These Probability NOTES Belong to:____________

Date	Topic	Notes	Questions
1.	Intro		
2.	And & Or		
3.	Dependant & Independent		
4.	Dependant & Independent		
5.	Conditional		
6.	Conditional		
7.	Combinations & Permutations		
8.	Binomial		
9.	Review		
10.	Review		
11.	TEST		

Record any questions that you find challenging.

How to use this booklet.

Do not do questions sequentially. Once you have got a concept, move on to the next set of questions. Once you have come to the end of a section, go back and complete the missing questions.

Memorize all definitions and terms:

Probability • Page 3	If an experiment has n equally likely outcomes of which r outcomes are favorable to event A, then the probability of event A is: $P(A)=\frac{r}{n}$. →P(Rolling a 1)= $\frac{r}{n}\frac{1}{6}\frac{option}{options}$ →Probabilities must be between zero and one.
Theoretical Probability • Page 3	• Theoretical Probability is what should happen. • These Probabilities are calculated using formulae.
Experimental Probability • Page 3	• Experimental Probability is what actually happens. • These probabilities are calculated by experiment.
Sample Space • Page 3→	• The sample space is the set or the list of all possible outcomes. • The sample space is "n" in $P(A)=\frac{r}{n}$
$0 \le P \le 1$ • Page 3→	• Probabilities are always between 0 and 1 • Probabilites can be written in decimal form or as fraction • Probabilities can be converted to percentages. 0.2→20% (20% is not a probability it is a percentage)
AND • Page 3→	• Means both • What is the probability that a randomly chosen student is a boy and in grade 12. Another way of saing this would be what is the probability that a Grade 12 boy is chosen. • Multiply probabilties
OR • Page 3→	• Means Either • What is the probability that a randomly chosen student is a boy or is in grade 12. Another way of saying this would be, what is the probability that the student chosen is a male student or a grade 12 girl. • Add probabilities
$P(a)$ vs $p(\overline{a})$ • Page 4,7→	• $P(a)$ →Probability that event "a" happens • $p(\overline{a})$ →Probability that event "a" does not happen • $P(a)$ + $p(\overline{a})$ =1 The complement of a is $\overline{a}$.
Complement • Page 11	If the probability of winning is 0.8, then the complement of winning is 0.2. The complement of winning is losing. $P(winning)=0.8$ & $P(\overline{winning})=P(NotWinning)=0.2$ • Using the complement is often helpful when solving at least or at most questions.

Conditional probability	$P(A \mid B) =$ This is read, the probability of A given B has already happened. The probability of A occurring under $P(A \mid B)$ will not be out of 10 but out of 5, the number of outcomes that are B.
Mutually exclusive • Page 7→	• Events are mutually exclusive if they have nothing in common • Playing card example. Hearts and diamonds are mutually exclusive. • Playing card example. Hearts and face cards are not mutually exclusive.
Independent events • Page 16	• Event A and B are independent events if they do not impact each other. • Examples of indepedent events: Rolling dice, Flipping coins, drawing cards with replacement • Probabilities do not change if the events are independent
Dependent events • Page	• Event A and B are dependent if they impact each other in any way • Example of dependent events: Drawing cards without replacement
Binomial Events & Binomial Distribution • Page 34	• An event/distribution where there are only 2 options. • Example of binomial events. Flipping a coin, Rolling a die and comparing rolling the number 1 vs everything else. • A binomial event always has a yes/no answer. Ie was the result a head? Was the result a 1. Was the result something other than 1? • ${}_nC_r(A)^r(A\text{ NOT})^{n-r}$

Introduction to Probability

Probability	If an experiment has n equally likely outcomes of which r outcomes are favorable to event A, then the probability of event A is: $P(A)=\frac{r}{n}$. →P(Rolling a 1)= $\frac{r}{n}\frac{1}{6}\frac{option}{options}$
Sample Space • Page 3→	• The sample space is the set or the list of all possible outcomes. • The sample space is "n" in $P(A)=\frac{r}{n}$
$0 \le P \le 1$ • Page 3→	• Probabilities are always between 0 and 1 • Probabilites can be written in decimal form or as fraction • Probabilities can be converted to percentages. 0.2→20% (20% is not a probability it is a percentage)
OR	Is inclusive.
And	Is exclusive.

A coin is flipped once. What is the probability that the result will be heads?

- P(Heads)=0.5 or $\frac{1}{2}$ → Probabilities are always between zero and one.
- We can convert probabilities to percentages. There is a 50% chance that a coin will land heads.

Theoretical versus Experimental Probabilities

A coin is tossed. Consider the event "a single head."

1. Determine the Theoretical probability
2. Determine the experimental probability with 400 tosses of the coin?

• Theoretical probability is what should happen.	• Experimental probability is what actually happen.

In Math 12 we will focus entirely on theoretical probabilities.

What is the sample space for a regular deck of cards?

Color	Suit	Non-Face Cards										Face Cards		
Red	Hearts	A	2	3	4	5	6	7	8	9	10	J	Q	K
Red	Diamonds	A	2	3	4	5	6	7	8	9	10	J	Q	K
Black	Clubs	A	2	3	4	5	6	7	8	9	10	J	Q	K
Black	Spades	A	2	3	4	5	6	7	8	9	10	J	Q	K

Determine the probabilities. Leave your answer as a fraction in lowest terms.

A single card is selected from a deck of 52 cards. What is the probability of the following event?

3. P(Heart)=
4. P(Red)=
5. P(Club)=
6. P(Red or Black)=
7. P(Red & Black)=
8. P(Club or face)=
9. P(Club & Face)=
10. P(Black 3)=
11. P(Not a heart)=
12. P(Not a King)=
13. P(Heart or 4)=
14. P(Jack)=
15. P(Ace through 10)=
16. P(Club or jack)=

Determine the sample space for each set of events. Use a tree diagram, a chart or a list to help. Leave your answer as a fraction in lowest terms.

17. A coin and a 6-sided dice are rolled. What is the sample space?
18. P(H and 6)=
19. P(T and odd)=
20. P(H or 6)=
21. P(T or odd)=

22. Two 4 sided dice are rolled. What is the sample space?
23. P(4 and 2)=
24. P(4 or 2)=
25. P(Not a 4)= $P\left(\overline{4}\right) =$
26. P(Sum is even)=

27. Three coins are flipped. What is the sample space?
28. P(At least 1 head)=
29. P(No heads)=
30. 1-P(No heads)=

Determine the probabilities and number of outcomes.

A biased coin is weighted so that it lands heads 70% of the time.

31. P(H)=

32. P(T)=

33. The biased coin is flipped 20 times. How many heads will result?

34. The biased coin is flipped 50 times. How many tails will result?

A biased coin weighted so that it lands tails 40% of the time.

35. P(H)=

36. P(T)=

37. The biased coin is flipped 80 times. How many heads will result?

38. The biased coin is flipped 60 times. How many tails will result?

A biased die is weighted so that it returns a 3 20% of the time.

39. P(3)=

40. P(6)=

41. The biased 6-sided die is rolled 50 times. How many 5s will result?

42. The biased 6-sided die is rolled 200 times. How many even numbers will result?

Calculate the following probabilities.

A card is drawn from a shuffled deck of 52 cards. What is the probability of each event?

43. What is the sample space?

44. A red card is drawn.

45. A face card is drawn.

46. A heart is drawn.

In the card game "In Between," 3 cards from a deck of 52 are drawn. To win, the 3rd card must be in between the first two cards. The player loses if the 3rd card is the same as the first two.

- Determine the probability of winning given the first two cards already drawn.

47. What is the sample space?

48. A 3 and a 7 are drawn.

49. A 5 and a Queen are drawn.

50. An 8 and a 9 are drawn.

Calculate the following probabilities.

2 six-sided dice are rolled. Determine the probability of each event

Sample space

11	12	13	14	15	16
21	22	23	24	25	26
31	32	33	34	35	36
41	42	43	44	45	46
51	52	53	54	55	56
61	62	63	64	65	66

51. The sum is odd.

52. The sum is 6 or 12.

53. A double is rolled.

54. The sum of the 2 dice is at least 3.

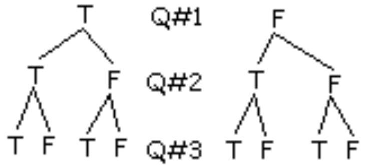
55. The sum of the two dice is at most 10.

Calculate the following probabilities.

