THEORETICAL AND EXPERIMENTAL MODAL ANALYSIS OF AN OPERATOR PROTECTION STRUCTURE

Ioan Catalin PERSU[1], Nicolae ENESCU[2], Ion MANEA[3]

Theoretical and experimental modal analysis of a protection structure represent two modern methods of optimizing their dynamic behaviour. The paper presents a method of theoretical modal analysis as well as its validation through experimental modal analysis, methods applied on an operator protection structure destined for self-propelled agricultural technical equipment. These methods are particularly useful for the achievement of the modal model of the protection structure, and by determining its own frequency modes, a complete analysis can be performed in order to optimize it beginning with the testing phases or the incipient testing stages.

Keywords: protection structure, modal analysis, frequency mode, operator.

1. Introduction

The study of structural dynamics is essential for understanding and assessing the dynamic performances of mechanical systems, for evaluating the structural response under specific design conditions and for determining the influence of structural changes on the dynamic response under given or imposed operating conditions. Modal analysis is a powerful tool for identifying the dynamic characteristics of structures, [1].

From the perspective of the dynamic study, any mechanical system can be mathematically described by a system of second order differential equations [2], but the associated mathematical model is complicated and for most systems is impossible to solve, both due to the complexity and to not exactly knowing the mechanisms of interaction and damping, both within the system, as well as between the system and the external environment.

Experimental modal analysis constitutes the set of theoretical and experimental procedures for the achievement of the mathematical model of the system, in terms of modal parameters, starting from the experimental determinations conducted on the mechanical system brought in a controlled vibrational state. Modal analysis is performed based on the own forms of vibration results obtained theoretically and experimentally, [3].

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[1] Eng., Testing Department, INMA Bucharest, Romania, e-mail: persucatalin@yahoo.com
[2] Prof., University POLITEHNICA of Bucharest, Romania, e-mail: nae_enescu@yahoo.com
[3] Prof., S.C Softronic, Craiova e-mail: ion.manea@softronic.ro

Through experimental modal analysis, the exact response of the system at the points where the excitation was applied in the experiment phase, or where the response was determined, can be determined, [4]. Based on this answer, an interpolation of the structural response can also be made at other points of interest.

Modal analysis can be performed both by analytical techniques (using finite element analysis), [5] [6] and by experimental techniques, both types of analysis practically leading to the same modal parameters. Theoretical or experimental modal models are not actual models of the system, but are rather models of the dynamic behaviour of the system constrained by a set of hypotheses and limit conditions.

The usual analytical model is elaborated through the finite element method (FEA), and the associated mathematical model assumes a set of coupled differential equations, which can only be solved by using advanced computing techniques, [7] [8].

2. Paper contents

Modal analysis with finite elements of an operator protection structure

SolidWorks software was used to elaborate the geometric model of the protection structure, and the geometric model elaborated in SolidWorks constituted the entry data for Ansys 15.1. The mesh of the cabin structure was achieved, the geometric model thus resulting being composed of 169124 knots and 150887 elements.

The geometric model was developed so that continuity in the network with finite elements is achieved, for the control of the mesh the control option for the size and types of finite elements being selected in the program.

Figure 1 shows the geometric model of the operator protection structure that was used in the subsequent analyses.
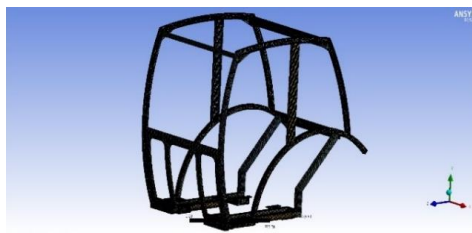

