This is what the leading indicators lead[*]

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Abstract

We propose an optimal filter to transform the Conference Board Composite Leading Index (CLI) into recession probabilities. We also analyze the CLI's accuracy at anticipating output growth. We compare the predictive performance of linear, VAR extensions of smooth transition regression and switching regimes, probit, nonparametric models and conclude that a combination of the switching regimes and nonparametric forecasts is the best strategy at predicting both the NBER business cycle schedule and GDP growth. This confirms the usefulness of CLI, even in a real-time analysis.

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1 Introduction

Consumption, savings and production decisions made by individual agents and monetary and fiscal policy made by policymakers are based on forecasts about the future developments of macroeconomic variables. The state of the business cycle is one of the key elements for the evolution of such variables. Hence, forecasting turning points is crucial for the optimality of the economic agents' decisions. .

An extensive literature exists which attempts to find the best forecasting tool for the business cycle turning points, from the early heuristic attempts by Mitchell and Burns (1938) to the more sophisticated of Stock and Watson (1989). Whatever approach we consider, the forecasting problem is twofold. First, we need to identify the group of variables that move in and out recessions before the rest of the economy. Second, we have to find the appropriate filter to extract the signal out of these series.

We focus on the second aspect of the forecasting problem by attempting to find an optimal signal extraction method to analyze the predictive power of the Composite Leading Index (CLI). This series, combination of several promising leading variables, is released by the Conference Board since October 1996 and by the Bureau of Economic Analysis prior to that date. We use the CLI because, even though it has suffered a number of important revisions, it has been published without interruption since 1968, allowing the researcher to analyze the predictive power of the leading index with information available in each time period.[1]

Studies that analyze the accuracy of the CLI for predicting turning points find contradictory results. Diebold and Rudebusch (1991) do not detect predictive power in a linear context. The probit model used by Estrella and Mishkin (1998) outline the poor performance of CLI, specially in the out-of-sample analysis. Hamilton and Perez-Quiros (1996) and Kim (1994) present evidence in favor of the usefulness of CLI. Filardo (1994) concludes that lag values of the CLI explains changes in the probability of switching from an expansion to a recession. Finally, Granger, Teräsvirta and Anderson (1993) find that the CLI is the driving factor in a Smooth Transition Regression (STR) model.

There are two main purposes for this paper. First, we want to formally compare these previous analysis and to propose a set of alternative filters.

[1] Only the experimental leading index (XLI) proposed by Stock and Watson (1989) would allow the same kind of analysis. However, a real-time evaluation of XLI is complicated since the number of observations is too small and this index only faces a recession in the early 1990s. Further research should go in the direction of comparing the predictive power of XLI versus CLI.

Only Filardo (1999), in independent work, has tried the same kind of approach. However, he uses a different set of models, conducts only a descriptive analysis and does not formally test the predictive power of each model. Second, a more ambitious goal of this paper, is to combine all the different approaches to propose a filter that transforms the data for the CLI into a probability of recession one quarter hence.

To our knowledge, this is the first attempt to address both issues. In fact, this paper is the first formal comparison of how the most popular time series filters analyze the predictive power of the CLI for forecasting turning points.[2] In particular, we compare the accuracy of the previously proposed linear, Markov switching, and probit models, a vector autoregressive extension of STR specifications, and a new nonparametric filter. Then, we combine the information contained in all of the models in order to obtain a "consensus" filter for transforming the CLI data into a recession probability.

In addition, we acknowledge that predicting turning points may not be the only goal of the CLI. Therefore, we repeat the approach to analyze the predictive power of the CLI on GDP movements.

We conclude that a combination of different models performs better in and out of sample than each of the single model proposed. Thus, the CLI is useful in anticipating both turning points and output growth, even in real-time analysis. Moreover, in contrast to Hess and Iwata (1997), we find that nonlinear specifications are better than simpler linear models at reproducing the business cycles features of real GDP.

The paper is organized as follows. Section 2 describes the data, section 3 outlines the different models, section 4 presents the empirical evidence, section 5 analyzes the combination of forecasts, and section 6 concludes.

2 Preliminary analysis of data

For the in-sample study, we use historically revised CLI series issued in January 1998. For the GDP, we use chained-weighted data. The data runs from the second quarter of 1960 to the fourth quarter of 1997. We transform the monthly CLI series into quarterly by choosing the last observation of each quarter. As a preliminary analysis, we test the stationarity properties of our series. The augmented Dickey-Fuller test can not reject the null hypothesis of a unit root for the log levels of the series, but it is consistent with a sta-

[2] Stock and Watson (1998) analyze a battery of models that includes linear, nonlinear, parametric nonparametric and a combination of them. However, they do not focus on predicting turning points and their study is so extensive that they cannot apply a formal comparison.

tionary specification for the differences of the logarithms. Thus, for now on, our series of interest will be the growth rates of GDP and CLI, denoted as y and x.

In addition, Johansen procedure fails to detect evidence of cointegration.[3]. Other authors as Hamilton and Perez-Quiros (1996) and Granger, Teräsvirta and Anderson (1993) have found that previous series of the CLI presented cointegration with GDP. However, as pointed out Harvey in a comment in Granger et al. (1993), there was not strong economic reason for GDP and CLI to be cointegrated. Thus, the absence of cointegration is an important characteristic of the last CLI revisions.

3 Models description

In order to quantify the accuracy of the CLI to predict both GDP movements and periods of recession in the US economy, we analyze different linear and nonlinear, parametric and nonparametric models. This section briefly describes these models.

