Today is Tuesday, November 12th

Distribution of Means and the Central Limit Theorem

Pages 108-112

Homework Notification

Due Today:

Hwk 4.2 Normal CDF and Inverse Normal

Assigned: Friday, November 8th

Assigned Today:

No Assignments were assigned

Due Tomorrow:

Hwk 4.2 Normal CDF and Inverse Normal

Due Tuesday, November 12th

Answers are provided so that you can check for understanding. Please do not just copy them. If you need help, ask.

Highly Recommended:

- You can begin working on Death Packet Problems 56-66
- You should be working on Death Packet Problems 46-56
- You should be finishing Death Packet Problems 28-45
- Death Packet Problems 1-27 should be complete

Name:__________________________ **HWK 4.2 FRQ Distributions** **Period:**_____

Due Friday, November 15th

1. **1998 Question 1** Consider the sampling distribution of a sample mean obtained by a random sampling from an infinite population. This population has a distribution that is highly skewed toward the large values.
   (a) How is the mean of the sampling distribution related to the mean of the population?
   (b) How is the standard deviation of the sampling distribution related to the standard deviation of the population?
   (c) How is the shape of the sampling distribution affected by the sample size?

2. **2004B Number 3** Trains carry bauxite ore from a mine in Canada to an aluminum process plant in northern New York state in hopper cars. Filling equipment is used to load ore into the hopper cars. When functioning properly, the actual weights of ore loaded into each car by the filling equipment at the mine are approximately normally distributed with a mean of 70 tons and a standard deviation of 0.9 ton. If the mean is greater than 70 tons, the load mechanism is overfilling.

   (a) If the filling equipment is functioning properly, what is the probability that the weight of the ore in a randomly selected car will be 70.7 tons or more? Show your work.
   (b) Suppose that the weight of the ore in a randomly selected car is 70.7 tons. Would that fact make you suspect that the loading mechanism is overfilling the cars? Justify your answer.
   (c) If the filling equipment is functioning properly, what is the probability that a random sample of 10 cars will have a mean ore weight of 70.7 tons or more? Show your work.
   (d) Based on your answer in part (c), if a random sample of 10 cars had a mean ore weight of 70.7 tons, would you suspect that the loading mechanism was overfilling the cars? Justify your answer.

3. **2017 Question 3** A grocery store purchases melons from only two distributors, J & K. Distributor J provides melons from organic farms. The distribution of diameters of the melons from Distributor J is approximately normal with mean 133 millimeters (mm) and standard deviation 5 mm.

   (a) For a melon selected at random from Distributor J, what is the probability that the melon will have a diameter greater than 137 mm?

   Distributor K provides melons from nonorganic farms. The probability is 0.8413 that a melon selected at random from Distributor K will have a diameter greater than 137mm. For all the melons at the grocery store, 70 percent of the melons are provided by Distributor J and 30 percent are provided by Distributor K.

   (b) For a melon selected at random from the grocery store, what is the probability that the melon will have a diameter greater than 137mm?

   (c) Given that a melon selected at random from the grocery store has a diameter greater than 137mm, what is the probability that the melon will be from distributor J?

4. 2016 Problem Number 4 A Company manufactures model rockets that require igniters to launch. Once an igniter is used to launch a rocket, the igniter cannot be reused. Sometimes an igniter fails to operate correctly, and the rocket does not launch. The company estimates that the overall failure rate, defined as the percent of all igniters that fail to operate correctly, is 15 percent.
A company engineer develop a new igniter, called the super igniter, with the intent of lowering the failure rate. To test the performance of the super igniters, the engineer uses the following process.

Step 1: One super igniter is selected at random and used in a rocket.

Step 2: If the rocket launches, another super igniter is selected at random and used in a rocket.

Step 2 is repeated until the process stops. The process stops when a super igniter fails to operate correctly or 32 super igniters have successfully launched rockets, whichever comes first. Assume that super igniter failures are independent.

