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# MANUAL

OF

# TOPOGRAPHIC METHODS

BY

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# MANUAL OF TOPOGRAPHIC METHODS.[a]

By Henry Gannett.

## CHAPTER I.

### INTRODUCTION.

#### OBJECT OF THIS MANUAL.

The object of this manual is a description of the topographic work, instruments, and methods of the United States Geological Survey, primarily for the information of the men engaged in this work. It is not intended to be an elementary treatise on surveying, as it presupposes a knowledge of the application of mathematics to surveying equivalent to that obtained in our professional schools. Neither is it intended to be a general treatise on topographic work, altho it may, to a certain extent, supply the need of such a work.

Surveys are made for two widely different purposes. One consists in laying down upon the ground certain geometric figures, such as a town plat or the alignment of a railroad or an aqueduct. The other purpose is the making of maps, or miniature representations of the country. While the instruments and operations are to some extent the same in both, the purpose and the results are very different.

Most books on surveying have in view mainly, if not entirely, the first of these two purposes, operations incident to the making of maps being more or less slighted. Herein the author will attempt to describe the most approved methods of surveying as applied to the production of topographic maps, whether on large or small scales and whether of a high or a low degree of accuracy, including rough reconnaissance as well as accurate and detailed surveys.

#### CLASSES OF MAPS.

A map is a representation, in plan, of a part of the earth's surface. The variety of maps is legion, depending upon the class of features or phenomena which they represent or which is made prominent on them. Thus there are geologic maps, zoologic maps, botanic maps,

[a] This manual was first published in 1893 as Monograph XXII of the United States Geological Survey.

cadastral maps (which represent, primarily, property lines), political maps, etc. For all these it is essential that there be a base upon which the specific phenomena can be represented. This base should show certain topographic features, as streams and other bodies of water, railroads, towns, and cities; while some sort of convention to represent the relief of the ground is usually a valuable adjunct.

From the point of view of scale, those maps which show only topographic features may be grouped in two classes: First, those on small scales, say 10 or more miles to an inch, which may be properly called geographic maps and which are usually compiled from other maps on larger scales; and, second, those on large scales, which may properly be called topographic maps. Maps of the latter class are often made directly from surveys, and being the first product of these surveys, are known as mother maps, because they constitute the source from which other maps are compiled. In this work are described the methods of surveying and the preparation of maps on large scales—maps which can be conveniently made directly from surveys.

## TOPOGRAPHIC MAPS.

A topographic map should represent rivers, creeks, lakes, ponds, and all other bodies of water, together with coast lines; all artificial features that are of sufficient prominence to be represented on a scale, such as roads, boundary lines, cities, towns, villages, houses, and, in case of large-scale maps, fences and other such objects, bridges, fords, dams, canals, aqueducts, etc., and the relief of the ground—its hills, valleys, mountains, canyons, etc.

The relief of features can be shown by different methods, which may be classed as qualitative and quantitive. Qualitative methods show relief by shading, either by means of crayon or color or by means of hachure lines—lines which are drawn in the direction of the slope and which by their relative heaviness and closeness produce the effect of shading. These methods merely show the degree of slope; they give no information regarding the absolute amount of relief, or elevation above sea level. They have been in extensive use thruout the world, but their use is now rapidly diminishing.

## CONTOURS.

The method of expressing relief now commonly in use, which has largely replaced the methods described above, is that known as the contour method. Contours are lines drawn at equal elevations above sea level, every point upon a contour being, or being supposed to be, at the elevation indicated by that line. These contours would become shore lines if the level of the sea were raised successively to the heights indicated by them. This method is confessedly not so graphic as the others, but it has this great advantage, that it expresses absolute elevations; the height above the datum (usually sea level) of any portion of the area represented can be read directly from the contours. Moreover, it expresses the slopes, since where they are steep the contours must necessarily be close together, and where they are gentle the contours must be far apart. Modern maps, as a rule, are contour maps.

The contour interval is the vertical scale of the map. This should be proportioned to the horizontal scale and to the degree of relief of the country. With the same horizontal scale the contour interval would naturally be large for a country of bold relief and small for one of slight relief, since in the former case the same difference of altitude is of much less importance than in the latter case.

## SCALES.

The scale of a map is the proportion which exists between the dimensions of the map and those of the area which it represents. It is designated in several ways:

By the number of miles or of feet represented by an inch on the map.

By what is called a fractional scale, the numerator representing a linear unit of the map and the denominator expressing the number of such units in a corresponding distance on the ground, as $\frac{1}{10000}$.

By a bar scale, wherein a measure on a map is marked with its corresponding measure on the ground.

On many maps all of these forms of designating the scale are given.

The scale of a map should be chosen so as to represent as much detail and as great accuracy as are needed for the purpose for which the map is made. It should be neither larger nor smaller than is required to suit this purpose. For a small amount of detail a large scale is unnecessary, while too small a scale does not allow the legible representation of the detail required. Thus the statement of the scale of a map should afford an idea of its accuracy and detail.

## CULTURAL FEATURES.

The list of cultural features, or works of man, which should be represented on a map depends upon the scale of the map and the purposes for which it is made. A general topographic map, which is designed for use for many years without revision, should contain comparatively few of these features—only those which are likely to be permanent and which are of importance to the community rather than to individuals. These features change with considerable rapidity, and keeping the map revised involves the expenditure of much labor and money if many such features are represented. For these reasons their introduction should be restricted so far as is compatible with the usefulness of the map. For maps on large scales and for temporary purposes the list may be enlarged to any extent and in any desired direction, but the matter of inclusion and exclusion of these features should be considered carefully in view of all attendant circumstances. It is very easy so to clutter a map with useless information as to obscure its really valuable features.

## PLAN OF THE MAP OF THE UNITED STATES.

### DIVERSITY OF CONDITIONS.

The field upon which the United States Geological Survey is at work is diversified. It comprizes broad plains, some of which are densely covered with forests, while on others trees are absent; high and rugged mountains, plateaus, and low, rolling hills. In some regions its topographic forms are on a grand scale; in other regions the entire surface is made up of minute detail. Some parts of the country are densely populated, as much so as any other region on the earth, while in other parts of the country great areas are almost without settlement. Geologically, portions of the country are extremely complex, requiring, for the elucidation of geologic problems, maps in great detail, while other areas are very simple.

It is obvious that, because of this diversity of conditions, maps of different areas should vary in scale, and that with such diversity in natural conditions and in scale there must be differences in the methods of work employed. The system which is found to work advantageously in the high mountain regions of the West is more or less inapplicable to the low, forested plains of the Mississippi Valley and the Atlantic Plain.

### SCALE.

The scales which have been adopted for the publication of the map are 1:62500, or very nearly 1 mile to an inch; 1:125000, or very nearly 2 miles to an inch; and 1:250000, or very nearly 4 miles to an inch.

The following considerations have determined the selection of the above scales: They are believed to be sufficiently large to admit of the representation of all the details necessary to a picture of the country which shall show the proper relations of its features and shall be of the greatest possible service for industrial and scientific uses consistent with other requirements to be mentioned hereafter. These scales are also sufficiently large to enable geologists to present the details of nearly all geologic phenomena. The map represents the country in sufficient detail to admit of the selection of general routes for railroads and other public works, and of the indication of boundary lines in such way that their positions may be recognized upon the ground. On the other hand, the scales are not so large as to prevent the representation of a considerable area on a single sheet, so that the relation between different regions can be seen at a glance.

A map on a larger scale would be more expensive and would require longer time for its completion. On the scales adopted the map of the United States, excluding Alaska, will cost about twenty-five million dollars and at the present rate of progress will require fifty years for its completion.

There is another objection to increasing the scale beyond that absolutely necessary, viz: Such a map, to be of value, must undergo revision at frequent intervals in order that changes in culture and possibly in natural features due to natural or artificial agencies may be incorporated. The larger the scale the more frequently such revision would be necessary, and hence the labor and expense of keeping such a map up to date would be greatly increased.

### CONTOUR INTERVAL.

The relief of the earth's surface is now represented on maps almost entirely by contour lines, or lines of equal elevation. Until a comparatively recent date this relief, secondary in importance only to the horizontal location, or the plan, has not been exprest quantitatively.

The contour intervals which have been adopted for the map of the United States are as follows: For the scale of 1:62500 the intervals range from 5 to 50 feet; for the scale of 1:125000 the range is from 10 to 100 feet; and for the scale of 1:250000 the interval is 100, 200, or 250 feet.

### FEATURES REPRESENTED.

The experience of European nations tends in the direction of reducing the number of features which should be placed on a map. In the preparation of the map of the United States it has been decided to go even beyond the present practise of European countries in this regard, and to show only natural features of such magnitude that the scale warrants their representation and cultural features of general or public importance. Under these limitations the map will show cities, towns, and villages; roads, railroads, and other means of communication; bridges, ferries, tunnels, fords, canals, and acequias; and boundaries of civil divisions. Fences, property lines, and other objects of kindred nature will not be represented. The reasons for excluding private culture are apparent: First, because such features are not of sufficient general interest to pay the cost of surveying or representing them; second, because they change rapidly, and in order to keep the maps up to date constant resurveys and republication would be required; and, third, because their number and complexity confuse the map and render its more important features less intelligible.

### SIZE OF SHEETS.

The atlas sheets are designed to be of approximately the same size—17½ inches in length by 12 to 15 inches in width, depending upon the latitude. Those on the same scale cover equal areas exprest in units of latitude and longitude; that is, each sheet on the 4-mile scale covers 1 degree of latitude by 1 degree of longitude; each sheet on the 2-mile scale, 30 minutes of latitude and 30 minutes of longitude; and each sheet on the 1-mile scale, 15 minutes of latitude and 15 minutes of longitude. The sheets are thus small enough to be conveniently handled, and if bound they form an atlas of suitable size. From the fact that each sheet covers either a full degree or a certain fractional part of a degree, its position with relation to adjacent sheets and to the area of the country may be readily ascertained.

# CHAPTER II.

## METHODS AND CLASSIFICATIONS OF WORK.

### GENERAL METHODS.

Every map is essentially a sketch controlled by locations. It follows that the work of making a map consists of two parts—that of fixing these locations, which is done by means of surveying instruments and is geometric, and that of sketching, which is done by the hand and eye and is artistic. This general description applies to the making of all maps, whatever their scales, their accuracy, their degree of detail, or the character of the organization making them. The methods and instruments employed in fixing the locations vary with different scales, kinds of country, and the ideas of the organizations engaged in the work.

Altho the sketching is all that appears on the finished map, a description of the methods of making the locations must of necessity form the bulk of a volume descriptive of surveying methods, for it is the essential part of the work. This volume will treat only of the most approved methods and instruments now used in topographic surveying; time and space will not be wasted upon methods or instruments that have been tried and discarded.

### CORRECTNESS OF MAPS.

The correctness of a map depends upon four elements: Accuracy of location, number of locations per square inch of map. distribution of locations, and quality of the sketching.

#### ACCURACY OF LOCATION.

The greatest accuracy attainable is not always desirable, because it would not be economical. The highest economy is in properly subordinating means to ends, and it is not economical to do work of greater accuracy than is needed—for instance, to execute triangulation of geodetic refinement for the control of maps on small scales. The quality of the work should be such as to insure against errors of sufficient magnitude to be appreciable on paper. While errors in triangulation tend to balance one another, still they are liable to accumulate, and this liability must be guarded against by maintaining a somewhat higher degree of accuracy than would be required for the location of any one point. It is not difficult to meet this first condition of accuracy of maps. The maximum allowable error of location may be set at 0.01 of an inch on the scale of publication. This admits of an error upon the ground, on a scale of 1:62500, not greater than 50 feet, and proportionally on other scales.

