



Thrust #3

**Active Information Exploitation
for Resource Management**

***Value of Information Sharing
In Networked Systems***

Doug Cochran





Synopsis of Research Activities



- Value of information sharing
 - Update on work on this topic area
- Quantification of information flow
 - Summary of topic area
- Synergistic activities
 - Interpolation on feature manifolds (with potential application to handoff problems)
 - ARL-ASU CRADA





Value of Information Sharing in Networked Signal Detection

Lauren Crider and Doug Cochran

- Multi-sensor signal detectors and estimators often use the eigenspectrum of a Gram matrix formed from vectors of sensor measurements
- Its elements are inner products from all sensor pairs
- In networked settings, the inner products can be formed locally by sharing of data between nodes in direct communication with each other
- When the network graph is incomplete, some elements of the Gram matrix cannot be formed locally

This work **quantifies the value of data sharing** between nodes in a sensor network in terms of the difference in detection performance that results from replacing the true data on the edge between the nodes with a maximum-entropy surrogate value

Reference: *L. Crider and D. Cochran*, "Effects of network topology on the conditional distributions of surrogated generalized coherence estimates," *Asilomar Conference on Signals, Systems, and Computers*, November 2014.





Gauge-Invariant Estimation in Networks*

Tenets



- The value of network structure in a sensor network is to enable sharing, fusion, and exploitation of data collected at different sensors
- Alignment of data across the nodes of a sensor network to a common coordinate system (“registration”) is usually essential to manifesting this value
 - Intrinsic data; e.g., clocks, platform orientation
 - Extrinsic data: collected by sensors

*Joint work with S. Howard and W. Moran
Appears in Proc. Fusion 2015





Gauge-Invariant Estimation in Networks

Nature of Data



- Data of interest often come from \mathbf{R}^n or \mathbf{C}^n , but may reside in a more general Lie group or homogeneous space; e.g.,
 - Real n -space \mathbf{R}^n (non-compact, abelian group): Time series segments from sampled transducers
 - Torus \mathbf{T}^n (compact, abelian group): Periodic clocks, phase sensors
 - Rotation group $\mathbf{SO}(3)$ (compact, non-abelian group): Object pose in 3-space
 - Grassmannian manifold $\mathbf{G}(n,k)$ (compact homogenous space under unitary group action): Signal and interference subspaces in a measurement space





Gauge-Invariant Estimation in Networks

Graph Model of Network



- Networks are modeled as directed graphs
 - Data to be registered reside at the vertices (“nodes”)
 - Each pair of nodes in direct communication are joined by an edge (“link”)
 - Information is shared along edges: each edge is labeled by a noisy measurement of the coordinate transformation mapping the value from the start node to the terminal node
- With this model, sensor network registration problems are naturally formulated in terms of gauge theory (theory of connections on principal bundles)
- Gauge invariance of statistical models can be exploited to quantify the statistical (information-geometric) limits of network registration





Gauge-Invariant Estimation in Networks

Gauge Concept



- Notation: directed graph Γ , vertex set $V(\Gamma)$, edge set $E(\Gamma)$
- Each sensor's data are represented in some local coordinate system
- A group G (“gauge group”) of transformations that convert between these coordinate systems is assumed
- There is no *a priori* no preferred choice of global coordinate system for the network
- Choosing a reference coordinate system at each node in the network is called choosing a *gauge*; thereafter...
 - The state of each node is the element of G that transforms the node's local coordinates to its reference coordinates
 - The network state is the corresponding element in $V(\Gamma) \times G$





Gauge-Invariant Estimation in Networks

Connections and Alignable Networks



- With no noise, each edge is labeled by the map taking the state of the start vertex to the state of the terminal vertex
- The collection of such maps/labels over $E(\Gamma)$ is a connection; i.e., an element of $G \times E(\Gamma)$
- The network is assumed to be alignable; i.e., there are choices of gauge such that, when each of the nodes is using its reference coordinate system, the value of the connection on all edges is the identity element in G
- When the network is not aligned, its state in this gauge is an element of $V(\Gamma) \times G$, where each node is associated with the transformation that takes the common reference coordinate system to the coordinate system of the node



