07/01/2020
Evening

Aakash
Medical | IIT-JEE | Foundations
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Time : 3 hrs.

Answers & Solutions

M.M. : 300

for

JEE (MAIN)-2020 Phase-1

(Physics, Chemistry and Mathematics)

Important Instructions :

1. The test is of **3 hours** duration.
2. The Test Booklet consists of **75** questions**.** The maximum marks are **300.**
3. There are **three** parts in the question paper A, B, C consisting of **Physics**, **Chemistry** and **Mathematics** having 25 questions in each part of equal weightage. Each part has two sections.

   (i) **Section-I :** This section contains 20 multiple choice questions which have only one correct answer. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer.

   (ii) **Section-II :** This section contains 5 questions. The answer to each of the questions is a numerical value. Each question carries **4 marks** for correct answer and there is no negative marking for wrong answer.

PHYSICS

SECTION - I

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. A stationary observer receives sound from two identical tuning forks, one of which approaches and the other one recedes with the same speed (much less than the speed of sound). The observer hears 2 beats/sec. The oscillation frequency of each tuning fork is ν_0 = 1400 Hz and the velocity of sound in air 350 m/s. The speed of each tuning fork is close to :

(1) $\frac{1}{4}$ m/s (2) $\frac{1}{2}$ m/s

(3) 1 m/s (4) $\frac{1}{8}$ m/s

Answer (1)

Sol. $f_1 = f_0 \frac{c}{c-v}$

$f_2 = f_0 \frac{c}{c+v}$

$\Rightarrow 2 = f_1 - f_2 = f_0 c\left[\frac{1}{c-v} - \frac{1}{c+v}\right]$

$= f_0 c \frac{2v}{c^2\left[1-\frac{v^2}{c^2}\right]}$

$\Rightarrow v = \frac{2c}{2f_0} = \frac{350}{1400} = \frac{1}{4}$ m/s

2. The electric field of a plane electromagnetic wave is given by

$\vec{E} = E_0 \frac{\hat{i}+\hat{j}}{\sqrt{2}} \cos(kz + \omega t)$

At t = 0, a positively charged particle is at the point (x, y, z) = $\left(0, 0, \frac{\pi}{k}\right)$. If its instantaneous velocity at (t = 0) is $v_0\hat{k}$, the force acting on it due to the wave is:

(1) Antiparallel to $\frac{\hat{i}+\hat{j}}{\sqrt{2}}$

(2) Zero

(3) Parallel to $\frac{\hat{i}+\hat{j}}{\sqrt{2}}$

(4) Parallel to $\hat{k}$

Answer (1)

Sol. $\vec{E}$ at t = 0 at z = πk is given by

$\vec{E} = \frac{E_0}{\sqrt{2}}(\hat{i}+\hat{j})\cos[\pi] = -\frac{E_0}{\sqrt{2}}(\hat{i}+\hat{j})$

$\vec{F}_E = q\vec{E}$

$\Rightarrow$ Force due to electric field, $\vec{F}_E$ is antiparallel to $\frac{\hat{i}+\hat{j}}{\sqrt{2}}$.

$\vec{F}_{mag} = q(\vec{v}\times\vec{B})$

$\vec{B}$ (at t = 0, z = πk) is $\frac{B_0}{\sqrt{2}}(-\hat{i}+\hat{j})$

$\Rightarrow \vec{F}_{mag} = q\, v_0\hat{k} \times \frac{B_0}{\sqrt{2}}(-\hat{i}+\hat{j})$ which is antiparallel to $\frac{(\hat{i}+\hat{j})}{\sqrt{2}}$

$\Rightarrow \vec{F}_{net} = \vec{F}_E + \vec{F}_B$ is Antiparallel to $\frac{\hat{i}+\hat{j}}{\sqrt{2}}$

3. In a Young's double slit experiment, the separation between the slits is 0.15 mm. In the experiment, a source of light of wavelength 589 nm is used and the interference pattern is observed on a screen kept 1.5 m away. The separation between the successive bright fringes on the screen is:

(1) 3.9 mm (2) 6.9 mm

(3) 5.9 mm (4) 4.9 mm

Answer (3)

Sol. Fringe-width, $\omega = \frac{D}{d}\lambda = \frac{1.5}{0.15\times10^{-3}}\times589\times10^{-9}$ m

$= 589 \times 10^{-2}$ mm $= 5.89$ mm

≈ 5.9 mm

4. Two ideal Carnot engines operate in cascade (all heat given up by one engine is used by the other engine to produce work) between temperatures, T_1 and T_2. The temperature of the hot reservoir of the first engine is T_1 and the temperature of the cold reservoir of the second engine is T_2. T is temperature of the sink of first engine which is also the source for the second engine. How is T related to T_1 and T_2, if both the engines perform equal amount of work?

(1) $T = \frac{T_1+T_2}{2}$ (2) $T = \sqrt{T_1T_2}$

(3) $T = \frac{2T_1T_2}{T_1+T_2}$ (4) $T = 0$

Answer (1)

Sol. Let Q_1 : Heat input to first engine

Q_C : Heat rejected by first engine

Q_2 : Heat rejected by second engine

T_C : Lower temperature of first engine

$W = Q_1 - Q_C = Q_C - Q_2$

$\Rightarrow 2Q_C = Q_1 + Q_2$

$\Rightarrow 2T_C = T_1 + T_2 \Rightarrow T_C = \frac{T_1+T_2}{2}$

5. An ideal fluid flows (laminar flow) through a pipe of non-uniform diameter. The maximum and minimum diameters of the pipes are 6.4 cm and 4.8 cm, respectively. The ratio of the minimum and the maximum velocities of fluid in this pipe is:

(1) $\frac{81}{256}$ (2) $\frac{9}{16}$

(3) $\frac{3}{4}$ (4) $\frac{\sqrt{3}}{2}$

Answer (2)

Sol. $A_1v_1 = A_2v_2$ (Equation of continuity)

$$\frac{v_{min}}{v_{max}} = \frac{v_1}{v_2} = \frac{A_2}{A_1} = \frac{(4.8)^2}{(6.4)^2} = \frac{9}{16}$$

6. An electron (of mass m) and a photon have the same energy E in the range of a few eV. The ratio of the de-Broglie wavelength associated with the electron and the wavelength of the photon is (c = speed of light in vacuum)

(1) $\frac{1}{c}\left(\frac{E}{2m}\right)^{\frac{1}{2}}$ (2) $\left(\frac{E}{2m}\right)^{\frac{1}{2}}$

(3) $c(2mE)^{\frac{1}{2}}$ (4) $\frac{1}{c}\left(\frac{2E}{m}\right)^{\frac{1}{2}}$

Answer (1)

Sol. $\lambda_e = \frac{h}{p_e} = \frac{h}{\sqrt{2mE}}$

$\frac{hc}{\lambda_P} = E \Rightarrow \lambda_p = \frac{hc}{E}$

$\Rightarrow \frac{\lambda_e}{\lambda_P} = \frac{1}{\sqrt{2mE}}\frac{E}{c} = \sqrt{\frac{E}{2m}}\cdot\frac{1}{c}$

7. Mass per unit area of a circular disc of radius a depends on the distance r from its centre as $\sigma(r) = A + Br$. The moment of inertia of the disc about the axis, perpendicular to the plane and passing through its centre is:

(1) $2\pi a^4\left(\frac{A}{4}+\frac{B}{5}\right)$ (2) $\pi a^4\left(\frac{A}{4}+\frac{aB}{5}\right)$

(3) $2\pi a^4\left(\frac{A}{4}+\frac{aB}{5}\right)$ (4) $2\pi a^4\left(\frac{aA}{4}+\frac{B}{5}\right)$

Answer (3)

Sol. $I = \int dmr^2 = \int \sigma\, 2\pi r dr\cdot r^2$

$\Rightarrow I = 2\pi\int_0^a (A+Br)r^3dr = 2\pi\left[\frac{Aa^4}{4}+\frac{Ba^5}{5}\right]$

$\Rightarrow I = 2\pi a^4\left[\frac{A}{4}+\frac{Ba}{5}\right]$

8. A mass of 10 kg is suspended by a rope of length 4 m, from the ceiling. A force F is applied horizontally at the mid-point of the rope such that the top half of the rope makes an angle of 45° with the vertical. Then F equals: (Take g = 10 ms^{-2} and the rope to be massless)

(1) 75 N

(2) 70 N

(3) 90 N

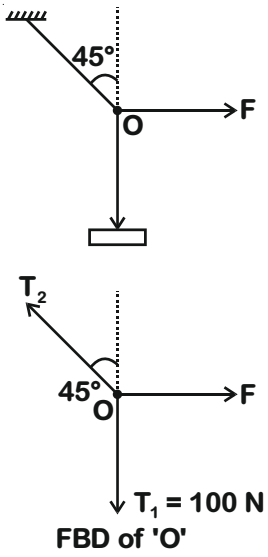
(4) 100 N

Answer (4)

Sol. $T_2 \cos 45° = 100$ N ...(i)

$T_2 \sin 45° = F$ ...(ii)

$\Rightarrow$ F = 100 N



FBD of 'O'

9. A thin lens made of glass (refractive index = 1.5) of focal length f = 16 cm is immersed in a liquid of refractive index 1.42. If its focal length in liquid is f_l, then the ratio $\frac{f_l}{f}$ is closest to the integer

(1) 17 (2) 1

(3) 5 (4) 9

Answer (4)

Sol. $\frac{1}{f_a} = (1.5-1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$ Lens Maker's formula

$$\frac{1}{f_e} = \left(\frac{1.5}{1.42} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\Rightarrow \frac{f_e}{f_a} = \frac{(1.5-1)1.42}{0.08} = \frac{1.42}{0.16} = \frac{142}{16} \approx 9$$

