

## Time Domain Simulation of Soil–Foundation–Structure Interaction in non–Uniform Soils

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### SUMMARY

Presented here is a numerical investigation of the influence of non–uniform soil conditions on a prototype concrete bridge with three bents (four span) where soil beneath bridge bents is varied between stiff sands and soft clay. A series of high fidelity models of the soil–foundation–structure system were developed and described in some details. Development of a series of high fidelity models was required to properly simulate seismic wave propagation (frequency up to 10 Hz) through highly nonlinear, elastic plastic soil, piles and bridge structure. Eight specific cases representing combinations of different soil conditions beneath each of the bents are simulated. It is shown that variability of soil beneath bridge bents has significant influence on bridge system (soil–foundation–structure) seismic behavior. Results also indicate that free field motions differ quite a bit from what is observed (simulated) under at the base of the bridge columns indicating that use of free field motions as input for structural only models might not be appropriate. In addition to that, it is also shown that usually assumed beneficial effect of stiff soils underneath a structure (bridge) cannot be generalized and that such stiff soils do not necessarily help seismic performance of structures. Moreover, it is shown that dynamic characteristics of all three components of a triad made up of of earthquake, soil and structure play crucial role in determining the seismic performance of the infrastructure (bridge) system.

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### 1. Introduction

Currently, for a vast majority of numerical simulations of the response of bridge structures to seismic ground motions, the input excitations are defined either from a family of damped

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response spectra or as one or more time histories of ground acceleration. These input excitations are usually applied simultaneously along the entire base of the structure, regardless of its dimensions and dynamic characteristics, the properties of the soil material in foundations, or the nature of the ground motions themselves. Application of ground motions like this does not account for spatial variations of the traveling seismic waves that control the ground shaking. In addition to that, ground motions applied in such a way neglect the soil–structure interaction (SSI) effects, that can significantly change ground motions that are actually developing in such SSI system. A number of papers in recent years have investigated the influence of the SSI on behavior of bridges.

Even though interest in SSI effects has grown significantly in recent years, Tyapin (2007) notes that after four decades of intensive studies there still exists a large gap in SSI simulation tools used between SSI specialists and practicing civil engineers. Results obtained using specialized SSI simulation tools match closer experimental and field data (validate better (Oberkampf et al., 2002)) than regular, general simulation tools. There is therefore a significant need to transfer advanced simulation technology (numerical tools, education...) to practicing engineers, so that SSI effects can be appropriately taken into account in designing structures. One of the first studies that has developed a three-dimensional, nonlinear model for complete soil – skew highway bridge system interacting with their surrounding soils during strong motion earthquakes was done by Chi Chen and Penzien (1977).

Due to a limitations of computer power, a number of studies were conducted with a variety of modeling simplifications that usually rely on closed form solutions for elastic material. We mention few such studies below. Makris et al. (1994) developed a simple integrated procedure to analyze soil-pile foundation-superstructure interaction, based on dynamic impedance and kinematic seismic response factors of pile foundations with a simple six-degree-of-freedom structural model. Sweet (1993) approximated geometry of pile groups to perform finite element analysis of a bridge system subjected to earthquake loads, while Dendrou et al. (1985) resorted to combining finite element and boundary integral methodology to resolve seismic wave propagation from soil to bridge structure.

It is very important to note that assumed beneficial role of not performing a full SSI analysis has been turned into dogma, particularly since the NEHRP-94 seismic code states that: *"These [seismic] forces therefore can be evaluated conservatively without the adjustments recommended in Sec. 2.5 [i.e. for SSI effects]"*. A number of studies have therefore investigated importance of performing SSI analysis. McCallen and Romstadt (1994) developed a detailed, 3D numerical simulation of dynamic response of a short-span overpass bridge system and showed that even when structure remains elastic, the complete soil–structure system is highly nonlinear due to soil interaction. SSI effects on cable stayed bridges together with effects of foundation depth were investigated by Zheng (1995). Gazetas and Mylonakis (1998) and Mylonakis et al. (2006) emphasized importance of proper SSI analysis on response of bridges and provided important insight on failure of Hanshin Expressway bridge during Kobe earthquake. Small (2001) developed SSI models showing how use of simple spring models for the soil behavior could lead to erroneous result and recommended that their use should be discontinued. In addition to that, they showed that the type of structure and its stiffness could have an effect on the deformation of the foundation. Tongaonkar and Jangid (2003) investigated SSI effects on peak response of three-span continuous deck bridge seismically isolated by the elastomeric bearings and found that bearing displacements at abutment locations may be underestimated if the SSI effects are not considered. Chow and Hao (2005) studied the effect of spatial

variations of ground motion with different wave propagation apparent velocities in soft and medium stiff soil, and revealed significant SSI effects. In addition to that, it was found that non-uniform ground excitation effects are significant, especially when a big difference between the fundamental frequency of the bridge frames and the dominant frequencies of the ground motions exists. Soneji and Jangid (2008) analyzed influence of dynamic SSI on behavior of seismically isolated cable-stayed bridge and observed that the soil had significant effects on the response of the isolated bridge. In addition to that, inclusion of SSI was found to be essential for effective design of seismically isolated cable-stayed bridge, especially when the towers are very rigid and the soil is soft to medium stiff. Elgamal et al. (2008) performed a very advanced 3D analysis of a full soil-bridge system, focusing on interaction of liquefied soil in foundation and bridge structure.

In addition to studies showing importance of SSI analysis, beneficial (as suggested by the code) and possibly detrimental effects of SSI were analyzed in a number of studies. For example Kappos et al. (2002) found that there are advantages in including SSI effects in the seismic design of irregular R/C bridges as seismic forces are typically lower when SSI is included in the analysis. This conclusion nicely reinforces recommendation of NEHRP-94 seismic code, mentioned above. On the other hand, Jeremić et al. (2004) found that SSI can have either beneficial or detrimental effects on structural behavior and is dependent on the dynamic characteristics of the earthquake motion, the foundation soil and the structure. Main conclusion was that while in some cases SSI can improve overall dynamic behavior of structural system, there are many cases where SSI is detrimental to such overall seismic response of the soil-structure system. However, due to computational limitations, Jeremić et al. (2004) had to analyze soil-pile system separately from the structure, thus reducing modeling accuracy. Present paper significantly improves on modeling, treating complete earthquake-soil-bridge system as tightly coupled triad, where interacting components (dynamic characteristics of the earthquake, soil and the bridge structure) control seismic response.

Based on the above (limited) literature overview, it seems that importance of full SSI analysis is well established in the research community. Purpose of this paper is to present a methodology for high fidelity modeling of seismic soil-structure interaction. This is done in Section 2. Presented methodology relies on rational mechanics and aims to reduce modeling uncertainty, by employing currently best available models and simulation procedures. In addition to presenting such state-of-the-art modeling, simulation results are used to illustrate influence of non-uniform soils on seismic response of a prototype bridge system. A number of interesting and sometimes perhaps counter-intuitive results, given in Section 3, emphasize the need for a full, detailed SSI analysis for each particular Earthquake-Soil-Structure triad.