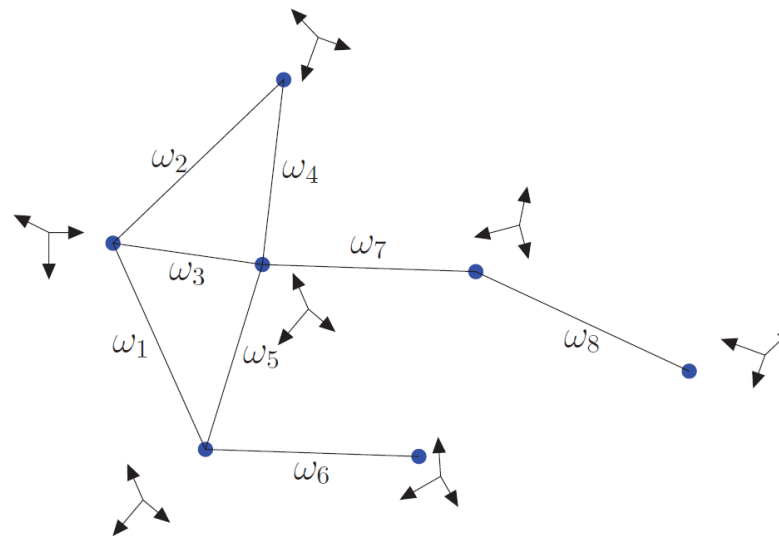


Gauge-Invariant Estimation in Networks



Estimation Goal

- A *gauge transformation* is separate change of coordinate system at each node in the network; i.e., an element of $V(\Gamma) \times G$



Goal: Estimate a gauge transformation $\gamma \in V(G) \times G$ that maps the given connection as closely as possible to the identity connection





Gauge-Invariant Estimation in Networks

Probability Model



- X a RV with sample space $(\mathcal{X}; \mu)$
- G a Lie group such that
 - There is a left action of G on \mathcal{X}
 - There is a left G -invariant measure on \mathcal{X}
- $S = \{f(x|g) \mid g \in G\}$ a parameterized set of possible statistical models for an observation of $x \in \mathcal{X}$
 - $f(x|g)$ are probability densities relative to μ
 - Assumed to be smooth as functions on G
 - Assumed to be G -invariant in that, given $g, h \in G$, $Y = hX$ has density $f(y|hg)$

The statistical problem is estimation of $g \in G$ from a realization of the data $x \in \mathcal{X}$





Gauge-Invariant Estimation in Networks

Summary of Theoretical Results



- Calculation of the Fisher information for this estimation problem
 - G invariance of the estimation problem yields Fisher metric that is left G -invariant
- Explicit ML estimators for gauge transformations that take the observed noisy connection to a connection near the identity
- Demonstration of how the Fisher information depends on the statistics of the noise *and* the topology of Γ

This provides a measure of value for each edge in Γ ; i.e., how will the Fisher information of the entire network suffer if a particular link is lost or benefit if it is added?



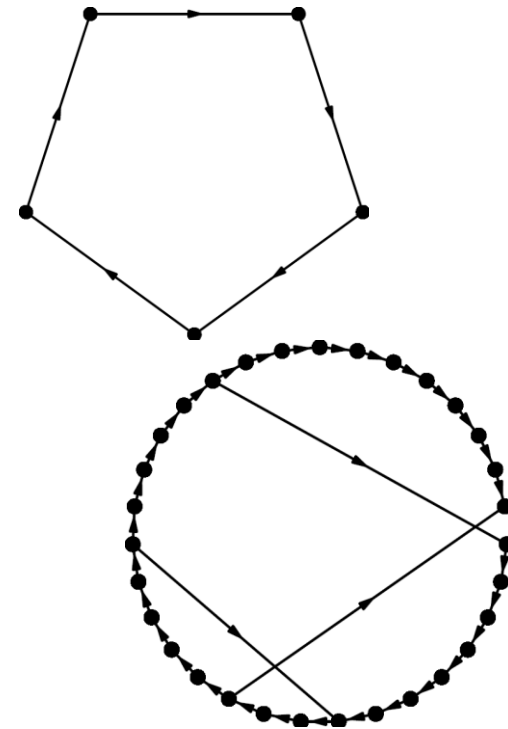


Gauge-Invariant Estimation in Networks

Some Empirical Results



- Local algorithms for network alignment
 - Compared to global results for a few networks using simulated data (from the model)
 - Preliminary tests at DST Group using multichannel RF sensor data
- Characterization of Fisher information in terms of network graph topology (using algebraic descriptors)



This provides a measure of value for each edge in Γ ; i.e., how will the Fisher information of the entire network suffer if a particular link is lost or benefit if it is added?





Synergistic Activities



- ARL and ASU are in the process of initiating a CRADA
 - Provides framework for cooperative educational activities
 - Builds on an established track record of placing students in DoD lab internships – particularly U.S. students from ASU's Barrett Honors College
 - Will leverage recently developed mechanisms:
 - ARL open campus
 - New NSF-sponsored research training group in the mathematical sciences
- Started preliminary work on interpolation on (feature) manifolds with guidance & data from AMRDEC

