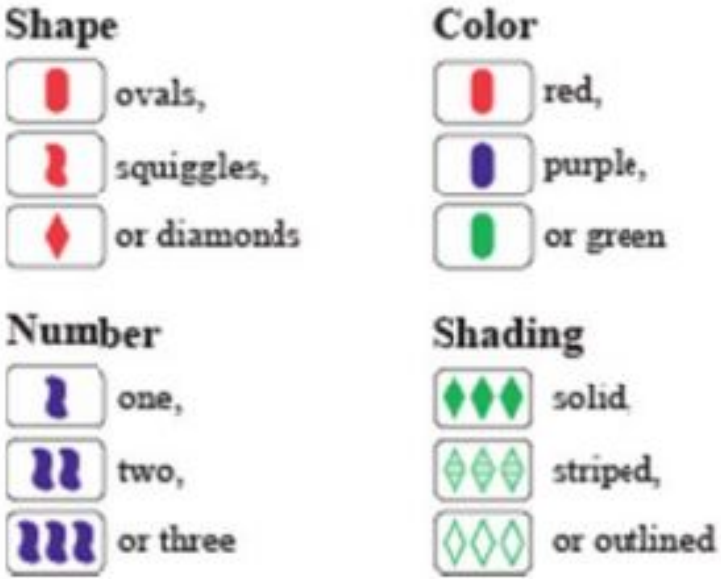
Tiles and Tessellations

Duke TIP Academic Adventures

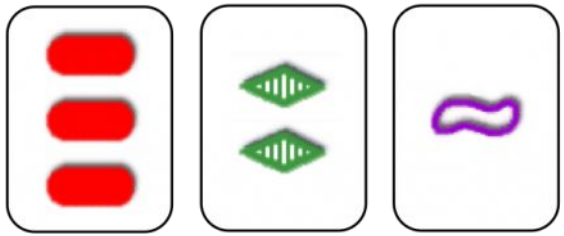
Game of SET



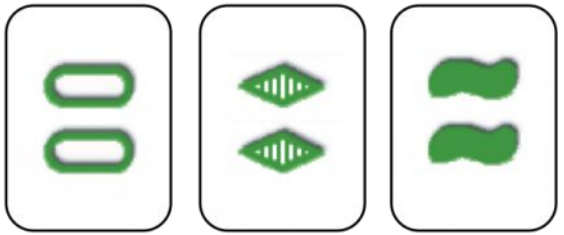
A set consists of 3 cards in which each of the cards features, looked at one-by-one, are the same on each card, or different on each card.

Game of SET

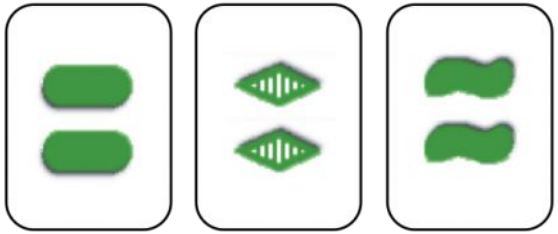
YES



YES



NO



Game of SET

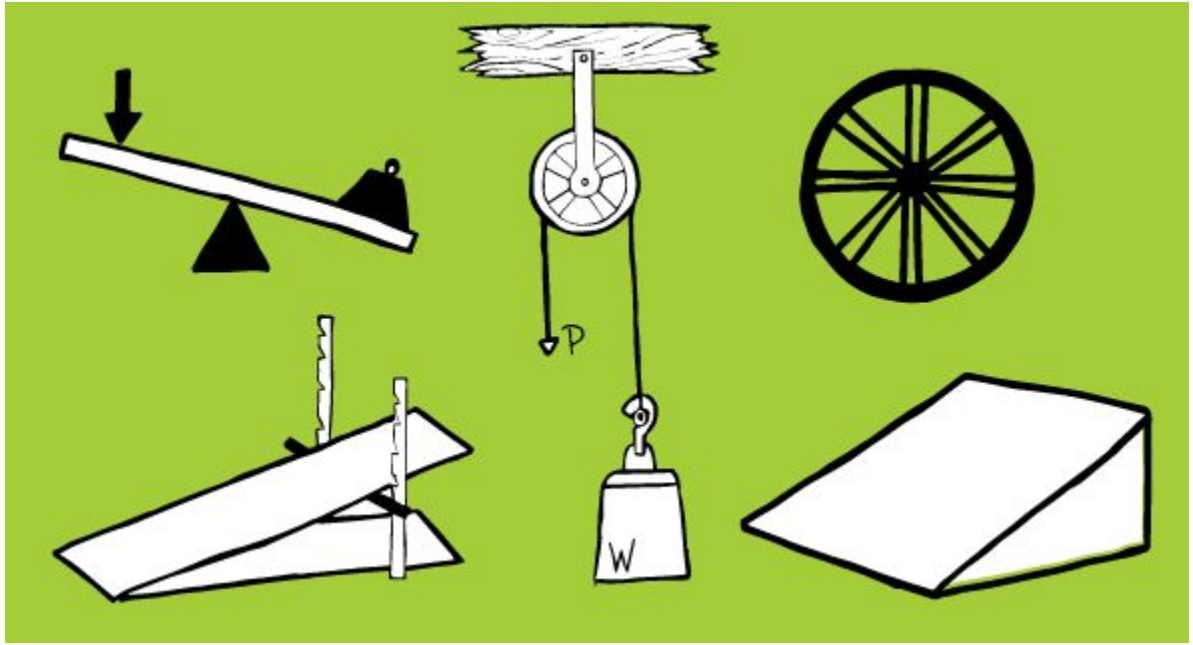
- At your table deal 12 cards face up.
- Players do not take turns, but pick up SETs as they see them. If everyone agrees there are no SETs, deal 3 more cards on the table.
- Replace the cards from the top of the deck when a SET is removed.
- The person with the most SETs wins!

Third Person Introductions

Introduce the person sitting next you by saying the following.

- Name
- Hometown
- Favorite Food

Human Machine



Rules and Expectations

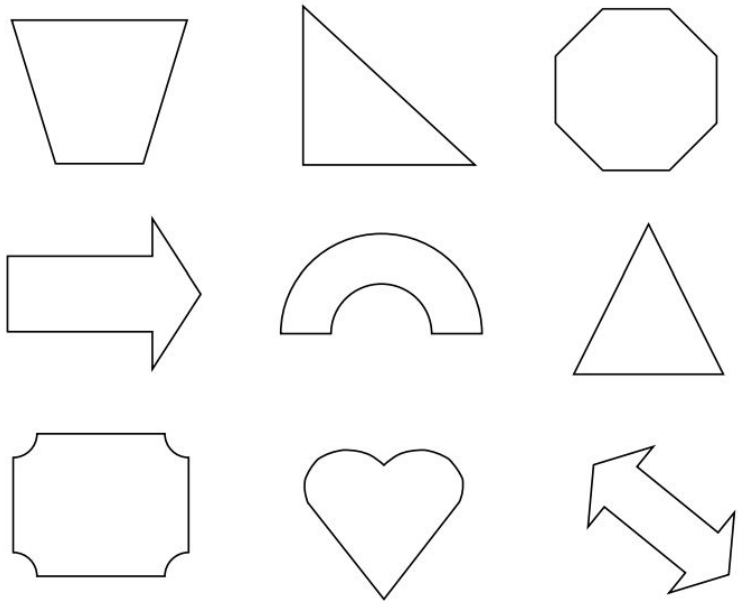
- Be Respectful
- Participate
- Don’t Talk When Someone Else Is
- Don’t Shout Out Answers
- Thinking > Answers
- Have Fun

Finishing Early

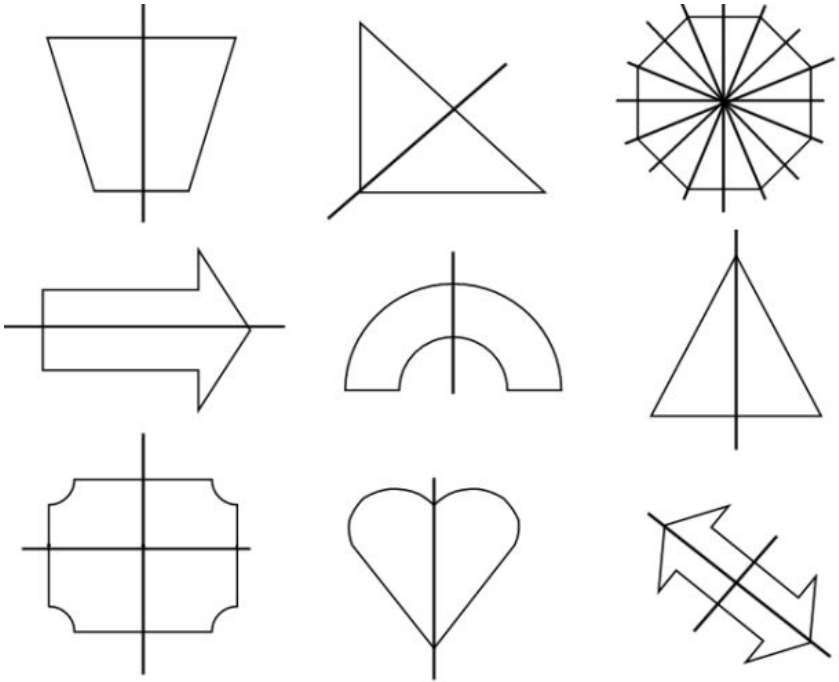
Everyone works at a different pace. If you you happen to finish something early, please raise your hand and let me know. You may get to be my assistant!