Analyzed bridge model represents prototype model that was devised as part of a grand challenge, pre-NEESR project, funded by NSF NEES program. Pre-NEESR project, titled "Collaborative Research: Demonstration of NEES for Studying Soil-Foundation-Structure Interaction" (PI Professor Wood from UT) brought together researchers from University of California at Berkeley, University of Texas at Austin, University of Nevada at Reno, University of Washington, University of Kansas, Purdue University and University of California at Davis, with the aim of demonstrating use of NEES facilities and use of existing and development of new simulations tools for studying SSI problems. Presented modeling, simulations and developed parallel simulations tools (used here and described by Jeremić and Jie (2007, 2008)) represented a small part of this large and ambitious project.

## 2. Model Development and Simulation Details

The finite element models used in this study have combined both solid elements, used for soils, and structural elements, used for concrete piles, piers, beams and superstructure. In this section described are material and finite element models used for both soil and structural components. In addition to that, described is the methodology used for seismic force application and staged construction of the model, followed by a brief description of a numerical simulation platform used for all simulations presented here.

### 2.1. Soil Model

Two types of soil were used in modeling. First type was based on soil found at the Capitol Aggregates site, a local quarry located south-east of Austin, Texas. This soil was chosen as part of modeling requirement for above mentioned pre-NEES project. Site characterization has been performed to collect information on soil by Kurtulus et al. (2005).

Based on stress-strain curve obtained from a triaxial test (Kurtulus et al., 2005), as shown in Figure 1(a), a nonlinear, kinematic hardening, elastic-plastic soil model has been developed using Template Elastic plastic framework (Jeremić and Yang, 2002). It should be noted that an isotropic hardening model would have been enough to fit monotonic lab test data. However, for cyclic loading, only kinematic hardening (in this case, rotational kinematic) is able to appropriately model Bauschinger effect. Developed model consists of a Drucker-Prager yield surface, Drucker-Prager plastic flow directions (potential surface) and a nonlinear Armstrong-Frederick (rotational) kinematic hardening rule (Armstrong and Frederick, 1966). Model calibration was performed using limited data set resulting in a very good match (see Figure 1(b)). Initial opening of a Drucker-Prager cone was set at approximately  $5^\circ$  only (in

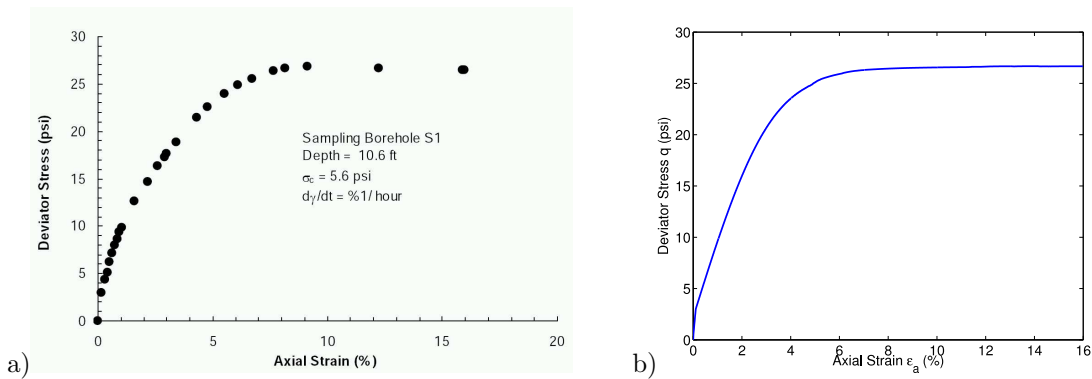


Figure 1. (a) Stress-strain curve obtained from triaxial compression test (b) Stress-strain curve by obtained by calibrated model (from Depth 10.6ft)

normal-shear stress space). This makes for a very sharp Drucker-Prager cone, with a very small elastic region (similar to Dafalias Manzari models Dafalias and Manzari (2004)). The actual deviatoric hardening is produced using Armstrong-Frederick nonlinear kinematic hardening with hardening constants  $a = 116.0$  and  $b = 80.0$ .

Second type of soil used in modeling was soft clay, Bay Mud). This type of soil was modeled using a total stress approach with an elastic perfectly plastic von Mises yield surface and plastic potential function. The shear strength for such (very soft) Bay Mud material was chosen to be  $C_u = 5.0$  kPa (Dames and Company, 1989). Since this soil is treated as fully saturated and there is not enough time during shaking for any dissipation to occur, elastic-perfectly plastic model provides enough modeling accuracy.

**Soil Element Size Determination** The accuracy of a numerical simulation of seismic wave propagation in a dynamic Soil-Structure-Foundation Interaction) (SFSI) problems is controlled by two main parameters Preisig (2005):

1. The spacing of nodes in finite element model  $\Delta h$
2. The length of time step  $\Delta t$ .

Assuming that numerical method converges toward exact solution as  $\Delta t$  and  $\Delta h$  tend toward zero, desired accuracy of solution can be obtained as long as sufficient computational resources are available.

In order to represent a traveling wave of a given frequency accurately about 10 nodes per wavelength are required. Fewer than 10 nodes can lead to numerical damping as discretization misses certain peaks of seismic wave. In order to determine appropriate maximum grid spacing the highest relevant frequency  $f_{max}$  that is present in model needs to be found by performing a Fourier analysis of input motion. Typically, for seismic analysis one can assume  $f_{max} = 10Hz$ . By choosing wavelength  $\lambda_{min} = v/f_{max}$ , where  $v$  is (shear) wave velocity, to be represented by 10 nodes, smallest wavelength that can still be captured with any confidence is  $\lambda = 2\Delta h$ , corresponding to a frequency of  $5f_{max}$ . The maximum grid spacing should therefore not be larger than

$$\Delta h \leq \frac{\lambda_{min}}{10} = \frac{v}{10f_{max}} \quad (1)$$

where  $v$  is smallest wave velocity that is of interest in simulation (usually wave velocity of softest soil layer).

In addition to that, mechanical properties of soil changes with (cyclic) loadings as plastification develops. In order to quantify those changes in soil stiffness, a number of laboratory and in situ tests were performed by Kurtulus et al. (2005). Moduli reduction curve ( $G/G_{max}$ ) and damping ratio relationship were then used to capture determine soil element size while taking into account soil stiffness degradation (plastification). Using shear wave velocity relation with shear modulus

$$v_{shear} = \sqrt{\frac{G}{\rho}} \quad (2)$$

one can readily obtain dynamic degradation of wave velocities. This leads to smaller element size required for detailed simulation of wave propagation in soils which have stiffness degradation (plastification). The addition of stiffness degradation effects (plastification) of soil on soil finite size are listed in Table I. Based on above soil finite element size determination, a three bent prototype finite element model has been developed and is shown in Figure 2. The model features 484,104 DOFs, 151,264 soil and beam-column elements, it is intended to model appropriately seismic waves of up to 10Hz, for minimal stiffness degradation of

Table I. Soil finite element size determination with shear wave velocity and stiffness degradation effects for assumed seismic wave with  $f_{max} = 10$  HZ, (minimal value of  $G/G_{max}$  corresponding to 0.2% strain level.)

