Topic 3: Sets, Logic and Probability
Sets & Venn Diagrams

IB Math Studies
Review Sheet - Sets and Venn Diagrams

You should be able to do the following things on the test:

Use the notation and vocabulary of sets
Represent sets using set notation and Venn diagrams
Using notation and Venn diagrams, identify elements, subsets, and complements
Using notation and Venn diagrams, identify the union and intersection of sets
Solve problems using Venn diagrams
Translate between verbal propositions, symbolic language, and Venn diagrams

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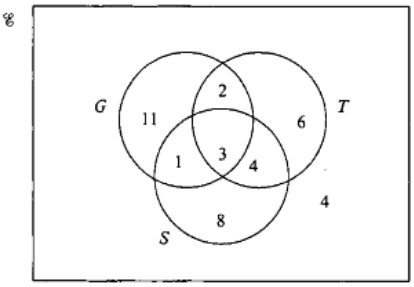
1. The sports offered at a retirement village are Golf (G), Tennis (T), and Swimming (S). The Venn diagram shows the numbers of people involved in each activity.

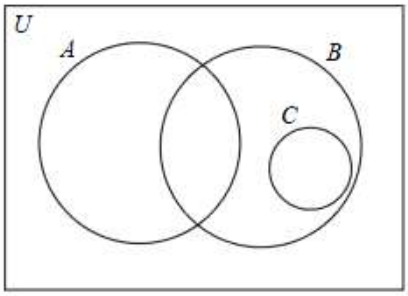


   a) How many people
      i) only play golf?
      ii) play both tennis and golf?
      iii) do not play golf?
      iv) are at the retirement village?

   b) Shade the part of the Venn diagram that represents the set $(G \cap S)'$.

   c) Describe in words the people who are represented by the set $(G \cap S)'$.

2. The following Venn Diagram shows the sets U, A, B, and C.



   State whether the following statements are true or false for the information illustrated in the Venn Diagram.

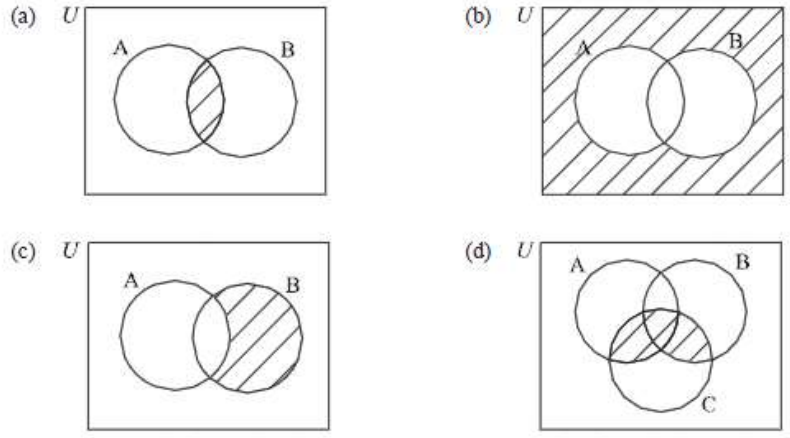
   a) $A \cap C = \emptyset$
   b) $C \cup B = C$
   c) $C \cup B = B$
   d) $C \subset A$
   e) $C \subset (A \cup B)$
   f) $A \subset C'$

3. The following results were obtained from a survey concerning the reading habits of students.

60% read magazine P	30% read magazines P and Q
50% read magazine Q	20% read magazines Q and R
50% read magazine R	30% read magazines P and R
10% read all three magazines	

a) Represent this information as a Venn diagram.
b) What percentage of students read exactly two magazines?
c) What percentage of students read at least two magazines?
d) What percentage of students do not read any of the magazines?

4. Write down an expression to describe the shaded area on the following Venn diagrams:



5.

A group of 30 children are surveyed to find out which of the three sports cricket (C), basketball (B) or volleyball (V) they play. The results are as follows :

3 children do not play any of these sports
2 children play all three sports
6 play volleyball and basketball
3 play cricket and basketball
6 play cricket and volleyball
16 play basketball
12 play volleyball.

(a) Draw a Venn diagram to illustrate the relationship between the three sports played. *[1 mark]*

(b) On your Venn diagram indicate the number of children that belong to each region. *[3 marks]*

(c) How many children play only cricket? *[2 marks]*

6.

Let $\mathscr{E} = \{x : 1 \leqslant x < 17 , x \in \mathbb{N}\}$.

P , Q and R are the subsets of $\mathscr{E}$ such that

P = {multiples of four} ;
Q = {factors of 36} ;
R = {square numbers} .

