

7-8 Recursive Formulas

Find the first five terms of each sequence.

1. $a_1 = 16, a_n = a_{n-1} - 3, n \geq 2$

SOLUTION:

Use $a_1 = 16$ and the recursive formula to find the next four terms.

$$\begin{aligned} a_2 &= a_{2-1} - 3 & n=2 \\ &= a_1 - 3 & \text{Simplify} \\ &= 16 - 3 \text{ or } 13 & a_1=16 \end{aligned}$$

$$\begin{aligned} a_3 &= a_{3-1} - 3 & n=3 \\ &= a_2 - 3 & \text{Simplify} \\ &= 13 - 3 \text{ or } 10 & a_2=13 \end{aligned}$$

$$\begin{aligned} a_4 &= a_{4-1} - 3 & n=4 \\ &= a_3 - 3 & \text{Simplify} \\ &= 10 - 3 \text{ or } 7 & a_3=10 \end{aligned}$$

$$\begin{aligned} a_5 &= a_{5-1} - 3 & n=5 \\ &= a_4 - 3 & \text{Simplify} \\ &= 7 - 3 \text{ or } 4 & a_4=7 \end{aligned}$$

The first five terms are 16, 13, 10, 7, and 4.

7-8 Recursive Formulas

2. $a_1 = -5, a_n = 4a_{n-1} + 10, n \geq 2$

SOLUTION:

Use $a_1 = -5$ and the recursive formula to find the next four terms.

$$\begin{aligned} a_2 &= 4a_{2-1} + 10 & n=2 \\ &= 4a_1 + 10 & \text{Simplify.} \\ &= 4(-5) + 10 \text{ or } -10 & a_1 = -5 \end{aligned}$$

$$\begin{aligned} a_3 &= 4a_{3-1} + 10 & n=3 \\ &= 4a_2 + 10 & \text{Simplify.} \\ &= 4(-10) + 10 \text{ or } -30 & a_2 = -10 \end{aligned}$$

$$\begin{aligned} a_4 &= 4a_{4-1} + 10 & n=4 \\ &= 4a_3 + 10 & \text{Simplify.} \\ &= 4(-30) + 10 \text{ or } -110 & a_3 = -30 \end{aligned}$$

$$\begin{aligned} a_5 &= 4a_{5-1} + 10 & n=5 \\ &= 4a_4 + 10 & \text{Simplify.} \\ &= 4(-110) + 10 \text{ or } -430 & a_4 = -110 \end{aligned}$$

The first five terms are $-5, -10, -30, -110$, and -430 .

Write a recursive formula for each sequence.

3. $1, 6, 11, 16, \dots$

SOLUTION:

Subtract each term from the term that follows it.

$$6 - 1 = 5; 11 - 6 = 5, 16 - 11 = 5$$

There is a common difference of 5. The sequence is arithmetic.

Use the formula for an arithmetic sequence.

$$a_n = a_{n-1} + d \quad \text{Recursive formula for arithmetic sequence.}$$

$$a_n = a_{n-1} + 5 \quad d = 5$$

The first term a_1 is 1, and $n \geq 2$.

A recursive formula for the sequence $1, 6, 11, 16, \dots$ is $a_1 = 1, a_n = a_{n-1} + 5, n \geq 2$.

7-8 Recursive Formulas

4. 4, 12, 36, 108, ...

SOLUTION:

Subtract each term from the term that follows it.

$$12 - 4 = 8; 36 - 12 = 24, 108 - 36 = 72$$

There is no common difference. Check for a common ratio by dividing each term by the term that precedes it.

$$\frac{12}{4} = 3; \frac{36}{12} = 3; \frac{108}{36} = 3$$

There is a common ratio of 3. The sequence is geometric.

Use the formula for a geometric sequence.

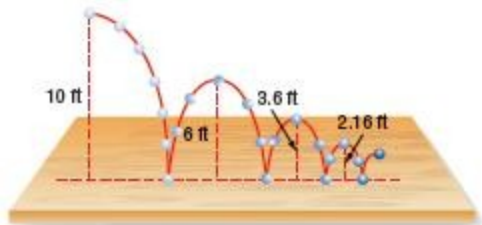
$$a_n = ra_{n-1} \quad \text{Recursive formula for geometric sequence.}$$

$$a_n = 3a_{n-1} \quad r = 3$$

The first term a_1 is 4, and $n \geq 2$. A recursive formula for the sequence 4, 12, 36, 108, ... is $a_1 = 4, a_n = 3a_{n-1}, n \geq 2$.

7-8 Recursive Formulas

5. **BALL** A ball is dropped from an initial height of 10 feet. The maximum heights the ball reaches on the first three bounces are shown.
- Write a recursive formula for the sequence.
 - Write an explicit formula for the sequence.



SOLUTION:

- a. The sequence of heights is 10, 6, 3.6, and 2.16. Subtract each term from the term that follows it.

$$6 - 10 = -4; 3.6 - 6 = -2.4, 2.16 - 3.6 = -1.44$$

There is no common difference. Check for a common ratio by dividing each term by the term that precedes it.

$$\frac{6}{10} = 0.6; \frac{3.6}{6} = 0.6; \frac{2.16}{3.6} = 0.6$$

There is a common ratio of 0.6. The sequence is geometric.

Use the formula for a geometric sequence.

$$a_n = ra_{n-1} \quad \text{Recursive formula for geometric sequence.}$$

$$a_n = 0.6a_{n-1} \quad r = 0.6$$

The first term a_1 is 10, and $n \geq 2$. A recursive formula for the sequence 10, 6, 3.6, and 2.16, ... is $a_1 = 10$, $a_n = 0.6a_{n-1}$, $n \geq 2$.

- b. Use the formula for the n th terms of a geometric sequence.

$$a_n = a_1 r^{n-1} \quad \text{Formula for the } n\text{th term}$$

$$= 10(0.6)^{n-1} \quad a_1 = 10 \text{ and } r = 0.6$$

The explicit formula is $a_n = 10(0.6)^{n-1}$.