Depth (ft)	Layer thick. (ft)	$v_s$ (fps)	$G/G_{max}$	$v_s^{min}$ (fps)	$\Delta h_{max}$ (ft)
0	1	320	0.36	192	1.92
1	1.5	420	0.36	252	2.52
2.5	4.5	540	0.36	324	3.24
7	7	660	0.36	396	3.96
14	7.5	700	0.36	420	4.20
21.5	17	750	0.36	450	4.50
38.5	half-space	2200	0.36	1320	13.20

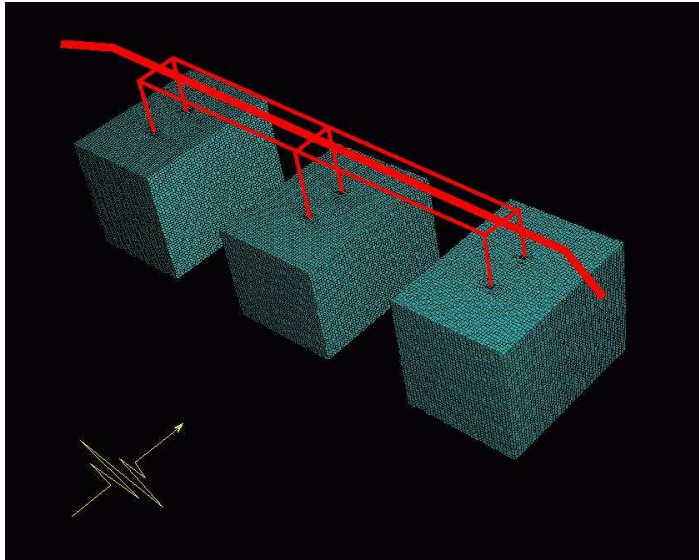


Figure 2. Detailed Three Bent Prototype SFSI Finite Element Model, 484,104 DOFs, 151,264 Elements.

$G/G_{max} = 0.08$ , maximum shear strain of  $\gamma = 1\%$  and with maximal element size  $\Delta h = 0.3$  m. It is noted that even larger set of models was created, that was able to capture 10 Hz motions, for  $G/G_{max} = 0.02$ , and maximum shear strain of  $\gamma = 5\%$ . This (our largest to date) set of models features over 1.6 million DOFs and over half a million finite elements. However, results from this very detailed model were almost same as results for model with half a million DOFs (484,104 to be precise) and it was decided to continue analysis with this smaller model. However, development of this more detailed model (featuring 1.6 million DOFs), that did not add much (anything) to our results brings another very important issue. It proves very important to develop a hierarchy of models that will, with refinement, improve our simulations. When model refinements (say mesh refinement) does not improve simulation results any more

(there is no observable difference), model can be considered mature (Oberkamp et al., 2002) and no further refinement is necessary. This maturation of model allows us use of immediate lower level (lower level of refinement) model for production simulations. It is therefore always advisable to develop a hierarchy of models, and to potentially settle for model that is one level below the most detailed model. This most detailed models is chosen as model which did not improve accuracy of simulation significantly enough to warrant its use. For our particular example, the most detailed model used, did not improve results (displacements, moments...) significantly (actually it almost did not change them at all) implying that accurate modeling of frequencies up to 10 Hz for this Earthquake-Soil-Structure system did not affect seismic response.

**Time Step Length Requirement** The time step  $\Delta t$  used for numerically solving nonlinear vibration or wave propagation problems has to be limited for two reasons (Argyris and Mlejnek, 1991). The stability requirement depends on time integration scheme in use and it restricts the size of  $\Delta t = T_n/10$ . Here,  $T_n$  denotes smallest fundamental period of the system. Similar to spatial discretization,  $T_n$  needs to be represented with about 10 time steps. While accuracy requirement provides a measure on which higher modes of vibration are represented with sufficient accuracy, stability criterion needs to be satisfied for all modes. If stability criterion is not satisfied for all modes of vibration, then the solution may diverge. In many cases it is necessary to provide an upper bound to frequencies that are present in a system by including frequency dependent damping to time integration scheme.

The second stability criterion results from the nature of finite element method. As a wave front progresses in space, it reaches one point (node) after the other. If time step in finite element analysis is too large, than wave front can reach two consecutive points (nodes) at the same time. This would violate a fundamental property of wave propagation and can lead to instability. The time step therefore needs to be limited to

$$\Delta t < \frac{\Delta h}{v} \quad (3)$$

where  $v$  is the highest wave velocity. Based on values determined in Table I, time step requirement can be written as

$$\Delta t < \frac{\Delta h}{v} = 0.00256s \quad (4)$$

thus limiting effective time step size used in numerical simulations of this particular soil-structure model.

## 2.2. Structural Model

The nonlinear structure model (piers and superstructure) used in this study were initially developed by Fenves and Dryden (2005). Model calibration was performed using experimental data from UNR shaking table tests (Dryden and Fenves, 2008). The original model was developed with fully fixed conditions at the base of piers. This choice of boundary conditions influences location of possible plastic hinges in piers. This is important as model predetermines location of plastic hinges by placing zero-length elements at bottom and top of piers. In reality, piers extend into piles and possible plastic hinges might form below ground surface in piles as well as at top of piers, and not necessarily at bottom of piers. The structural model was

subsequently updated to reflect this more realistic condition. In addition to that, beam elements used for piles were modeled using nonlinear fiber beam element which allows for development of (distributed) plastic hinges.

**Concrete Modeling.** Concrete material was modeled using Concrete01 uniaxial material as available in OpenSees framework and is fully described by Fenves and Dryden (2005); Dryden (2006). Basic description is provided here for completeness. Concrete model is based on work by Kent and Park (1971) and includes linear unloading/reloading stiffness that degrades with increasing strain. No tensile strength is included in the model. The peak strength for unconfined concrete model is based on test of concrete cylinders performed at UNR. Material model parameters used for unconfined concrete in simulation models are  $f'_{co} = 5.9$  ksi,  $\epsilon_{co} = 0.002$ ,  $f'_{cu} = 0.0$  ksi, and  $\epsilon_{cu} = 0.006$ , while material parameters for confined concrete used are  $f'_{co} = 7.5$  ksi,  $\epsilon_{co} = 0.0048$ ,  $f'_{cu} = 4.8$  ksi, and  $\epsilon_{cu} = 0.022$ .

**Steel Modeling.** Hysteretic uniaxial material model available within OpenSees framework used to model response of steel reinforcement to match the monotonic response observed during the steel coupon tests. Parameters included in this model are  $F_1 = 67$  ksi,  $\epsilon_1 = 0.0023$ ,  $F_2 = 92$  ksi,  $\epsilon_2 = 0.028$ ,  $F_3 = 97$  ksi, and  $\epsilon_3 = 0.12$ . No allowance for pinching or damage under cyclic loading has been made ( $pinchX = pinchY = 1.0$ ,  $damage1 = damage2 = 0.0$ ,  $beta = 0$ ).

**Pier and Pile Modeling** The finite element model for piers and piles features a nonlinear fiber beam-column element (Spacone et al., 1996). In addition to that, a zero-length elements is introduced at top of piers in order to capture effect of rigid body rotation at joints due to elongation of anchored reinforcement.