(a) If the failure rate of the super igniters is 15 percent, what is the probability that the first 30 super igniters selected using the testing process successfully launch rockets?
(b) Given that the first 30 super igniters successfully launch rockets, what is the probability that the first failure occurs on the thirty-first of thirty-second super igniter tested if the failure rate of the super igniters is 15 percent.
(c) Given that the first 30 super igniters successfully launch rockets, is it reasonable to believe that the failure rate of the super igniters is less than 15 percent? Explain.
(d) Assume the failure rate for the rocket igniters is 15% and the company decides to fire the rocket 30 times and then count the number of failures. What is the likelihood that there are 6 to 15 failures inclusive?

5. 2014 Question 3 Schools in a certain state receive funding based on the number of students who attend the school. To determine the number of students who attend a school, one school day is selected at random and the number of students in attendance that day is counted and used for funding purposes. The daily number of absences at High School A in the state is approximately normally distributed with mean of 120 students and standard deviation of 10.5 students.

(a) If more than 140 students are absent on the day the attendance count is taken for funding purposes, the school will lose some of its state funding in the subsequent year. Approximately what is the probability that High School A will lose some state funding?

(b) The principals' association in the state suggests that instead of choosing one day at random, the state should choose 3 days at random. With the suggested plan, High School A would lose some of its state funding in the subsequent year if the mean number of students absent for the 3 days is greater than 140. Would High School A be more likely, less likely, or equally likely to lose funding using the suggested plan compared to the plan described in part (a)? Justify your choice.

(c) A typical school week consists of the days Monday, Tuesday, Wednesday, Thursday, and Friday. The principal at High School A believes that the number of absences tends to be greater on Mondays and Fridays, and there is concern that the school will lose state funding if the attendance count occurs on a Monday or Friday. If one school day is chosen at random from each of 3 typical school weeks, what is the probability that none of the 3 days chosen is a Tuesday, Wednesday, or Thursday?

(d) The school board has decided to randomly select one day from each of five weeks. What is the likelihood that more than 2 of the days selected are either a Monday or Friday?

Name:_________________________ HWK 4.2 FRQ Distributions Period:_____

Due Friday, November 15th

1. **1998 Question 1** Consider the sampling distribution of a sample mean obtained by a random sampling from an infinite population. This population has a distribution that is highly skewed toward the large values.

   (a) How is the mean of the sampling distribution related to the mean of the population?

   The mean of the Sampling Distribution is equal to the mean of the population

   (b) How is the standard deviation of the sampling distribution related to the standard deviation of the population?

   The standard deviation of the sampling distribution is equal to the standard deviation of the population divided by the square root of the sample size. As the sample size increases the standard deviation of the sampling distribution decreases. Note: the standard deviation of the population is not affected by sample size.

   (c) How is the shape of the sampling distribution affected by the sample size?

   For small sample sizes the sampling distribution is skewed but becomes closer and closer to the normal as the sample size increases.

2. **2004B Number 3** Trains carry bauxite ore from a mine in Canada to an aluminum process plant in northern New York state in hopper cars. Filling equipment is used to load ore into the hopper cars. When functioning properly, the actual weights of ore loaded into each car by the filling equipment at the mine are approximately normally distributed with a mean of 70 tons and a standard deviation of 0.9 ton. If the mean is greater than 70 tons, the load mechanism is overfilling.

   (a) If the filling equipment is functioning properly, what is the probability that the weight of the ore in a randomly selected car will be 70.7 tons or more? Show your work.

   (Draw a picture)

   P(x > 70.7) = $P\left(Z > \frac{x-\mu}{\sigma}\right) = P\left(Z > \frac{70.7-70}{0.9}\right)$ **= P(Z > .778) = .218**

   (use 2nd Vars Normal CDF)

   The probability of a randomly selected ore cart weighing 70.7 tons or more is .218.

   (b) Suppose that the weight of the ore in a randomly selected car is 70.7 tons. Would that fact make you suspect that the loading mechanism is overfilling the cars? Justify your answer.

   No, a weight of 70.7 for a single ore car would not lead me to believe that the ore cars were being overfilled. I would expect to have 21.8% of the ore cars with weights greater than or equal to 70.7 tons.

   (c) If the filling equipment is functioning properly, what is the probability that a random sample of 10 cars will have a mean ore weight of 70.7 tons or more? Show your work.