7-8 Recursive Formulas

For each recursive formula, write an explicit formula. For each explicit formula, write a recursive formula.

6. $a_1 = 4, a_n = a_{n-1} + 16, n \geq 2$

SOLUTION:

The common difference is 16.

Use the formula for the n th terms of an arithmetic sequence.

$$\begin{aligned} a_n &= a_1(n-1)d && \text{Formula for the } n\text{th term} \\ &= 4 + (n-1)16 && a_1 = 4 \text{ and } d = 16 \\ &= 4 + 16n - 16 && \text{Distributive Property} \\ &= 16n - 12 && \text{Simplify} \end{aligned}$$

The explicit formula is $a_n = 16n - 12$.

7. $a_n = 5n + 8$

SOLUTION:

Write out the first 4 terms. 13, 18, 23, 28

Subtract each term from the term that follows it.

$$18 - 13 = 5; 23 - 18 = 5; 28 - 23 = 5$$

There is a common difference of 5. The sequence is arithmetic.

Use the formula for an arithmetic sequence.

$$\begin{aligned} a_n &= a_{n-1} + d && \text{Recursive formula for arithmetic sequence.} \\ a_n &= a_{n-1} + 5 && d = 5 \end{aligned}$$

The first term a_1 is 13, and $n \geq 2$. A recursive formula for the explicit formula $a_n = 5n + 8$ is $a_1 = 13, a_n = a_{n-1} + 5, n \geq 2$.

8. $a_n = 15(2)_{n-1}$

SOLUTION:

Write out the first 4 terms. 15, 30, 60, 120

Subtract each term from the term that follows it.

$$30 - 15 = 15; 60 - 30 = 30; 120 - 60 = 60$$

There is no common difference. Check for a common ratio by dividing each term by the term that precedes it.

$$\frac{30}{15} = 2; \frac{60}{30} = 2; \frac{120}{60} = 2$$

There is a common ratio of 2. The sequence is geometric.

Use the formula for a geometric sequence.

$$\begin{aligned} a_n &= ra_{n-1} && \text{Recursive formula for geometric sequence.} \\ a_n &= 2a_{n-1} && r = 2 \end{aligned}$$

The first term a_1 is 15, and $n \geq 2$. A recursive formula for the explicit formula $a_n = 15(2)^{n-1}$ is $a_1 = 15, a_n = 2a_{n-1}, n \geq 2$.

7-8 Recursive Formulas

9. $a_1 = 22, a_n = 4a_{n-1}, n \geq 2$

SOLUTION:

The common ratio is 4.

Use the formula for the n th terms of a geometric sequence.

$$\begin{aligned} a_n &= a_1 r^{n-1} && \text{Formula for the } n\text{th term} \\ &= 22(4)^{n-1} && a_1 = 22 \text{ and } r = 4 \end{aligned}$$

The explicit formula is $a_n = 22(4)^{n-1}$.

Find the first five terms of each sequence.

10. $a_1 = 23, a_n = a_{n-1} + 7, n \geq 2$

SOLUTION:

Use $a_1 = 23$ and the recursive formula to find the next four terms.

$$\begin{aligned} a_2 &= a_{2-1} + 7 && n = 2 \\ &= a_1 + 7 && \text{Simplify.} \\ &= 23 + 7 \text{ or } 30 && a_1 = 23 \end{aligned}$$

$$\begin{aligned} a_3 &= a_{3-1} + 7 && n = 3 \\ &= a_2 + 7 && \text{Simplify.} \\ &= 30 + 7 \text{ or } 37 && a_2 = 30 \end{aligned}$$

$$\begin{aligned} a_4 &= a_{4-1} + 7 && n = 4 \\ &= a_3 + 7 && \text{Simplify.} \\ &= 37 + 7 \text{ or } 44 && a_3 = 37 \end{aligned}$$

$$\begin{aligned} a_5 &= a_{5-1} + 7 && n = 5 \\ &= a_4 + 7 && \text{Simplify.} \\ &= 44 + 7 \text{ or } 51 && a_4 = 44 \end{aligned}$$

The first five terms are 23, 30, 37, 44, and 51.

7-8 Recursive Formulas

11. $a_1 = 48, a_n = -0.5a_{n-1} + 8, n \geq 2$

SOLUTION:

Use $a_1 = 48$ and the recursive formula to find the next four terms.

$$\begin{aligned}a_2 &= -0.5a_{2-1} + 8 & n=2 \\&= -0.5a_1 + 8 & \text{Simplify.} \\&= -0.5(48) + 8 \text{ or } -16 & a_1=48\end{aligned}$$

$$\begin{aligned}a_3 &= -0.5a_{3-1} + 8 & n=3 \\&= -0.5a_2 + 8 & \text{Simplify.} \\&= -0.5(-16) + 8 \text{ or } 16 & a_2 = -16\end{aligned}$$

$$\begin{aligned}a_4 &= -0.5a_{4-1} + 8 & n=4 \\&= -0.5a_3 + 8 & \text{Simplify.} \\&= -0.5(16) + 8 \text{ or } 0 & a_3=16\end{aligned}$$

$$\begin{aligned}a_5 &= -0.5a_{5-1} + 8 & n=5 \\&= -0.5a_4 + 8 & \text{Simplify.} \\&= -0.5(0) + 8 \text{ or } 8 & a_4=0\end{aligned}$$

The first five terms are 48, -16, 16, 0, and 8.

12. $a_1 = 8, a_n = 2.5a_{n-1}, n \geq 2$

SOLUTION:

Use $a_1 = 8$ and the recursive formula to find the next four terms.

$$\begin{aligned}a_2 &= 2.5a_{2-1} & n=2 \\&= 2.5a_1 & \text{Simplify.} \\&= 2.5(8) \text{ or } 20 & a_1=8\end{aligned}$$

$$\begin{aligned}a_3 &= 2.5a_{3-1} & n=3 \\&= 2.5a_2 & \text{Simplify.} \\&= 2.5(20) \text{ or } 50 & a_2=20\end{aligned}$$

$$\begin{aligned}a_4 &= 2.5a_{4-1} & n=4 \\&= 2.5a_3 & \text{Simplify.} \\&= 2.5(50) \text{ or } 125 & a_3=50\end{aligned}$$

$$\begin{aligned}a_5 &= 2.5a_{5-1} & n=5 \\&= 2.5a_4 & \text{Simplify.} \\&= 2.5(125) \text{ or } 312.5 & a_4=125\end{aligned}$$

The first five terms are 8, 20, 50, 125, and 312.5.