Cross section of both piers and piles was discretized using  $4 \times 16$  subdivisions of core and  $2 \times 16$  subdivisions of cover for radial and tangential direction respectively. Additional deformation that can develop at the upper pier end results from elongation of steel reinforcement at beam-column joint with superstructure. To model this phenomenon, a simplified hinge model is developed (Mazzoni et al., 2004). In that model, bar slip occurs in two modes: elongation due to variation in strain along length of anchored bar resulting from bond to surrounding concrete, and rigid body slip of bar that is resisted by friction from surrounding concrete. A bi-uniform bond stress distribution was assumed along length of anchored bar based on simplified model developed by Lehman and Moehle (1998). Two sets of parameters were considered for this bond stress distribution, namely  $u_e = 12\sqrt{f'_c}$  and  $u_e = 6\sqrt{f'_c}$  for assuming bond stress distribution, and  $u_e = 8\sqrt{f'_c}$  and  $u_e = 6\sqrt{f'_c}$  determined based on strain gauge data from tests. First set is based on recommendations given by Lehman and Moehle (1998) while second set is based on a calibration done by Ranf (2006) to match bond stress distribution to strain gauge data recorded along length of anchored reinforcement during shaking table tests at UNR.

**Bent Cap** The bent cap beams are modeled as linear elastic beam-column elements with geometric properties developed using effective width of cap beam and reduction of its stiffness due to cracking. The cap beams at all bents are assumed to have a depth of 15 in. and an effective width of 15 in. The effective width is selected based on observation that the amount of longitudinal reinforcement outside this effective width is small. A reduction factor is applied



to gross stiffness to account for cracking in a member. Based on recommendations of Seismic Design Criteria (SDC) developed by Caltrans (2004), value of this reduction factor is selected within the range of 0.5-0.75, where 0.5 corresponds to a lightly-reinforced section, and 0.75 corresponds to a heavily-reinforced section.

**Superstructure** The superstructure consists of prismatic prestressed concrete members, which are prestressed in both longitudinal and transverse directions. Each segment of superstructure is modeled with a linear elastic beam-column element. No stiffness reduction has been done for these elements in accordance with recommendations of SDC. In addition to that, no reduction of torsional moment of inertia is done since this bridge meets Ordinary Bridge requirements of SDC (Caltrans, 2004). Superstructure ends were left free, as it was assumed that structure was disconnected from approach abutments.

### 2.3. Coupling of Structural and Soil Models

In order to create a model of a complete soil-structure system, it was necessary to couple structural and soil (solid) finite elements. Figure 3 shows schematically how was this coupling performed, between bridge foundation (piles) and surrounding soil. The volume that would

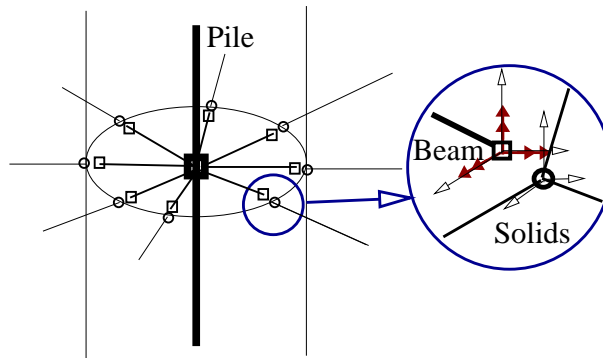


Figure 3. Schematic description of coupling of structural elements (piles) with solid elements (soil).

be physically occupied by pile is left open within solid mesh that models foundation soil. This opening (hole) is excavated during a staged construction process (described later in section 2.6). Beam-column elements (representing piles) are then placed in middle of this opening. Beam-column elements representing pile are connected to surrounding solid (soil) elements by means of stiff short elastic beam-column elements. These short "connection" beam-column elements extend from each pile beam-column node to surrounding nodes of solids (soil) elements. The connectivity of short, connection beam-column element nodes to nodes of soil (solids) is done only for translational degrees of freedom (three of them for each node), while rotational degrees of freedom (three of them) from beam-column element are left unconnected. Connecting piles to soil using above described method has a number of advantages and disadvantages. On a positive side, geometry of soil-pile system is modeled very accurately. A thin layer of elements next to pile is used to mimic frictional behavior soil-

pile interface. In addition to that, deformation modes of a pile (axial, bending, shearing) are accurately transferred to surrounding soil by means of connection beam–column elements. In addition to that, both pile and soil are modeled using best available finite elements (nonlinear beam–column for pile and elastic–plastic solids for soil). On a negative side, discrepancy of displacement approximation fields between pile ( a nonlinear beam–column) and soil (a linear solid brick elements) will lead to incompatibility of displacements between nodes of pile–soil system. However, this incompatibility was deemed acceptable in view of advantages described above.

#### *2.4. Domain Reduction Method*

Seismic ground motions were applied to finite element model of SSI system using Domain Reduction Method (Bielak et al., 2003; Yoshimura et al., 2003). The DRM represents the only method that can consistently (analytically) apply free field ground motions to finite element model. The method features a two-stage strategy for complex, realistic three dimensional earthquake engineering simulations. The first is an auxiliary problem that simulates earthquake source and propagation path effects with a model that encompasses source and a background structure from which soil–structure system has been removed. The second problem models local, soil-structure effects. Its input is a set of equivalent forces (so called effective forces) derived from the first step. These forces act only within a single layer of elements adjacent to interface between exterior region and geological feature of interest. While DRM allows for application of arbitrary, 3D wave fields to the finite element model, in this study a vertically propagating wave field was used. Given surface, free field ground motions were de-convoluted to a depth of 100 m. Then, a vertically propagating wave field was (re-) created and used to create effective forces for DRM (Bielak et al., 2003; Yoshimura et al., 2003). Deconvolution and (back) propagation of vertically propagating wave field was performed using closed form solution as implemented in Shake program (Idriss and Sun, 1992).

#### *2.5. Time Integration*

Numerical integration of equations of motion was done using Hilber-Hughes-Taylor (Hilber et al., 1977; Hughes and Liu, 1978a,b) algorithm. Proper algorithmic treatment for nonlinear analysis follows methodology described by Argyris and Mlejnek (1991). No Rayleigh damping was used here, and modeling completely relies on displacement proportional damping (Argyris and Mlejnek, 1991), provided by elastic–plastic behavior of material (soil, piles and structure) while small amount of numerical damping was used to damp out response in higher frequencies that are introduced by spatial finite element discretization (Hughes, 1987).

#### *2.6. Staged Simulations*

A very important modeling issue is that of staged construction. Initial state of stress in soil significantly affects its response. Similar observation can be made about concrete structures as well.

In general, nonlinear finite element analysis can be split up in three nested loops (levels). This is true for both geometric and/or material nonlinear finite element analysis (Felippa, 1993). Top loop comprises load stages, which represent realistic loading sequence on solids and structures. Within loading stage loop is an incremental loading loop, which splits load in each

stage into increments. Split into increments is not only important from numerical stability standpoint, but is also essential from proper modeling of elastic-plastic materials. Within each increment, equilibrium iterations are advisable but not necessary for advancement of solution. Simulations presented in this studies were performed in three main stages, number of increments and equilibrium iterations.

**Soil Self-Weight Stage.** During this stage, finite element model for soil (only, no structure) is loaded with soil self-weight. The finite element model for this stage excludes any structural elements, and opening (hole) where the pile will be placed, is full of soil. Displacement boundary conditions on sides of three soil blocks are allowing vertical displacement, and allow horizontal in boundary plane displacement, while they prohibit out of boundary plane displacement of soil. All displacements are suppressed at bottom of all three soil blocks (for a model shown in Figure 2). The soil self weight is in our case applied in 10 incremental steps. While such small number of steps is not advisable in general, initial finite element model was simple enough (three soil blocks without any interactions between them) that only ten increments of load were sufficient to obtain initial state of stress, strain and internal variables for soil.