Warm Up

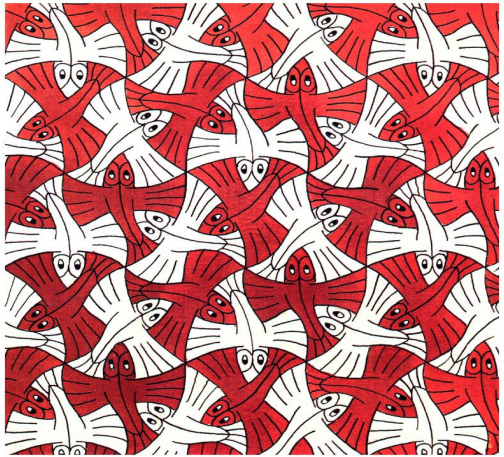
Draw the line(s) of symmetry on each figure.



Warm Up



Tiles and Tessellations

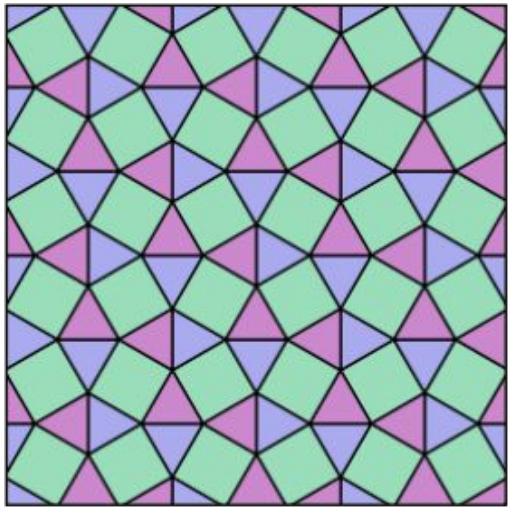


Tiles and Tessellations

A **tessellation** of a flat surface is the covering of a plane using one or more geometric shapes, called **tiles**, with no overlaps and no gaps.

Create a Tessellation

Create your own tessellations using the shapes provided. You may use as many different shapes as you like.



Create a Tessellation

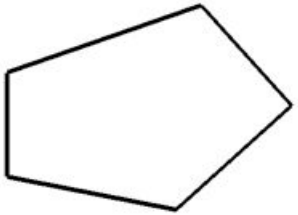
- How did you create your tessellations?

- Could you come up with a way to create a tessellation, if given a few shapes?

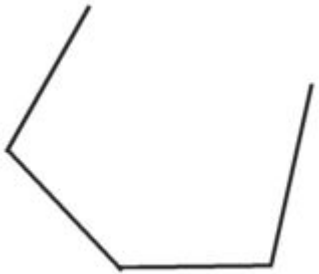
Polygons

A **polygon** is a closed geometric figure.

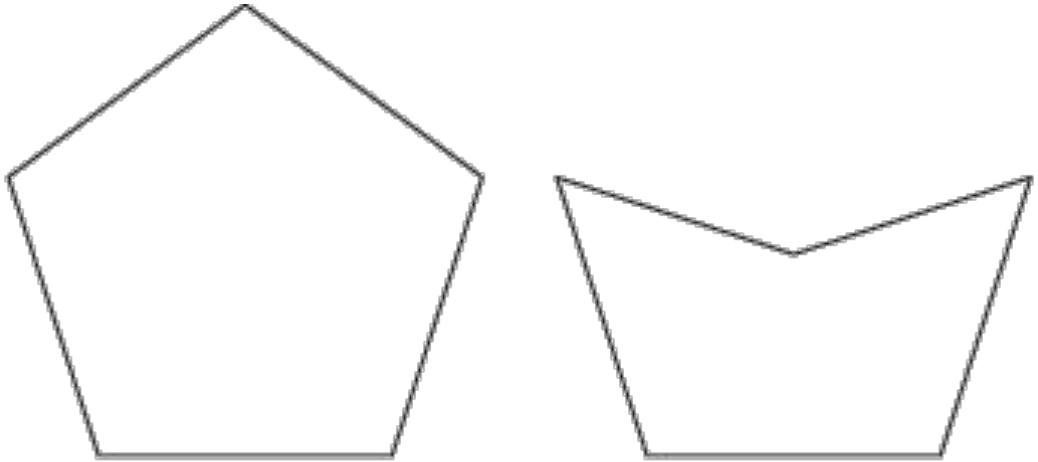
Polygon



Not a Polygon



Convex and Concave Polygons



convex polygon

concave polygon

Polygon Angle Sum

What is the sum of the angle measures of a triangle?

Polygon Angle Sum

1. Draw a triangle on a sheet of paper using a ruler.

2. Carefully cut out the triangle with the scissors provided.

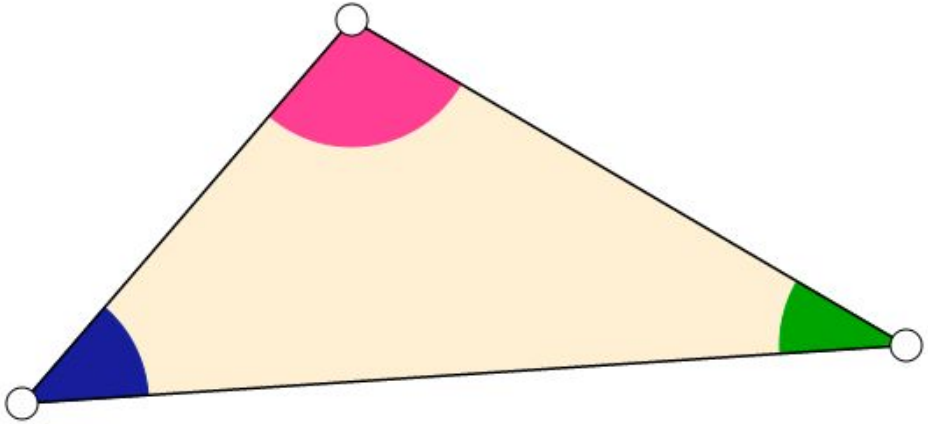
3. Rip off two of the corners.

4. Place them next to the remaining corner so that they all share a vertex.

5. What do you notice?

Polygon Angle Sum

The sum of the angle measures of a triangle is 180 degrees!



Polygon Angle Sum

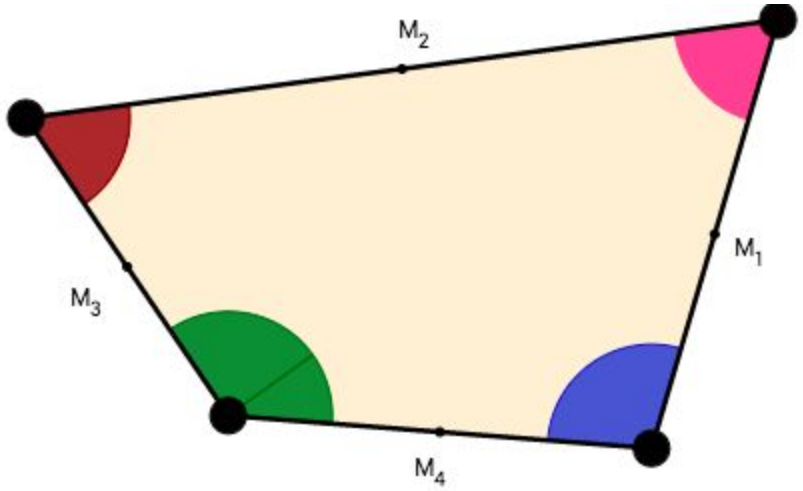
What is the sum of the angle measures of a quadrilateral (rectangle, square, rhombus, parallelogram, trapezoid, kite, etc.)?

Polygon Angle Sum

Hint: Separate the quadrilateral into two triangles.

Polygon Angle Sum

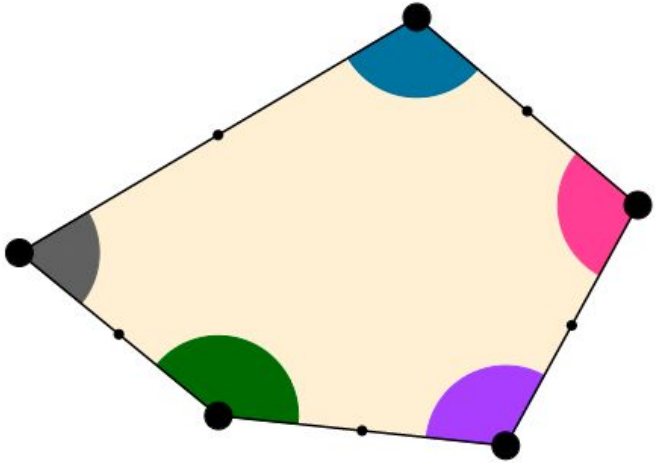
The sum of the angle measures of a quadrilateral is 360 degrees!

