Thrust equation of Jet propulsion

2.1 Introduction:-

The designer of an aircraft engine must recognize the differing requirements for take-off, climb, cruise, and maneuvering, the relative importance of these being different for civil and military applications and for long- and short-haul aircrafts.

In the early aircrafts, it was common practice to focus on the take-off thrust. This is no longer adequate for later and present day aircrafts. For long-range civil transports like Boeing 747, 777, 787 and Airbus A 340, A380 (the world's truly double-deck airliner), A350 XWB (extra wide body), the fuel consumption through some 10 or more flight hours is the dominant parameter.

Military aircrafts have numerous criteria like

1. The rate of climb
2. Maneuverability for fighters
3. Short take-off distance for aircrafts operating from air carriers
4. Maximum ceilings for high altitude reconnaissance aircrafts like SR-71 Blackbird aircrafts.

For civil and military freighter airplanes, the maximum payload is its main requirement.

The jet engine is a device that takes in air at essentially the free-stream velocity V_∞, heats it by combustion of fuel inside the duct, and then blasts the hot mixture of air and combustion products out the back end at a much higher velocity Ve.

The jet engine creates a change in momentum of the gas by taking a small mass of air and giving it a large increase in velocity (hundreds of meters per second). By Newton's third law, the equal and opposite reaction produces a thrust.

In all types of aircrafts, the engines are requested to provide efficiently the thrust force necessary for their propelling during different flight phases and at different operating conditions including hottest/coldest ambient temperature and rainy/ windy/snowing weather.

2.2 Thrust Force

Thrust force is the force responsible for propelling the aircraft in its different flight regimes.

It is in addition to the lift, drag, and weight represents the four forces that govern the aircraft motion. During the cruise phase of flight, where the aircraft is flying steadily at a constant speed and altitude, each parallel pair of the four forces are in equilibrium (lift and weight as well as thrust and drag). During landing, thrust force is either fully or partially used in braking of the aircraft through a thrust reversing mechanism.

2.3 Thrust Equation

The true fundamental source of the thrust of a jet engine is the net force produced by the pressure and shear stress distributions exerted over the surface of the engine.

Fig. (2.1), illustrates the distribution of pressure p_s over the internal surface of the engine duct, and the ambient pressure, essentially p_∞, over the external engine surface. Shear stress, which is generally secondary in comparison to the magnitude of the pressures, is ignored here. let X denote the flight direction

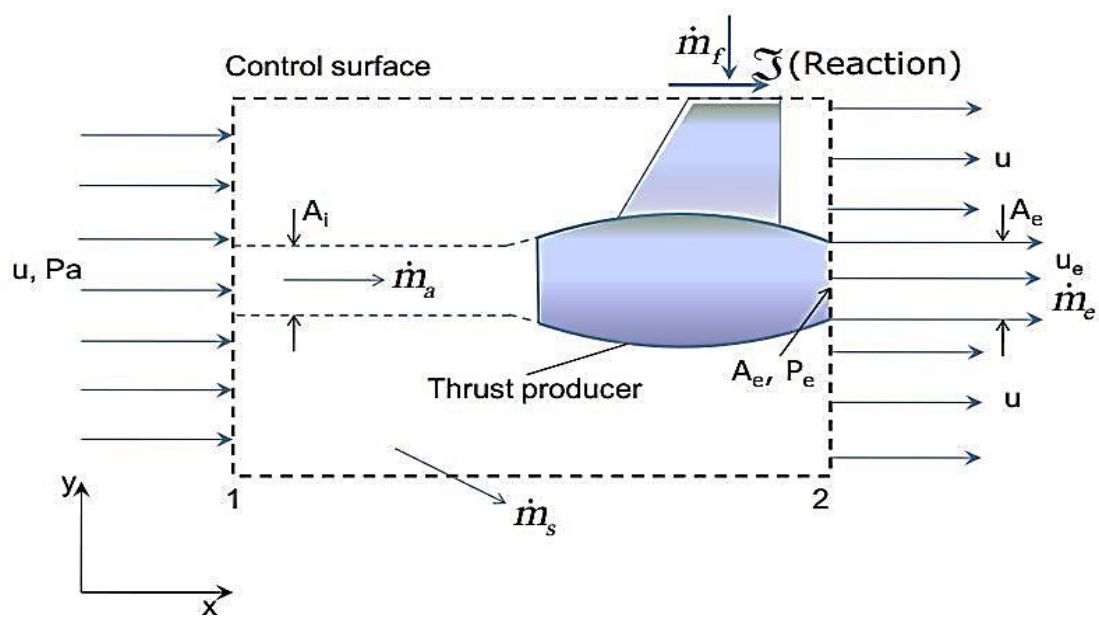

Figure (2): Illustration the Change in momentum of the flow through the engine.