**Piles, Columns and Superstructure Self-Weight Stage.** In this, second stage, number of modeling changes happen to occur. Firstly, soil elements where piles will be placed are removed (excavated), then concrete piles (beam-column elements) are placed in holes (while appropriately connecting structural and solids degrees of freedom, see section 2.3), columns are placed on top of piles and finally superstructure is placed on top of columns. All of this construction is done at once. With all components in place, self weight analysis of piles-columns-superstructure system is performed. Ideally, it would have been better to perform "construction" process in few stages, but even by adding all structural elements at once and performing their self weight analysis in (this) one stage (using 100 increment of load) an accurate initial state of section forces (stress) and deformation (strains) has been obtained for prototype bridge model.

**Seismic Shaking Stage.** The last stage in our analysis consists of applying seismic shaking, by means of effective forces through use of DRM. It is important to note that seismic shaking is applied to already deformed model, with all stresses, internal variables and deformation that resulted from first two stages of loading.

### 2.7. Simulation Platform

Numerical simulations described in this paper were done using a parallel computer program based on Plastic Domain Decomposition (PDD) method (Jeremić and Jie, 2007, 2008). Program was developed using a number of publicly available numerical libraries. They are briefly described below. Graph partitioning is achieved using ParMETIS libraries (Karypis et al., 1998)). Parts of OpenSees framework (McKenna, 1997) were used to connect the finite element domain. In particular, Finite Element Model Classes from OpenSees (namely, class abstractions Node, Element, Constraint, Load, Domain and set of Analysis classes) were used to describe finite element model and to store results of analysis performed on a model. An excellent adoption of Actor model (Hewitt et al., 1973; Agha, 1984) and addition of a Shadow, Chanel, MovableObject, ObjectBroker, MachineBroker classes within OpenSees framework

(McKenna, 1997) also provided an excellent basis for our development. On a lower level, a set of Template3Dep numerical libraries (Jeremić and Yang, 2002) were used for constitutive level integrations, nDarray numerical libraries (Jeremić and Sture, 1998) were used to handle vector, matrix and tensor manipulations, while FEMtools element libraries from UCD CompGeoMech toolset (Jeremić, 2004) were used to supply other necessary libraries and components. Parallel solution of system of equations has been provided by PETSc set of numerical libraries (Balay et al., 2001, 2004, 1997)). Large part of simulation was carried out on our local parallel computer GeoWulf. Only the largest models (too big to fit on GeoWulf system) were simulated on TeraGrid machine at SDSC and TACC.

### 3. Seismic Simulation Results

Effects of varying soil conditions under prototype soil–bridge system are main focus of this study. Since one of concerns with prototype bridge structure was the effect higher frequencies have on bridge subcomponents (like short structural elements...), Northridge (1994) earthquake motions at Century City were chosen for this study (shown in Figure 4). This particular ground motion contains frequencies that were deemed potentially detrimental to parts of the structure. Ground motions were propagated in a vertical direction, and were polarized in a plane transversal to main bridge axes. That is, incident motions are perpendicular to main bridge axis, thus exciting mainly transversal motions of bridge structure. Such transversal motions put highest demand on foundation–bridge system, particularly in non–uniform soils. It should be noted that DRM can be used to apply any 3D seismic motions to soil–foundation–bridge (SFB) system, however for this study 1D, vertically propagating motions were chosen for analysis. It is also important to note that while 1D vertically propagating transversal motions were used as input for DRM, SFB system has been subjected to a full 3D motions, as difference in soil conditions, difference in pier length and difference in soil profile depth, create conditions for incoherence, which results in full 3D, incoherent bridge input motions (including longitudinal and vertical components).

#### 3.1. Simulation Scenarios

In order to investigate effects variable soil condition beneath bridge bents have on seismic behavior a parametric study was performed. Base soil finite element model (to which structural components are added in loading stage 2, as described in section 2.6) is shown in Figure 5. Boundary conditions for first and second loading stages (soil self weight and structure construction/self weight) are full support at the bottom of a model, while vertical sides are allowed to slide down but not to displace perpendicular to the plane. For a dynamic loading stage, applied using DRM, and due to analytic nature of DRM, support condition outside single layer of elements that is used to apply effective DRM forces will not have much effect on behavior of the model (Bielak et al., 2003; Yoshimura et al., 2003). Nevertheless, minimal amount of support is needed to remove rigid body modes for a model. In our case, for this particular stage of loading (seismic), same support conditions were used as for the first two stages.

Soil beneath each of the bents was varied between stiff sand and soft clay (models for which were described in section 2.1) There were total of 8 soil condition scenarios, as described in

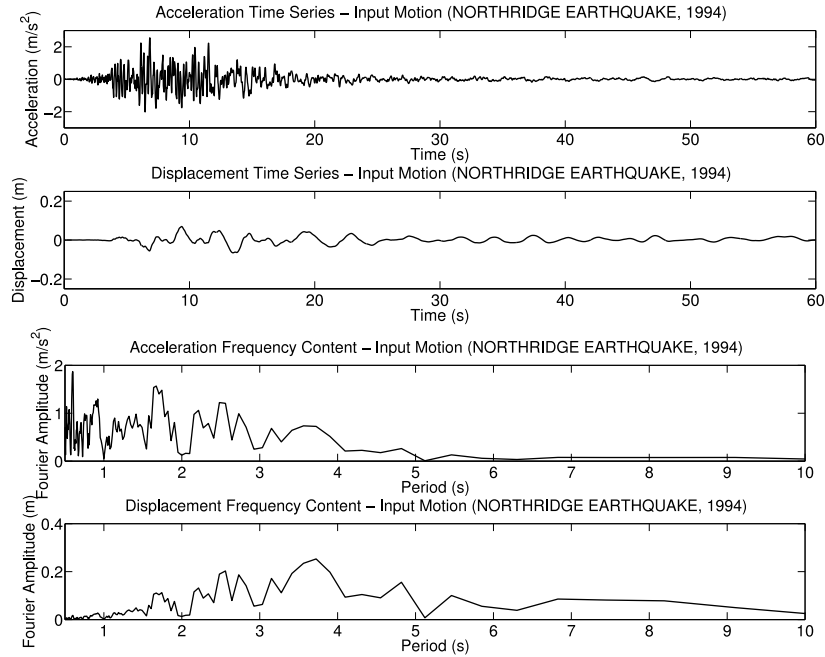


Figure 4. Input Motion - Century City, Northridge Earthquake 1994

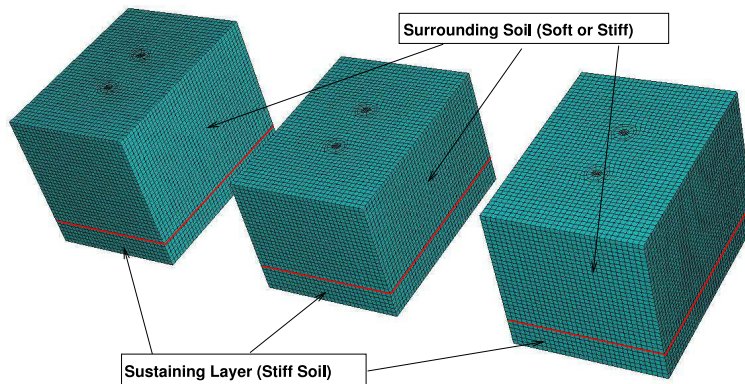


Figure 5. Finite Element Model for 3 Bent Prototype Bridge System

Table II. It is important to note that in each of eight cases, there was a stiff soil layer at the