3.1 Univariate and bivariate linear models

Linear models have been widely developed in the earlier forecasting literature. However, these models have been applied just to generate a forecast of the explained variable, let's say, rate of growth of GDP, rate of growth of industrial production or some coincident indicator. It is not common to use them to forecast a non-linear phenomena such as a turning point. In the literature, Stock and Watson (1993) propose a filter to extract turning points forecasts from a linear model. This is used by Hamilton and Perez-Quiros (1996) to successfully describe the predictive power of the CLI over the business cycles. Their basic findings are the following. Let the linear processes for the GDP be either an AR(p)

$$y_t = \mu + a(L)(y_{t-1} - \mu) + \varepsilon_t, \tag{1}$$

or a VAR(p)

$$\begin{aligned} y_t &= \mu + a(L)(y_{t-1} - \mu) + b(L)(x_{t-1} - \eta) + \varepsilon_t \\ x_t &= \eta + c(L)(y_{t-1} - \mu) + d(L)(x_{t-1} - \eta) + u_t, \end{aligned} \tag{2}$$

[3]The likelihood ratio test of the null hypothesis of no cointegration against the alternative of one cointegrating relation is 14.48

where L is the lag-operator.[4] Errors in (1) are i.i.d. gaussian with zero mean and variance σ_{11}. Errors in (2) follow the usual assumptions:

$$\begin{pmatrix} e_t \\ u_t \end{pmatrix} \sim i.i.d.N\left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}; \begin{pmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{pmatrix}\right].$$

Under the previous hypotheses, one and two quarter ahead output growth predictions form the random vector

$$\begin{pmatrix} y_{t+1}/y_t, y_{t-1}, \ldots, y_1 \\ y_{t+2}/y_t, y_{t-1}, \ldots, y_1 \end{pmatrix} \sim i.i.d.N\left[g_t; Q\right]. \qquad (3)$$

For the AR case

$$g_t = \begin{pmatrix} \mu + a(L)(y_t - \mu) \\ \mu + a^2(L)(y_t - \mu) \end{pmatrix},$$

and

$$Q = \begin{bmatrix} \sigma_{11} & a_1\sigma_{11} \\ a_1\sigma_{11} & (1 + a^*(1))\sigma_{11} \end{bmatrix}.$$

However, for the VAR case

$$g_t = \begin{pmatrix} \mu + a(L)(y_t - \mu) + b(L)(x_t - \eta) \\ \mu + (a^2(L) + b(L)c(L))(y_t - \mu) + (a(L)b(L) + b(L)d(L))(x_t - \eta) \end{pmatrix},$$

and

$$Q = \begin{bmatrix} \sigma_{11} & a_1\sigma_{11} + b_1\sigma_{12} \\ a_1\sigma_{11} + b_1\sigma_{12} & (1 + a^*(1))\sigma_{11} + b^*(1)\sigma_{22} + a'b\sigma_{12} \end{bmatrix}.$$

We adopt Okun's rule of thumb that a recession occurs whenever the real GDP falls for at least two consecutive periods. Therefore, the probability of being in recession at $t+1$ depends on the actual value of y_t. If $y_t < 0$, the forecasted probability of recession is the probability that y_{t+1} was less than zero. On the other hand, the probability that the downturn starts at $t+1$ when $y_t > 0$, coincides with the probability that both y_{t+2} and y_{t+1} were less than zero. Thus, given observations until t, these probabilities can be easily calculated from probability tables or Monte Carlo simulations on (3).

[4] For now on, we define $h = (h_1; h_2; \ldots; h_p)'$, $h(L) = (h_1 + h_2L + \ldots + h_pL^{p-1})$; $h^2(L) = \sum_i h_i^2 \left(L^{i-1}\right)^2 + 2\sum_{i<j} h_i h_j L^{i-1}L^{j-1}$, and $h^*(1) = (h_1^2 + h_2^2 + \ldots + h_p^2)$; with h being a, b, c and d.

The results for the linear AR and VAR specifications are presented in the first and second row of Table 1. The optimal number of lags, applying Schwarz criteria is 1 in both cases. In addition, as in Hamilton and Perez-Quiros, we find that lagged growth of GDP does not help in forecasting neither current growth rates of GDP nor current growth rates of CLI.

3.2 Vector Smooth Transition Regression (VSTR)

We extend the STR models proposed by Granger and Teräsvirta (1993) to a VAR context. These were developed to capture the fact that may exist two (or more) data generating processes that change with the state of the economy. The probability of being in each state is determined by the transition function. To study how these models work, we start from the following VSTR model:

$$
\begin{aligned}
y_t &= \mu + a(L)y_{t-1} + b(L)x_{t-1} + \left[\theta + \alpha(L)y_{t-1} + \beta(L)x_{t-1}\right] F_y + e_t \\
x_t &= \nu + c(L)y_{t-1} + d(L)x_{t-1} + \left[\eta + \gamma(L)y_{t-1} + \delta(L)x_{t-1}\right] F_x + u_t \qquad (4)
\end{aligned}
$$

where lag-operators and errors hold the same assumptions than in (2). Note that there are as possible VSTR specifications as different explanatory variables and functional forms are considered in the transition function F. In order to select among them, we use linearity and model selection tests based on maximum likelihood principles as follows. We first specify a linear VAR and choose the optimal lag length p. Second, we apply linearity tests for each selected candidate to be explanatory variable in F. Third, for each of them that rejects linearity, we carry out model selection tests to obtain one of the possible VSTR forms. Finally, we perform in-sample and out-of-sample model evaluation techniques to select one final specification from the set of possible VSTR models.[5]. From this analysis, we find that the best specification for y_t and x_t is a Logistic-VSTR with the following functional form for F_y and F_x:

$$
F_i(y_{t-2}) = \frac{1}{1 + e^{-\gamma_i(y_{t-2} - g_i)}}; \qquad (5)
$$

where $i = y; x$. Variables γ_i and g_i are called smoother parameter and threshold, respectively.

These models implicitly contain information about recession probabilities as follows. For simplicity in the exposition assume both, that parameters in

[5] Details of the selection process and results for the estimation of each of the possible models considered can be found in Camacho (1998).

$\tilde{a}(L)$ and $\tilde{b}(L)$ are zero and that γ_y and $\tilde{\gamma}$ are positive. In extreme contractions, y_{t-2} takes a much lower value than the threshold. Hence, the higher is the smoother parameter, the closer to zero is the transition function value. Likewise, great expansions can be associated with transition function values near to one. Hence, the transition function locates the model either near to or far from recessions depending on the values of y_{t-2} relative to the threshold. Thus, once (4) is estimated with information until t, $F(y_{t-1})$ can be interpreted as a one quarter ahead forecasted recession probability.

The results for this estimation are presented in the third row of Table 1. As in the linear case, we get an optimal lag length equal to 1 and we find that lagged growth of GDP does not help to forecast neither y nor x. In addition, we accept the null that the constant is the only changing parameter. Therefore, $\tilde{a}, \tilde{b}; \tilde{c}$ and $\tilde{d}$ are statistically insignificant on this model.