(a) List the elements of

(i) $\mathscr{E}$;

(ii) $P \cap Q \cap R$. *[2 marks]*

(b) Describe in words the set $P \cup Q$. *[1 mark]*

(c) (i) Draw a Venn diagram to show the relationship between sets P, Q and R . *[2 marks]*

(ii) Write the elements of $\mathscr{E}$ in the appropriate places on the Venn diagram. *[3 marks]*

IB Math Studies

3.1 Introduction to Set Theory & Subsets and Complements of Sets

A set is a grouping of objects or ideas. You are already familiar with some sets of numbers:

Symbol	Name	List {…} or Examples	Describe in words
	The Natural Numbers		
		$\{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$	
$\mathbb{Q}$			All numbers that can be expressed a quotient of two integers: $\frac{a}{b}$
			The union of all rational and irrational numbers; all numbers that can be found on a number line

Elements

If an element is a member of a set, we use this symbol: $\in$

If an element is NOT a member of a set, we use this symbol: $\notin$

Consider the set L of all letters in the English alphabet. Fill in the correct symbol:

C ____ L f ____ L é ____ L Ж ____ L e _____ L L ____ e

Consider the set $\mathbb{Z}$. Fill in the correct symbol:

3 ____ $\mathbb{Z}$ 0.3 ____ $\mathbb{Z}$ 3.2×10^{15} ____ $\mathbb{Z}$ 3×10^{-5} ____ $\mathbb{Z}$ $\mathbb{Z}$____ -14

Subsets

If a grouping of objects forms a smaller set within a larger set, we call it a subset. The set A is a subset of the set B if every element of A is also an element of B.

$$A \subseteq B$$

Are these statements true or false? Explain each answer.

{ a, e, i, o, u } $\subseteq$ { letters in the English language} T F

Explain:_____

{ Brazil, Argentina, Mexico } $\subseteq$ { countries in South America} T F

Explain:_____

$\mathbb{R} \subseteq \mathbb{Q}$ T F

Explain:_____

{ -1, 3, π, 12 } $\subseteq \mathbb{Z}$ T F

Explain:_____

Universal Sets and Complements

The universal set U is the set of all elements under discussion, such as all countries in the world, all students in a class, or all real numbers. We are often interested in a set of elements in U that are <u>not</u> in some specified subset A.

This idea is called the complement, and its symbol is A'.
The complement A' is the set of all elements that are in U, but not in A.

If U is the set of students in this classroom, and B is the subset of boys in this classroom, how do we represent the set of girls in this classroom?

If U is $\mathbb{Z}$, and $B = \{\dots, -4, -2, 0, 2, 4, \dots\}$, then what B' ?

Challenge

Some set notation is much more complicated… It takes some work to figure it out.

List the elements of $\{x: x \in \mathbb{Z}, 1 \leq x < 10\}$

List the elements of $\{x: x \in \mathbb{Z}^+, x < 15\}$

IB Math Studies
3.1/3.2 Union and Intersection of Sets / Introduction to Venn Diagrams

Hypoglycemia (too little sugar)	**Hyperglycemia (too much sugar)**
Nausea	Headache
Visual disturbances	Stomach cramps
Trembling	Nausea
Headache	Rapid breathing

Venn Diagram:

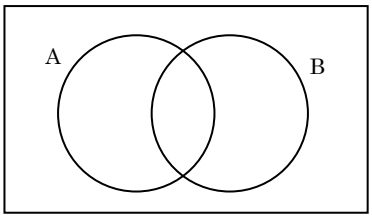
What does the set { nausea, headache } represent?

What does the set { nausea, visual disturbances, trembling, headache, stomach cramps, rapid breathing } represent?

The activity on the previous page leads us to two important ideas related to sets:

Intersection and Union

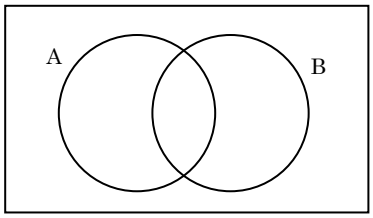
If A and B are sets, the <u>intersection</u> of A and B, denoted by $A \cap B$, is the set of all elements that are <u>common</u> to both A and B.



$A \cap B$

The intersection set contains only the elements that overlap the two sets.

If A and B are sets, the <u>union</u> of A and B, denoted by $A \cup B$, is the set of all elements that are <u>either</u> in A or in B or in <u>both</u> A and B.