Table II. Simulation scenarios for prototype soil–bridge system study

Simulation Cases	Soil Block 1	Soil Block 2	Soil Block 3
Case 1 (SSS)	Stiff Sand	Stiff Sand	Stiff Sand
Case 2 (SSC)	Stiff Sand	Stiff Sand	Soft Clay
Case 3 (SCS)	Stiff Sand	Soft Clay	Stiff Sand
Case 4 (SCC)	Stiff Sand	Soft Clay	Soft Clay
Case 5 (CSS)	Soft Clay	Stiff Sand	Stiff Sand
Case 6 (CSC)	Soft Clay	Stiff Sand	Soft Clay
Case 7 (CCS)	Soft Clay	Soft Clay	Stiff Sand
Case 8 (CCC)	Soft Clay	Soft Clay	Soft Clay

base of piles. This stiff layer was needed in order to provide enough carrying capacity for cases where soft clay was used beneath bridge bents.

### 3.2. Displacement Response

The study presented here produced a very rich dataset. While space restriction prevents us from showing all results, a subset of results is used to emphasize main findings related to effects of soil variability on seismic response of the soil–bridge system. In order to illustrate main findings, results for top of the first (left most) bent and top of first soil block (next to pile/pier) are used. Figure (6) shows displacement time histories for that first bent and soil block, for all eight cases. In addition to that, Figure (7) shows displacement response spectra for the same, first bent and soil block. A number of observations can be made.

The first observation is that displacement time histories for bent and soil response are quite a bit different from each other. These differences are present irrespective of what are local soil condition beneath bent 1. For example, Figure 6 shows that even for four cases when soil beneath observed bent number 1 is stiff (all full lines) or soft (all dot–dashed lines), response is very variable. This is true for both displacement time histories of top of bent and for top of soil (near pier/pile connection). Both amplitude (Figure (6)) and frequency (Figure (7)) show great variability. This is particularly true for structural response, while soil response (lower set of time histories in Figure (6)) is somewhat less variable. Smaller variability of soil is understandable as dynamic characteristic of soil at soil block 1 is very much dependent on its own stiffness, while structure is significantly affected by soil conditions at other soil block as well.

Another important observation is that free field motions (that are often used as input motions for structural only models) that are shown in lower part of Figures (6) and (7) are also quite a bit different than what is actually recorded (simulated) at same location with structure present. In particular, displacement time history of free field motions (seen as gray line in Figure (6)) has significantly lower amplitudes. In some cases (for example Clay–Sand–Clay (CSC) case) free field amplitude is only half of what is observed beneath the structure. This difference between free field motions and the ones observed (simulated) with SSI effects is also obvious in displacement response spectra in Figure (7). The difference between free field motions and the ones observed (simulated) is even more apparent for two cases with all stiff

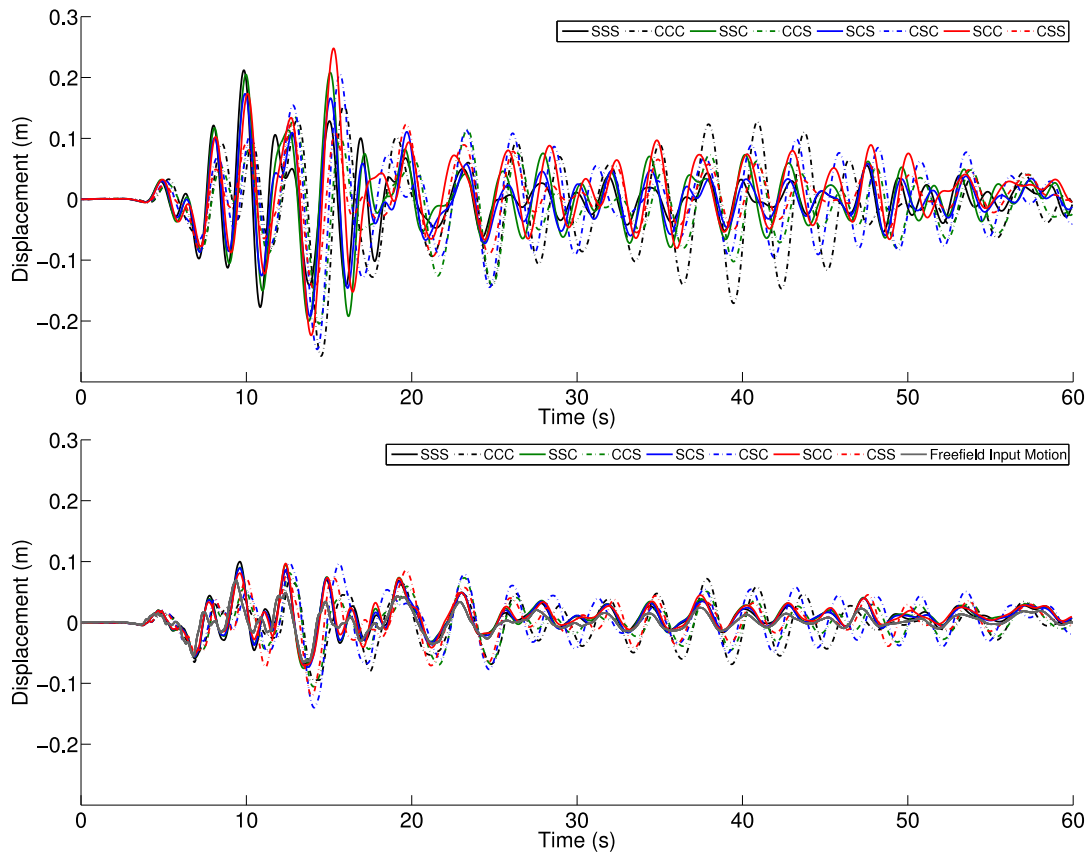


Figure 6. Simulated Displacement Time Series, Northridge 1994, Century City input motions, Comparison of Eight Cases (upper: structure, top of bent # 1; lower: top of soil block, next to pier/pile # 1).

soil (Case 1, SSS) and all soft soil (Case 8, CCC). Such comparison is shown for the first 25 seconds of shaking in Figure (8). It is interesting to note a very large discrepancy of free field motions with those that were simulated in soil under the structure, as shown in Figure (8). This discrepancy emphasizes the importance of soil–structure interaction (SSI) on response of bridge and other infrastructure objects. It is also apparent from displacement response spectra (Figure 7) that amplification of ground motions (those measured with SSI effects) can be significant. In some cases (for example for Sand–Clay–Clay (SCC) case), amplification at period of 2.5 seconds is almost five.