56. If you guess on every question, what is the probability of getting 100% on a 3 question true false test?

Solution:
Determine the sample space first

Question #1→2 options
Question #2→2 options
Question #3→2 options



- There are 8 possible answer keys $2 \times 2 \times 2 = 2^3 = 8$

There is one correct answer key
P(100%)=$\frac{1}{8}$=0.125

57. If you guess on every question, what is the probability of getting 100% on a 5 question true-false test?

0.03125

58. If you guess on every question, what is the probability of getting 100% on an 8 question true-false test?

0.0039

59. If you guess on every question, what is the probability of getting 100% on a 5 question multiple choice test?(ABCD)

0.000977

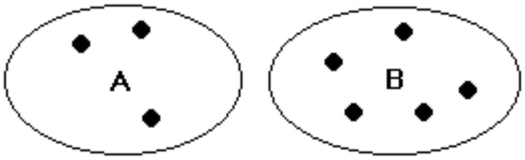
60. If you guess on every question, what is the probability of getting 100% on a 10 question multiple choice test?(ABCD)

0.000000953

"OR", "AND" and Probability

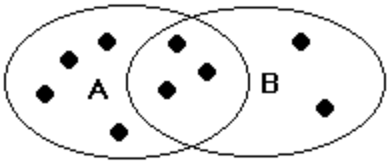
AND	• Means both • What is the probability that a randomly chosen student is a boy and in grade 12. Another way of saying this would be what is the probability that a Grade 12 boy is chosen. • Multiply probabilties
OR	• Means Either • What is the probability that a randomly chosen student is a boy or is in grade 12. Another way of saying this would be, what is the probability that the student chosen is a male student or a grade 12 girl. • Add probabilities
$P(a)$ vs $p(\bar{a})$	• $P(a)$ →Probability that event "a" happens • $p(\bar{a})$ →Probability that event "a" does not happen • $P(a) + p(\bar{a}) = 1$ The complement of a is $\bar{a}$.
Complement	If the probability of winning is 0.8, then the complement of winning is 0.2. The complement of winning is losing.
Conditional probability	$P(A \mid B) =$ This is read, the probability of A given B has already happened. The probability of A occurring under $P(A \mid B)$ will not be out of 10 but out of 5, the number of outcomes that are B.

Events A and B are *mutually exclusive.*



There is no overlap of A and B
P(A and B)=0

Events A and B are not *mutually exclusive.*



There is overlap of A and B.
P(A and B)=3/9

Calculate the probabilities .

61. $P(A) =$

62. $P(B) =$

63. $P(\overline{A}) =$
(Not A)

64. $P(\overline{B}) =$

65. $P(A \,\&\, B) =$
Both A and B

66. $P(A or B) =$
Either A or B or both

67. $p(\overline{A or B}) =$

68. $P(\overline{A \,\&\, B}) =$

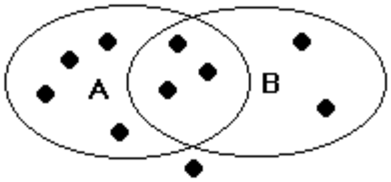
69. P(Neither)=

70. P(Only one)=
Only A or only B

71. $P(A \mid B) =$

72. $P(B \mid A) =$

Venn Diagram



$P(A \mid B) =$ This is read, the probability of A given B has already happened. The probability of A occurring under $P(A \mid B)$ will not be out of 10 but out of 5, the number of outcomes that are B.

Calculate the probabilities.

73. $P(A \,\&\, B) =$

74. $P(A \mid B) =$

75. P(Neither)=

76. $P(A or B) =$

77. $P(A) =$

78. $p(\overline{A or B}) =$

79. $P(B \mid A) =$

80. P(only one)=

81. $P(B) =$

82. $P(\overline{A}) =$

83. $P(\overline{A \,\&\, B}) =$

84. $P(\overline{B}) =$

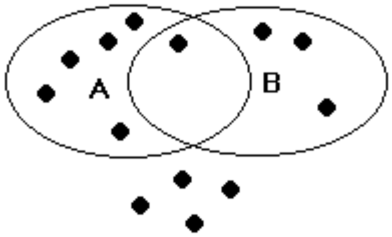
85. $P(B or \overline{B}) =$

86. $P(B and \overline{B}) =$

87. $P(A or \overline{B}) =$

88. $P(A and \overline{B}) =$

Venn Diagram



Challenge #1: A market study found that 50% of a neighborhood likes Japanese food while 60% likes Italian food. 30% like both. Determine the following probabilities:

- P(likes at least 1)=
- P(likes only one type of food)=

Challenge #2: A TV station determined that 30% of boys watch sports and 60% watch soaps. 20% watch neither. Determine the following probabilities:

- P(Boy watches at most 1 show)=
- P(Boy watches at least 1 show)=

Use Venn diagrams to calculate the following probabilities.

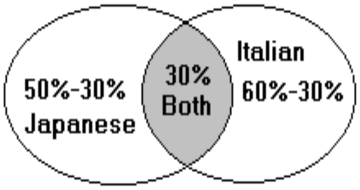
A market study found that 50% of a neighborhood like Japanese food while 60% like Italian food. 30% like both.

89. P(likes at least 1)=

90. P(likes only one type of food)=

Solution:

Draw and label a Venn diagram
30% goes in the overlap
Fill out the rest of the diagram by subtracting 30% from 50% and 60%



Use the diagram to calculate the above probabilities.

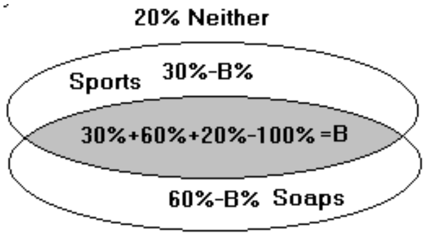
A TV station determined that 30% of boys watch sports and 60% watch soaps. 20% watch neither.

91. P(Boy watches at most 1 show)=

92. P(Boy watches at least 1 show)=

Solution:

All percentages must add to 100%. Determine the overlap by adding all percentages and then subtracting by 100%.



Use the diagram to calculate the above probabilities.

64% of girls want to go into business and 44% want to go into education. 14% want neither.

93. P(Girl pursue at most one career)=

A market Study found that 40% of a neighborhood likes Japanese food while 50% likes Italian food. 30% like both.

94. P(likes at least 1)=

95. P(likes only one type of food)=

A TV station determined that 20% of boys watch sports and 50% watch the news. 40% watch neither.

96. P(Boy watches only sports)=

97. P(Boy watches the news or nothing)=

80% of girls want to go into business and 30% want to go into education. 20% want neither.

98. P(Girl pursues both careers)=

99. P(Girl pursue at most one career)=

Challenge #3: A card is drawn from a shuffled deck of 52 cards. Determine the probability that the card is a heart or a spade.

Challenge #4: A card is drawn from a shuffled deck of 52 cards. Determine the probability that the card is a heart or a face card.

Calculate the following probabilities. Leave your answer as a fraction in lowest terms.

A card is drawn from a shuffled deck of 52 cards. Determine the probability of each event.

100. The card is a heart or a spade.

Solution: These events are mutually exclusive:

$$P(H)+P(S)= \frac{13}{52}+\frac{13}{52}=\frac{1}{2}$$

101. The card is an ace or a face card.

102. The card is a 10 or a face card.

103. The card is a spade or a red 4.

104. The card is a heart or a face card.

Solution: These events are not mutually exclusive

$$P(H)+P(F)-P(H\&F)= \frac{13}{52}+\frac{12}{52}-\frac{3}{52}=\frac{22}{52}=\frac{11}{26}$$

105. The card is an ace or a spade.

106. The card is a spade or a black 8.

107. The card is a club or a black card.

Challenge #5: A survey of 200 people indicated that 60 learn from the newspaper, 50 from the TV and 30 from both sources. Determine the following probabilities:

- P(A randomly selected person learns from the newspaper and TV)=
- P(A randomly selected person learns from at least one of the sources)=
- P(A randomly selected person learns from exactly one of the sources)=

Challenge #6: A study of hand-eye coordination tested people on how quickly they could respond to a moving object on a screen. 30% of the people responded in less than 0.3s; 60% in 0.5s or less; and 5% took more than 0.8s. Determine the following probabilities:

- P(A randomly selected person from this group will take 0.8s or less)
- P(A randomly selected person from this group will take longer than 0.5)
- P(A randomly selected person from this group will take between 0.3 and 0.5 inclusive)

A survey of 200 people indicated that 60 learn from the newspaper, 50 from the TV and 30 from both sources. What is the probability of each event?

Venn Diagram

108. P(A randomly selected person learns from the newspaper and TV)=

109. P(A randomly selected person learns from at least one of the sources)=

110. P(A randomly selected person learns from exactly one of the sources)=

A study of hand-eye coordination tested people on how quickly they could respond to a moving object on a screen. 30% of the people responded in less than 0.3s; 60% in 0.5s or less; and 5% took more than 0.8s. What is the probability of each event?

Venn Diagram

111. P(A randomly selected person from this group will take 0.8s or less)

112. P(A randomly selected person from this group will take longer than 0.5)

113. P(A randomly selected person from this group will take between 0.3 and 0.5 inclusive)

A study of hand-eye coordination tested people on how quickly they could respond to a moving object on a screen. 25% of the people responded in less than 0.3s; 55% in 0.5s or less; and 10% took more than 0.8s. What is the probability of each event?

Venn Diagram

114. P(A randomly selected person from this group will take 0.8s or less)=

115. P(A randomly selected person from this group will take longer than 0.5)=

116. P(A randomly selected person from this group will take between 0.3 and 0.5 inclusive)

Challenge #7: In a recent survey of grade 12 students, it was found that 70% took math and 50% took chemistry. Determine the chance of each event. If 80% took math or chemistry, what percent of students took math only?

Calculate the percentage.

Venn Diagram

117. In a recent survey of grade 12 students, it was found that 70% took math and 50% took chemistry. Determine the chance of each event. If 80% took math or chemistry, what percent of students took math only?

118. Chemistry only?

119. In a recent survey of grade 12 students, it was found that 70% took math and 50% took chemistry. Determine the chance of each event. If 80% took math or chemistry, what percent of students took both math and chemistry?

120. Neither?

Venn Diagram

121. In a recent survey of grade 12 students, it was found that 65% took math and 45% took chemistry. If 30% took math and chemistry, what percent of students took chemistry only?

122. Math Only?

123. In a recent survey of grade 12 students, it was found that 65% took math and 45% took chemistry. If 30% took math and chemistry, what percent of students took neither math nor chemistry?

