THEORETICAL ANALYSIS OF FLUID FLOW AND ENERGY TRANSPORT IN HYDROTHERMAL SYSTEMS

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THEORETICAL ANALYSIS OF

FLUID FLOW AND ENERGY TRANSPORT

IN HYDROTHERMAL SYSTEMS

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Charles R. Faust and James W. Mercer

ABSTRACT

A mathematical derivation for fluid flow and energy transport
in hydrothermal systems is presented. Specifically, the mathematical
model describes the three-dimensional flow of both single- and two-phase,
single-component water and the transport of heat in porous media. The
derivation begins with the point balance equations for mass, momentum,
and energy. These equations are then averaged over a finite volume
to obtain the macroscopic balance equations for a porous medium. The
macroscopic equations are combined by appropriate constitutive
relationships to form two simplified partial differential equations
posed in terms of fluid pressure and enthalpy. A two-dimensional formulation
of the simplified equations is also derived by partial integration in the
vertical dimension.

Errata:

Replace on
page 11

$$\langle \psi_s \gamma_s \rangle = \varepsilon_s \langle \psi_s \rangle^s \langle \gamma_s \rangle^s + \langle \hat{\psi}_s \hat{\gamma}_s \rangle \tag{8}$$

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To proceed any further with these equations, a constitutive relationship must be assumed for the two surface integrals and the divergence of the extra stress tensor. Such a relationship must describe the behavior

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$$\frac{\partial (\rho U)}{\partial t} + \nabla \cdot \underline{v} U \rho + \nabla \cdot \underline{\lambda}_c + p \, (\nabla \cdot \underline{v}) + \underline{\underline{\tau}} : \, \nabla \underline{v} - q'(U + p/\rho)' = 0, \tag{43}$$

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$$\frac{\partial (\rho h)}{\partial t} + \nabla \cdot \rho h \underline{v} + \nabla \cdot \underline{\lambda}_c - q'h' - \frac{\partial p}{\partial t} - \underline{v} \cdot \nabla p = 0. \tag{46}$$

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$$\langle \frac{\partial (\rho_s h_s)}{\partial t} \rangle - \langle \frac{\partial p_s}{\partial t} \rangle = \frac{\partial \langle \rho_s h_s \rangle}{\partial t} - \frac{\partial \langle p_s \rangle}{\partial t} - \frac{1}{V} \int_{A_{sw}} (p_s - \rho_s h_s) \underline{w}_{sw} \cdot \underline{n}_s dA$$

$$- \frac{1}{V} \int_{A_{sr}} (p_s - \rho_s h_s) \underline{w}_{sr} \cdot \underline{n}_s dA \, . \tag{49}$$

$$\langle \underline{v}_s \cdot \nabla p_s \rangle = \langle \underline{v}_s \rangle \cdot \nabla \langle p_s \rangle^s + \langle \underline{v}_s \rangle \cdot \left(\frac{1}{V} \int_{A_{sw}} \hat{p}_s \underline{n}_s dA + \frac{1}{V} \int_{A_{sr}} \hat{p}_s \underline{n}_s dA \right). \tag{52}$$

Combining all the terms gives,

$$\frac{\partial \langle \rho_s h_s \rangle}{\partial t} - \frac{\partial \langle p_s \rangle}{\partial t} + \nabla \cdot \langle \rho_s \underline{v}_s h_s \rangle + \frac{1}{V} \int_{A_{sw}} (p_s - \rho_s h_s)(\underline{w}_{sw} - \underline{v}_s) \cdot \underline{n}_s dA$$

$$+ \frac{1}{V} \int_{A_{sr}} (p_s - \rho_s h_s)(\underline{w}_{sr} - \underline{v}_s) \cdot \underline{n}_s dA + \frac{1}{V} \int_{A_{sw}} \underline{\lambda}_{cs} \cdot \underline{n}_s dA + \frac{1}{V} \int_{A_{sr}} \underline{\lambda}_{cs} \cdot \underline{n}_s dA$$

$$+ \langle \underline{v}_s \rangle \cdot \nabla \langle p_s \rangle^s + \langle \underline{v}_s \rangle \cdot \left(\frac{1}{V} \int_{A_{sw}} \hat{p}_s \underline{n}_s dA + \frac{1}{V} \int_{A_{sr}} \hat{p}_s \underline{n}_s dA \right) + \frac{1}{V} \int_{A_{sw}} p_s \underline{v}_s \cdot \underline{n}_s dA$$

$$+ \frac{1}{V} \int_{A_{sr}} p_s \underline{v}_s \cdot \underline{n}_s dA$$

$$+ \nabla \cdot \langle \underline{\lambda}_{cs} \rangle - \langle q_s' h_s' \rangle = 0. \tag{53}$$

The first two surface integrals in equation 53 represent transfer of
internal energy due to phase changes between water and steam, and rock and
steam, respectively. Since it is assumed that no phase change occurs with
the rock, the second surface integral may be eliminated. The next two
integrals in the equation represent the conductive energy flux between
the steam and water, and the steam and rock. The dot product of the phase
velocity and ~~the next~~ and the last two surface integrals ~~represent~~ the flux of pressure
' forces across the phase boundaries. In this development the surface

$$Q''_{sw} = \langle \underline{v}_s \rangle \cdot \frac{1}{V} \int_{A_{sw}} \hat{p}_s \underline{n}_s dA + \frac{1}{V} \int_{A_{sw}} p_s \underline{v}_s \cdot \underline{n}_s dA, \tag{56}$$

$$Q''_{sr} = \langle \underline{v}_s \rangle \cdot \frac{1}{V} \int_{A_{sr}} \hat{p}_s \underline{n}_s dA + \frac{1}{V} \int_{A_{sr}} p_s \underline{v}_s \cdot \underline{n}_s dA. \tag{58}$$

replace on
page 29

$$\langle \rho_s \underline{v}_s \hat{h}_s \rangle = \langle \underline{v}_s \rangle^s \langle \rho_s h_s \rangle + \epsilon_s \langle \hat{\underline{v}}_s \hat{\rho}_s \hat{h}_s \rangle^s . \tag{62}$$

$$\frac{\partial}{\partial t}(\epsilon_s \langle \rho_s \rangle^s \langle h_s \rangle^s) + \frac{\partial}{\partial t}(\epsilon_s \langle \hat{\rho}_s \hat{h}_s \rangle) - \frac{\partial}{\partial t}(\epsilon_s \langle p_s \rangle^s) + \nabla \cdot \langle \underline{v}_s \rangle \langle \rho_s \rangle^s \langle h_s \rangle^s$$

$$+ \nabla \cdot \langle \underline{v}_s \rangle \langle \hat{\rho}_s \hat{h}_s \rangle^s + \nabla \cdot \epsilon_s \langle \hat{\underline{v}}_s \hat{\rho}_s \hat{h}_s \rangle^s - \langle \underline{v}_s \rangle \cdot \nabla \langle p_s \rangle^s + \nabla \cdot \langle \underline{\lambda}_{cs} \rangle + Q_{sw} + Q'_{sw} + Q'_{sr}$$

$$- Q''_{sw} - Q''_{sr} - \langle q'_s h'_s \rangle = 0. \tag{64}$$

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Note that the heat of vaporization terms and the interphase conduction
and pressure terms in equations 77 and 78 have been eliminated in equation
85. This is the result of the jump energy balance at a phase interface
(see Truesdell and Toupin, 1960, p. 610). If the mechanical energy is
neglected, this balance requires that the thermal energy interface flux terms
in the steam, water and rock equations sum to zero. Finally, the last term
in equation 85 is the pressure material derivative, which
comprises part of the compressible work term.
Moench (1976) points out that the compressible
work terms is negligible except for conditions
of low water saturations. The same characteristics
can be shown for the rest of the pressure
material derivative

on page 47

$$q'_h = q'_s h'_s + q'_w h'_w , \tag{94}$$

INTRODUCTION

The primary objective of this report is to present a rational
mathematical description of fluid (liquid water and steam) flow and
energy transport in porous hydrothermal systems. Specifically, we will
first derive the macroscopic balance equations by suitably averaging the
point balance equations for mass, momentum, and energy in porous media.
This procedure will yield macroscopic balance equations that will then be
combined with appropriate constitutive relationships resulting in simplified
partial differential equations for fluid flow and energy transport in single-
and two-phase hydrothermal systems. We will then present and discuss the
consistent boundary and initial conditions required to solve these equations.
Finally, we will derive the two-dimensional formulation of these equations
by partial integration in the vertical dimension.

The simplified partial differential equations for hydrothermal systems
are nonlinear and, in general, are not amenable to analytical solution.
Numerical techniques have been employed to solve the two-dimensional formulations
and are the subjects of other reports (Mercer and Faust, 1975; Faust and
Mercer, 1976). In this report the mathematical derivation will be presented
in much greater detail and rigor than given in our earlier hueristic description
(Mercer, Faust, and Pinder, 1974). This extension is necessary in order
to more fully account for the explicit and implicit assumptions inherent
in that derivation. The mathematical development presented in this report
follows closely that given by Faust (1976); however, more attention is
devoted in this report to interpretation.

We must emphasize that it is difficult to describe the behavior of hydrothermal systems without describing the coupled chemical and physical behavior. The mathematical model presented in this report is thus subject to certain limitations due to its assumption of pure water. This does not, however, invalidate the basic mathematical model which may be extended to describe chemical behavior if it becomes necessary and when required chemical data are available.

It should also be indicated that other mathematical models for multi-phase flow in hydrothermal systems have been developed (Donaldson, 1968; Toronyi, 1974; Brownell, Garg, and Pritchett, 1975; Lasseter, Witherspoon, and Lippmann, 1975). In general, these models invoke the same basic assumptions as ours (Mercer and others, 1974) and mainly differ in the specification of the unknown dependent variables.

HYDROTHERMAL SYSTEMS AND GEOTHERMAL ENERGY

The ultimate source of all geothermal energy is the heat energy stored and generated within the earth. To be economically significant, a geothermal resource must have high temperatures and be located at shallow depths within the earth's crust. Since current technology does not permit economical extraction of heat directly from dry rock, geothermal resources must also contain a fluid (steam and/or water) to transfer heat from the geothermal reservoir to the surface. In addition, the reservoir must have sufficient volume, porosity, and permeability to yield adequate flow rates to wells, over a long enough time.

Hydrothermal systems are geothermal reservoirs that contain fluid, and are classified by the dominant fluid phase contained within the reservoir. White, Muffler, and Truesdell (1971) have described the characteristics of two types of systems: vapor-dominated and hot-water. These two systems may be subdivided further on the basis of the geological conditions, the amount of dissolved species and non-condesable gases contained within the fluid, and the heat content (enthalpy) of the fluid.

Although geothermal fluids contain impurities, many reservoirs may be treated as pure water systems. Making this assumption, consider the pressure-enthalpy diagram for pure water in figure 1. Since all the known geothermal reservoirs exist at temperatures below the critical point of water (the temperature above which two phases cannot exist), this diagram may be divided into three regions. The first of these is the compressed water region, which is the condition existing in hot-water geothermal systems. The second is the two-phase (steam-water) region in which temperature is a function of pressure only. The third region contains super-heated steam. The vapor-dominated system described by White, Muffler, and Truesdell (1971) is believed to exist in the two-phase region, although the lower part of these systems may have a water table below which the fluid exists as compressed water. In the vapor-dominated systems, it is also possible, especially when influenced by production, that portions of the system contain super-heated steam.

From the above description it is apparent that any mathematical model of fluid flow and energy transport in hydrothermal systems will be complex. It must account for the flow behavior of single- and two-phase fluids and heat transport in complex natural systems. The remainder of this report is an attempt to offer a simplified yet realistic description of this behavior subject to appropriate assumptions.
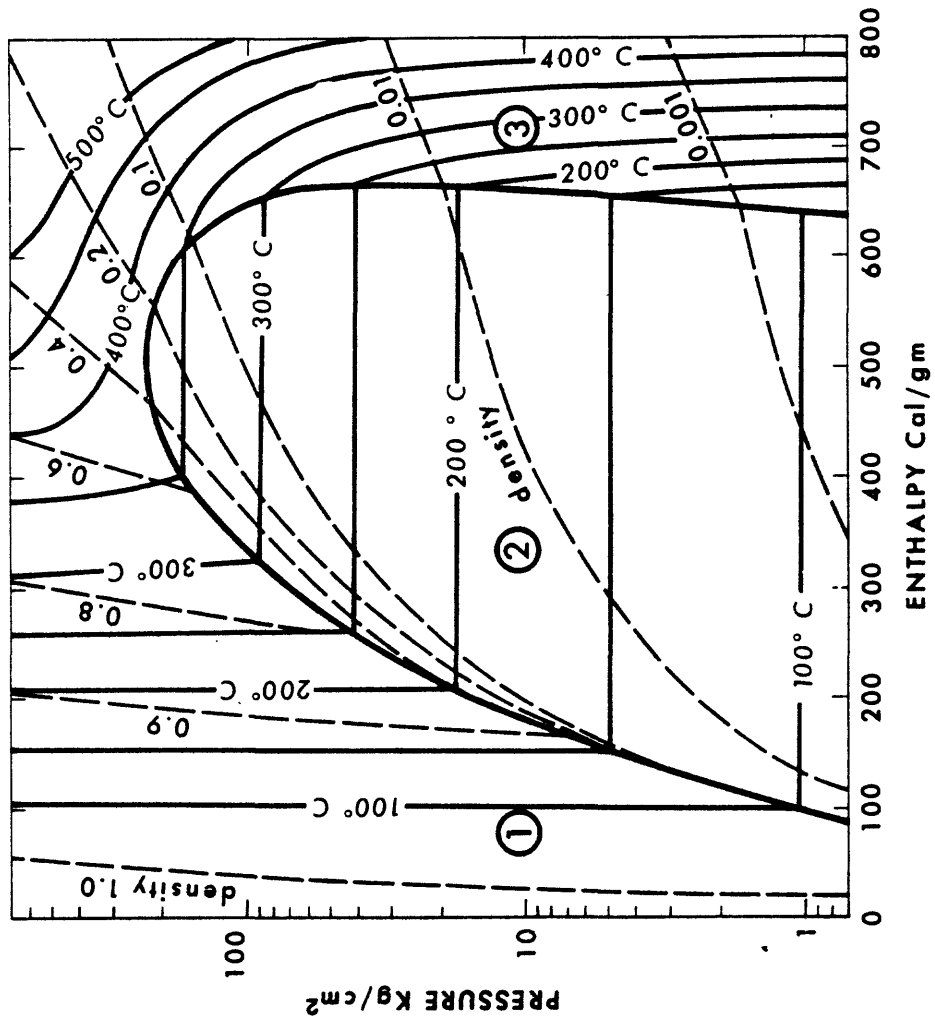
Figure 1.--Pressure-enthalpy diagram for pure water and vapor showing three thermodynamic regions below the critical point: 1) compressed water, 2) two-phase, steam and water, and 3) superheated steam (modified from White, Muffler, and Truesdell, 1971).

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MACROSCOPIC BALANCE EQUATIONS

Considering only the fluid phase, it is possible, in principle, to arrive at a complete set of equations describing an initial boundary value problem in which the boundaries are the surfaces separating the fluid and the rock matrix. The geometrical complexity of these surfaces, however, makes it impossible to define the boundary conditions, and thus impossible to solve the initial boundary value problem for the fluid on a size scale equivalent to the dimensions of pores. Therefore, as the first step in developing our mathematical model of hydrothermal systems the macroscopic balance equations for mass, momentum, and energy are derived for a single component, two-phase fluid (water and steam) in a porous medium. These equations are obtained by averaging the corresponding microscopic balance equations over a suitably defined averaging volume. The purpose of averaging is to increase the size scale of the volume element of interest in order to remove microscopic inhomogeneities at rock-fluid boundaries and to relate the thermodynamic variables to macroscopic, experimentally accessible quantities. It is also necessary to make sure that large-scale inhomogeneities in the porous media are not included within the enlarged volume element.

The application of averaging techniques to balance equations for porous media is not new; Whitaker (1969) and (1973), Bear (1972), Gray (1975), Gray and O'Neill (1976), Blake and Garg (1976), and Carnahan (1976) have demonstrated the use of averaging techniques for momentum and mass transport in porous media. Recently, Whiterspoon, Neumann, Sorey and Lippman (1975) and Lee, Gray, and Pinder (written communication, 1975) have applied averaging techniques to equations for energy transport in porous media.

Rules, Conventions, and Basic Theorems

The averaging procedure used in this derivation is basically the one that has been applied to the general transport equations for multi-phase flow in porous media by Whitaker (1973). The first step of the analysis is to associate with every point in space an averaging volume, V. This volume must be large enough so that the average values of quantities (scalers, vectors, or tensors) are "well behaved". Whitaker (1969) has shown that an averaging procedure leads to meaningful results if the characteristic length of the averaging volume is much less than the characteristic length of the entire porous medium, but also much greater than the characteristic length of the pores. The characteristic length of the sperical averaging volume illustrated in figure 2 would be the diameter. The volume shown in figure 2 is the sum of the volumes associated with the rock, liquid water, and steam, that is,

$$V = V_r(t) + V_s(t) + V_w(t), \qquad (1)$$

where V_r, V_s, and V_w may be functions of time, t, even though the averaging volume, V, is constant.