3.3 Switching regimes model

Our statistical definition of the switching regime model is described in detail in Hamilton and Perez-Quiros (1996). As in the previous case, two regimes are considered. Let s_t be an unobserved latent variable which takes a value equals to 1 when the economy is in an expansion and 2 when the economy is in a contraction. In the former case, GDP and CLI are expected to grow by amounts μ_1 and η_1. However, in a contractions they grow at a lower rates μ_2 and η_2. In switching regimes models, the changes between regimes do not follow a logistic function (which depends upon observable variables). Their law of motion is governed by the unobservable state variable s_t, that evolves according to a homogeneous Markov chain that is independent of past observations on y_t and x_t. This implies that the probability that s_t equals some particular value j depends on the past only through the most recent value s_{t-1}:

$$p\left(s_t = j | s_{t-1} = i; s_{t-2} = k; \ldots; \chi_{t-1}\right) = p(s_t = j | s_{t-1} = i) = p_{ij};$$

where $\chi_t = (y_t; x_t; y_{t-1}; x_{t-1}; \ldots)$.

Thus, our time-series model is

$$\begin{aligned} y_t &= \mu_{s_t} + a(L)(y_t - \mu_{s_{t-1}}) + b(L)(x_t - \eta_{s_{t+r-1}}) + e_t \\ x_t &= \eta_{s_{t+r}} + c(L)(y_t - \mu_{s_{t-1}}) + d(L)(x_t - \eta_{s_{t+r-1}}) + u_t; \end{aligned} \quad (6)$$

with lag-operators and errors following the same assumptions as in (2).

After testing, we impose the restriction that the CLI and the GDP "share" the state of the business cycle, as in Hamilton and Perez-Quiros (1996).[6] In particular, as they suggest, the CLI moves r periods before GDP. Thus, the conditional expectation of CLI depends on s_{t+r}.

With this kind of specification, recessions are predicted as follows. First, we define s^*_t as a latent variable which summarizes the values of s_{t-p} through s_{t+r}, and the transition probabilities matrix P^*. Second, we estimate the model and calculate the vector $\hat{\xi}_{t|t}$, whose ith element gives the probability that state i occurs, given the observed values of y and x until t. A forecast of whether the economy will be in a recession one quarter from now is obtained by summing those elements of $\hat{\xi}_{t+1|t} = P^* \hat{\xi}_{t|t}$ corresponding to $s_t = 2$.

We find the same kind of results as in Hamilton and Perez-Quiros (1996). Schwarz criterion has selected $p = 1$ in (6). The highest value for the likelihood function is reached by $r = 1$. Furthermore, like in previous cases, the coefficients for the lagged GDP growth are not significant in any of the equations. The fourth panel of Table 1 presents the results for this model.

3.4 Probit model

The fifth model that we analyze follows the lines of the probit model proposed in Estrella and Mishkin (1998). These authors develop a filter for quantifying the predictive recessions power of the variables contained in a d-dimensional vector z_{t-1}.

Let z_{t-1} be lagged CLI growth rates. Let r_t be an unobservable variable that determines the occurrence of a recession at time t. The model is defined in reference to the theoretical relation

$$r_t = \beta' z_{t-1} + \varepsilon_t, \tag{7}$$

where ε_t follows a standard normal distribution. Since r_t is unobservable, the estimation is based in a dichotomous recession indicator d_t that equals one if the economy is in recession in quarter t, and zero otherwise. If the model is correct, r_t should be greater than zero whenever d_t was equal to one. This implies that

$$P(d_t = 1) = P(r_t > 0) = \Phi(\beta' z_{t-1}), \tag{8}$$

where Φ is the cumulative normal distribution function. The estimation of the parameter uses standard maximum likelihood procedures on the logarithmic likelihood function of probit models.[7]

[6] This somehow coincides with the result that a similar transition function locates both GDP and CLI between regimes in VSTR models.

[7] The reader will find in (18) an explicit expresion.

In practice, we rely on the NBER recession indicator for determining d_t. To examine the CLI's usefulness at predicting recessions, we try current and lagged values of x_t in (7). For each, we calculate the pseudo R^2

$$PR^2 = 1 - \left(\frac{\log L_u}{\log L_c}\right)^{-(2/T)\log L_c}; \qquad (9)$$

where L_u and L_c are the unconstrained maximum value of the likelihood function, and such value under the constraint that all coefficients are zero except for the constant. Intuitively, this corresponds to the coefficient of determination in linear regression. The maximum value is achieved by x_{t-1}.[8] The results for this model are shown in the last row of Table 1.

3.5 Nonparametric gaussian kernel

Smoothing methods provide a powerful methodology for gaining insights into the data since they avoid the problem of specifying a closed form for the density function. However, a search for the optimal non-parametric specification using all possible set of explanatory variables could be costly. Therefore, we use some results from the previous analysis. In particular, we have learned that all parametric models show a common characteristic: CLI is a turning point predictor in the short run. Specifically, we find a relation between current GDP growth and current recessions with CLI growth during the previous quarter. Then, we will use x_{t-1} as explanatory variable in nonparametric models.

In forecasting growth, we approximate the relation

$$y_t = m(x_{t-1}) + e_t;$$

using the standard Nadaraya-Watson estimator in line with Härdle (1989).[9]

We focus on the question of how recessions can be predicted nonparametrically. Keeping in mind the Okun's rule of thumb, we propose the following methodology for predicting probabilities of recession in real-time. Assume that $y_t < 0$: First, we construct the conditional density function, depending

[8] This value is 0.280 and declines within a year.