124. Took both?

Challenge #8: Determine the probability of flipping three heads in three flips.

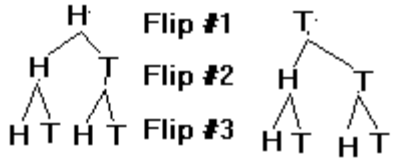
Challenge #9: Determine the probability of flipping one head in three flips.

Dependent and Independent Probabilities

Complement	The complement of an event happening is the probability that it does not happen • $P(winning) = 0.8$ & $P(\overline{winning}) = P(NotWinning) = 0.2$ • Using the complement is often helpful when solving an "at least" or "at most question".
$P(a)$ vs $p(\overline{a})$	• $P(a)$ →Probability that event "a" happens • $p(\overline{a})$ →Probability that even "a" does not happen • $P(a)\ +\ p(\overline{a})\ =1$ OR $P(a)\ =1 -\ p(\overline{a})$

A coin is flipped 3 times. Determine the probabilities as fractions in lowest terms.

Here are the possible outcomes:



H	H	H	H	T	T	T	T
H	H	T	T	H	H	T	T
H	T	H	T	H	T	H	T

125. What is the probability of flipping 3 heads?

126. What is the probability of flipping 1 head in 3 flips?

127. What is the probability of flipping 2 heads in 3 flips?

128. What is the probability of flipping at least 1 head?

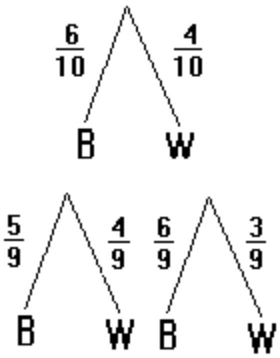
Challenge #10: Timmy has a bag full of marbles. There are 6 black marbles and 4 white marbles in the bag. Determine the following probabilities if he removes 2 marbles from the bag without replacement. Determine the probability that both marbles are black.

Challenge #11: Timmy has a bag full of marbles. There are 6 black marbles and 4 white marbles in the bag. He takes out one marble, looks at it, puts it back in the bag and then randomly draws another marble. Determine the probability that exactly one of marbles is black.

Dependent probabilities

Timmy has a bag full of marbles. There are 6 black marbles and 4 white marbles in the bag. Determine the following probabilities if he removes 2 marbles from the bag without replacement.

Tree diagram:



129. P(Both are black)=

0.333

130. P(Exactly 1 is black)=

0.533

131. What is the probability that at least 1 marble will be black?

0.867

Independent probabilities

Timmy has a bag full of marbles. There are 6 black marbles and 4 white marbles in the bag. He takes out one marble, looks at it, puts it back in the bag and then randomly draws another marble.

Tree diagram:

132. P(Both are black)=

0.36

133. P(Exactly 1 is black)=

0.48

134. What is the probability that at least 1 marble will be black?

0.84

Calculate the following probabilities. Round your answer to 3 decimals.

Timmy has a bag full of marbles. There are 3 black marbles, 4 white marbles and 3 striped marble in the bag. He removes 2 marbles one at a time without replacement.

135. P(Both are black)=

0.067

136. P(Exactly 1 is black)=

0.467

137. P(1B & 1S)=

0.2

138. P(1W & 1S)=

0.267

Timmy has a bag full of marbles. There are 3 black marbles, 4 white marbles and 3 striped marble in the bag. He takes out one marble, looks at it, puts it back and then randomly draws another marble.

139. P(Both are black)=

0.09

140. P(Exactly 1 is black)=

0.42

141. P(1B & 1S)=

0.18

142. P(1W & 1S)=

0.24

2 cards are removed from a deck of 52 cards without replacement. Determine the following.

143. P(1 heart & 1 spade)=

Remember:
P(1H & 1 S) =P(1H & 1S) or P(1S & 1H)

0.127

144. P(1 five & 1 four)=

0.012

145. P(1 face card & 1 ace)=

0.036

146. P(1 ten & 1 two)=

0.012

147. P(1 red & 1 black)=

0.510

148. P(1 face card & 1 six)=

0.036

Calculate the following probabilities. Round your answer to 3 decimals.

A biased coin with P(Heads)=0.7 was tossed 3 times. Determine the following.

149. Tree diagram

150. P(H,T,T in that order)=

0.063

151. P(H,T,H in that order)=

0.147

152. P(T,T,T)=

0.027

153. P(1H&2T)=

0.189

154. P(2H &1T)=

0.441

155. What is the probability of flipping at least 1 head?

0.973

Challenge #13: The probability that you are late for class is 0.2. The probability that your teacher is late is 0.1. Determine the following probabilities:

- P(both on time)=
- P(1 of you is late)=

Challenge #14: In 5 tosses of a coin, the first 2 tosses resulted in 2 heads. What is the probability that the 5 tosses will produce exactly 3 heads?

Calculate the following probabilities.

The probability that you are late for class is 0.2. The probability that your teacher is late is 0.1. .If these events are independent, determine the following probabilities.

156. Tree Diagram

157. P(both on time)=

0.72

158. P(1 of you is late)=

0.26

The probability that a student completes her math HW is 0.6. The probability that she completes her French HW is 0.3. If these events are independent, determine the following probabilities.

159. Tree Diagram

160. P(No HW done)=

0.28

161. P(Only 1 complete)=

0.54

162. P(both complete)=

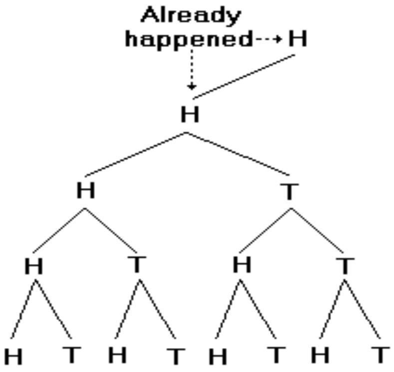
0.18

Calculate the following probabilities. Round your answer to 3 decimals.

163. In 5 tosses of a coin, the first 2 tosses resulted in 2 heads. What is the probability that the 5 tosses will produce exactly 3 heads?

Solution:

P(1H,2T|2H)=What is the probability that a third head will be flipped after 2 have already occurred. Because two heads have already occurred, we will solve this as a 3 flip question rather than a 5 flip question→P(1H, 2T).



P(1H, 2T|HH)=

=P(HTT|HH) or P(THT|HH) or P(TTH|HH)=

$$= \frac{1 \cdot 1 \cdot 1}{2 \cdot 2 \cdot 2} + \frac{1 \cdot 1 \cdot 1}{2 \cdot 2 \cdot 2} + \frac{1 \cdot 1 \cdot 1}{2 \cdot 2 \cdot 2} = \frac{3}{8}$$

164. In 7 tosses of a coin, the first 5 tosses resulted in 3 tails and 2 heads. What is the probability that the 7 tosses will produce exactly 3 heads?

0.5

165. In 80 tosses of a coin, the first 78 tosses resulted in 78 tails. What is the probability that the 80 tosses will produce exactly 2 heads?

0.25

166. Timmy is planning to role a die 6 times. After he rolled 2 fives and 2 threes, he wonders what the probability that the 6 tosses will result in 3 fives and 3 threes.

0.056

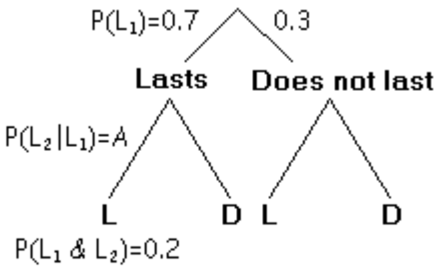
167. Timmy is planning to role a die 6 times. After he rolled 2 fives and 2 threes, he wonders what the probability that the 6 tosses will result in 2 fives and 4 threes.

0.0278

Challenge #15: The probability that a battery will last one month is 0.7 and that it will last 2 months is 0.2. At the end of the first month, what is the probability that the battery will also last until the end of the 2nd month?

Challenge #16: There is a 5% chance that a car will malfunction in the 1st year and a 10% chance in the first 2 years. What is the probability that the car lasts the first year but breaks down in the 2nd year?

168. The probability that a battery will last one month is 0.7 and that it will last 2 months is 0.2. At the end of the first month, what is the probability that the battery will also last until the end of the 2nd month?



P(Lasts 2 months)=P(Lasts 1st &2nd)

$=P(L_1)$ and $P(L_2|L_1)$

$0.2=(0.7)(A)$

$A=0.2857$

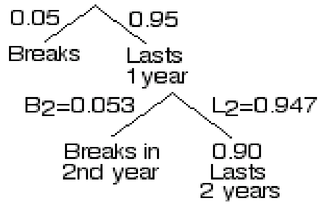
169. The probability that a battery will last one month is 0.8 and that it will last 3 months is 0.4. At the end of the first month, what is the probability that the battery will also last until the end of the 3rd month?

0.5

170. The probability that your new I-Book lasts 3 years is 0.9 and that it will last 5 years is 0.3. After 3 years, what is the chance that your I-Book will last another 2 years?

0.333

171. There is a 5% chance that a car will malfunction in the 1st year and a 10% chance in the first 2 years. What is the probability that the car lasts the first year but breaks down in the 2nd year?