10. A box weighs 196 N on a spring balance at the north pole. Its weight recorded on the same balance if it is shifted to the equator is close to (Take g = 10 ms^{-2} at the north pole and the radius of the earth = 6400 km)

(1) 194.66 N

(2) 195.66 N

(3) 195.32 N

(4) 194.32 N

Answer (3)

Sol. Weight at equator = mg′

$= m(g - \omega^2 R)$

mg = 196 N $\Rightarrow$ m = 19.6 kg

$$\Rightarrow mg' = 19.6\left[10 - \left(\frac{2\pi}{24\times 3600}\right)^2 \times 6400 \times 10^3\right] \text{ N}$$

$= 19.6[10 - 0.034] = 195.33$ N

11. An elevator in a building can carry a maximum of 10 persons, with the average mass of each person being 68 kg. The mass of the elevator itself is 920 kg and it moves with a constant speed of 3 m/s. The frictional force opposing the motion is 6000 N. If the elevator is moving up with its full capacity, the power delivered by the motor to the elevator ($g = 10\ m/s^2$) must be at least

(1) 56300 W

(2) 66000 W

(3) 48000 W

(4) 62360 W

Answer (2)

Sol. F = (10 m + M)g + f

where, m = 68 kg, M = 920 kg, f = 6000 N

$\Rightarrow$ F = 22000 N

$\Rightarrow$ P = FV = 22000 × 3 = 66000 W

12. In a building there are 15 bulbs of 45 W, 15 bulbs of 100 W, 15 small fans of 10 W and 2 heaters of 1 kW. The voltage of electric main is 220 V. The minimum fuse capacity (rated value) of the building will be

(1) 15 A

(2) 10 A

(3) 20 A

(4) 25 A

Answer (3)

Sol. P = VI

$$\Rightarrow I_{main} = 15\times\frac{45}{220} + 15\times\frac{100}{220} + 15\times\frac{10}{220} + 2\times\frac{10^3}{220}$$

$$\Rightarrow I_{main} = \frac{15\times 155 + 2000}{220} = 19.66 \text{ A}$$

Answer is 20 A

13. Under an adiabatic process, the volume of an ideal gas gets doubled. Consequently the mean collision time between the gas molecule changes from τ_1 to τ_2. If $\frac{C_P}{C_v}=\gamma$ for this gas then a good estimate for $\frac{\tau_2}{\tau_1}$ is given by

(1) $\left(\frac{1}{2}\right)^{\frac{\gamma+1}{2}}$ (2) $\left(\frac{1}{2}\right)^{\gamma}$

(3) 2 (4) $\frac{1}{2}$

Answer (Bonus)

Sol. $\tau \propto \frac{1}{n<v>}, <v> \propto \sqrt{T}$

$\Rightarrow \tau \propto \frac{1}{n\sqrt{T}} \Rightarrow \frac{\tau_2}{\tau_1}=\frac{n_1}{n_2}\sqrt{\frac{T_1}{T_2}}$

$=2\sqrt{\frac{T_1}{T_2}}$

$T_1V_1^{\gamma-1} = T_2(2V_1)^{\gamma-1} \Rightarrow \frac{T_1}{T_2}=2^{\gamma-1}$

$\Rightarrow \frac{\tau_2}{\tau_1}=2\times 2^{\frac{(\gamma-1)}{2}}=2^{\left(\frac{\gamma+1}{2}\right)}$

Answer does not match with given options.

14. An emf of 20 V is applied at time t = 0 to a circuit containing in series 10 mH inductor and 5 Ω resistor. The ratio of the currents at time t = ∞ and at t = 40 s is close to

(Take $e^2 = 7.389$)

(1) 1.06 (2) 1.15

(3) 1.46 (4) 0.84

Answer (1)

Sol. $I=I_0\left(1-e^{-\frac{tR}{L}}\right)$

$I_\infty = I\,(t=\infty) = I_0$

$I_{40}=I(t=40\text{ s})=I_0\left(1-e^{\frac{-40\times 5}{10\times 10^{-3}}}\right)=I_0\left(1-e^{-20,000}\right)$

$\Rightarrow \frac{I_\infty}{I_{40}}=\frac{1}{1-e^{-20,000}}$, which is slightly greater than 1

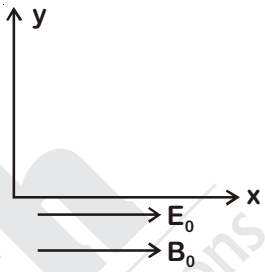
15. A particle of mass m and charge q has an initial velocity $\vec{v}=v_0\hat{j}$. If an electric field $\vec{E}=E_0\hat{i}$ and magnetic field $\vec{B}=B_0\hat{i}$ act on the particle, its speed will double after a time :

(1) $\frac{3mv_0}{qE_0}$ (2) $\frac{\sqrt{3}mv_0}{qE_0}$

(3) $\frac{2mv_0}{qE_0}$ (4) $\frac{\sqrt{2}mv_0}{qE_0}$

Answer (2)

Sol.



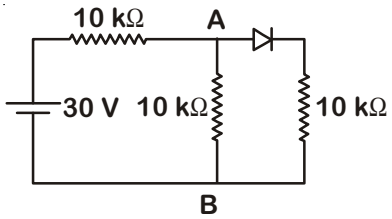
$a_x=\frac{E_0q}{m}$

$V_0^2+V_x^2=(2V_0)^2$

$\Rightarrow V_x=\sqrt{3}\,V_0=a_xt$

$\Rightarrow t=\frac{\sqrt{3}\,V_0m}{E_0q}$

16. In the figure, potential difference between A and B is :

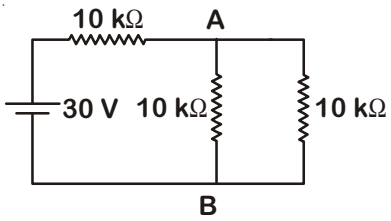


(1) Zero (2) 5 V

(3) 10 V (4) 15 V

Answer (3)

Sol.



Here the diode is in forward bias. So we replace it by a connecting wire.

$V_a-V_b=\frac{I}{2}\times 10$

$=\frac{30}{15\times 2}\times 10\text{ V}=10\text{ V}$

17. The activity of a radioactive sample falls from 700 s^{-1} to 500 s^{-1} in 30 minutes. Its half life is close to

(1) 66 min

(2) 52 min

(3) 62 min

(4) 72 min

Answer (3)

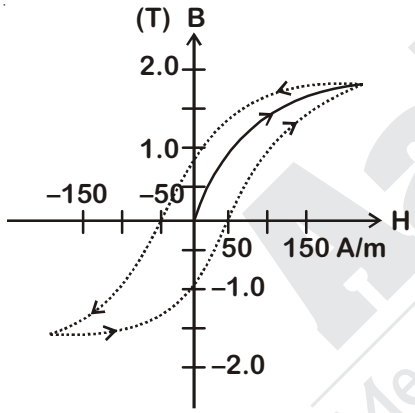
Sol. $A = A_0 e^{-t\ln 2/T_{1/2}}$

$\Rightarrow 500 = 700\, e^{-t\ln 2/T_{1/2}}$

$\Rightarrow \ln\frac{7}{5} = \frac{30\ln 2}{T_{1/2}} \Rightarrow T_{1/2} = 30\frac{\ln 2}{\ln 1.4} = 61.8 \text{ minute}$

$\Rightarrow T_{1/2} \approx 62$ minutes

18.



The figure gives experimentally measured B vs. H variation in a ferromagnetic material. The retentivity, co-ercivity and saturation, respectively, of the material are

(1) 1.0 T, 50 A/m and 1.5 T

(2) 1.5 T, 50 A/m and 1.0 T

(3) 1.5 T, 50 A/m and 1.0 T

(4) 150 A/m, 1.0 T and 1.5 T

Answer (1)

Sol. Theoretical

The values can be directly taken from the graph.

19. A planar loop of wire rotates in a uniform magnetic field. Initially, at t = 0, the plane of the loop is perpendicular to the magnetic field. If it rotates with a period of 10 s about an axis in its plane then the magnitude of induced emf will be maximum and minimum, respectively at

(1) 2.5 s and 7.5 s

(2) 5.0 s and 7.5 s

(3) 2.5 s and 5.0 s

(4) 5.0 s and 10.0 s

Answer (3)

Sol. $\phi(t) = AB\cos\omega t$

$E = \frac{-d\phi}{dt} = AB\,\omega\sin\omega t = AB\omega\sin\left(\frac{2\pi}{T}.t\right)$

Induced emf, $|\varepsilon|$ is maximum when $\frac{2\pi t}{T} = \frac{\pi}{2}, \frac{3\pi}{2}$

$\Rightarrow t = \frac{T}{4}$ or $\frac{3T}{4}$ i.e. 2.5 s or 7.5 s.

For induced emf to be minimum i.e zero

$\frac{2\pi t}{T} = n\pi \Rightarrow t = n\frac{T}{2},$

$\Rightarrow$ Induced emf is zero at t = 5 s, 10 s

20. The dimension of $\frac{B^2}{2\mu_0}$, where B is magnetic field and μ_0 is the magnetic permeability of vaccum, is

(1) $ML^{-1}T^{-2}$

(2) ML^2T^{-2}

(3) ML^2T^{-1}

(4) MLT^{-2}

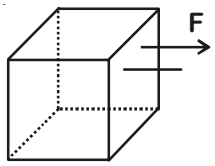
Answer (1)

Sol. The quantity $\frac{B^2}{2\mu_0}$ is the energy density of magnetic field.

$\Rightarrow \left[\frac{B^2}{2\mu_0}\right] = \left[\frac{ML^2T^{-2}}{L^3}\right] = ML^{-1}T^{-2}$

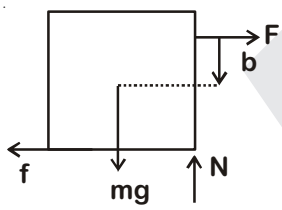
SECTION - II

Numerical Value Type Questions: This section contains 5 questions. The answer to each of the questions is a numerical value. Each question carries 4 marks for correct answer and there is no negative marking for wrong answer.