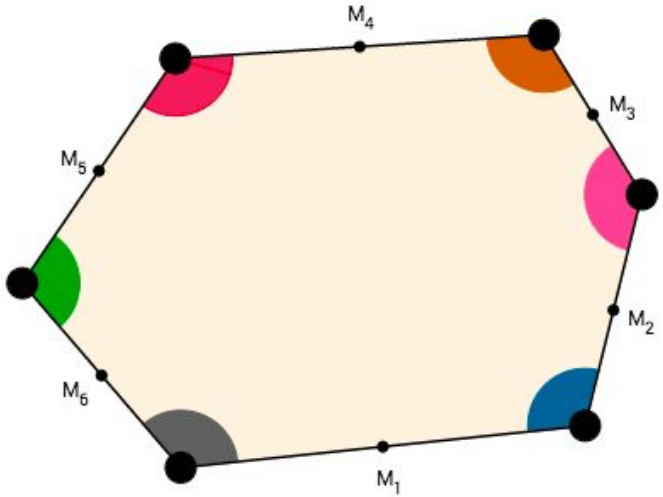


Polygon Angle Sum

- What is the sum of the angle measures of pentagon (5 sided polygon)? Hexagon (6 sided polygon)?
- Create a table and record these values.
- Can you predict the sum of the angle measures a polygon with any number of sides?

Polygon Angle Sum





Polygon Angle Sum

Polygon	Number of Sides	Sum of Angle Measures
Triangle	*3*	
Quadrilateral	*4*	
Pentagon	*5*	
Hexagon	*6*	
Heptagon	*7*	
Octagon	*8*	
Nonagon	*9*	
Decagon	*10*	
n-gon	*n*	
100-gon	*100*	

Polygon Angle Sum

Polygon	Number of Sides	Sum of Angle Measures
Triangle	*3*	*180*
Quadrilateral	*4*	*360*
Pentagon	*5*	
Hexagon	*6*	
Heptagon	*7*	
Octagon	*8*	
Nonagon	*9*	
Decagon	*10*	
n-gon	*n*	
100-gon	*100*	

Polygon Angle Sum

Polygon	Number of Sides	Sum of Angle Measures
Triangle	*3*	*180*
Quadrilateral	*4*	*360*
Pentagon	*5*	*540*
Hexagon	*6*	*720*
Heptagon	*7*	
Octagon	*8*	
Nonagon	*9*	
Decagon	*10*	
n-gon	*n*	
100-gon	*100*	

Polygon Angle Sum

Polygon	Number of Sides	Sum of Angle Measures
Triangle	*3*	*180*
Quadrilateral	*4*	*360*
Pentagon	*5*	*540*
Hexagon	*6*	*720*
Heptagon	*7*	*900*
Octagon	*8*	*1080*
Nonagon	*9*	*1260*
Decagon	*10*	*1440*
n-gon	*n*	
100-gon	*100*	

Polygon Angle Sum

Polygon	Number of Sides	Sum of Angle Measures
Triangle	*3*	*180*
Quadrilateral	*4*	*360*
Pentagon	*5*	*540*
Hexagon	*6*	*720*
Heptagon	*7*	*900*
Octagon	*8*	*1080*
Nonagon	*9*	*1260*
Decagon	*10*	*1440*
n-gon	*n*	*180(n-2)*
100-gon	*100*	

Polygon Angle Sum

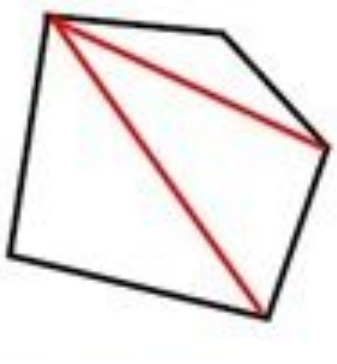
Polygon	Number of Sides	Sum of Angle Measures
Triangle	*3*	*180*
Quadrilateral	*4*	*360*
Pentagon	*5*	*540*
Hexagon	*6*	*720*
Heptagon	*7*	*900*
Octagon	*8*	*1080*
Nonagon	*9*	*1260*
Decagon	*10*	*1440*
n-gon	*n*	*180(n-2)*
100-gon	*100*	*17640*

Polygon Angle Sum

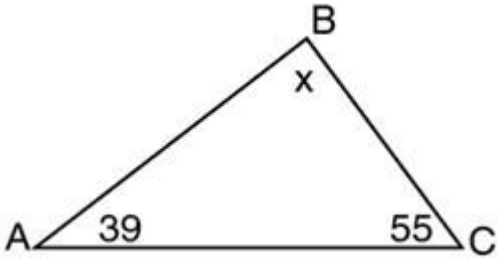
Why does the formula $S = 180(n-2)$ make sense?

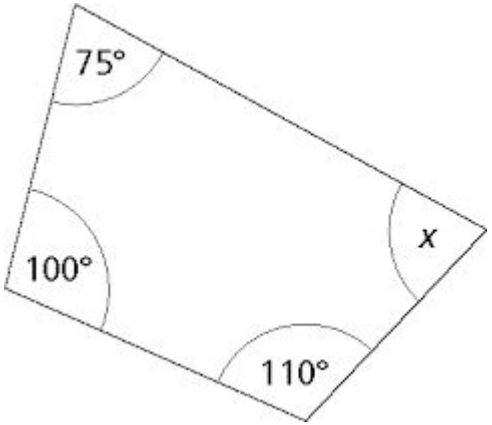
Polygon Angle Sum

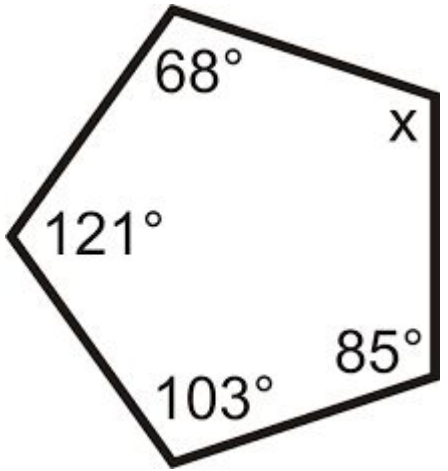
Hint: How many triangles can you create inside of a polygon with *n* sides?



Find the Measure of the Missing Angle







Find the Measure of the Missing Angle

$$39 + 55 + x = 180$$

$$94 + x = 180$$

$$x = 86$$

$$75 + 100 + 110 + x = 360$$

$$285 + x = 360$$

$$x = 75$$

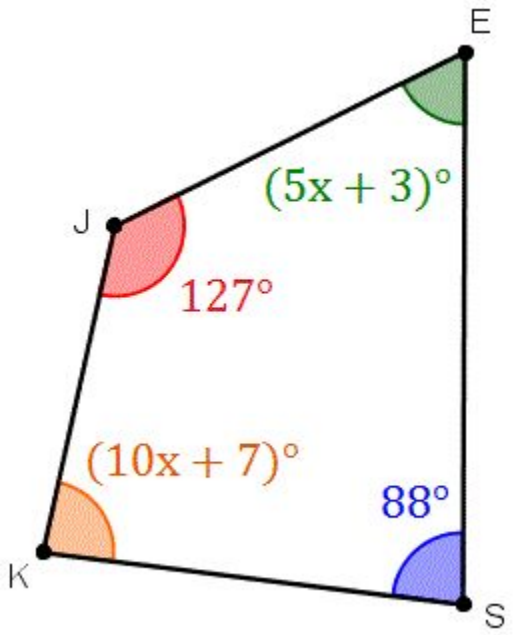
$$68 + 121 + 103 + 85 + x = 540$$

$$377 + x = 540$$

$$x = 163$$

Challenge Problem

Find the value of x, then find the measures of the missing angles.



Challenge Problem

$$5x + 3 + 88 + 10x + 7 + 127 = 360$$

$$15x + 225 = 360$$

$$15x = 135$$

$$x = 9$$

The two missing angle measures are $5(9) + 3 = 48$ and $10(9) + 7 = 97$.

Polygon Angle Sum

Is there a polygon with an angle sum of 1980 degrees? If so, how many sides does it have? If not, explain why.

Polygon Angle Sum

Yes, it has 13 sides!

$$180(n-2) = 1980$$

$$180n - 360 = 1980$$

$$180n = 2340$$

$$n = 13$$

Polygon Angle Sum

Is there a polygon with an angle sum of 2960 degrees? If so, how many sides does it have? If not, explain why.

Polygon Angle Sum

No!