7-8 Recursive Formulas

13. $a_1 = 12, a_n = 3a_{n-1} - 21, n \geq 2$

SOLUTION:

Use $a_1 = 12$ and the recursive formula to find the next four terms.

$$\begin{aligned} a_2 &= 3a_{2-1} - 21 & n=2 \\ &= 3a_1 - 21 & \text{Simplify.} \\ &= 3(12) - 21 \text{ or } 15 & a_1=12 \end{aligned}$$

$$\begin{aligned} a_3 &= 3a_{3-1} - 21 & n=3 \\ &= 3a_2 - 21 & \text{Simplify.} \\ &= 3(15) - 21 \text{ or } 24 & a_2=15 \end{aligned}$$

$$\begin{aligned} a_4 &= 3a_{4-1} - 21 & n=4 \\ &= 3a_3 - 21 & \text{Simplify.} \\ &= 3(24) - 21 \text{ or } 51 & a_3=24 \end{aligned}$$

$$\begin{aligned} a_5 &= 3a_{5-1} - 21 & n=5 \\ &= 3a_4 - 21 & \text{Simplify.} \\ &= 3(51) - 21 \text{ or } 132 & a_4=51 \end{aligned}$$

The first five terms are 12, 15, 24, 51, and 132.

7-8 Recursive Formulas

14. $a_1 = 13, a_n = -2a_{n-1} - 3, n \geq 2$

SOLUTION:

Use $a_1 = 13$ and the recursive formula to find the next four terms.

$$\begin{aligned}a_2 &= -2a_{2-1} - 3 && n=2 \\&= -2a_1 - 3 && \text{Simplify.} \\&= -2(13) - 3 \text{ or } -29 && a_1=13\end{aligned}$$

$$\begin{aligned}a_3 &= -2a_{3-1} - 3 && n=3 \\&= -2a_2 - 3 && \text{Simplify.} \\&= -2(-29) - 3 \text{ or } 55 && a_2 = -29\end{aligned}$$

$$\begin{aligned}a_4 &= -2a_{4-1} - 3 && n=4 \\&= -2a_3 - 3 && \text{Simplify.} \\&= -2(55) - 3 \text{ or } -113 && a_3 = 55\end{aligned}$$

$$\begin{aligned}a_5 &= -2a_{5-1} - 3 && n=5 \\&= -2a_4 - 3 && \text{Simplify.} \\&= -2(-113) - 3 \text{ or } 223 && a_4 = -113\end{aligned}$$

The first five terms are 13, -29, 55, -113, and 223.

7-8 Recursive Formulas

15. $a_1 = \frac{1}{2}, a_n = a_{n-1} + \frac{3}{2}, n \geq 2$

SOLUTION:

Use $a_1 = \frac{1}{2}$ and the recursive formula to find the next four terms.

$$\begin{aligned} a_2 &= a_{2-1} + \frac{3}{2} & n=2 \\ &= a_1 + \frac{3}{2} & \text{Simplify.} \\ &= \frac{1}{2} + \frac{3}{2} \text{ or } 2 & a_1 = \frac{1}{2} \end{aligned}$$

$$\begin{aligned} a_3 &= a_{3-1} + \frac{3}{2} & n=3 \\ &= a_2 + \frac{3}{2} & \text{Simplify.} \\ &= 2 + \frac{3}{2} \text{ or } \frac{7}{2} & a_2 = 2 \end{aligned}$$

$$\begin{aligned} a_4 &= a_{4-1} + \frac{3}{2} & n=4 \\ &= a_3 + \frac{3}{2} & \text{Simplify.} \\ &= \frac{7}{2} + \frac{3}{2} \text{ or } 5 & a_3 = \frac{7}{2} \end{aligned}$$

$$\begin{aligned} a_5 &= a_{5-1} + \frac{3}{2} & n=5 \\ &= a_4 + \frac{3}{2} & \text{Simplify.} \\ &= 5 + \frac{3}{2} \text{ or } \frac{13}{2} & a_4 = 5 \end{aligned}$$

The first five terms are $\frac{1}{2}, 2, \frac{7}{2}, 5$, and $\frac{13}{2}$.

Write a recursive formula for each sequence.

16. $12, -1, -14, -27, \dots$

SOLUTION:

Subtract each term from the term that follows it.

$$-1 - 12 = -13; -14 - (-1) = -13, -27 - (-14) = -13$$

There is a common difference of -13 . The sequence is arithmetic.

Use the formula for an arithmetic sequence.

$$\begin{aligned} a_n &= a_{n-1} + d & \text{Recursive formula for arithmetic sequence.} \\ a_n &= a_{n-1} - 13 & d = -13 \end{aligned}$$

The first term a_1 is 12, and $n \geq 2$. A recursive formula for the sequence $12, -1, -14, -27, \dots$ is $a_1 = 12, a_n = a_{n-1} - 13, n \geq 2$.

7-8 Recursive Formulas

17. 27, 41, 55, 69, ...

SOLUTION:

Subtract each term from the term that follows it.

$$41 - 27 = 14; 55 - 41 = 14, 69 - 55 = 14$$

There is a common difference of 14. The sequence is arithmetic.

Use the formula for an arithmetic sequence.

$$a_n = a_{n-1} + d \quad \text{Recursive formula for arithmetic sequence .}$$

$$a_n = a_{n-1} + 14 \quad d = 14$$

The first term a_1 is 27, and $n \geq 2$. A recursive formula for the sequence 27, 41, 55, 69, ... is $a_1 = 27, a_n = a_{n-1} + 14, n \geq 2$.