P(lasts 2 years)=P(lasts 1st) & P(lasts 2nd)

$0.9=(0.95)(L_2)$

$L_2=0.947$

P(Breaks in the 2nd year)=1-P(lasts 2nd)

$B_2=1-0.947$

0.053

172. The probability that your new I-Book last 3 years is 0.9 and that it will last 5 years is 0.3. After 3 years what is the chance that your I-Book will break down in the next two years?

0.667

173. The probability that a battery will last one month is 0.6 and that it will last 2 months is 0.3 At the end of the first month, what is the probability that the battery will also last until the end of the 2nd month?

0.5

Determine the following probabilities. Round your answer to 3 decimals.

If a fair six-sided die is tossed twice, determine the following probabilities:

174. Determine the probability that the first toss is greater than 2 and the second is less than 2.

0.111

175. Determine the probability that the first toss is greater than 5 and the second is less than 5.

0.111

176. Determine the probability that the first toss is greater than 4 and the second is less than 4.

0.167

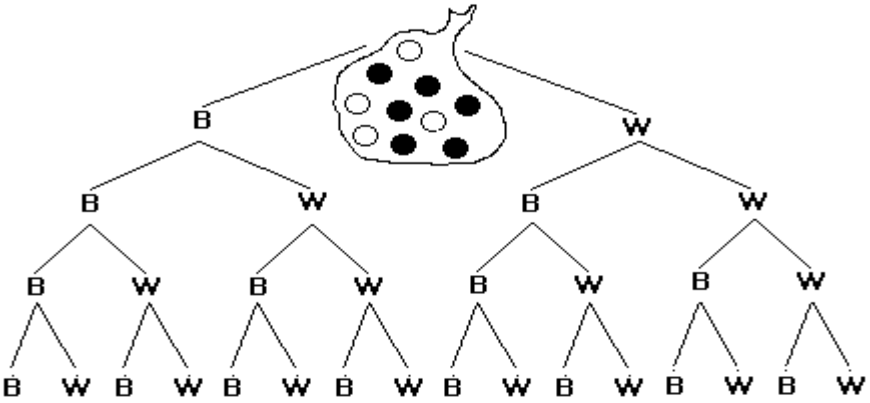
177. A bag contains 5 red balls and 10 black balls. Two balls are drawn from the bag, one after the other, without replacement. Determine the probability that one of each is chosen.

178. A bag contains 5 red balls and n black balls. Two balls are drawn from the bag, one after the other, without replacement. Determine an expression to represent the probability that one of each is chosen.

179. A bag contains x red balls and 10 black balls. Two balls are drawn from the bag, one after the other, without replacement. Determine an expression to represent the probability that one of each is chosen.

$_nC_r$, Tree Diagrams and Probabilities.

Timmy takes 4 marbles out of the bag at the same time. Determine the probability for the following events: P(4B), P(3B), P(2B), P(1B), P(0B).



	B B B B	B B B W	B B W B	B B W W	B W B B	B W B W	B W W B	B W W W	W B B B	W B B W	W B W B	W B W W	W W B B	W W B W	W W W B	W W W W	
# OF BLACK	4	3	3	2	3	2	2	1	3	2	2	1	2	1	1	0	**# OF BLACK**

How many pathways to 4 Bs?	*How many pathways to 3 Bs?*	*How many pathways to 2 Bs?*	*How many pathways to 1 B?*	*How many pathways to 0 Bs?*
$_4C_4=1$	$_4C_3=4$	$_4C_2=6$	$_4C_1=4$	$_4C_0=1$
BBBB $\frac{6}{10}\times\frac{5}{9}\times\frac{4}{8}\times\frac{3}{7}=\frac{1}{14}$	*BBBW* $\frac{6}{10}\times\frac{5}{9}\times\frac{4}{8}\times\frac{4}{7}=\frac{2}{21}$ *BBWB* $\frac{6}{10}\times\frac{5}{9}\times\frac{4}{8}\times\frac{4}{7}=\frac{2}{21}$ *BWBB* $\frac{6}{10}\times\frac{4}{9}\times\frac{5}{8}\times\frac{4}{7}=\frac{2}{21}$ *WBBB* $\frac{4}{10}\times\frac{6}{9}\times\frac{5}{8}\times\frac{4}{7}=\frac{2}{21}$ *The probability of each pathway is the same regardless of the order of the fractions.*	*BBWW* $\frac{6}{10}\times\frac{5}{9}\times\frac{4}{8}\times\frac{3}{7}=\frac{1}{14}$ *BWBW* $\frac{6}{10}\times\frac{4}{9}\times\frac{5}{8}\times\frac{3}{7}=\frac{1}{14}$ *BWWB* $\frac{6}{10}\times\frac{4}{9}\times\frac{3}{8}\times\frac{5}{7}=\frac{1}{14}$ *WBBW* $\frac{4}{10}\times\frac{6}{9}\times\frac{5}{8}\times\frac{3}{7}=\frac{1}{14}$ *WBWB* $\frac{4}{10}\times\frac{6}{9}\times\frac{3}{8}\times\frac{5}{7}=\frac{1}{14}$ *WWBB* $\frac{4}{10}\times\frac{3}{9}\times\frac{6}{8}\times\frac{5}{7}=\frac{1}{14}$	*BWWW* $\frac{6}{10}\times\frac{4}{9}\times\frac{3}{8}\times\frac{2}{7}=\frac{1}{35}$ *WBWW* $\frac{4}{10}\times\frac{6}{9}\times\frac{3}{8}\times\frac{2}{7}=\frac{1}{35}$ *WWBW* $\frac{4}{10}\times\frac{3}{9}\times\frac{6}{8}\times\frac{2}{7}=\frac{1}{35}$ *WWWB* $\frac{4}{10}\times\frac{3}{9}\times\frac{2}{8}\times\frac{6}{7}=\frac{1}{35}$ *The probability of each pathway is the same regardless of order of the fractions.*	*WWWW* $\frac{4}{10}\times\frac{3}{9}\times\frac{2}{8}\times\frac{1}{7}=\frac{1}{210}$
$_4C_4\left(\frac{6}{10}\frac{5}{9}\frac{4}{8}\frac{3}{7}\right)=\frac{1}{14}$	$_4C_3\left(\frac{6}{10}\frac{5}{9}\frac{4}{8}\frac{4}{7}\right)=\frac{8}{21}$	$_4C_2\left(\frac{6}{10}\frac{5}{9}\frac{4}{8}\frac{3}{7}\right)=\frac{3}{7}$	$_4C_1\left(\frac{6}{10}\frac{4}{9}\frac{3}{8}\frac{2}{7}\right)=\frac{4}{35}$	$_4C_0\left(\frac{4}{10}\frac{3}{9}\frac{2}{8}\frac{1}{7}\right)=\frac{1}{210}$

Remember: The order of fractions does not matter.

Challenge #17: Little Timmy removed 4 cards without replacement from a deck of 52 cards. Determine the probability of drawing exactly 3 out of 4 kings.

Challenge #18: A biased coin with P(Heads)=0.7 was tossed 5 times. Determine the probability that exactly 4 out of the 5 flips will land heads.

Calculate the following probabilities. Round your answer to 4 decimals.

Little Timmy removed 4 cards without replacement from a deck of 52 cards.

Tree Diagram

180. P(3Kings)=

$$_4C_3(KKK,0) =$$

$$_4C_3\,\frac{4}{52}\cdot\frac{3}{51}\cdot\frac{2}{50}\cdot\frac{48}{49} =$$

0.0007

181. P(4 Kings)=

0.0000(037)

182. P(2 Kings)=

0.0260

Little Timmy removed 4 cards without replacement from a deck of 52 cards.

183. What is the probability of picking at least 1 five?

184. omit

0.997

185. What is the probability of picking at least 1 heart?

0.696

Calculate the following probabilities. Round your answer to 3 decimals.

A biased coin with P(Heads)=0.7 was tossed 5 times. Calculate each probability.

Tree Diagram:

186. P(5H)=

0.168

187. P(4H)=

Solution:

$_nC_r$(HHHHT)=

5 flips choose 4 heads

$_5C_4$(0.7)(0.7)(0.7)(0.7)(0.3)=

=0.36015

0.360

188. P(3T)=

0.132

189. P(4T)=

0.028

190. What is the probability of flipping at least 2 heads?

0.969

191. What is the probability of flipping at least 1 head?

0.998

Calculate the following probabilities. Round your answer to 3 decimals.

5 balls are removed without replacement from a bag containing 5 red balls and 6 yellow balls.

192. Tree Diagram

193. P(3 Red)=

0.325

194. P(4 red)=

0.0649

195. P(5 red)=

0.002

196. What is the probability of picking at least 3 red?

0.39177?

197. What is the probability of picking at least 1 red?

0.987

Challenge #19: Two cards are drawn without replacement from a shuffled deck of 52 cards. What is the probaility that the second card will be an ace? Round your answer to 3 decimals.

Challenge #20: Two cards are drawn without replacement from a shuffled deck of 52 cards. What is the probaility that the first card is a heart and the second card is the king of hearts.

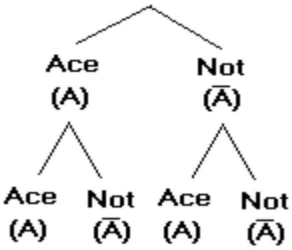
Conditional Probability

Two cards are drawn without replacement from a shuffled deck of 52 cards. What is the probaility that the second card will be the following card? Round your answer to 3 decimals.