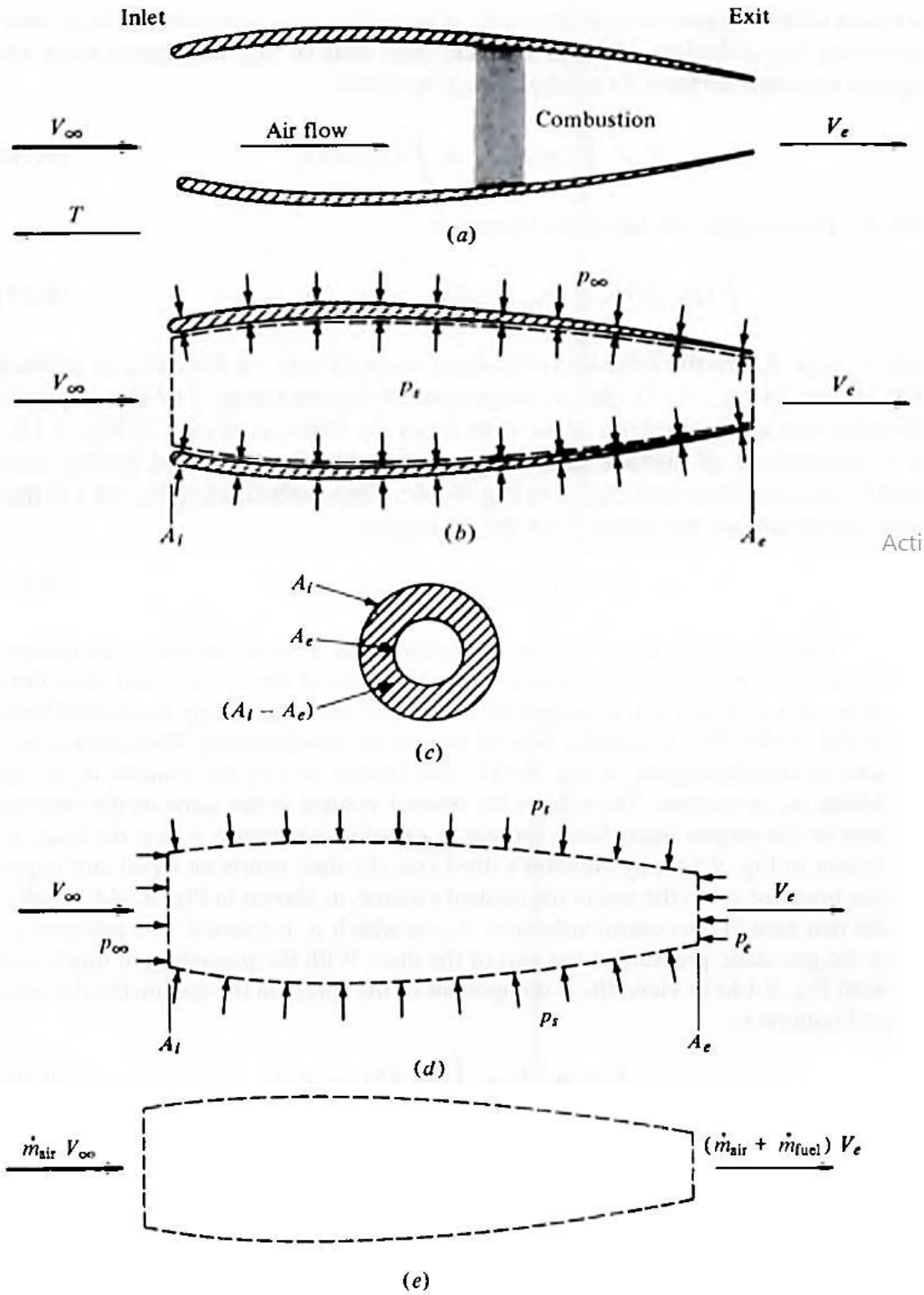

Figure (2-1): Illustration of the principle of jet propulsion, (a) Jet propulsion engine. (b) Surface pressure on inside and outside surfaces of duct, (c) Front view, illustrating inlet and exit areas. (d) Control volume for flow through duct, (e) Change in momentum of the flow through the engine.

The thrust of the engine in this direction is equal to the *X* component of p_s integrated over the complete internal surface, plus that of p_∞ integrated over the complete external surface. In mathematical symbols,

$$T = \int (p_s\, dS)_x + \int (p_\infty\, dS)_x \quad \dots\dots\dots\dots\dots\dots Eq.(2-1)$$

Since p_∞ is constant, the last term becomes

$$\int (p_\infty\, dS)_x = p_\infty \int (\, dS)_x = p_\infty (A_i - A_e) \quad \dots\dots\dots\dots\dots\dots Eq.(2-2)$$

Where A_i and A_e are the inlet and exit areas, respectively, of the duct, as defined in Fig. 2.1-b. In Eq. (2.2), the *x* component of the duct area, $\int (dS)_x$, is physically what you see by looking at the duct from the front, as shown in Fig. 2.1-c.

The *x* component of surface area is geometrically the projected frontal area shown by the crosshatched region in Fig. 2.1-c. Thus, substituting Eq. (2.2) into (2.1), we obtain for the thrust *T* of the jet engine.

$$T = \int (p_s\, dS)_x + p_\infty (A_i - A_e) \quad \dots\dots\dots\dots\dots\dots eq.(2-3)$$

The integral in Eq. (2.3) is not particularly easy to handle in its present form. Let us proceed to couch this integral in terms of the velocity and mass flow of gas through the duct.