21. 

Consider a uniform cubical box of side a on a rough floor that is to be moved by applying minimum possible force F at a point b above its centre of mass (see figure). If the coefficient of friction is $\mu = 0.4$, the maximum possible value of $100\times\frac{b}{a}$ for box not to topple before moving is _____.

Answer (75)

Sol. 

When the block slides,

$F = f = 0.4\ mg$

For the block not to topple

$$F\left(\frac{a}{2}+b\right) < mg\frac{a}{2}$$

$$\Rightarrow 0.4\,mg\left(\frac{a}{2}+b\right) < mg\frac{a}{2}$$

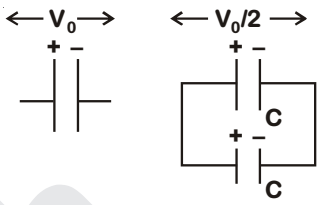
$$\Rightarrow 0.2\,a + 0.4\,b < 0.5\,a$$

$$\Rightarrow \frac{b}{a} < \frac{3}{4}$$

$$\Rightarrow \text{Maximum possible value of } \frac{100\,b}{a} \text{ is } 75$$

22. A 60 pF capacitor is fully charged by a 20 V supply. It is then disconnected from the supply and is connected to another uncharged 60 pF capacitor in parallel. The electrostatic energy that is lost in this process by the time the charge is redistributed between them is (in nJ) _____.

Answer (6)

Sol. 

$$U_i = \frac{1}{2}CV_0^2$$

$$U_f = \frac{1}{2}2C.\left(\frac{V_0}{2}\right)^2$$

$$H = U_i - U_f = \frac{1}{4}CV_0^2 = \frac{1}{4}\times 60\times 10^{-12}\times 400\times 10^{9}\,nJ$$

$= 6$ nJ

23. The balancing length for a cell is 560 cm in a potentiometer experiment. When an external resistance of 10 Ω is connected in parallel to the cell, the balancing length changes by 60 cm. If the internal resistance of the cell is $\frac{N}{10}\Omega$, where N is an integer then value of N is _____.

Answer (12)

Sol. $\varepsilon = \phi\,560$ ...(i)

$$\frac{\varepsilon}{r+10}10 = \phi\,500 \ldots(ii)$$

$$\Rightarrow \frac{r+10}{10} = \frac{56}{50} \Rightarrow 50r + 500 = 560$$

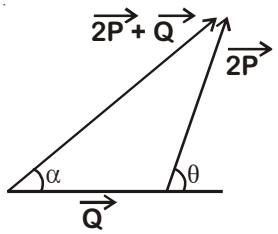
$$\Rightarrow r = \frac{6}{5}\Omega = \frac{N}{10}\Omega$$

$$\Rightarrow N = 12$$

24. The sum of two forces $\vec{P}$ and $\vec{Q}$ is $\vec{R}$ such that $|\vec{R}| = |\vec{P}|$. The angle θ (in degrees) that the resultant of $2\vec{P}$ and $\vec{Q}$ will make with $\vec{Q}$ is, _____.

Answer (90)

Sol.



$P^2 = P^2 + Q^2 + 2PQ.\cos\theta$

$\Rightarrow \cos\theta = -\frac{Q}{2P}$...(i)

$\tan\alpha = \frac{2P\sin\theta}{Q + 2P\cos\theta} = \infty$

$\Rightarrow \alpha = 90°$

25. M grams of steam at 100°C is mixed with 200 g of ice at its melting point in a thermally insulated container. If it produces liquid water at 40°C [heat of vaporization of water is 540 cal/g and heat of fusion of ice is 80 cal/g], the value of M is _____.

Answer (40)

Sol. M 540 + M × 1 × (100 – 40)

= 200 × 80 + 200 × 1 × 40

$\Rightarrow$ 600 M = 24000

$\Rightarrow$ M = 40

❑ ❑ ❑

CHEMISTRY

SECTION - I

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. In the following reactions, products (A) and (B), respectively, are

$$\underset{\text{(hot and conc.)}}{NaOH} + Cl_2 \rightarrow (A) + \text{side products}$$

$$\underset{\text{(dry)}}{Ca(OH)_2} + Cl_2 \rightarrow (B) + \text{side products}$$

(1) $NaOCl$ and $Ca(OCl)_2$

(2) $NaClO_3$ and $Ca(ClO_3)_2$

(3) $NaOCl$ and $Ca(ClO_3)_2$

(4) $NaClO_3$ and $Ca(OCl)_2$

Answer (4)

Sol. $NaOH + Cl_2 \rightarrow \underset{(A)}{NaCl + NaClO_3} + H_2O$

$$Ca(OH)_2 + Cl_2 \rightarrow \underset{(B)}{CaCl_2 + Ca(OCl)_2} + H_2O$$

2. For the reaction

$2H_2(g) + 2NO(g) \rightarrow N_2(g) + 2H_2O(g)$

the observed rate expression is,

rate = $k_f[NO]^2[H_2]$. The rate expression for the reverse reaction is :

(1) $k_b[N_2][H_2O]^2/[NO]$ (2) $k_b[N_2][H_2O]$

(3) $k_b[N_2][H_2O]^2$ (4) $k_b[N_2][H_2O]^2/[H_2]$

Answer (4)

Sol. $k_{eq} = \dfrac{k_f}{k_b} = \dfrac{[N_2][H_2O]^2}{[H_2]^2[NO]^2}$

Rearranging

$$k_f[NO]^2[H_2] = \frac{k_b[N_2][H_2O]^2}{[H_2]}$$

on comparing R_f and R_b at equilibrium,

$$R_b = k_b\frac{[N_2][H_2O]^2}{[H_2]}$$

3. Among the statements (a)-(d), the incorrect ones are

(a) Octahedral Co(III) complexes with strong field ligands have very high magnetic moments

(b) When $\Delta_0 < P$, the d-electron configuration of Co(III) in an octahedral complex is $t_{eg}^4 e_g^2$

(c) Wavelength of light absorbed by $[Co(en)_3]^{3+}$ is lower than that of $[CoF_6]^{3-}$

(d) If the Δ_0 for an octahedral complex of Co(III) is 18,000, cm^{-1} the Δ_t for its tetrahedral complex with the same ligand will be 16,000 cm^{-1}

(1) (c) and (d) only

(2) (a) and (d) only

(3) (a) and (b) only

(4) (b) and (c) only

Answer (2)

Sol. (a) Co^{3+} with strong field complex forms low magnetic moment complex

(b) If $\Delta_0 < P$ configuration of Co^{3+} will be $t_g^6 e_g^2$

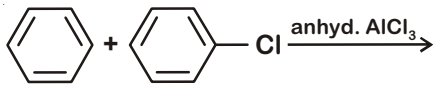
(c) CFSE of $[Co(en)_3]^{3+}$ is more than $[CoF_6]^{3-}$

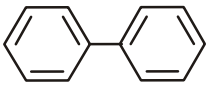
$\Rightarrow \lambda_{absorbed}$ of $[Co(en)_3]^{3+}$ is less than $[CoF_6]^{3-}$

(d) $\Delta_t = \dfrac{4}{9}\Delta_0 = \dfrac{4}{9} \times 18000 = 8000\ cm^{-1}$

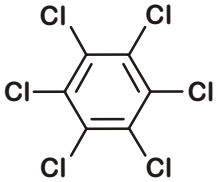
Hence, (a) and (d) are incorrect

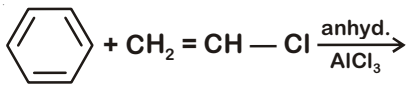
4. Consider the following reactions

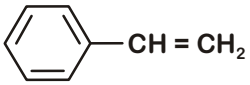
(a) 

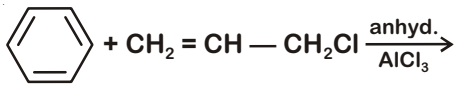


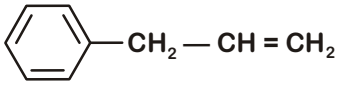
(b) 



(c)  + $CH_2 = CH - Cl$ $\xrightarrow[AlCl_3]{\text{anhyd.}}$



(d)  + $CH_2 = CH - CH_2Cl$ $\xrightarrow[AlCl_3]{\text{anhyd.}}$



Which of these reactions are possible?

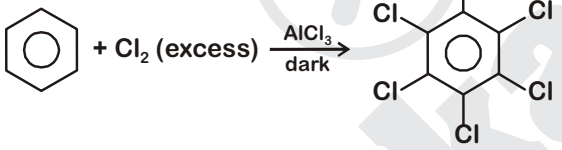
(1) (b) and (d) (2) (a) and (d)

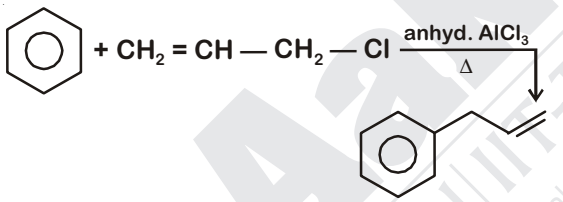
(3) (a) and (b) (4) (b), (c) and (d)

Answer (1)

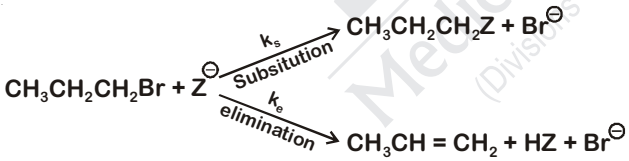
Sol. Vinyl halide and aryl halide do not give Friedel Craft's reaction.