Two types of averages are used in this analysis. The <u>phase average</u> is given by,

$$<\psi_s> = \frac{1}{V} \int_{V_s(t)} \psi_s dV. \qquad (2)$$

The quantity, ψ_s, represents the value of ψ in the steam phase and is taken to be zero in the water and rock. Similar averages may be defined for the water and rock. The average, $<\psi_s>$, is obtained by averaging ψ_s over the entire volume, V, but since ψ_s is zero in the other phases, integration is performed over only the volume of the steam phase contained in V. Darcy velocity is an example of the type of average that arises from equation 2.

A more convenient average for many phase properties (such as pressure, temperature, and density) is the <u>intrinsic phase average</u>, defined as,

$$<\psi_s>^s = \frac{1}{V_s(t)} \int_{V_s(t)} \psi_s dV. \qquad (3)$$

The average $<\psi_s>^s$ is obtained by averaging ψ_s over only the steam phase volume. Again, similar averages may be defined for the water and rock.
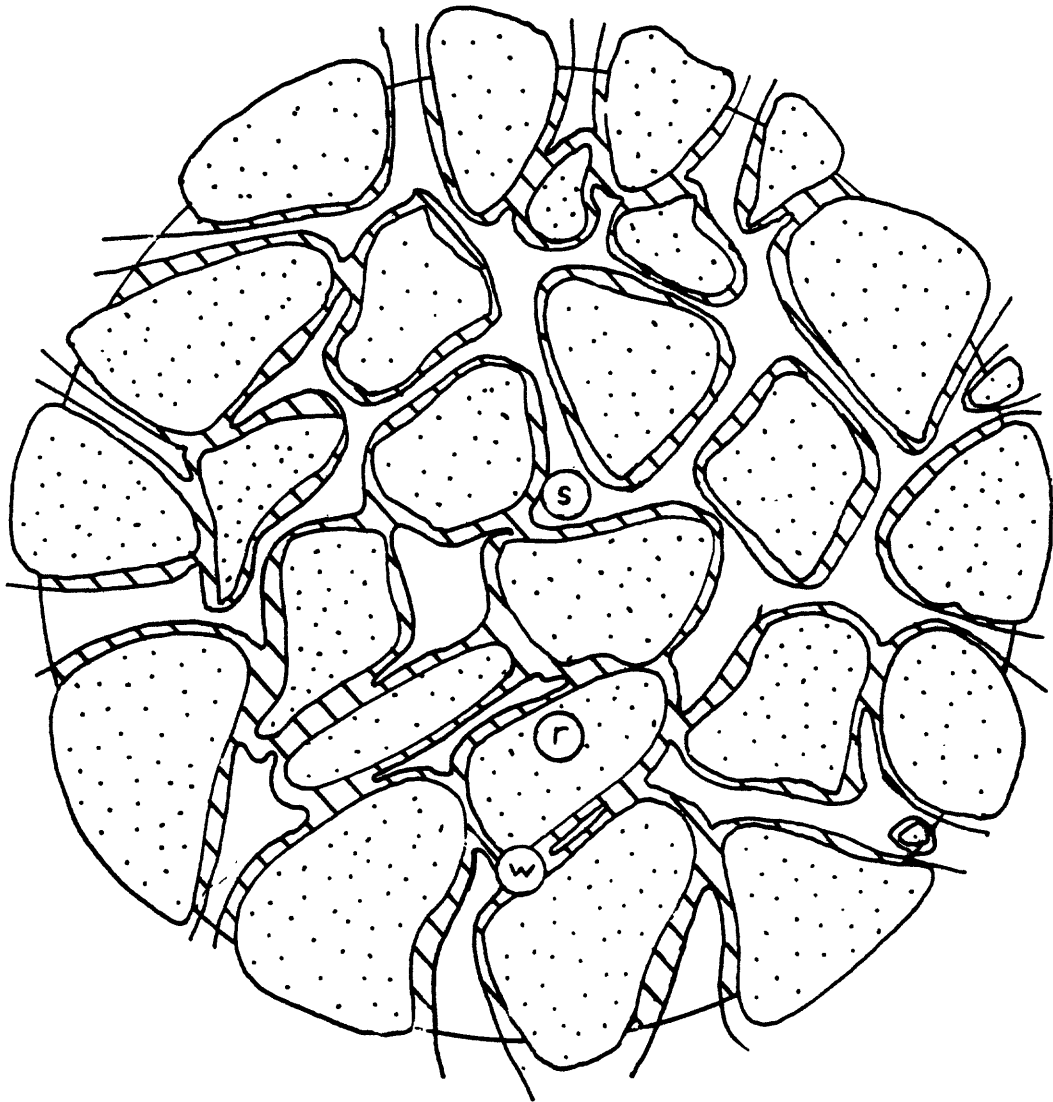
Figure 2.--Averaging volume for a two-phase fluid in a porous medium:
r, rock; w, water; and s, steam.

Both $\langle\psi_s\rangle^s$ and $\langle\psi_s\rangle$ are defined everywhere in space. They represent functions evaluated at the point associated with the averaging volume, and they are assumed to be continuous functions of time and space. The intrinsic phase average, $\langle\psi_s\rangle^s$, is related to the phase average, by,

$$\langle\psi_s\rangle = \varepsilon_s \langle\psi_s\rangle^s, \tag{4}$$

where ε_s is the ratio, $V_s(t)/V$.

For "well behaved" averages the definition for the deviation of a quantity from its phase average is given by Gray (1975) as,

$$\hat{\psi}_s = \psi_s - \langle\psi_s\rangle^s. \tag{5}$$

Just as the property, ψ_s, is zero in the rock and water, so is its deviation, $\hat{\psi}_s$, that is,

$$\psi_s = \hat{\psi}_s = 0. \tag{6}$$

Additionally,

$$\langle\hat{\psi}_s\rangle = \langle\hat{\psi}_s\rangle^s = 0. \tag{7}$$

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Other important relationships that may be derived are

$$<\psi_s \gamma_s> = \varepsilon_s \ <\psi_s>^s \ <\psi_s>^s + <\hat{\psi}_s \hat{\gamma}_s> \tag{8}$$

$$= \varepsilon_s \ <\psi_s>^s \ <\gamma_s>^s + \varepsilon_s <\hat{\psi}_s \hat{\gamma}_s>^s \tag{9}$$

$$= <\psi_s>^s \ <\gamma_s> + <\hat{\psi}_s \hat{\gamma}_s> \ . \tag{10}$$

The term $<\hat{\psi}_s \hat{\gamma}_s>$ represents the covariance between ψ_s and γ_s. In the development that follows similar covariance terms will arise frequently. It will often be desirable to neglect them whenever possible. This may occur in one of two ways. First, the variables, ψ_s and γ_s may be uncorrelated, that is, fluctuations in ψ_s are independent of fluctuations in γ_s (in this case $<\hat{\psi}_s \hat{\gamma}_s>$ is zero). The covariance term may also be neglected if $\varepsilon_s <\psi_s> <\gamma_s>^s$ is much greater than $<\hat{\psi}_s \hat{\gamma}_s>$. This situation exists when fluctuations, ψ_s and γ_s, are very small within the averaging volume.

An averaging theorem (Whitaker, 1969) that relates the average of a derivative to the derivative of the average is given as,

$$<\nabla \psi_s> = \nabla <\psi_s> + \frac{1}{V} \int_{A_{sw}} \psi_s \underline{n}_s dA + \frac{1}{V} \int_{A_{sr}} \psi_s \underline{n}_s dA, \tag{11}$$

where $\underline{n}_s$ is the outward normal vector to the steam phase and A_{sw} and A_{sr} are the steam-water and steam-rock interface surface areas, respectively. Gray (1975) has derived a "modified averaging theorem" which is also useful. This theorem is written as,

$$<\nabla \psi_s> = \varepsilon_s \nabla <\psi_s>^s + \frac{1}{V} \int_{A_{sw}} \hat{\psi}_s \underline{n}_s dA + \frac{1}{V} \int_{A_{sr}} \hat{\psi}_s \underline{n}_s dA. \qquad (12)$$

The general transport theorem (see, for example, Slattery, 1972) is given as,

$$<\frac{\partial \psi_s}{\partial t}> = \frac{\partial <\psi_s>}{\partial t} - \frac{1}{V} \int_{A_{sw}} \psi_s \underline{w}_{sw} \cdot \underline{n}_s dA - \frac{1}{V} \int_{A_{sr}} \psi_s \underline{w}_{sr} \cdot \underline{n}_s dA, \qquad (13)$$

where $\underline{w}_{sw}$ and $\underline{w}_{sr}$ are the velocities of the steam-water and steam-rock interfaces, respectively. These three theorems are important in the analysis that follows and will be used frequently.

Although the relationships and theorems given above are for the steam phase similar expressions hold for the water and rock. These expressions may also be easily extended for vector and tensor quantities.

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Mass Balance

The general, differential, microscopic mass balance equation (see for example, Bird, Stewart and Lightfoot, 1960) may be written for the steam phase as,

$$\frac{\partial \rho_s}{\partial t} + \nabla \cdot (\rho_s \underline{v}_s) - q_s' = 0, \tag{14}$$

where ρ is the density, $\underline{v}$ is the velocity, and q' is the mass source rate. In the above equation the first term represents the time rate gain of mass per unit volume; the second term is the rate of mass input per unit volume due to convection; and the last term is the rate of mass supply per unit volume.

The phase average of equation 14 is,

$$<\frac{\partial \rho_s}{\partial t}> + < \nabla \cdot (\rho_s \underline{v}_s)> - < q_s'>= 0. \tag{15}$$

Application of the transport theorem to the first term in equation 15 yields,

$$<\frac{\partial \rho_s}{\partial t}> = \frac{\partial <\rho_s>}{\partial t} - \frac{1}{V} \int_{A_{sw}} \rho_s \underline{w}_{sw} \cdot \underline{n}_s dA - \frac{1}{V} \int_{A_{sr}} \rho_s \underline{w}_{sr} \cdot \underline{n}_s dA. \tag{16}$$

The averaging theorem permits the second term to be written as,

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$$\langle \nabla \cdot (\rho_s \underline{v}_s) \rangle = \nabla \cdot \langle \rho_s \underline{v}_s \rangle + \frac{1}{V} \int_{A_{sw}} \rho_s \underline{v}_s \cdot \underline{n}_s dA + \int_{A_{sr}} \rho_s \underline{v}_s \cdot \underline{n}_s dA. \tag{17}$$

Substituting equations 16 and 17 into equation 15 gives,

$$\frac{\partial \langle \rho_s \rangle}{\partial t} + \nabla \cdot \langle \rho_s \underline{v}_s \rangle - \frac{1}{V} \int_{A_{sw}} \rho_s (\underline{w}_{sw} - \underline{v}_s) \cdot \underline{n}_s dA \tag{18}$$

$$- \frac{1}{V} \int_{A_{sr}} \rho_s (\underline{w}_{sr} - \underline{v}_s) \cdot \underline{n}_s dA - \langle q_s' \rangle = 0.$$

The velocity of the steam, $\underline{v}_s$, and velocity of the steam-rock interface, $\underline{w}_{sr}$, normal to the interface are assumed to be equal and thus the second surface integral in equation 18 vanishes. The first surface integral, in general, is nonzero and represents the mass transfer due to vaporation of liquid water to steam. The second term in equation 18 may be expanded using equation 10. With that substitution and the elimination of the second integral, we obtain,

$$\frac{\partial \varepsilon_s \langle \rho_s \rangle^s}{\partial t} + \nabla \cdot (\langle \underline{v}_s \rangle \langle \rho_s \rangle^s + \langle \hat{\underline{v}}_s \hat{\rho}_s \rangle) - \frac{1}{V} \int_{A_{sw}} \rho_s (\underline{w}_{sw} - \underline{v}_s) \cdot \underline{n}_s dA$$

$$- \langle q_s' \rangle = 0, \tag{19}$$

where the density is expressed as an intrinsic phase average, $\langle\rho_s\rangle^s$, and the velocity is expressed as a phase average, $\langle\underline{v}_s\rangle$.

For the liquid-water phase the analogous mass balance may be written as,

$$\frac{\partial\varepsilon_w \langle\rho_w\rangle^w}{\partial t} + \nabla\cdot(\langle\underline{v}_w\rangle \langle\rho_w\rangle^w + \langle\hat{\underline{v}}_w\hat{\rho}_w\rangle) - \frac{1}{V}\int_{A_{ws}} \rho_w(\underline{w}_{ws}-\underline{v}_w)\cdot\underline{n}_w dA$$

$$- \langle q_w'\rangle = 0. \qquad (20)$$

For the rock,

$$\frac{\partial\varepsilon_r \langle\rho_r\rangle}{\partial t} + \nabla\cdot(\langle\underline{v}_r\rangle\langle\rho_r\rangle^r + \langle\hat{\underline{v}}_r\hat{\rho}_r\rangle) = 0, \qquad (21)$$

where both surface integrals vanish and no mass source is specified.

If it is assumed that the covariance terms $\langle\hat{\underline{v}}_s\hat{\rho}_s\rangle$, $\langle\hat{\underline{v}}_w\hat{\rho}_w\rangle$ and $\langle\hat{\underline{v}}_r\hat{\rho}_r\rangle$ are uncorrelated then these terms may be neglected in equations 19,20, and 21. The surface integral terms in 19 and 20 are equivalent but with opposite signs, because the mass of steam gained by vaporization in the steam phase is equal to the mass of liquid lost to vaporization in the water phase. Representing the mass vaporization terms by (dv) and neglecting the convariance terms, one has,

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$$\frac{\partial \epsilon_s <\rho_s>^s}{\partial t} + \nabla \cdot (<\underline{v}_s> <\rho_s>^s) - dv - <q'_s> = 0, \tag{22}$$

$$\frac{\partial \epsilon_w <\rho_w>^w}{\partial t} + \nabla \cdot (<\underline{v}_w> <\rho_w>^w) + dv - <q'_w> = 0, \tag{23}$$

and,

$$\frac{\partial \epsilon_r <\rho_r>^r}{\partial t} + \nabla \cdot (<\underline{v}_r> <\rho_r>^r) = 0. \tag{24}$$

Momentum Balance

The microscopic momentum balance equation (see for example Slattery, 1972) for the steam phase may be written as,

$$\frac{\partial (\rho_s \underline{v}_s)}{\partial t} + \nabla \cdot (\rho_s \underline{v}_s \underline{v}_s) - \nabla \cdot \underline{\underline{\sigma}}_s - \rho_s \underline{f}_b - q'_s \underline{v}'_s = 0, \tag{25}$$

where $\underline{\underline{\sigma}}_s$ is the partial stress tensor and $\underline{f}_b$ is the body force vector. In this equation the first term represents the rate of increase in momentum per unit volume; the second term is the rate of momentum gain per unit volume due to convection; the third term is rate of momentum gain by viscous transfer and pressure forces per unit volume; the fourth term is the momentum gain due to body forces; and the final term is the time rate supply of momentum per unit volume. If the first two terms are expanded, the above equation may be given as,

$$\underline{v}_s \left(\frac{\partial \rho_s}{\partial t} + \nabla \cdot \rho_s \underline{v}_s - q'_s \right) + \left(\underline{v}_s - \underline{v}'_s \right) q'_s + \rho_s \left(\frac{\partial \underline{v}_s}{\partial t} + \nabla \cdot \underline{v}_s \underline{v}_s \right)$$

$$- \nabla \cdot \underline{\underline{\sigma}}_s - \rho_s \underline{f}_b = 0. \tag{26}$$

Substitution of the microscopic mass balance equation into the first term in equation 26 eliminates this term. The second term may be neglected if it is assumed that the velocity of the source is the same as that of the fluid. The third term represents the inertial forces. Polubarinova-Kochina (1962) has shown that the inertial forces tend towards zero very rapidly after the onset of motion, and thus may be justifiably neglected for many applications. If it is also assumed that the only body force

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acting on the steam is gravity, $\underline{g}$, then equation 26 reduces to,

$$- \nabla \cdot \underline{\underline{\sigma}}_s - \rho_s \underline{g} = 0. \tag{27}$$

The partial stress tensor, $\underline{\underline{\sigma}}_s$, for the steam phase may be expressed in terms of the extra stress tensor, $\underline{\underline{\tau}}_s$, and the thermodynamic phase pressure p_s, as,

$$\underline{\underline{\sigma}}_s = - p_s \underline{\underline{\delta}} + \underline{\underline{\tau}}_s , \tag{28}$$

where $\underline{\underline{\delta}}$ is the identity tensor. Equation 26 may then be given as,

$$\nabla p_s - \nabla \cdot \underline{\underline{\tau}}_s - \rho_s \underline{g} = 0. \tag{29}$$

The phase average of equation 29 is,

$$\langle \nabla p_s \rangle - \langle \nabla \cdot \underline{\underline{\tau}}_s \rangle - \langle \rho_s \underline{g} \rangle = 0. \tag{30}$$