   P(x > 70.7) for n= 10 is $P\left(Z > \frac{x-\mu}{\frac{\sigma}{\sqrt{n}}}\right) = P\left(Z > \frac{70.7-70}{\frac{0.9}{\sqrt{10}}}\right)$ **= P(Z > 2.46) = .007**

   (d) Based on your answer in part (c), if a random sample of 10 cars had a mean ore weight of 70.7 tons, would you suspect that the loading mechanism was overfilling the cars? Justify your answer.

   Yes, a mean weight of 70.7 for ten ore cars would lead me to believe that ore cars were being overfilled. The probability of getting a sample like this is less than 1 percent given that the cars were not being overfilled.

3. **2017 Question 3** A grocery store purchases melons from only two distributors, J and K. Distributor J provides melons from organic farms. The distribution of diameters of the melons from Distributor J is approximately normal with mean 133 millimeters (mm) and standard deviation 5 mm.

(a) For a melon selected at random from Distributor J, what is the probability that the melon will have a diameter greater than 137 mm?

$$z = \frac{x-\mu}{\sigma}$$

P(x>137)= $P\left(z > \frac{137-133}{5}\right)$ = P(Z>8) =.2119

The probability that a randomly selected melon from distributor J has a diameter greater than 137mm is .2119 *Note: you will need this number below.*

Distributor K provides melons from nonorganic farms. The probability is 0.8413 that a melon selected at random from Distributor K will have a diameter greater than 137mm. For all the melons at the grocery store, 70 percent of the melons are provided by Distributor J and 30 percent are provided by Distributor K.

(b) For a melon selected at random from the grocery store, what is the probability that the melon will have a diameter greater than 137mm?

	J	K	Total
> 137	.14833 (.70)(.2119)	.25239 (.30) (.8413)	.40072
< 137	.55167 (.70)(1-.2119)	.04761 (.30) (1-.8413)	.59928
Total	.70	.30	1.0

There is a .40072 probability that a melon selected at random will have a probability greater than .40072

(c) Given that a melon selected at random from the grocery store has a diameter greater than 137mm, what is the probability that the melon will be from distributor J?

P(diameter > 137) =.40072

P(diameter > 137 and from J) =.40072

P(J | >137) = $\frac{P(J \cap >137)}{P(>137)}$

$= \frac{.14833}{.40072}$ = .37

There is a .37 probability that a melon with greater than a 137 diameter random will be from store J.

4. **2006 Question 4** A Company manufactures model rockets that require igniters to launch. Once an igniter is used to launch a rocket, the igniter cannot be reused. Sometimes an igniter fails to operate correctly, and the rocket does not launch. The company estimates that the overall failure rate, defined as the percent of all igniters that fail to operate correctly, is 15 percent.

   A company engineer develop a new igniter, called the super igniter, with the intent of lowering the failure rate. To test the performance of the super igniters, the engineer uses the following process.

   Step 1: One super igniter is selected at random and used in a rocket.

   Step 2: If the rocket launches, another super igniter is selected at random and used in a rocket.

   Step 2 is repeated until the process stops. The process stops when a super igniter fails to operate correctly or 32 super igniters have successfully launched rockets, whichever comes first. Assume that super igniter failures are independent.

   (a) If the failure rate of the super igniters is 15 percent, what is the probability that the first 30 super igniters selected using the testing process successfully launch rockets?

   P(x = 30) **0 1 2 3 6 . . . 29 30**

   P(failure) =.15 so the probability of success = 1 - .15 or .85

   $\binom{30}{30}(.85^{30})(30^{0})$ **Calculator:** (BinomPDF 30 trials, p=.85 x=30)

   = .00763

   (b) Given that the first 30 super igniters successfully launch rockets, what is the probability that the first failure occurs on the thirty-first **or** thirty-second super igniter tested if the failure rate of the super igniters is 15 percent.
   Events are independent so the probability of the 1st failure occurring on the 31st or 32nd launch given that there have been 30 successful launches is the same as the probability of the first failure occurring on the 1st or 2nd launch

   P(failure on the 1st) + P(failure on the 2nd)

   = (.15) + (.85)(.15)

   =.2775 **Calculator:** (geomCDF p=.15 x=2)

   (c) Given that the first 30 super igniters successfully launch rockets, is it reasonable to believe that the failure rate of the super igniters is less than 15 percent? Explain.
   Yes it is reasonable to believe that the failure rate is less than 15%. There is less than a 1% chance of having 30 out of 30 rockets launch successfully.