The reactions which are possible are :

(b) 

(d) 

5. For the following reactions



where,

$Z^{\ominus} = CH_3CH_2O^{\ominus}$ (A) or $H_3C - \underset{CH_3}{\overset{CH_3}{\underset{|}{\overset{|}{C}}}} - O^{\ominus}$ (B),

k_s and k_e, are, respectively, the rate constants for substitution and elimination, and $\mu = \frac{k_s}{k_e}$, the correct option is _____ .

(1) $\mu_B > \mu_A$ and $k_e(A) > k_e(B)$

(2) $\mu_A > \mu_B$ and $k_e(A) > k_e(B)$

(3) $\mu_B > \mu_A$ and $k_e(B) > k_e(A)$

(4) $\mu_A > \mu_B$ and $k_e(B) > k_e(A)$

Answer (4)

Sol. $CH_3 - CH_2 - CH_2 - Br + Z^- \xrightarrow{k_s} \mu = \frac{k_s}{k_e}$ $\xrightarrow{k_e}$

when

$Z^{\ominus} = CH_3CH_2O^-$ (A)

$Z^{\ominus} =$  (B)

$\Rightarrow \mu_A > \mu_B$ and $k_e(B) > k_e(A)$

6. A chromatography column, packed with silica gel as stationary phase, was used to separate a mixture of compounds consisting of (A) benzanilide (B) aniline and (C) acetophenone. When the column is eluted with a mixture of solvents, hexane : ethyl acetate (20 : 80), the sequence of obtained compounds is

(1) (A), (B) and (C) (2) (C), (A) and (B)

(3) (B), (C) and (A) (4) (B), (A) and (C)

Answer (2)

Sol. The component which is more strongly adsorbed will be obtained later upon elution

It also depends upon the strength of interaction between compound and mobile phase.

(A) Benzanilide

(B) Aniline

(C) Acetophenone

So, (C) will be obtained first then (A) then (B).

7. The number of possible optical isomers for the complexes MA_2B_2 with sp^3 and dsp^2 hybridized metal atom, respectively, is

Note : A and B are unidentate neutral and unidentate monoanionic ligands, respectively.

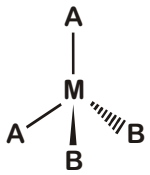
(1) 2 and 2 (2) 0 and 2

(3) 0 and 1 (4) 0 and 0

Answer (4)

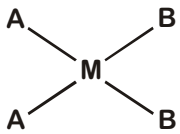
Sol. $[MA_2B_2]$

Tetrahedral



has plane of symmetry

Square planar



planar

Total number of optical isomer is zero in both the cases.

8. Which of the following statements is correct?

(1) Gluconic acid is a dicarboxylic acid

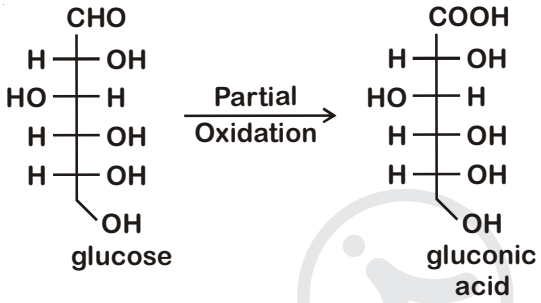
(2) Gluconic acid can form cyclic (acetal/ hemiacetal) structure

(3) Gluconic acid is a partial oxidation product of glucose

(4) Gluconic acid is obtained by oxidation of glucose with HNO_3

Answer (3)

Sol.



Gluconic acid is partial oxidation product of glucose and does not form hemiacetal or acetal.

9. The bond order and the magnetic characteristics of CN^- are

(1) $2\frac{1}{2}$, paramagnetic (2) 3, diamagnetic

(3) $2\frac{1}{2}$, diamagnetic (4) 3, paramagnetic

Answer (2)

Sol. CN^-

Total number of electron = 14

Bond order = 3

It is diamagnetic in nature.

10. Among statements (a)-(d) the correct ones are:

(a) Decomposition of hydrogen peroxide gives dioxygen.

(b) Like hydrogen peroxide, compounds, such as $KClO_3$, $Pb(NO_3)_2$ and $NaNO_3$ when heated liberate dioxygen.

(c) 2-Ethylanthraquinone is useful for the industrial preparation of hydrogen peroxide.

(d) Hydrogen peroxide is used for the manufacture of sodium perborate.

(1) (a) and (c) only (2) (a), (b) and (c) only

(3) (a), (b), (c) and (d) (4) (a), (c) and (d) only

Answer (3)

Sol. $H_2O_2 \xrightarrow{\Delta} H_2O + \frac{1}{2}O_2$

$KClO_3$, $Pb(NO_3)_2$, $NaNO_3$ on heating will release O_2 gas.

$$2NaBO_2 + 2H_2O_2 + 6H_2O \longrightarrow \underset{\text{Sodium peroxoborate}}{Na_2\left[(OH)_2B(O-O)_2B(OH)_2\right].6H_2O}$$

(Sodium metaborate)

11. Two open beakers one containing a solvent and the other containing a mixture of that solvent with a non volatile solute are together sealed in a container. Over time:

(1) The volume of the solution and the solvent does not change

(2) The volume of the solution increases and the volume of the solvent decreases

(3) The volume of the solution does not change and the volume of the solvent decreases

(4) The volume of the solution decreases and the volume of the solvent increases.

Answer (2)

Sol. There will be lowering in vapour pressure for solution containing non-volatile solute. So, there will be transfer of solvent molecules from pure solvent to solution and hence, volume of beaker containing solvent (pure) will decrease and volume of beaker containing solution will increase.

12. The redox reaction among the following is:

(1) Reaction of $[Co(H_2O)_6]Cl_3$ with $AgNO_3$

(2) Formation of ozone from atmospheric oxygen in the presence of sunlight.

(3) Combination of dinitrogen with dioxygen at 2000 K

(4) Reaction of H_2SO_4 with NaOH

Answer (3)

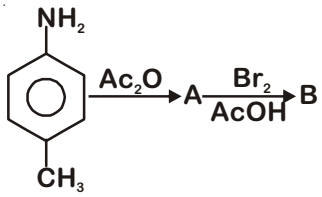
Sol. The redox reaction is

$$N_2 + O_2 \underset{}{\overset{2000\,K}{\rightleftharpoons}} 2NO$$

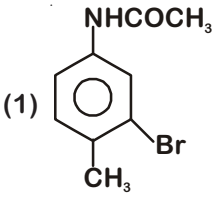
Nitrogen is oxidised while oxygen is reduced.

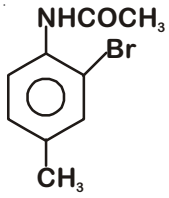
Reaction of $[CO(H_2O)_6]Cl_3$ with $AgNO_3$ is not redox reaction. It is a precipitation reaction.

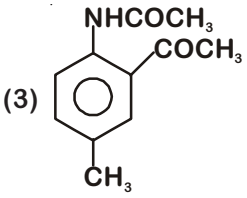
13. In the following reaction sequence:

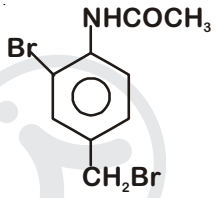


the major product B is:

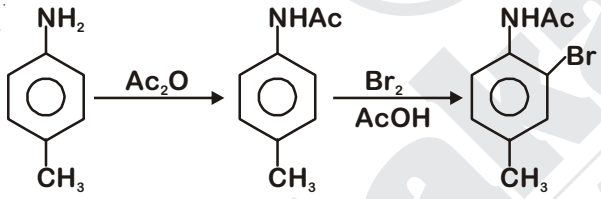
(1) 

(2) 

(3) 

(4) 

Answer (2)

Sol. 

14. Within each pair of elements F & Cl, S & Se, and Li & Na, respectively, the elements that release more energy upon an electron gain are:

(1) F, S and Li

(2) F, Se and Na

(3) Cl, S and Li

(4) Cl, Se and Na

Answer (3)

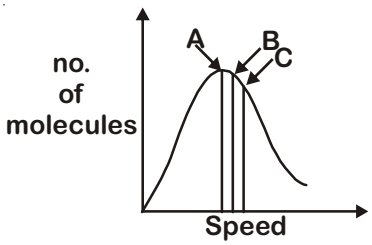
Sol. Order of energy released upon electron gain

F < Cl

S > Se

Li > Na

15. Identify the correct labels of A, B and C in the following graph from the options given below:



Root mean square speed (V_{rms}); most probable speed (V_{mp}); Average speed (V_{av})

(1) A – V_{av}; B – V_{rms}; C – V_{mp}

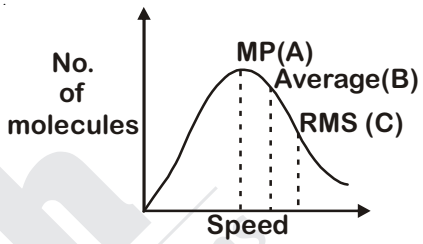
(2) A – V_{rms}; B – V_{mp}; C –V_{av}

(3) A – V_{mp}; B – V_{rms}; C – V_{av}

(4) A – V_{mp}; B – V_{av}; C – V_{rms}

Answer (4)

Sol.

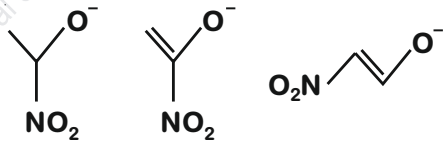


A – V_{mp}

B – V_{av}

C – V_{rms}

16. The correct order of stability for the following alkoxides is

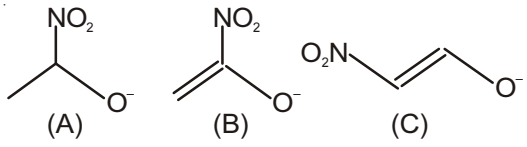


(A) (B) (C)

(1) (B) > (A) > (C) (2) (C) > (B) > (A)

(3) (B) > (C) > (A) (4) (C) > (A) > (B)

Answer (2)

Sol. 