If the gravity term is assumed to be constant, the modified averaging theorem (equation 12) is applied to the first term, and the averaging

theorem (equation 11) is applied to the second term; then,

$$\epsilon_s \nabla <p_s>^s + \frac{1}{V} \int_{A_{sw}} \hat{p}_s \underline{\delta} \cdot \underline{n}_s dA + \frac{1}{V} \int_{A_{sr}} \hat{p}_s \underline{\delta} \cdot \underline{n}_s dA - \nabla \cdot <\underline{\tau}_s>$$

$$- \frac{1}{V} \int_{A_{sw}} \underline{\tau}_s \cdot \underline{n}_s dA - \frac{1}{V} \int_{A_{sr}} \underline{\tau}_s \cdot \underline{n}_s dA - \underline{g}\epsilon_s <\rho_s>^s = 0. \qquad (31)$$

The gravity vector, $\underline{g}$, may be expressed in terms of the scaler potential, D, as,

$$\underline{g} = g\nabla D, \qquad (32)$$

where g is the gravitational acceleration constant.   This relationship can be used to simplify equation 31, that is,

$$\epsilon_s \nabla <p_s>^s - \epsilon_s g <\rho_s>^s \nabla D - \frac{1}{V} \int_{A_{sw}} (\underline{\tau}_s - \hat{p}_s \underline{\delta}) \cdot \underline{n}_s dA$$

$$- \frac{1}{V} \int_{A_{sr}} (\underline{\tau}_s - \hat{p}_s \underline{\delta}) \cdot \underline{n}_s dA - \nabla \cdot <\underline{\tau}_s> = 0. \qquad (33)$$

For the water phase, a similar momentum balance may be derived, that is,

$$\varepsilon_w \nabla < p_w >^w - \varepsilon_w g \; <\rho_w>^w \; \nabla D - \frac{1}{V} \int_{A_{ws}} (\underline{\underline{\tau}}_w - \hat{p}_w \; \underline{\underline{\delta}}) \cdot \underline{n}_w dA$$

$$- \frac{1}{V} \int_{A_{wr}} (\underline{\underline{\tau}}_w - \hat{p}_w \; \underline{\underline{\delta}}) \cdot \underline{n}_w dA - \nabla \cdot <\underline{\underline{\tau}}_w> = 0 \; . \tag{34}$$

To proceed any further with these equations, a constitutive relationship must be assumed for the two surface integrals and the divergence of the partial stress tensor. Such a relationship must describe the behavior of the material (steam and water) and must be independent of the observer. Raats and Klute (1968) have suggested relationships for liquid and gas phases in porous media. Using the notation specified in this report, these relationships for steam and water are:

$$\frac{1}{V} \int_{A_{sw}} (\underline{\underline{\tau}}_s - \hat{p}_s \underline{\underline{\delta}}) \cdot \underline{n}_s dA + \frac{1}{V} \int_{A_{sr}} (\underline{\underline{\tau}}_s - \hat{p}_s \; \underline{\underline{\delta}}) \cdot \underline{n}_s dA$$

$$+ \nabla \cdot <\underline{\underline{\tau}}_s> = \underline{\underline{\Omega}}_s = \underline{\underline{\Omega}}_s \; [(< \underline{v}_s > - <\underline{v}_r>), (< \underline{v}_s > - <\underline{v}_w>), \underline{p}_s, t] \; , \tag{35}$$

and,

$$\frac{1}{V} \int_{A_{ws}} (\underline{\underline{\tau}}_w - \hat{p}_w \underline{\underline{\delta}}) \cdot n_w dA + \frac{1}{V} \int_{A_{wr}} (\underline{\underline{\tau}}_w - \hat{p}_w \underline{\underline{\delta}}) \cdot \underline{n}_w dA$$

$$+ \nabla \cdot < \underline{\underline{\tau}}_w > = \underline{\Omega}_w = \underline{\Omega}_w [(< \underline{v}_w > - <\underline{v}_r>), (< \underline{v}_w > - <v_s>), \underline{P}_w, t] , \qquad (36)$$

that is, the surface integrals and the divergence of $<\underline{\underline{\tau}}>$ are equal to $\underline{\Omega}$ which is a function of the relative velocity terms $(<\underline{v}_s> - <\underline{v}_r>)$, the fluid particle reference, $\underline{P}$, and time; all of which are independent of the observer. Dependency upon the particle references and time includes the effect of the variations in porous-medium and fluid properties.

To be useful, a constitutive assumption must be correlated with observed behavior. The constitutive relationships that have received the most experimental verification are simpler than those given above. They are of the form,

$$\underline{\Omega}_s = \underline{\Omega}_s [(<\underline{v}_s> - <\underline{v}_r>), \underline{P}_s, t] , \qquad (37)$$

and,

$$\underline{\Omega}_w = \underline{\Omega}_w [(<\underline{v}_w> - <\underline{v}_r>), \underline{P}_w, t] . \qquad (38)$$

Raats and Klute (1968) discussed the alternative constitutive expressions and pointed out the theoretical discrepancies in equations 37 and 38. Unfortunately, expressions such as equations 35 and 36 have not been sufficiently correlated with observed behavior.

For any practical purposes it is necessary to use the simplified constitutive relationships (of the form given in equations 37 and 38) such as those suggested by Wykoff and Botset (1936) and Muskat and Meres (1936). These may be expressed as,

$$\underline{\Omega}_s - \underline{\underline{k}}^{-1} \frac{\varepsilon_s \mu_s}{k_{rs}} \cdot (<\underline{v}_r> - <\underline{v}_s>) , \tag{39}$$

and

$$\underline{\Omega}_w = \underline{\underline{k}}^{-1} \frac{\varepsilon_w \mu_w}{k_{rw}} \cdot (<\underline{v}_r> - <\underline{v}_w>) , \tag{40}$$

where $\underline{\underline{k}}$ is the local intrinsic permeability tensor, k_r is the dimensionless relative permeability, and μ is the dynamic viscosity. If it is assumed that the rock velocity is negligible, equations 39 and 40 may be substituted into equations 33 and 34 to obtain,

$$\nabla <p_s>^s - g<\rho_s>^s \nabla D = -\underline{\underline{k}}^{-1} \frac{\mu_s}{k_{rs}} \cdot <\underline{v}_s> , \tag{41}$$

and,

$$\nabla <p_w>^w - g<\rho_w>^w \nabla D = -\underline{\underline{k}}^{-1} \frac{\mu_w}{k_{rw}} \cdot <\underline{v}_w> . \tag{42}$$

Energy Balance

The general, differential, microscopic thermal energy balance equation in terms of internal energy (see for example, Bird, and others, 1960) is given as,

$$\frac{\partial(\rho U)}{\partial t} + \nabla \cdot \underline{v} U + \nabla \cdot \underline{\lambda}_c + p\ (\nabla \cdot \underline{v}) + \underline{\underline{\tau}} : \nabla \underline{v} - q'(U + p/\rho)' = 0, \tag{43}$$

where U is the internal energy and $\underline{\lambda}_c$ is the conduction vector. In this equation the first term represents the time rate gain of internal energy per unit volume. The second and third terms are the rate of energy gain per unit volume by convection and conduction, respectively. The fourth term is sometimes called the compressible work term, which stands for the reversible rate of internal energy increase per unit volume by compression. The fifth term represents the irreversible rate of internal energy increase per unit volume by viscous dissipation. The last term represents the rate of energy supply from external sources per unit volume.

Noting that internal energy is related to enthalpy by,

$$U = h - p/\rho, \tag{44}$$

permits the thermal energy equation to be written as,

$$\frac{\partial(\rho h)}{\partial t} + \nabla \cdot \rho \underline{v} h + \nabla \cdot \underline{\lambda}_c + \underline{\underline{\tau}} : \nabla \underline{v} - q'h' - \frac{\partial p}{\partial t} - \underline{v} \cdot \nabla p = 0. \tag{45}$$

If it is assumed that the viscous dissipation term in equation 45 is small in comparison to the other terms, then,

$$\frac{\partial(\rho h)}{\partial t} + \nabla \cdot \rho h \underline{v} + \nabla \cdot \underline{\lambda}_c - q'h' - \frac{\partial p}{\partial t} - \underline{v} \cdot \nabla \rho = 0. \tag{46}$$

Brownell, Garg, and Pritchett (1975) demonstrated by using nondimensional analysis that for properties typical of liquid-dominated hydrothermal systems the viscous dissipation term is neglible. Further, if one uses appropriate values for steam in the analysis of Brownell and others, this term is also negligible for typical vapor-dominated flow problems. For the steam phase equation 46 may be written as,

$$\frac{\partial(\rho_s h_s)}{\partial t} + \nabla \cdot \rho_s \underline{v}_s h_s + \nabla \cdot \underline{\lambda}_{cs} - q_s'h_s' - \frac{\partial p_s}{\partial t} - \underline{v}_s \cdot \nabla p_s = 0. \tag{47}$$

The phase average of the above equation is,

$$\langle\frac{\partial(\rho_s h_s)}{\partial t}\rangle + \langle\nabla\cdot\rho_s\underline{v}_s h_s\rangle + \langle\nabla\cdot\underline{\lambda}_{cs}\rangle - \langle q_s' h_s'\rangle - \langle\frac{\partial p_s}{\partial t}\rangle - \langle\underline{v}_s\cdot\nabla p_s\rangle = 0. \quad (48)$$

The transport theorem (equation 13) may be applied to the first and fifth terms to yield,

$$\langle\frac{\partial(\rho_s h_s)}{\partial t}\rangle - \langle\frac{\partial p_s}{\partial t}\rangle = \frac{\partial\langle\rho_s h_s\rangle}{\partial t} - \frac{\partial\langle p_s\rangle}{\partial t} - \frac{1}{V}\int_{A_{sw}} (p_s - \rho_s h_s)\underline{w}_{sw}\cdot\underline{n}_s dA$$

$$- \frac{1}{V}\int_{A_{sr}} (p_s - \rho_s h_s)\underline{w}_{sr}\cdot\underline{n}_s dA . \quad (49)$$

The averaging theorem permits the second term in equation 48 to be written as,

$$\langle\nabla\cdot\rho_s\underline{v}_s h_s\rangle = \nabla\cdot\langle\rho_s\underline{v}_s h_s\rangle + \frac{1}{V}\int_{A_{sw}} \rho_s h_s\underline{v}_s\cdot\underline{n}_s dA + \frac{1}{V}\int_{A_{sr}} \rho_s h_s\underline{v}_s\cdot\underline{n}_s dA. \quad (50)$$

Likewise the averaged conduction term is,

$$\langle\nabla\cdot\underline{\lambda}_{cs}\rangle = \nabla\cdot\langle\underline{\lambda}_{cs}\rangle + \frac{1}{V}\int_{A_{sw}} \underline{\lambda}_{cs}\cdot\underline{n}_s dA + \frac{1}{V}\int_{A_{sr}} \underline{\lambda}_{cs}\cdot\underline{n}_s dA . \quad (51)$$

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If it is assumed that the correlation between the phase velocity, $\underline{v}_s$, and the gradient of the phase pressure, ∇p_s, is negligible, then,

$$\langle \underline{v}_s \cdot \nabla p_s \rangle = \langle \underline{v}_s \rangle \cdot \nabla \langle p_s \rangle^s + \langle \underline{v}_s \rangle \cdot \left(\frac{1}{V} \int_{A_{sw}} \hat{p}_s \underline{n}_s dA + \frac{1}{V} \int_{A_{sr}} \hat{p}_s \underline{n}_s dA \right). \tag{52}$$

Combining all the terms gives,

$$\frac{\partial \langle \rho_s h_s \rangle}{\partial t} - \frac{\partial \langle p_s \rangle}{\partial t} + \nabla \cdot \langle \rho_s \underline{v}_s h_s \rangle + \frac{1}{V} \int_{A_{sw}} (p_s - {}_s h_s)(\underline{w}_{sw} - \underline{v}_s) \cdot \underline{n}_s dA$$

$$+ \frac{1}{V} \int_{A_{sr}} (p_s - \rho_s h_s)(\underline{w}_{sr} - \underline{v}_s) \cdot \underline{n}_s dA + \frac{1}{V} \int_{A_{sw}} \lambda_{cs} \cdot \underline{n}_s dA + \frac{1}{V} \int_{A_{sr}} \lambda_{cs} \cdot \underline{n}_s dA$$

$$+ \langle \underline{v}_s \rangle \cdot \nabla \langle p_s \rangle^s + \langle \underline{v}_s \rangle \cdot \left(\frac{1}{V} \int_{A_{sw}} \hat{p}_s \underline{n}_s dA + \frac{1}{V} \int_{A_{sr}} \hat{p}_s \underline{n}_s dA \right)$$

$$+ \nabla \cdot \langle \underline{\lambda}_{cs} \rangle - \langle q_s' h_s' \rangle = 0. \tag{53}$$

The first two surface integrals in equation 53 represent transfer of internal energy due to phase changes between water and steam, and rock and steam, respectively. Since it is assumed that no phase change occurs with the rock, the second surface integral may be eliminated. The next two integrals in the equation represent the conductive energy flux between the steam and water, and the steam and rock. The dot product of the phase velocity and the last two surface integrals represents the flux of pressure-work forces across the phase boundaries. In this development the surface

integrals, will not be evaluated, so let,

$$Q_{sw} = \frac{1}{V} \int_{A_{sw}} (p_s - \rho_s h_s) \, (\underline{w}_{sw} - \underline{v}_s) \cdot \underline{n}_s \, dA, \tag{54}$$

$$Q'_{sw} = \frac{1}{V} \int_{A_{sw}} \underline{\lambda}_{cs} \cdot \underline{n}_s \, dA, \tag{55}$$

$$Q''_{sw} = <\underline{v}_s> \cdot \frac{1}{V} \int_{A_{sw}} \hat{p}_s \underline{n}_s \, dA, \tag{56}$$

$$Q'_{sr} = \frac{1}{V} \int_{A_{sr}} \underline{\lambda}_{cs} \cdot \underline{n}_s \, dA, \tag{57}$$

and,

$$Q''_{sr} = <\underline{v}_s> \cdot \frac{1}{V} \int_{A_{sr}} \hat{p}_s \underline{n}_s \, dA. \tag{58}$$

Equation 53 may then be given as,

$$\frac{\partial <\rho_s h_s>}{\partial t} - \frac{\partial <p_s>}{\partial t} + \nabla \cdot <\rho_s \underline{v}_s h_s> - <\underline{v}_s> \cdot \nabla <p_s>^s + \nabla \cdot <\underline{\lambda}_{cs}>$$

$$+ Q_{sw} + Q'_{sw} + Q'_{sr} - Q''_{sw} - Q''_{sr} - <q'_s h'_s> = 0. \tag{59}$$

To eliminate the average of the product in the time derivative, equation 8 may be used. This yields,

$$\frac{\partial \langle \rho_s h_s \rangle}{\partial t} = \frac{\partial (\varepsilon_s \langle \rho_s \rangle^s \langle h_s \rangle^s)}{\partial t} + \frac{\partial (\varepsilon_s \langle \hat{\rho}_s \hat{h}_s \rangle^s)}{\partial t} . \tag{60}$$

For the product average in the convective term it is necessary to expand the product as follows:

$$\langle \rho_s \underline{v}_s h_s \rangle = \langle (\langle \underline{v}_s \rangle^s + \hat{\underline{v}}_s)(\langle \rho_s h_s \rangle^s + \hat{\rho}_s \hat{h}_s) \rangle. \tag{61}$$

Then using equation 8 yields,

$$\langle \rho_s \underline{v}_s h_s \rangle = \langle \underline{v}_s \rangle^s \langle \rho_s h_s \rangle + \varepsilon_s \langle \hat{\underline{v}}_s \hat{\rho}_s \hat{h}_s \rangle . \tag{62}$$

It follows that,

$$\langle \rho_s \underline{v}_s h_s \rangle = \langle \underline{v}_s \rangle \langle \rho_s \rangle^s \langle h_s \rangle^s + \langle \underline{v}_s \rangle \langle \hat{\rho}_s \hat{h}_s \rangle^s + \varepsilon_s \langle \hat{\underline{v}}_s \hat{\rho}_s \hat{h}_s \rangle^s. \tag{63}$$

Substituting equations 60 and 63 into equation 59 gives,

$$\frac{\partial}{\partial t}(\varepsilon_s \langle \rho_s \rangle^s \langle h_s \rangle^s) + \frac{\partial}{\partial t}(\varepsilon_s \langle \hat{\rho}_s \hat{h}_s \rangle) + \frac{\partial}{\partial t}(\varepsilon_s \langle p_s \rangle^s) + \nabla \cdot \langle \underline{v}_s \rangle \langle \rho_s \rangle^s \langle h_s \rangle^s$$

$$+ \nabla \cdot \langle \underline{v}_s \rangle \langle \hat{\rho}_s \hat{h}_s \rangle + \nabla \cdot \varepsilon_s \langle \hat{\underline{v}}_s \hat{\rho}_s \hat{h}_s \rangle^s - \langle \underline{v}_s \rangle \cdot \nabla \langle p_s \rangle^s + \nabla \cdot \langle \underline{\lambda}_{cs} \rangle + Q_{sw} + Q'_{sw} + Q'_{sr}$$

$$- Q''_{sw} - Q''_{sr} - \langle q'_s h'_s \rangle = 0. \tag{64}$$

Although the covariance $\varepsilon_s \langle \hat{\rho}_s \, \hat{h}_s \rangle^s$ is certainly not zero, it is reasonable to assume that in comparison to $\varepsilon_s \langle \rho_s \rangle^s \langle h_s \rangle^s$ it is small. This assumption may be used to eliminate the second and fourth terms in equation 61. This leaves two terms, $\nabla \cdot \langle \underline{\lambda}_{cs} \rangle$ and $\nabla \cdot \varepsilon_s \langle \underline{v}_s \, \hat{\rho}_s \hat{h}_s \rangle^s$, that require further evaluation.