If this were true we would have

$$180(n-2) = 2960$$

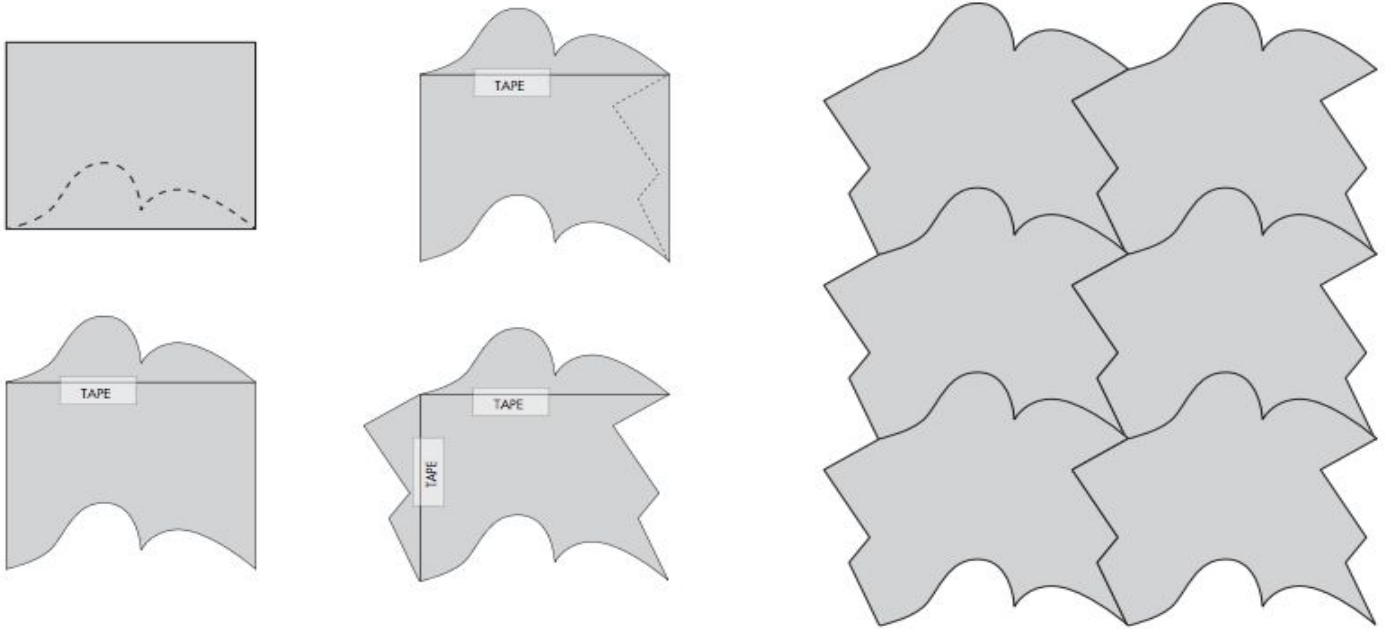
$$180n - 360 = 2960$$

$$180n = 3320$$

$$n \approx 18.4$$

Since n is the number of sides, n must be a whole number.

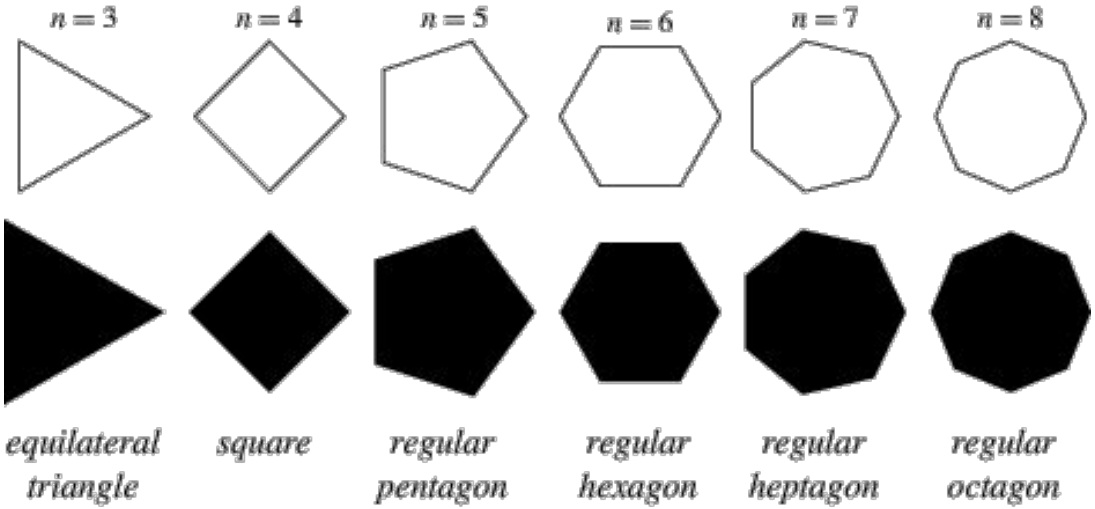
Create a Tessellation



Regular Polygons

A **regular polygon** is a polygon with congruent sides and angles. This means all of the sides have the same length and all of the angles have the same measure.

Regular Polygons



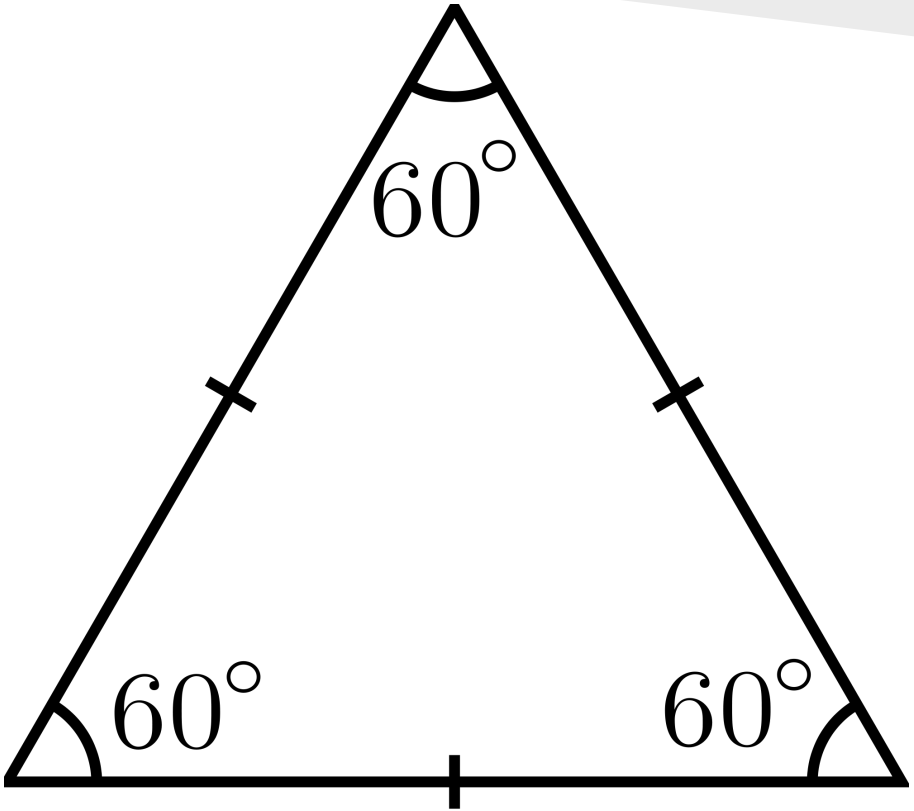
Regular Polygons

Now that we know the sum of the angle measures of any polygon, let's find the measure of each angle in a regular polygon. We will call this angle the **corner angle**.

Regular Polygons

What is the measure of each angle in an equilateral triangle?

Regular Polygons



Regular Polygons

- What about a square? Regular pentagon? Regular Hexagon?
- Create a table and record these values.
- Can you come up with a formula for the corner angle of a regular *n*-gon?

Regular Polygon Corner Angle

Regular Polygon	Number of Sides	Measure of Corner Angle
Equilateral Triangle	*3*	
Square	*4*	
Pentagon	*5*	
Hexagon	*6*	
Heptagon	*7*	
Octagon	*8*	
Nonagon	*9*	
Decagon	*10*	
n-gon	*n*	
100-gon	*100*	

Regular Polygon Corner Angle

Regular Polygon	Number of Sides	Measure of Corner Angle
Equilateral Triangle	*3*	*60*
Square	*4*	
Pentagon	*5*	
Hexagon	*6*	
Heptagon	*7*	
Octagon	*8*	
Nonagon	*9*	
Decagon	*10*	
n-gon	*n*	
100-gon	*100*	

Regular Polygon Corner Angle

Regular Polygon	Number of Sides	Measure of Corner Angle
Equilateral Triangle	*3*	*60*
Square	*4*	*90*
Pentagon	*5*	*108*
Hexagon	*6*	*120*
Heptagon	*7*	
Octagon	*8*	
Nonagon	*9*	
Decagon	*10*	
n-gon	*n*	
100-gon	*100*	

Regular Polygon Corner Angle

Regular Polygon	Number of Sides	Measure of Corner Angle
Equilateral Triangle	*3*	*60*
Square	*4*	*90*
Pentagon	*5*	*108*
Hexagon	*6*	*120*
Heptagon	*7*	*128.6*
Octagon	*8*	*135*
Nonagon	*9*	*140*
Decagon	*10*	*144*
n-gon	*n*	
100-gon	*100*	