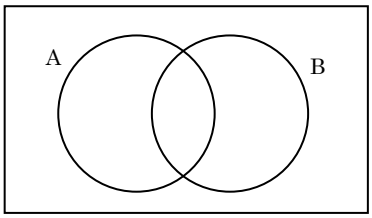


$A \cup B$

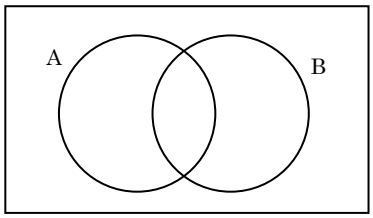
The union set contains all of the elements from the two sets.

Shade each of these:

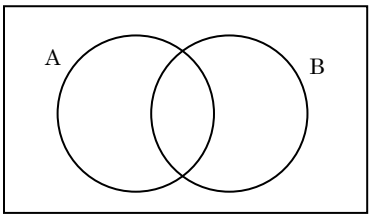
A'



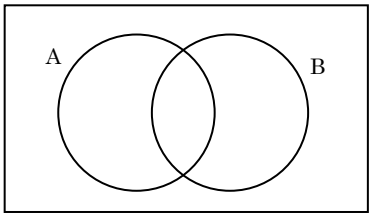
B'



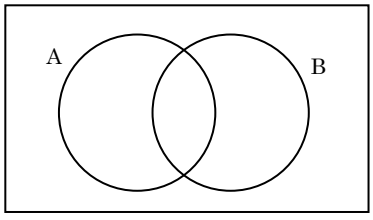
$(A \cap B)'$



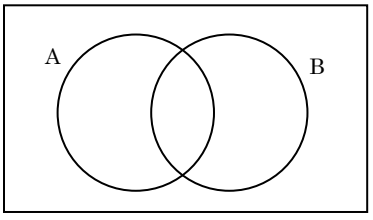
$(A \cup B)'$



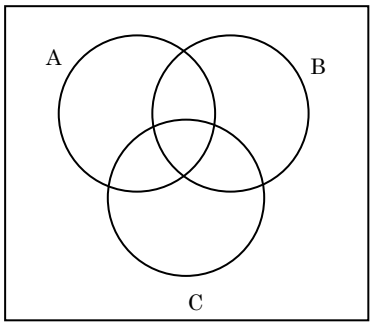
$A' \cap B$



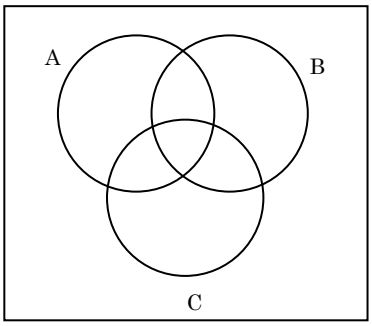
U



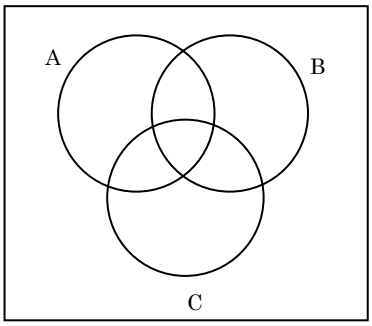
$B \cap C$



$A \cup (B \cap C)$

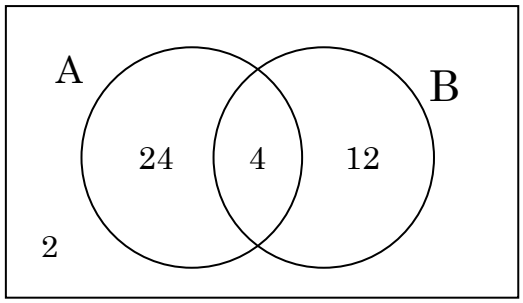


$(A \cup B) \cap C'$



Venn diagrams can also be used to show the number of elements in different sets. The notation n(A) means "the number of elements in set A"

Given the Venn diagram below:



a) Find n(U).

b) Find n($A \cap B$).

c) Find n($A \cup B$).

d) Find n(A').

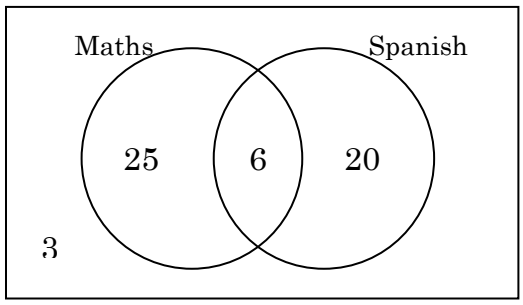
IB Math Studies
3.2 Applications of Venn Diagrams
3.2 Venn Diagrams Involving Three Sets

Venn diagrams can be really helpful in sorting information and solving problems.