Fig.1. Geometric model of the operator protection structure

The modal analytical analysis was conducted with the *Modal* mode of the Ansys program, the frequency analysis being performed in the range of 20 - 100Hz. The modal shapes resulted by finite element analysis, FEA, were identified through the method mentioned above, a number of 11 independent modes of vibration being identified. To avoid erroneous results caused by thin

sheet elements, the FEM model presented contains only beams. Figure 2 shows the protection structure in the two extreme positions of a complete vibration cycle, for the first mode.
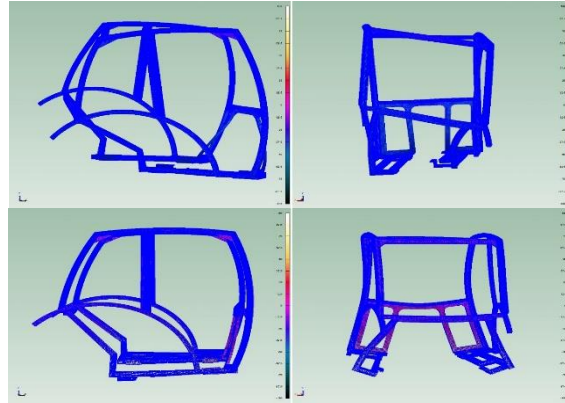

Fig.2. Anays – Protection structure in vibration mode 1 mode at a frequency of 26.739 Hz

To analyse the interdependence of the modal forms of the structure in its own modes, a correlation analysis of the analytical model made in Ansys was performed. Figure 3 shows the result of the MAC (*Modal Assurance Criterion*) analysis. From the analysis of the values of the terms in the MAC matrix it is observed that the modes are totally independent, the terms on the diagonal having the value 1 and the extra diagonal terms being practically 0.
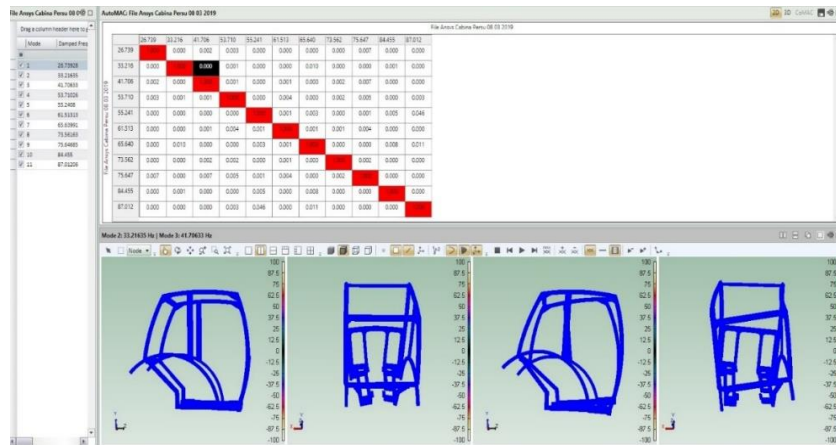

Fig.3. Correlation analysis of Anays own modes for the protection structure

The most general mathematical model is one in which the elements of the mass, rigidity, and damping matrices are estimated based on the measurement of excitation forces and vibrational response. The mathematical model is based on a

system of differential equations adapted to the computation domain, which can be the time domain or the frequency domain.

The general form for time domain is:

$$[M]\{\ddot{x}(t)\}+[C]\{\dot{x}(t)\}+[K]\{x(t)\}=\{F(t)\} \tag{1}$$

The general form for the frequency range is:

$$-\omega^2 [M]\{X(\omega)\}+i\omega[C]\{X(\omega)\}+[K]\{X(\omega)\}=\{Q(\omega)\} \tag{2}$$

For most mathematical models describing the dynamics of an elastic system with N degrees of freedom, the structural response can be arranged in a convenient form, so that it is expressed by the following relations defined in the time or frequency domains:

for the time domain:

$$h_{pq}(t) = \sum_{r=1}^{N} A_{pqr} e^{\lambda_r t} + A_{pqr}^* e^{\lambda_r^* t} \tag{3}$$

for the frequency domain::

$$H_{pq}(\omega) = \sum_{r=1}^{N} \frac{A_{pqr}}{i\omega - \lambda_r} + \frac{A_{pqr}^*}{i\omega - \lambda_r^*} \tag{4}$$

where the notations were used:

ω – pulsation (rad/sec);

p – the degree of freedom measured as response;

q –the reference degree of freedom;

r – the order of the mode;

N – number of frequencies included in the analysis;