   Given that the igniters have a failure rate of 15 percent, there is only a .00763 chance that the rockets would launch successfully 30 times in a row.

   (d) Assume the failure rate for the rocket igniters is 15% and the company decides to fire the rocket 30 times and then count the number of failures. What is the likelihood that there are 6 to 15 failures inclusive?
   Let X = the number of successful launches

   P(6 ≤ x ≤ 15) = P(6) + P(7) + … + P(4) + P(5) **0 1 2 . . . 5 6 7. . .14 15 16. . .30**

   $\binom{30}{6}(.15^{6})(.85^{24}) + \cdots + \binom{30}{15}(.15^{15})(.85^{15})$ = **.2894**

   Calculator: (BinomCDF 30 trials, p=.15, x=15) - (BinomCDF 30 trials, p=.15, x=5)

5. **2014 Question 3** Schools in a certain state receive funding based on the number of students who attend the school. To determine the number of students who attend a school, one school day is selected at random and the number of students in attendance that day is counted and used for funding purposes. The daily number of absences at High School A in the state is approximately normally distributed with mean of 120 students and standard deviation of 10.5 students.

(a) If more than 140 students are absent on the day the attendance count is taken for funding purposes, the school will lose some of its state funding in the subsequent year. Approximately what is the probability that High School A will lose some state funding?

$z = \frac{x-\mu}{\sigma}$

P(x>140)= P$\left(z > \frac{140-120}{10.5}\right)$ = P(Z>1.9) =.0284

2[nd] Vars Normal CDF
Lower: 1.9
Upper: Infinity
μ: 0 & σ: 1

The probability that they will lose some state funding is about .0284

(b) The principals' association in the state suggests that instead of choosing one day at random, the state should choose 3 days at random. With the suggested plan, High School A would lose some of its state funding in the subsequent year if the mean number of students absent for the 3 days is greater than 140. Would High School A be more likely, less likely, or equally likely to lose funding using the suggested plan compared to the plan described in part (a)? Justify your choice.

The school would be less likely to lose funding using the 3 day plan as opposed to looking at a single day because the distribution of the sample means has less variability than the single day.

P(x>140)= P$\left(z > \frac{140-120}{\frac{10.5}{\sqrt{3}}}\right)$= P(Z > 3.3) =.00048

2[nd] Vars Normal CDF
Lower: 140
Upper: Infinity
μ: 120 & σ: $\frac{10.5}{\sqrt{3}}$

(c) A typical school week consists of the days Monday, Tuesday, Wednesday, Thursday, and Friday. The principal at High School A believes that the number of absences tends to be greater on Mondays and Fridays, and there is concern that the school will lose state funding if the attendance count occurs on a Monday or Friday. If one school day is chosen at random from each of 3 typical school weeks, what is the probability that none of the 3 days chosen is a Tuesday, Wednesday, or Thursday?

The probability that in given week that A Monday or Friday is chosen is 2/5 which is equal to the probability of not choosing a Tuesday, Wednesday or Thursday.

$\left(\frac{2}{5}\right)\left(\frac{2}{5}\right)\left(\frac{2}{5}\right)$**=.064 is the probability that none are Tuesday, Wednesday or Thursday or all are Monday and Friday**

(d) The school board has decided to randomly select one day from each of five weeks. What is the likelihood that more than 2 of the days selected are either a Monday or Friday?

P(D > 2) → P(D = 3) + . . . + P(D = 5)

0 1 2 3 4 5

$\binom{5}{3}(.4)^3(.6)^3 + \ldots + \binom{5}{5}(.4)^5(.6)^0$ **= .31744**

Note: *Calculator 1 – BinomCDF of 2*