One key skill is to remember that the overlap/intersection part is included in the count of each of the sets, but only once in the union of the sets.

Another key skill is to recognize the language of the question and notice when the words "only", "both", "or", and "neither" are used.

Students in 11th Grade



How many students are in 11th grade? _____

How many students study Maths? _____

How many students study Spanish? _____

How many students study only Maths? _____

How many students study only Spanish? _____

How many students study Maths or Spanish? _____

How many students study Maths and Spanish? _____

How many students study Maths or Spanish, but not both? _____

When you are given information to create a Venn diagram, it is usually best to start with the overlap of the sets and work your way out.

Example 1: There are two artist guilds in a city: the ABC and the PAA. 300 artists in the city are members of both guilds and 200 are members of neither. There are 1500 total members of the ABC guild and 2000 total members of the PAA. How many artists are in the city?

Example 2: There are 65 golf players at a charity tournament. 45 of these players will play 9 holes of golf and 40 will play 18 holes. There are 5 people at the tournament who have decided not to play at all. How many people will play both 9 holes and 18 holes of golf?

Try these in your notebook:

1. A platform diving squad of 25 members has 18 members who dive from 10 meters and 17 who dive from 4 meters. How many dive from both platforms?

2. A badminton club has 31 playing members. 28 play singles and 16 play doubles. How many play both singles and doubles?

3. In a group of 120 students, 75 know how to use a Macintosh, 65 know how to use a PC, and 20 do not know how to use either. How many students know how to use both kinds of computers

Venn Diagrams Involving Three Sets

When there are three sets, the strategy is the same: Start in the middle and work your way out.

In a survey of children who saw three different shows at Walt Disney World, the following information was gathered:

39 children liked The Little Mermaid
43 children liked 101 Dalmatians
56 children liked Mickey Mouse
7 children liked The Little Mermaid and 101 Dalmatians
10 children liked The Little Mermaid and Mickey Mouse
16 children liked 101 Dalmatians and Mickey Mouse
4 children liked The Little Mermaid, 101 Dalmatians, and Mickey Mouse
6 children did not like any of the shows

How many children were surveyed?

A web software company decides to do a survey of its 220 employees to find out which web programming languages they know. They discover:

20 of their employees know no programming languages
40 know PHP, JavaScript and Flash
60 know PHP and JavaScript
50 know PHP and Flash
80 know JavaScript and Flash
120 know PHP
120 know JavaScript
110 know Flash

How many of their employees know exactly two languages?

Try these in your notebook:

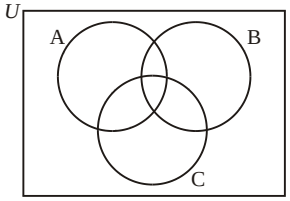
4. A city has three newspapers A, B and C. Of the adult population, 1% read none of these newspapers, 36% read A, 40% read B, 52% read C, 8% read A and B, 11% read B and C, 13% read A and C and 3% read all three papers. What percentage of the adult population read newspaper A only?

5. In a certain population, 87 people like raspberries, with 9 liking only raspberries. 91 people like strawberries, with 10 liking strawberries only. 91 people like blueberries, with 12 liking blueberries only. If 40 of these people like all three berries, how many people like strawberries and blueberries, but not raspberries?

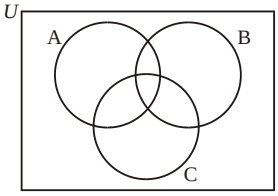
IB Check for Understanding

1. Shade the given region on the corresponding Venn Diagram.

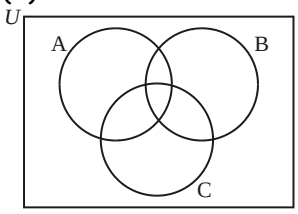
(a) $A \cap B$



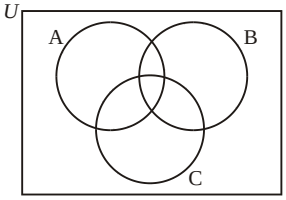
(c) $(A \cup B \cup C)'$



(b) $C \cup B$



(d) $A \cap C'$



2. In a survey of 52 students it was found that 30 study Spanish and 15 have computers. Seven of the students who study Spanish also have computers.

(a) **Copy** and complete this table.

