


Reason

1. Given

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Prove: $\angle \mathrm{BAD} \cong \angle \mathrm{CAD}$
Proof
Statement

1. Isosceles $\triangle \mathrm{ABC}$ with vertex $\angle A$
(S) 2. $\overline{\mathrm{AB}} \cong \overline{\mathrm{AC}}$

Reason


1. Given
2. Legs of isos. $\Delta$ are $\cong$.

The median to the base of an isosceles triangle bisects the vertex angle.
Given: Isosceles $\triangle \mathrm{ABC}$ with vertex $\angle A$ and median $\overline{\mathrm{AD}}$

Prove: $\angle \mathrm{BAD} \cong \angle \mathrm{CAD}$
Proof
Statement

1. Isosceles $\triangle \mathrm{ABC}$ with vertex $\angle A$
(S) 2. $\overline{\mathrm{AB}} \cong \overline{\mathrm{AC}}$
(A) 3. $\angle B \cong \angle C$

Reason


1. Given
2. Legs of isos. $\Delta$ are $\cong$.
3. If $A$, then $\Delta$.

The median to the base of an isosceles triangle bisects the vertex angle.
Given: Isosceles $\triangle \mathrm{ABC}$ with vertex $\angle A$ and median $\overline{\mathrm{AD}}$

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Prove: \(\angle B A D \cong \angle C A D\)
Proof
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Statement
Reason

1. Isosceles $\triangle \mathrm{ABC}$
2. Given
with vertex $\angle A$
(S) 2. $\overline{\mathrm{AB}} \cong \overline{\mathrm{AC}}$
3. Legs of isos. $\Delta$ are $\cong$.
4. If $A$, then $\Delta$.
5. Given

The median to the base of an isosceles triangle bisects the vertex angle.
Given: Isosceles $\triangle \mathrm{ABC}$ with vertex $\angle A$ and median $\overline{\mathrm{AD}}$

Prove: $\angle B A D \cong \angle C A D$
Proof
$\frac{\text { Statement }}{\text { 1. Isosceles } \triangle \mathrm{ABC}}$
 with vertex $\angle A$
(S) 2. $\overline{\mathrm{AB}} \cong \overline{\mathrm{AC}}$
(A) 3. $\angle B \cong \angle C$
4. $\overline{\mathrm{AD}}$ is a median
5. $D$ is mdpnt. of $\overline{\mathrm{AD}}$
2. Legs of isos. $\Delta$ are $\cong$.
3. If $A$, then $\Delta$.
4. Given
5. Def. of median

The median to the base of an isosceles triangle bisects the vertex angle.

Given: Isosceles $\triangle \mathrm{ABC}$ with vertex $\angle A$ and median $\overline{\mathrm{AD}}$

Prove: $\angle B A D \cong \angle C A D$ Proof Statement

$\frac{\text { Statement }}{\text { 1. Isosceles } \triangle \mathrm{ABC}}$
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5. D is mdpnt. of $\overline{\mathrm{AD}}$
(S) $6 . \overline{\mathrm{BD}} \cong \overline{\mathrm{CD}}$
2. Legs of isos. $\Delta$ are $\cong$.
3. If $A$, then $\Delta$.
4. Given
5. Def. of median
6. Def. of midpoint

The median to the base of an isosceles triangle bisects the vertex angle.

Given: Isosceles $\triangle \mathrm{ABC}$ with vertex $\angle A$ and median $\overline{\mathrm{AD}}$

Prove: $\angle B A D \cong \angle C A D$ Proof


| Statement1. Isosceles $\triangle \mathrm{ABC}$(S) 2. $^{\text {with vertex } \triangle \mathrm{A}} \cong \overline{\mathrm{AC}}$(A)3. $\angle \mathrm{B} \cong \angle \mathrm{C}$4. $\overline{\mathrm{AD}}$ is a median5. D is mdpnt. of $\overline{\mathrm{AD}}$(S) 6. $\overline{\mathrm{BD}} \cong \overline{\mathrm{CD}}$7. $\triangle \mathrm{ABD} \cong \triangle \mathrm{ACD}$ |  |
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Reason

1. Given
2. Legs of isos. $\Delta$ are $\cong$.
3. If $A$, then $\Delta$.
4. Given
5. Def. of median
6. Def. of midpoint
7. SAS

The median to the base of an isosceles triangle bisects the vertex angle.

Given: Isosceles $\triangle \mathrm{ABC}$ with vertex $\angle A$ and median $\overline{\mathrm{AD}}$

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Prove: \(\angle B A D \cong \angle C A D\)
    Proof
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Statement

1. Isosceles $\triangle \mathrm{ABC}$ with vertex $\angle A$
(S) 2. $\overline{\mathrm{AB}} \cong \overline{\mathrm{AC}}$
(A) 3. $\angle B \cong \angle C$
2. $\overline{\mathrm{AD}}$ is a median
3. D is mdpnt. of $\overline{\mathrm{AD}}$
(S) 6. $\overline{\mathrm{BD}} \cong \overline{\mathrm{CD}}$
4. $\triangle \mathrm{ABD} \cong \triangle \mathrm{ACD}$
5. $\angle \mathrm{BAD} \cong \angle C A D$


Reason

1. Given
2. Legs of isos. $\Delta$ are $\cong$.
3. If $A$, then $\Delta$.
4. Given
5. Def. of median
6. Def. of midpoint
7. SAS
8. СРСТС

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