198. An ace is the second card

Solution:

- This a conditional probability question because probabilities are conditional based on the first card chosen.
- There are 2 options for the first card, an ace or not an ace.



$$P(A_2) = P(A,A) or P(\overline{A},A)$$

$$= \frac{4}{52}\times\frac{3}{51}+\frac{48}{52}\times\frac{4}{51}$$

$$= \frac{4}{52}=\frac{1}{13}\approx 0.0769$$

199. A red five is the 2nd card

0.038

200. The king of spades is the 2nd card.

0.019

201. A face card is the 2nd card.

0.231

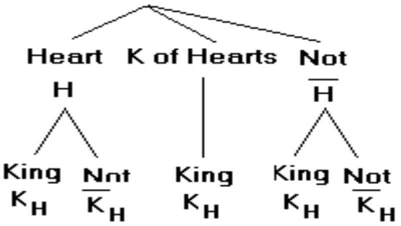
202. The 10 of clubs is the second card

0.019

Two cards are drawn without replacement from a shuffled deck of 52 cards. Determine the following probabilities. Round your answer to 3 decimals.

203. 1st →Heart, 2nd →King of Hearts

Solution:



$$P(H\ 1^{st}\rightarrow K_H\ 2^{nd}) = \frac{12}{52}\cdot\frac{1}{51}=\frac{12}{2652}$$

The first card can not be the king of hearts. That is why there are 12 options and not 13.

204. 1st →Red, 2nd →8 of Hearts

0.009

205. 1st →face Card, 2nd →K of Hearts

0.004

Color	Suit	Non-Face Cards										Face Cards		
Red	Hearts	A	2	3	4	5	6	7	8	9	10	J	Q	K
Red	Diamonds	A	2	3	4	5	6	7	8	9	10	J	Q	K
Black	Clubs	A	2	3	4	5	6	7	8	9	10	J	Q	K
Black	Spades	A	2	3	4	5	6	7	8	9	10	J	Q	K

Two cards are drawn without replacement from a shuffled deck of 52 cards. What is the probability that the second card is each card? Round your answer to 3 decimals.

206. A 10 is the 2nd card.

0.077

207. A face card is the 2nd card.

0.231

208. A red card is the 2nd card.

0.5

209. An even number is the 2nd card.

0.385

210. The 5 of clubs is the 2nd card.

0.019

211. A diamond is the 2nd card.

0.25

Two cards are drawn without replacement from a shuffled deck of 52 cards. Determine the following probabilities. Round your answer to 3 decimals.

212. 1st →Face, 2nd →King

0.017

213. 1st →Red, 2nd →Red Face

0.057

214. 1st →Club, 2nd →8 of Clubs

0.005

Understanding Bayes Law

Challenge #19: 100 grade 12 students were surveyed. 60 were boys and 40 were girls. 80% of the boys and 70% of the girls who were surveyed said they love chocolate ice cream.

Examine the following two questions and calculate each probability below.

- Determine the probability that a randomly selected student likes chocolate ice cream.(215)
- A student is chosen who likes chocolate ice cream, determine the probability that selected student is a boy.(219)

Can you see the difference between these two questions?

215. What is the probability that a randomly selected math 12 student loves math 12?

0.76

216. How many boys surveyed love chocolate ice cream?

217. How many girls surveyed love chocolate ice cream?

218. How many of the students surveyed love chocolate ice cream?

219. A student is chosen who likes chocolate ice cream, determine the probability that selected student is a boy.

This is Bayes Law

A student is randomly chosen and loves math. What is the probability that it is a boy?

$$\frac{\text{Number of boys who love chocolate ice cream=}}{\text{Number of students who love chocolate ice cream=}}$$

220. P(Student who love chocolate ice cream is a boy)=0.63158

221. A computer supply store buys 40% of their computer chips from Bigchips and 60% from Fastchips. On average, 6% of the Bigchips are faulty and 5% of the Fastchips are faulty. If a randomly selected chip is faulty, what is the probability that Bigchips made it?

0.444

Calculate the following probabilities. Round your answer to 3 decimals.

There are 100 boys and 120 girls in the grade 12 year. 20 boys and 30 girls have no siblings. A student is randomly selected.

222. Tree Diagram

223. What is the probability that the student has no siblings?

0.227

224. A student is chosen who has no siblings. What is the probability that the student is a girl?

0.6

There are 100 boys and 50 girls in the grade 12 year. 40 boys and 10 girls have no siblings. A student is randomly selected.

225. Tree Diagram

226. What is the probability that the student has no siblings?

0.333

227. A student is chosen who has no siblings. What is the probability that the student is a girl?

0.2

Calculate the following probabilities.

A new medical test for tsprayitis is 95% accurate. Suppose 8% of the population have tsprayitis. What is the probability of each event?

228. Tree Diagram

229. A randomly selected person will test negative.

0.878

230. A person tests negative, calculate the probability that they have actually have tsprayitis.

0.005

231. A person tests positive, caculate the probability that they do not have tsprayitis.

0.377

A new medical test for bad hair is 90% accurate. Suppose 20% of the population has bad hair. What is the probability of each event?

232. Tree Diagram

233. A randomly selected person will test negative.

0.74

234. A person tests negative, calculate the probability that they actually have bad hair.

0.027

235. A person tests positive, calculate the probability that they actually have good hair.

0.308

A new medical test for Simonsonrea is 80% accurate. Suppose 30% of the population has Siminsonrea. What is the probability of each event?

236. Tree Diagram

237. A randomly selected person will test negative.

0.62

238. A person tests negative, caclulate the probablity that they do not have Simonsonrea.

0.903

239. A person tests positive, calculate the probability that they actually have Simonsonrea.

0.632

240. There are two bags full of marbles. One bag is black and one bag is green. The black bag contains 6 white balls and 2 red balls. The green bag contains 2 red balls and 6 white balls. A marble is randomly selected from the black bag and placed in the green bag. A marble is then randomly selected from the green bag.

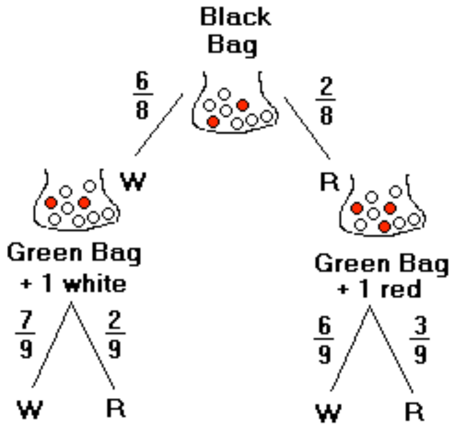
Challenge #21: What is the probability that the ball selected from the green bag is white?

Challenge #22: If a white ball is selected from the green bag, what is the probability that a red ball was transferred from the black bag to the green bag?

Calculate the following probabilities.

Little Timmy has a black bag and a green bag. The black bag contains 6 white balls and 2 red balls. The green bag contains 2 red balls and 6 white balls. A marble is randomly selected from the black bag and placed in the green bag. A marble is then randomly selected from the green bag.

241. What is the probability that the ball selected from the green bag is white?



Solution:
There are 2 options:
1→Both balls are white
2→1st is not white, but 2nd is

P(2nd ball is white)=

P(W|W) or P(W|R)=

$$\frac{6}{8}\cdot\frac{7}{9}+\frac{2}{8}\cdot\frac{6}{9}=\frac{54}{72}=\frac{3}{4}$$

242. If a white ball is selected from the green bag, what is the probability that a red ball was transferred from the black bag to the green bag?

0.222

243. What is the probability that the ball selected from the green bag is red?

0.25

244. If a red ball is selected from the green bag, what is the probability that a red ball was transferred from the black bag to the green bag?

0.333?

245. If a white ball is selected from the green bag, what is the probability that a white ball was transferred from the black bag to the green bag?

0.778

Little Timmy has Black Bag and a Green bag. The black bag contains 4 red ball and 6 white balls. The green bag contains 3 red balls and 2 white balls.

246. A marble is randomly selected from the black bag and placed in the green bag. A marble is then randomly selected from the green bag. What is the probability that the ball selected from the green bag is white?

0.433

247. If a white ball is selected from the green bag, what is the probability that a white marble was transferred from the black bag to the green bag?

0.692

Calculate the following probabilities. Round your answer to 3 decimals.

Little Timmy has black bag and a green bag. The black bag contains 7 red marbles and 3 white marbles. The green bag contains 3 red marbles and 1 white marbles.

248. A marble is randomly selected from the black bag and placed in the green bag. A marble is then randomly selected from the green bag. What is the probability that the marble selected from the green bag is white?

0.26

249. If a white marble is selected from the green bag, what is the probability that a white marble was transferred from the black bag to the green bag?

0.462

250. A black bag contains 6 red balls and 4 green balls. A white bag contains 4 red balls and 1 green ball. A regular die is rolled and if a 1 comes up, a ball is selected randomly from the black bag. Otherwise, a ball will be selected randomly from the white bag. If a green ball is selected, what is the probability that it came from the white bag?

0.714

Permutations, Combinations and Probabilities

Color	Suit	Non-Face Cards										Face Cards		
Red	Hearts	A	2	3	4	5	6	7	8	9	10	J	Q	K
Red	Diamonds	A	2	3	4	5	6	7	8	9	10	J	Q	K
Black	Clubs	A	2	3	4	5	6	7	8	9	10	J	Q	K
Black	Spades	A	2	3	4	5	6	7	8	9	10	J	Q	K

Calculate the number of possible 5 card hands.