A_{pqr} – residue for the r order mode;

$$A_{pqr} = Q_r \Psi_{pr} \Psi_{qr} \tag{5}$$
$$A_{pqr} = \Psi_{pr} L_{qr} \tag{6}$$

Q_r – complex modal coefficient of scaling in the r order mode;

Ψ_{pr} – p component of the complex vector of modal displacements in the r order mode;

L_{qr} – factor of modal participation of the reference degree of freedom q in the r order mode;

λ_r – r order pole;

$$\lambda_r = \mu_r + i v_{d,r} \qquad cu \qquad \mu_r < 0 \quad si \quad v_{d,r} > 0 \tag{7}$$

$v_{d,r}$- the damped natural pulsation of the r order mode, it is always positive and is responsible for the oscillatory properties of the system;

μ_r- the damping factor of the r order mode, or exponential drop rate, it is always negative and is responsible for the dissipation of energy in the external environment and for the damping of free oscillations.

Considering, by similarity with the viscous damping system, with a single degree of freedom, defining the natural non-damped pulse, v_n, the damping factor, μ, and the critical damping ratio, ζ through the relations:

$$v_n = \sqrt{k/m} \text{ - natural undamped pulsation} \tag{8}$$
$$\mu = c/2m \text{ - damping factor} \tag{9}$$
$$\zeta = \mu/v_n = \mu\sqrt{k/m} \text{ - critical damping ratio} \tag{10}$$

The following connection relation between the damped natural frequency and the undamped natural frequency of the r order mode results:

$$v_{d,r} = v_{n,r}\sqrt{1 - \zeta^2} \text{ - natural damped pulsation of } r \text{ order mode} \tag{11}$$

where:

$v_{n,r}$ - natural undamped pulsation of r order mode.

By similarity, the notion of damped / undamped natural frequency is introduced:

$$f_{d/n} = \frac{v_{d/n}}{2\pi} \tag{12}$$

In the experimental modal analysis, adequate techniques are used to bring the analysed system into a controlled vibrational state and by experimentally determining the excitation conditions and the response in accelerations, speeds or displacements, it is sought to obtain, through functions, the dynamic response of the unit impulse (relation 3) or through frequency response functions (relation 4). The vast majority of modal parameter estimation techniques assume that the investigated system is linear and invariable over time. In reality, depending on the system analysed and on the conditions under which the excitation was performed, these hypotheses are valid to a greater or lesser extent, but are almost never fully true.

Experimental modal analysis of the protection structure

The experiments were conducted, on an operator protection structure that was not equipped with windows. In order to correctly apply the experimental modal analysis techniques, the aim was to ensure pseudo-free vibration conditions of the cabin, this being suspended on rubber pads that ensure a sufficiently high elasticity to be considered that, at the low level of the vibrational movements, the cabin can be considered as a free system, without restricting the degrees of freedom.

For the excitation of the cab structure, the single point excitation technique was employed, using an 086D20 type impact hammer with a mass of 1.1 kg, the excitation being achieved by the successive application of force impulses in the vertical, transversal and longitudinal directions.

The paper presents the case when for estimation the method Rational Fraction Polynomial-Z (RFP-Z) was selected and the frequency range was set at 20 - 100 Kz.

Figure 4 shows the assembly for conducting the modal analysis of the protective structure, as well as a detail on the positioning of accelerometers.

In the *Structural Dynamic Test Consultants (SDTC)* module, running under the *PULSE LabShop* platform, the 3D geometric model of the protection structure was made, by importing the geometric model made in Ansys, selecting the representative points for the structure, in order to obtain a correct animation. The measurement sequences were designed on the geometric model thus achieved, taking into account the fact that there are 6 input channels in the LAN-XI data acquisition modules, 5 accelerometers and an 086D20 impact hammer.