It is also very interesting to observe that response of structure in stiffer soil is much larger than response of structure founded on soft soil. This might seem to contradict usual assumption that structures founded on soft soils are much more prone to large deformation, and subsequently more damage. This particular amplification of response for structure founded on stiff soil is due to (positive, amplifying) interaction of dynamic characteristics of earthquake,

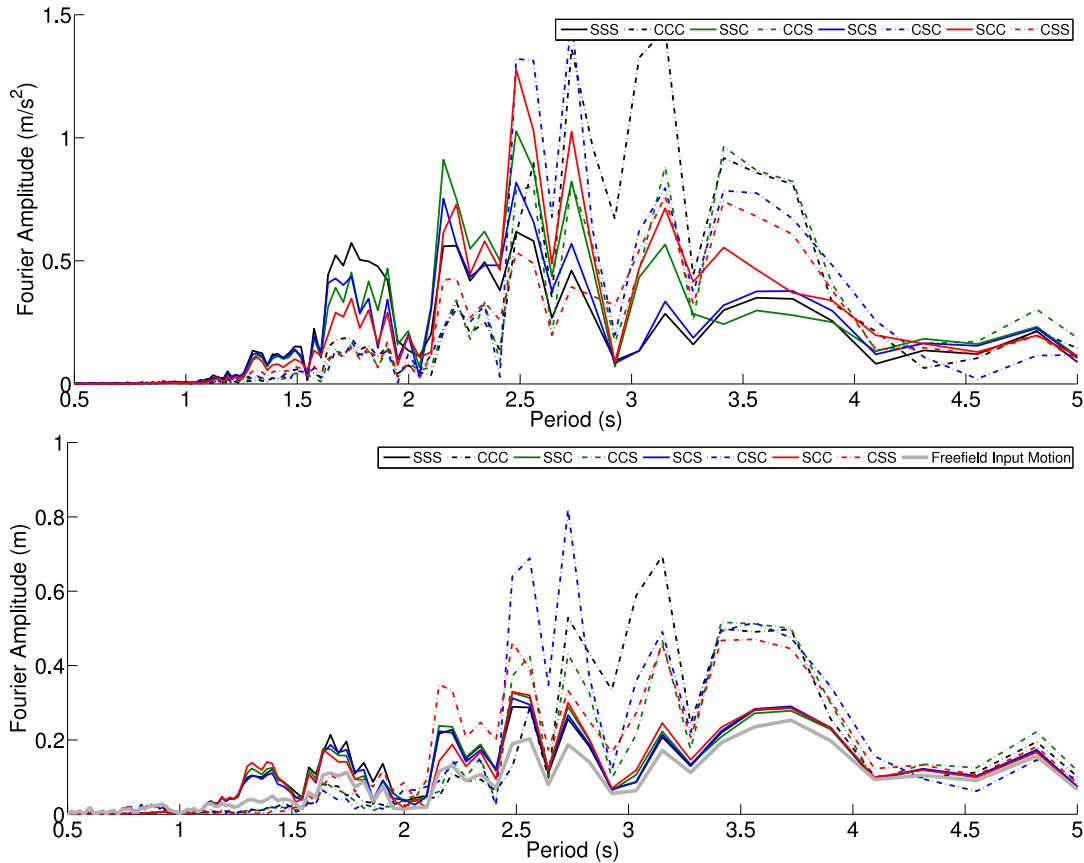


Figure 7. Simulated Displacement Response Spectra, Northridge 1994, Century City input motions, Comparison of Eight Cases (upper: structure, top of bent # 1; lower: top of soil block, next to pier/pile # 1).

stiff soil and stiff structure (both will have natural periods in somewhat higher frequency range). Positive interaction of dynamic characteristics of earthquake, soil and structure are producing larger amplifications and are thus detrimental to behavior of the structural system. Similar observations can be made about responses of other bents and soil blocks.

### 3.3. Bent Bending Moments

Time domain bending moment history, recorded at top of bent number 1, shown in Figure (9) presents another set of interesting results. It is important to note that bending moments presented in Figure 9 are resulting for seismic shaking stage of loading, which comes after previous structural self weight stage. The effects of previous loading stage is observed as initial bending moment of approximately  $\pm 750$  kNm.

A number of interesting observation can be made. First observation is that, similar to displacement results, bending moments are quite variable for different local soil conditions.



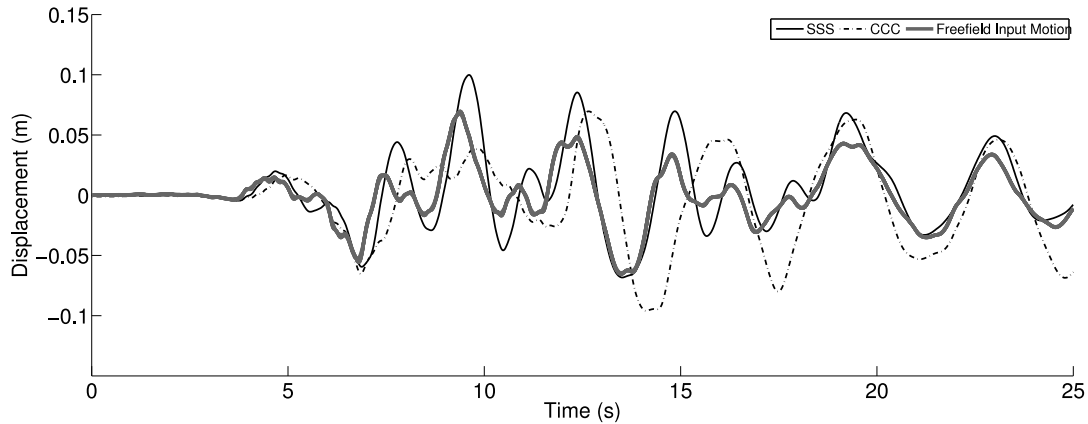


Figure 8. Simulated Displacement Time Series, Northridge 1994, Century City input motions, Comparison of Two Cases (SSS and CCC) - First 25s (Soil Block 1)

This variability is expected as support condition beneath bents change with changes in local soil, however, what is striking is that the magnitude of variability is quite large. For example, moment differences at 10 seconds (time of highest moments for most cases, see Figure (9)) between cases SCC ( $M = 3,750 \text{ kN/m}^2$ ) and CSS ( $M = 750 \text{ kN/m}^2$ ) are on the order of 5 times.

It is also noted that in some, but not all, cases top of piers reach yield plateau. This is observed for a number of response curves between 8 s and 17 s marks. It is important to note that these yielding points are reached first by structure that is founded on stiff soil. This observation accentuates previous observation about larger displacements in those cases with stiff soil underneath bridge. There are two main reasons this attraction of large moments happens. First, when soil is stiffer beneath bridge bent number 1, that bent attracts more forces, since bridge bent – pile – soil in foundation system is stiffer than other bents (all or some of them). This force attraction naturally results as stiff components of bridge system will resist more seismic demand from earthquake motions, and will thus produce stronger shaking and larger moments at bent number 1. Secondly, for an earthquake with fairly short period motions, combined with a fairly stiff structure and a stiff soil (sand), bent number 1 might be experiencing condition close to resonance. Occurrence of (or close to) resonance amplifies response (motions and bending moments) significantly and contribute to observed stronger response. The only reason resonance is not amplifying response even more is that displacement proportional damping (through elasto-plastic behavior of soil, pile and structure, cf. (Argyris and Mlejnek, 1991)) is dissipating enough seismic (input) energy thus damping out motions.