### NUMBER OF LOCATIONS PER SQUARE INCH OF MAP.

The second condition of correctness for the proper control of the work is not easily defined. The requirements differ with the character of the country. A region of great detail and of abrupt features requires more control than one of greater uniformity, of gentle slopes, or of large features, so that no general rule can be laid down. Furthermore, it depends upon the quality of the sketching. With indifferent sketching a greater number of locations is required in order to bring the map to the requisite quality. Examples of the amount of control upon maps are scarce, since little has been published on this subject.

There are two general methods of locating stations and points for the correction of the sketch—one by angular measurements (triangulation) and the other by measurement of direction and distance, or what is known popularly as the traverse or meander method. In ordinary practise work may be done by either of these methods, or they may be used in conjunction. The former method can be carried on with the plane table, with various forms of the theodolite, with a compass, or, indeed, with any angle-reading instrument. The latter method can be pursued with the same instruments, supplemented by various forms of odometers, chain, steel tape, stadia, etc., for the measurement of distance. The first method, whenever it can be used economically, is the more accurate and, as a rule, the more rapid, and locations thus made are likely to be of greater service and to be distributed more uniformly than if made by the other method. It can be used economically where the country presents relief and where either natural or artificial points for location exist in sufficient numbers and are well distributed. These conditions are satisfied almost everywhere in the western mountain regions, where peaks, summits of hills, plateau points, buttes, etc., furnish an abundance of natural points for stations and locations. It can be used to a considerable extent, tho not with the same ease and economy, in the Appalachian Mountains; but in this region it is necessary to supplement it extensively by traverse lines, especially in tracing the courses of streams in valleys. It can be used in the hill country of New England, where objects of culture, such as churches, houses, etc., furnish plenty of signals. On the other hand, thruout the whole extent of the Atlantic and Gulf plains and the Mississippi Valley, where the country is level, or nearly so, and is covered with forests, the traverse method of surveying must be resorted to. In these parts of the country there are no sharp natural objects, and extended views can not be obtained. For many reasons the second method of obtaining locations is inferior to the former. It is inferior not only in accuracy but in the facilities which it affords for sketching the country, and it should be so regarded and should be adopted only when it becomes necessary or when the former method can not be applied economically. For convenience traverse lines are generally run along roads or trails and thus the best points for commanding views of the country are avoided rather than sought, and there is danger lest the topographer neglect the areas lying between the roads. On account of the errors incident to running a traverse, it is necessary that in this class of work frequent locations be made by triangulation for checking and thereby eliminating the errors.

As a rule, triangulation locations are selected points chosen because each controls positions in a certain area. On the other hand, traverse locations, as a rule, are not chosen for their control value, but only for intervisibility on roads. Furthermore, the great majority of traverse stations are of no service whatever beyond carrying the line forward, so that in estimating the total amount of control in a certain area where the control is made up wholly or in part of traverse lines less weight should be given to them than to locations by triangulation.

### DISTRIBUTION OF LOCATIONS.

The third element of accuracy is a point concerning which it is equally difficult to speak definitely. Other things being equal, the distribution should be uniform over the area, but it will necessarily vary with the character of the surface. In general, in mountain regions, locations by angular measurements are frequent and follow the ranges or ridges, but in valleys such locations are few in number, being supplemented by traverses.

The subject of locations may be treated from another point of view—that of the features to be represented. All points for locations should be chosen for the purpose of guiding the sketch of certain features. Streams, roads, and other linear features ordinarily can be best and most economically located by means of traverse lines; that is, by establishing by traverse a series of points along their courses. Houses can usually be best located by intersection, either in connection with the secondary triangulation or from a traverse.

The location of contour lines requires more than a passing mention. At the outset a very common impression should be corrected. It is generally supposed by those unacquainted with the subject that contours are located by finding and fixing points directly on each contour and sketching the contour between these points. This method, however, is very rarely employed, and then only in the most detailed surveys. In the topographic surveys of all nations, except the English Ordnance Survey, a different method is employed, or, rather, a different selection of points is made. For location and height measurement salient points are selected, such as the summit of a hill, the shoulder of a spur, the foot of a slope, the bed of a valley. The contours are then sketched with reference to these points. By judicious selection of such points comparatively few locations and height measurements serve to locate the intervening contours with sufficient accuracy. On the whole, the results are quite as good as if points had been located immediately on the contours, and the work of location is vastly reduced.

### QUALITY OF SKETCHING.

The fourth of the elements of the correctness of the map depends on the artistic sense of the topographer, on his ability to see things in their proper relations, and on his facility in transferring his impressions to paper.

Every map, whatever its scale, is a reduction from nature and is generalized, and consequently departs from the original in greater or less degree. The smaller the scale the greater the generalization, and the further the map must depart from nature the less can it resemble the ground which it represents. Bends in streams, petty gulches, canyons, and hills must be omitted. Contours must cross such omitted gulches instead of turning up and down their courses. A searching examination of the map would therefore show countless places where the contours are widely out of place, owing simply to this generalization. To specify that the contours of a map must nowhere be in error more than a contour interval, for instance, is to require an impossibility.

The education of the topographer therefore consists of two parts—the mathematical and the artistic. The first may be acquired largely from books, but this book knowledge must be supplemented by practise in the field. The second, if not inherent, can be acquired only by long experience in the field, and by many can never be acquired in high degree. In fact, the sketching makes the map, and it should be done by the best topogragher in the party, usually the chief, whenever practicable.

### CLASSIFICATION OF WORK.

The work involved in making a map usually comprizes several operations, which in practise may be more or less distinct from one another. They are as follows:

(1) The location of the map upon the earth's surface, by means of astronomic observations.

(2) The horizontal location of points. This is usually of three grades of accuracy: Primary triangulation, or primary traverse in cases where triangulation is not feasible; secondary triangulation for the location of numerous points within the primary triangulation; and ordinary traverse, for the location of details.

(3) The measurement of heights, which usually accompanies the horizontal location, and which may, similarly, be divided into three classes, in accordance with the degree of accuracy.

(4) The sketching of the map.

Nearly all of the geometric work of surveying—the work of location—is executed with few instruments, as follows:

Theodolites, of a powerful and compact form, used in the primary control.

Wye levels.

Plane tables, with telescopic alidades of the best type, used for secondary triangulation and height measurements.

Plane tables of simple form, with ruler alidades, compasses, or solar attachment, used for traversing and minor triangulation.

Aneroids, for the measurement of details of heights.

These instruments, with which nine-tenths of the work is done, will be described in their proper places with such fulness of detail as seems necessary.

Other instruments, such as transits, surveyor's theodolites, compasses, hand and Abney levels, telemeters, chains, and tapes, are occasionally used. Most of these instruments, which are figured and described in nearly all works on surveying, are assumed to be well known to the readers of this manual, and will therefore receive no special attention.

# CHAPTER III.

## ASTRONOMIC DETERMINATION OF POSITION.

### OBJECT AND IMPORTANCE OF ASTRONOMIC DETERMINATION.

The object of astronomic determinations of position is to locate the map upon the earth's surface. They are made also for the purpose of checking and correcting positions determined by primary triangulation and primary traverse.

With regard to the checking of primary triangulation by astronomic determinations, it should be understood that in the case of a single determination the work by triangulation is far more accurate than that by the astronomic method, even when made under the best circumstances. It is, therefore, desirable to introduce checks of this kind upon primary triangulation only when the latter has been carried for a long distance—200 or 300 miles, for instance—in the course of which there may have accumulated errors greater than those incident to astronomic work.

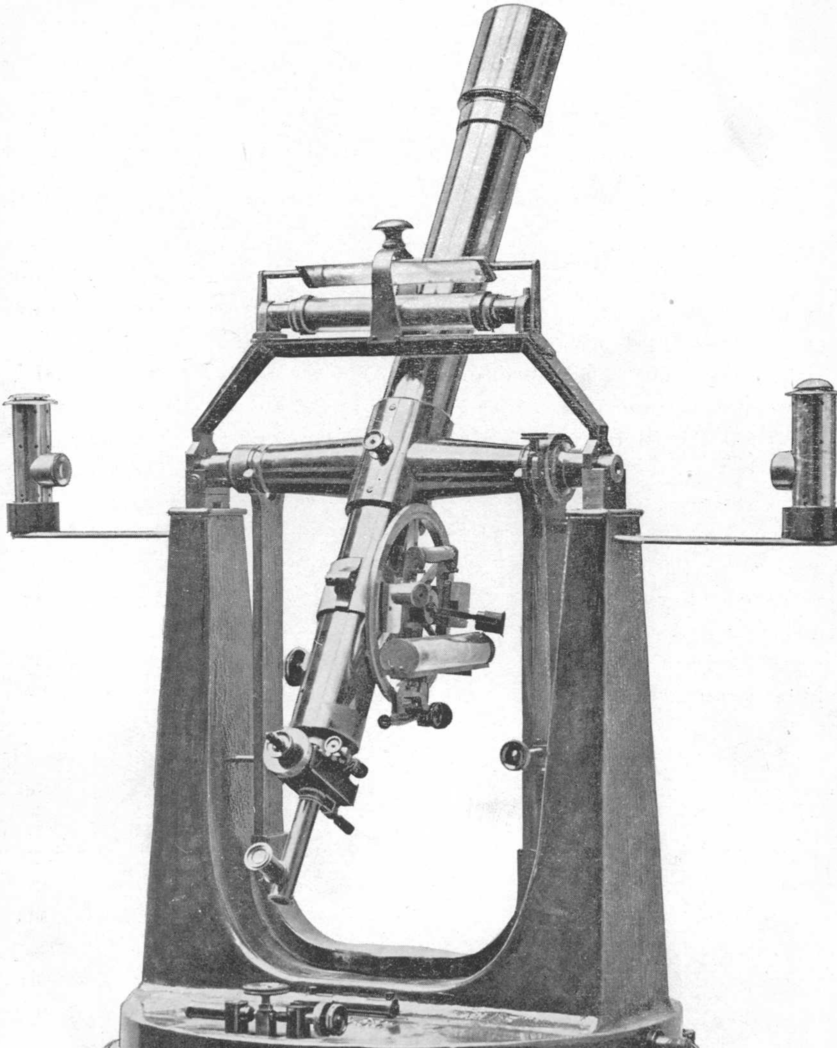
The case is different with primary traverse. The great number of courses required in this work affords an opportunity for the accumulation of error much greater than can arise in triangulation, and consequently it is desirable to introduce more frequent checks. It may be said that, in general, such work should be checked at every 100 miles.

As has been suggested, the best astronomic determinations are none too good for the control of maps. Indeed, certain errors incident to this work, some of which as yet can not be corrected, may be of magnitude sufficient to show upon the map. It is necessary, therefore, in these determinations to use the best instruments and the most refined methods known to modern science, in order to reduce all avoidable errors to a minimum.

Determinations of position that have been made by the United States Coast and Geodetic Survey, the United States Lake Survey, or the Mississippi River Commission, whether by astronomic work or by triangulation, may be utilized for the above purposes.

### DEFINITIONS.

Sidereal time is the time indicated by the stars, a sidereal day being the time which elapses between two passages of the vernal equinox across the meridian. Solar or apparent time is the time



ASTRONOMIC TRANSIT AND ZENITH TELESCOPE.

measured by the sun's apparent movement, or the revolution of the earth with reference to the sun; and since the earth revolves at a differing rate in different portions of its orbit, the solar days are not of equal length. A mean day is the average solar day; mean time differs from solar time by an amount which varies with the time of year, and which, under the name "equation of time," is given in the Nautical Almanac. Mean time differs from sidereal time by about a day in the course of a year, or about four minutes in each day, the mean day being longer than the sidereal day. To convert a given date of mean time into sidereal time it is necessary to obtain, from the Nautical Almanac, the sidereal time at noon immediately preceding the date in question. Then the interval after noon, exprest in mean time, is converted into sidereal time, and the result added to the sidereal time of mean noon. Local time, whether sidereal, solar, or mean, is the time of the locality as distinguished from the time of any other locality. It must be distinguished from standard time, which is the local time of only certain meridians.

