

THEORY OF PLATES AND SHELLS

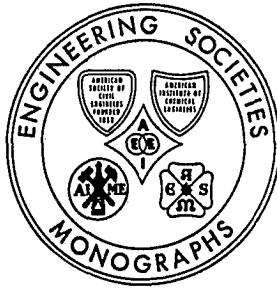
S. TIMOSHENKO

*Professor Emeritus of Engineering Mechanics
Stanford University*

S. WOINOWSKY-KRIEGER

*Professor of Engineering Mechanics
Laval University*

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