Fourier's law is used as the constitutive expression for the conductive flux, that is,

$$\underline{\lambda}_{cs} = - \, \underline{\underline{K}}_{cs} \cdot \nabla T_s.$$

(65)

It then follows that,

$$\langle \underline{\lambda}_{cs} \rangle = - \, \langle \underline{\underline{K}}_{cs} \cdot \nabla T_s \rangle = - \, \langle \underline{\underline{K}}_{cs} \rangle^s \langle \nabla T_s \rangle + \varepsilon_s \, \langle \underline{\underline{K}}_{cs} \, \widehat{\nabla T}_s \rangle^s.$$

(66)

If it is assumed that the covariance, $\varepsilon_s \langle \underline{\underline{K}}_{cs} \, \widehat{\nabla T}_s \rangle^s$, is negligible, then,

$$\langle \underline{\lambda}_{cs} \rangle = -\varepsilon_s \, \langle \underline{\underline{K}}_{cs} \rangle^s \cdot \nabla \langle T_s \rangle^s - \langle \underline{\underline{K}}_{cs} \rangle^s \cdot \frac{1}{V} \int_{A_{sw}} \hat{T}_s \cdot \underline{n}_s \, dA$$

$$-\langle \underline{\underline{K}}_{cs} \rangle^s \cdot \frac{1}{V} \int_{A_{sr}} \hat{T}_s \cdot \underline{n}_s \, dA,$$

(67)

where the two surface integrals represent the decreased conduction rate due to the tortuosity of the system. These integrals, for convenience, may be represented by a tortuosity vector defined as,

$$\theta_s = -<\underset{=}{K}_{cs}>^S \cdot (\frac{1}{V}\int_{A_{sw}} \hat{T}_s \cdot \underset{-}{n}_s dA + \frac{1}{V}\int_{A_{sr}} \hat{T}_s \cdot \underset{-}{n}_s dA) . \tag{68}$$

Gray (1975) has defined a similar tortuosity vector for species transport for porous media, containing a single fluid phase. Bear (1972) has pointed out that for energy transport in porous media (where heat conduction in the solid phase occurs) the tortuosity effect is not important.

The term, $\nabla \cdot \varepsilon_s <\hat{v}_s \hat{\rho}_s h_s>^S$, may be expanded using equation 44, so that,

$$\nabla \cdot \varepsilon_s <\hat{v}_s \hat{\rho}_s h_s>^S = \nabla \cdot \varepsilon_s <\hat{v}_s \hat{\rho}_s U_s>^S + \hat{v}_s \hat{p}_s>^S$$

If it is assumed that the phase pressure and velocity are not correlated, then the second part of the above term may be neglected.

The term, $\nabla \cdot \varepsilon_s <\hat{v}_s \hat{\rho}_s U_s>^S$, represents the divergence of the dispersion vector. A constitutive relationship for the dispersion vector may be obtained by assuming that dispersion is mathematically equivalent to a diffusion process (see Bear, 1961; Scheidegger, 1961; Gray, 1975; Witherspoon and others, 1975). This relationship may be expressed,

$$\langle \hat{\underline{v}}_s \hat{\rho}_s \hat{U}_s \rangle^s = - \underline{\underline{K}}_{ds} \cdot \nabla \langle T_s \rangle^s. \tag{69}$$

Substituting equations 67 thru 69 into equation 64 and neglecting the covariance, $\varepsilon_s \langle \hat{\rho}_s \hat{h}_s \rangle^s$, yields,

$$\frac{\partial}{\partial t}(\varepsilon_s \langle \rho_s \rangle^s \langle h_s \rangle^s) - \frac{\partial}{\partial t}(\varepsilon_s \langle p_s \rangle^s) + \nabla \cdot \langle \underline{v}_s \rangle \langle \rho_s \rangle^s \langle h_s \rangle^s$$

$$- \nabla \cdot (\varepsilon_s \underline{\underline{K}}_{ds} \cdot \nabla \langle T_s \rangle^s) - \langle \underline{v}_s \rangle \cdot \nabla \langle p_s \rangle^s$$

$$- \nabla \cdot [\varepsilon_s \langle \underline{\underline{K}}_{cs} \rangle^s \cdot (\nabla \langle T_s \rangle^s + \underline{\theta}_s)]$$

$$+ Q_{sw} + Q'_{sw} + Q'_{sr} - Q''_{sw} - Q''_{sr} - \langle q'_s h'_s \rangle = 0. \tag{70}$$

The similar equation for the water phase is given as,

$$\frac{\partial}{\partial t}(\varepsilon_w \langle \rho_w \rangle^w \langle h_w \rangle^w) - \frac{\partial}{\partial t}(\varepsilon_w \langle p_w \rangle^w) + \nabla \cdot \langle \underline{v}_w \rangle \langle \rho_w \rangle^w \langle h_w \rangle^w - \nabla \cdot (\varepsilon_w \underline{\underline{K}}_{dw} \cdot \nabla \langle T_w \rangle^w)$$

$$- \langle \underline{v}_w \rangle \cdot \nabla \langle p_w \rangle^w$$

$$- \nabla \cdot [\varepsilon_w \langle \underline{\underline{K}}_{cw} \rangle^w \cdot (\nabla \langle T_w \rangle^w + \underline{\theta}_w)]$$

$$+ Q_{ws} + Q'_{ws} + Q'_{wr} - Q''_{ws} - Q''_{wr} - \langle q'_w h'_w \rangle = 0. \tag{71}$$

For the rock, the velocity and the source term are assumed to be negligible, so that,

$$\frac{\partial}{\partial t} (\varepsilon_r <\rho_r>^r <h_r>^r) - \nabla \cdot [\varepsilon_r <\underline{\underline{K}}_{cr}>^r \cdot (\nabla <T_r>^r + \theta_r)]$$

$$+ Q_{rw} + Q_{rs} = 0 \ . \tag{72}$$

GENERAL EQUATIONS AND CONSTITUTIVE RELATIONSHIPS

The phase balance equations derived in the previous section serve as the starting point for the general mathematical model presented in this section. These equations can be combined with appropriate assumptions (constitutive relationships) to yield two simplified equations posed in terms of fluid pressure and enthalpy. Formulation of the final equations in terms of pressure and enthalpy permits one set of equations to be used for both single- and two-phase hydrothermal systems.

Balance Equations

The mass balance equations given by equations 22 and 23 may be written as,

$$\frac{\partial(\phi S_s \rho_s)}{\partial t} + \nabla \cdot (\rho_s \underline{v}_s) - q_s' - dv = 0, \tag{73}$$

and

$$\frac{\partial(\phi S_w \rho_w)}{\partial t} + \nabla \cdot (\rho_w \underline{v}_w) - q_w' + dv = 0, \tag{74}$$

where ϕ is the porosity and S is the volume saturation, and the brackets representing the averaged terms have been removed. Recall that velocity is represented by a phase average while other quantities are represented by intrinsic phase averages. Furthermore, ϕS_w and ϕS_s are substituted for ϵ_w and ϵ_s, respectively. The rock mass balance equation is neglected in this development. The implicit assumption is that rock velocity is very small. The major effect of the small velocity, however, is incorporated in an approximation relating porosity to the pressure in the fluid phases.

The momentum balance equations (equations 41 and 42) may be rearranged to give:

$$\underline{v}_s = - \frac{k k_{rs}}{\mu_s} \cdot (\nabla p_s - \rho_s g \nabla D) \ , \tag{75}$$

and

$$\underline{v}_w = - \frac{k k_{rw}}{\mu_w} \cdot (\nabla p_w - \rho_w g \nabla D), \tag{76}$$

where the averaging brackets have been removed.

If the averaging brackets are removed, and ϕS_s, ϕS_w, and $(1-\phi)$ substituted for ϵ_s, ϵ_w, ϵ_r in equations 70, 71, and 72 respectively, one has,

$$- \frac{\partial}{\partial t}(\phi S_s P_s) - \underline{v}_s \cdot \nabla P_s + \frac{\partial(\phi S_s \rho_s h_s)}{\partial t} + \nabla \cdot (\rho_s h_s \underline{v}_s) - \nabla \cdot (\phi S_s \underline{\underline{K}}_{ds} \cdot \nabla T_s) \qquad (77)$$

$$- \nabla \cdot [\phi S_s \underline{\underline{K}}_{cs} \cdot (\nabla T_s + \theta_s)] + Q_{sw} + Q'_{sw} + Q'_{sr} - Q''_{sw} - Q''_{sr} - q'_s h'_s = 0,$$

$$- \frac{\partial}{\partial t}(\phi S_w P_w) - \underline{v}_w \cdot \nabla P_w + \frac{\partial(\phi S_w \rho_w h_w)}{\partial t} + \nabla \cdot (\rho_w h_w \underline{v}_w) - \nabla \cdot (\phi S_w \underline{\underline{K}}_{ds} \cdot \nabla T_w) \qquad (78)$$

$$- \nabla \cdot [\phi S_w \underline{\underline{K}}_{cw} \cdot (\nabla T_w + \theta_w)] + Q_{ws} + Q'_{ws} + Q_{wr} - Q''_{ws} - Q''_{wr} - q'_w h'_w = 0,$$

and

$$\frac{\partial[(1-\phi)\rho_r h_r]}{\partial t} - \nabla \cdot [(1-\phi)\underline{\underline{K}}_{cr} \cdot \nabla T_r] + Q'_{rs} + Q'_{rw} = 0. \qquad (79)$$

Constitutive Relationships

The balance equations (equations 73 thru 79) are not sufficient to describe the system and consequently additional relationships are required. These are in the form of constitutive relationships that are formulated under the following basic assumptions of the model:

1. Capillary pressure effects are negligible.

2. Temperature equilibrium exists among the steam, water and rock.

3. The reservoir fluid is single-component pure water consisting of either one or two phases.

4. Relative permeability is a function of water saturation.

5. Viscosities are considered as functions of temperature.

6. Porosity is a function of space and pressure.

7. Reservoir thickness, rock density, and intrinsic permeability are functions of space.

8. Rock enthalpy is a function of temperature.

Capillary Pressure

An expression relating phase pressures is given by,

$$p_c = p_s - p_w,$$ (80)

where p_c is the capillary pressure. Capillary pressure has the effect of lowering the vapor pressure of water. Ramey, Kruger, and Raghavan (1973) point out that vapor-pressure data found in steam tables (Meyer, McClintock, Silvestri, and Spencer, 1967; Keenan, Keyes, Hill, and Moore, 1969) are based on flat steam-water interfaces, whereas the interface in porous media is curved. The amount the vapor-pressure curve is lowered

in a geothermal reservoir is not completely understood. The work of
Calhoun, Lewis, and Newman (1949) on consolidated rock does show a lowering
of the vapor-pressure curve with decreased fluid saturation. The efforts
of Cady (1969) and Bilhartz (1971), however, indicate no significant vapor
pressure lowering in their experiments using unconsolidated sands. An
important difference in the two results is that the experiments of
Calhoun, and others were made at a temperature of $36^{o}C$, and those conducted
by Cady and Bilhartz ranged from approximately $121^{o}C$ to $240^{o}C$. Further work
on the importance of capillary pressure in geothermal reservoirs is
required. For this development capillary pressure is assumed negligible,
which implies that fluid pressures in the steam and water phase are equal.
With these assumptions, equations 73 and 74 may be combined,

$$\frac{\partial(\phi\rho)}{\partial t} + \nabla\cdot(\rho_s \underline{v}_s) + \nabla\cdot(\rho_w \underline{v}_w) - q_s^{'} - q_w^{'} = 0, \tag{81}$$

where ρ is the density of the total steam-water mixture, defined as,

$$\rho = S_w \rho_w + S_s \rho_s, \tag{82}$$

and the volume saturations are defined so that they sum to one,

$$S_s + S_w = 1. \tag{83}$$

Temperature Equilibrium

The movement of steam and water through porous media is sufficiently slow, and the surface areas of all phases are sufficiently large, so that it is reasonable to assume local thermal equilibrium among phases is achieved instantaneously. This assumption permits the energy equations for rock, steam, and water to be combined and the medium conduction-dispersion term to be posed in terms of one temperature, T, for all three phases. In this development the lumped conduction-dispersion term is greatly simplified; the combined conduction-dispersion term is defined as,

$$\nabla \cdot (\phi S_w \underline{K}_{dw} \cdot \nabla T_w) + \nabla \cdot (\phi S_s \underline{K}_{ds} \cdot \nabla T_s) + \nabla \cdot [\phi S_w \underline{K}_{cw} \cdot (\nabla T_w + \theta_w)]$$

$$+ \nabla \cdot [\phi S_s \underline{K}_{cs} \cdot (\nabla T_s + \theta_s)] + \nabla \cdot [(1-\phi) \underline{K}_{cr} \cdot (\nabla T_r + \theta_r)] = \nabla \cdot (K_m \nabla T). \quad (84)$$

where the medium conduction-dispersion coefficient, K_m, is isotopic. In addition to the limitation of combining the effects of conduction and dispersion, equation 84 neglects the important effect of temperature on thermal conductivity. In this regard, Somerton, Keese and Chu (1974) point out that the thermal conductivity of a porous medium is a function of temperature, porosity, and water saturation. This effect may be important in a purely conductive system, however, in this study these effects are neglected.

Invoking these assumptions concerning thermal equilibrium and dispersion, the energy balance equations may be combined, yielding,

$$\frac{\partial}{\partial t}[\phi\rho h + (1-\phi)\rho_r h_r] + \nabla\cdot(\rho_s h_s \underline{v}_s) + \nabla\cdot(\rho_w h_w \underline{v}_w)$$
$$- \nabla\cdot(K_m \nabla T) - q_s' h_s' - q_w' h_w' - [\frac{\partial}{\partial t}\phi(S_w p_w + S_s p_s) + \underline{v}_s\cdot\nabla\ p_s + \underline{v}_w\cdot\nabla p_w] = 0 \quad (85)$$

in which, h, is the enthalpy of the steam-water mixture defined as,

$$h = \frac{S_s\rho_s h_s + S_w\rho_w h_w}{\rho} . \quad (86)$$

Note that the heat of vaporization terms and the interphase conduction and pressure terms in equations 77 and 78 have been eliminated in equation 85. This is the result of the jump energy balance at a phase interface (see Truesdell and Toupin, 1960, p. 610). If the mechanical energy is neglected, this balance requires that the thermal energy interface flux terms in the steam, water and rock equations sum to zero. Finally, the last term in equation 85 is the compressible-work term. Moench (1976) points out that this term is negligible except for conditions of low water saturation.

Thermodynamic Properties

As previously indicated it is assumed that the hydrothermal fluid is virtually pure water. Just how restrictive this assumption can be is demonstrated in figure 3. In this figure the effect of a NaCl concentration is shown in the pressure-enthalpy phase diagram. This effect is seen as an enlargment of the two-phase region (data from Haas, 1975a, Hass, 1975b). It is apparent that with weight concentrations greater than 2 percent a pure water assumption would not be valid. Although geothermal reservoirs contain other dissolved solids in addition to NaCl, in a qualitative sense, the effects would be similar for other impurities. For geothermal reservoirs such as those at Wairakei, New Zealand, Larderello, Italy, and Geysers, Calif. in which the salinities are about 1 percent or less (Koenig, 1973), the effects of dissolved solids would be small.