The right ascension of the sun or a star is the sidereal time which has elapsed between the passage of the vernal equinox and the star across the meridian. It is commonly exprest in hours, minutes, and seconds.

Declination is the angular distance of a heavenly body north or south of the equator. It is plus when north and minus when south of the equator.

The zenith distance of a heavenly body equals its declination minus the latitude of the place of observation.

Latitude is determined by what is known as Talcott's method, by measuring the differences of zenith distance at culmination of two stars which culminate on opposite sides of the zenith.

Longitude is determined by telegraphic comparison of local time at two stations, the longitude of one of which is known. This involves the determination of the errors of the clocks or chronometers used, which is done by observation of transits of stars across the meridians of the places of observation.

## ASTRONOMIC TRANSIT AND ZENITH TELESCOPE.

A single instrument is used for the determination of both latitude and time. This is a combination of the transit and the zenith telescope. The instruments in use by the United States Geological Survey were made by Saegmüller and embody the latest improvements. (See Pl. I.) The circular base rests upon three leveling screws. Upon this circular base the whole instrument can be made to revolve when using it as a zenith telescope. A circle is graduated around the base, having a micrometer screw for slow motion, for making settings and adjusting the instrument in azimuth. The frame of the instrument is cast in one piece, and the standards are hollow in order to reduce the weight of the upper part. The telescope has a focal distance of 27 inches and a clear aperture of 2.5 inches. Its magnifying power with diagonal eyepiece is 74 diameters. The length of the axis of the telescope is 16 inches. For use as a zenith telescope the instrument is equipped with a vertical circle reading by vernier to 20 seconds, attached to which is a delicate level. In the focus of the object glass there is, besides the ordinary reticule for use in transit work, a movable thread, which is moved by means of a micrometer screw, by which measurements of differences of zenith distances are made. It is furnished with direct and diagonal eyepieces, the latter of which is commonly used in astronomic work.

For use as a transit instrument the telescope is equipped with a delicate striding level for measuring the inclination of the pivots and a reversing apparatus for turning the telescope in the wyes. The reticule, as the stationary threads in the focus of the instrument are called, consists of five threads for observing the transits of stars. The reticule is illuminated by means of bull's-eye lamps, the light from which comes thru the hollow axis of the telescope and is reflected by a mirror placed at the intersection of the telescope with its axis.

### CHRONOGRAPH.

The chronograph is used for the purpose of recording the time of transits of stars as observed with the transit instrument. It may be popularly characterized as an instrument for measuring time by the yard. It consists essentially of a drum upon which is wound a strip of paper and which is kept in revolution by a train of clockwork controlled by an escapement. A pen carried on a small car, which is moved very slowly in a direction parallel to the axis of the cylinder, traces a spiral line upon the paper on the drum. This pen is held in place by a magnet carried on the car, and as long as the current from the battery passes thru the coil and thus holds the armature, the pen traces an unbroken spiral line. If the current is suddenly broken and restored, the armature is set free for an instant and a jog is made in the line traced. The battery commonly used in connection with this outfit is the ordinary zinc copper, and copper sulfate battery, of which four cells are usually required. The ordinary dry battery can also be used and is much more convenient. With this apparatus break-circuit chronometers are used. These differ from ordinary chronometers in that they are arranged to break an electric circuit automatically at regular intervals. Those in use by the United States Geological Survey break the circuit every two seconds, and the end of the minute is indicated by breaking at the fifty-ninth as well as the fifty-eighth and sixtieth seconds. When one of these chronometers is connected with a battery and a chronograph is put in the same circuit, the beginning of every even second is recorded upon the chronograph by a jog on the paper, and the distance between the jogs in each case therefore represents two seconds. The observer at the instrument is provided with a telegraph key, which may also be put in the circuit with the chronometer and chronograph, and as a star

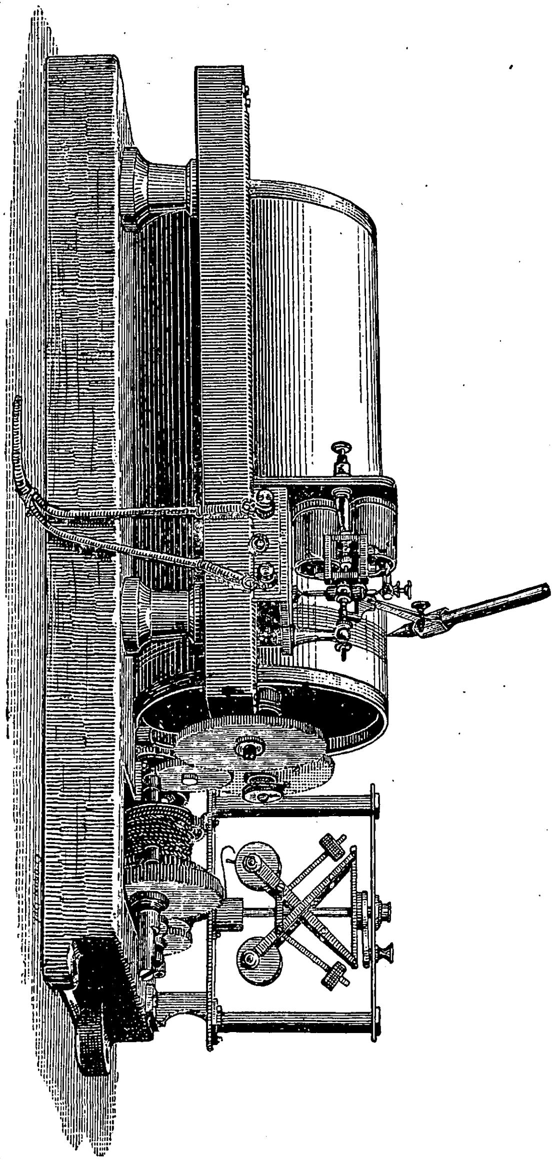


Fig. 1.—Chronograph.

near the meridian crosses a thread in the telescope he records that fact by pressing on the key, which makes a record upon the chronograph along with the record of the chronometer.

An illustration of the form of chronograph in use by the United States Geological Survey is shown in fig. 1.

### FIELD WORK.

Since the observations for latitude and longitude, tho different, are made with the same instrument, at the same time, and by the same party, certain parts of the work apply equally to both determinations and may be described once for all.

In the selection of a station care must be taken to avoid a locality where for any cause the ground is liable to be seriously jarred, as, for instance, proximity to a railroad track or to a street over which heavy wagons pass. The station should have a clear view from the southern horizon thru the zenith to the northern horizon. It is desirable to locate at a convenient distance from a telegraph office, as it is necessary to bring in a wire from such office for the purpose of comparing chronometers. If possible, the station should be selected on a public reservation, in order that the permanence of the monument which is to be erected to mark the spot may be assured. In any event a locality in which such a monument is likely to be disturbed should be avoided.

The support of the instrument should consist of a brick, stone, or concrete pier sunk fully 3 feet in the ground and rising to the requisite height. Upon this should be placed a block of stone well set in mortar for the immediate support of the instrument. The chronograph may be set up on an ordinary table. Over all should be erected a wall tent with a slit in the roof, closed by flaps, which can be opened when observing. The instrument should be set up on the pier, collimated, and leveled, the verticality of the threads should be tested as accurately as possible, and then it should be pointed upon the pole star. This places it somewhere near the meridian. Then taking the time of transit of a star which culminates close to the zenith, and comparing this time with the right ascension of the star, a sufficiently close approximation to the clock error is obtained for use in placing the instrument in the meridian. The instrument should then be turned in azimuth to point upon some close circumpolar star approaching upper or lower culmination, the instrument being moved in azimuth with the tangent screw, so as to keep the star under the middle wire to the instant of culmination. If this is done accurately at the first attempt the instrument is placed nearly in the meridian and is ready for work, but it commonly happens that more than one trial is required before the meridian is reached. In any case the result should be verified by a second star before proceeding with the observations.

## OBSERVATIONS FOR LATITUDE.

As preliminary to this work it is necessary to prepare a list of pairs of stars, the two stars of each pair having such zenith distances that they will culminate at nearly equal distances from the zenith, one to the north and the other to the south of it. Such a list can be prepared from the Safford Catalog of the Wheeler Survey. For this it is necessary to know the approximate latitude of the station and the right ascensions and the declinations of the stars. The zenith distance of a star is equal to its declination minus the latitude of the place of observation. The stars of each pair must be observed consecutively, and therefore those stars should be selected which culminate as nearly as possible together, leaving only a sufficient interval of time between them for setting the instrument.

On the approach of the first star of the pair to the meridian the instrument should be set for it, using the vertical circle for that purpose, and setting the spirit level upon the vertical circle as nearly level as possible. Then, as the star traverses the field of the telescope, the movable thread in the reticule should be kept upon it by means of the micrometer screw until it crosses the middle vertical thread. Then the micrometer and the two ends of the level bubble should be read and recorded. Without disturbing in the slightest degree the setting of the telescope, the entire instrument should be turned 180 degrees upon its bedplate, when it will point north of the zenith at the same angle at which it formerly pointed south, or vice versa, as the case may be, and will be set for the other star on the opposite side of the zenith. As this approaches culmination it should be followed with the micrometer as before until it reaches the middle thread, then the readings of the micrometer and of the level should be recorded as before, whether changed or not.

This constitutes the observations upon a single pair of stars. For the determination of latitude twenty such pairs of stars should be observed each evening, if possible, and the same pairs of stars should also be observed on other evenings, assuming it to be possible. The following example, taken from observations at Rapid, S. Dak., shows a portion of the star list and the form of record:

*List of stars for observation with zenith telescope.*

[Station: Rapid, S. Dak. Approximate latitude: 44° 05′.]

| Name or number. Safford's Catalog. | Mag. | Class. | R. A. | Dec. | Zen. dist. | Setting. |
|---|---|---|---|---|---|---|
| | | | h. m. | ° ′ | ° ′ | ° ′ |
| 7 Lacertæ.... | 4.0 | A A | 22 27 | 49 43 | 5 38 N. | 5 37 N. |
| 10 Lacertæ... | 5.0 | A A | 22 34 | 38 29 | 5 36 S. | |
| 1539.......... | 6.5 | B | 22 41 | 45 37 | 1 32 N. | 1 27 N. |
| 1551.......... | 6.5 | A | 22 47 | 42 42 | 1 23 S. | |
| 1565.......... | 6.5 | C | 22 52 | 38 42 | 5 23 S. | 5 22 S. |
| 1579.......... | 5.0 | A | 22 59 | 49 26 | 5 21 N. | |
| 1600.......... | 6.0 | A | 23 08 | 56 34 | 12 29 N. | 12 19 N. |
| 1633.......... | 6.7 | B | 23 18 | 31 56 | 12 09 S. | |
| 1676.......... | 5.6 | A | 23 42 | 67 12 | 23 07 N. | 23 05 N. |
| 1686.......... | 6.5 | A | 23 47 | 21 03 | 23 02 S. | |
| 1702.......... | 4.5 | A | 23 52 | 24 32 | 19 33 S. | 19 31 S. |
| 1722.......... | 6.5 | B | 24 00 | 63 35 | 19 30 N. | |

*Example of record.*

[Station: Rapid, S. Dak. Date: November 9, 1890. Instrument: Fauth combined transit and zenith telescope No. 534. Observer: S. S. G. Recorder: A. F. D.]