[9] Specifically, we estimate

$$\hat{y}_h = \left[\sum_{j=1} K\left(\frac{x_{t-1} - x_j}{h}\right) y_j\right] \Big/ \left[\sum_{j=1} K\left(\frac{x_{t-1} - x_j}{h}\right)\right];$$

where K is the gaussian kernel and h is selected by leave-one-out cross-validation.

upon the unknown value y_{t+1}, given the growth of the CLI at t, that is

$$f(y_{t+1}/x_t) = \frac{f(y_{t+1}, x_t)}{f(x_t)}. \tag{10}$$

Second, we calculate the expectation that it takes values less than zero:

$$p_t(y_{t+1} < 0/x_t) = \int_{y_{t+1}<0} f(y_{t+1}/x_t)\,dy_{t+1}. \tag{11}$$

On the other hand, when $y_t > 0$, the probability at t for a recession at $t+1$ coincides with the probability that both y_{t+1} and y_{t+2} would be less than zero. Following the same methodology we propose

$$p_t(y_{t+2} < 0, y_{t+1} < 0/x_t) = \int_{y_{t+2}<0}\int_{y_{t+1}<0} f(y_{t+2}, y_{t+1}/x_t)\,dy_{t+2}\,dy_{t+1}. \tag{12}$$

Standard smoother techniques suffer from a slight drawback when applied to multidimensional data with long-tailed distribution. This is precisely the case of predicting recessions.[10] In order to avoid this problem, we have used adaptive kernel estimation, which consists of finding kernel estimators with bandwidth varying from one point to another. In particular, the estimated joint distribution of any d-dimensional variable z at any point j is given by

$$\hat{f}(z_j) = \frac{1}{T}\sum_{t=1}^{T}\prod_{i=1}^{d}\frac{1}{h^d\lambda_t^d}\,k\left(\frac{z_{j,i}-z_{t,i}}{h\lambda_t}\right), \tag{13}$$

where h is the bandwidth, and λ_t is the local bandwidth factor at time t.

The procedure that we use to get h and λ in (13) is the following. First, we define the local bandwidth factor as

$$\lambda_t = \left(\frac{\tilde{f}(z_t)}{g}\right)^{-\alpha},$$

where $\tilde{f}(z_t)$ is a pilot estimation of (13), with λ_r equals to one, and the bandwidth chosen by reference to a standard distribution.[11] Parameter g is the geometric mean of $\tilde{f}(z_t)$. We set α equal to $1/2$, following Abramson (1982).

[10] GDP growth observations are usually non-negative, but estimating their density treating them as observations on $(-\infty, \infty)$. This leads to noisy density estimation in the right-hand tail.

[11] See Silverman 1986, page 87.

Second, we select the bandwidth that maximizes the likelihood cross-validation function

$$LCV(h) = \frac{1}{T}\sum_{j=1}^{T} \log \hat{f}_{-j}(z_j);$$

with $\hat{f}_{-j}(z_j)$ defined as in (13), and where the sum does not include values of t equal to j.[12]

4 Empirical evidence

The ability of any leading indicator to anticipate events depends on using the appropriate technique to extract the information contained in the predictor. We apply two different statistics to measure the accuracy of the different specifications at forecasting growth and recessions. First, to analyze the accuracy at forecasting growth, we use the Mean Square Error:

$$MSE = \frac{1}{T}\sum_{t=1}^{T} (y_t - \hat{y}_t)^2; \tag{14}$$

where y_t and $\hat{y}_t$ are actual and estimated GDP[13].

On the other hand, to compare the power of such models at anticipating turning points, we construct the Turning Points Error, a measure of the squared deviation from the NBER schedule:

$$TPE = \frac{1}{T}\sum_{t=1}^{T} (d_t - \hat{d}_t)^2; \tag{15}$$

where d_t is a dichotomous variable which equals 1 if, according to the NBER, the economy is in recession at time t; and 0 otherwise. Variable $\hat{d}_t$ is the forecasted probability of being in recession at time t[14]:

[12] In fact, real-time predictions in nonparametric models follows the same strategy as in parametric models. At any period t; parameters h and λ are estimated in (11) from the relation between y_t and x_{t-1}, whereas they are estimated in (12) from the relationship among y_t, y_{t-1} and x_{t-2}. Once these values are approximated, we use them for anticipating recessions for $t+1$.

[13] For the out of sample exercise, we define Mean Square Forecasting Error (MSFE) defined with the same formula, where $\hat{y}_t$ is the estimated value for y_t with information up to period $t-1$.

[14] For the out of sample exercise, we define Turning Point Forecasting Error (TPFE) with the same formula, where $\hat{d}_t$ is the estimated value for d_t with information up to period $t-1$.

In order to test if the differences between each pair of models are significant, we use the test proposed by Diebold and Mariano (1995), henceforth DM. Specifically, consider two different specifications, model i and model j. Let $\{\hat{g}_{it}\}_{t=1}^{T}$ and $\{\hat{g}_{jt}\}_{t=1}^{T}$ be their respective forecasts.[15] Let E_t be either $(y_t - \hat{y}_{it})^2 - (y_t - \hat{y}_{jt})^2$ at forecasting growth, or $\left(d_t - \hat{d}_{it}\right)^2 - \left(d_t - \hat{d}_{jt}\right)^2$ at anticipating recessions. Finally, let $\overline{E}$ be equal to $\frac{1}{T}\sum_{t=1}^{T} E_t$. Under the null hypothesis of no difference in the accuracy of these two competing forecasts, the large-sample statistic

$$DM = \frac{\overline{E}}{\sqrt{\frac{2\pi \hat{f}_d(0)}{T}}}; \qquad (16)$$

where

$$2\pi \hat{f}_d(0) = \frac{1}{T} \sum_{\tau=-(T-1)}^{T-1} 1\left(\frac{\tau}{S(T)}\right) \sum_{t=|\tau|+1}^{T} (E_t - \overline{E})(E_{t-|\tau|} - \overline{E});$$

the indicator function

$$1\left(\frac{\tau}{S(T)}\right) \begin{cases} 1 & \text{for } \left|\frac{\tau}{S(T)}\right| \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

is the lag window, and $S(T)$ is the truncation lag, follows a $N(0;1)$ random variable.

The first two columns of Table 2 display the in-sample MSE and TPE for the whole set of models.[16] For each model, the first entry refers to the entire sample. The second and third entries exclusively refer to recessionary and expansionary periods, according to the NBER schedule. The in-sample results show that the Markov switching model performs better than any other specification in both the GDP and turning points forecasts. In addition, most of the gains come from the reduction in the mean square error in recessions (in the case of the switching versus the AR model, 58% of the reduction comes from recessions whereas only 26% comes from expansions). Therefore, in sample we can conclude that the CLI has predictive power over the business cycle and GDP movements. These are better captured with a non-linear Markov switching specification that allows the forecaster to take into account

[15] Note that g is either y (in 14) or d (in 15). Also that forecast errors may be non-gaussians, nonzero mean, and serially and contemporaneously correlated.

[16] Models COMB and RTCOMB will be treated in detail in the next section.

the changes in the data generating process of both GDP and CLI due to the phenomena of expansions and recessions.