Stability order C > B > A

17. The equation that is incorrect is

(1) $(\Lambda_m^\circ)_{NaBr} - (\Lambda_m^\circ)_{NaCl} = (\Lambda_m^\circ)_{KBr} - (\Lambda_m^\circ)_{KCl}$

(2) $(\Lambda_m^\circ)_{H_2O} = (\Lambda_m^\circ)_{HCl} + (\Lambda_m^\circ)_{NaOH} - (\Lambda_m^\circ)_{NaCl}$

(3) $(\Lambda_m^\circ)_{NaBr} - (\Lambda_m^\circ)_{NaI} = (\Lambda_m^\circ)_{KBr} - (\Lambda_m^\circ)_{NaBr}$

(4) $(\Lambda_m^\circ)_{KCl} - (\Lambda_m^\circ)_{NaCl} = (\Lambda_m^\circ)_{KBr} - (\Lambda_m^\circ)_{NaBr}$

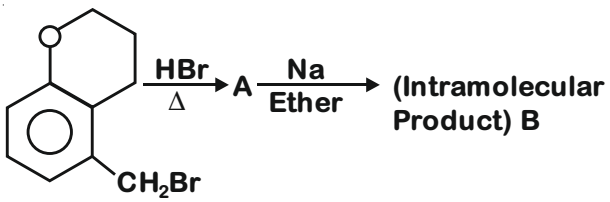
Answer (3)

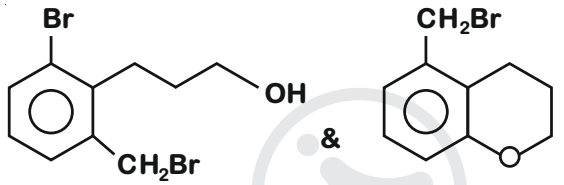
Sol. $\Lambda_m^\circ NaBr - \Lambda_m^\circ NaI = \Lambda_m^\circ\left(Br^-\right) - \Lambda_m^\circ\left(I^-\right)$ ...(1)

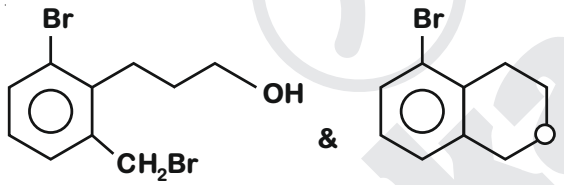
$\Lambda_m^\circ(KBr) - \Lambda_m^\circ(NaBr) = \Lambda_m^\circ\left(K^+\right) - \Lambda_m^\circ\left(Na^+\right)$ ...(2)

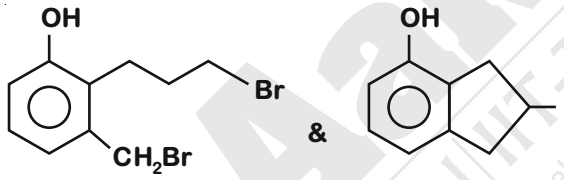
(1) and (2) are not equal.

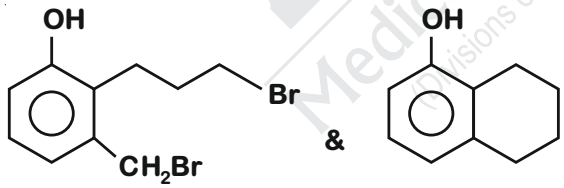
18. In the following reaction sequence, structures of A and B, respectively will be



(1) 

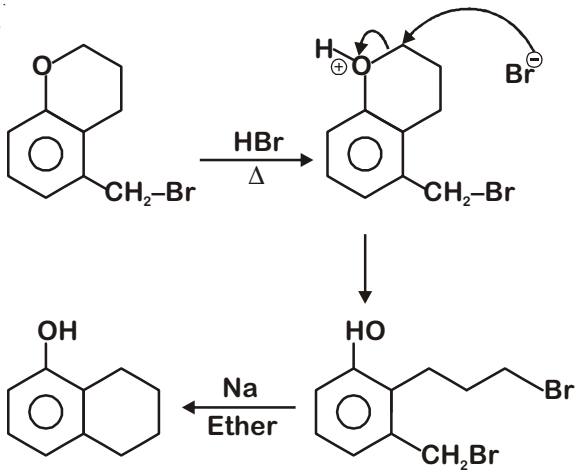
(2) 

(3) 

(4) 

Answer (4)

Sol.



19. The refining method used when the metal and the impurities have low and high melting temperatures, respectively, is

(1) Zone refining

(2) Vapour phase refining

(3) Liquation

(4) Distillation

Answer (3)

Sol. Liquation is applicable for metals which have low melting points as compared to their impurities.

20. The ammonia (NH_3) released on quantitative reaction of 0.6 g urea (NH_2CONH_2) with sodium hydroxide (NaOH) can be neutralized by

(1) 200 ml of 0.4 N HCl

(2) 100 ml of 0.1 N HCl

(3) 200 ml of 0.2 N HCl

(4) 100 ml of 0.2 N HCl

Answer (4)

Sol. $NH_2-\overset{\overset{O}{\|}}{C}-NH_2 \xrightarrow{NaOH} 2NH_3$

2 moles of NH_3 will react with 2 mole of HCl.

0.6 g of urea give $= \frac{0.6}{60} \times 2 = 0.02$ mol of NH_3

100 × 0.2 N HCl = 0.02 mol of HCl

SECTION - II

Numerical Value Type Questions: This section contains 5 questions. The answer to each of the questions is a numerical value. Each question carries 4 marks for correct answer and there is no negative marking for wrong answer.

21. Consider the following reactions:

$NaCl + K_2Cr_2O_7 + H_2SO_4 \rightarrow (A) + \text{side products}$ (Conc.)

$(A) + NaOH \rightarrow (B) + \text{Side products}$

$(B) + H_2SO_4 + H_2O_2 \rightarrow (C) + \text{Side products}$ (dilute)

The sum of the total number of atoms in one molecule each of (A), (B) and (C) is _____.

Answer (18)

Sol. $NaCl \xrightarrow[conc.H_2SO_4]{K_2Cr_2O_7} \underset{(A)}{CrO_2Cl_2} \xrightarrow{NaOH} \underset{(B)}{Na_2CrO_4}$

$\underset{(B)}{Na_2CrO_4} \xrightarrow[H_2O_2]{dil\ H_2SO_4} \underset{(C)}{CrO_5}$

Total number of atoms in A, B and C are 18.

22. The flocculation value of HCl for arsenic sulphide sol. is 30 m mol L^{-1}. If H_2SO_4 is used for the flocculation of arsenic sulphide, the amount, in grams, of H_2SO_4 in 250 ml required for the above purpose is _____.

(molecular mass of H_2SO_4 = 98 g/mol)

Answer (0.37)

Sol. Arsenic sulphide sol is negatively charged, so H^+ brings about precipitation

for 1 L, 30 m mol of HCl is required

for 250 ml, $\frac{30}{4}$ m mol of HCl is required

for 250 ml, $\frac{30}{4\times2}$ m mol of H_2SO_4 is required

$\therefore$ Weight of H_2SO_4 required $= \frac{30}{4\times2}\times10^{-3}\times98$ g

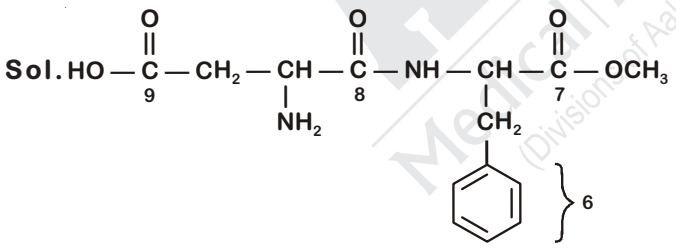
$= 0.3675$ g

$\simeq 0.37$ g

23. The number of sp^2 hybridised carbons present in "Aspartame" is _____.

Answer (9)

Sol.



Number of sp^2 hybridized carbon are '9'.

24. 3 g of acetic acid is added to 250 mL of 0.1 M HCl and the solution made up to 500 mL. To 20 mL of this solution $\frac{1}{2}$mL of 5 M NaOH is added. The pH of the solution is_____.

[Given : pK_a of acetic acid = 4.75, molar mass of acetic acid = 60 g/mol, log3 = 0.4771]

Neglect any changes in volume.

Answer (5.23)

Sol.

	CH_3COOH	HCl
Moles :	$\frac{3}{60}$mol	250 × 0.1 m mol
Concentration in 500 ml	0.1 M	0.05 M
Moles in 20 ml	2 m mol	1 m mol

When $\frac{1}{2}$ ml of 5 M NaOH is added then solution contains 0.5 m moles of CH_3COOH and 1.5 m moles of CH_3COONa

$$pH = pK_a + \ln\left(\frac{salt}{acid}\right) = 4.75 + \log(3)$$

$= 4.75 + 0.48$

$= 5.23$

25. The standard heat of formation $\left(\Delta_f H^0_{298}\right)$ of ethane (in kJ/mol), if the heat of combustion of ethane, hydrogen and graphite are –1560, –393.5 and –286 kJ/mol, respectively is_____.