Regular Polygon Corner Angle

Regular Polygon	Number of Sides	Measure of Corner Angle
Equilateral Triangle	*3*	*60*
Square	*4*	*90*
Pentagon	*5*	*108*
Hexagon	*6*	*120*
Heptagon	*7*	*128.6*
Octagon	*8*	*135*
Nonagon	*9*	*140*
Decagon	*10*	*144*
n-gon	*n*	*180(n-2)/n*
100-gon	*100*	

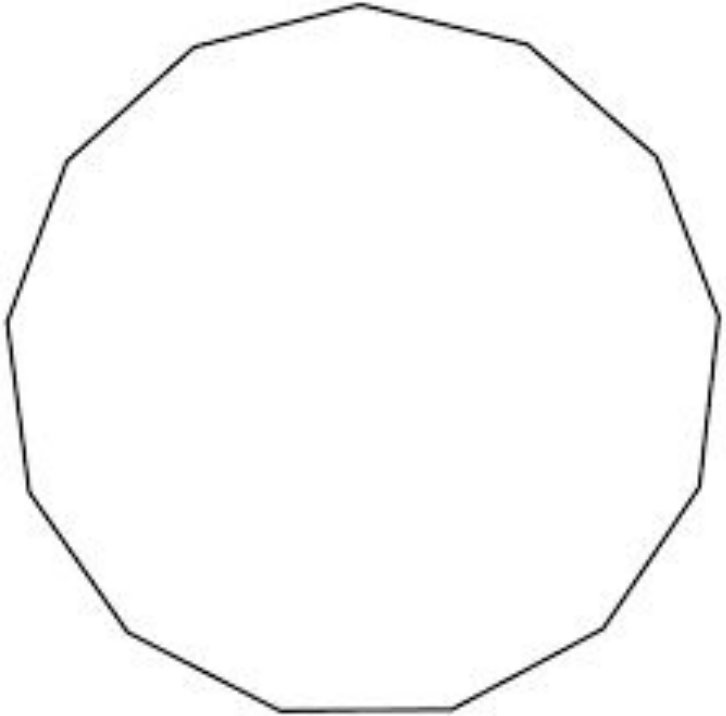
Regular Polygon Corner Angle

Regular Polygon	Number of Sides	Measure of Corner Angle
Equilateral Triangle	*3*	*60*
Square	*4*	*90*
Pentagon	*5*	*108*
Hexagon	*6*	*120*
Heptagon	*7*	*128.6*
Octagon	*8*	*135*
Nonagon	*9*	*140*
Decagon	*10*	*144*
n-gon	*n*	*180(n-2)/n*
100-gon	*100*	*176.4*

Regular Polygon Corner Angle

How big can a corner angle of a regular polygon get? Why?

Regular Polygon Corner Angle



Regular Polygon Corner Angle

A corner angle can not exceed 180 degrees, since this would be a straight line.

Regular Polygon Corner Angle

If a polygon has a corner angle measure of 150, can it be a regular polygon? If so, how many sides does it have? If not, explain why.

Regular Polygon Corner Angle

Yes, it has 12 sides!

$$180(n-2)/n = 150$$

$$180(n-2) = 150n$$

$$180n-360 = 150n$$

$$30n - 360 = 0$$

$$30n = 360$$

$$n = 12$$

Regular Polygon Corner Angle

If a polygon has a corner angle measure of 100, can it be a regular polygon? If so, how many sides does it have? If not, explain why.

Regular Polygon Corner Angle

No!

$$180(n-2)/n = 100$$

$$180(n-2) = 100n$$

$$180n-360 = 100n$$

$$80n - 360 = 0$$

$$80n = 360$$

$$n = 4.5$$

Since n is the number of sides, n must be a whole number.

Lunch

Regular Tessellations

Regular Polygon	Tile (Yes or No)	Explanation	Picture (If Possible)
Equilateral Triangle	Yes		
Square	Yes		
Pentagon	No		
Hexagon	Yes		
Heptagon	No		
Octagon	No		

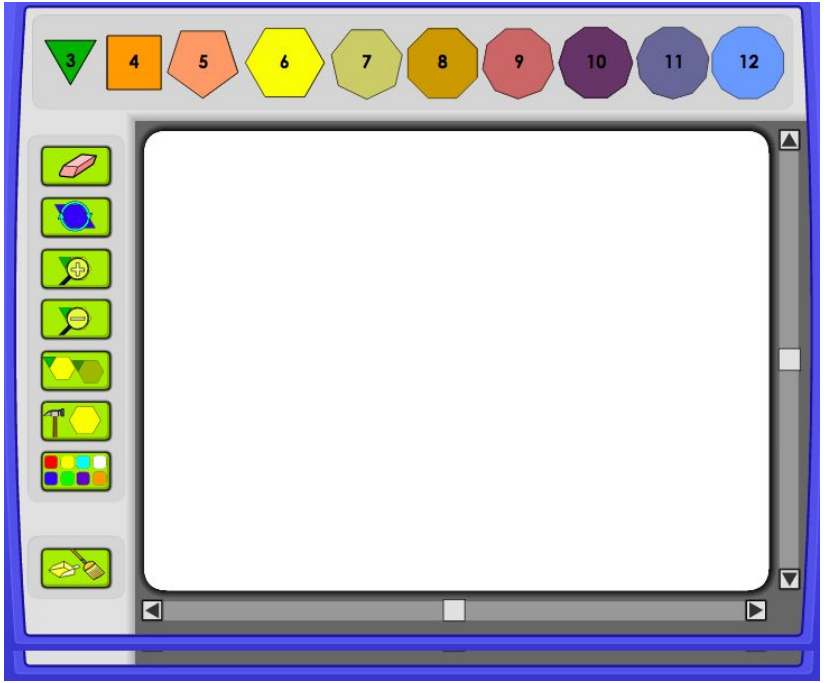
Regular Tessellations

Can we tile the plane with any regular polygon?

Regular Tessellations

- In the table you just created, there are only three regular polygons that can be used to tile the plane.
- Determine which three can be used to tile the plane.
- Explain why those three can be used to tile the plane and why the other five cannot.

Regular Tessellations



Regular Tessellations

Corners of the tiles need to fit together around a point, which means the corner angle of the regular polygon must evenly divide 360.

Regular Tessellations

Regular Polygon	Tile (Yes or No)	Explanation	Picture (If Possible)
Equilateral Triangle	Yes	$360/60 = 6$	
Square	Yes	$360/90 = 4$	
Pentagon	No	$360/108 \approx 3.33$	
Hexagon	Yes	$360/120 = 3$	
Heptagon	No	$360/128.6 \approx 2.8$	
Octagon	No	$360/135 \approx 2.66$	

Regular Tessellations

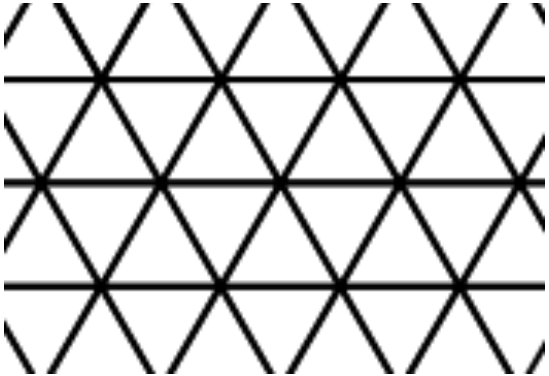
Can we tile the plane with any polygon with more than 6 sides? Why or why not?

Regular Tessellations

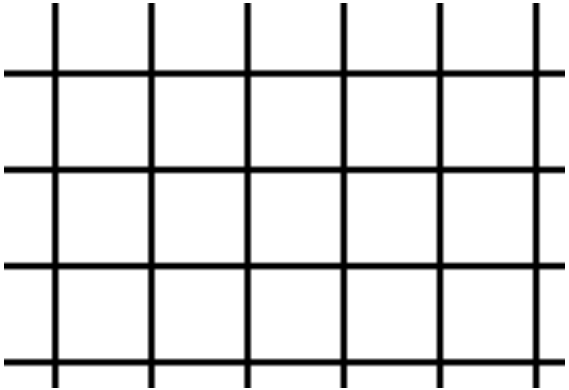
- No!
- A regular polygon with more than 6 sides has a corner angle larger than 120 degrees and smaller than 180 degrees.
- Is there any number between 120 and 180 that divides into 360 evenly?