Relationships expressing the thermodynamic properties of pure water and steam as functions of enthalpy and pressure may be determined from data in steam tables such as Meyer and others (1967) or Keenan and others (1969). The necessary relationships for this development are as follows:

1. Steam enthalpy, h_s, and water enthalpy, h_w, are treated as functions of pressure.

2. Temperature is treated as a function of pressure and enthalpy for the compressed-water and superheated-steam regions.
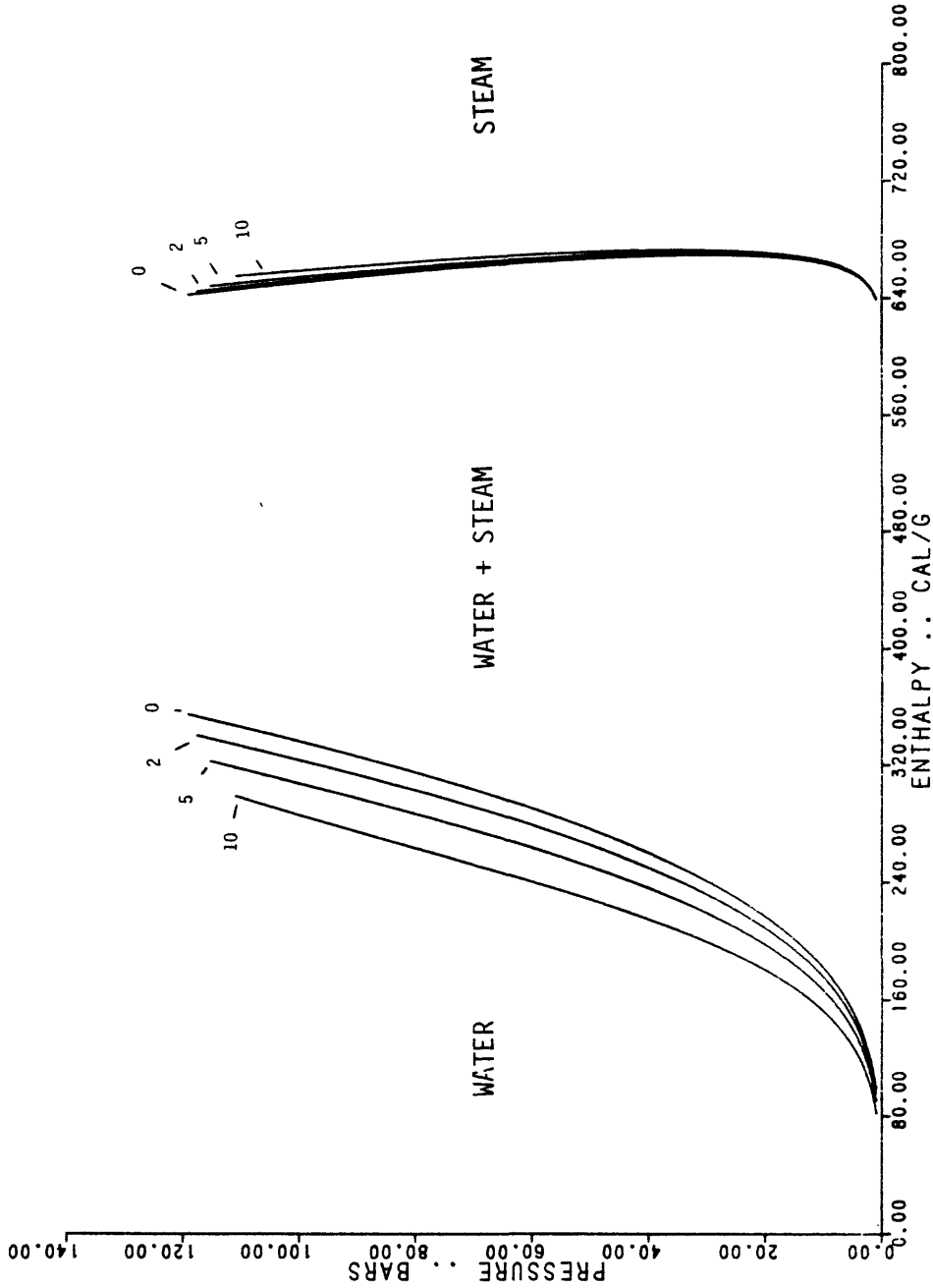
Figure 3.--Pressure-enthalpy diagram for pure water and NaCl solutions showing the boundaries between single-and two-phase regions. The two-phase region is enlarged with increasing NaCl concentration (pure water, 2, 5, 10 percent weight concentration's shown based on data from Haas 1975a and Haas 1975b).

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3. Total density, ρ, steam and water densities, ρ_s and ρ_w, are considered functions of pressure and enthalpy.

4. Phase saturations are functions of enthalpy and pressure. Water saturation in the compressed water region is assumed one, and in the superheated steam region zero. In the steam-water region saturations are obtained using

$$ S_w = \frac{\rho_s(h_s - h)}{h(\rho_w - \rho_s) - (h_w\ \rho_w - h_s\ \rho_s)} , \qquad (87) $$

and S_s is determined using equation 83.

5. Viscosities, μ_s, and μ_w, are considered functions of temperature. Additional relationships required for treating problems involving two-phase flow in porous media include:

6. Relative permeability is treated as a function of saturation and relationships similar to those in Corey (1954) may be used.

7. Porosity, ϕ, is a function of pressure.

Finally, two important properties of the rock that appear in the balance equations are rock density, ρ_r, and enthalpy, h_r. For this study:

8. Rock density is assumed to be a function of space only.

9. Rock enthalpy is a function of temperature and may be described using relationships similar to those presented in Dew and Martin (1965).

Combined Equations

With the assumptions of capillary pressure and thermal equilibrium the number of balance equations was reduced to four (equation 75,76,81 and 85). These can be further reduced by substituting equations 75 and 76 into equations 81 and 85 to yield the following:

$$\frac{\partial(\phi\rho)}{\partial t} - \nabla \cdot [\frac{\underline{k}k_{rs}\rho_s}{\mu_s} \cdot (\nabla p - \rho_s g \nabla D)]$$

$$- \nabla \cdot [\frac{\underline{k}k_{rw}\rho_w}{\mu_w} \cdot (\nabla p - \rho_w g \nabla D)] - q_w' - q_s' = 0, \tag{88}$$

and

$$\frac{\partial}{\partial t} [\phi\rho h + (1-\phi)\rho_r h_r] - \nabla \cdot [\frac{\underline{k}k_{rs}\rho_s h_s}{\mu_s} \cdot (\nabla p - \rho_s g \nabla D)]$$

$$- \nabla \cdot [\frac{\underline{k}k_{rw}\rho_w h_w}{\mu_w} \cdot (\nabla p - \rho_w g \nabla D)] - \nabla \cdot (K_m \nabla T) - q_s' h_s' - q_w' h_w'$$

$$- [\frac{\partial(\phi p)}{\partial t} + (\underline{v}_s + \underline{v}_w) \cdot \nabla p] = 0 \tag{89}$$

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where for convenience, we have not substituted for velocity in the compressible-work term. The temperature derivative in the conduction-dispersion term in equation 89 can be expressed in terms of the unknown dependent variables, pressure and enthalpy, by using the chain rule of differentiation yielding,

$$\frac{\partial}{\partial t} [\phi \rho h + (1-\phi)\rho_r h_r] - \nabla \cdot [\frac{\underline{\underline{k}} k_{rs} \rho_s h_s}{\mu_s} \cdot (\nabla p - \rho_s g \nabla D)]$$

$$- \nabla \cdot [\frac{\underline{\underline{k}} k_{rw} \rho_w h_w}{\mu_w} \cdot (\nabla p - \rho_w g \nabla D)] - \nabla \cdot [K_m (\frac{\partial T}{\partial p})_h \nabla p$$

$$+ K_m (\frac{\partial T}{\partial h})_p \nabla h] - q_s' h_s' - q_w' h_w' - [\frac{\partial (\phi p)}{\partial t} + (\underline{v}_s + \underline{v}_w) \cdot \nabla p] = 0. \tag{90}$$

Also, employing the chain rule of differentiation the time derivatives in equations 88 and 90 can be expanded in terms of pressure and enthalpy to give,

$$[\rho \frac{d\phi}{dp} + \phi (\frac{\partial \rho}{\partial p})_h] \frac{\partial p}{\partial t} + \phi (\frac{\partial \rho}{\partial h})_p \frac{\partial h}{\partial t} - \nabla \cdot [\frac{\underline{\underline{k}} k_{rs} \rho_s}{\mu_s} \cdot (\nabla p - \rho_s g \nabla D)]$$

$$- \nabla \cdot [\frac{\underline{\underline{k}} k_{rw} \rho_w}{\mu_w} \cdot (\nabla p - \rho_w g \nabla D)] - q_w' - q_s' = 0, \tag{91}$$

and,

$$\left[(\rho h - \rho_r h_r) \frac{d\phi}{dp} + \phi h \left(\frac{\partial \rho}{\partial p}\right)_h + (1-\phi)\rho_r \frac{dh_r}{dT} \left(\frac{\partial T}{\partial p}\right)_h \right] \frac{\partial p}{\partial t}$$

$$+ \left[\phi h \left(\frac{\partial \rho}{\partial h}\right)_p + (1-\phi)\rho_r \frac{dh_r}{dT} \left(\frac{\partial T}{\partial h}\right)_p + \phi \rho \right] \frac{\partial h}{\partial t}$$

$$- \nabla \cdot \left[\frac{\underline{k} k_{rs} \rho_s h_s}{\mu_s} \cdot (\nabla p - \rho_s g \nabla D) \right] - \nabla \cdot \left[\frac{\underline{k} k_{rw} \rho_w h_w}{\mu_w} (\nabla p - \rho_w g \nabla D) \right]$$

$$- \nabla \cdot \left[K_m \left(\frac{\partial T}{\partial p}\right)_h \nabla p + K_m \left(\frac{\partial T}{\partial h}\right)_p \nabla h \right] - q_s' h_s' - q_w' h_w' - \left[\frac{\partial (\phi p)}{\partial t} + (\underline{v}_s + \underline{v}_w) \cdot \nabla p \right] \qquad (92)$$

$$= 0.$$

Equations 91 and 92 describe the two-phase flow of heat in a steam-water-rock system; however, with minor modification, these equations also describe the flow of heat in a water-rock or a steam-rock system. When either steam or water is absent, the saturation of the absent phase is zero and that for the existing phase is one. Further, it is assumed that the relative permeability of the absent phase is zero and that for the existing phase

is one. Therefore, equations 91 and 92 reduce to the appropriate equations for either the compressed-water region or the superheated-steam region. A solution for these equations will determine whether a specified location contains compressed water, a steam-water mixture or super-heated steam.

Source Terms

Mass and energy source terms appear in the combined equations (equations 91 and 92). These represent the amount of mass and heat lost (or gained) to wells. In the two-phase region, the amount of mass rate lost to a well is defined as,

$$q'_m = q'_s + q'_w, \qquad (93)$$

and the total heat rate lost to the well as,

$$q'_h = h'_s h'_s + h'_w h'_w, \qquad (94)$$

where a negative rate indicates a loss from the reservoir. The steam production rate may be determined by the fractional flow of the steam phase on a mass transport basis as follows:

$$q'_s = \alpha_s q'_m, \qquad (95)$$

where

$$\alpha_s = k_{rs}/(k_{rs} + \frac{\rho_w \mu_s}{\rho_s \mu_w} k_{rw}).$$

Since h'_s and h'_w are known functions of pressure and the total mass flux, q'_m, is specified, q'_s is calculated using equation 95, and q'_h is calculated using equation 94.

<center>Boundary Conditions</center>

The combined mass and energy equations 91 and 92 together comprise a pair of nonlinear, second-order, partial-differential equations. As such, two boundary conditions (one in terms of pressure and one in terms of enthalpy) are required at the boundaries. For both variables three types of boundary conditions are admissible. These are as follows:

1) Pressure or enthalpy may be specified at the boundary surface.

2) Spatial derivatives of pressure or enthalpy may be specified at the boundary. These boundary conditions incorporate the flux terms for mass and energy.

3) Certain combinations of pressure and enthalpy and their respective spatial derivatives may be specified at the boundary. This is the so-called mixed boundary condition.

Only a few types of the above boundary conditions are necessary for actual reservoir simulation problems. Perhaps the most common of these is the specification of fluxes at the boundaries. Frequently the flux is specified as zero. For specification of a mass flux,

$$q_m^* = - (\frac{\underline{\underline{k}}k_{rs}\rho_s}{\mu_s} + \frac{\underline{\underline{k}}k_{rw}\rho_w}{\mu_w}) \frac{\partial(p-\rho g D)}{\partial \underline{n}} \quad , \tag{96}$$

where q_m^* is the specified mass flux at the boundary. If a mass flux is specified, a convective energy flux must also be specified according to an equation analogous to equation 94,

$$q_h^* = q_s^* h_s^* + q_w^* h_w^* , \tag{97}$$

where h_s^*, and h_w^* are the pressure-dependent saturated water and steam enthalpies and q_s^* and q_w^* are the fractional steam and water fluxes at the boundary. The total energy flux for the general case consists of two parts,

$$q_{th}^* = q_h^* + q_h^{**} \quad , \tag{98}$$

where q_h^{**} represents the conductive heat flux at the boundary and is

determined by

$$q_h^{**} = - K_r \left. \frac{\partial T}{\partial \underline{n}} \right|_{\text{boundary}} \qquad (99)$$

A constant pressure boundary condition may also be encountered.
Since this implies a mass flux at the boundary, it also implies a
convective-energy flux. To determine the convective-energy flux, the mass
flux is calculated from equation 96 and used in equation 97.

TWO-DIMENSIONAL FORMULATION OF THE GENERAL EQUATIONS

In this section the three-dimensional combined flow and combined energy equations (equations 88 and 90) are partially integrated in the z-dimension. The resulting two-dimensional equations are defined in terms of quantities averaged in the z-dimension. This rigorous development is necessary in order to gain insight into the nature and the adequacy of the assumptions used to obtain the simplified two-dimensional equations.

Basic Rules and Conventions

A quantity averaged in the z-dimension, for the analysis in this section, is given by,

$$<\psi> = \frac{1}{b} \int_{z_1}^{z_2} \psi dz , \tag{100}$$

where $z_1 = z_1(x,y,t)$ is the bottom of the reservoir, $z_2 = z_2(x,y,t)$ is the top, $b = b(x,y,t) = z_2 - z_1$ is the thickness, and the brackets, $< >$, signify a quantity averaged in the z-dimension.

In the development that follows Leibnitz' rule is frequently used to reverse the order of integration and differentiation. This rule has the form,

$$\int_{z_1}^{z_2} \frac{\partial u}{\partial x}\, dz = \frac{\partial}{\partial x} \int_{z_1}^{z_2} u\, dz + u(x,y,z,t) \left|_{z_1} \frac{\partial z_1}{\partial x} - u(x,y,z,t) \right|_{z_2} \frac{\partial z_2}{\partial x} . \qquad (101)$$

Reynolds' operational rule for defining the average, $<\psi>$, and deviation, $\hat{\psi}$, of a quantity, ψ, is given as,

$$\psi = <\psi> + \hat{\psi} . \qquad (102)$$

Throughout the rest of this report the brackets, $< \; >$, and superscript, $\hat{}$, will refer to quantities averaged in the z-dimension and their deviations defined by equations 100 and 102. A graphical illustration of these quantities is provided in figure 4. It then follows that,

$$<\hat{\psi}> = \frac{1}{b} \int_{z_1}^{z_2} (\psi - <\psi>)dz = 0. \qquad (103)$$

Given these definitions, other useful relationships that may be derived are,

$$<\psi + \gamma> = <\psi> + <\gamma>, \qquad (104)$$

$$<<\psi>\cdot\gamma> = <\psi> \cdot <\gamma>, \qquad (105)$$

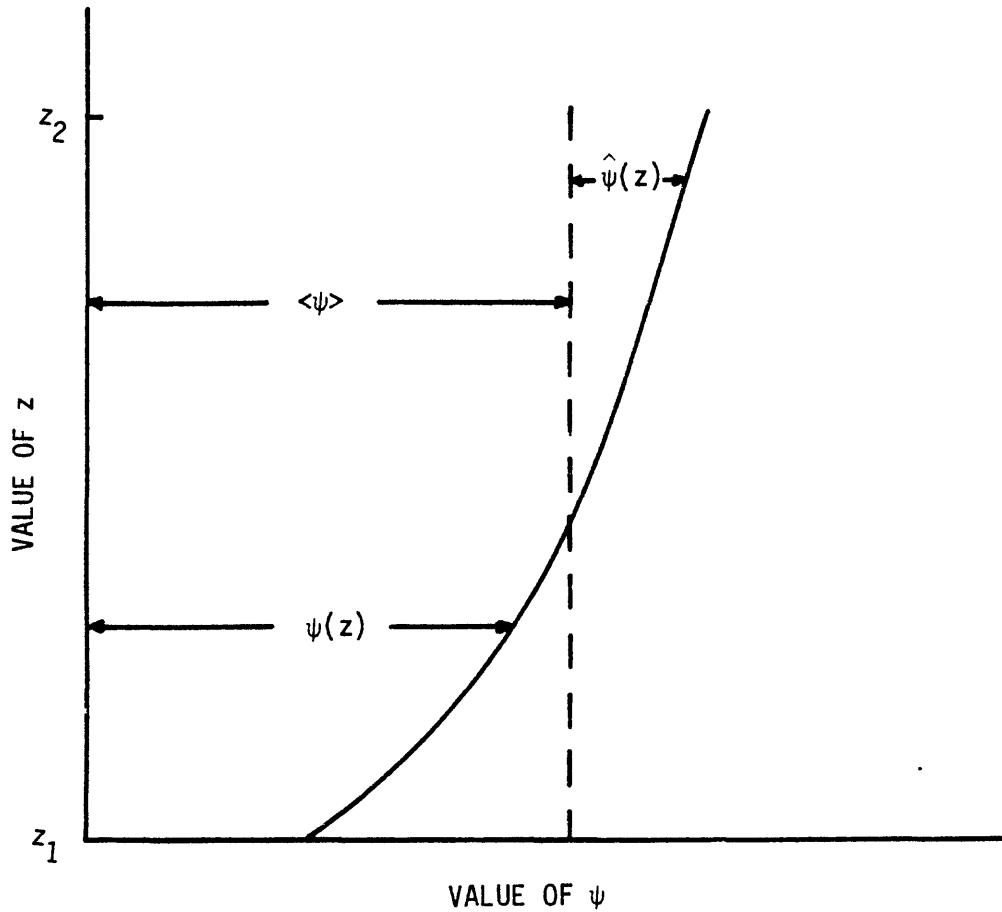$$<\psi\cdot\gamma> = <\psi> \cdot <\gamma> + <\hat{\psi} \cdot \hat{\gamma}>, \qquad (106)$$