18. 2, 11, 20, 29, ...

SOLUTION:

Subtract each term from the term that follows it.

$$11 - 2 = 9; 20 - 11 = 9, 29 - 20 = 9$$

There is a common difference of 9. The sequence is arithmetic.

Use the formula for an arithmetic sequence.

$$a_n = a_{n-1} + d \quad \text{Recursive formula for arithmetic sequence .}$$

$$a_n = a_{n-1} + 9 \quad d = 9$$

The first term a_1 is 2, and $n \geq 2$. A recursive formula for the sequence 2, 11, 20, 29, ... is $a_1 = 2, a_n = a_{n-1} + 9, n \geq 2$.

19. 100, 80, 64, 51.2, ...

SOLUTION:

Subtract each term from the term that follows it.

$$80 - 100 = -20; 64 - 80 = -16, 51.2 - 64 = 12.8$$

There is no common difference. Check for a common ratio by dividing each term by the term that precedes it.

$$\frac{80}{100} = 0.8; \frac{64}{80} = 0.8; \frac{51.2}{64} = 0.8$$

There is a common ratio of 0.8. The sequence is geometric.

Use the formula for a geometric sequence.

$$a_n = ra_{n-1} \quad \text{Recursive formula for geometric sequence .}$$

$$a_n = 0.8a_{n-1} \quad r = 0.8$$

The first term a_1 is 100, and $n \geq 2$. A recursive formula for the sequence 100, 80, 64, 51.2, ... is $a_1 = 100, a_n = 0.8a_{n-1}, n \geq 2$.

7-8 Recursive Formulas

20. 40, -60, 90, -135, ...

SOLUTION:

Subtract each term from the term that follows it.

$$-60 - 40 = -100; 90 - (-60) = 30, -135 - 90 = -225$$

There is no common difference. Check for a common ratio by dividing each term by the term that precedes it.

$$\frac{-60}{40} = -1.5; \frac{90}{-60} = -1.5; \frac{-135}{90} = -1.5$$

There is a common ratio of -1.5. The sequence is geometric.

Use the formula for a geometric sequence.

$$a_n = ra_{n-1} \quad \text{Recursive formula for geometric sequence.}$$

$$a_n = -1.5a_{n-1} \quad r = -1.5$$

The first term a_1 is 40, and $n \geq 2$. A recursive formula for the sequence 40, -60, 90, -135, ... is $a_1 = 40$, $a_n = -1.5a_{n-1}$, $n \geq 2$.

21. 81, 27, 9, 3, ...

SOLUTION:

Subtract each term from the term that follows it.

$$27 - 81 = -54; 9 - 27 = -18, 3 - 9 = -6$$

There is no common difference. Check for a common ratio by dividing each term by the term that precedes it.

$$\frac{27}{81} = \frac{1}{3}; \frac{9}{27} = \frac{1}{3}; \frac{3}{9} = \frac{1}{3}$$

There is a common ratio of $\frac{1}{3}$. The sequence is geometric.

Use the formula for a geometric sequence.

$$a_n = ra_{n-1} \quad \text{Recursive formula for geometric sequence.}$$

$$a_n = \frac{1}{3}a_{n-1} \quad r = \frac{1}{3}$$

The first term a_1 is 81, and $n \geq 2$. A recursive formula for the sequence 81, 27, 9, 3, ... is $a_1 = 81$, $a_n = \frac{1}{3}a_{n-1}$, $n \geq 2$.

7-8 Recursive Formulas

22. **CCSS MODELING** A landscaper is building a brick patio. Part of the patio includes a pattern constructed from triangles. The first four rows of the pattern are shown.



- Write a recursive formula for the sequence.
- Write an explicit formula for the sequence.

SOLUTION:

For each recursive formula, write an explicit formula. For each explicit formula, write a recursive formula.

23. $a_n = 3(4)^{n-1}$

SOLUTION:

Write out the first 4 terms: 3, 12, 48, 192.

Subtract each term from the term that follows it.

$$12 - 3 = 9; 48 - 12 = 36, 192 - 48 = 144$$

There is no common difference. Check for a common ratio by dividing each term by the term that precedes it.

$$\frac{12}{3} = 4; \frac{48}{12} = 4; \frac{192}{48} = 4$$

There is a common ratio of 4. The sequence is geometric.

Use the formula for a geometric sequence.

$$a_n = ra_{n-1} \quad \text{Recursive formula for geometric sequence.}$$

$$a_n = 4a_{n-1} \quad r = 4$$

The first term a_1 is 3, and $n \geq 2$. A recursive formula for the explicit formula $a_n = 3(4)^{n-1}$ is $a_1 = 3, a_n = 4a_{n-1}, n \geq 2$.

24. $a_1 = -2, a_n = a_{n-1} - 12, n \geq 2$

SOLUTION:

The common difference is -12 . Use the formula for the n th terms of an arithmetic sequence.

$$a_n = a_1(n-1)d \quad \text{Formula for the } n\text{th term}$$

$$= -2 + (n-1)(-12) \quad a_1 = -2 \text{ and } d = -12$$

$$= -2 - 12n + 12 \quad \text{Distributive Property}$$

$$= -12n + 10 \quad \text{Simplify.}$$

The explicit formula is $a_n = -12n + 10$.

7-8 Recursive Formulas

25. $a_1 = 38, a_n = \frac{1}{2}a_{n-1}, n \geq 2$

SOLUTION:

The common ratio is $\frac{1}{2}$. Use the formula for the n th terms of a geometric sequence.

$$\begin{aligned} a_n &= a_1 r^{n-1} && \text{Formula for the } n\text{th term} \\ &= 38 \left(\frac{1}{2} \right)^{n-1} && a_1 = 38 \text{ and } r = \frac{1}{2} \end{aligned}$$

The explicit formula is $a_n = 38 \left(\frac{1}{2} \right)^{n-1}$.