Find the number of 5 card hands made up of:	Formula	Number
251. 1 five, 1 seven, 3 other cards→	${}_4C_1 \times {}_4C_1 \times {}_{44}C_3$ 4 "5"s→choose 1, 4 "7"s→ choose 1, 44 cards that are not 5s or 7s→ choose 3	211904
252. Any cards		2598960
253. Hearts only		1287
254. 2 fours, 3 fives		24
255. 4 kings, 1 ace		4

Calculate the following probabilities. Round your answer to 3 decimals.

Find the probability of the following 5 card hands:	Formula	Probability
256. 3 fives, 2 other cards	$\frac{{}_4C_3 \times {}_{48}C_2}{{}_{52}C_5}$	0.002
257. 3 black,2 hearts		0.078
258. 1 heart,3 clubs, 5 of spades		0.001
259. 4 red, king of clubs		0.0057
260. AH,KH,QH,JH,10H		0.000(0004)
261. 2 fours, 2 fives, any other card		0.001
262. 4 Sevens, any other card		0.000(02)?

4 people are randomly selected from a group of 8 boys and 6 girls to represent the school in the debate championships.

Challenge #20: How many unique groups can be created from the group of 14 students?

1001

How many unique groups can be created with exactly 3 girls?

160

Challenge #21: Determine the probability that exactly 3 of the four students chosen are girls.

Calculate the number of possibilities. Round your answer to 3 decimals.
A bowling team is made up of 6 boys and 4 girls.

263. How many different groups of 4 can be sent to the city championships if 2 boys and 2 girls have to go?

Solution

Choose 2 boys from 6→ ${}_6C_2$

Choose 2 girls from 4→ ${}_4C_2$

Multiply the results

${}_6C_2 \times {}_4C_2 =$

90

264. How many different groups of 4 can be sent to the city championships?

210

265. How many different groups of 4 can be sent to the city championships if 1 boy and 3 girls have to go?

24

Calculate the following probabilities.
4 people are to be randomly selected from a group of 8 boys and 6 girls.

266. P(Exactly 3 girls)=

0.160

267. P(All boys)=

0.070

268. P(Alternate in gender)=

0.1399

6 people are randomly selected from a group of 10 boys and 12 girls.

269. P(Exactly 5 boys)=

0.041

270. P(All boys)=

0.003

271. P(Alternate in gender)=

0.035

Calculate the following probabilities.

Of the 20 students on this year's student council, 14 are girls. Five students from the council are to be randomly selected to participate in a student exchange to Huddy-Huddy-ville.

272. What is the probability that at least 2 girls are selected?

0.986

273. What is the probability that at least 1 girl is selected?

1.000← 0.9999355

274. What is the probability that at most 4 girls are selected?

0.871

Of the 15 students on this year's student council, 8 are girls. 6 students from the council are to be randomly selected to participate in a student exchange to Huddy-H-STATE.

275. What is the probability that at least 2 girls are selected?

0.965

276. What is the probability that at least 1 girl is selected?

0.999

277. What is the probability that at most 5 girls are selected?

0.994

Color	Suit	Non-Face Cards										Face Cards		
Red	Hearts	A	2	3	4	5	6	7	8	9	10	J	Q	K
Red	Diamonds	A	2	3	4	5	6	7	8	9	10	J	Q	K
Black	Clubs	A	2	3	4	5	6	7	8	9	10	J	Q	K
Black	Spades	A	2	3	4	5	6	7	8	9	10	J	Q	K

Use the following 5 card hand descriptions to help you answer the questions below.

Royal Flush	A,K,Q,J,10 same suit	Full House	a pair and 3 of a kind	3 of a kind	3 cards same rank
Straight Flush	Run of 5 same suit	Flush	All 5 same suit	2 pair	Two pairs
Four of a kind	4 cards same rank	Straight	A run of 5, any suit	1 pair	2 cards same rank

Calculate the following probabilities and round your answer to 6 decimals.

5 cards are dealt from a shuffled deck of 52 cards. What is the probability of each event?

278. A full house made up of a pair of aces and 3 kings.

0.000009

279. A flush made up of all hearts.

0.000495

280. A royal flush made up of Hearts. (A,K,Q,J,10)

0.000000(4)

281. Four of a kind made up of 4 aces.

0.000018

282. Four of a kind made up of 4 queens and an ace.

0.000002

Calculate the following probabilities.

A pizza store offers 15 different toppings on its pizzas.

283. Suppose a pizza has 5 toppings. How many different pizzas can be made?

3003

284. What is the probability that the 5 randomly selected toppings will include ham, pineapple and feta cheese?

Solution:
3 specific toppings choose 3
→${}_3C_3$
12 other toppings choose 2
→${}_{12}C_2$

$$P(H,PA,F)=\frac{{}_3C_3\times{}_{12}C_2}{{}_{15}C_5}=$$

0.022

285. What is the probability that the 5 randomly selected toppings will include pepperoni and bacon?

0.095

A pizza store offers 12 different toppings on its pizzas.

286. Suppose a pizza has 4 toppings. How many different pizzas can be made?

495

287. What is the probability that the 4 randomly selected toppings will include turkey, beets and spinach?

0.018

288. What is the probability that the 4 randomly selected toppings will include ground beef and sausage?

0.091

Calculate the following probabilities.

Allie, Betsy, Colleen, Debby and Edith are finalists in radio prize giveaway. 2 of them will be randomly selected to win prizes. What is the probability of each event?

289. Allie is one of the winners.

0.4

290. Allie is a winner but Betsy is not a winner.

0.3

291. Debby and Edith do not win

0.3

Detailed solutions are available in the answer key.

Enriched: Determine the probabilities and round your answer to 3 decimals.

292. P(4 of a kind in a 5 card hand)=

Solution

Picking the 4 equal cards

Pick Rank→13 options pick 1 → $_{13}C_1$

Pick Suit→4 options pick 4 → $_4C_4$

Picking other 1 card→ must be different

Pick Rank→12 options pick 1 → $_{12}C_1$

Pick suit →4 options pick 1 → $_4C_1$

P(4 of a kind)= $\dfrac{_{13}C_1 \times {_4C_4} \times {_{12}C_1} \times {_4C_1}}{_{52}C_5} = 0.00024$

293. P(Full house)= P(3 of a kind and 2 of a kind)=

0.00144

294. P(2 pair)=

0.04754

295. P(3 of a kind)=

0.0211

296. P(1 pair)=

0.4226

Calculate the following probabilities.

Mr. Spray has 5 different Probabilities Retests to give randomly to his students. Two boys and 3 girls want to retest. Determine the probability of each event.

297. The 3 girls take the same test

0.04

298. Exactly 3 people take the same test.

0.256

299. All 5 people take a different test.

0.0384

300. All 5 of them take the same test.

0.002

301. All the boys write the same test. All the girls write the same test but it is different than the boys.

0.0064

302. Exactly 2 people take the same test and the 3 remaining take a different test.

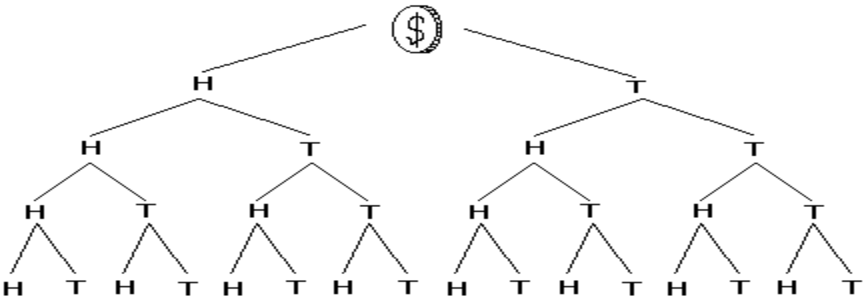
0.384

Binomial Theorem and Probabilities

The Binomial Theorem ${}_nC_r(a)^r(\bar{a})^{n-r}$	n is the # of options, r is the # chosen, a is the probability that an even occurs.

Calculate the following probabilities.

Timmy flips a biased coin and flips it 4 times. The probability of heads is 0.6. Determine the following probabilities P(4H), P(3H), P(2H), P(1H), P(0H).