Answer (–192.5)

Sol. (1) $C + O_2 \longrightarrow CO_2$ $\quad \Delta_C H° = -286$ kJ/mol

(2) $H_2 + \frac{1}{2}O_2 \longrightarrow H_2O$ $\quad \Delta_C H° = -393.5$ kJ/mol

(3) $C_2H_6 + \frac{7}{2}O_2 \longrightarrow 2CO_2 + 3H_2O$

$\Delta_C H° = -1560$ kJ/mol

$2 \times (1) + 3 \times (2) - (3)$

$2C + 3H_2 \longrightarrow C_2H_6$

$\Delta H_f^\circ(C_2H_6) = 2(-286) + 3(-393.5) + 1560$

$= -192.5$ kJ/mol

❑ ❑ ❑

MATHEMATICS

SECTION - I

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. The value of c in the Lagrange's mean value theorem for the function $f(x) = x^3 - 4x^2 + 8x + 11$, when $x \in [0,1]$ is

(1) $\frac{4-\sqrt{7}}{3}$ (2) $\frac{4-\sqrt{5}}{3}$

(3) $\frac{2}{3}$ (4) $\frac{\sqrt{7}-2}{3}$

Answer (1)

Sol. $\because \quad f'(c) = \frac{f(b)-f(a)}{b-a}$

$\therefore \quad 3c^2 - 8c + 8 = \frac{f(1)-f(0)}{1-0}$

$3c^2 - 8c + 8 = 16 - 11$

$3c^2 - 8c + 3 = 0$

$\therefore \quad c = \frac{8 \pm 2\sqrt{7}}{6}$

$\therefore \quad c = \frac{4-\sqrt{7}}{3}$ as $c \in (0, 1)$

2. Let y= y(x) be the solution curve of the differential equation, $(y^2 - x)\frac{dy}{dx} = 1$, satisfying y(0) = 1. This curve intersects the x-axis at a point whose abscissa is

(1) 2 – e (2) 2 + e

(3) – e (4) 2

Answer (1)

Sol. $\because \quad (y^2 - x)\frac{dy}{dx} = 1$

$\therefore \quad y^2dy - xdy = dx$

$\Rightarrow \quad e^y \cdot y^2dy = e^ydx + xe^ydy$

$\Rightarrow \quad e^y \cdot y^2dy = d(e^y \cdot x)$

On integrating both sides we get

$\int e^y \cdot y^2dy = \int d(e^y \cdot x)$

$y^2 \cdot e^y - \int 2y \cdot e^ydy = e^y \cdot x$

$y^2 \cdot e^y - 2\{y \cdot e^y - \int e^ydy\} = e^y \cdot x$

$\therefore \quad y^2 \cdot e^y - 2ye^y + 2e^y = e^y \cdot x + c$

$\therefore \quad y(0) = 1$

$\Rightarrow \quad c = e$

$\therefore \quad y^2 - 2y + 2 = x + e \cdot e^{-y}$

$\therefore \quad$ when y = 0 then x = 2 – e

3. Let y = y(x) be a function of x satisfying $y\sqrt{1-x^2} = k - x\sqrt{1-y^2}$ where k is a constant and $y\left(\frac{1}{2}\right) = -\frac{1}{4}$. Then $\frac{dy}{dx}$ at $x = \frac{1}{2}$, is equal to

(1) $-\frac{\sqrt{5}}{2}$ (2) $\frac{2}{\sqrt{5}}$

(3) $\frac{\sqrt{5}}{2}$ (4) $-\frac{\sqrt{5}}{4}$

Answer (1)

Sol. $\because \quad y\sqrt{1-x^2} = k - x\sqrt{1-y^2}$ ...(i)

On differentiating both side of eq. (i) w.r.t. x we get,

$\frac{dy}{dx}\sqrt{1-x^2} - y\frac{2x}{2\sqrt{1-x^2}} = 0 - \sqrt{1-y^2} + \frac{x \cdot y}{\sqrt{1-y^2}}\frac{dy}{dx}$

Put $x = \frac{1}{2}$ and $y = -\frac{1}{4}$ we get

$\frac{dy}{dx} \cdot \frac{\sqrt{3}}{2} - \left(-\frac{1}{4}\right) \cdot \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = -\frac{\sqrt{15}}{4} + \frac{-\frac{1}{8}}{\frac{\sqrt{15}}{4}} \cdot \frac{dy}{dx}$

$\therefore \quad \frac{dy}{dx} = -\frac{\sqrt{5}}{2}$

4. Let $A = [a_{ij}]$ and $B = [b_{ij}]$ be two 3 × 3 real matrices such that $b_{ij} = (3)^{(i+j-2)}a_{ji}$, where i, j = 1, 2, 3. If the determinant of B is 81, then the determinant of A is

(1) 1/9 (2) 1/81

(3) 3 (4) 1/3

Answer (1)

Sol. $B=\left[b_{ij}\right]_{3\times3}=\begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}$

$\therefore \quad (B)=\begin{bmatrix} 3^0\cdot a_{11} & 3\cdot a_{21} & 3^2\cdot a_{31} \\ 3\cdot a_{12} & 3^2\cdot a_{22} & 3^3\cdot a_{32} \\ 3^2\cdot a_{13} & 3^3\cdot a_{23} & 3^4\cdot a_{33} \end{bmatrix}$

$$\det(B)=3.3^2\cdot3.3^2\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

$\therefore \quad 3^4 = 3^6\cdot\det(A)$

$\therefore \quad \det(A) = \frac{1}{3^2}=\frac{1}{9}$

5. Let α and β be the roots of the equation $x^2-x-1=0$. If $p_k=(\alpha)^k+(\beta)^k$, $k\geq1$, then which one of the following statements is not true?

(1) $p_3 = p_5 - p_4$

(2) $(p_1 + p_2 + p_3 + p_4 + p_5) = 26$

(3) $p_5 = 11$

(4) $p_5 = p_2\cdot p_3$

Answer (4)

Sol. $\because$ α, β are roots of $x^2 - x - 1 = 0$ ...(i)

$\therefore \quad \alpha^2 - \alpha - 1 = 0$

$\Rightarrow \quad \alpha^{n+2} - \alpha^{n+1} - \alpha^n = 0$ ...(ii)

Similarly, $\beta^{n+2} - \beta^{n+1} - \beta^n = 0$ ...(iii)

From eq. (ii) + (iii), we get

$$\alpha^{n+2} + \beta^{n+2} = \left(\alpha^{n+1}+\beta^{n+1}\right)+\left(\alpha^n+\beta^n\right)$$

$\therefore \quad p_{n+2} = p_{n+1} + p_n$

For n = 0, $p_0 = \alpha^0 + \beta^0 = 2$

For n = 1, $p_1 = \alpha + \beta = 1$

and $p_2 = p_0 + p_1 = 2 + 1 = 3$

$p_3 = p_2 + p_1 = 3 + 1 = 4$

$p_4 = p_3 + p_2 = 4 + 3 = 7$

$p_5 = p_4 + p_3 = 7 + 4 = 11$

6. Let A, B, C and D be four non-empty sets. The contrapositive statement of "If $A \subseteq B$ and $B \subseteq D$, then $A \subseteq C$" is

(1) If $A \not\subseteq C$, then $A \subseteq B$ and $B \subseteq D$

(2) If $A \not\subseteq C$, then $A \not\subseteq B$ and $B \subseteq D$

(3) If $A \not\subseteq C$, then $A \not\subseteq B$ or $B \not\subseteq D$

(4) If $A \subseteq C$, then $B \subset A$ or $D \subset B$

Answer (3)

Sol. Consider the statements;

p : $A \subseteq B$ and $B \subseteq D$

q : $A \subseteq C$

Given statement is "If p then q". It's contrapositive will be "If not q then not p"

$\Rightarrow$ If $A \not\subseteq C$ then $A \not\subseteq B$ or $B \not\subseteq D$.

7. The area (in sq. units) of the region $\{(x, y)\in R^2 \mid 4x^2 \leq y \leq 8x+12\}$ is

(1) $\frac{128}{3}$ (2) $\frac{125}{3}$

(3) $\frac{127}{3}$ (4) $\frac{124}{3}$

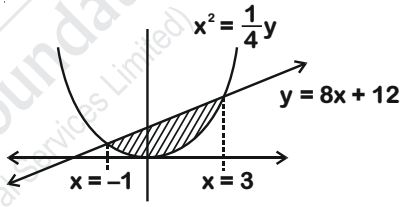
Answer (1)

Sol. For point of intersections

$4x^2 = 8x + 12$

$x^2 - 2x - 3 = 0$

$\therefore \quad x = -1, 3$



The required area = $\int_{-1}^{3}(8x+12-4x^2)dx$

$$=4\left(2\cdot\frac{x^2}{2}+3x-\frac{x^3}{3}\right)_{-1}^{3}$$

$$=4\left\{(9+9-9)-\left(1-3+\frac{1}{3}\right)\right\}$$

$$=\frac{128}{3}\text{ square units.}$$

8. Let the tangents drawn from the origin to the circle, $x^2 + y^2 - 8x - 4y + 16 = 0$ touch it at the points A and B. The $(AB)^2$ is equal to

(1) $\frac{64}{5}$

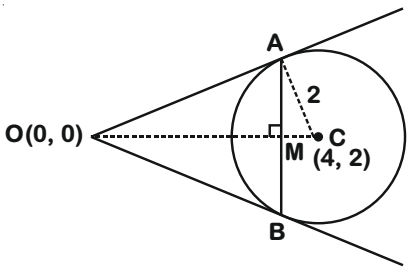
(2) $\frac{52}{5}$

(3) $\frac{56}{5}$

(4) $\frac{32}{5}$

Answer (1)

Sol. Equation of chord of contact is



$x\cdot 0 + y\cdot 0 - 4(x + 0) - 2(y + 0) + 16 = 0$

$\therefore$ $2x + y - 8 = 0$

$\therefore$ Length of $CM = \left[\frac{2\cdot 4 + 2 - 8}{\sqrt{2^2 + 1^2}}\right] = \frac{2}{\sqrt{5}}$ units.

$\therefore$ $AM = BM = \sqrt{4 - \frac{4}{5}} = \sqrt{\frac{16}{5}}$

$\therefore$ Length of chord of contact $(AB) = \frac{8}{\sqrt{5}}$.