Figure 4.--Graphical representation of typical quantity, ψ, as a
function of the z-dimension showing the average value,
$\langle\psi\rangle$ and the deviation, $\hat{\psi}$.

and,

$$< \frac{\partial \psi}{\partial x} > = \frac{\partial <\psi>}{\partial x} \cdot \tag{107}$$

Development of the General Two-Dimensional Equations

If the Cartesian coordinate system is aligned with the principle directions of the permeability tensor, $\underline{k}$, and if the conduction-dispersion, K_m, is assumed to be isotropic, equations 88 and 90 may be written as

$$\frac{\partial(\phi\rho)}{\partial t} - \frac{\partial}{\partial x} \left(\omega_x \frac{\partial p}{\partial x} - \omega_{gx} \frac{\partial D}{\partial x} \right) - \frac{\partial}{\partial y} \left(\omega_y \frac{\partial p}{\partial y} - \omega_{gy} \frac{\partial D}{\partial y} \right)$$

$$- \frac{\partial}{\partial z} \left(\omega_z \frac{\partial p}{\partial z} - \omega_{gz} \frac{\partial D}{\partial z} \right) - q_m' = 0 , \tag{108}$$

and

$$\frac{\partial}{\partial t} \left[\phi\rho h + (1-\phi)\rho_r h_r \right] - \frac{\partial}{\partial x} \left[(\omega_{hx} + \omega_{cp}) \frac{\partial p}{\partial x} - \omega_{hgx} \frac{\partial D}{\partial x} \right]$$

$$- \frac{\partial}{\partial y} \left[(\omega_{hy} + \omega_{cp}) \frac{\partial p}{\partial y} - \omega_{hgy} \frac{\partial D}{\partial y} \right] - \frac{\partial}{\partial z} \left[(\omega_{hz} + \omega_{cp}) \frac{\partial p}{\partial z} \right.$$

$$\left. - \omega_{hgz} \frac{\partial D}{\partial z} \right] - \frac{\partial}{\partial x} \left(\omega_{ch} \frac{\partial h}{\partial x} \right) - \frac{\partial}{\partial y} \left(\omega_{ch} \frac{\partial h}{\partial y} \right) - \frac{\partial}{\partial z} \left(\omega_{ch} \frac{\partial h}{\partial z} \right) - q_h' = 0, \tag{109}$$

where for the x-direction (terms for the y- and z-directions are similar):

$$\omega_x = \frac{k_x k_{rw} \rho_w}{\mu_w} + \frac{k_x k_{rs} \rho_s}{\mu_s} \; ,$$

$$\omega_{hx} = \frac{k_x k_{rw} \rho_w h_w}{\mu_w} + \frac{k_x k_{rs} \rho_s h_s}{\mu_s} \; ,$$

$$\omega_{gx} = \frac{k_x k_{rw} \rho_w g \rho_w}{\mu_w} + \frac{k_x k_{rs} \rho_s g \rho_s}{\mu_s} \; ,$$

$$\omega_{hgx} = \frac{k_x k_{rw} \rho_w g \rho_w h_w}{\mu_w} + \frac{k_x k_{rs} \rho_s g \rho_s h_s}{\mu_s} \; ,$$

$$\omega_{cp} = K_m \left(\frac{\partial T}{\partial p}\right)_h \; ,$$

and,

$$\omega_{ch} = K_m \left(\frac{\partial T}{\partial h}\right)_p \; .$$

Also, note that for this development we have assumed that the compressible-work term is negligible.

Flow Equation

Equation 108 may be integrated in the z-dimension to give,

$$\int_{z_1}^{z_2} \left[\frac{\partial(\phi\rho)}{\partial t} - \frac{\partial}{\partial x}\left(\omega_x \frac{\partial p}{\partial x} - \omega_{gx}\frac{\partial D}{\partial x}\right) - \frac{\partial}{\partial y}\left(\omega_y \frac{\partial p}{\partial y} - \omega_{gy}\frac{\partial D}{\partial y}\right) \right.$$

$$\left. - \frac{\partial}{\partial z}\left(\omega_z \frac{\partial p}{\partial z} - \omega_{gz} \frac{\partial D}{\partial z}\right) - q_m' \right] \; dz = 0. \qquad (110)$$

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Applying Leibnitz' rule to the first term in the integral yields,

$$\int_{z_1}^{z_2} \frac{\partial}{\partial t}(\phi\rho)dz = \frac{\partial}{\partial t}\int_{z_1}^{z_2}\phi\rho dz + \phi\rho\bigg|_{z_1}\frac{\partial z_1}{\partial t} - \phi\rho\bigg|_{z_2}\frac{\partial z_2}{\partial t} , \tag{111}$$

which in terms of averaged quantities becomes,

$$\int_{z_1}^{z_2} \frac{\partial}{\partial t}(\phi\rho)dz = \frac{\partial}{\partial t}(b<\phi\rho>) + \phi\rho\bigg|_{z_1}\frac{\partial z_1}{\partial t} - \phi\rho\bigg|_{z_2}\frac{\partial z_2}{\partial t} . \tag{112}$$

Integration of the x-component term in equation 110 gives,

$$\int_{z_1}^{z_2}[\frac{\partial}{\partial x}(\omega_x\frac{\partial p}{\partial x} - \omega_{gx}\frac{\partial D}{\partial x})]\ dz = \frac{\partial}{\partial x}[b(<\omega_x\frac{\partial p}{\partial x}> - <\omega_{gx}\frac{\partial D}{\partial x}>)]$$

$$+ (\omega_x\frac{\partial p}{\partial x} - \omega_{gx}\frac{\partial D}{\partial x})\bigg|_{z_1}\frac{\partial z_1}{\partial x} - (\omega_x\frac{\partial p}{\partial x} - \omega_{gx}\frac{\partial D}{\partial x})\bigg|_{z_2}\frac{\partial z}{\partial x}z. \tag{113}$$

The first term on the right side of equation 113 may be expanded by using equation 106 to yield,

$$\int_{z_1}^{z_2}[\frac{\partial}{\partial x}(\omega_x\frac{\partial p}{\partial x} - \omega_{gx}\frac{\partial D}{\partial x})]\ dz = \frac{\partial}{\partial x}[b(<\omega_x><\frac{\partial p}{\partial x}> + <\hat{\omega}_x\frac{\partial\hat{p}}{\partial x}> - <\omega_{gx}><\frac{\partial D}{\partial x}>$$

$$- <\hat{\omega}_{gx}\frac{\partial\hat{D}}{\partial x}>)] + (\omega_x\frac{\partial p}{\partial x} - \omega_{gx}\frac{\partial D}{\partial x})\bigg|_{z_1}\frac{\partial z_1}{\partial x} - (\omega_x\frac{\partial p}{\partial x} - \omega_{gx}\frac{\partial D}{\partial x})\bigg|_{z_2}\frac{\partial z_2}{\partial x} . \tag{114}$$

Similarly, the integrated y-component in equation 110 may be written as,

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$$\int_{z_1}^{z_2} \left[\frac{\partial}{\partial y}\left(\omega_y\frac{\partial p}{\partial y} - \omega_{gy}\frac{\partial D}{\partial y}\right) \right] dz = \frac{\partial}{\partial y}\left[b\left(<\omega_y><\frac{\partial p}{\partial y}> + <\hat{\omega}_y\frac{\hat{\partial p}}{\partial y}> - <\omega_{gy}><\frac{\partial D}{\partial y}>\right.\right.$$

$$\left.\left. - <\hat{\omega}_{gy}\frac{\hat{\partial D}}{\partial y}>\right) \right] + \left(\omega_y\frac{\partial p}{\partial y} - \omega_{gy}\frac{\partial D}{\partial y}\right)\Bigg|_{z_1}\frac{\partial z_1}{\partial y} - \left(\omega_y\frac{\partial p}{\partial y} - \omega_{gy}\frac{\partial D}{\partial y}\right)\Bigg|_{z_2}\frac{\partial z_2}{\partial y} \qquad (115)$$

For the z-component term, integration leads to,

$$\int_{z_1}^{z_2} \left[\frac{\partial}{\partial z}\left(\omega_z\frac{\partial p}{\partial z} - \omega_{gz}\frac{\partial D}{\partial z}\right) \right] dz = \left(\omega_z\frac{\partial p}{\partial z} - \omega_{gz}\frac{\partial D}{\partial z}\right)\Bigg|_{z_2} - \left(\omega_z\frac{\partial p}{\partial z} - \omega_{gz}\frac{\partial D}{\partial z}\right)\Bigg|_{z_1}. \qquad (116)$$

Finally, the mass source term may be given as,

$$\int_{z_1}^{z_2} q_m' dz = b<q_m'>. \qquad (117)$$

With all terms in equation 108 considered, the integrated flow equation is,

① ② ③

$$\frac{\partial}{\partial t}(b<\phi\rho>) + (\phi\rho)\Bigg|_{z_1}\frac{\partial z_1}{\partial t} - (\phi\rho)\Bigg|_{z_2}\frac{\partial z_2}{\partial t} - \frac{\partial}{\partial x}\left[b\left(<\omega_x><\frac{\partial p}{\partial x}> + <\hat{\omega}_x\frac{\hat{\partial p}}{\partial x}>\right.\right.$$

④

$$\left.\left. - <\omega_{gx}><\frac{\partial D}{\partial x}> - <\hat{\omega}_{gx}\frac{\hat{\partial D}}{\partial x}>\right) \right] - \left(\omega_x\frac{\partial p}{\partial x} - \omega_{gx}\frac{\partial D}{\partial x}\right)\Bigg|_{z_1}\frac{\partial z_1}{\partial x}$$

⑤ ⑥

$$+ \left(\omega_x\frac{\partial p}{\partial x} - \omega_{gx}\frac{\partial D}{\partial x}\right)\Bigg|_{z_2}\frac{\partial z_2}{\partial x} - \frac{\partial}{\partial y}\left[b\left(<\omega_y><\frac{\partial p}{\partial y}> + <\hat{\omega}_y\frac{\hat{\partial p}}{\partial y}> - <\omega_{gy}><\frac{\partial p}{\partial y}>\right.\right.$$

$$\overset{\textcircled{7}}{} \qquad\qquad \overset{\textcircled{8}}{}$$

$$- <\hat{\omega}_{gy}\tfrac{\partial \hat{D}}{\partial y}>)\,] \; - \; (\omega_y\tfrac{\partial p}{\partial y} - \omega_{gy}\tfrac{\partial D}{\partial y})\Big|_{z_1}\tfrac{\partial z_1}{\partial y} + (\omega_y\tfrac{\partial p}{\partial y} - \omega_{gy}\tfrac{\partial D}{\partial y})\Big|_{z_2}\tfrac{\partial z_2}{\partial y}$$

$$\overset{\textcircled{9}}{} \qquad\qquad \overset{\textcircled{10}}{} \quad \overset{\textcircled{11}}{}$$

$$- (\omega_z\tfrac{\partial p}{\partial z} - \omega_{gz}\tfrac{\partial D}{\partial z})\Big|_{z_2} + (\omega_z\tfrac{\partial p}{\partial z} - \omega_{gz}\tfrac{\partial D}{\partial z})\Big|_{z_1} - b<q'_m> = 0, \tag{118}$$

where the circled numbers above the major terms will be used later in this development for easy reference.

Energy Equation

The integrated combined energy equation (equation 109) is written as,

$$\int_{z_1}^{z_2} \{ \tfrac{\partial}{\partial t} [\, \phi\rho h + (1 - \phi)\rho_r h_r \,] - \tfrac{\partial}{\partial x}[\, (\omega_{hx} + \omega_{cp})\tfrac{\partial p}{\partial x} - \omega_{hgx}\tfrac{\partial D}{\partial x}]$$

$$- \tfrac{\partial}{\partial y}[(\omega_{hy} + \omega_{cp})\tfrac{\partial p}{\partial y} - \omega_{hgy}\tfrac{\partial D}{\partial y}] \; - \tfrac{\partial}{\partial z}[(\omega_{hz} + \omega_{cp})\tfrac{\partial p}{\partial z} - \omega_{hgz}\tfrac{\partial D}{\partial z}]$$

$$- \tfrac{\partial}{\partial x}(\omega_{ch}\tfrac{\partial h}{\partial x}) - \tfrac{\partial}{\partial y}(\omega_{ch}\tfrac{\partial h}{\partial y}) - \tfrac{\partial}{\partial z}(\omega_{ch}\tfrac{\partial h}{\partial z}) - q'_h\}dz = 0 \tag{119}$$

Application of Leibnitz' rule allows the first term in equation 119 to be given as,

$$\int_{z_1}^{z_2}\tfrac{\partial}{\partial t}[\phi\rho h + (1 - \phi)\rho_r h_r] = \tfrac{\partial}{\partial t}[\, b(<\phi\rho h> + <\rho_r h_r> - <\phi\rho_r h_r>)]$$

$$+ \; [\phi\rho h + (1 - \phi)\rho_r h_r] \; \Big|_{z_1} \frac{\partial z_1}{\partial t} - [\phi\rho h + (1 - \phi)\rho_r h_r] \; \Big|_{z_2} \frac{\partial z_2}{\partial t} . \tag{120}$$

The averaged x-components are given by,

$$\int_{z_1}^{z_2} \frac{\partial}{\partial x}(\omega_{hx}\frac{\partial p}{\partial x})dz = \frac{\partial}{\partial x} \; [b(<\omega_{hx}><\frac{\partial p}{\partial x}> + <\hat{\omega}_{hx}\frac{\hat{\partial p}}{\partial x}>)]$$
$$+ \; \omega_{hx}\frac{\partial p}{\partial x}\Big|_{z_1} \frac{\partial z_1}{\partial x} - \omega_{hx}\frac{\partial p}{\partial x}\Big|_{z_2} \frac{\partial z_2}{\partial x} , \tag{121}$$

$$\int_{z_1}^{z_2} \frac{\partial}{\partial x}(\omega_{hgx}\frac{\partial D}{\partial x})dz = \frac{\partial}{\partial x} \; \left[b(<\omega_{hgx}><\frac{\partial D}{\partial x}> + <\hat{\omega}_{hgx}\frac{\hat{\partial D}}{\partial x}>)\right]$$
$$+ \; \omega_{hgx}\frac{\partial D}{\partial x}\Big|_{z_1} \frac{\partial z_1}{\partial x} - \omega_{hgx}\frac{\partial D}{\partial x}\Big|_{z_2} \frac{\partial z_2}{\partial x} , \tag{122}$$

$$\int_{z_1}^{z_2} \frac{\partial}{\partial x}(\omega_{cp}\frac{\partial p}{\partial x})dz = \frac{\partial}{\partial x} \; [b(<\omega_{cp}><\frac{\partial p}{\partial x}> + <\hat{\omega}_{cp}\frac{\hat{\partial p}}{\partial x}>)]$$
$$+ \; \omega_{cp}\frac{\partial p}{\partial x}\Big|_{z_1} \frac{\partial z_1}{\partial x} - \omega_{cp}\frac{\partial p}{\partial x}\Big|_{z_2} \frac{\partial z_2}{\partial x} , \tag{123}$$

and

$$\int_{z_1}^{z_2} \frac{\partial}{\partial x}(\omega_{ch}\frac{\partial h}{\partial x})dz = \frac{\partial}{\partial x} \; [b(<\omega_{ch}><\frac{\partial h}{\partial x}> + <\hat{\omega}_{ch}\frac{\hat{\partial h}}{\partial x}>)] + \omega_{ch}\frac{\partial h}{\partial x}\Big|_{z_1} \frac{\partial z_1}{\partial x}$$
$$- \; \omega_{ch}\frac{\partial h}{\partial x}\Big|_{z_2} \frac{\partial z_2}{\partial x} . \tag{124}$$