Fig.4. Assembly for conducting the modal analysis experiments of the tractor cab. Detail on positioning the accelerometers for measuring the vibrational response

Figure 5 presents the geometric model and the measurement sequences designed in SDTC for the modal analysis of the operator protection structure. The excitation of the structure was performed in point 10 on the geometric model, located at the base of the cab, successively in the vertical, transversal and longitudinal directions. The excitation points and directions are represented by the three hammers, and the vibrational response measurement points are represented by arrows.
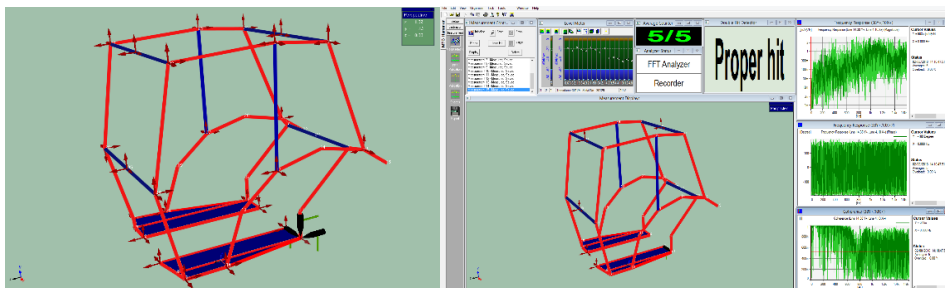

Fig.5. Measurement sequences designed in SDTC for the modal analysis of the tractor cab

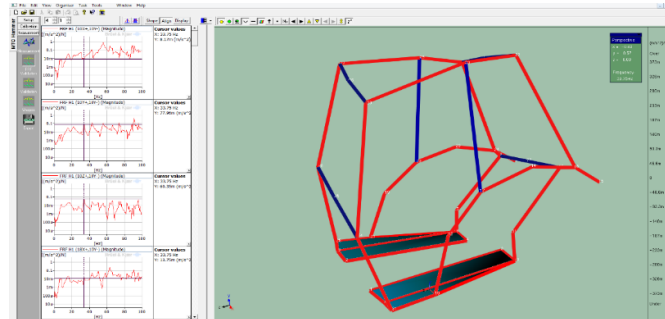Fig. 6 shows the validation sequence of the data acquired under SDTC.

Fig.6 Validation of the data acquired under SDTC for the experimental modal analysis of the protection structure

Fig. 7 shows the data acquisition technique used to apply the modal analysis under SDTC-PulseLabshop.

Fig.7 Data acquisition equipment, during a measurement sequence under SDTC

Fig. 8 shows the mode selection panel from Puls Reflex - Modal Analysis. In the upper left side, the Stability Diagram is shown in which the stable modes are presented by red rhombus, corresponding to each iteration cycle. Stable modes are aligned along vertical lines, corresponding to modal frequencies.
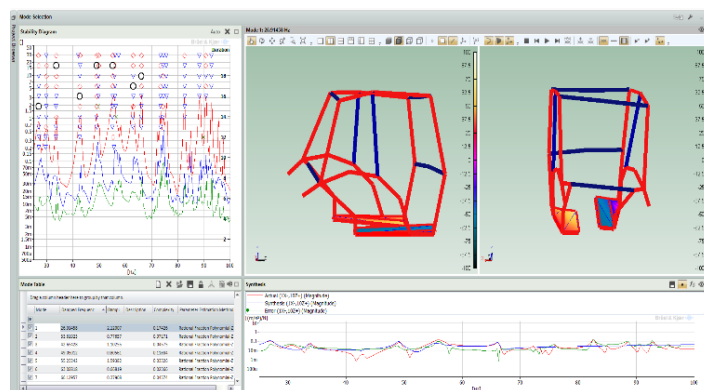

Fig.8. Mode selection panel of the PulseReflex mode - Modal Analysis

Each mode is individually analysed, on the top right side being represented the animation of the structure in the selected vibration mode, and the graph on the lower right side showing the synthesized response.

In the paper, the selection of the final modes remaining for post-analysis (correlation analysis) was made by comparison with the analytical model with finite elements. In the paper, in the experimental modal model, a number of 5 modes were selected.