| Star name or number. | N. or S. | Micrometer reading. | Diff. | Level. N. | Level. S. | (N+S) −(N′+S′). | Remarks. |
|---|---|---|---|---|---|---|---|
| | | Rev. | Rev. | Div. | Div. | Div. | |
| 7 Lacertæ.... | N. | 26.256 | | 39.9 | 16.7 | +56.6 | |
| 10 Lacertæ... | S. | 24.052 | − 2.204 | 26.5 | 49.7 | −76.2 | |
| | | | | | | −19.6 | |
| 1539.......... | N. | 30.432 | | 42.0 | 18.7 | +60.7 | |
| 1551.......... | S. | 20.095 | −10.337 | 21.9 | 45.0 | −66.9 | |
| | | | | | | − 6.2 | |
| 1565.......... | S. | 25.164 | | 14.1 | 37.6 | −51.7 | Faint. |
| 1579.......... | N. | 26.703 | + 1.539 | 38.1 | 15.0 | +53.1 | Distinct. |
| | | | | | | + 1.4 | |
| 1600.......... | N. | 32.214 | | 37.5 | 14.1 | +51.6 | |
| 1633.......... | S. | 16.033 | −16.181 | 19.9 | 43.1 | −63.0 | Faint. |
| | | | | | | −11.4 | |
| 1676.......... | N. | 26.656 | | 51.0 | 28.0 | +79.0 | |
| 1686.......... | S. | 17.684 | − 8.972 | 17.0 | 39.6 | −56.6 | |
| | | | | | | −22.4 | |
| 1702.......... | S. | 25.345 | | 18.0 | 40.9 | −58.9 | |
| 1722.......... | N. | 23.722 | + 1.623 | 36.0 | 13.2 | +49.2 | |
| | | | | | | − 9.7 | |

## REDUCTION OF LATITUDE OBSERVATIONS.

Before proceeding with the reduction of latitude observations it is necessary to investigate the constants of the instrument and to ascertain the value of a division of the head of the micrometer screw and of a division of the latitude level.

### MEASUREMENT OF A DIVISION OF THE HEAD OF THE MICROMETER SCREW.

The value of a division of the head of the micrometer screw is measured by observing the transits of some close circumpolar star, when near elongation, across the movable thread, setting the thread repeatedly at regular intervals in advance of the star, and taking the time of its passage with the reading of the micrometer. The precaution should be taken to read the latitude level occasionally and correct for it if necessary. This correction, which is to be applied to the observed time, is equal to one division of the level, in seconds of time, divided by the cosine of the declination of the star and multiplied by the level error, the average level reading being taken as the standard.

*Determination of value of one revolution of micrometer belonging to Z. T. No. 534, by observations on 51 Cephei near eastern elongation, November 15, 1890, Rapid, S. Dak.*

| | | |
|---|---|---|
| log cot δ....87° 12′ 50″.8 | =8.6871826 | |
| log tan φ...44 04 45 | =9.9860380 | |
| log cos $t_o$...87 17 57 | =8.6732206 | |
| log 1 div. (=1 33)......... | =0.12385 | |
| a. c. log cos δ............ | =1.31334 | |
| a. c. log 15.............. | =8.82391 | |
| log 1.82 s. .................. | =0.26110 | |

| | h. | m. | s. |
|---|---|---|---|
| $t_o$= | 5 | 49 | 11.8 |
| $a$= | 6 | 49 | 20.4 |
| $T_o$= | 1 | 00 | 08.6 |
| $\delta t$= | | | −8.0 |
| chron. time elong.= | 1 | 00 | 00.6 |

| Time of observation (recorded on chronograph sheet). | Micrometer revolutions. | Level. N. | Level. S. | Level. N+S. | Reduction to mean level. | Correction for change of level. | Time from elongation. =T. | Correction for T. | Reduced time. | Revolutions. | Time for nine revolutions. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| *h. m. s.* | | *Div.* | *Div.* | *Div.* | *Div. s.* | *s.* | *m. s.* | *s.* | *h. m. s.* | | *m. s.* |
| 00 40 19.7 | 31.0 | 37.0 | 16.0 | 53 0 | +0.8×1.82= | +1.5 | −19 40.9 | +1.4 | 00 40 22.6 | 31.0 and 22.0 | 15 37.7 |
| 41 12.3 | 30.5 | | | | | +1.3 | 18 48.3 | +1.3 | 41 14.9 | 30.5 21.5 | 36.6 |
| (lost on chronograph sheet.) | 30.0 | 37.1 | 16.1 | 53.2 | +0 6 | +1.1 | | | | 29.5 20.5 | 34.4 |
| 42 53 7 | 29 5 | | | | | +0.8 | 17 06.9 | +0.9 | 42 55.4 | 29.0 20.0 | 39.2 |
| 43 44.6 | 29.0 | | | | | +0.5 | 16 16.0 | +0.8 | 43 45.9 | 28.5 19.5 | 40.2 |
| 44 35.9 | 28 5 | | | | | +0.3 | 15 24.7 | +0.7 | 44 36.9 | 28.0 19.0 | 36.9 |
| 45 31.9 | 28 0 | | | | | +0.0 | 14 28.7 | +0.5 | 45 32.4 | 26.5 17.5 | 36.8 |
| (mist) | 27.5 | | | | | −0.3 | | | | 26.0 17.0 | 37.5 |
| (mist) | 27.0 | | | | | −0.5 | | | | 25.5 16.5 | 36.2 |
| 48 08.0 | 26.5 | | | | | −0.8 | 11 52.6 | +0.3 | 48 07.5 | 25.0 16.0 | 29.6 |
| 48 57.1 | 26.0 | 38.3 | 17.5 | 55.8 | −0 6 | −1.1 | 11 03 5 | +0.2 | 48 56.2 | 24.5 15.5 | 33.1 |
| 49 50.2 | 25.5 | | | | | −1.1 | 10 10.4 | +0.2 | 49 49.3 | 23.5 14.5 | 36.5 |
| 50 47.0 | 25 0 | | | | | −1.0 | 9 13.6 | +0.1 | 50 46.1 | 23.0 14.0 | 31.3 |
| 51 42.3 | 24.5 | | | | | −1.0 | 8 18.3 | +0.1 | 51 41.4 | 22.5 13.5 | 35.2 |
| (mist) | 24.0 | | | | | −0.9 | | | | | 15 35.80=mean |

*Determination of value of one revolution of micrometer belonging to Z. T. No. 534, by observations on 51 Cephei near eastern elongation, November 15, 1890, Rapid, S. Dak.*—Continued.

| Time of observation (recorded on chronograph sheets.) | Micrometer revolutions. | Level. N. | Level. S. | Level N + S. | Reduction to mean level. | Correction for change of level. | Time from elongation. = T. | Correction for T. | Reduced time. |
|---|---|---|---|---|---|---|---|---|---|
| *h. m. s.* | | *Div.* | *Div.* | *Div.* | *Div. s.* | *s.* | *m. s.* | *s.* | *h. m. s.* |
| 53 22.4 | 23.5 | 38.3 | 17.3 | 55.6 | −0.5 | −0.9 | 6 38.2 | +0.1 | 53 21.6 |
| 54 18.8 | 23.0 | | | | | −0.9 | 5 41.8 | +0.0 | 54 17.9 |
| 55 07.1 | 22.5 | | | | | −0.9 | 4 53.5 | 0.0 | 55 06.2 |
| 56 01.2 | 22.0 | | | | | −0.9 | 3 59.4 | 0.0 | 56 00.3 |
| 56 52.4 | 21.5 | 38.3 | 17.3 | 55.6 | −0.5 | −0.9 | 3 08.5 | 0.0 | 56 51.5 |
| 57 41.2 | 21.0 | | | | | −0.9 | 2 19.4 | 0.0 | 57 40.3 |
| 58 30.7 | 20.5 | | | | | −0.9 | 1 29.9 | 0.0 | 58 29.8 |
| 59 25.9 | 20.0 | | | | | −0.8 | − 0 34.7 | +0.0 | 59 25.1 |
| 1 00 17.9 | 19.5 | | | | | −0.8 | + 0 17.3 | −0.0 | 1 00 17.1 |
| 01 10.0 | 19.0 | | | | | −0.7 | 1 09.4 | 0.0 | 01 09.3 |
| (mist) | 18.5 | | | | | −0.6 | | 0.0 | |
| 02 52.8 | 18.0 | | | | | −0.6 | 2 52.2 | 0.0 | 02 52.2 |
| 03 44.8 | 17.5 | 38.0 | 17.0 | 55.0 | −0.3 | −0.5 | 3 44.2 | 0.0 | 03 44.3 |
| 04 34.2 | 17.0 | | | | | −0.5 | 4 33.6 | 0.0 | 04 33.7 |
| 05 25.9 | 16.5 | | | | | −0.4 | 5 25.3 | 0.0 | 05 25.5 |
| 06 16.0 | 16.0 | | | | | −0.3 | 6 15.4 | 0.0 | 06 15.7 |
| 07 14.8 | 15.5 | | | | | −0.2 | 7 14.2 | −0.1 | 07 14.5 |
| 08 05.1 | 15.0 | | | | | −0.1 | 8 04.5 | −0.1 | 08 04.9 |
| 08 58.2 | 14.5 | 37.8 | 16.8 | 54.6 | −0.0 | −0.0 | 8 57.6 | −0.1 | 08 58.1 |
| 09 49.1 | 14.0 | | | | | +0.3 | 9 48.5 | −0.2 | 09 49.2 |
| 10 41.0 | 13.5 | | | | | +0.6 | 10 40.4 | −0.2 | 10 41.4 |
| 11 35.0 | 13.0 | 37.2 | 16.2 | 53.4 | +0.5 | +0.9 | 11 34.4 | −0.3 | 11 35.6 |
| Mean level | | | | =54.5 | | | | | |

log . . . . . . . . . . . . . . . . 935.80 s. = 2.97118
log cos δ . . . . . . . . . . . . 8.68666
log 15 . . . . . . . . . . . . . . 1.17659
a. c. log 9 . . . . . . . . . . 9.04576

log $R$ . . . . . . . . . . . . . . 75″.804 = 1.87969
refraction . . . . . . . . . . . − .044

one revolution . . . = 75 .760

DIFF. REFRACTION.

log . . . . . . . . . . . . . . . . . . 57″.7 = 1.7612
log sin . . . . . . . . . . . . . . . 75 .8 = 6.5652
log sec $Z$ . . . . . . . . . . . . . 0.1568
log sec $Z$ . . . . . . . . . . . . . 0.1568

diff. refr . . . . . . . . = ″.044 log. = 8.6400

The time from elongation of the star requires a correction in order to reduce the curve in which the star apparently travels to a vertical line. The hour angle of the star is first obtained from the equation,

$$\cos t_0 = \cot \delta \tan \varphi,$$

$\delta$ being the star's declination and $\varphi$ the latitude.

The chronometer time of elongation, $T_0 = \alpha - t_0 - \delta t$, $\alpha$ being the right ascension of the star obtained from the Nautical Almanac, and $\delta t$ the error of the chronometer.

Having thus obtained the chronometric time of elongation, the correction in question is obtained from the observed interval of time of each observation before or after elongation, from tables in Appendix No. 14, United States Coast and Geodetic Survey Report for 1880, pp. 256 and 257. A discussion of this subject will be found in the appendix above referred to and in Chauvenet's Practical Astronomy, vol. 2, pp. 360 to 364.

The times of observation thus corrected for level, and distance from elongation, are then grouped in pairs, selected as being a certain number of revolutions of the micrometer apart, and the time intervals between the members of each pair are obtained. The mean of these, divided by the number of revolutions which separate the members of each pair, is yet to be corrected for differential refraction, which is derived from the equation,

$$\text{refraction} = 57''.7 \sin R \sec^2 Z,$$

$R$ being the value of a division of the micrometer and $Z$ the zenith distance of the star. Four-place logarithms are sufficient for computing this correction, as it is small. Herewith is an example of record and computation of the value of a revolution of the micrometer of combined instrument No. 534, one of the two in possession of the United States Geological Survey.

