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# Thermodynamics in Earth and Planetary Sciences

*Dedicated to the Pioneers who led the Way  
and the Students, Colleagues and Mentors  
who helped me along the Way.*

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