Modal Assurance Criterion (MAC) is a statistical indicator that, based on the analysis of the vector differences between the corresponding points of the analysed modal models, establishes the degree of correlation, or similarity of the modal forms. MAC presents, in matrix form, the results of the correlation calculation in the form of indicators with values between 0 and 1, where values close to 0 indicate that the modes are not correlated, and the values close to 1 indicate very close modal forms (1 for identical modal forms).

Fig. 9 shows the analysis panel of the experimental modal model of the protection structure. In the table on the left side, the remaining modes for analysis are presented, and in the upper right side, the matrix of MAC indicators is presented. It is observed that the terms on the diagonal have the value 1 and the extra-diagonal terms have values very close to zero, indicating that the modes are totally independent, and the experimental modal model is a correctly determined one. This is, moreover, a confirmation of the correctness of performing the experimental modal analysis experiment.
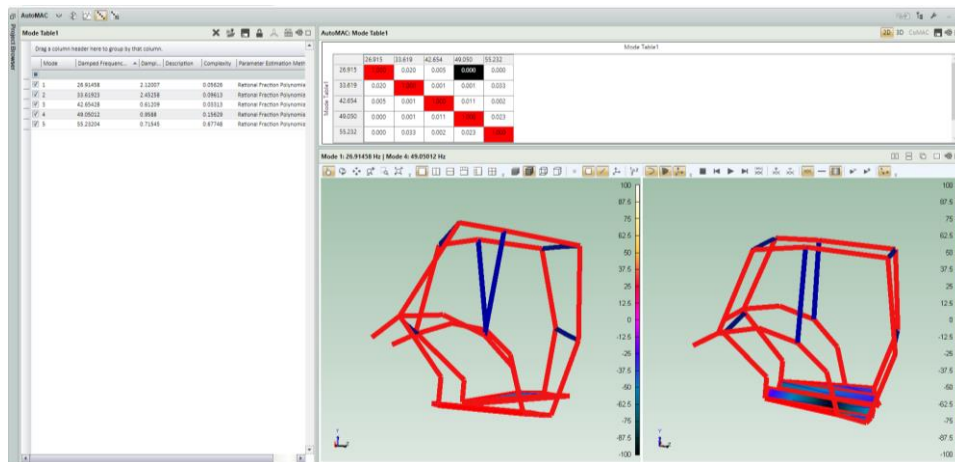

Fig.9 Analysis of the experimental modal model of the protection structure

Table 1 presents the modal parameters of the experimental model, for each mode and for each vibrational response point.

Modal parameters of the protection structure determined using the
Rational Fraction Polynomial-Z analysis method

Order Mode	Unamortized frequency, f_n (Hz)	Amortized frequency, f_d (Hz)	Damping report, ζ (%)	Complexity
1	26.9206	26.9146	2.1201	0.0510
2	33.6293	33.6192	2.4526	0.0434
3	42.6551	42.6543	0.6121	0.0575
4	49.0524	49.0501	0.9588	0.0821
5	55.2335	55.2320	0.7154	0.0079

Figure 10 shows the modal form associated with the first own mode of vibration selected for post-analysis (the correlation analysis).
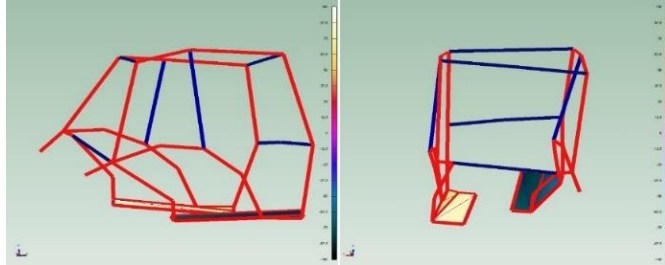

Fig.10. Reflex – Protection structure in vibration mode 1 at the frequency of 26.914 Hz

Correlation analysis of the analytical and experimental models

The correlation analysis is a powerful tool for verifying and validating the analytical modal model, achieved with finite elements, through real data provided by the system through a procedure of identifying the experimental modal model, similar to the analytical modal model, and of correlating the corresponding modal parameters. In the paper, the experimental modal model is elaborated through experimental modal analysis, and the parameters to correlate are: the undamped natural frequencies and the modal forms.