$\therefore$ Square of length of chord of Contact $= \left(\frac{8}{\sqrt{5}}\right)^2 = \frac{64}{5}$.

9. Let $a_1, a_2, a_3 \ldots$ be a G.P. such that $a_1 < 0$, $a_1 + a_2 = 4$ and $a_3 + a_4 = 16$. If $\sum_{i=1}^{9} a_i = 4\lambda$, then λ is equal to

(1) –513 (2) –171

(3) $\frac{511}{3}$ (4) 171

Answer (2)

Sol. Let the G.P. be a, ar, ar^2, ar^3, ... and $a < 0$.

$\because$ $a_1 + a_2 = 4 \Rightarrow a(1 + r) = 4$ ...(i)

$a_3 + a_4 = 16 \Rightarrow ar^2(1 + r) = 16$...(ii)

$\therefore$ from (i) and (ii), $r = \pm 2$.

if $r = 2$, then $a = \frac{4}{3}$

if $r = -2$, then $a = -4$.

$\therefore$ $\sum_{i=1}^{9} a_i = \frac{a(r^9 - 1)}{r - 1} = 4\lambda$

$= \frac{-4\cdot\left((-2)^9 - 1\right)}{-2 - 1} = 4\lambda$

$\therefore$ $\lambda = -171$

10. In a workshop, there are five machines and the probability of any one of them to be out of service on a day is $\frac{1}{4}$. If the probability that at most two machines will be out of service on the same day is $\left(\frac{3}{4}\right)^3 k$, then k is equal to

(1) 4 (2) $\frac{17}{4}$

(3) $\frac{17}{8}$ (4) $\frac{17}{2}$

Answer (3)

Sol. Probability that a machine is faulted $= \frac{1}{4} = P$

Probability that a machine is not faulted

$= 1 - \frac{1}{4} = \frac{3}{4} = q$

$\therefore$ Probability that atmost two machine is faulted = P(X = 0) + P(X = 1) + P(X = 2)

$\therefore$ ${}^5C_0\left(\frac{1}{4}\right)^0\cdot\left(\frac{3}{4}\right)^5 + {}^5C_1\left(\frac{1}{4}\right)^1\cdot\left(\frac{3}{4}\right)^4 + {}^5C_2\left(\frac{1}{4}\right)^2\cdot\left(\frac{3}{4}\right)^3$

$= \left(\frac{3}{4}\right)^3\cdot k$

$\Rightarrow$ $\left(\frac{3}{4}\right)^2 + 5\cdot\frac{1}{4}\cdot\frac{3}{4} + 10\cdot\left(\frac{1}{4}\right)^2 = K$

$\therefore$ $k = \frac{10 + 15 + 9}{16} = \frac{34}{16} = \frac{17}{8}$

11. The number of ordered pairs (r, k) for which $6\cdot{}^{35}C_r = (k^2 - 3)\cdot{}^{36}C_{r+1}$, where k is an integer, is

(1) 3 (2) 6

(3) 2 (4) 4

Answer (4)

Sol. $\because$ ${}^{36}C_{r+1}\cdot(k^2 - 3) = {}^{35}C_r \times 6$

$\frac{36!}{(r+1)!(35-r)!}\cdot(k^2 - 3) = \frac{35!}{r!(35-r)!}\times 6$

$6(k^2 - 3) = r + 1$

$\therefore$ $k^2 = 3 + \frac{r+1}{6}$

$\therefore$ r can be 5 and 35

When r = 5 then $k = \pm 2$

and when r = 35, then $k = \pm 3$.

$\therefore$ Total number of ordered pairs = 4

12. The locus of the mid-points of the perpendiculars drawn from points on the line, x = 2y to the line x = y is

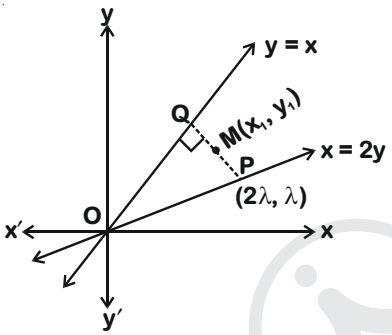
(1) 5x – 7y = 0 (2) 2x – 3y = 0

(3) 3x – 2y = 0 (4) 7x – 5y = 0

Answer (1)

Sol. Let coordinate of P is $(2\lambda, \lambda)$

and coordinate of mid-point M is (x_1, y_1).



$\therefore$ Coordinate of Q

$= (2x_1 - 2\lambda, 2y_1 - \lambda)$

$\because$ Q lies on line y = x

$\therefore \quad \lambda = 2x_1 - 2y_1 \qquad ...(i)$

(Slope of line PQ) · (Slope of line y = x) = –1

$\therefore \quad \dfrac{\lambda - y_1}{2\lambda - x_1} = -1$

$\therefore \quad \lambda = \dfrac{x_1 + y_1}{3} \qquad ...(ii)$

From equation (i) and (ii) : $5x_1 = 7y_1$

$\therefore$ Required locus is 5x = 7y.

13. Let f(x) be a polynomial of degree 5 such that x = ±1 are its critical points. If $\lim\limits_{x\to 0}\left(2 + \dfrac{f(x)}{x^3}\right) = 4$, then which one of the following is not true?

(1) f is an odd function

(2) x =1 is a point of minima and x = –1 is a point of maxima of f.

(3) f(1) – 4f(–1) = 4

(4) x=1 is a point of maxima and x = –1 is a point of minimum of f.

Answer (2)

Sol. $\because$ f(x) is a five degree polynomial such that

$\lim\limits_{x\to 0}\left(2 + \dfrac{f(x)}{x^3}\right) = 4$ then

let $f(x) = ax^5 + bx^4 + cx^3$

$\lim\limits_{x\to 0}\left(2 + \dfrac{ax^5 + bx^4 + cx^3}{x^3}\right) = 4$

$\Rightarrow 2 + c = 4 \Rightarrow c = 2.$

Now, $f'(x) = 5ax^4 + 4bx^3 + 3cx^2$

$= x^2\left(5ax^2 + 4bx + 3c\right)$

$\because \quad f'(1) = 0 \Rightarrow 5a + 4b + 6 = 0$

and $f'(-1) = 0 \Rightarrow 5a - 4b + 6 = 0$

$\therefore$ b = 0, $a = -\dfrac{6}{5}$

$\therefore \quad f(x) = -\dfrac{6}{5}x^5 + 2x^3$

$f'(x) = -6x^4 + 6x^2 = -6x^2(x+1)(x-1)$

It is clear that maxima at x = 1 and minima at x = –1.

and f(1) – 4f(–1) = 4

14. If θ_1 and θ_2 be respectively the smallest and the largest values of θ in $(0, 2\pi) - \{\pi\}$ which satisfy the equation, $2\cot^2\theta - \dfrac{5}{\sin\theta} + 4 = 0$, then $\int\limits_{\theta_1}^{\theta_2} \cos^2 3\theta\, d\theta$, is equal to

(1) $\dfrac{\pi}{3} + \dfrac{1}{6}$ (2) $\dfrac{\pi}{3}$

(3) $\dfrac{2\pi}{3}$ (4) $\dfrac{\pi}{9}$

Answer (2)

Sol. $\because \quad 2\cot^2\theta - \dfrac{5}{\sin\theta} + 4 = 0$

$2 + 2\operatorname{cosec}^2\theta - 5\operatorname{cosec}\theta = 0$

$2\operatorname{cosec}^2\theta - 4\operatorname{cosec}\theta - \operatorname{cosec}\theta + 2 = 0$

$\therefore \quad (2\operatorname{cosec}\theta - 1)(\operatorname{cosec}\theta - 2) = 0$

$\therefore \quad \operatorname{cosec}\theta = \dfrac{1}{2}$ or 2.

$\therefore \quad \sin\theta = 2$ or $\dfrac{1}{2}$. $\qquad \because \theta \in (0, 2\pi)$

$\therefore \quad \theta_1 = \dfrac{\pi}{6}$ and $\theta_2 = \dfrac{5\pi}{6}$, $\qquad \because \theta_1 < \theta_2$

$\therefore \quad I = \int\limits_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \cos^2 3\theta\, d\theta = \int\limits_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \dfrac{1 + \cos 6\theta}{2}\, d\theta$

$= \dfrac{1}{2}\left[\theta + \dfrac{\sin 6\theta}{6}\right]_{\frac{\pi}{6}}^{\frac{5\pi}{6}} = \dfrac{\pi}{3}$

15. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three unit vectors such that $\vec{a}+\vec{b}+\vec{c}=\vec{0}$. If $\lambda=\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}$ and $\vec{d}=\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}$ then the ordered pair, $(\lambda, \vec{d})$ is equal to

(1) $\left(\frac{3}{2}, 3\vec{a}\times\vec{c}\right)$ (2) $\left(-\frac{3}{2}, 3\vec{c}\times\vec{b}\right)$

(3) $\left(-\frac{3}{2}, 3\vec{a}\times\vec{b}\right)$ (4) $\left(\frac{3}{2}, 3\vec{b}\times\vec{c}\right)$

Answer (3)

Sol. $\because \ |\vec{a}|=|\vec{b}|=|\vec{c}|=1$

and $\vec{a}+\vec{b}+\vec{c}=\vec{0}$

On squaring both sides

$$|\vec{a}|^2+|\vec{b}|^2+|\vec{c}|^2+2(\vec{a}.\vec{b}+\vec{a}.\vec{c}+\vec{b}.\vec{c})=0$$

$$\therefore \ \lambda=\vec{a}.\vec{b}+\vec{a}.\vec{c}+\vec{b}.\vec{c}=-\frac{3}{2}$$

and $\vec{d}=\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}$

$$=\vec{a}\times\vec{b}+\vec{b}\times(-\vec{a}-\vec{b})+(-\vec{a}-\vec{b})\times\vec{a}$$

$$=\vec{a}\times\vec{b}-\vec{b}\times\vec{a}-0-0-\vec{b}\times\vec{a}$$

$$=3(\vec{a}\times\vec{b})$$

$$\therefore \ (\lambda, \vec{d})=\left(-\frac{3}{2}, 3(\vec{a}\times\vec{b})\right)$$