Using the DM test, Table 3 presents statistical evidence of the significativity of these gains. Comparing linear models, the inclusion of the CLI in the GDP equation gives an statistically significant improvement in the MSE (DM test of 2.47) but not in the TPE (DM test of 1.02). However, in a nonlinear context, we reject the null of no gain with respect to the univariate linear model in both the MSE and TPE and we reject also the null of no gain with respect to the multivariate VAR.[17]

Nevertheless, such promising results in-sample do not necessarily imply that the CLI is useful for real-time predictions. First, it is well-known that very flexible nonlinear models have a poor performance in out-of-sample exercises. Second, the CLI series is revised very frequently, and therefore, the in-sample analysis contains information not available for prediction at each period of time.

The out-of-sample analysis predicts in real-time 104 values. The first data point for which predictions are made is the second quarter of 1972.[18] For each period of time t, we estimate each model with data from the beginning of the sample up to period t, using the revision of the CLI available in that period of time. The transformation from monthly to quarterly observations is done as in the in-sample analysis. Then, with the coefficient estimates, a one period ahead forecast is computed through the first quarter of 1998. This procedure mimics what a statistical model would have predicted with the information available at any point in the past.[19]

It is important to mention that sometimes the release of the NBER decision about the state of the economy in period t may be delayed for almost two years. This leads to a serious problem when real-time analysis is applied to the probit model, since d_t is usually unknown at time t. To solve this, at any time t, we estimate β in (8) with observations until $t-8$, to ensure that d is available. This estimation is then used to predict a probability at t of

[17] DM test comparing the in-sample accuracy of SWITCH versus VAR is 4.9 for MSE and 3.2 for TPE.

[18] We select this date because we want to have enough number of observations to estimate the different models and to capture in the out-of-sample analysis the recession in the early 1970s.

[19] In order to forecast for quarter $t+1$ with the information up to period t, we need the CLI in period t, which is not know until one month after the end of quarter t. However, this first number is usually strongly revised. Thus, we use the first published revision of this data, made two months after the end of the period. Therefore, for example, to forecast the GDP in the first quarter (figures available in may), we use the CLI in december (the revision published in February).

being in recession at $t+1$ as follows:

$$P_t(d_{t+1} = 1) = F(\hat{b}'_{t-8} z_t):$$

The last two columns of Table 2 present the results for the real-time analysis. Looking at the results, we observe that even in the out-of-sample exercise, there is still gain from using the CLI and, again, the best model is the Markov-switching. In addition, we can conclude that all the gains from using the CLI come from the recessionary periods. However, as shown in Table 3, even though the bivariate specifications' MSFE are numerically lower than in the case of the univariate linear model, not even the best of them is statistically significant according to the DM tests.[20] Hence, while the CLI appears useful in forecasting GDP within the historical sample, it seems not be as useful in a real-time exercise. The reader can find similar conclusions at anticipating recessions.

5 Combination of forecasts

As shown in Table 2, different models have different predictive power depending on the state of the business cycle. For example, the nonparametric estimator presents the best TPFE in expansions but it holds the worst record in recession times among the bivariate specifications. Filardo (1999) also finds that the performance of the different models change with the sample period considered. Therefore, he proposes that the best way to improve their reliability is by continuously monitoring their performance, thereby learning about when they are likely to predict correctly and when they are likely to fail. This is precisely what we allow by using encompassing methods. Hence, we suggest that a combination of the forecasts may draw more leading information from the CLI than any of the individual forecasting models.

In order to combine growth's forecasts, we apply the linear combination rule proposed by Granger and Ramanathan (1984). To combine in-sample forecasts, weights are obtained by simple linear squares techniques on

$$y_t = \beta' f_t + u_t; \tag{17}$$

where y_t is output growth at t; $\beta = \left(\beta_0; \beta_1; \ldots; \beta_{m+1}\right)$; m is the number of different forecasting methods, $f_t = (1; f_{t1}; \ldots; f_{tm+1})$; and f_{ti} is the forecast for time t that corresponds to AR, VAR, LVSTR, SWITCH and KERNEL, respectively. To combine out-of-sample forecasts, even though individual

[20] This result is similar to Diebold and Rudebusch (1991) conclusions and contradicts Hamilton and Perez-Quiros (1996) results.

models predict growth for $t+1$; the dependent variable y_{t+1} is not actually available at any time t. We solve this problem by using real-time combination. More specifically, we fit (17) with in-sample predictions until t; and we use these weighs to combine the out-of-sample forecasts $f_{t+1;i=t}$ to obtain an estimation of y_{t+1}.

In the case of forecasting recessions, it is not clear that such a rule would imply an output lying between zero and one. Instead, in the spirit of Li and Dorfman (1996), we propose an encompassing strategy based upon discrete choice analysis. To combine in-sample probabilities of recession, consider the following relation

$$r_t = \alpha p_t + e_t;$$

where $\alpha = (\alpha_0; \alpha_1; \dots; \alpha_{m+1})$, $p_t = (1; p_{t,1}; \dots; p_{t,m+1})$; r_t and m have been defined above, and $p_{t,i}$ is the in-sample forecasted recession probabilities for time t from AR, VAR, LVSTR, SWITCH, PROBIT and KERNEL models. Let d_t be the NBER indicator variable presented in section 3.4. We conclude that

$$P(d_t = 1) = F(\alpha p_t);$$

where F is the cumulative normal distribution function as in the probit model. Weighs are obtained by applying maximum likelihood principles to the objective function

$$L(\alpha) = \sum_{t=1}^{T} \{d_t \ln[F(\alpha p_t)] + (1 - d_t) \ln[1 - F(\alpha p_t)]\}: \qquad (18)$$

Combining forecasts in real-time, we find the same problem that in out-of-sample estimation from the probit model. The delay on which d_t is known has been solved using real-time combination as before. Thus, to combine forecasts for any time $t+1$, we estimate the α in (18) that combines in-sample forecasts until $t-8$.[21] Then, we use such estimation for combining the out-of-sample probabilities of recession for $t+1$. Note that the real-time combination uses changing weights for each period of time.