Consider the volume of gas bounded by the dashed lines in Fig. 2.1-b. This is called a control volume in aerodynamics. The frontal area of the volume is A_i, on which p_∞ is exerted. The side of the control volume is the same as the internal area of the engine duct. Since the gas is exerting a pressure p_s, on the duct, as shown in Fig. 2.1-b, by Newton's third law, the duct exerts an equal and opposite pressure p_s on the gas in the control volume, as shown in Fig. 2.1-d.

Finally, the rear area of the control volume is A_e, on which p_e is exerted. The pressure p_e is the gas static pressure at the exit of the duct. With the preceding in mind, and with Fig. 2.1-d in view,

The X component of the force on the gas inside the control volume is

$$F = p_\infty A_i + \int (p_s\, dS)_x - p_e A_e \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2-4)$$

From Newton's second law, namely, (F = ma). This can also be written as F = d (mV)/d t, that is, the force equals the time rate of change of momentum

The mass flow of air (kg/s) entering the duct is $\dot{m}_{air}$; its momentum is $\dot{m}_{air}\, V_\infty$ .The mass flow of gas leaving the duct (remember that fuel has been added and burned inside) is $m_{air} + m_{fuel}$; its momentum is $(m_{air} + m_{fuel})V_e$. Thus, the time rate of change of momentum of the airflow through the control volume is the difference between what comes out and what goes in $(m_{air} + m_{fuel})V_e — m_{air}\, V_\infty$. From Newton's second law, this is equal to the force on the control volume,

$$F = (m_{air} + m_{fuel})V_e - m_{air}\, V_\infty \ldots\ldots\ldots\ldots\ldots (2-5)$$

Combining Eqs. (2.4) and (2.5) yields

$$(\dot{m}_{air} + \dot{m}_{fuel})V_e - \dot{m}_{air}V_\infty = p_\infty A_i + \int (p_s\, dS)_x - p_e A_e \ldots\ldots\ldots\ldots (2-6)$$

Solving Eq. (2-6) for the integral term, we obtain

$$\int (p_s\, dS)_x = (\dot{m}_{air} + \dot{m}_{fuel})V_e - \dot{m}_{air}V_\infty + p_e A_e - p_\infty A_i \ldots\ldots\ldots\ldots (2-7)$$

We now have the integral in the original thrust equation Eq. (2.3), in terms of velocity and mass flow, as originally desired. The final result for the engine thrust is obtained by substituting Eq. (2.7) into Eq. (2.3):

$$T = (\dot{m}_{air} + \dot{m}_{fuel})V_e - \dot{m}_{air}V_\infty + p_e A_e - p_\infty A_i + (A_i - A_e)p_\infty \ldots\ldots\ldots\ldots (2-8)$$

The terms involving A_i cancel, and we have

$$T = (\dot{m}_{air} + \dot{m}_{fuel})V_e - \dot{m}_{air}V_\infty + (p_e - p_\infty)A_e \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2-9)$$

Equation (2.9) is the fundamental thrust equation for jet propulsion.

$$T = \dot{m}_{air}\left[\left(1+\frac{\dot{m}_{fuel}}{\dot{m}_{air}}\right)V_e - V_\infty\right] + (p_e - p_\infty)A_e \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2-10)$$

$$T = \dot{m}_{air}[(1+f)V_e - V_\infty] + (p_e - p_\infty)A_e \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2-11)$$

<u>Where</u>

Net thrust =T

The other types of thrusts are:-

Gross thrust = $\dot{m}_{air}[(1+f)V_e] + (p_e - p_\infty)A_e$

Momentum thrust = $\dot{m}_{air}[(1+f)V_e]$

Pressure thrust = $(p_e - p_\infty)A_e$

Momentum drag = $\dot{m}_{air}\, u$

Thus:

Net thrust = Gross thrust – Momentum drag

Or in other words, Net thrust = Momentum thrust + Pressure thrust – Momentum drag

If the nozzle is *<u>unchoked</u>*, then ($P_e = P_a$), the pressure thrust cancels in Eq. (2.11).
The thrust is then expressed as

$$T = \dot{m}_{air}[(1+f)V_e - V_\infty] \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2-12)$$

In many cases the fuel to air ratio is negligible, thus the thrust force equation is reduced to the simple form:

$$T = \dot{m}_{air}[V_e - V_\infty] \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2-13)$$

In a similar way, the thrust force for two stream engines like turbofan (Fig. 2.2) and prop fan engines can be derived. It will be expressed as

$$T = \dot{m}_h[(1+f)V_{eh} - V_\infty] + \dot{m}_c(V_{ec} - V) + (p_{eh} - p_\infty)A_{eh} + (p_{ec} - p_\infty)A_{ec}. (2-14)$$

<u>Where</u>

f = $\frac{\dot{m}_{fuel}}{\dot{m}_{hot\ air}}$: fuel to air ratio

$\dot{m}_{hot\ air}$: Air mass flow passing through the hot section of engine; turbine(s)

$\dot{m}_{eh} = \dot{m}_h[(1+f)]$: Mass of hot gases leaving the engine
$\dot{m}_c$: Air mass flow passing through the fan
u_{eh}: Velocity of hot gases leaving the turbine nozzle
u_{ec}: Velocity of cold air leaving the fan nozzle
P_{eh}: Exhaust pressure of the hot stream
P_{ec}: Exhaust pressure of the cold stream
A_{eh}: Exit area for the hot stream
A_{ec}: Exit area for the cold stream
The specific thrust is defined as the thrust per unit air mass flow rate (T/m_a),which can be obtained from Eq. (2.14). It has the dimensions of a velocity (say m/s).