Figure (10) is used to emphasize the effect of uniform sand (stiff) and clay (soft) soil on bending moments in one of piers of bent number 1. The absolutely largest damage (longest occurrence of plastic bending moment) is observed for a case where bridge is founded on stiff soil (SSS case). For this case, interplay of dynamic characteristics of earthquake, soil and structure creates close to resonance condition, thus amplifying response. On the other hand, when bridge is founded on soft clay, response is smaller and in fact, only partial yielding occurs at time  $t = 14\text{s}$ . These results emphasize the importance of ESS interaction and can be used as

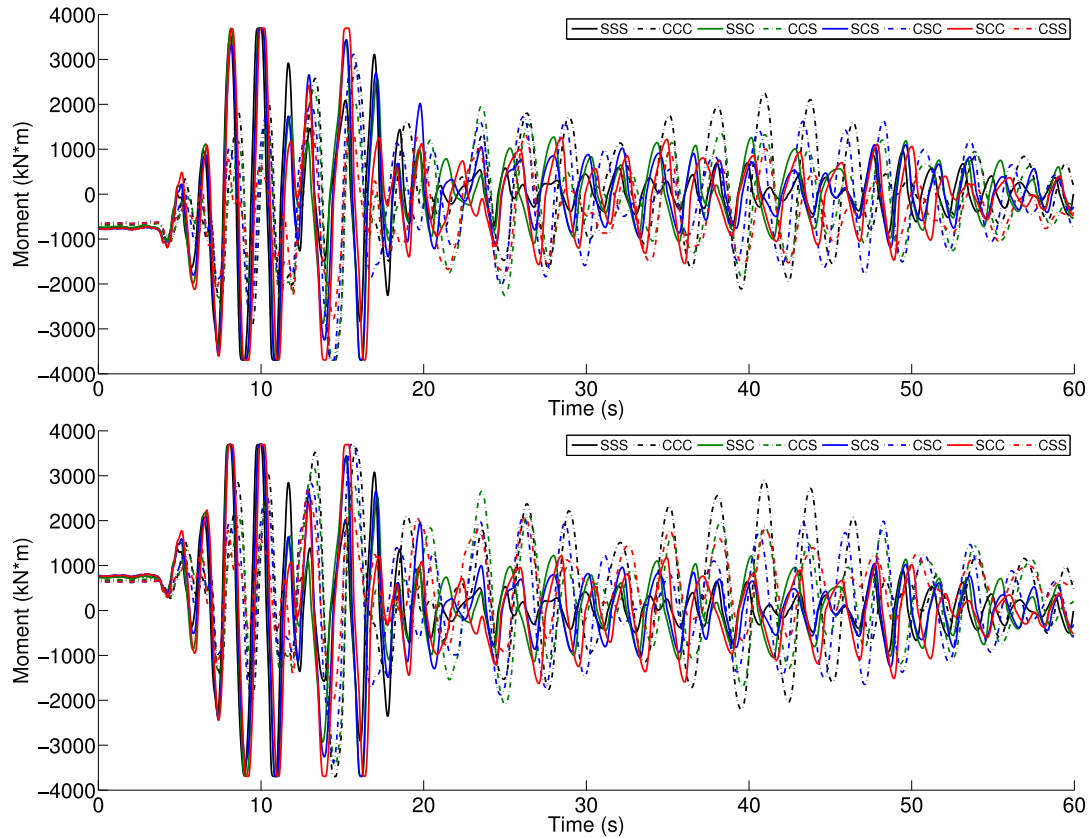


Figure 9. Simulated Maximum Moment Time Series, Northridge 1994, Century City input motions, Comparison of Eight Cases (Structure Bent 1, upper: column # 1; lower: column # 2).

counterargument against usually made claim that bridges founded in soft soils are experiencing more damage than those founded in stiff soils during seismic loading.

In addition to dynamic effects discussed above, bending moments are redistributed as a result of partial or full yielding (plastic hinge formation) during main shaking event. This is observed toward the end of bending moment time history, as bending moment results tend to oscillate around zero moment instead around their respective initial values ( $\pm 750$  kNm). This redistribution of bending moments becomes important as it indicates change of structural system for bent and thus a bridge structure. That is, at the beginning of loading stage 3 (seismic shaking) bent behaves as monolithic, full moment bearing frame. After seismic shaking and consequent yielding (formation of full or partial plastic hinges) monolithic structural system has changed to a couple of piers (consoles) with a simple beam on top, representing cross beam. This change of structural system might significantly affect response of bridge system during next seismic event or aftershock. In addition to that, dynamic characteristics of foundation soil during future seismic events (aftershocks or new earthquakes) might change as soil might have become denser and thus stiffer.

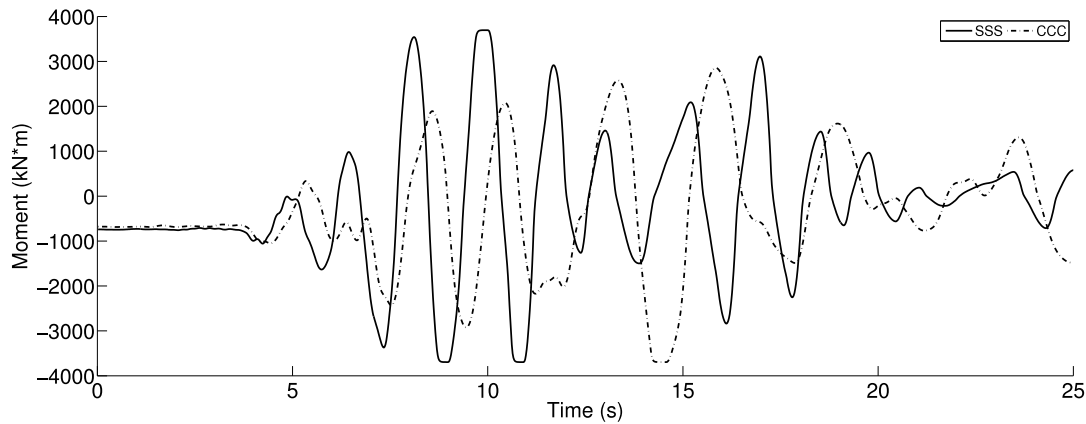


Figure 10. Simulated moment time series, Northridge 1994, Century City input motions, comparison of stiff sand (SSS) and soft clay (CCC) cases – First 25s (bridge bent number 1).

#### 4. Summary and Conclusions

Presented in this paper was simulation methodology and numerical investigation of seismic soil–structure interaction (SSI) for bridge structure on variable soil. Number of high fidelity models were developed and used to assess the influence of SSI on response of a prototype soil–bridge system. Detailed description was provided of high fidelity modeling approach that emphasizes low modeling uncertainty. Number of interesting conclusions were made. First, SSI effects cannot be neglected and should be modeled and simulated as much as possible. This observation becomes even more important as it seems that a triad of dynamic characteristic of earthquake, soil and structure (ESS) plays crucial role in determining the seismic behavior of infrastructure objects. In addition to that, results show that stiffer soil does not necessarily help seismic behavior of the structure. This emphasizes above mentioned interplay of ESS as the main mechanism that ultimately controls seismic performance of any object. It is also important to observe that nonlinear response of the soil–structure system changes the dynamics characteristics of its components where soil might become denser, stiffer, while the structure might become softer. This change, that is happening during shaking, might significantly affect the response of soil–structure system for possible aftershocks and future seismic events. On a final note, since tools (both software and hardware) are available and affordable, it is our hope that professional practice will use the opportunity and start using advanced, detailed models in assessing seismic performance of infrastructure objects in order to make them safer

and more economical.

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