Regular Tessellations

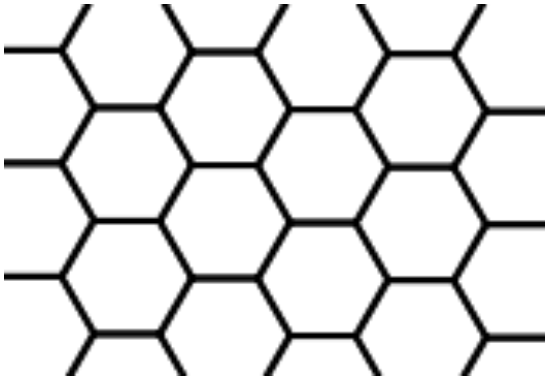
Equilateral Triangle



Square

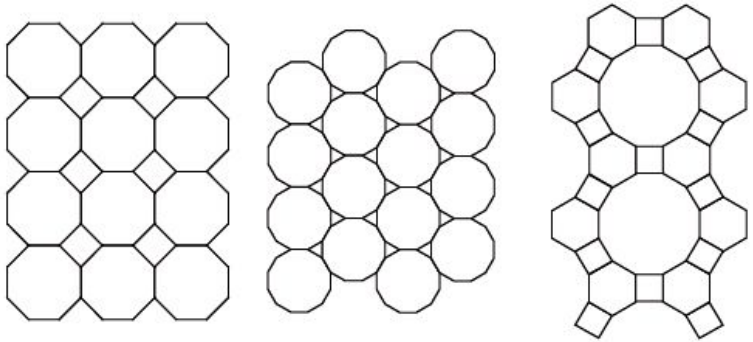


Regular Hexagon



Semi-Regular Tessellations

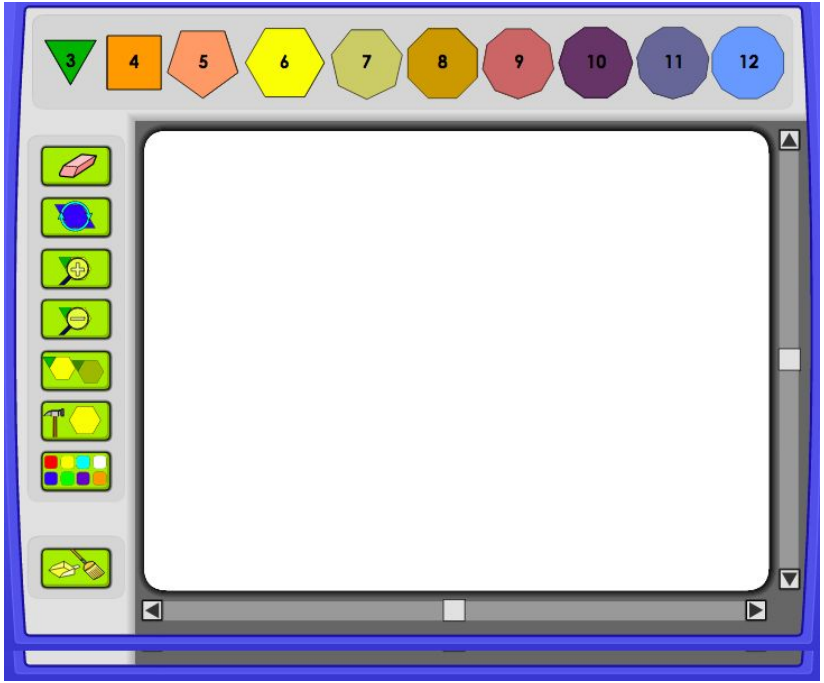
A **semi-regular** tessellation is a tiling of the plane made up of two or more regular polygons such that the same polygons are in the same order surrounding each vertex.



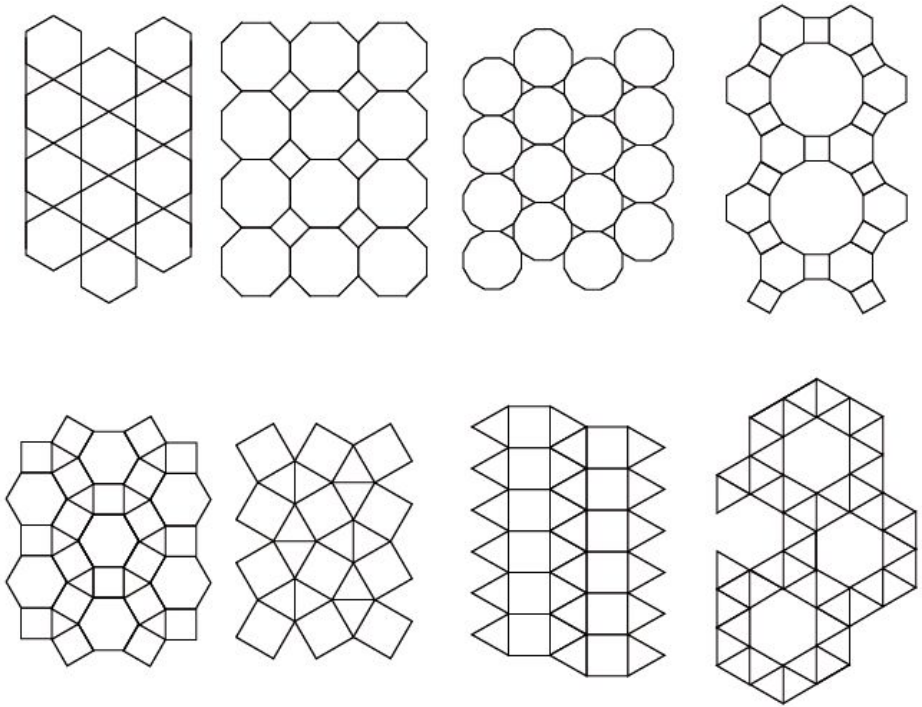
Semi-Regular Tessellations

- There are 5 other semi-regular tessellations.
- Create them using the shapes provided and sketch what each one looks like.
- Explain why there are no other semi-regular tessellations.

Semi-Regular Tessellations



Semi-Regular Tessellations



How can we move shapes?

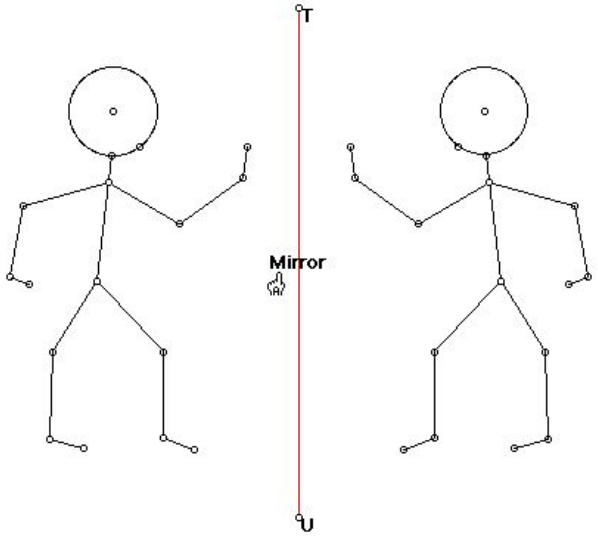
How can we move shapes?

- Flips
- Turns
- Slides

Reflection (Flip)

- A figure can be **reflected** (flipped) across a **line of symmetry** (mirror line).

- Can you think of any real life examples?

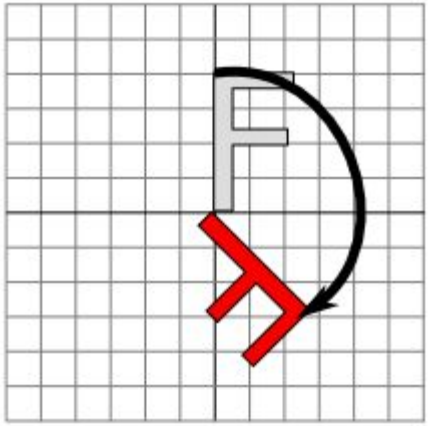


Reflection (Flip)

Rotation (Turn)

- A figure can be **rotated** (turned) about a **point of rotation** (center point).

- Can you you think of any real life examples?

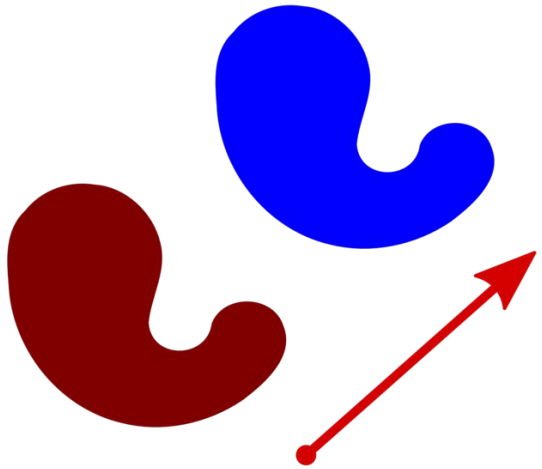


Rotation (Turn)

Translation (Slide)

- A figure can be **translated** (slid) along a **vector**.

- Can you you think of any real life examples?



Translation (Slide)

Transformations

- Reflections
- Rotations
- Translations

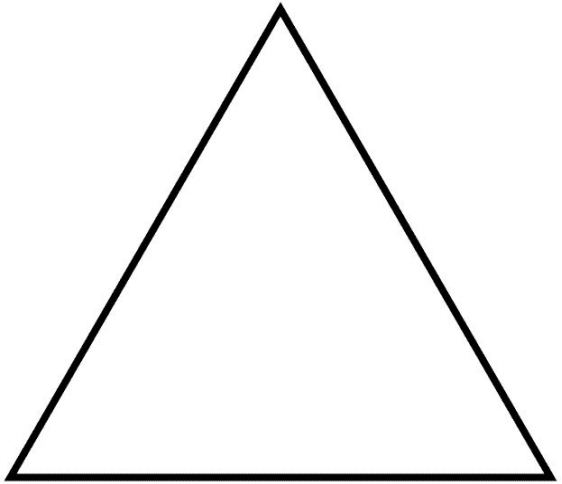
Symmetries

- A symmetry is a transformation that moves a figure onto itself.