	H	H	H	H	H	H	H	H	T	T	T	T	T	T	T	T	
	H	H	H	H	T	T	T	T	H	H	H	H	T	T	T	T	
	H	H	T	T	H	H	T	T	H	H	T	T	H	H	T	T	
	H	T	H	T	H	T	H	T	H	T	H	T	H	T	H	T	
# OFHEADS	4	3	3	2	3	2	2	1	3	2	2	1	2	1	1	0	# OF HEADS

P(4 HEADS)= ${}_4C_4\ (0.6)^4=0.1296$	*P(3 HEADS)=* ${}_4C_3\ (0.6)^3(0.4)^1=0.3456$	*P(2 HEADS)=* ${}_4C_2\ (0.6)^2(0.4)^2=0.3456$	*P(1 HEAD)=* ${}_4C_1(0.6)^1(0.4)^3=0.1536$	*P(0 HEADS)=* ${}_4C_0\ (0.4)^4=0.0256$
How many pathways to 4 Hs ${}_4C_4=1$ *HHHH* *(0.6)(0.6)(0.6)(0.6)=0.1296* *NOTICE:* $(0.6)^4=0.1296$	*How many pathways to 3 Hs* ${}_4C_3=4$ *HHHT* *(0.6)(0.6)(0.6)(0.4)=0.0864* *HHTH* *(0.6)(0.6)(0.4)(0.6)= 0.0864* *HTHH* *(0.6)(0.4)(0.6) (0.6)=0.0864* *THHH* *(0.4)(0.6)(0.6) (0.6)= 0.0864* *NOTICE:* $(0.6)^3(0.4)^1=0.0864$	*How many pathways to 2 Hs* ${}_4C_2=6$ *HHTT* *(0.6)(0.6)(0.4)(0.4)=0.0576* *HTHT* *(0.6)(0.4)(0.6)(0.4)=0.0576* *HTTH* *(0.6)(0.4)(0.4)(0.6)=0.0576* *THHT* *(0.4)(0.6)(0.6)(0.4)=0.0576* *THTH* *(0.4)(0.6)(0.4)(0.6)=0.0576* *TTHH* *(0.4)(0.4)(0.6)(0.6)=0.0576* *NOTICE:* $(0.6)^2(0.4)^2=0.0576$	*How many pathways to 1 H* ${}_4C_1=4$ *HTTT* *(0.6)(0.4)(0.4)(0.4)=0.0384* *THTT* *(0.4)(0.6)(0.4)(0.4)=0.0384* *TTHT* *(0.4)(0.4)(0.6)(0.4)=0.0384* *TTTH* *(0.4)(0.4)(0.4)(0.6)=0.0384* *NOTICE:* $(0.6)^1(0.4)^3=0.0384$	*How many pathways to 0 Hs* ${}_4C_0=1$ *TTTT* *(0.4)(0.4)(0.4)(0.4)=0.0256* *NOTICE:* $(0.4)^4=0.0256$
	Remember: Order of fractions does not matter.			

Calculate the following probabilities. Round your answer to 3 decimals.

A fair coin is tossed 5 times. Determine the probability of each event.

303. P(0 heads)=	304. P(1 heads)=	305. P(2 heads)=	306. P(3 heads)=	307. P(4 head)=	308. P(5 heads)=
0.031	0.156	0.313	0.313	0.156	0.031

309. P(at least 1 head)=	310. P(at most 4 heads)=
0.969	0.969

••How can I get my calculator to do the same thing••

A coin is tossed 5 times. Determine the probability of each event.

***Pdfbin(N,P,S)=** N=Number of events P=Probability of success S=Number of successful events Calculates probability of a single event.	311. P(4 heads)= Pdfbin(5,0.5,4)= 0.156
	312. P(3 Heads)= 0.313
	313. P(0 Heads)= 0.031
***Cdfbin(N,P,S)=** N=Number of events P=Probability of success S=The sum of the first S successful events Calculates sum of probabilities.	314. P(At most 3 heads)= Cdfbin(5,0.5,3)= 0.813
	315. P(At most 4 heads)= 0.969
	316. P(At least 4 least 4 heads)= 0.188

*Press STAT→ Choose F DISTRIBUTION→ Choose (10)pdfbin or (11)cdfbin.
IMPORTANT→These options can only be used to calculate probabilities for a binomial distribution.

A biased coin is tossed 5 times, where P(T)=.6. Determine the probability of each event.

317. 5 tails	318. 4 tails	319. 3 tails	320. 3 heads	321. 4 heads	322. 5 heads
0.078	0.259	0.346	0.230	0.077	0.010

323. If you have at least 4 tails, what is the probability that you have 5?	324. omit
0.231	

Challenge #22: The probability of the Vancouver Canucks winning any game against any opponent is 0.60 Determine the probability that the Canucks win 6 of the next seven games (Any order).

Challenge #23: The probability of a Canucks' win in any game of a best of seven game series is 60%. Determine the probability that the Canucks win a best of 7 series in exactly 6 games.

Calculate the following probabilities. Round your answer to 3 decimals.

The probability of the Vancouver Canucks winning any game against any opponent is 60%.

325. P(Canucks win 6 of 7 games)=

0.131

326. P(Canucks win 5 of 7games)=

0.261

327. P(Canucks win 7 of 7games)=

0.0279

The probability of a Canucks' win in any game of a best of seven game series is 60%.

328. P(Canucks win in the 6th game)=

Solution: This is a tricky one

- To win in exactly 6 games means that the 6th game must be a win.
- This means that the 1st 5 games resulted in 3W & 2L
- P(3W,2L) and P(Win 6th)

$_5C_3(0.6)^3(0.4)^2 \times (0.6) =$

0.207

329. P(Canucks win the best of 7 series in the 5th game)=

0.207

330. P(Canucks win the best of 7 series in the 7th game)=

0.166

The probability of a Canucks' win in any game of a best of seven game series is 60%.

331. P(Canucks lose in 4 games)=

0.026

332. P(Canucks lose in 7 games)=

0.111

333. P(Canucks lose in 6 games)=

0.092

Questions 340-360 are good review for the test!

334. A biased coin with P(Heads)=0.7 was tossed 5 times. Find P(3H)=

335. 2 regular die are rolled. Find P(sum is at most 10)=

336. A biased coin with P(Heads)=0.7 was tossed 3 times. Find P(2H & 1T)=

337. The probability that your new I-Book last 3 years is 0.9 and that it will last 5 years is 0.3. After 3 years what is the chance that your I-Book will break down in the next two years?

338. In a recent survey of grade 12 students, it was found that 70% took math and 50% took chemistry. If 80% took math or chemistry, what percent of students took math only?

339. Timmy is planning to role a die 6 times. After he rolled 2 fives and 2 threes, he wonders what the probability will be that the 6 roles will result in 2 fives and 4 threes.

340. A regular die is rolled twice. Determine the probability that the first toss is greater than 4 and the second is less than 4.

341. 80% of girls want to go into business and 30% want to go into education. 20% want neither. Find P(Girl pursue at most one career)=

342. A certain experiment has only 4 outcomes: A, B, C, D. The probability of each outcome is twice the probability of the following outcome. Find P(B)=

343. The probability of the Vancouver Canucks winning any game against any opponent is 60%. Find P(Canucks win 5 of 7games)=

344. A pizza store offers 12 different toppings on its pizzas. What is the probability that the 4 randomly selected toppings will include ground beef and sausage?

345. (optional) A six-sided die is rolled 100 times. Write a formula using cdfbin or pdfbin for P(at most 99 sixes)

346. There are 3 black marbles, 4 white marbles and 3 striped marble in the bag. He takes out one marble, looks at it, puts it back and then randomly draws another marble. Find P(1W &1S)=

347. Of the 15 students on this year's student council, 8 are girls. 6 students from the council are to be randomly selected to participate in a student exchange to Huddy-H-STATE. Find P(at least 1 girl is selected).

348. A computer supply store buys 40% of computer chips from X-Chips and 60% from Y-Chips. On average 6% of the X-Chips are faulty and 5% of the Y-Chips are faulty. If a randomly selected chip is faulty, what is the probability that X-Chips made it?

349. A single card is selected from a deck of 52 cards. Determine P(Club or jack)=

350. A biased 4-sided die is weighted so that it returns a 1, 40% of the time. Determine P(2)=

351. Find P(2 fours, 2 fives, any other card) when 5 cards are drawn from a regular deck.

352. Two cards are drawn without replacement from a shuffled deck of 52 cards. Find P(1st →Club, 2nd →8 of Clubs).

353. The probability of the Vancouver Canucks winning any game against any opponent is 60%. Find P(Canucks win a best of 7 game series in 7 games)=

354. Two cards are drawn without replacement from a shuffled deck of 52 cards. Find P(King of Spades is the 2nd card).

Answers can be found in the answer key.

Answers to Probabilities Notes

1) 0.5	2) Answers will vary with each experiment, but the probability will be approximately 0.5.						
3) $\frac{1}{4}$	4) $\frac{1}{2}$	5) $\frac{1}{4}$	6) 1	7) 0	8) $\frac{11}{26}$	9) $\frac{3}{52}$	
10) $\frac{1}{26}$	11) $\frac{3}{4}$	12) $\frac{12}{13}$	13) $\frac{4}{13}$	14) $\frac{1}{13}$	15) $\frac{10}{13}$	16) $\frac{4}{13}$	
	17) h1,h2,h3,h4,h5,h6 t1,t2,t3,t4,t5,t6	18) $\frac{1}{12}$	19) $\frac{1}{4}$	20) $\frac{7}{12}$	21) $\frac{3}{4}$		
22) 11,12,13,14,21,22,23,24 31,32,33,34,41,42,43,44	23) $\frac{1}{8}$	24) $\frac{3}{4}$	25) $\frac{9}{16}$	26) $\frac{1}{2}$	27) hhh,hht,hth,thh ttt,tth,tht,htt		
28) $\frac{7}{8}$	29) $\frac{1}{8}$	30) $\frac{7}{8}$		31) 0.7	32) 0.3	33) 14	34) 15
35) 0.6	36) 0.4	37) 48	38) 24	39) 0.2	40) 0.16	41) 8	42) 96
				43) deck of cards	44) $\frac{1}{2}$	45) $\frac{3}{13}$	46) $\frac{1}{4}$
	47) 50 cards since 2 cards have already been used		48) $\frac{6}{25}$	49) $\frac{12}{25}$	50) 0		51) $\frac{1}{2}$
52) $\frac{1}{6}$	53) $\frac{1}{6}$		54) $\frac{35}{36}$		55) $\frac{11}{12}$	56) 0.125	
		57) 0.03125	58) 0.0039	59) 0.0010	60) 0.0000	61) $\frac{7}{10}$	62) $\frac{5}{10}$
63) $\frac{3}{10}$	64) $\frac{5}{10}$	65) $\frac{3}{10}$	66) $\frac{9}{10}$	67) $\frac{1}{10}$	68) $\frac{7}{10}$	69) $\frac{1}{10}$	70) $\frac{6}{10}$
71) $\frac{3}{5}$	72) $\frac{3}{7}$	73) $\frac{1}{13}$	74) $\frac{1}{4}$	75) $\frac{4}{13}$	76) $\frac{9}{13}$	77) $\frac{6}{13}$	78) $\frac{4}{13}$
79) $\frac{1}{6}$	80) $\frac{8}{13}$	81) $\frac{4}{13}$	82) $\frac{7}{13}$	83) $\frac{12}{13}$	84) $\frac{9}{13}$	85) 1	86) 0
87) $\frac{10}{13}$	88) $\frac{5}{13}$	89) 0.8	90) 0.5	91) 0.9	92) 0.8	93) 0.78	
94) 0.6	95) 0.3	96) 0.1	97) 0.9	98) 0.3	99) 0.7	100) $\frac{1}{2}$	101) $\frac{4}{13}$
102) $\frac{4}{13}$	103) $\frac{15}{52}$	104) $\frac{11}{26}$	105) $\frac{4}{13}$	106) $\frac{7}{26}$	107) $\frac{1}{2}$	108) 0.15	109) 0.4
110) 0.25	111) 0.95	112) 0.4	113) 0.3	114) 0.9	115) 0.45	116) 0.3	117) 30%
118) 10%	119) 40%	120) 20%	121) 15%	122) 35%	123) 20%	124) 30%	125) $\frac{1}{8}$
126) $\frac{3}{8}$	127) $\frac{3}{8}$	128) $\frac{7}{8}$					
				129) 0.333	130) 0.533	131) 0.867	