26. $a_n = -7n + 52$

SOLUTION:

Write out the first 4 terms. 45, 38, 31, 24

Subtract each term from the term that follows it.

$$38 - 45 = -7; 31 - 38 = -7, 24 - 31 = -7$$

There is a common difference of -7 . The sequence is arithmetic.

Use the formula for an arithmetic sequence.

$$\begin{aligned} a_n &= a_{n-1} + d && \text{Recursive formula for arithmetic sequence.} \\ a_n &= a_{n-1} - 7 && d = -7 \end{aligned}$$

The first term a_1 is 45, and $n \geq 2$. A recursive formula for the explicit formula $a_n = -7n + 52$ is $a_1 = 45, a_n = a_{n-1} - 7, n \geq 2$.

7-8 Recursive Formulas

27. **TEXTING** Barbara received a chain text that she forwarded to five of her friends. Each of her friends forwarded the text to five more friends, and so on.
- Find the first five terms of the sequence representing the number of people who receive the text in the n th round.
 - Write a recursive formula for the sequence.
 - If Barbara represents a_1 , find a_8 .

SOLUTION:

a.

$$\begin{aligned} a_n &= a_1 r^{n-1} && \text{Formula for the } n\text{th term} \\ &= 1(5)^{n-1} && a_1 = 1 \text{ and } r = 5 \end{aligned}$$

Then the first 5 terms of the sequence would be 1, 5, 25, 125, 625.

- b.** The first term a_1 is 1, and $n \geq 2$. A recursive formula is $a_1 = 1, a_n = 5a_{n-1}, n \geq 2$.

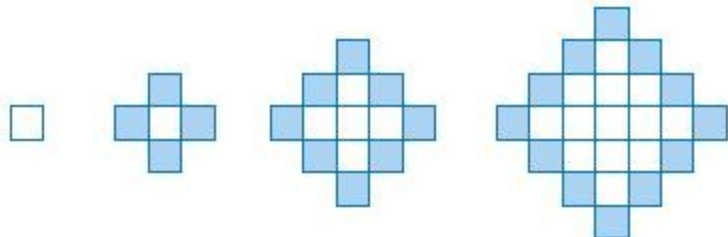
c.

$$\begin{aligned} a_n &= a_1 r^{n-1} && \text{Formula for the } n\text{th term} \\ a_8 &= 1(5)^{8-1} && a_1 = 1, n = 8 \text{ and } r = 5 \\ a_8 &= 5^7 \text{ or } 78,125 && \text{Simplify.} \end{aligned}$$

On the 8th round, 78,125 would receive the chain text message.

7-8 Recursive Formulas

28. **GEOMETRY** Consider the pattern below. The number of blue boxes increases according to a specific pattern.



- a. Write a recursive formula for the sequence of the number of blue boxes in each figure.
b. If the first box represents a_1 , find the number of blue boxes in a_8 .

SOLUTION:

- a. The sequence of blue boxes is 0, 4, 8, and 12.

Subtract each term from the term that follows it.

$$4 - 0 = 4; 8 - 4 = 4, 12 - 8 = 4$$

There is a common difference of 4. The sequence is arithmetic.

Use the formula for an arithmetic sequence.

$$a_n = a_{n-1} + d \quad \text{Recursive formula for arithmetic sequence.}$$

$$a_n = a_{n-1} + 4 \quad d = 4$$

The first term a_1 is 0, and $n \geq 2$. A recursive formula is $a_1 = 0$, $a_n = a_{n-1} + 4$, $n \geq 2$.

- b. Use the formula for the n th terms of an arithmetic sequence.

$$a_n = a_1(n-1)d \quad \text{Formula for the } n\text{th term}$$

$$a_8 = 0 + (8-1)4 \quad a_1 = 0, d = 4, \text{ and } n = 8$$

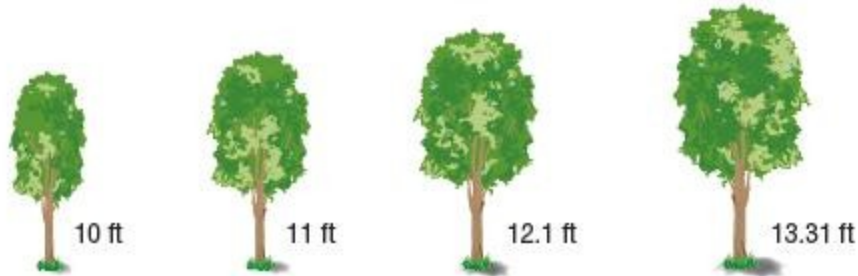
$$= 0(7)4 \quad \text{Subtraction}$$

$$= 28 \quad \text{Simplify.}$$

When $n = 8$, there will be 28 blue boxes.

7-8 Recursive Formulas

29. **TREE** The growth of a certain type of tree slows as the tree continues to age. The heights of the tree over the past four years are shown.



- a. Write a recursive formula for the height of the tree.
b. If the pattern continues, how tall will the tree be in two more years? Round your answer to the nearest tenth of a foot.

SOLUTION:

- a. The sequence of heights is 10, 11, 12.1, and 13.31.

Subtract each term from the term that follows it.

$$11 - 10 = 1; 12.1 - 11 = 1.1; 13.31 - 12.1 = 1.21$$

There is no common difference. Check for a common ratio by dividing each term by the term that precedes it.

$$\frac{11}{10} = 1.1; \frac{12.1}{11} = 1.1; \frac{13.31}{12.1} = 1.1$$

There is a common ratio of 1.1. The sequence is geometric.

Use the formula for a geometric sequence.

$$a_n = r a_{n-1} \quad \text{Recursive formula for geometric sequence.}$$

$$a_n = 1.1 a_{n-1} \quad r = 1.1$$

The first term a_1 is 10, and $n \geq 2$. A recursive formula for the sequence 10, 11, 12.1, 13.31, ... is $a_1 = 10$, $a_n = 1.1 a_{n-1}$, $n \geq 2$.