For turboprop engines (Fig. 2.2), the high value of thrust is achieved by the very large quantity of the airflow rate, though the exhaust and flight speeds are very close. An analogous formula to Eq. (3.4) may be employed as follows:

$$T = \dot{m}_c[(1+f)V_e - V_1] + \dot{m}_0(V_1 - V_0) \ldots\ldots\ldots\ldots (2-15)$$

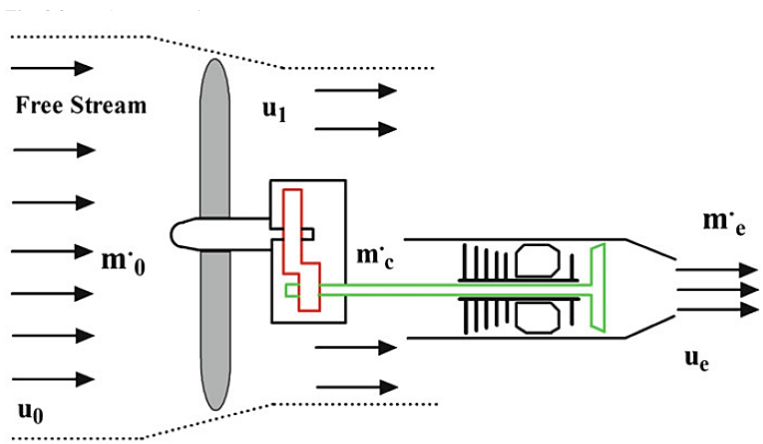

Figure (2.2): Turboprop engine.

<u>Where</u>

m_0 : is the air mass flow sucked by the propeller.
m_c is a part of the air flow crossed the propeller and then entered the engine through its intake
u_0, u_1, and u_e : are air speed upstream and downstream the propeller and gases speed at the engine exhaust. The exhaust nozzle is normally unchoked.

2.4 Factors Affecting Thrust.

As seen from Eq. (2.14) for a single stream aero engine (ramjet or turbojet engine), the thrust force depends on

1. The inlet and outlet air mass flow rates.
2. Fuel-to-air ratio.
3. Flight speed.
4. Exhaust speed.
5. Exhaust and ambient pressures.

Factors listed above each of them are dependent on several parameters. For example

The inlet air mass flow rate influencing both of the momentum thrust and momentum drag is dependent on several variables including the flight speed, ambient temperature and pressure, humidity, altitude, and rotational speed of the compressor.

The outlet gas mass flow rate is dependent on the fuel added, air bleed, and water injection.

The pressure thrust term depends on the turbine inlet temperature, flight altitude, and the nozzle outlet area and pressure.

The momentum thrust is also dependent on the jet nozzle velocity. These parameters can be further explained as below:

2.4.1 Jet Nozzle

Pressure thrust has finite values only for choked nozzles, where the exit pressure is greater than the ambient pressure. Nozzles are either of the convergent or convergent–divergent (C–D) type. Only convergent nozzles may be choked. Fig (2.3).

For a choked convergent nozzle, the pressure thrust depends on both of the area of the exhaust nozzle and also on the difference between the exit and ambient pressures.

Moreover, the exhaust speed is equal to the sonic speed which is mainly influenced by the exhaust gas temperature. If a convergent nozzle is unchoked, then the jet velocity will attain subsonic values. For a convergent divergent (CD) nozzle, the jet speed may attain supersonic values. CD nozzles are seen only in supersonic aircrafts.

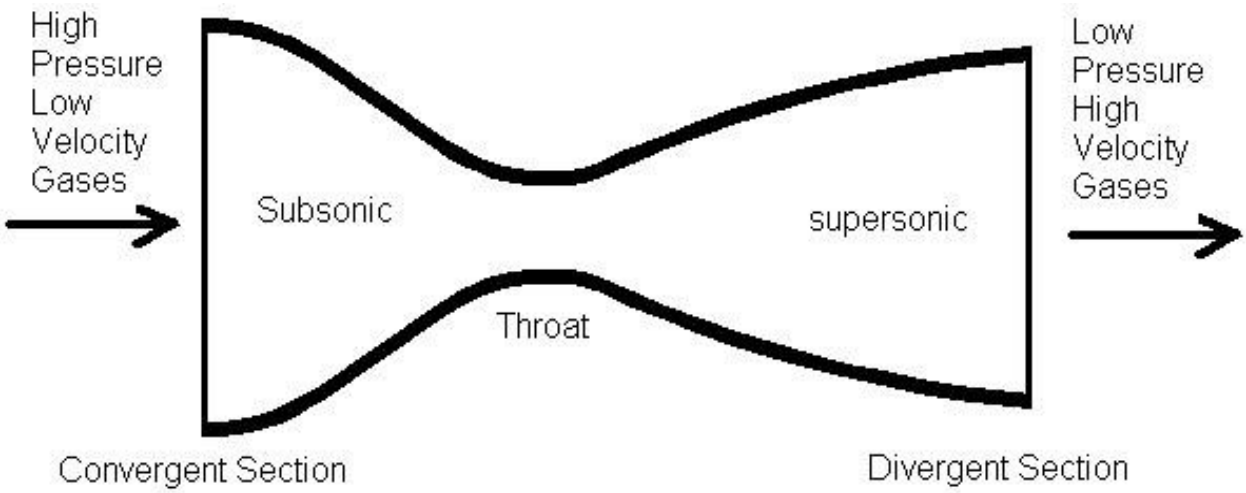

Figure (2.4): Convergent–Divergent nozzle.