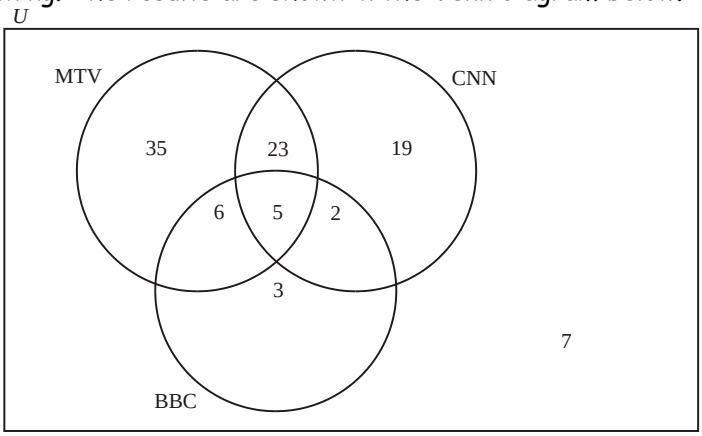
	Study Spanish	Do not study Spanish	Total
Have computers			
Do not have computers			
Total			52

(b) Draw and **label fully** a Venn diagram to illustrate this information. Use U to represent the set of all students surveyed, S the set of students who study Spanish and C the set of students who have computers.

(c) Describe, in words, the set represented by $C \cup S'$.

(d) Find $n(C \cup S')$.

3. 100 students were asked which television channel (MTV, CNN or BBC) they had watched the previous evening. The results are shown in the Venn diagram below.



From the information in the Venn diagram, write down the number of students who watched

(a) both MTV and BBC;

(b) MTV or BBC;

(c) CNN and BBC but not MTV;

(d) MTV or CNN but not BBC.

4. A poll was taken of the leisure time activities of 90 students.
60 students watch TV (*T*), 60 students read (*R*), 70 students go to the cinema (*C*).
26 students watch TV, read **and** go to the cinema.
20 students watch TV and go to the cinema only.
18 students read and go to the cinema only.
10 students read and watch TV only.

(a) Draw a Venn diagram to illustrate the above information.

(b) Calculate how many students

(i) only watch TV;

(ii) only go to the cinema.

5. Let $U = \{-4, -\frac{2}{3}, 1, \pi, 13, 26.7, 69, 10^{33}\}$.

A is the set of all the integers in U.

B is the set of all the rational numbers in U.

(a) List all the prime numbers contained in U.

(b) List all the members of A.

(c) List all the members of B.

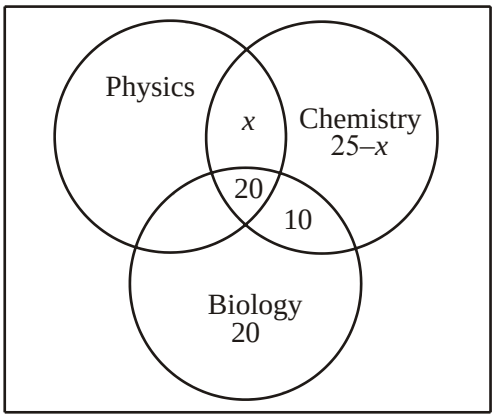
(d) List all the members of the set $A \cap B$.

6. At a certain school there are 90 students studying for their IB diploma. They are required to study at **least one** of the subjects: Physics, Biology or Chemistry.

50 students are studying Physics,
60 students are studying Biology,
55 students are studying Chemistry,
30 students are studying both Physics and Biology,
10 students are studying both Biology and Chemistry but not Physics,
20 students are studying all three subjects.

Let x represent the number of students who study both Physics and Chemistry but not Biology. Then $25-x$ is the number who study Chemistry only.

The figure below shows some of this information and can be used for working.



U with $n(U) = 90$

(a) Express the number of students who study Physics only, in terms of x.

(b) Find x.

(c) Determine the number of students studying **at least two** of the subjects.

7. A school offers three activities, basketball (B), choir (C) and drama (D). Every student must participate in at least one activity.

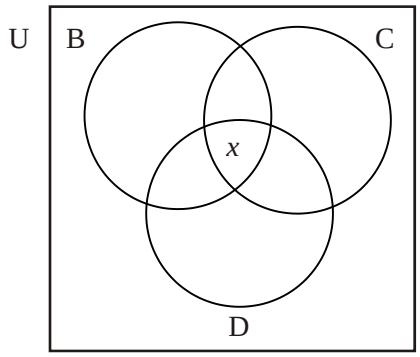
16 students play basketball only.

18 students play basketball and sing in the choir but do not do drama.

34 students play basketball and do drama but do not sing in the choir.

27 students are in the choir and do drama but do not play basketball.

(a) Enter the above information on the Venn diagram below.



99 of the students play basketball, 88 sing in the choir and 110 do drama.

(b) Calculate the number of students x participating in all three activities.

(c) Calculate the total number of students in the school.