Similar expressions for the y-components are,

$$\int_{z_1}^{z_2} \frac{\partial}{\partial y}(\omega_{hy}\frac{\partial p}{\partial y})dz = \frac{\partial}{\partial y}\left[b(<\omega_{hy}><\frac{\partial p}{\partial y}> + <\hat{\omega}_{hy}\frac{\hat{\partial p}}{\partial y}>) \right]$$

$$+ \omega_{hy}\frac{\partial p}{\partial y}\bigg|_{z_1}\frac{\partial z_1}{\partial y} - \omega_{hy}\frac{\partial p}{\partial y}\bigg|_{z_2}\frac{\partial z_2}{\partial y} \quad , \tag{125}$$

$$\int_{z_1}^{z_2} \frac{\partial}{\partial y}(\omega_{hgy}\frac{\partial D}{\partial y})dz = \frac{\partial}{\partial y}\left[b(<\omega_{hgy}><\frac{\partial D}{\partial y}> + <\hat{\omega}_{hgy}\frac{\hat{\partial D}}{\partial y}>) \right]$$

$$+ \omega_{hgy}\frac{\partial D}{\partial y}\bigg|_{z_1}\frac{\partial z_1}{\partial y} - \omega_{hgy}\frac{\partial D}{\partial y}\bigg|_{z_2}\frac{\partial z_2}{\partial y} \quad , \tag{126}$$

$$\int_{z_1}^{z_2} \frac{\partial}{\partial y}(\omega_{cp}\frac{\partial p}{\partial y})dz = \frac{\partial}{\partial y}\left[b(<\omega_{cp}><\frac{\partial p}{\partial y}> + <\hat{\omega}_{cp}\frac{\hat{\partial p}}{\partial y}>) \right]$$

$$+ \omega_{cp}\frac{\partial p}{\partial y}\bigg|_{z_1}\frac{\partial z_1}{\partial y} - \omega_{cp}\frac{\partial p}{\partial y}\bigg|_{z_2}\frac{\partial z_2}{\partial y} \quad , \tag{127}$$

and

$$\int_{z_1}^{z_2} \frac{\partial}{\partial y}(\omega_{ch}\frac{\partial h}{\partial y})dz = \frac{\partial}{\partial y}\left[b(<\omega_{ch}><\frac{\partial h}{\partial y}> + <\hat{\omega}_{ch}\frac{\hat{\partial h}}{\partial y}>) \right]$$

$$+ \omega_{ch}\frac{\partial h}{\partial y}\bigg|_{z_1}\frac{\partial z_1}{\partial y} - \omega_{ch}\frac{\partial h}{\partial y}\bigg|_{z_2}\frac{\partial z_2}{\partial y} \quad . \tag{128}$$

For the z-components,

$$\int_{z_1}^{z_2} \frac{\partial}{\partial z}(\omega_{hz}\frac{\partial p}{\partial z})dz = \omega_{hz}\frac{\partial p}{\partial z}\bigg|_{z_2} - \omega_{hz}\frac{\partial p}{\partial z}\bigg|_{z_1} \quad , \tag{129}$$

$$\int_{z_1}^{z_2} \frac{\partial}{\partial z}(\omega_{hgz}\frac{\partial D}{\partial z})dz = \omega_{hgz}\frac{\partial D}{\partial z}\bigg|_{z_2} - \omega_{hgz}\frac{\partial D}{\partial z}\bigg|_{z_1} \quad , \tag{130}$$

$$\int_{z_1}^{z_2} \frac{\partial}{\partial z}(\omega_{cp}\frac{\partial p}{\partial z})dz = \omega_{cp}\frac{\partial p}{\partial z}\bigg|_{z_2} - \omega_{cp}\frac{\partial p}{\partial z}\bigg|_{z_1} \quad , \tag{131}$$

and

$$\int_{z_1}^{z_2} \frac{\partial}{\partial z}(\omega_{ch}\frac{\partial h}{\partial z})dz = \omega_{ch}\frac{\partial h}{\partial z}\bigg|_{z_2} - \omega_{ch}\frac{\partial h}{\partial z}\bigg|_{z_1} \quad , \tag{132}$$

The averaged source term is given by,

$$\int_{z_1}^{z_2} q_h^! dz = b<q_h^!> . \tag{133}$$

Combining all terms yields

①
$$\frac{\partial}{\partial t}[b(<\phi\rho h> + <\rho_r h_r> - <\phi\rho_r h_r>)] +$$

②
$$\{[\phi\rho h + (1-\phi)\rho_r h_r]\bigg|_{z_1}\frac{\partial z_1}{\partial t}$$

$$- [\phi\rho h + (1-\phi)\rho_r h_r]\bigg|_{z_2}\frac{\partial z_2}{\partial t}\} - $$

③
$$\frac{\partial}{\partial x}[b(<\omega_{hx}><\frac{\partial p}{\partial x}> - <\omega_{hgx}><\frac{\partial D}{\partial x}>$$

④
$$+<\hat{\omega}_{hx}\frac{\partial p}{\partial x}> - <\hat{\omega}_{hgx}\frac{\partial D}{\partial x}>)] - (\omega_{hx}\frac{\partial p}{\partial x} - \omega_{hgx}\frac{\partial D}{\partial x})\bigg|_{z_1}\frac{\partial z_1}{\partial x}$$

⑤
$$+ (\omega_{hx}\frac{\partial p}{\partial x} - \omega_{hgx}\frac{\partial D}{\partial x})\bigg|_{z_2}\frac{\partial z_2}{\partial x} - $$

⑥
$$\frac{\partial}{\partial y}[b(<\omega_{hy}><\frac{\partial p}{\partial y}> - <\omega_{hgy}><\frac{\partial D}{\partial y}>$$

⑦
$$+ <\hat{\omega}_{hy}\frac{\partial p}{\partial y}> - <\hat{\omega}_{hgy}\frac{\partial D}{\partial y}>)] - (\omega_{hy}\frac{\partial p}{\partial y} - \omega_{hgy}\frac{\partial D}{\partial y})\bigg|_{z_1}\frac{\partial z_1}{\partial y}$$

$$+ (\omega_{hy}\frac{\partial p}{\partial y} - \omega_{hgy}\frac{\partial D}{\partial y})\bigg|_{z_2}\frac{\partial z_2}{\partial y} - \frac{\partial}{\partial x}[\,b(<\omega_{cp}><\frac{\partial p}{\partial x}> + <\omega_{ch}><\frac{\partial h}{\partial x}>$$

$$+ <\hat{\omega}_{cp}\frac{\partial p}{\partial x}> + <\hat{\omega}_{ch}\frac{\partial h}{\partial x}>)\,] - (\omega_{cp}\frac{\partial p}{\partial x} + \omega_{ch}\frac{\partial h}{\partial x})\bigg|_{z_1}\frac{\partial z_1}{\partial x}$$

$$+ (\omega_{cp}\frac{\partial p}{\partial x} + \omega_{ch}\frac{\partial h}{\partial x})\bigg|_{z_2}\frac{\partial z_2}{\partial x} - \frac{\partial}{\partial y}[\,b(<\omega_{cp}><\frac{\partial p}{\partial y}> + <\omega_{ch}><\frac{\partial h}{\partial y}>$$

$$+ <\hat{\omega}_{cp}\frac{\partial p}{\partial y}> + <\hat{\omega}_{ch}\frac{\partial h}{\partial y}>)\,] - (\omega_{cp}\frac{\partial p}{\partial y} + \omega_{ch}\frac{\partial h}{\partial y})\bigg|_{z_1}\frac{\partial z_1}{\partial y}$$

$$+ (\omega_{cp}\frac{\partial p}{\partial y} + \omega_{ch}\frac{\partial h}{\partial y})\bigg|_{z_2}\frac{\partial z_2}{\partial y} - (\omega_{hz}\frac{\partial p}{\partial z} - \omega_{hgz}\frac{\partial D}{\partial z})\bigg|_{z_2}$$

$$+ (\omega_{hz}\frac{\partial p}{\partial z} - \omega_{hgz}\frac{\partial D}{\partial z})\bigg|_{z_1} - (\omega_{cp}\frac{\partial p}{\partial z} + \omega_{ch}\frac{\partial h}{\partial z})\bigg|_{z_2}$$

$$+ (\omega_{cp}\frac{\partial p}{\partial z} + \omega_{ch}\frac{\partial h}{\partial z})\bigg|_{z_1} - b<q_h'> = 0, \tag{134}$$

where the major terms are numbered for later reference.

Analysis of the General Two-Dimensional Equations

In order to apply the general, two-dimensional equations developed in the preceding section, it is necessary to specify the boundary conditions at the top and base of the reservoir and make certain assumptions concerning the variations in flow, reservoir properties, and thermodynamic properties normal to the bedding plane.

Boundary Conditions at Top and Base of the Reservoir

The boundary conditions that require evaluation in the flow and energy equations (equations 118 and 134) concern the flux of mass and energy from the confining strata and the change in mass and energy due to the moving boundaries of the confining beds. The terms numbered 2 in equations 118 and 134 account for the changes in mass and energy due to the moving boundaries of the confining beds. In the flow equation the terms numbered 4, 7, and 10 represent the mass flow terms at the base of the reservoir. The x- and y-flow terms (4 and 7) account for the slope of the reservoir bedding and the spatial change in reservoir thickness. The tenth term is the mass flux in the z-direction. Also in equation 118 the terms numbered 5, 8 and 9 are the corresponding mass flux terms at the top of the reservoir.

In the general two-dimensional energy equation (equation 134) energy boundary fluxes occur due to two distinctly different physical processes, conductive and convective transport. The terms numbered 4, 7, 16, and 10,13, 18 represent, respectively, the convection and conduction boundary conditions at the base of the reservoir. For the top of the reservoir terms 5, 8, 15 and 11, 14, 17 are the corresponding convection and conduction boundary conditions. As with the flow equation the x- and y-components (4, 5, 10, 11; 7, 8, 13 and 14) account for the fluxes due to the slope and spatial variations in thickness of the reservoir. If we expand the time derivative in equation 118 and recall that $b = z_2 - z_1$, we obtain,

$$\frac{\partial}{\partial t} (b<\phi\rho>) = b \frac{\partial}{\partial t} (<\phi\rho>) + \frac{\partial z_1}{\partial t} [(\phi\rho)\Big|_{z_1} - <\phi\rho>] + \frac{\partial z_2}{\partial t}[<\phi\rho> - (\phi\rho)\Big|_{z_2}] \quad (135)$$

The first term to the right of the equal sign in this equation is much larger than the other terms, for example if,

b = 100 ft.	$\rho_s\big|_{z_2}$ = 0.05 (steam)
ϕ_i = 0.2	
ρ_i = 0.8 $^g/cm^3$	$\rho_w\big|_{z_2}$ = 0.8 (water)
Δt = 1 year	
Δb = 1 ft.	
$\Delta\phi$ = 0.01	$< \rho >$ = 0.2 (S_w = 0.2)

then,

$$b \frac{\partial}{\partial t} (<\phi\rho>) = \frac{100}{1} (0.2 \times 0.8 - 0.19 \times 0.2) = 12.2$$

and

$$\frac{\partial z_2}{\partial t} [<\phi\rho> - (\phi\rho) \Big|_{z_2}] = \frac{1}{1} (0.19 \times 0.2 - 0.19 \times .05) = 0.0285.$$

The last two terms in 135 are very small and may be neglected. The analogous terms in 134 may also be neglected, thus our equations become,

$$b \frac{\partial}{\partial t} (<\phi\rho>) - \frac{\partial}{\partial x} [b(<\omega_x> <\frac{\partial p}{\partial x}> + <\hat{\omega}_x \frac{\partial \hat{p}}{\partial x}> - <\omega_{gx}> <\frac{\partial D}{\partial x}> - <\hat{\omega}_{gx} \frac{\partial \hat{D}}{\partial x}>)]$$

$$- \frac{\partial}{\partial y} [b(<\omega_y> <\frac{\partial p}{\partial y}> + <\hat{\omega}_y \frac{\partial \hat{p}}{\partial y}> - <\omega_{gy}> <\frac{\partial D}{\partial y}> - <\hat{\omega}_{gy} \frac{\partial \hat{D}}{\partial y}>)]$$

$$- b <q'_m> + \underline{v} \Big|_{z_1} \cdot \nabla (z-z_1) - \underline{v} \Big|_{z_2} \cdot \nabla (z-z_2) = 0, \qquad (136)$$

where,

$$\underline{v}\Big|_{z_1} \cdot \nabla(z-z_1) = - \left(\omega_x \frac{\partial p}{\partial x} - \omega_{gx} \frac{\partial D}{\partial x}\right)\Big|_{z_1} \frac{\partial z_1}{\partial x}$$

$$- \left(\omega_y \frac{\partial p}{\partial y} - \omega_{gy} \frac{\partial D}{\partial y}\right)\Big|_{z_1} \frac{\partial z_1}{\partial y} + \left(\omega_z \frac{\partial p}{\partial z} - \omega_{gz} \frac{\partial D}{\partial z}\right)\Big|_{z_1} \tag{137}$$

and

$$\underline{v}\Big|_{z_2} \cdot \nabla(z-z_2) = - \left(\omega_x \frac{\partial p}{\partial x} - \omega_{gx} \frac{\partial D}{\partial x}\right)\Big|_{z_2} \frac{\partial z_2}{\partial x}$$

$$- \left(\omega_y \frac{\partial p}{\partial y} - \omega_{gy} \frac{\partial D}{\partial y}\right)\Big|_{z_2} \frac{\partial z_2}{\partial y} + \left(\omega_z \frac{\partial p}{\partial z} - \omega_{gz} \frac{\partial D}{\partial z}\right)\Big|_{z_2} \tag{138}$$

and

$$b\frac{\partial}{\partial t}\left[(\langle \phi\rho h\rangle + \langle \rho_r h_r\rangle - \langle \phi\rho_r h_r\rangle)\right] - \frac{\partial}{\partial x}\left[b(\langle \omega_{hx}\rangle\langle\frac{\partial p}{\partial x}\rangle - \langle \omega_{hgx}\rangle\langle\frac{\partial D}{\partial x}\rangle\right.$$

$$+ \langle \hat{\omega}_{hx}\frac{\hat{\partial p}}{\partial x}\rangle + \langle \hat{\omega}_{hgx}\frac{\hat{\partial D}}{\partial x}\rangle)] - \frac{\partial}{\partial y}[b\langle \omega_{hy}\rangle \langle\frac{\partial p}{\partial y}\rangle - \langle \omega_{hgy}\rangle \langle\frac{\partial D}{\partial y}\rangle$$

$$+ \langle \hat{\omega}_{hy}\frac{\hat{\partial p}}{\partial y}\rangle + \langle \hat{\omega}_{hgy}\frac{\hat{\partial D}}{\partial y}\rangle)] - \frac{\partial}{\partial x}[b(\langle \omega_{cp}\rangle \langle\frac{\partial p}{\partial x}\rangle + \langle \omega_{ch}\rangle \langle\frac{\partial h}{\partial x}\rangle$$

$$+ \langle \hat{\omega}_{cp}\frac{\hat{\partial p}}{\partial x}\rangle + \langle \hat{\omega}_{ch}\frac{\hat{\partial h}}{\partial x}\rangle)] - \frac{\partial}{\partial y}[b(\langle \omega_{cp}\rangle \langle\frac{\partial p}{\partial y}\rangle + \langle \omega_{ch}\rangle \langle\frac{\partial h}{\partial y}\rangle$$

$$+ \langle \hat{\omega}_{cp}\frac{\hat{\partial p}}{\partial y}\rangle + \langle \hat{\omega}_{ch}\frac{\hat{\partial h}}{\partial y}\rangle)] - b\langle q_h'\rangle$$

$$+ h\underline{v}\Big|_{z_1} \cdot \nabla(z-z_1) - h\underline{v}\Big|_{z_2} \cdot \nabla(z-z_2) + \underline{\lambda}_m\Big|_{z_1} \cdot \nabla(z-z_1)$$

$$- \underline{\lambda}_m\Big|_{z_2} \cdot \nabla(z-z_2) = 0 \tag{139}$$

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where

$$h\underline{v} \bigg|_{z_1} \cdot \nabla(z-z_1) = - \left(\omega_{hx}\frac{\partial p}{\partial x} - \omega_{hgx}\frac{\partial D}{\partial x}\right) \bigg|_{z_1} \frac{\partial z_1}{\partial x}$$

$$-\left(\omega_{hy}\frac{\partial p}{\partial y} - \omega_{hgy}\frac{\partial D}{\partial y}\right) \bigg|_{z_1} \frac{\partial z_1}{\partial y} + \left(\omega_{hz}\frac{\partial p}{\partial z} - \omega_{hgz}\frac{\partial D}{\partial z}\right) \bigg|_{z_1} \tag{140}$$

and

$$h\underline{v} \bigg|_{z_2} \cdot \nabla(z-z_2) = - \left(\omega_{hx}\frac{\partial p}{\partial x} - \omega_{hgx}\frac{\partial D}{\partial x}\right) \bigg|_{z_2} \frac{\partial z_2}{\partial x}$$

$$-\left(\omega_{hy}\frac{\partial p}{\partial y} - \omega_{hgy}\frac{\partial D}{\partial y}\right) \bigg|_{z_2} \frac{\partial z_2}{\partial y} + \left(\omega_{hz}\frac{\partial p}{\partial z} - \omega_{hgz}\frac{\partial D}{\partial z}\right) \bigg|_{z_2} \tag{141}$$

and

$$\underline{\lambda_m} \bigg|_{z_1} \cdot \nabla(z-z_1) = - \left(\omega_{cp}\frac{\partial p}{\partial x} + \omega_{ch}\frac{\partial h}{\partial x}\right) \bigg|_{z_1} \frac{\partial z_1}{\partial x}$$

$$-\left(\omega_{cp}\frac{\partial p}{\partial y} + \omega_{ch}\frac{\partial h}{\partial y}\right) \bigg|_{z_1} \frac{\partial z_1}{\partial y} + \left(\omega_{cp}\frac{\partial p}{\partial z} + \omega_{ch}\frac{\partial h}{\partial z}\right) \bigg|_{z_1} \tag{142}$$

and finally,

$$\underline{\lambda}_m \Big|_{z_2} \cdot \nabla(z-z_2) = - \left(\omega_{cp}\frac{\partial p}{\partial x} + \omega_{ch}\frac{\partial h}{\partial x} \right) \Big|_{z_2} \frac{\partial z_2}{\partial x}$$

$$- \left(\omega_{cp}\frac{\partial p}{\partial y} + \omega_{ch}\frac{\partial h}{\partial y} \right) \Big|_{z_2} \frac{\partial z_2}{\partial y} + \left(\omega_{cp}\frac{\partial p}{\partial z} + \omega_{ch}\frac{\partial h}{\partial z} \right) \Big|_{z_2} \qquad (143)$$

Averaged Reservoir and Thermodynamic Properties

Equations 136 and 139 are our final equations, however, to be applied, it is necessary to evaluate the vertically integrated terms. In general, these terms may be evaluated under two conditions: (1) the fluids have no segregation, and (2) the fluids are completely segregated.