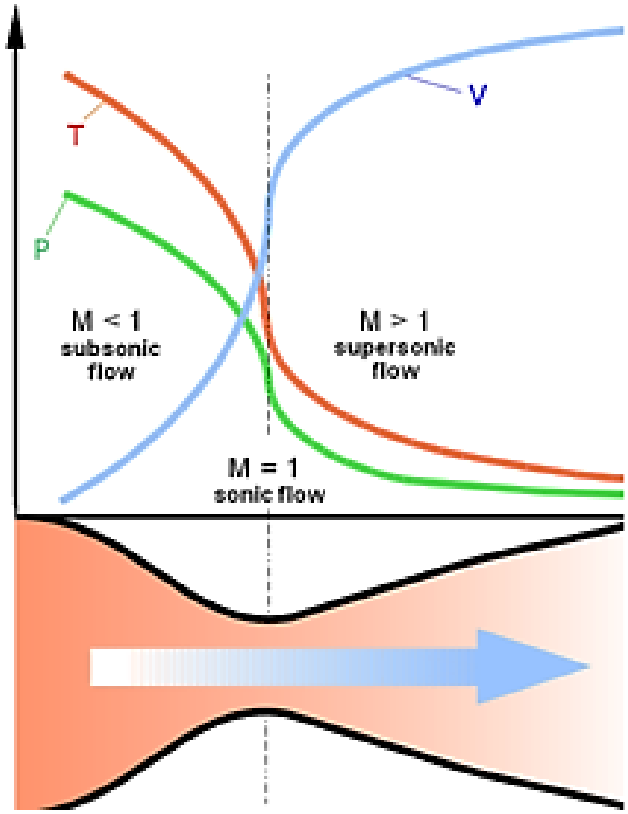

Figure (2.5): Flow properties of convergent–divergent nozzle.

2.4.2 Air Speed

The air speed, sometimes denoted as the approach speed, is equal to the flight speed in the thrust force; Eq. (2.11). Such a parameter has a direct effect on the net thrust. If the exhaust gas velocity is constant and the air velocity is increased, then the difference between both velocities $[(1 + f) u_e - u]$ is decreased leading to a decrease also in the net thrust. If the air mass flow and the fuel to air ratio are assumed constants, then a linear decrease in the net thrust is enhanced (Fig. 2.6).

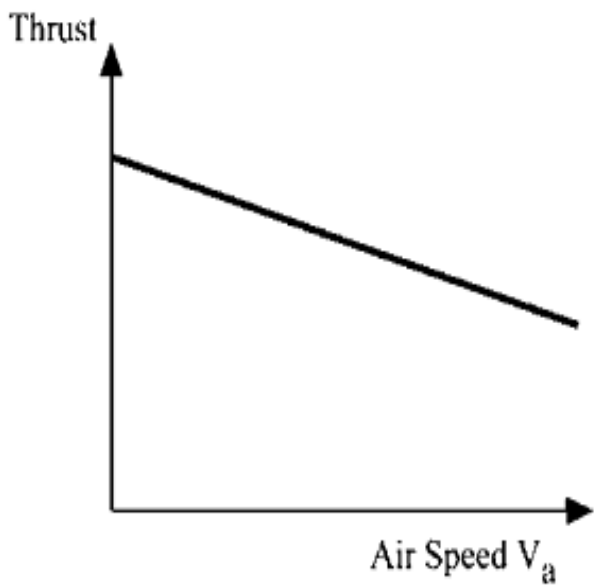

Figure (2.6): Variation of thrust force with air speed.

2.4.3 Mass Air Flow

The mass air flow m_a is the most significant parameter in the thrust equation. It depends on the air temperature and pressure as both together determine the density of the air entering the engine.

In free air, a rise in temperature will decrease the density. Thus air density and mass flow rate is inversely proportional with the air temperature. On the contrary, an increase in the pressure of a free air increases its density and, consequently, its thrust increases. The effect of both of air temperature and pressure is illustrated in Fig. 2.7. *In brief, the density affects the inlet air mass flow and it directly affects thrust.*