As a first approximation, we made a combination of the six (five, for forecasting growth) alternative models in in-sample and out-of-sample forecasts. As we expected, these forecasts are highly correlated, which suggests that the combination uses redundant information. Since SWITCH and KERNEL are

[21] As in the case of the out-of-sample forecasts from the probit model, we are assuming that the delay on the release of the NBER decision about the state of the economy is of at most two years.

the best models within recession and expansion data, we try an encompassing method that combines these two specifications. In terms of PR^2 and TPE, this combination is as good as the combination that contains the whole set of models. In-sample and out-of-sample combinations of switching regimes and nonparametric forecasts are called COMB and RTCOMB respectively.

The out-of sample results presented in Tables 2 and 3 reveal one the most important findings of this paper. RTCOMB presents the lowest MSFE and TPFE. Moreover, DM tests (Table 3) confirm that this combination significantly improves the linear model's results. This implies that the CLI is useful in anticipating both recessions and GDP growth, even in real-time.[22] Figures 1 and 2 present the in-sample and out-of-sample probabilities of recession predicted from SWITCH and KERNEL. They also show how well the in-sample and real-time combinations mimic the NBER schedule.

We are now ready to propose a filter that transforms the CLI releases into probabilities of recession next quarter. The CLI was originally designed as a tool to predict business cycle turning points. However, every month the Conference Board only releases the rate of growth of such leading index. Alternatively, we construct a filtering rule to extract the CLI's leading information about turning points, by transforming the growth rate of CLI into probabilities of recession. Based on previous results, we propose a real-time combination of the switching regimes model and the nonparametric specification.

To see how it works, we present the following empirical exercise. Suppose we are in the last quarter of 1997, and we want a filtering rule for the CLI release. We simulate the possible outcomes of the CLI growth rate from -2% to +2%. and we predict in real-time the probability of recession for 1998.1 (belonging to a wide expansionary period). Figure 3 displays the predicted probability of recession, associated to each CLI growth rate value, that are the outcomes of SWITCH, KERNEL and RTCOMB. Furthermore, we present in Figure 4 the results of a similar analysis, but we predict probability of recessions for 1990.1 (just after a recession) using exclusively the information available at 1997.4.

Since we proved that the best filter is the real-time combination, let us concentrate in the analysis of RTCOMB results. As we can see from the pictures, the same CLI growth rate contains very different information about the probability of an imminent recession depending on the period that we consider. Specifically, in 1990.4, a CLI growth rate of 0% would be associated to a probability of recession next quarter of almost 1. However, in 1997.4,

[22] DM tests to compare the accuracy of SWITCH and RTCOMB are 1.7 for MSE and 1.2 for TPE. This implies that there exists some evidence in favor of RTCOMB.

the same CLI growth rate would have implied a recession probability next period close to 0. The intuition is clear. In order to predict that a recession is coming, we need stronger evidence in the CLI behavior after 9 years of expansions that just after a recession to believe that a recession is imminent. Our filter efficiently uses the information about the state of the economy to interpret the rate of growth of the CLI in each period of time.

6 Conclusions

Conference Board's CLI is released to anticipate turning points. However, the ability of a predictor depends upon our model's accuracy at extracting its leading information about future events. Thus, we have evaluated how well the most standard specifications predict recessions. We propose a methodology to combine different forecasted probabilities. We conclude that a combination of a switching VAR model and a nonparametric system is the best approach to anticipate recessions. This kind of approach uses the CLI to reproduce the US business cycle data fairly well, compared with the ex-post NBER schedule. Hence, we find that the CLI is statistically useful at anticipating recessions, even in real-time analysis.

We conclude that CLI is also useful in forecasting US GDP growth, even in out-of-sample exercise. Again, a combination is the best approach in real-time, confirming the power of the combination of forecasts at extracting the leading information from the CLI.

Thus, we propose a filtering rule to extract the CLI's leading information about turning points. Our proposition transforms the rate of growth of the CLI in accordance with the state of the economy in the period of time in which it is released.

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Table 1. Maximum likelihood estimates of parameters

	Model estimation	VARCOV
AR	$\hat{y}_t = \underset{(0.09)}{0.76} + \underset{(0.07)}{0.28}(y_{t-1} - \underset{(0.09)}{0.76})$	$\hat\sigma_{11} = \underset{(0.09)}{0.78}$
VAR	$\hat{y}_t = \underset{(0.07)}{0.78} + \underset{(0.06)}{0.61}(x_{t-1} - \underset{(0.07)}{0.24})$ $\hat{x}_t = \underset{(0.07)}{0.24} + \underset{(0.05)}{0.43}(x_{t-1} - \underset{(0.07)}{0.24})$	$\hat\sigma_{11} = \underset{(0.02)}{0.60}$ $\hat\sigma_{22} = \underset{(0.02)}{0.52}$ $\hat\sigma_{12} = \underset{(0.03)}{0.09}$
LVSTR	$\hat{y}_t = \underset{(0.12)}{0.91}\hat{F}_x + \underset{(0.05)}{0.60}x_{t-1}$ $\hat{x}_t = \underset{(0.18)}{0.42} + \underset{(0.13)}{-0.31}\hat{F}_y + \underset{(0.05)}{0.43}x_{t-1}$ $\hat{F}_y = \left[1 + \exp\left(\underset{(0.26)}{-1.85}(y_{t-2} - \underset{(0.01)}{0.13})\right)\right]^{-1}$ $\hat{F}_x = \left[1 + \exp\left(\underset{(15.66)}{-87.57}(y_{t-2} + \underset{(0.47)}{0.33})\right)\right]^{-1}$	$\hat\sigma_{11} = \underset{(0.02)}{0.55}$ $\hat\sigma_{22} = \underset{(0.02)}{0.51}$ $\hat\sigma_{12} = \underset{(0.03)}{0.11}$
SWITCH	$\hat{y}_t = \hat\mu_{s_t} + \underset{(0.01)}{0.43}\left(x_{t-1} - \hat\mu_{s_{t-1}}\right)$ $\hat{x}_t = \hat\nu_{s_t} + \underset{(0.01)}{0.35}\left(x_{t-1} - \hat\nu_{s_{t-1}}\right)$ $\hat\mu_1 = \underset{(0.01)}{1.00}; \quad \hat\mu_2 = \underset{(0.08)}{-0.23}$ $\hat\nu_1 = \underset{(0.01)}{0.42}; \quad \hat\nu_2 = \underset{(0.08)}{-0.57}$ $p_{11} = \underset{(0.02)}{0.95}; \quad p_{22} = \underset{(0.10)}{0.79}$	$\hat\sigma_{11} = \underset{(0.01)}{0.53}$ $\hat\sigma_{22} = \underset{(0.01)}{0.44}$ $\hat\sigma_{12} = \underset{(0.01)}{0.07}$
PROBIT	$\hat{r}_t = \underset{(0.20)}{-0.97} - \underset{(0.22)}{1.19}$	$\{1\}$

Note. This estimation uses the sample 1960.2-1997.4. Variables x_t and y_t are growth of GDP and CLI respectively. Variable r_t determines how probable it is that a recession will occur at time t. Parameters σ_{11} and σ_{22} are variances of GDP and CLI errors, whereas parameter σ_{12} is the covariance between them. Standard errors are in parentheses. Note the joint uncertainty in the estimation of smoother parameter and threshold when the former is large. Following Estrella and Mishkin, the probit model's standard errors are estimated by using the Nadaraya-Watson estimator.