In this paper, *PULSE Reflex Correlation Analysis* mode was used for conducting the correlation, which is a post-processing application that allows the correlation of two modal models, that can be of the same type, or of different types such as an analytical model with finite elements and another one obtained by experimental modal analysis. The program allows the import of models with finite elements. The correlation is achieved through an intuitive workflow, which consists in aligning the geometries, mapping the degrees of freedom, graphical and vector comparison through MAC analysis and analysis of the pairs of modes.

Fig. 11 shows the mapping of the degrees of freedom of the modal model with finite elements, FEM, and the experimental modal model, EMA, of the protection structure. The experimental model (with the red trace) follows the analytical model
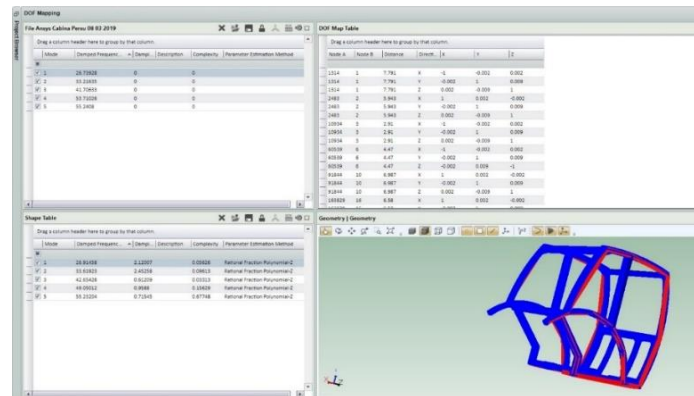
Fig11. Mapping the degrees of freedom of the FEM and EMA
models of the protective structure

The vector comparison is made by calculating the *AutoMAC* and *CrossMAC* tables, similar to the experimental modal analysis, and presenting the results in 2D or 3D dimensional table form. *AutoMAC* correlates the model with itself, and *CrossMAC* correlates two different models, analytical and experimental.

AutoMAC and *CrossMAC* offer the possibility for quantitative and qualitative comparison of all possible combinations of the modal shapes. The value 1.0 corresponds to the pairs of modes with identical modal forms (totally correlated), and the value 0 corresponds to those pairs of modes that are totally independent.

For the correlation analysis, the first five own modes were selected. From the analysis of the correlation matrix it is observed that the first three own modes, both analytical and experimental, are very well correlated, having a MAC correlation index greater than 0.6. Mode 4 has a correlation index of 0.5, but is considered an acceptable mode. Starting with mode 5, it can be considered that the analytical model is no longer correlated with the experimental model.

Figure 12 shows the *CrossMAC* vector comparison panel for the FEM and EMA models of the protection structure.
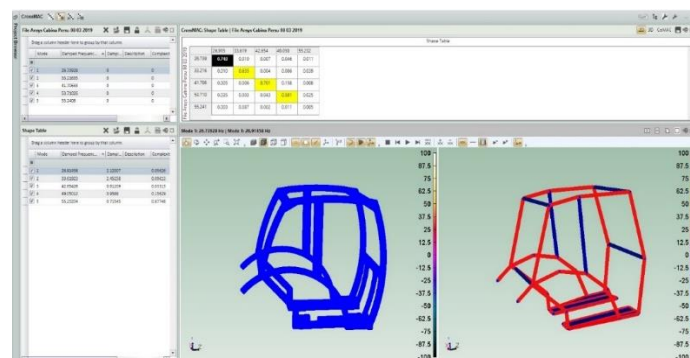

Fig.12 *CrossMAC* vector comparison of the FEM and EMA
models of the protection structure

Table 2 presents the result of the correlation analysis for the analytical, FEA, and experimental, EMA, models of the protection structure.

The FEA model is characterized only by the natural undamped frequency. The EMA model is characterized by the natural undamped frequency, the depreciation ratio, as well as the complexity of the modes.