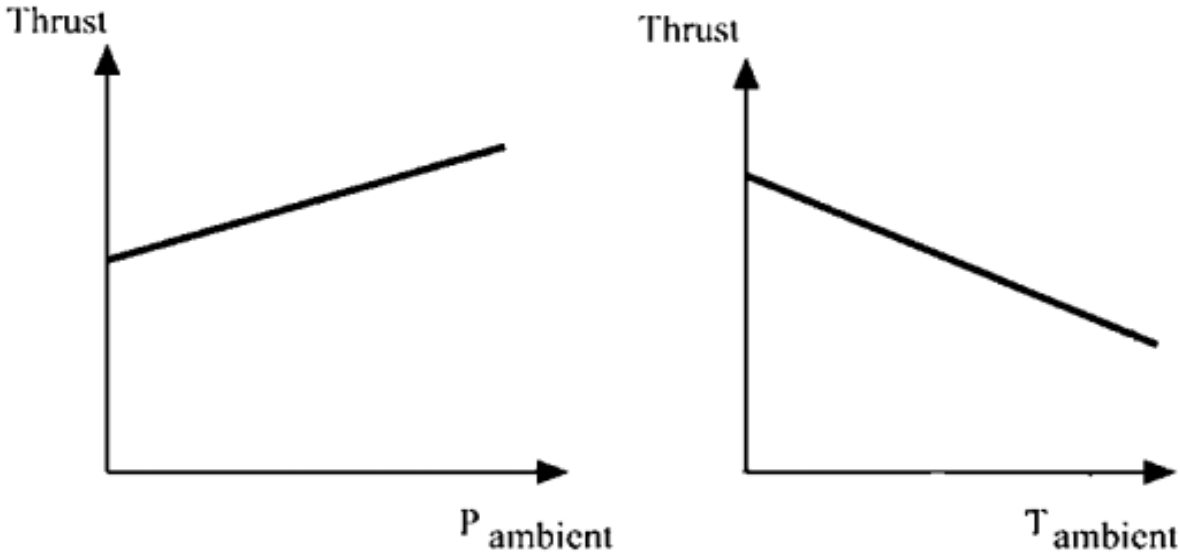

Figure (2.7): Variation of the thrust force with air temperature and pressure.

2.4.4 Altitude

Since the air temperature and pressure have significant effects on the thrust. in the International Standard Atmosphere (ISA) temperature decreases by about 3.2 K per 500 m of altitude up to nearly 11,000 m (36,089 ft). The variations of ambient temperature and pressure are given by Eqs. (2.16) and (2.17). These relations are repeated here, but with altitude expressed in meter:

$$\boldsymbol{T = T_1 + a(h - h_1)} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \boldsymbol{(2-16)}$$

$$\boldsymbol{\frac{P}{P_1} = \left(\frac{T(h)}{T_1}\right)^{-\frac{g_0}{R\,a}}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \boldsymbol{(2-17)}$$

After 11,000 m, the temperature stops falling, but the pressure continues to drop steadily with increasing altitude.

Consequently, above 11,000 m (36,089 ft), the thrust will drop off more rapidly (Fig. 2.8). This makes the 11,000 m an optimum altitude for long-range cruising at nominal speed. *It may be concluded that the effect of altitude on thrust is really a function of density.*

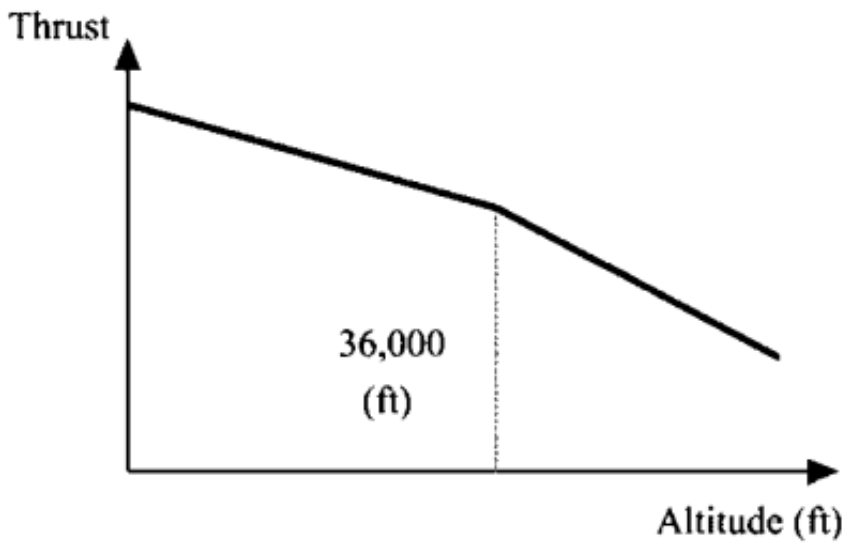

Figure (2.8): Variation of the thrust force with altitude.