Table 2. MSE and TPE in-sample and out-of-sample.

	MSE in	TPE in	MSFE out	TPFE out
AR	0.78 1.73 0.57	0.10 0.57 0.005	0.79 2.19 0.49	0.11 0.62 0.008
VAR	0.60 1.09 0.49	0.11 0.62 0.006	0.75 1.82 0.53	0.09 0.47 0.011
LVSTR	0.55 0.88 0.48	0.16 0.30 0.133	0.70 1.46 0.54	0.17 0.55 0.091
SWITCH	0.48 0.72 0.42	0.05 0.22 0.011	0.68 1.56 0.50	0.09 0.27 0.060
PROBIT	...	0.10 0.40 0.035	...	0.09 0.35 0.038
KERNEL	0.60 1.18 0.47	0.11 0.61 0.002	0.73 1.80 0.48	0.10 0.57 0.006
COMB	0.55 0.90 0.47	0.03 0.10 0.009	...	...
RTCOMB	...	...	0.60 1.44 0.48	0.05 0.24 0.007

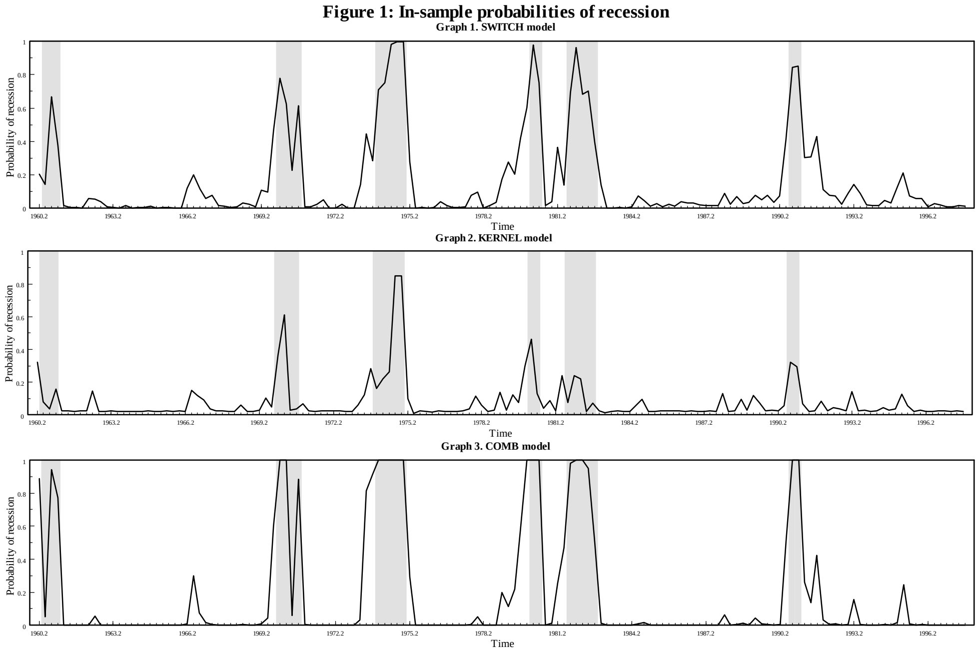
Note. "In" refers to 1960.2-1997.4. "Out" refers to 1972.2-1998.1. For each model, first entry have been calculated from the entire forecasting sample. Second and third entries only refers to recessionary and expansionary data, using the NBER schedule. MSE and TPE are defined in (14) and (15). COMB and RTCOMB are the combination of KERNEL and SWITCH as Section 5 describes.

Table 3. Diebold and Mariano tests.

		AR		COMB		RTCOMB	
		MSE	TPE	MSE	TPE	MSE	TPE
SWITCH	IN	3.79	3.12	2.45	0.83	...	...
	OUT	1.30	0.54	1.80	2.03	1.71	1.19
VAR	IN	2.47	1.02	2.15	2.94	...	...
	OUT	0.42	1.83	...	...	2.50	1.89
COMB	IN	2.79	2.82	...		...	
	OUT	...	...				
RTCOMB	IN	...	...	...		...	
	OUT	1.98	2.48				

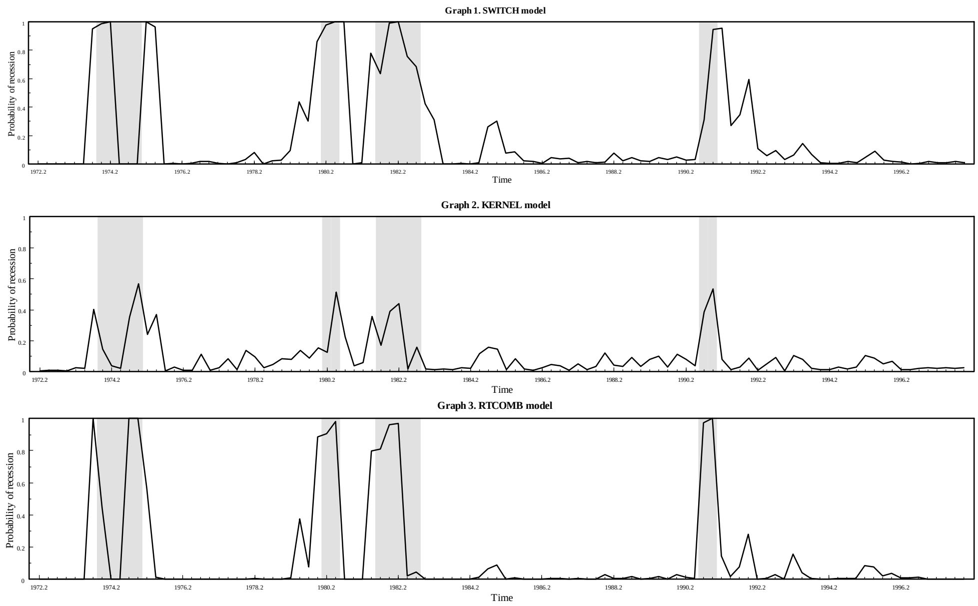
Note. "In" refers to 1960.2-1997.4. "Out" refers to 1972.2-1998.1. MSE and TPE are defined in (14) and (15). COMB and RTCOMB are the combination of KERNEL and SWITCH as Section 5 describes. All the entries refers to the absolute value of the DM statistic which is calculated for in-row and in-column models as (16) describes. For example, 3.79 (3.12) is the absolute value of the DM statistics under the hypothesis of no difference in the accuracy of models SWITCH and AR at anticipating in-sample growth (recessions).