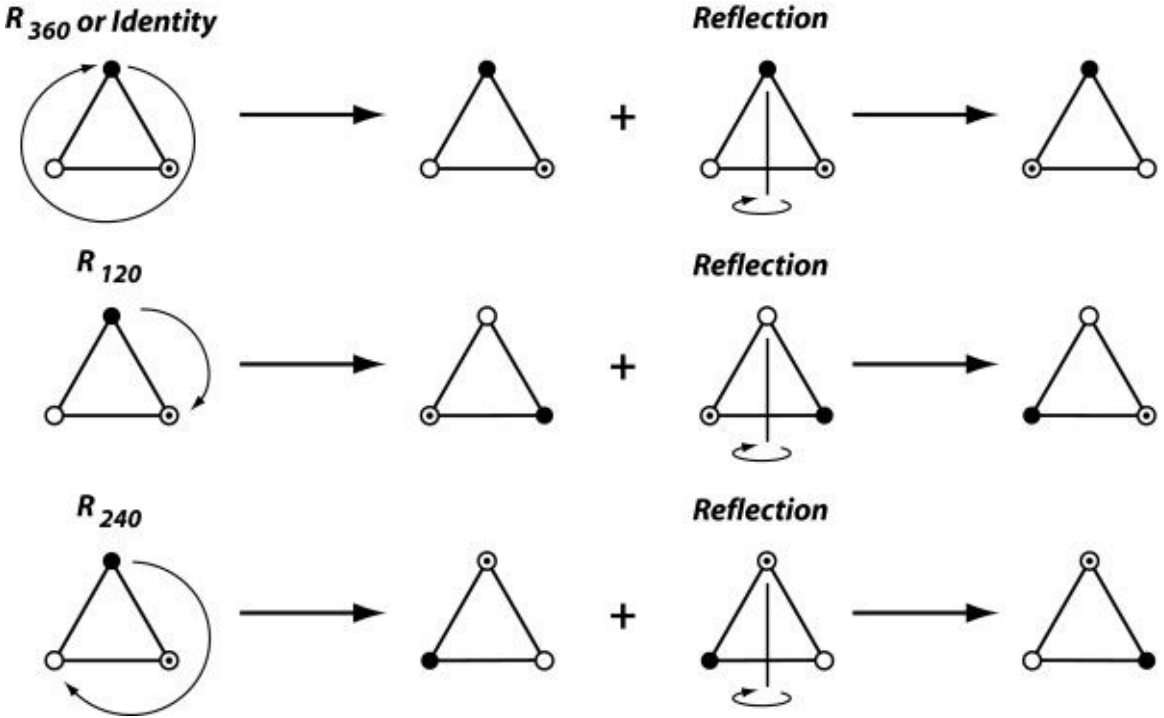
- A figure can have either reflectional or rotational symmetry.

Symmetries

How many symmetries does an equilateral triangle have?

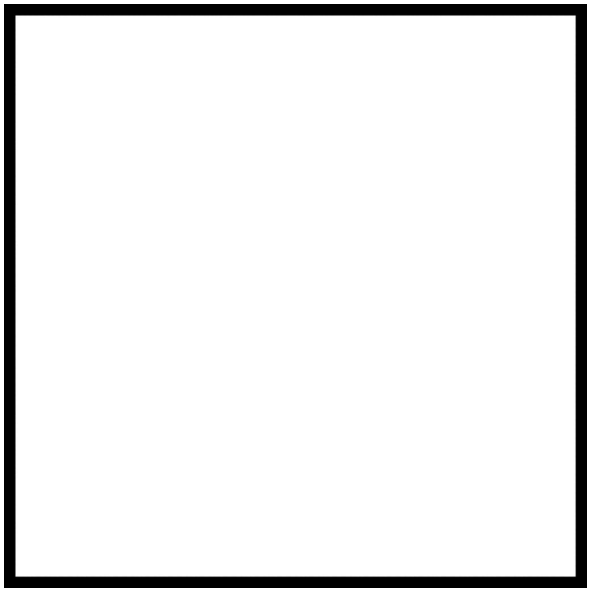


Symmetries

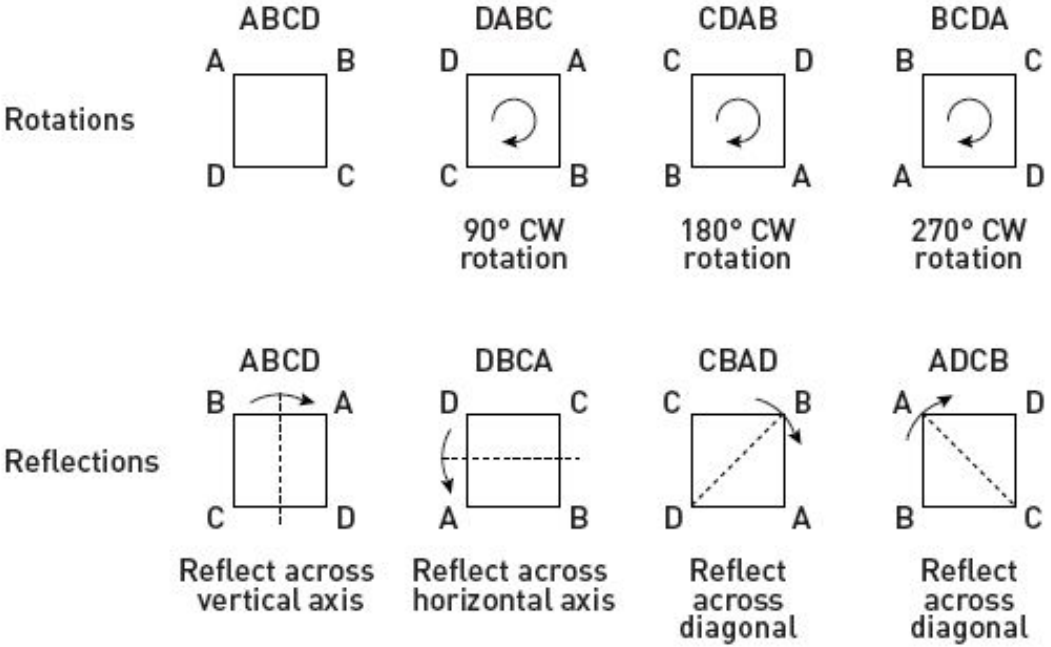

R 360 or Identity
Reflection
R 120
Reflection
R 240
Reflection

Symmetries

How many symmetries does a square have?