## MEASUREMENT OF A LEVEL DIVISION.

The value of a division of the level is commonly measured with a level trier. The latitude level, however, may be easily tested by means of the micrometer, the value of a revolution being obtained by the following method:

Point the telescope upon some well-defined terrestrial mark and set the level at an extreme reading near one end of the tube. Set the movable thread upon the object and read the micrometer and the level.

Now move the telescope and level until the bubble is near the other end of the tube. Again set the movable thread upon the object and again read both micrometer and level. It is evident that the micrometer and the level have measured the same angle, and that the ratio between these readings equals that between a revolution of the micrometer and a level division.

An example illustrative of this is appended.

*Determination of value of 1 division of latitude level No. 534.*

[By comparison with micrometer screw 534.]

| Micrometer. | Level. | | Difference. | | aa. | ab. |
|---|---|---|---|---|---|---|
| | N. | S. | Microm. | Level. | | |
| *r.* | *d.* | *d.* | *b.* | *a.* | | |
| 8.025 | 47.3 | 29.2 | *d.* | *d.* | | |
| 8.508 | 20.7 | 02.7 | 48.3 | 26.55 | 704.9 | 1283. |
| 8.509 | 18.9 | 01.0 | | | | |
| 7.984 | 49.8 | 31.0 | 52.5 | 30.45 | 927.2 | 1599. |
| 8.511 | 18.5 | 00.6 | | | | |
| 8.045 | 47.2 | 29.1 | 46.6 | 28.60 | 818.0 | 1333. |
| 9.076 | 18.7 | 00.8 | | | | |
| 8.604 | 46.0 | 28.0 | 47.2 | 27.25 | 742.6 | 1286. |
| 9.442 | 23.7 | 06.0 | | | | |
| 9.009 | 48.0 | 30.0 | 43.3 | 24.15 | 583.2 | 1046. |
| 10.055 | 21.8 | 04.0 | | | | |
| 9.574 | 48.0 | 30.1 | 48.1 | 26.15 | 683.8 | 1258. |
| 10.661 | 24.0 | 06.1 | | | | |
| 10.212 | 50.7 | 33.0 | 44.9 | 26.80 | 718.2 | 1203. |
| 11.771 | 18.3 | 00.7 | | | | |
| 11.252 | 48.3 | 31.9 | 51.9 | 30.60 | 936.4 | 1588. |
| 12.328 | 20.0 | 02.3 | | | | |
| 11.872 | 46.1 | 28.5 | 45.6 | 26.15 | 683.8 | 1192. |
| 12.869 | 22.2 | 04.6 | | | | |
| 12.438 | 47.7 | 30.0 | 43.1 | 25.45 | 647.7 | 1097. |
| 13.468 | 23.0 | 05.3 | | | | |
| 13.080 | 44.5 | 26.9 | 38.8 | 21.55 | 464.4 | 836. |
| 14.146 | 20.1 | 02.4 | | | | |
| 13.702 | 45.4 | 27.8 | 44.4 | 25.35 | 642.6 | 1125. |
| 14.758 | 22.3 | 04.8 | | | | |
| 14.282 | 48.6 | 31.0 | 47.6 | 26.25 | 689.1 | 1249. |
| Sum | ........ | ........ | .......... | .......... | 9241.9 | 16095. |

log.....................16095. =4.20669.
a. c. log.............. 9241.9=6.03424.
log 1 div. micrometer.........=9.87966.

1 div. level......=1″.320 log.=0.12059.

## COMPUTATION OF APPARENT DECLINATION OF STARS.

After the determination of the constants of the instrument used, the next step is to obtain the apparent declinations of the stars used. Whenever possible, these should be taken from the Nautical Almanac or the Berliner Jahrbuch. In other cases they must be computed. The positions of stars are given in Safford's Catalog for the epoch 1875.0, together with the annual precession and proper motion. The declinations there given should be revised by the aid of more recent catalogs, particularly with reference to stars of class C. The annual precession and proper motion multiplied by the number of years which have elapsed, and applied, together with the effect of secular variation in precession, give the declination at the beginning of the year. Further corrections to bring the positions down to the date of observation are exprest by the symbols $Aa'$, $Bb'$, $Cc'$, $Dd'$. Logarithms of $a'$, $b'$, $c'$, $d'$ are given in Safford's Catalog, and A, B, C, and D are given in the Nautical Almanac. A slight additional correction is also to be made for proper motion for the elapsed portion of the year. This reduction is illustrated below:

*Example of reduction. Computation of apparent declination of star 1539.*

[From Safford's Catalog, p. 40.]

| | Declination 1875.0 | Annual precession. | Proper motion. |
|---|---|---|---|
| | ° ′ ″ | ″ | ″ |
| Star No. 1539 | 45 33 29.20 | +18.87 | −.03 |

| | ° ′ ″ | |
|---|---|---|
| (1890–1875) 15×18″.87= | + 4 43.05 | = Precession for 15 years. |
| 15×−″.03= | − 0 00.45 | = Proper motion for 15 years. |
| | + 0 00.07 | = Secular variation in precession. |
| | 45 38 11.87 | = Declination 1890. |
| | + 9.38 | = $Aa'$ |
| | − 0.78 | = $Bb'$ |
| | + 6.88 | = $Cc'$ |
| | +10.16 | = $Dd'$ |
| | − 0.03 | = Proper motion, Jan. 1—Nov. 9, 1890. |
| | 45 38 37.48 | = Declination Nov. 9, 1890. |

| | | | | |
|---|---|---|---|---|
| | $\log a'=1.2757$ | $\log b'=9.5294$ | $\log c'=9.7367$ | $\log d'=9.8273$ |
| Nov. 9. | $\log A=9.6966$ | $\log B=0.3649_n$ | $\log C=1.1006$ | $\log D=1.1796$ |
| | 0.9723 | $9.8943_n$ | 0.8373 | 1.0096 |
| | ″ | ″ | ″ | ″ |
| | $Aa'=+9.38$ | $Bb'=-0.78$ | $Cc'=+6.88$ | $Dd'=+10.16$ |

With all this preliminary work done, the reduction proper of latitude observations is comparatively simple. The observations being grouped by pairs, the mean declination of each pair is obtained, the corrections for difference of micrometer readings and levels are applied, with a small correction for differential refraction, and the result is the desired latitude.

Following is an example of the reduction of six pairs of stars observed for latitude at Rapid, S. Dak.:

*Example of reduction.*

[Station: Rapid, S. Dak. November 9, 1890. Half rev. micrometer=37″.900. One div. level=1″.33.]

| Date. | Star Nos. | $\delta_1$ | $\delta_2$ | $\frac{1}{2}(\delta_1+\delta_2)$ | Corrections. Microm. | Corrections. Level. | Corrections. Refr. | Latitude n. | Weight p. | p. n. |
|---|---|---|---|---|---|---|---|---|---|---|
| | | ° ′ ″ | ° ′ ″ | ° ′ ″ | ′ ″ | ″ | ″ | ° ′ ″ | | ″ |
| Nov. 9. | 7 Lacert and 10 Lacert. | 49 42 87.33 | 38 29 04.60 | 44 06 15.97 | − 1 23.53 | −6.51 | −0.03 | 44 04 45.90 | 0.98 | 5.78 |
| | 1539 1551 | 45 38 37.48 | 42 44 04.65 | 11 21.06 | − 6 31.77 | −2.06 | − .11 | 47.12 | .90 | 6.41 |
| | 1565 1579 | 38 43 39.78 | 49 27 41.04 | 05 40.41 | − 0 58.33 | +0.46 | − .03 | 42.51 | .79 | 1.98 |
| | 1600 1633 | 56 34 06.66 | 31 55 56.91 | 15 01.78 | −10 13.25 | −3.78 | − .19 | 44.56 | .90 | 4.10 |
| | 1676 1686 | 67 12 10.93 | 21 03 54.02 | 08 02.48 | − 3 08.43 | −7.44 | − .07 | 46.54 | .93 | 6.08 |
| | 1702 1722 | 24 32 09.04 | 63 35 27.34 | 03 48.19 | + 1 01.51 | −3.22 | + .02 | 46.50 | .90 | 5.85 |
| | | | | | | | | | 5.40 | 30.20 |

November 9. Weighted mean=44° 04′ 45″.59.

## OBSERVATIONS FOR TIME.

With the transit mounted, leveled, and adjusted in the meridian, the chronograph set up and running and connected in a circuit with the battery, and the chronometer and observing key connected in the same circuit, the observer is prepared to begin time observations.

The list of stars which should be used is that given in the Berliner Jahrbuch, as the list is fuller and more accurate than that in any other catalog which gives day places. Stars should be so selected north and south of the zenith that the azimuth errors will balance one another as nearly as possible, as is explained hereafter. On the approach of the selected star to the meridian, the telescope should be set by means of the vertical circle upon the altitude of the star above the horizon, deduced from the declination and the latitude. As the star crosses each thread in the reticule, the fact should be recorded by pressing the observing key, which produces, as previously described, a record upon the chronograph sheet. In this way four time stars, as stars between the equator and zenith are designated, and one circumpolar star, or a star so near the pole that it is constantly in sight, should be observed. Then the telescope should be reversed in the wyes and a similar set of stars observed. Between observations upon any two stars the striding level should be placed upon the pivots of the instrument and readings taken to ascertain the departure of the axis from a horizontal position.

In order to avoid unequal expansion of the pivots from unequal heating both bull's-eye lamps must be lighted and placed in their stands.

After the comparison of chronometers at the two stations, to be hereafter described, a similar set of stars should be observed, if possible.

## REDUCTION OF TIME OBSERVATIONS.

Certain constants of the transit should be measured before proceeding with the reduction of time observations. The value of a division of the striding level should be measured by means of a level trier. The equatorial interval of time between each of the threads and the mean of all the threads should be obtained, as it is not infrequently needed in utilizing broken or imperfect observations. These can best be obtained from observations on slow-moving stars, but any stars may be used for the purpose. The intervals as observed are reduced to the equator by multiplying them by the cosine of the declination of the star observed.

The object of these observations is specifically the determination of the error of the chronometer. This error equals the right ascension of a star minus its observed time of transit, corrected for certain instrumental errors. These errors are as follows:

### CORRECTION FOR ERROR OF LEVEL.

The level error, designated by $b$, is ascertained from the readings of the striding level. The value of a division of the level in seconds of time must have been previously ascertained by means of a level trier. The effect of the level error is greatest at the zenith and diminishes to zero at the horizon. The correction in seconds of time is given by the equation,

$$\text{cor.} = b \cos(\varphi - \delta) \sec \delta = b\text{B}.$$

When the declination is north it is to be regarded as having a plus sign for upper and a minus sign for lower culmination; when south it is negative.

### CORRECTION FOR INEQUALITY OF PIVOTS.

This correction can be made a part of the level correction.

Let $p$ = the inequality of pivots,

B = inclination of axis given by level for clamp west,

B′ = inclination of axis given by level for clamp east,

$b$ = true inclination of axis for clamp west,

$b'$ = true inclination of axis for clamp east;

then $p = \dfrac{\text{B}' - \text{B}}{4}$,

$b = \text{B} + p$ for clamp west,

$b' = \text{B}' - p$ for clamp east.

(Chauvenet, vol. 2, p. 155.)