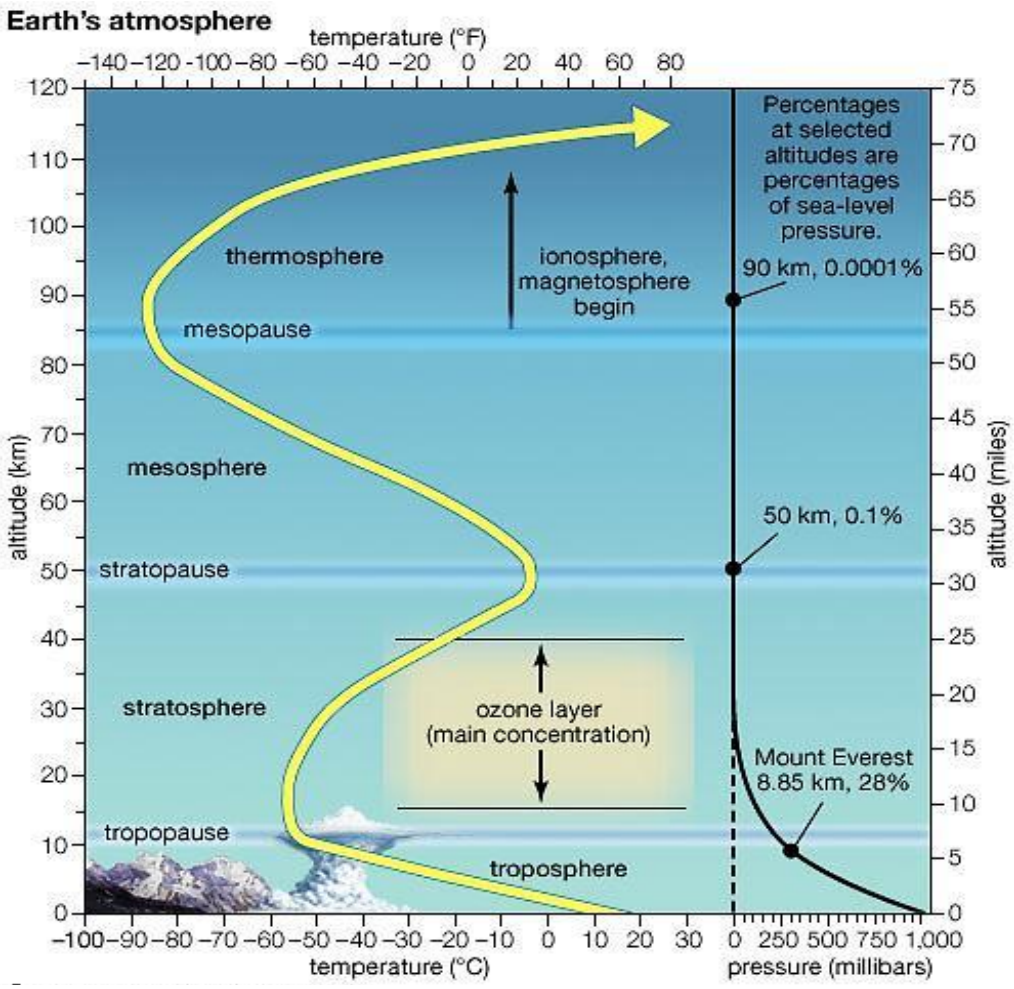

Figure (2.9): Variation of temperature and pressure with altitude.

2.4.5 Ram Effect

The movement of the aircraft relative to the outside air causes air to be rammed into the engine inlet duct.

Ram effect increases the airflow to the engine, which in turn, increases the gross thrust. However, it is not as easy, ram effects combine two factors, namely, the air speed increase and in the same time increases the pressure of the air and the airflow into the engine.

As described earlier, the increase of air speed reduces the thrust, which is sketched in Fig. 2.10 as the 'A' curve. Moreover, the increase of the airflow will increase the thrust, which is sketched by the 'B' curve in the same figure.

The 'C' curve is the result of combining curves 'A' and 'B'. The increase of thrust due to ram becomes significant as the air speed increases, which will compensate for the loss in thrust due to the reduced pressure at high altitude. Ram effect is thus important in high speed fighter aircrafts. Also modern subsonic jet-powered aircraft fly at high subsonic speeds and higher altitudes to make use of the ram effect.

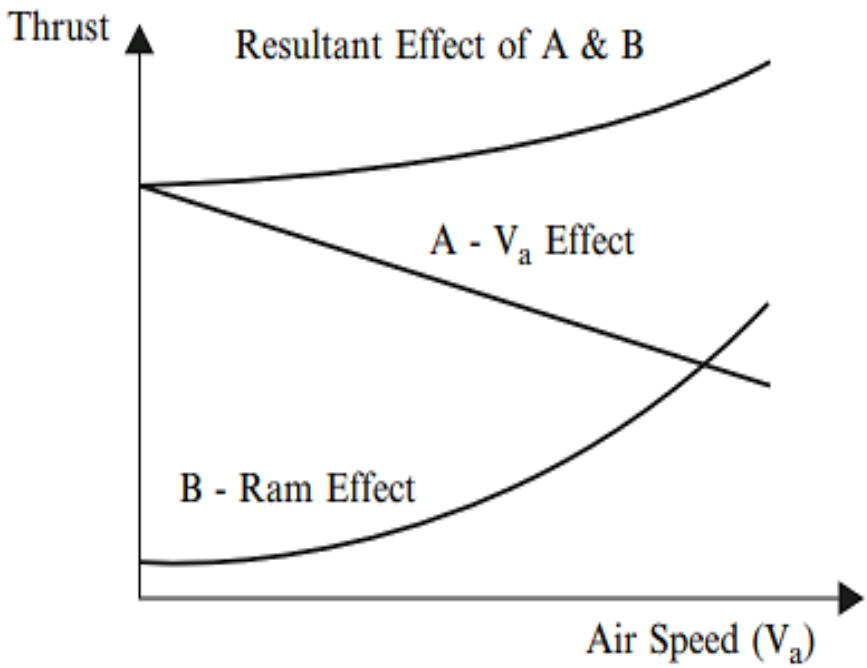

Figure (2.10): Effect of ram pressure on thrust.