177) $\left(\frac{5}{15}\times\frac{10}{14}\right)+\left(\frac{10}{15}\times\frac{5}{14}\right)$	178) $\left(\frac{5}{5+n}\times\frac{n}{4+n}\right)+\left(\frac{n}{5+n}\times\frac{5}{4+n}\right)$	179) $\left(\frac{x}{x+10}\times\frac{10}{x+9}\right)+\left(\frac{10}{x+10}\times\frac{9}{x+9}\right)$

Alli, Betsy, Colleen, Debby and Edith are finalists in radio prize giveaway. 2 of them will be randomly selected to win prizes. What is the probability of each event?

290) Alli is one of the winners	291) Alli is a winner but Betsy is not	292) Debby and Edith do not win
1 A one way to pick A 4 other options pick 1 $P(A)=\frac{{}_1C_1\times{}_4C_1}{{}_5C_2}=\frac{4}{10}=0.4$	1 A one way to pick A 1 B do not pick it 3 other options pick 1 $P(AnotB)=\frac{{}_1C_1\times{}_3C_1}{{}_5C_2}=\frac{3}{10}=0.3$	1 E do not pick it 1 D do not pick it 3 other options pick 2 $P(AnotB)=\frac{{}_3C_2}{{}_5C_2}=\frac{3}{10}=0.3$

NOTE: This Question can be done by listing the sample set and counting. Much easier!!

AB,AC,AD,AE BC,BD,BE CD,CE DE	AB,AC,AD,AE ~~BC,BD,BE~~ ~~CD,CE~~ ~~DE~~	~~AB~~,AC,AD,AE ~~BC,BD,BE~~ ~~CD,CE~~ ~~DE~~	AB,AC,~~AD,AE~~ BC,~~BD,BE~~ ~~CD,CE~~ ~~DE~~
Sample set	4 out of 10	3 out of 10	3 out of 10

293) Full House:

Picking 3 equal cards

Pick Rank→13 options pick 1 → ${}_{13}C_1=13$

Pick Suit→4 options pick 3 → ${}_4C_3=4$

Picking 2 equal cards→ Different than the 3 above

Pick Rank→12 options pick 1 → ${}_{12}C_1=12$

Pick suit →4 options pick 2 → ${}_4C_2=6$

$$\frac{{}_{13}C_1\cdot{}_4C_3\cdot{}_{12}C_1\cdot{}_4C_2}{{}_{52}C_5}=0.00144$$

294) 2 Pairs

Picking the 2 pairs

Pick Rank→13 options pick2 → ${}_{13}C_2=78$

Pick Suit of 1st pair→4 options pick 2 → ${}_4C_2=6$

Pick Suit of 2nd pair→4 options pick 2 → ${}_4C_2=6$

Picking the 5th card →must be different than pairs

Pick Rank→11 options pick 1 → ${}_{11}C_1=11$

Pick Suit→4 options pick 1 → ${}_4C_1=4$

$$\frac{{}_{13}C_2\cdot{}_4C_2\cdot{}_4C_2\cdot{}_{11}C_1\cdot{}_4C_1}{{}_{52}C_5}=0.4754$$

295) 3 of a kind in a 5 card hand

Picking the 3 or a kind

Pick Rank→13 options pick 1 → ${}_{13}C_1=13$

Pick Suit of pair→4 options pick 3 → ${}_4C_3=4$

Pick 2 other cards→must be different

Pick rank→12 options pick 2 → ${}_{12}C_2=66$

Pick suit→4 options pick 1 → ${}_4C_1=4$

Pick suit→4 options pick 1 → ${}_4C_1=4$

$$\frac{{}_{13}C_1\cdot{}_4C_3\cdot{}_{12}C_2\cdot{}_4C_1\cdot{}_4C_1}{{}_{52}C_5}=0.0211$$

296) 1 pair in a 5 card hand

Picking the 1 pair

Pick Rank→13 options pick1→ ${}_{13}C_1=13$

Pick Suit of pair→4 options pick 2→ ${}_4C_2=6$

Pick 3 different cards→ must be different

Pick rank→12 options pick 3→ ${}_{12}C_3=220$

Pick suit of 1st card→4 options pick 1→ ${}_4C_1=4$

Pick suit of 2nd card→4 options pick 1→ ${}_4C_1=4$

Pick suit of 3rd card→4 options pick 1→ ${}_4C_1=4$

$$\frac{{}_{13}C_1\cdot{}_4C_2\cdot{}_{12}C_3\cdot{}_4C_1\cdot{}_4C_1\cdot{}_4C_1}{{}_{52}C_5}=0.4226$$

Mr. Spray has 5 different Probabilities Retests to give randomly to his students. Two boys and three girls want to retest. Determine the probability of each event.

297) The 3 girls take the same test

Solution:
3 girls pick 3 girls
5 tests pick 1test
2 boys pick 2 boys
1st woman 5 tests pick 1test
2nd woman 5 tests Pick 1 test

P(3girls take the same test)=

$$= \frac{\left({}_3C_3 \cdot {}_5C_1\right)\left({}_2C_2 \cdot {}_5C_1 \cdot {}_5C_1\right)}{5^5} = \frac{1}{25}$$

298) Exactly 3 people take the same test.
Solution:
5 people pick 3 people
5 tests pick 1 test

2 people left over pick 2 people
(2 tests can be the same but must be different that the one pick above)

1st person 4 tests pick 1 test
2nd person 4 tests Pick 1 test

P(3 people write the same test)=

$$= \frac{\left({}_5C_3 \cdot {}_5C_1\right)\left({}_2C_2 \cdot {}_4C_1 \cdot {}_4C_1\right)}{5^5} = \frac{800}{3125}$$

299) All 5 people take a different test.

Solution:
1st person has 5 choice
2nd person has 4 choices
3rd person has 3 choices
4th person has 2 choices
5th person has 1 choice

P(5 people write 5 different tests)=

$$\frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{5^5} = \frac{120}{3125}$$

Or

$$\frac{5}{5} \times \frac{4}{5} \times \frac{3}{5} \times \frac{2}{5} \times \frac{1}{5} = \times \frac{120}{3125}$$

300) All 5 of them take the same test.
Solution:
5 people pick 5 people
5 tests pick 1 test

P(5 people write the same test)=

$$= \frac{{}_5C_5 \cdot {}_5C_1}{5^5} = \frac{5}{3125}$$

301) All the boys write the same test. All the girls write the same test but it is different than the boys.

Solution:
2 boys pick 2 boys
5 tests pick 1
3 girls pick 3 girls
4 tests left pick 1

$$= \frac{\left({}_2C_2 \cdot {}_5C_1\right)\left({}_3C_3 \cdot {}_4C_1\right)}{5^5} = \frac{20}{3125}$$

302)Exactly 2 people take the same test.

Solution:
5 people pick 2 people
5 tests pick 1 test

3 people left over pick 3. Each of these people must write different tests so that only 2 people write the same test.

1st person 4 tests pick 1 test
2nd person 3 tests Pick 1 test
3rd person 2 tests pick 1 test

P(2 people write the same test)=

$$= \frac{\left({}_5C_2 \cdot {}_5C_1\right)\left({}_3C_3 \cdot {}_4C_1 \cdot {}_3C_1 \cdot {}_2C_1\right)}{5^5} = \frac{}{3125}$$

Answers to mixing it up.

340. 0.309	341. 11/12	342. 0.441	343. 0.667	344. 30%	345. 0.028	346. 0.167
347. 0.7	348. 4/15	349. 0.261	350. 0.091	351. cdfbin(100, $\frac{1}{6}$,99)	352. 0.24	353. 0.999
354. 0.444	355. 4/13	356. 0.2	357. 0.0006	358. 0.0045	359. 0.0006	360. 0.019