### CORRECTION FOR ERROR OF COLLIMATION.

This correction, designated by $c$, is the departure of the mean of the threads from the optical axis of the telescope. For stars at upper culmination, with clamp west it is plus when the mean of the threads is east of the axis, and minus when it is west of it; for stars at lower culmination the reverse is the case. The value of $c$ is one-half the difference between the clock error indicated by stars observed before and after reversal of the instrument, divided by the mean secant of the declinations of the stars. This is slightly complicated with the azimuth, altho the effect of that is largely eliminated by the proper selection of stars; consequently it is to be obtained by approximations in conjunction with the azimuth errors. The correction to be applied to each star equals $c \sec \delta = c\text{C}$, which is plus for a star at upper culmination and minus for a star at lower culmination. It is least for equatorial stars and increases with the secant of the declination.

### CORRECTION FOR DEVIATION IN AZIMUTH.

This correction, designated by $a$, represents the error in the setting of the instrument in the meridian. Its effect is zero at the zenith and increases toward the horizon. Since the instrument is liable to be disturbed during the operation of reversal, it is necessary to determine the azimuth error, both before and after reversal, separately. A comparison of the clock error, determined from observations upon north and south stars, will furnish the data necessary for the determination of azimuth. Practically, it is determined by elimination from equations involving the mean of all these stars observed in each of the two positions of the instrument, after correcting for level, and as it is slightly complicated with collimation it must be reached by two or more approximations. The error is essentially positive when the telescope points east of south, and negative when west of south. The correction applicable to any star is exprest in the equation,

$$\text{cor.} = a \sin (\varphi - \delta) \sec \delta = a\text{A}.$$

It must be understood that the declination when north is positive for upper and negative for lower culmination, and that with south declination it is negative.

### CORRECTION FOR DIURNAL ABERRATION.

The right ascension of stars, as taken from the Berliner Jahrbuch, must be corrected for diurnal aberration, which equals $0^s.021 \cos \varphi \sec \delta$. This correction is positive for upper and negative for lower culmination.

These corrections are summarized in the equation,

$$\delta t = \alpha - (t + a\text{A} + b\text{B} + c\text{C}).$$

A, B, C, as seen above, are constants, depending upon the latitude of the place of observation and the declination of the star. Tables for these quantities will be found in a special publication of the United States Coast and Geodetic Survey for 1874.

### EXAMPLE OF REDUCTION OF TIME OBSERVATIONS.

The following is an example of the form for record of observation and reduction of time observations, taken from a campaign for the determination of the position of Rapid, S. Dak.

*Time determination: Example of record.*

[Rapid, S. Dak., November 20, 1890. Fauth transit, No. 534. Sidereal chronometer: Bond & Sons, No. 187. 1 division of level = $0^s.118$. Hourly rate of chronometer = $0^s.133$.]

| Star | γ Cephei. | φ Pegasi. | ω Piscium. | 33 Piscium. | α Androm. | |
|---|---|---|---|---|---|---|
| Clamp | W. | W. | W. | W. | W. | W. |
| Level | Telescope north.<br>*W.* *Sum.* *E.*<br>19.8 −88.1 68.3<br>68.2 +87.6 19.4 | Telescope south.<br>*W.* *Sum.* *E.*<br>68.0 +87.1 19.1<br>20.2 −89.2 69.0 | Telescope south.<br>*W.* *Sum.* *E.*<br>20.0 −89.5 69.5<br>68.8 +87.2 18.4 | Telescope south.<br>*W.* *Sum.* *E.*<br>68.2 +86.9 18.7<br>19.9 −89.4 69.5 | Telescope south.<br>*W.* *Sum.* *E.*<br>19.8 −89.3 69.5<br>68.3 +86.8 18.5 | Telescope north.<br>*W.* *Sum.* *E.*<br>19.7 −89.5 69.8<br>68.8 +87.3 18.5 |
| Difference | − 0.5 | − 2.1 | − 2.3 | −2 .5 | − 2.5 | − 2.2 |
| | *h.* *m.* *s.* | *h.* *m.* *s.* | *h.* *m.* *s.* | *h.* *m.* *s.* | *h.* *m.* *s.* | |
| Thread I | 23 34 52.25 | 23 47 24.00 | 23 54 10.89 | 00 00 13.33 | 00 03 12.00 | |
| II | 35 11.40 | 28.55 | 14.88 | 17.96 | 16.83 | |
| III | 29.41 | 32.72 | 19.22 | 21.94 | 21.32 | |
| IV | 46.78 | 36.75 | 23.14 | 25.95 | 26.00 | |
| V | 36 05.00 | 41.09 | 27.20 | 29.83 | 30.85 | |
| Sum | 4.84 | 3.11 | 5.33 | 9.01 | 7.00 | |
| Mean | 23 35 28.97 | 23 47 32.62 | 23 54 19.07 | 00 00 21.80 | 00 03 21.40 | |
| Aberration | −.07 | −.02 | −.02 | −.02 | −.02 | |
| Correction for level ($b$ B) | −.22 | −.06 | −.05 | −.04 | −.06 | |
| Correction for rate | +.05 | +.03 | +.01 | +.00 | +.00 | |
| Reduced transit ($t$) | 23 35 28.83 | 23 47 32.57 | 23 54 19.01 | 00 00 21.74 | 00 03 21.32 | |
| Tabular R. A ($a$) | 23 34 53.13 | 46 55.67 | 53 41.98 | 23 59 44.61 | 00 02 44.42 | |
| $a-t$ | −35.70 | −36.90 | −37.03 | −37.13 | −36.90 | |

$$\text{Mean of levels} = -\frac{2.02^{div.}}{4} \times .118^{s.} = -.0596^{s.} = b. \quad \text{Inequality of pivots} = .00^{s.}$$

*Time determination: Example of record*—Continued.

| | | | | | |
|---|---|---|---|---|---|
| Star | γ Pegasi. | Br. 6. | ι Ceti. | 44 Piscium. | 12 Ceti. |
| Clamp | E. | E. | E. | E. | E. |
| Level | Telescope south.<br>*W.* *Sum.* *E.*<br>19.2 −88.3 69.1<br>68.9 +87.8 18.9 | Telescope south.<br>*W.* *Sum.* *E.*<br>68.7 +87.3 18.6<br>19.4 −88.7 69.3 | Telescope south.<br>*W.* *Sum.* *E.*<br>19.2 −88.4 69.2<br>68.5 +86.7 18.2 | Telescope north.<br>*W.* *Sum.* *E.*<br>68.9 +87.8 18.9<br>18.9 −87.9 69.0 | |
| Difference | − 0.5 | − 1.4 | − 1.7 | − 0.1 | |
| | *h.* *m.* *s.* | *h.* *m.* *s.* | *h.* *m.* *s.* | *h.* *m.* *s.* | *h.* *m.* *s.* |
| Thread V | 00 08 05.25 | 00 10 05.00 | 00 14 20.70 | 00 20 17.35 | 00 24 56.85 |
| IV | 09.30 | 22.81 | 24.68 | 20.84 | 25 00.73 |
| III | 13.54 | 39.30 | 28.52 | 24.93 | 05.37 |
| II | 17.65 | 56.90 | 32.90 | 29.16 | 09.15 |
| I | 22.00 | 11 15.49 | 37.23 | 33.42 | 13.07 |
| Sum | 7.74 | 9.50 | 4.03 | 5.70 | 5.17 |
| Mean | 00 08 13.55 | 39.90 | 28.81 | 25.14 | 05.03 |
| Correction for aberration | −.02 | −.06 | −.02 | −.02 | −.02 |
| Correction for level (*b* B) | −.02 | −.09 | −.02 | −.02 | −.02 |
| Correction for rate | −.02 | −.02 | −.03 | −.04 | −.05 |
| Reduced transit (*t*) | 00 08 13.49 | 00 10 39.73 | 00 14 28.74 | 00 20 25.06 | 00 25 04.94 |
| Tabular R. A (*a*) | 00 07 36.59 | 10 03.56 | 00 13 51.75 | 00 19 48.17 | 00 24 27.91 |
| $a-t$ | −36.90 | −36.17 | −36.99 | −36.89 | −37.03 |

Mean of levels $= -\frac{.925}{4}\overset{div.}{} \times \overset{s.}{.118} = -\overset{s.}{.027} = b$. Inequality of pivots $= \overset{s.}{.00}$.

*Example of reduction.*

[Rapid, S Dak., November 20, 1890. After exchange of clock signals.]

| Clamp. | 1. | 2. Star. | 3. A. | 4. C. | 5. a−t | 6. Cc. | 7. a−t Corrected for Cc. | 8. Aa$_w$. | 9. Corrected for Aa$_w$. | 10. Cc′. | 11. | 12. Aa′$_w$. | 13. | V. | t. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Wt: $\delta t_o$ | | | | s. | s. c=−.019 s. | s. | s. | s. | s. C′+.015 s. | s. | | s. | s. | h. |
| W. | 1 | γ Cephei | −2.42 | +4.45 | − 35.70 | −.08 | −35.78 | −1.00 | −36.78 | +.07 | −36.71 | −.05 | −36.76 | .00 | 23.58 |
| | 1 | φ Pegasi | +0.45 | 1.05 | 36.90 | −.02 | 36.92 | +0.19 | .73 | .01 | .73 | +.01 | .71 | −.05 | .78 |
| | 1 | ω Piscium | +0.62 | 1.01 | 37.03 | −.02 | 37.05 | +0.26 | .79 | .02 | .77 | +.01 | .76 | .00 | .89 |
| | 1 | 33 Piscium | +0.77 | 1.01 | 37.13 | −.02 | 37.15 | +0.32 | .83 | .01 | .82 | +.02 | .80 | +.04 | .99 |
| | 1 | α Androm | +0.30 | +1.14 | − 36.90 | −.02 | −36.92 | +0.12 | −36.80 | +.02 | .78 | +.01 | .77 | +.01 | 24.01 |
| | | Sum = | +2.14 | | | | | | | | | | 3.80 | | |
| | | Sum = | −2.42 | +8.66 | −183.66 | | | | 3.93 | | | | | | |
| | | (1) Mean = | −0.06 Aw | +1.73c | − 36.732 | | | | −36.786 | | | | −36.760 | | |
| | | | | | | | | Aa. | | | | Aa′$_w$. | | | |
| E. | 1 | γ Pegasi | +0.51 | −1.03 | − 36.90 | +.02 | −36.88 | +0.15 | −36.73 | −.01 | −36.74 | −.01 | −36.75 | −.01 | 24.13 |
| | 1 | Bradley 6 | −2.26 | 4.23 | 36.17 | .08 | 36.09 | − .64 | .73 | .06 | .79 | +.03 | .76 | .00 | .17 |
| | 1 | ι Ceti | +0.81 | 1.01 | 36.99 | .02 | 36.97 | + .23 | .74 | .02 | .76 | −.01 | .77 | +.01 | .23 |
| | 1 | 44 Piscium | +0.68 | 1.00 | 36.89 | .02 | 36.87 | + .19 | .68 | .01 | .69 | −.01 | .70 | −.06 | .33 |
| | 1 | 12 Ceti | +0.75 | −1.00 | − 37.03 | +.02 | −37.01 | + .21 | −36.80 | −.02 | .82 | −.01 | .83 | +.07 | .41 |
| | | Sum = | +2.75 | | | | | | | | | | 3.81 | | |
| | | Sum = | −2.26 | −8.27 | −183.98 | | | | 3.68 | | | | | Mean | =24.05 |
| | | (2) Mean = | +0.10 Ae | −1.65c | − 36.796 | | | | −36.736 | | | | −36.762 | | |

Subtracting (2) from (1), ignoring azimuth terms which are small, we have:

$$+3.38c + .064^{s} = 0$$

Approx .....$c = -.019^{s}$

From below.$c' = +.015^{s}$ Adopted..$c = -.004^{s}$

$$\begin{array}{ll} -2.42\,Aw - 35.78^{s} = 0 & -2.26\,Ae - 36.09^{s} = 0 \\ +0.54 \quad\;\; -37.01 = 0 & +0.69 \quad\;\; -36.93 = 0 \\ \hline +2.96\,Aw - 1.23^{s} = 0 & +2.95\,Ae - 0.84^{s} = 0 \\ Aw = +.415^{s} & Ae = +.285^{s} \end{array}$$