- b. Use the formula for the n th terms of a geometric sequence.

$$a_n = a_1 r^{n-1} \quad \text{Formula for the } n\text{th term}$$

$$a_6 = 10(1.1)^{6-1} \quad a_1 = 10 \text{ and } r = 1.1$$

$$= 16.1051 \quad \text{Simplify.}$$

In two more years, the tree will be 16.1 feet tall.

7-8 Recursive Formulas

30. **MULTIPLE REPRESENTATIONS** The Fibonacci sequence is neither arithmetic nor geometric and can be defined by a recursive formula. The first terms are 1, 1, 2, 3, 5, 8, ...

- a. Logical Determine the relationship between the terms of the sequence. What are the next five terms in the sequence?
- b. Algebraic Write a formula for the n th term if $a_1 = 1$, $a_2 = 1$, and $n \geq 3$.
- c. Algebraic Find the 15th term.
- d. Analytical Explain why the Fibonacci sequence is not an arithmetic sequence.

SOLUTION:

a. Sample answer: The first two terms are 1. Starting with the third term, the two previous terms are added together to get the next term. So, the next 5 terms after 8 is $5 + 8$ or 13, $8 + 13$ or 21, $13 + 21$ or 34, $21 + 34$ or 55, and $34 + 55$ or 89.

b. The first term a_1 is 1 and the second term a_2 is 1, and $n \geq 3$. A recursive formula for Fibonacci sequence is $a_1 = 1$, $a_2 = 1$, $a_n = a_{n-2} + a_{n-1}$, $n \geq 3$.

c. First, find the 13th and 14th terms by writing out 14 terms of the sequence: 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377.

Then, $a_{13} = 233$ and $a_{14} = 377$. Thus, $a_{15} = a_{13} + a_{14}$ or 610.

d. Sample answer: Fibonacci sequence is not an arithmetic sequence since there is no common difference.

31. **ERROR ANALYSIS** Patrick and Lynda are working on a math problem that involves the sequence 2, -2, 2, -2, 2, Patrick thinks that the sequence can be written as a recursive formula. Lynda believes that the sequence can be written as an explicit formula. Is either of them correct? Explain.

SOLUTION:

Both; sample answer: The sequence can be written as the recursive formula $a_1 = 2$, $a_n = (-1)a_{n-1}$, $n \geq 2$. The sequence can also be written as the explicit formula $a_n = 2(-1)^{n-1}$.

7-8 Recursive Formulas

32. **CHALLENGE** Find a_1 for the sequence in which $a_4 = 1104$ and $a_n = 4a_{n-1} + 16$.

SOLUTION:

Find a_3 first.

$$a_n = 4a_{n-1} + 16 \quad \text{Formula for } n\text{th term}$$

$$a_4 = 4a_{4-1} + 16 \quad n=4$$

$$1104 = 4a_3 + 16 \quad a_4 = 1104$$

$$1088 = 4a_3 \quad \text{Subtract 16 from each side.}$$

$$272 = a_3 \quad \text{Divide each side by 4.}$$

Find a_2 next.

$$a_n = 4a_{n-1} + 16 \quad \text{Formula for } n\text{th term}$$

$$a_3 = 4a_{3-1} + 16 \quad n=3$$

$$272 = 4a_2 + 16 \quad a_3 = 272$$

$$256 = 4a_2 \quad \text{Subtract 16 from each side.}$$

$$64 = a_2 \quad \text{Divide each side by 4.}$$

Find a_1 .

$$a_n = 4a_{n-1} + 16 \quad \text{Formula for } n\text{th term}$$

$$a_2 = 4a_{2-1} + 16 \quad n=2$$

$$64 = 4a_1 + 16 \quad a_2 = 64$$

$$48 = 4a_1 \quad \text{Subtract 16 from each side.}$$

$$12 = a_1 \quad \text{Divide each side by 4.}$$

Therefore, a_1 is 12.

33. **CCSS ARGUMENTS** Determine whether the following statement is *true* or *false*. Justify your reasoning.

There is only one recursive formula for every sequence.

SOLUTION:

False; sample answer: A recursive formula for the sequence 1, 2, 3, ... can be written as $a_1 = 1$, $a_n = a_{n-1} + 1$, $n \geq 2$ or as $a_1 = 1$, $a_2 = 2$, $a_n = a_{n-2} + 2$, $n \geq 3$.

7-8 Recursive Formulas

34. **CHALLENGE** Find a recursive formula for 4, 9, 19, 39, 79, ...

SOLUTION:

Subtract each term from the term that follows it.

$$9 - 4 = 5; 19 - 9 = 10, 39 - 19 = 20$$

There is no common difference. Check for a common ratio by dividing each term by the term that precedes it.

$$\frac{9}{4} = 2.25; \frac{19}{9} \approx 2.1; \frac{39}{19} \approx 2.1$$

There is no common ratio. Therefore the sequence must be a combination of both.

From the difference above, you can see each is twice as big as the previous. So r is 2. From the ratios, if each numerator was one less, the ratios would be 2. Thus, the common difference is 1. So, if the first term a_1 is 4, and $n \geq$

2 a recursive formula for the sequence 4, 9, 19, 39, 79, ... is $a_1 = 4, a_n = 2a_{n-1} + 1, n \geq 2$.

35. **WRITING IN MATH** Explain the difference between an explicit formula and a recursive formula.

SOLUTION:

Sample answer: In an explicit formula, the n th term a_n is given as a function of n . In a recursive formula, the n th term a_n is found by performing operations to one or more of the terms that precede it.

36. Find a recursive formula for the sequence 12, 24, 36, 48,

- A $a_1 = 12, a_n = 2a_{n-1}, n \geq 2$
B $a_1 = 12, a_n = 4a_{n-1} - 24, n \geq 2$
C $a_1 = 12, a_n = a_{n-1} + 12, n \geq 2$
D $a_1 = 12, a_n = 12a_{n-1} + 12, n \geq 2$

SOLUTION:

Subtract each term from the term that follows it.

$$24 - 12 = 12; 36 - 24 = 12, 48 - 36 = 12$$

There is a common difference of 12. The sequence is arithmetic.