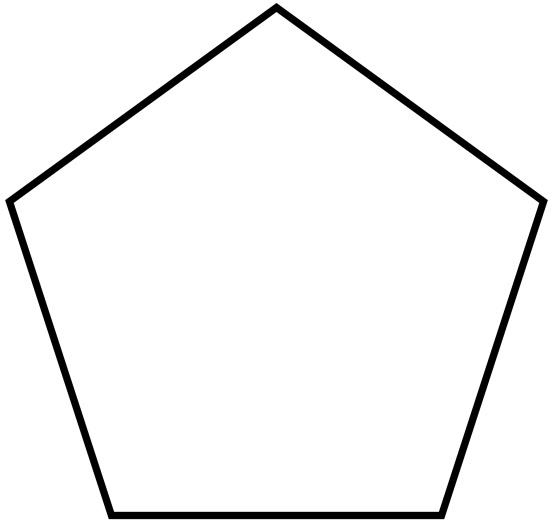


Symmetries

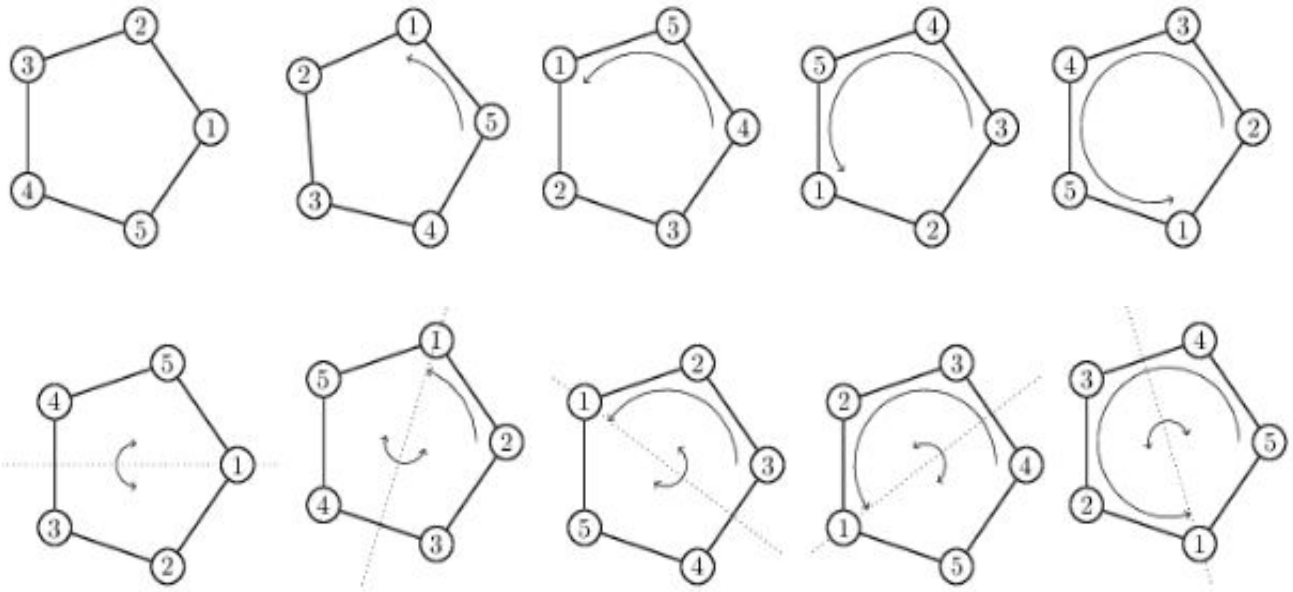

Rotations
ABCD
A B
D C
DABC
D A
C B
90° CW rotation
CDAB
C D
B A
180° CW rotation
BCDA
B C
A D
270° CW rotation
Reflections
ABCD
B A
C D
Reflect across vertical axis
DBCA
D C
A B
Reflect across horizontal axis
CBAD
C B
D A
Reflect across diagonal
ADCB
A D
B C
Reflect across diagonal

Symmetries

How many symmetries does regular pentagon have?



Symmetries


2 3 1 4 5
1 2 5 3 4
5 1 4 2 3
4 5 3 1 2
3 4 2 5 1
5 4 1 3 2
1 5 2 4 3
2 1 3 5 4
3 2 4 1 5
4 3 5 2 1

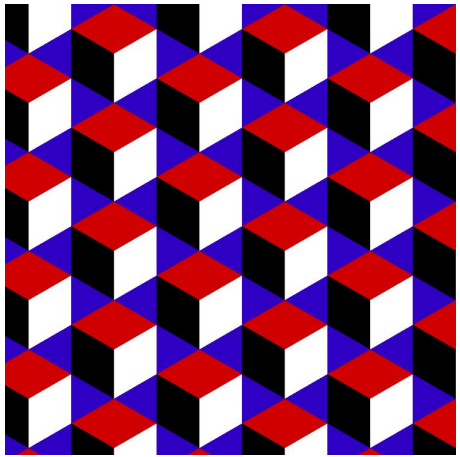
Symmetries

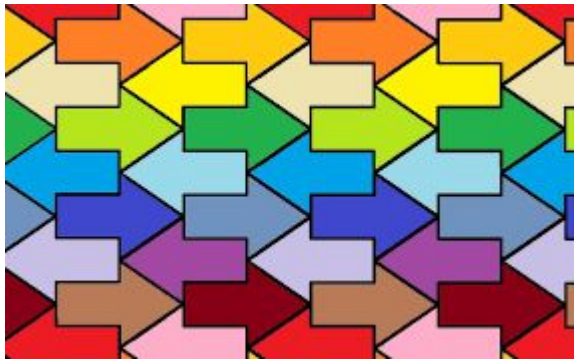
Can you predict the number of rotations that an n sided regular polygon has?

Symmetries

An n sided regular polygon has n reflectional symmetries and n rotational symmetries, a total of $2n$ symmetries.

How are transformations and symmetries related to tessellations?



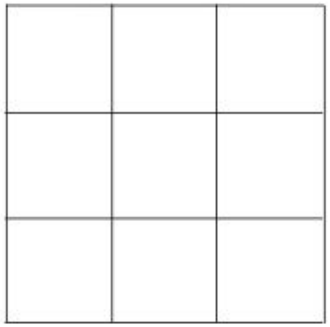


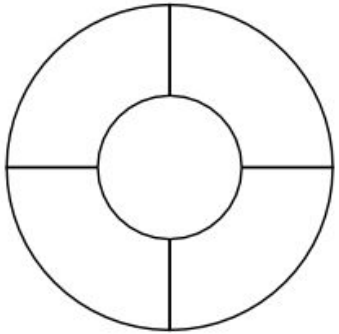
Map Coloring

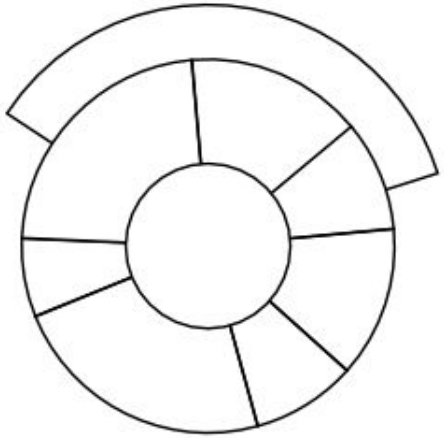
What is the minimum number of colors required to color any “map” so that adjacent edges are not the same color?

Map Coloring

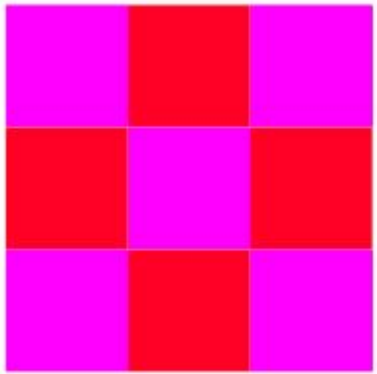
Try to color these using the least amount of colors, where adjacent edges must be different colors.

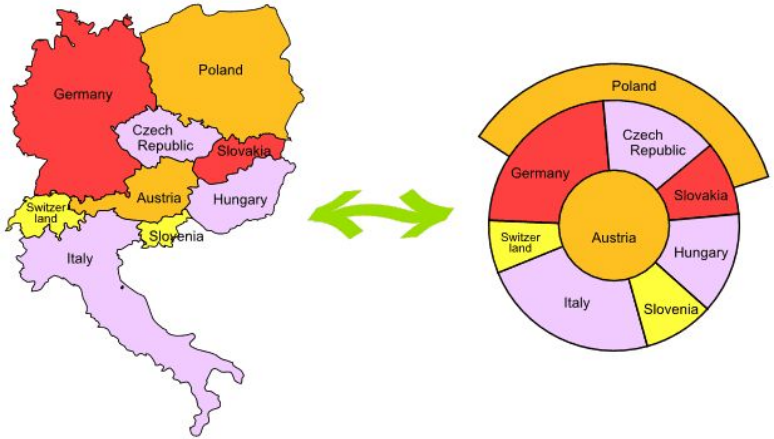






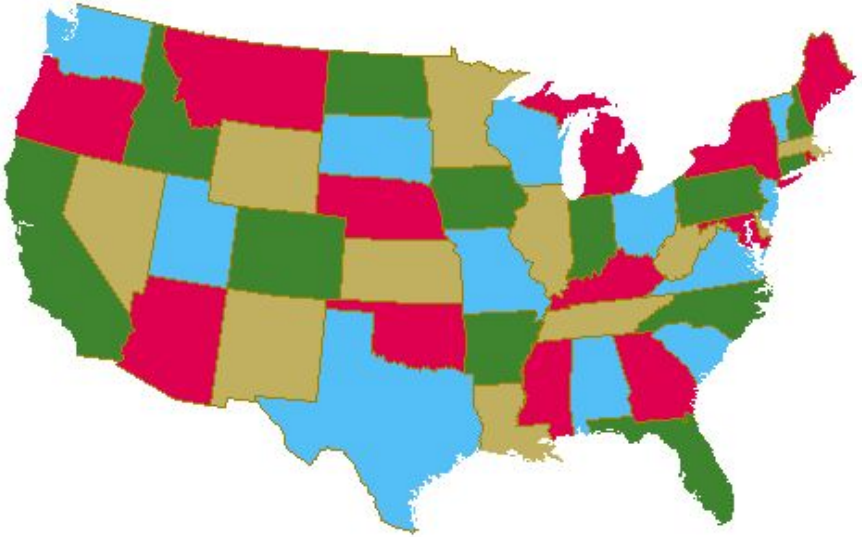
Map Coloring





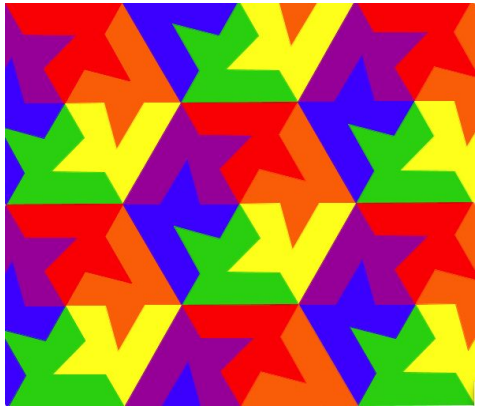
Four Color Theorem

It turns out that any “map” can be colored with four colors or less.



Tessellation Coloring

Color the tessellation you created earlier so that no adjacent sides are the same color.



Evaluations and Certificates

1. Fill out the evaluation for Duke TIP.
2. Write down what you liked and didn't like about the class on a blank sheet of paper.
3. Receive your certificate.
4. Celebrate!