Forming equations to determine c′ from columns 4 and 9:

$$\begin{array}{l} +1.73c' - 36.786^{s} = 0 \\ -1.65 \quad - 36.736 = 0 \\ +3.38c' - 0.050^{s} = 0 \\ c' = +.015^{s} \end{array}$$

Forming equations from columns 3 and 11:

$$\begin{array}{ll} -2.42\,A'w - 36.710^{s} = 0 & -2.26\,A'e - 36.790^{s} = 0 \\ +0.54\,A'w - 36.772 = 0 & +0.69 \quad\;\; -36.752 = 0 \\ \hline +2.96\,A'w - 0.062^{s} = 0 & +2.95\,A'e + 0.038^{s} = 0 \\ A'w = +.021^{s} & A'e = -.013^{s} \\ Aw = +.415 & Ae = +.285 \end{array}$$

Adopted.$Aw = +.436^{s}$ Adopted.$Ae = +.272^{s}$
Adopted...$C = -.004$

Normal equation, formed from columns 1, 3, 4, and 5

$$10\delta t - .28Aw + .49Ae + .39c + 367.64^{s} = 0$$

$$\delta t - .028Aw + .049Ae + .039c + 36.764^{s} = 0$$

$$\delta t = -36.76^{s} \quad \text{at } 24.05^{h}$$

$$\begin{array}{rr} \alpha - t = & -36.764^{s} \\ -.028 \times +.436^{s} = & -.012 \\ +.049 \times +.272^{s} = & +.013 \\ +.039 \times -.004^{s} = & -.000 \\ \hline \delta t = & -36.763 \end{array}$$

## COMPARISON OF TIME.

After time has been thus observed the chronometers at the two stations should be compared by telegraph.

Chronometers are compared in the following manner: The chronometer at one station being in circuit with the chronograph and recording upon it, the chronometer at the other station is switched into the general telegraphic circuit, by which it is brought to the first station and switched into the local circuit there, so that the two chronometers register upon the same chronograph, their beats being marked side by side by the same pen. After this has gone on for a minute or more the operation is reversed, the chronometer at the first station being switched into the telegraphic circuit and made to record upon the chronograph with the chronometer at the second station. Of course the observers are informed of the hour and minute at which the joint record upon the several chronographs begins.

This method constitutes what is known as the *automatic* exchange of signals.

The *arbitrary* exchange of signals is made as follows:

Each chronometer recording on its own chronograph as usual, and each local circuit being connected with the main-line circuit, the observer at one station breaks the circuit by means of the main-line talking key, which break is recorded on the chronograph sheets at both stations. The breaks are repeated every two seconds for at least one full minute. The operation is then reversed by the observer at the second station making the breaks, which are recorded at both stations as before.

The differences of time between the chronometers at the two stations are read from the chronograph sheets at each station and corrected for error of the chronometers. The results from the two chronograph sheets differ by an amount equal to twice the time occupied in transmission of signals. The mean of the two is therefore the approximate difference of longitude.

This result is yet to be corrected for personal equation, or the difference between the errors of observing of the two observers. Every observer has the habit of recording a transit a little too early or too late, the difference between two observers not infrequently being as great as a fourth of a second. To measure this difference, the observers usually meet, preferably at the known station, both before and after the campaign, and observe for a time each with his own instrument or with one similar in all respects to that used in the campaign. A comparison of the time determinations made by the two observers gives an approximation to the personal equation.

A better method, but one not always practicable, is for the observers, having completed half of the observations for time and longitude, to exchange stations for the remainder of the work. The mean of the results before and after exchange of stations will eliminate personal equation.

There is one error incident to this work which can not be eliminated. This is the unequal attraction of gravity, or local attraction, or, as it is sometimes called, station error. A neighboring mountain mass will attract the plumb line and deflect the spirit level to such an extent as to cause serious errors in astronomic determinations of latitude and time. The same result is frequently produced by a difference in density of the underlying strata of rock, so that station errors of magnitude often appear where they are not expected. Indeed, the station error can not be predicted with any certainty as to amount or direction.

The only practical method of even partially eliminating this error is to select a number of stations for astronomic location under conditions as widely diverse as possible, connect them by triangulation, and by this means reduce all these astronomic determinations to one point, thus obtaining for this point a number of determinations each having a different station error. The mean of these gives for this point a position from which—in part, at least—station error has been eliminated, and this mean position can be transferred back to the several astronomic stations by means of the triangulation, thus giving each of them a position similarly comparatively free from station error.

## OBSERVATIONS FOR AZIMUTH.

The initial direction from which the directions of other lines in primary triangulation and in primary traversing are computed is obtained by means of astronomic observations. Such observations should be taken not only upon the initial line, but at intervals thruout the work for its verification. Such intervals should not exceed 100 miles in the primary triangulation and 10 to 20 miles in primary traversing.

Azimuth observations are made with the theodolite used in primary triangulation or traverse. The observations consist in the measurement of the horizontal angle between some close circumpolar star, usually Polaris, and a terrestrial mark, generally a lantern set at a distance of half a mile to a mile from the observing station. The time of observation on the star should be noted by a chronometer or a good watch. As the star is at a much higher angle of elevation than the lamp, it is necessary not only to level the instrument carefully but to measure the error of level and to correct for it. It is therefore essential that the value of a division of the level bulb be known. These observations for azimuth may be made at any time of the night, but it is very important that they be made at or near the time of elongation of the star, as it is then moving most slowly in azimuth, and any error in the time of observation has the least effect on the resulting azimuth. If such observations be taken at elongation, the reduction of the observations is simplified. When such observations are made at any other time than at elongation, the time must be noted, as it forms an element in the reduction. The error of the clock or watch used can be obtained by comparison with standard time, making correction for the difference in longitude between the station and the meridian of the standard time. A form of observation and record is appended.

*Example of record of azimuth observations.*

[Station: West base, near Little Rock, Ark. Fauth 8″ theod. No. 300. December 27, 1888. 1 div. micr.=2″. 1 div. level=3″.]

| Object. | Time P. M. | Level. | | Micrometer. | | Mean. | Angle. |
|---|---|---|---|---|---|---|---|
| | | West end. | East end. | A. | B. | | |
| | | | | Telescope direct. | | | |
| | *h. m. s.* | *Div.* | *Div.* | ° ′ *Div.* | ° ′ *Div.* | ° ′ ″ | |
| Polaris | 11 00 18 | 13.9 | 47.1 | 346 00 14.8 | 165 58 25.1 | 345 59 39.9 | |
| | | 50.5 | 10.2 | | | | ° ′ ″ |
| | | 64.4 | 57.3 | | | | 115 32 30.0 |
| | | +7.1 | | | | | |
| East base (mark) | ........ | ........ | | 101 32 18.1 | 281 31 21.8 | 101 32 09.9 | |
| East base (mark) | ........ | ........ | | 101 32 19.8 | 281 31 19.7 | 101 32 09.5 | 115 34 16.1 |
| Polaris | 11 09 20 | 50.4 | 10.3 | 345 58 22.0 | 165 57 01.4 | 345 57 53.4 | |
| | | 13.8 | 46.5 | | | | |
| | | 64.2 | 56.8 | | | | |
| | | +7.4 | | | | | |
| | | | | Telescope reverse. | | | |
| Polaris | 11 17 14 | 50.5 | 10.1 | 211 28 29.0 | 31 27 23.4 | 211 28 22.4 | |
| | | 12.9 | 46.6 | | | | |
| | | 63.4 | 56.7 | | | | 115 35 53.8 |
| | | +6.7 | | | | | |
| East base (mark) | ........ | ........ | | 327 05 06.7 | 147 03 09.5 | 327 04 16.2 | |
| East base (mark) | ........ | ........ | | 327 04 26.3 | 147 03 00.6 | 327 03 56.9 | 115 37 08.8 |
| Polaris | 11 26 22 | 14.3 | 46.3 | 211 27 10.7 | 31 26 07.4 | 211 26 48.1 | |
| | | 50.1 | 10.5 | | | | |
| | | 64.4 | 56.8 | | | | |
| | | +7.6 | | | | | |

*Summary of results of azimuth observations.*

[Station: West base, Arkansas. December 27, 1888.]

| Individual results. | | | | | Combined results. | | |
|---|---|---|---|---|---|---|---|
| | ° | ′ | ″ | ″ | ° | ′ | ″ |
| First set | 294 | 10 | 34.2<br>36.3 | 35.25 D. | | | 38.80 |
| | | | 49.9<br>34.8 | 42.35 R. | | | |
| | | | 35.9<br>46.3 | 41.10 R. | | | 39.38 |
| | | | 41.8<br>33.5 | 37.65 D. | | | |
| Second set | | | 42.4<br>45.4 | 43.90 D. | | | 38.75 |
| | | | 26.4<br>40.8 | 33.60 R. | | | |
| | | | 49.1<br>45.0 | 47.05 R. | | | 40.10 |
| | | | 40.3<br>26.0 | 33.15 D. | | | |
| Grand mean | | | | | 294 | 10 | 39.26 |

## REDUCTION OF OBSERVATIONS FOR AZIMUTH.

The time of observation of a star should be first corrected for the difference in longitude, assuming that standard time has been used, and for the error of the watch. It is then reduced from mean to sidereal time. From the sidereal time of observation should be subtracted the right ascension of Polaris, if that star is used, which is given in the Nautical Almanac, the result being the hour angle or the sidereal time which has elapsed since it past the meridian of the place of observation, given in hours, minutes, and seconds. This result should be converted into degrees, minutes, and seconds.

$$\text{Then } \tan A = -\frac{a \sin t}{1 - b \cos t};$$

where $a = \sec \varphi \cot \delta$ ($\varphi$ = the latitude),

$$b = \frac{\tan \varphi}{\tan \delta} (\delta = \text{the declination of star}),$$

$t$ = hour angle,

and A = angle between true north and the star.

This angle should be corrected for level as follows:

$$\text{level correction} = -\frac{d}{4}\{w+w')-(e+e')\}\tan h;$$

$d$ being the value of a division of the level;
$w+w'$, readings of west end of level bubble;
$e+e'$, readings of east end of level bubble;
$h$, the angular elevation of pole star.