Example :1/

Air flows through a turbojet engine at the rate of 50.0 kg/s and the fuel flow rate is 1.0 kg/s. The exhaust gases leave the jet nozzle with a relative velocity of 600 m/s. Compute the velocity of the airplane, if the thrust power is 1.5 MW in the following two cases:

1. Pressure equilibrium exists over the exit plane
2. If the pressure thrust is 8 kN

Solution

1. When the nozzle is unchoked, pressure equilibrium exists over the exit plane. Then, thrust force is expressed as

$$T = (\dot{m}_a + \dot{m}_f)u_e - \dot{m}_a u$$

$$\text{Thrust power} = T \times u$$

$$\text{Thrust power} = (\dot{m}_a + \dot{m}_f)u_e u - \dot{m}_a u^2$$

$$1.5 \times 10^6 = (51)(600)\, u - 50\, u^2$$

$$50\, u^2 - 30,600\, u + 1.5 \times 10^6 = 0$$

$$\text{or} \qquad u = \frac{30,600 \pm 10^3\sqrt{936.36 - 300}}{100}$$

Thus, either $u = 558.26$ m/s or $u = 53.74$ m/s

2. When the exit pressure is greater than the ambient pressure, a pressure thrust (T_p) is generated. The thrust equation with pressure thrust is then

$$T = (\dot{m}_a + \dot{m}_f)u_e - \dot{m}_a u + T_p$$

Thus, the thrust power is

$$T \times u = (\dot{m}_a + \dot{m}_f)u_e u - \dot{m}_a u^2 + T_p \times u = [(\dot{m}_a + \dot{m}_f)u_e + T_p] \times u - \dot{m}_a u^2$$

$$1.5 \times 10^6 = [51 \times 600 + 8000] \times u - 50u^2 = 38,600\, u - 50\, u^2$$

$$50\, u^2 - 38,600\, u + 1.5 \times 10^6 = 0$$

$$u = \frac{38,600 \pm 10^3\sqrt{1490 - 300}}{100} = \frac{38,600 \pm 34,495}{100}$$

Thus either $u = 731$ m/s or 41 m/s

<u>Example 2/</u>

A fighter airplane is powered by two turbojet engines. It has the following characteristics during cruise flight conditions:

Wing area (S) = 49.24 m^2
Engine inlet area A_i = 0.06 m^2
Cruise speed V_f = 243 m/s
Flight altitude = 35,000 ft
Drag and lift coefficients are C_D = 0.045, C_L = 15 C_D
Exhaust total temperature T_0 = 1005 K
Specific heat ratio and specific heat at exit are γ = 1.3, C_p = 1100 J/(kgK)

It is required to calculate:
1. Net thrust
2. Gross thrust
3. Weight
4. Jet speed assuming exhaust pressure is equal to ambient pressure if $P_e = P_a$
5. Static temperature of exhaust T_e
6. Exhaust Mach number M_e

Solution

At 35,000 m altitude, the properties of ambient conditions are

Temperature $T = -54.3$ °C, pressure $P = 23.84$ kPa, and density 0.3798 kg/m^3

The mass flow rate is $\dot{m} = \rho_a V_f A_i = 0.3798 \times 243 \times 0.6 = 55.375$ kg/m^3

1. During cruise flight segment, the thrust and drag force (D) are equal. Thus for two engines and (T) is the net thrust of each engine, then

$$2\,T = D = \rho V^2 A C_D/2$$

$$T = 0.3798 \times (243)^2 \times 49.24 \times 0.045/4 = 12,423\,N = 12.423 \text{ kN}$$

2. Gross thrust = Net thrust + Ram drag

$$T_{\text{gross}} = T + \dot{m}\,V_f = 12,423 + 55.3 \times 243 = 25,879 \text{ N} = 25.879 \text{ kN}$$

3. Since Weight = Lift, thus $L = W$.
   Moreover, lift and drag are correlated by the relation:

$$C_L = 15\,C_D = 0.675$$

$$L = W = 15D = 30T = 37.2690\,N = 372.69 \text{ kN}$$

4. Assuming negligible fuel flow ratio, and since $P_e = P_a$, then the net thrust is expressed by the relation:

$$T = \dot{m}\left(V_J - V_f\right)$$

$$V_J = V_f + \frac{T}{\dot{m}} = 243 + \frac{12,423}{55.375} = 467.3 \text{ m/s}$$

5. Exhaust static temperature is expressed by the relation:

$$T_e = T_{0e} - \frac{V_j^2}{2C_p} = 1005 - \frac{(467)^2}{2 \times 1100} = 905.9 \text{ K}$$

6. Sonic speed at exit $a_e = \sqrt{\gamma R T} = \sqrt{1.3 \times 287 \times 905.7} = 581.3$ m/s
   Exhaust Mach number is $M_e = \frac{V_j}{a_e} = 0.804$

Effect of water injection on thrust

When used in a turbine engine water injection normally preventing detonation is not the primary goal. Water is normally injected either at the compressor inlet or in the diffuser just before the combustion chambers. Adding water increases the mass being accelerated out of the engine, increasing thrust, but it also serves to cool the turbines. Since temperature is normally the limiting factor in turbine engine performance at low altitudes, the cooling effect lets the engine run at higher RPM with more fuel injected and more thrust created without overheating.

The drawback of the system is that injecting water quenches the flame in the combustion chambers somewhat, as there is no way to cool the engine parts without coincidentally cooling the flame. This leads to unburned fuel out the exhaust and a characteristic trail of black smoke.