Figure 1: In-sample probabilities of recession



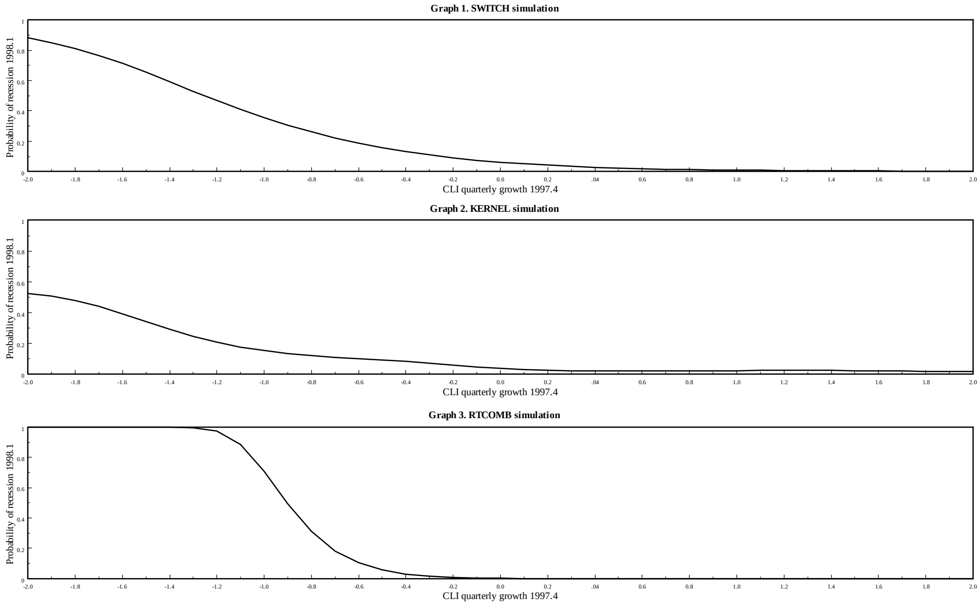
Note: Graph 1 and Graph 2 represent in-sample probabilities of recession from the switching regimes and the nonparametric specifications respectively. Graph 3 shows in-sample probabilities of recession using a combination of the first two models as Section 5 describes. "In-sample" refers to the period 1960.2-1997.4. Shaded areas correspond to the NBER recessions.

Figure 2: Out-of-sample probabilities of recession



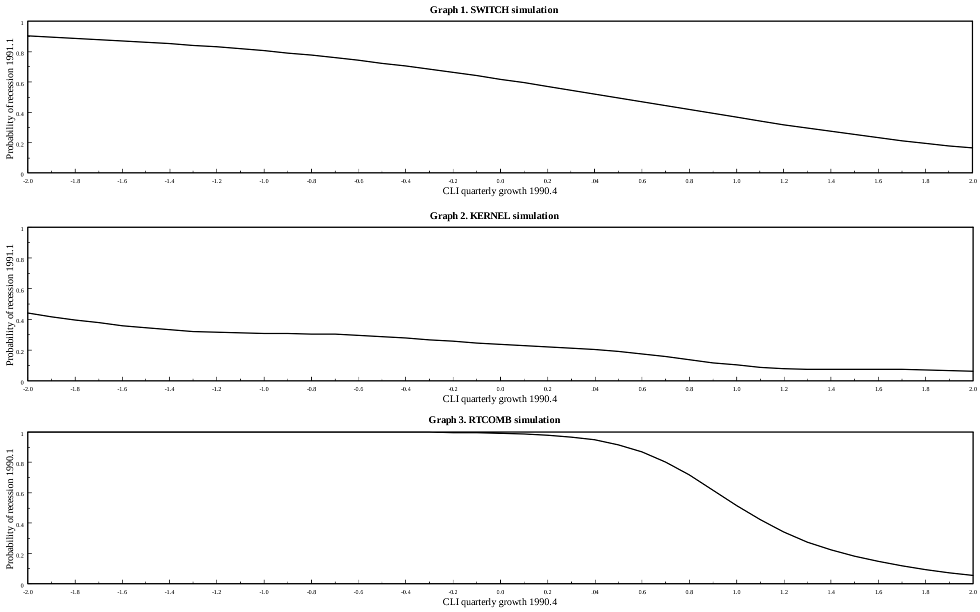
Note: Graph 1 and Graph 2 represent out-of-sample probabilities of recession from the switching regimes and the nonparametric specifications respectively. Graph 3 shows out-of-sample probabilities of recession using a real-time combination of the first two models as Section 5 describes. "Out-of-sample" refers to the period 1972.2-1998.1. Shaded areas correspond to the NBER recessions.

Figure 3: Simulation for 1998.1



Note: Horizontal axes represent simulated CLI quarterly growth values for 1997.4. Vertical axes show the real-time forecasts of the probability of recession in 1998.1 from the switching regimes model (Graph 1), the nonparametric specification (Graph 2), and the real-time combination of them (Graph 3) as Section 5 describes.

Figure 4: Simulation for 1991.1



Note: Horizontal axes represent simulated CLI quarterly growth values for 1990.4. Vertical axes show the real-time forecasts of the probability of recession in 1991.1 from the switching regimes model (Graph 1), the nonparametric specification (Graph 2), and the real-time combination of them (Graph 3) as section 5 describes.