Use the formula for an arithmetic sequence.

$$a_n = a_{n-1} + d \quad \text{Recursive formula for arithmetic sequence.}$$

$$a_n = a_{n-1} + 12 \quad d = 12$$

The first term a_1 is 12, and $n \geq 2$. A recursive formula for the sequence 12, 24, 36, 48, ... is $a_1 = 12, a_n = a_{n-1} + 12, n \geq 2$.

Therefore, the correct choice is C.

7-8 Recursive Formulas

37. **GEOMETRY** The area of a rectangle is $36m^4n^6$ square feet. The length of the rectangle is $6m^3n^3$ feet. What is the width of the rectangle?

F $216m^7n^9$ ft

G $6mn^3$ ft

H $42m^7n^3$ ft

J $30mn^3$ ft

SOLUTION:

Use the area formula for a rectangle to determine the measure of the width.

$$A = \ell \cdot w \quad \text{Area of a rectangle}$$

$$36m^4n^6 = (6m^3n^3)w \quad \ell = 6m^3n^3, A = 36m^4n^6$$

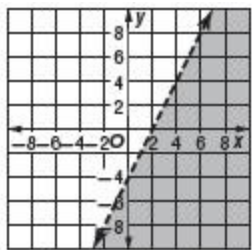
$$\frac{36m^4n^6}{6m^3n^3} = w \quad \text{Divide each side by } 6m^3n^3.$$

$$\left(\frac{36}{6}\right)\left(\frac{m^4}{m^3}\right)\left(\frac{n^6}{n^3}\right) = w \quad \text{Group powers with the same base.}$$

$$6mn^3 = w \quad \text{Simplify.}$$

Therefore, the correct choice is G.

38. Find an inequality for the graph shown.



A $y > 2x - 4$

B $y \geq 2x - 4$

C $y < 2x - 4$

D $y \leq 2x - 4$

SOLUTION:

The y -intercept of the line is -4 . The slope is 2 . The graph is shaded below, so the inequality should use a $<$ or \leq symbol. Since the line is dashed, the inequality does not include the equals. Therefore, the correct choice is C.

7-8 Recursive Formulas

39. Write an equation of the line that passes through $(-2, -20)$ and $(4, 58)$.

F $y = 13x + 6$

G $y = 19x - 18$

H $y = 19x + 18$

J $y = 13x - 6$

SOLUTION:

First find the slope.

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} && \text{Slope formula} \\ &= \frac{58 - (-20)}{4 - (-2)} && (x_1, y_1) = (-2, -20) \text{ and } (x_2, y_2) = (4, 58) \\ &= \frac{78}{6} && \text{Simplify.} \\ &= 13 && \text{Divide.} \end{aligned}$$

Use the point-slope formula to find the equation.

$$\begin{aligned} y - y_1 &= m(x - x_1) && \text{Point-slope formula} \\ y - (-20) &= 13[x - (-2)] && (x_1, y_1) = (-2, -20); m = 13 \\ y + 20 &= 13x + 26 && \text{Distributive Property} \\ y &= 13x + 26 - 20 && \text{Subtract 20 from each side.} \\ y &= 13x + 6 && \text{Simplify.} \end{aligned}$$

Therefore, the correct choice is F.

Find the next three terms in each geometric sequence.

40. 675, 225, 75, ...

SOLUTION:

Find the common ratio.

$$\frac{225}{675} = \frac{1}{3}, \frac{75}{225} = \frac{1}{3}$$

The common ratio is $\frac{1}{3}$. Multiply by the common ratio to find next three terms. They are $25, \frac{25}{3}, \frac{25}{9}$.

41. 16, -24, 36, ...

SOLUTION:

Find the common ratio.

$$\frac{-24}{16} = -\frac{3}{2}, \frac{36}{-24} = -\frac{3}{2}$$

The common ratio is $-\frac{3}{2}$. Multiply by the common ratio to find next three terms. They are -54, 81, -121.5.

7-8 Recursive Formulas

42. 6, 18, 54, ...

SOLUTION:

Find the common ratio.

$$\frac{18}{6} = 3; \frac{54}{18} = 3$$

The common ratio is 3. Use the common ratio to find next three terms. They are 162, 324, 972.

43. 512, -256, 128, ...

SOLUTION:

Find the common ratio.

$$\frac{-256}{512} = -\frac{1}{2}, \frac{128}{-256} = -\frac{1}{2}$$

The common ratio is $-\frac{1}{2}$. Use the common ratio to find next three terms. They are -64, 32, -16.

44. 125, 25, 5, ...

SOLUTION:

Find the common ratio.

$$\frac{25}{125} = \frac{1}{5}, \frac{5}{25} = \frac{1}{5}$$

The common ratio is $\frac{1}{5}$. Use the common ratio to find next three terms. They are $\frac{1}{5}, \frac{1}{25}, \frac{1}{125}$.

45. 12, 60, 300, ...

SOLUTION:

Find the common ratio.

$$\frac{60}{12} = 5; \frac{300}{60} = 5$$

The common ratio is 5. Multiply by the common ratio to find next three terms. They are 1500, 7500, 37,500.

46. **INVESTMENT** Nicholas invested \$2000 with a 5.75% interest rate compounded monthly. How much money will Nicholas have after 5 years?

SOLUTION:

$$A = P\left(1 + \frac{r}{n}\right)^{nt} \quad \text{Equation for compound interest}$$

$$A = 2000\left(1 + \frac{0.0575}{12}\right)^{12(5)} \quad P = 2000, r = 0.0575, n = 12, t = 5$$

$$A \approx 2000(1.004792)^{60} \quad \text{Simplify.}$$

$$A \approx 2664.35 \quad \text{Use a calculator.}$$

After 5 years, Nicholas will have \$2664.35.

7-8 Recursive Formulas

47. **TOURS** The Snider family and the Rollins family are traveling together on a trip to visit a candy factory. The number of people in each family and the total cost are shown in the table below. Find the adult and children's admission prices.