16. If $3x+4y=12\sqrt{2}$ is a tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{9}=1$ for some $a \in R$, then the distance between the foci of the ellipse is

(1) $2\sqrt{5}$ (2) $2\sqrt{7}$

(3) 4 (4) $2\sqrt{2}$

Answer (2)

Sol. A line $y=mx+c$ be a tangent to ellipse

$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, if $c^2=a^2m^2+b^2$

Here, eq. of tangent is : $4y=-3x+12\sqrt{2}$

$$\therefore \ y=-\frac{3}{4}x+3\sqrt{2}$$

$$\therefore \ (3\sqrt{2})^2=a^2.\left(-\frac{3}{4}\right)^2+9$$

$$\therefore \ a^2=9\times\frac{16}{9}=16$$

$\therefore$ Eccentricity of ellipse = $e=\sqrt{1-\frac{9}{16}}=\frac{\sqrt{7}}{4}$

$\therefore$ Distance between foci = 2ac = $2.4.\frac{\sqrt{7}}{4}$

$= 2\sqrt{7}$

17. If the sum of the first 40 terms of the series, 3 + 4 + 8 + 9 + 13 + 14 + 18 + 19 + ... is (102)m, then m is equal to

(1) 5 (2) 20

(3) 25 (4) 10

Answer (2)

Sol. S = 3 + 4 + 8 + 9 + 13 + 14 + 18 + ... 40 terms.

= 7 + 17 + 27 + ... 20 terms

$$=\frac{20}{2}\{2\times7+(20-1).10\}$$

= 2040

(102)m = 2040

$\therefore$ m = 20

18. If $\frac{3+i\sin\theta}{4-i\cos\theta}, \theta\in[0,2\pi]$, is a real number, then an argument of $\sin\theta+i\cos\theta$ is

(1) $\pi-\tan^{-1}\left(\frac{3}{4}\right)$ (2) $\pi-\tan^{-1}\left(\frac{4}{3}\right)$

(3) $-\tan^{-1}\left(\frac{3}{4}\right)$ (4) $\tan^{-1}\left(\frac{4}{3}\right)$

Answer (2)

Sol. $\because \ Z=\frac{3+i\sin\theta}{4-i\cos\theta}\times\frac{4+i\cos\theta}{4+i\cos\theta}$

$$=\frac{(12-\sin\theta\cos\theta)+i(4\sin\theta+3\cos\theta)}{16+\cos^2\theta}$$

$\because$ Z is purely real

$\therefore \ 4\sin\theta+3\cos\theta=0$

$$\tan\theta=-\frac{3}{4}$$

if $\theta\in\left(\frac{\pi}{2}, \pi\right)$, then

$\arg(\sin\theta+i\cos\theta) = \pi-\tan^{-1}\left(\frac{4}{3}\right)$

if $\theta\in\left(\frac{3\pi}{2}, 2\pi\right)$, then

$\arg(\sin\theta+i\cos\theta)=-\tan^{-1}\frac{4}{3}$

19. The coefficient of x^7 in the expression $(1 + x)^{10} + x(1 + x)^9 + x^2 (1 + x)^8 + \ldots + x^{10}$ is

(1) 120

(2) 330

(3) 420

(4) 210

Answer (2)

Sol. $(1 + x)^{10} + x(1 + x)^9 + x^2(1 + x)^8 + \ldots + x^{10}$

$$= \frac{(1+x)^{10}\left(1-\left(\frac{x}{1+x}\right)^{11}\right)}{1-\frac{x}{1+x}}$$

$= (1 + x)^{11} - x^{11}$

$\therefore$ Coeff. of $x^7 = {}^{11}C_7 = {}^{11}C_4$

$$= \frac{11\times10\times9\times8}{4\times3\times2\times1}$$

$= 330$

20. The value of α for which $4\alpha\int_{-1}^{2} e^{-\alpha|x|}\,dx = 5$, is

(1) $\log_e\left(\frac{4}{3}\right)$

(2) $\log_e\left(\frac{3}{2}\right)$

(3) $\log_e 2$

(4) $\log_e \sqrt{2}$

Answer (3)

Sol. $\because \quad 4\alpha\int_{-1}^{2} e^{-\alpha|x|}dx = 5$

$$\Rightarrow \quad 4\alpha\left\{\int_{-1}^{0} e^{\alpha x}dx + \int_{0}^{2} e^{-\alpha x}dx\right\} = 5$$

$$4\alpha\left\{\left(\frac{e^{\alpha x}}{\alpha}\right)_{-1}^{0} + \left(\frac{e^{-\alpha x}}{-\alpha}\right)_{0}^{2}\right\} = 5$$

$4(1 - e^{-\alpha} - e^{-2\alpha} + 1) = 5$

$4(2 - e^{-\alpha} - e^{-2\alpha}) = 5$

$4e^{-2\alpha} + 4e^{-\alpha} - 3 = 0$

$(2e^{-\alpha} + 3)(2e^{-\alpha} - 1) = 0$

$\therefore \quad e^{-\alpha} = \frac{1}{2} \Rightarrow \alpha = \ln 2$

SECTION - II

Numerical Value Type Questions: This section contains 5 questions. The answer to each of the questions is a numerical value. Each question carries 4 marks for correct answer and there is no negative marking for wrong answer.

21. If the system of linear equations,

$x + y + z = 6$

$x + 2y + 3z = 10$

$3x + 2y + \lambda z = \mu$

has more than two solutions, then $\mu - \lambda^2$ is equal to _____.

Answer (13)

Sol. Given system of equation more than 2 solutions.

Hence system of equation has infinite many solution.

$\therefore \quad D = D_1 = D_2 = D_3 = 0$

$$\therefore \begin{vmatrix}1&1&1\\1&2&3\\3&2&\lambda\end{vmatrix} = \begin{vmatrix}6&1&1\\10&2&3\\\mu&2&\lambda\end{vmatrix} = \begin{vmatrix}1&6&1\\1&10&3\\3&\mu&\lambda\end{vmatrix} =$$

$$\begin{vmatrix}1&1&6\\1&2&10\\3&2&\mu\end{vmatrix} = 0$$

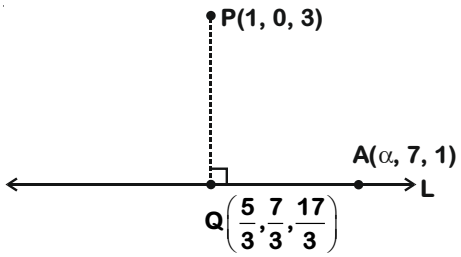
$\therefore \quad \lambda = 1$ and $\mu = 14$.

$\therefore \quad \mu - \lambda^2 = 13$

22. If the foot of the perpendicular drawn from the point (1, 0, 3) on a line passing through $(\alpha, 7, 1)$ is $\left(\frac{5}{3}, \frac{7}{3}, \frac{17}{3}\right)$, then α is equal to _____.

Answer (4)

Sol.



Direction Ratio of PQ are

$$< \frac{5}{3} - 1, \frac{7}{3} - 0, \frac{17}{3} - 3 >$$

$$= < \frac{2}{3}, \frac{7}{3}, \frac{8}{3} > = < 2, 7, 8 >$$

Direction ratio of line L are

$$<\alpha-\frac{5}{3},7-\frac{7}{3},1-\frac{17}{3}>$$

$= <3\alpha - 5, 14, -14>$

$\therefore$ PQ is perpendicular to line L.

$\therefore$ $2(3\alpha - 5) + 7.14 + (-14).8 = 0$

$\therefore$ $\alpha = 4$

23. If the mean and variance of eight numbers 3, 7, 9, 12, 13, 20, x and y be 10 and 25 respectively, then x.y is equal to _____.

Answer (54)

Sol. $\frac{x+y+64}{8}=10$

$\Rightarrow$ $x + y = 16$ ...(1)

Also $25=\frac{\Sigma x_i^2}{8}-100$

$\Rightarrow$ $\Sigma x_i^2 = 1000$

$x^2 + y^2 = 148$ ...(2)

From (1) & (2); $\Rightarrow$ $xy = 54$

24. If the function f defined on $\left(-\frac{1}{3},\frac{1}{3}\right)$ by

$$f(x)=\begin{cases}\frac{1}{x}\log_e\left(\frac{1+3x}{1-2x}\right), & \text{when } x\neq 0\\ k, & \text{when } x=0\end{cases}$$

is continuous, then k is equal to _____.

Answer (5)

Sol. $K=\lim\limits_{x\to 0^+}\frac{1}{x}\ln\left(\frac{1+3x}{1-2x}\right)$

$$=\lim_{x\to 0^+}\left(\frac{1-2x}{1+3x}\right)\left(\frac{(1-2x)3-(1+3x)(-2)}{(1-2x)^2}\right)$$

$= 3 + 2 = 5$

25. Let $X = \{n \in N : 1 \le n \le 50\}$. If $A = \{n \in X : n$ is a multiple of 2$\}$ and $B = \{n \in X : n$ is a multiple of 7$\}$, then the number of elements in the smallest subset of X containing both A and B is _____.

Answer (29)

Sol. $\because$ $X = \{1, 2, 3, 4, ..., 50\}$

$A = \{2, 4, 6, 8, ..., 50\}$

$B = \{7, 14, 21, 28, 35, 42, 49\}$

Here $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$= 29.$

$\therefore$ Number of elements in smallest subset of X containing both A and B is 29.

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