If the fluids are not segregated, then their properties are uniform throughout the thickness of the reservoir. This implies that the covariance terms (the average of the product of deviations, such as $\langle \hat{\omega}_x \frac{\hat{\partial p}}{\partial x} \rangle$) are zero, and therefore the average value $\langle \omega_x \frac{\partial p}{\partial x} \rangle$ is equal to the quantity $\langle \omega_x \rangle \langle \frac{\partial p}{\partial x} \rangle$. Also, for this condition, laboratory relative permeability and capillary pressure curves may be used in the areal calculation. Thus, this leads to the easiest evaluation of the vertically integrated terms, but for two-phase systems it is also a very restrictive assumption, being limited to very thin reservoirs. However, for single phase reservoirs the uniform property assumption is normally the one used. For such systems equations 136 and 139 (with covariance terms set to zero) offer a good approximation for variable thickness, sloping reservoirs.

A less restrictive condition, and one that conforms with our assumption concerning the absence of significant capillary pressure, is that of complete segregation. Under this condition, it is common to assume vertical equilibrium. This concept was first introduced in the petroleum literature by Coats, Nielsen, Terhune, and Weber (1967) for reservoirs having a large capillary transition zone. It was later modified by Coats, Dempsey, and Henderson

(1971) for reservoirs with a small transition zone, similar to our condition of complete segregation. Details of the vertical equilibrium concept are given in these references and only a brief outline is presented here.

It is assumed when applying the concept of vertical equilibrium that the fluid potentials (p- gD) are uniform throughout the reservoir thickness. This corresponds to a gravity segregated fluid distribution with the potential of each fluid being uniform in the portion of the column occupied by that fluid. This condition requires that the reservoir have good vertical communication (that is, an infinite vertical flow rate). Using this assumption, vertically averaged liquid saturations are related to pressure at some reference level by employing pseudo capillary pressure and pseudo relative permeability curves. Basically, the pseudo function approach gives equaivalent results to those that would be obtained if vertically averaged pressures were used.

For our problem, many of our thermodynamic properties are strongly-dependent functions of pressure, and, an analogous approach would require many pseudo functions. Instead, we use the concept of vertical equilibium to vertically integrate the terms in equations 136 and 139.

For the vertical integration, we need to make certain a priori assumptions on the vertical characteristics of the variables. As exploitation of a reservoir progresses, steam will form in the reservoir. Based on our previous assumptions, the steam and water will separate by gravity segregation producing a steam cap with a water saturation equal to the residual water saturation; below will be water with a water saturation of 1.0 (see figure 5).
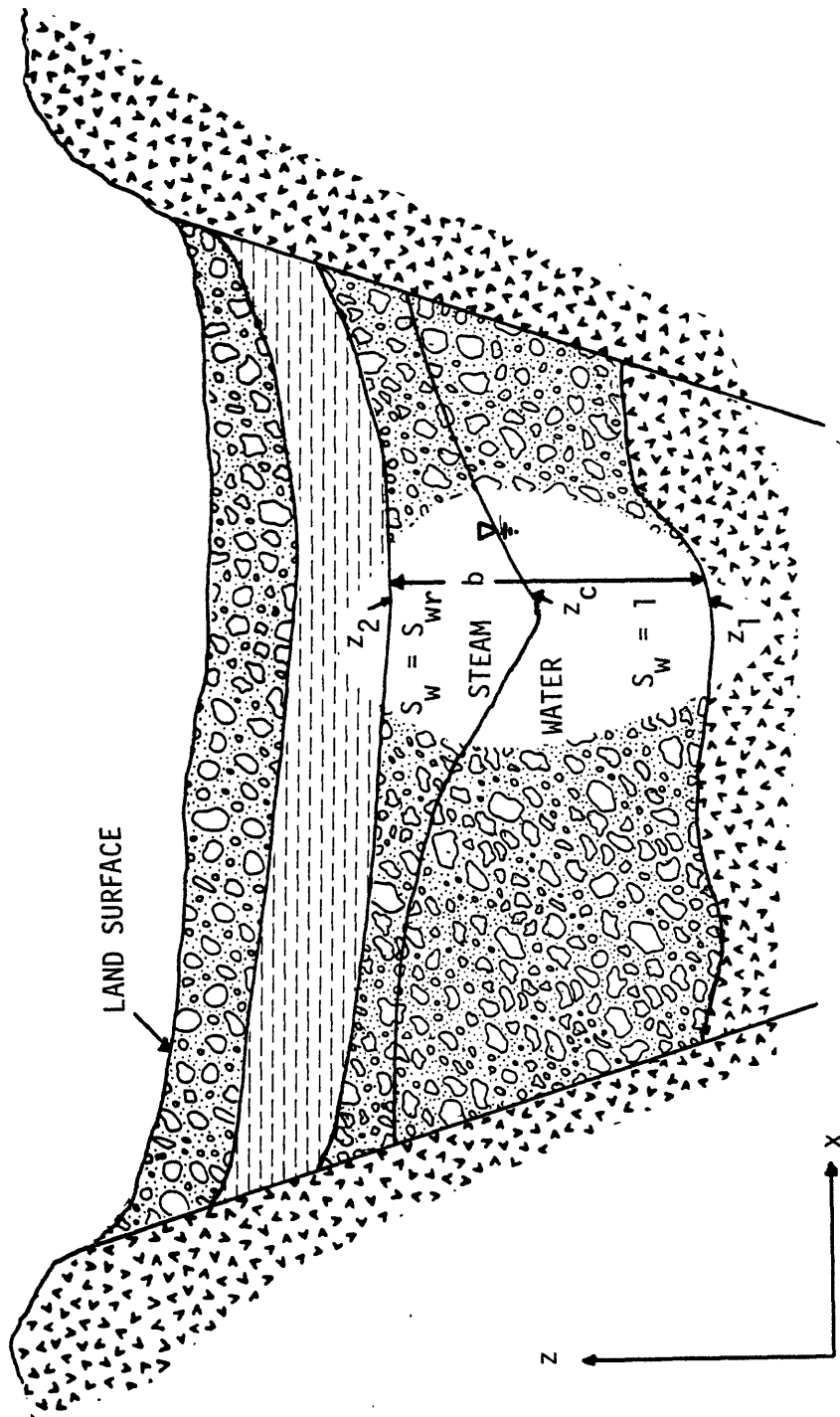
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Figure 5.--Idealized reservoir cross section showing steam cap; z_c is the steam-water contact and S_{wr} is the residual water saturation. Note that the residual steam saturation is assumed to be zero and, therefore, only water exists below the steam cap.

Since the pressure varies hydrostatically in each phase, the vertically averaged pressure distribution is,

$$<p> = [<p_w> z_c + <p_{sw}>(b-z_c)] /b, \qquad (144)$$

which is shown graphically in figure 6. In equation (144) the vertically averaged pressure in the saturated region (below the interface at z_c),$<p_w>$, defined by,

$$<p_w> = p_{z_c} + \rho_w g z_c/2, \qquad (145)$$

and the vertically averaged pressure in the two-phase region (above the interface at z_c),$<p_{sw}>$ is defined by,

$$<p_{sw}> = p_{z_c} - \rho_x g(b-z_c)/2, \qquad (146)$$

where ρ_x is the density in the two-phase region defined by,

$$\rho_x = \rho_s(1-S_{wr}) + \rho_w S_{wr}. \qquad (147)$$

Note that the steam and water densities in equations 145-147 are considered functions of the interface pressure, p_{z_c}.
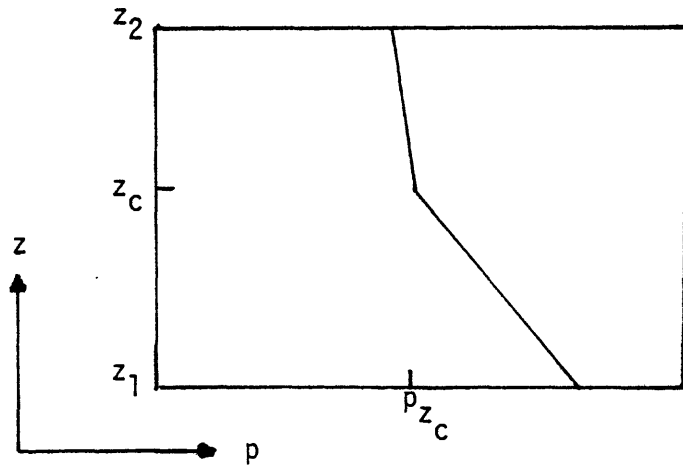
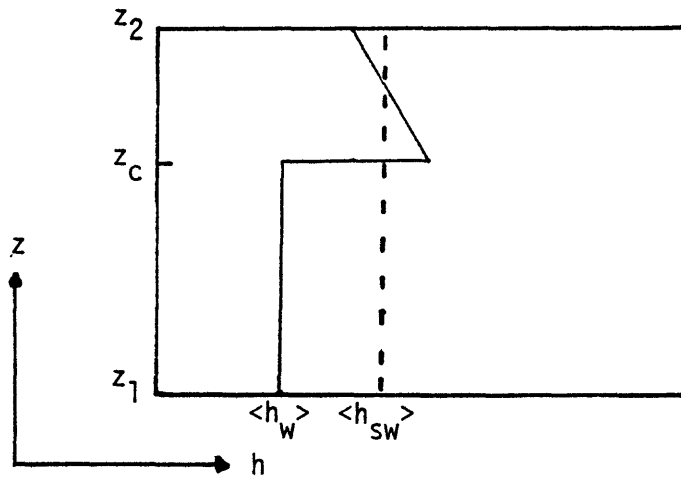Figure 6.--Pressure profile through idealized reservoir; p_{z_c} is the pressure at the steam-water contact.

Figure 7.--Enthalpy profile through idealized reservoir, showing average water enthalpy, $\langle h_w \rangle$, and average steam cap enthalpy, $\langle h_{sw} \rangle$.

The vertically averaged enthalpy distribution is shown in figure 7 and is defined as,

$$<h> = [<h_w> z_c + <h_{sw}> (b-z_c)]/b,$$ (148)

where $<h_w>$ is the vertically averaged enthalpy in the saturated region and is a function of the interface pressure. The vertically averaged enthalpy in the two-phase region is defined by,

$$<h_{sw}> = \frac{\rho_s h_s (1-S_{wr}) + \rho_w h_w S_{wr}}{\rho_s (1-S_{wr}) + \rho_w S_{wr}} ,$$ (149)

where the water and steam densities and enthalpies in equation (149) are considered functions of $<p_{sw}>$.

The assumptions concerining the vertical distribution of pressure and enthalpy expressed explicitly in equations 144-149 permit the determination of all pressure and enthalpy dependent parameters in equations 136 and 139. The procedure used to obtain these parameters is straight forward. Equations 144-149 are solved simultaneously (by Newton-Raphson iteration, for example) to obtain the elevation of the interface contact, z_c, and the fluid pressure, p_{z_c}, at the contact as functions of x-y space. Required for these calculations are the average pressure, $<p>$, and enthalpy, $<h>$, and the top, z_2, and bottom, z_1, elevations of the reservoir also as functions of x-y space. With this information, $<p_{sw}>$ is computed using equation 146;

$<p_w>$ is computed using equation 144; p_{z_c} is computed using equation 145;

$<h_{sw}>$ is computed using equation 149; and z_c is computed using equation 148.

If steam is not present at a point (x,y), the averaged pressure and enthalpy are used to calculate the thermodynamic properties.

Finally, relative permeabilities are given by,

$$<k_{rw}> = [z_c - z_1]/b,$$ (150)

$$<k_{rs}> = (z_2 - z_c)k_{rs}(S_{wr})/b,$$ (151)

where $k_{rs}(S_{wr})$ is the rock relative permeability of steam at the residual water saturation.

CONCLUSIONS

The theoretical analysis of fluid flow and energy transport in hydrothermal systems presented in this report serves two general purposes. Mainly, it provides a better understanding of the implicit and explicit assumptions that are necessary to derive tractable governing equations that describe hydrothermal systems. But as it emphasizes these assumptions it also reveals the need for further work to eliminate weak assumptions.

For two-phase hydrothermal systems the need for experimental data is most evident. Essentially, no suitable experimental work has been done on thermal dispersion in steam-water porous systems. Although some very limited data for relative permeability of steam and water are available they are less than sufficient for general applications. Additional studies of capillary pressure effects and thermal effects on intrinsic permeability also would be useful.

From practical considerations it is necessary to keep the mathematical models as simple (yet realistic) as possible. It is apparent that general analytical solutions to the multiphase equations for hydrothermal systems are not likely to be obtained due to their complex nonlinear nature. Numerical solutions are also difficult. For three-dimensional problems the expense of numerical solutions is often excessive. A rigorous two-dimensional treatment also requires more study. Specifically, alternative assumptions for averaging quantities in the vertical dimension must be investigated and the significance of covariance terms that arise in the partial integration must be determined.

NOMENCLATURE

Upper Case Roman Letters

A	= area
A_{sw}, A_{wr}, A_{sr}	= interphase surface areas
D	= depth
$\underline{\underline{K}}$	= thermal conductivity tensor
$\underline{\underline{K}}_m$	= medium thermal conductivity and dispersion
$\underline{P}$	= particle reference vector
Q_{sw}, Q_{ws}	= interphase vaporization energy terms
Q'_{sw}, Q'_{wr}, Q'_{sr}, Q'_{ws}, Q'_{rw}, Q'_{rs}	= interphase conduction terms
Q''_{sw}, Q''_{sr}, Q''_{ws}, Q''_{wr}	= interphase pressure-work terms
S	= volume saturation
S_{wr}, S_{sr}	= residual volume saturation (water and steam)
T	= temperature
U	= internal energy
V	= volume

Lower Case Roman Letters

b	= reservoir thickness
c	= heat capacity
dv	= mass vaporization term
$\underline{f}_b$	= body force vector
g, $\underline{g}$	= gravitational acceleration constant and gravitational force vector

h	= enthalpy
h'	= injected fluid enthalpy
$\underline{\underline{k}}$	= local intrinsic permeability tensor
k_r	= relative permeability
$\underline{n}$	= unit outward normal vector
p	= pressure
q'	= mass source term
q'_h	= energy source term
q'_m	= total mass source term
q^*_h	= conductive-boundary, energy source term
q^{**}_h	= convective-boundary, energy source term
q^*_{th}	= total boundary energy source term
q'	= vertical conductive energy source term
r	= radial dimension
t	= time
$\underline{v}$	= phase-average velocity
$\underline{w}_{sw}, \underline{w}_{wr}, \underline{w}_{sr}$	= interphase surface velocities
x	= horizontal dimension
y	= horizontal dimension
z	= vertical dimension
z_c	= elevation of steam-water contact
z_1	= base of the reservoir
z_2	= top of the reservoir

Upper Case Greek Letters

$\underline{\Omega}$ = resistance function

Lower Case Greek Letters

α_s = steam mobility ratio

β = vertical reservoir compressibility

γ = example variable

$\underline{\underline{\delta}}$ = identity tensor

ε = ratio of phase volume to total volume

$\underline{\lambda}_c$ = thermal conduction vector

$\underline{\lambda}_d$ = thermal dispersion vector

μ = dynamic viscosity

ρ = density

$\underline{\underline{\sigma}}$ = stress tensor

$\underline{\underline{\tau}}$ = extra stress tensor

ϕ = porosity

ψ = example variable

$\omega_h, \omega_g, \omega_{hg}, \omega_{cp},$ and ω_{ch} = various lumped parameters used in vertical integration

Subscripts

i,j = summation indices

r = rock

s = stream

w = water

x,y, and z = coordinate directions

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Subscripts (cont.)

$-$	= vector quantity
$=$	= tensor quantity

Superscripts

r	= rock
s	= steam
w	= water
$< >$	= average
$\wedge$	= deviation

Other Notation

∇	= gradient operator
$\bullet$	= dot product
$:$	= double dot product

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