Family	Number of Adults	Number of Children	Total Cost
Snider	2	3	\$58
Rollins	2	1	\$38

SOLUTION:

Let a be the admissions cost for adults and c the admission cost of children.

Then $2a + 3c = 58$ and $2a + c = 38$.

$$2a + 3c = 58 \quad \text{Snider Family equation}$$

$$2a + c = 38 \quad \text{Rollins Family equation}$$

$$2c = 20 \quad \text{Subtract 2nd equation from first.}$$

$$c = 10 \quad \text{Divide each side by 2 and simplify.}$$

$$2a + 10 = 38 \quad \text{Substitute 10 into 2nd equation for } c.$$

$$2a = 28 \quad \text{Subtract 10 from each side.}$$

$$a = 14 \quad \text{Divide each side by 2 and simplify.}$$

So, tickets for adults cost \$14 and tickets for children cost \$10.

Write each equation in standard form.

48. $y + 6 = -3(x + 2)$

SOLUTION:

$$y + 6 = -3(x + 2) \quad \text{Original equation}$$

$$y + 6 = -3x - 6 \quad \text{Distributive Property}$$

$$3x + y + 6 = -6 \quad \text{Add } 3x \text{ to each side.}$$

$$3x + y = -12 \quad \text{Subtract 6 from each side.}$$

49. $y - 12 = 4(x - 7)$

SOLUTION:

$$y - 12 = 4(x - 7) \quad \text{Original equation}$$

$$y - 12 = 4x - 28 \quad \text{Distributive Property}$$

$$-4x + y - 12 = -28 \quad \text{Subtract } 4x \text{ from each side.}$$

$$-4x + y = -16 \quad \text{Add 12 to each side.}$$

$$4x - y = 16 \quad \text{Multiply each side by } -1.$$

7-8 Recursive Formulas

50. $y + 9 = 5(x - 3)$

SOLUTION:

$$y + 9 = 5(x - 3) \quad \text{Original equation}$$

$$y + 9 = 5x - 15 \quad \text{Distributive Property}$$

$$-5x + y + 9 = -15 \quad \text{Subtract } 5x \text{ from each side.}$$

$$-5x + y = -24 \quad \text{Subtract 9 from each side.}$$

$$5x - y = 24 \quad \text{Multiply each side by } -1.$$

51. $y - 1 = \frac{1}{3}(x + 15)$

SOLUTION:

$$y - 1 = \frac{1}{3}(x + 15) \quad \text{Original equation}$$

$$y - 1 = \frac{1}{3}x + 5 \quad \text{Distributive Property}$$

$$-\frac{1}{3}x + y - 1 = 5 \quad \text{Subtract } \frac{1}{3}x \text{ from each side.}$$

$$-\frac{1}{3}x + y = 6 \quad \text{Add 1 to each side.}$$

$$x - 3y = -18 \quad \text{Multiply each side by } -3.$$

52. $y + 10 = \frac{2}{5}(x - 6)$

SOLUTION:

$$y + 10 = \frac{2}{5}(x - 6) \quad \text{Original equation}$$

$$y + 10 = \frac{2}{5}x - \frac{12}{5} \quad \text{Distributive Property}$$

$$-\frac{2}{5}x + y + 10 = -\frac{12}{5} \quad \text{Subtract } \frac{2}{5}x \text{ from each side.}$$

$$-\frac{2}{5}x + y = -\frac{62}{5} \quad \text{Subtract 10 from each side.}$$

$$2x - 5y = 62 \quad \text{Multiply each side by } -5$$

53. $y - 4 = -\frac{2}{7}(x + 1)$

SOLUTION:

$$y - 4 = -\frac{2}{7}(x + 1) \quad \text{Original equation}$$

$$y - 4 = -\frac{2}{7}x - \frac{2}{7} \quad \text{Distributive Property}$$

$$\frac{2}{7}x + y - 4 = -\frac{2}{7} \quad \text{Add } \frac{2}{7}x \text{ to each side.}$$

$$\frac{2}{7}x + y = \frac{26}{7} \quad \text{Add 4 to each side.}$$

$$2x + 7y = 26 \quad \text{Multiply each side by 7.}$$

7-8 Recursive Formulas

Simplify each expression. If not possible, write simplified.

54. $8x + 3y^2 + 7x - 2y$

SOLUTION:

$$\begin{aligned}8x + 3y^2 + 7x - 2y &= 3y^2 + 8x + 7x - 2y && \text{CommutativeProperty} \\&= 3y^2 + (8x + 7x) - 2y && \text{AssociativeProperty} \\&= 3y^2 + 15x - 2y && \text{Substitution}\end{aligned}$$

55. $4(x - 16) + 6x$

SOLUTION:

$$\begin{aligned}4(x - 16) + 6x &= 4x - 64 + 6x && \text{DistributiveProperty} \\&= 4x + 6x - 64 && \text{CommutativeProperty} \\&= (4x + 6x) - 64 && \text{AssociativeProperty} \\&= 10x - 64 && \text{Substitution.}\end{aligned}$$

56. $4n - 3m + 9m - n$

SOLUTION:

$$\begin{aligned}4n - 3m + 9m - n &= 4n - n - 3m + 9m && \text{CommutativeProperty} \\&= (4n - n) + (-3m + 9m) && \text{AssociativeProperty} \\&= 3n + 6m && \text{Substitution}\end{aligned}$$

57. $6r^2 + 7r$

SOLUTION:

There are no like terms, so the expression is simplified.

58. $-2(4g - 5h) - 6g$

SOLUTION:

$$\begin{aligned}-2(4g - 5h) - 6g &= -8g + 10h - 6g && \text{DistributiveProperty} \\&= -8g - 6g + 10h && \text{CommutativeProperty} \\&= (-8g - 6g) + 10h && \text{AssociativeProperty} \\&= -14g + 10h && \text{Substitution.}\end{aligned}$$

59. $9x^2 - 7x + 16y^2$

SOLUTION:

There are no like terms, so the expression is simplified.