Table 2

The result of the correlation analysis for the FEM and EMA models of the protection structure

Ord. mode	FEA Model Undamped frequency, f_n (Hz)	EMA Model Undamped frequency, f_n (Hz)	Damped frequency, f_d (Hz)	Damping report, ζ (%)	Complexity	CrossMAC	Undamped frequency error (%)
1	26.739	26.920	26.914	2.120	0.051	0.748	0.674
2	33.216	33.629	33.619	2.452	0.043	0.635	1.228
3	41.706	42.655	42.654	0.612	0.057	0.701	2.225
4	53.710	49.052	49.050	0.958	0.082	0.501	-9.495
5	55.240	55.233	55.232	0.715	0.007	-	-0.011

Figure 13 shows, by comparison, the forms of the model associated to the FEA and EMA modal models in their own vibration modes. On the left side are presented the modal FEA forms, and on the right, the EMA modal forms.
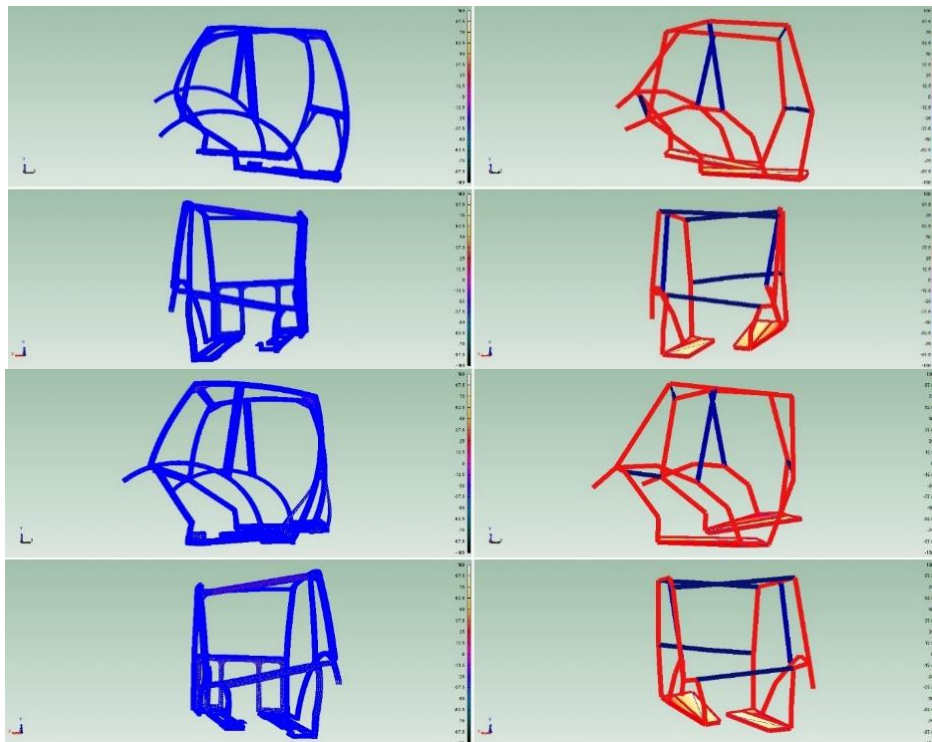

Fig. 13. Vibration mode 1. FEA: 26.739 Hz; EMA: 26.9206 Hz, CrossMac: 0.748

Conclusions

The correlation of the modal forms is done by vector statistical analysis and by calculating the AutoMAc and CrossMAC coefficients of the FEM and EMA models.

From the analysis of the correlation matrix and the modal forms, the following conclusions are drawn:

- For the first three own modes, the evaluation error of the undamped frequency is below the value of 2.5%, and the CrossMAC correlation index is greater than 0.63.

- For mode 4, the evaluation error of the undamped frequency is below the value of 9.5%, and the CrossMAC correlation index greater than 0.5.

- Starting with mode 5, it can be considered that the modes are no longer correlated, although the evaluation error of the undamped frequencies is relatively small.

- Taking into account the significance of the vector correlation analysis and the fact that the experimental model is mainly built on the measured vibrational response points from the stage of applying the experimental modal analysis, it is natural that from a certain frequency, the experimental modal forms are no longer correlated with the analytical modal forms.

R E F E R E N C E S

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