An example of reduction is as follows:

*Example of reduction of azimuth observations.*

[Station: West base; December 27, 1888. Observer, S. S. G. Latitude=34° 45′ 26″.8. Longitude 92° 13′ 31″.5.]

| | | h. | m. | s. |
|---|---|---|---|---|
| Time of observation | $Tw=$ | 11 | 00 | 18 |
| Correction; ninetieth meridian time to 92°.215 | $=-$ | | 8 | 54 |
| Watch slow; ninetieth meridian time | + | | | 02 |
| Local mean time | $Tm=$ | 10 | 51 | 26 |
| Correction; mean to sidereal time | = | | +1 | 47 |
| Right ascension mean sun | | 18 | 26 | 36 |
| Sidereal time of observation | = | 29 | 19 | 49 |
| R. A. Polaris | − | 1 | 18 | 25 |
| Hour angle | $t=$ | 28 | 01 | 24 |
| | | −24 | | |
| | | h. | m. | s. |
| | $t$ (time)= | 4 | 01 | 24 |
| | | ° | ′ | ″ |
| | $t$ (arc) = | 60 | 21 | 00 |

$$\tan A = -\frac{a \sin t}{1-b\cos t} \text{ where } a=\sec\phi\cot\delta \text{ and } b=\frac{\tan\phi}{\tan\delta}$$

| | ° ′ ″ | | | |
|---|---|---|---|---|
| $\phi=34$ 45 26.8 | | log sec =0.0853539 | log tan..34° 45′ 26″.8 | =9.8413076 |
| $\delta=88$ 43 11.9 | | log cot.=8.3491690 | log tan..88 43 11.9 | =1.6508310 |
| log $a$ | | =8.4345229 | log $b$ | =8.1904766 |
| log sin $t$ | 60 21 00 | =9.9390515 | log cos $t$ | =9.6943423 |
| log $a$ sin $t$ | | =8.3735744 | log−.0076704 | =7.8848189 |
| log $(1-b\cos t)$ | | =9.9966559 | + 1.0000000 | |
| log tan A | 178 38 08.0 | =8.3769185 | 0.9923296 | $=1-b\cos t$ |
| angle to mark | +115 32 30.0 | | | |
| Level correction | −3.8 | level corr.$=-\frac{d}{4}\{(w+w')-(e-e')\}\tan h.$ | | |
| Az. of mark | =294 10 34.2 | $=\frac{3''.1}{4}\times 7.1 \text{ Div.} \times .694 = -3''.8$ | | |

When observations for azimuth are to be made at elongation, it is necessary to know the mean time of elongation. This is computed by the following method:

The hour angle at elongation is obtained from the equation,

$$\cos t_e = \tan \varphi \cot \delta.$$

The hour angle plus the right ascension of the star gives the sidereal time of its western elongation, which, reduced to mean time, gives the local mean time in question.

The azimuth of a pole star at elongation is determined by the use of the equation,

$$\sin A = \sec \varphi \cos \delta.$$

The following is an example of these computations:

*Example of the computation of the azimuth at elongation, and the local mean times of both elongations of Polaris.*

[Latitude $=\phi=40°$. Meridian of Washington. November 28, 1891.]

Sine azimuth at elongation $=\sec\phi\cos\delta$.

| | ° | ′ | ″ | |
|---|---|---|---|---|
| log sec 40° | | | | =0.1157460 |
| log cos δ | 88 | 44 | 05.5 | =8.3439803 |
| log sin A | 1 | 39 | 05.8 | =8.4597263 |

Cos hour angle at elongation, $t_e$, $=\tan\phi\cot\delta$.

| | ° | ′ | ″ | |
|---|---|---|---|---|
| log tan 40° | | | | =9.9238135 |
| log cot δ | 88 | 44 | 05.5 | =8.3440862 |
| log cos $t_e$ | 88 | 56 | 17.5 | =8.2678997 |

$t_e = 5^h\ 55^m\ 45^s.2$.

Sidereal time, western elongation, $T_s$=R. A. Polaris$+t_e$.

| | h. | m. | s. | |
|---|---|---|---|---|
| R. A. Polaris | = 1 | 19 | 35.2 | |
| $t_e$ | = 5 | 55 | 45.2 | |
| Sidereal time, western elongation, $T_s$= | 7 | 15 | 20.4 | |
| R. A. mean sun= $a_s$= | 16 | 29 | 14.4 | |
| Sidereal interval before noon, | = 9 | 13 | 54.0 | |
| Correction sidereal to mean interval = | | 1 | 30.7 | |
| Mean interval before noon | = 9 | 12 | 23.3 | Nov. 28. |
| Local mean time, western elongation= | 2 | 47 | 36.7 | A. M., Nov. 28. |

| | h. | m. | s. | |
|---|---|---|---|---|
| Sidereal time, eastern elongation $=24^h+a-t_e$= | 19 | 23 | 50.0 | |
| $a_s$= | 16 | 29 | 14.4 | |
| Sidereal interval after noon, | = 2 | 54 | 35.6 | |
| Correction sidereal to mean interval | = | 0 | 28.6 | |
| Local mean time, eastern elongation | = 2 | 54 | 07.0 | P. M., Nov. 28. |
| Local mean time, western elongation | = 2 | 47 | 36.7 | A. M., Nov. 28. |

For longitudes west of Washington, decrease times of elongation 0.66 second for each degree.

# CHAPTER IV.

## BASE LINE, PRIMARY CONTROL, AND ELEVATIONS.

The primary control or geometric work is, in the ordinary case, effected by triangulation. Wherever this is not practicable or not economical, resort is had to what is known as primary traversing, but wherever the country presents sufficient relief for the purpose triangulation is employed, as it is more accurate and cheaper. In some parts of the country triangulation of sufficiently accurate character for controlling the topographic map of the United States has been executed by other organizations, notably by the Coast and Geodetic Survey and the Lake Survey. Wherever such triangulation is available the results should be utilized.

### PARTY ORGANIZATION.

The primary triangulation is generally carried on by a special party. Under certain circumstances, however, it is economical and advisable for one party to do all the work. The disadvantage is that it divides the time and attention of the topographer, requiring him to turn from one thing to another; the advantage, that it insures the selection of such points as are needed by the topographer. If the work is done by a special party, the points selected are more likely to be chosen on account of their forming good figures in the triangulation than on account of their convenience and usefulness to the topographer. The secondary triangulation, the traversing, and the sketching should be carried on by different men, but under a single party organization. The sketching should be done by the chief of party, the secondary triangulation and height measurement by his most experienced assistant, and the traversing, with height measurement, by the other assistants.

### BASE-LINE MEASUREMENT.

#### SITE.

The measurement of the base line is ordinarily the first of the preparatory steps toward map making. Upon the proper selection of its site and its correct measurement depends all the subsequent work of triangulation. The site must be reasonably level. It is not essential that it be absolutely so, but the more closely it approaches a plane the less difficulty will be experienced in making an accurate measurement. The site should afford sufficient room for the measurement of a base about 5 miles in length. A base much less than 5 miles in length is not always an economical one, inasmuch as it is less costly to extend the base than to complicate the expansion. A much greater length than 5 miles is unnecessary, because this length permits of easy expansion, and if the length be greater than this it may be difficult to construct intervisible signals at the ends of the base.

The ends of the base must be intervisible, and they must be so situated with regard to suitable points for expansion and triangulation as to form well-proportioned figures. Whenever possible, the base line should form a side or diagonal of a closed quadrilateral or pentagonal figure.

While it is unnecessary to devote time to obtaining extreme accuracy in the measurement of a base, this measurement should be so nearly accurate that its errors can not affect the map altho multiplied many times in the associated triangulation. All necessary precaution should be taken to secure this result.

### METHODS AND INSTRUMENTS USED.

Various methods and instruments have been employed in the measurement of base lines by the United States Geological Survey. At first wooden rods were employed, varnished and tipt with metal. When used in measuring, these were supported upon trestles and contacts were made between them with considerable refinement. The advantage of using these rods consisted in the fact that their length was but slightly affected by temperature, which is the main source of error in base-line measurement, and being thoroly varnished they were not greatly affected by moisture.

Subsequently bars of metal were employed of the pattern known as the Coast Survey secondary bars. These consisted each of a steel rod between two zinc tubes. As the two metals expand at different rates under changes of temperature, their relative lengths at any temperature as compared to the relative lengths at a normal temperature was theoretically an indication of the temperature of the bars at any time. The arrangement for indicating their relative lengths formed part of the apparatus, and was intended to indicate the temperature of the bars, and thus to afford means of reducing their lengths to a normal temperature. These bars have not been found, however, to work well in practise. Besides, there are other objections to the use of bars of any kind, which may be summarized as follows: First, their use is expensive; a considerable number of men are needed, and as the measurement proceeds slowly it often requires from a month to six weeks to measure and remeasure a base 5 miles in length. Again, since these bars are but 4 to 6 meters in length, there are many contacts to be made in each mile of measurement, and each contact offers the possibility of a slight error.

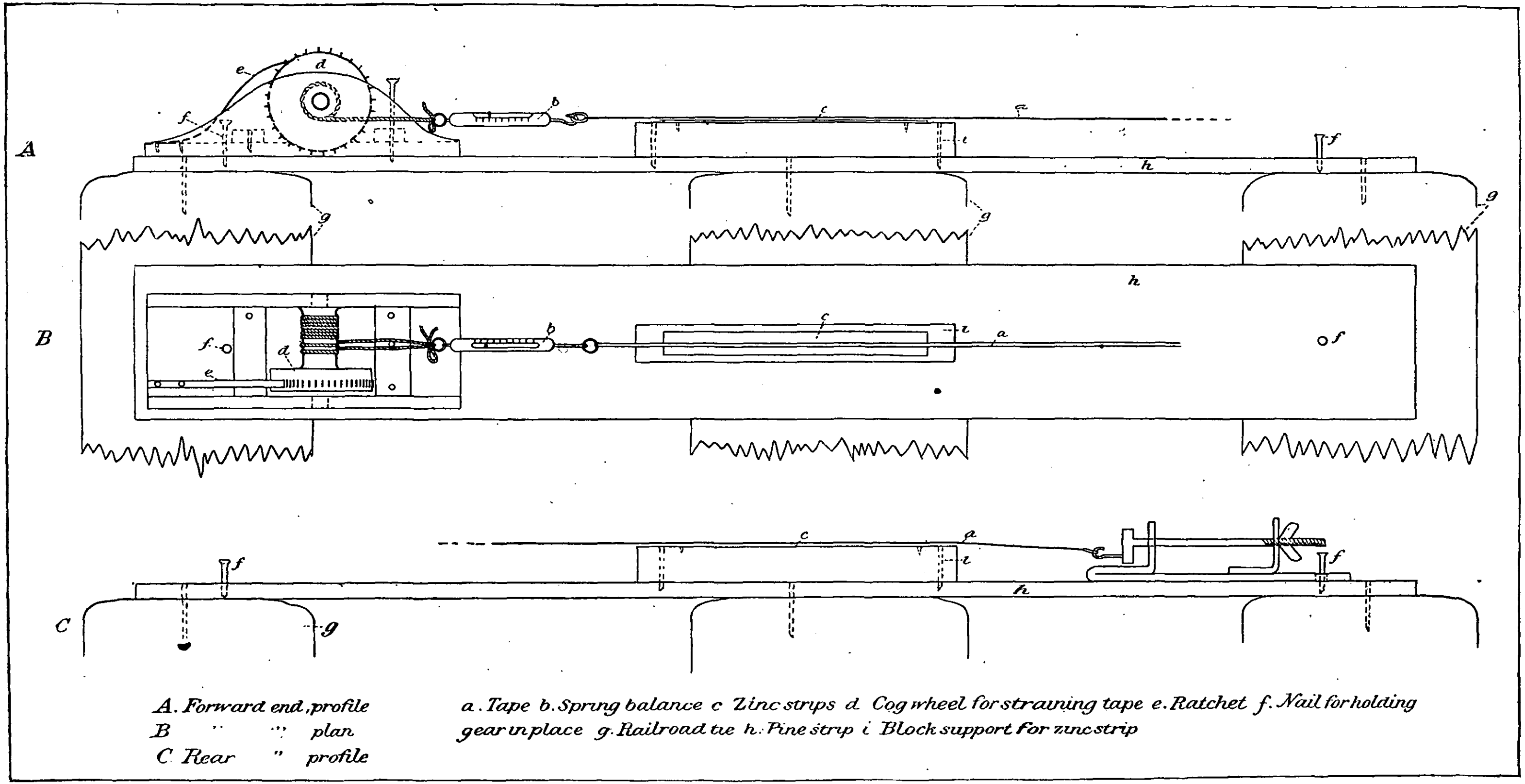
In view of these objections and of certain positive advantages which the change would produce, it was decided to drop the use of bars in the measurement of base lines and to adopt long steel tapes. By their use it has been found easy to attain the required degree of accuracy in measurement, inasmuch as the number of contacts is much smaller than when bars are used, while the uncertainty in regard to the temperature of the measuring apparatus is reduced to a minimum by carrying on the measurement at night or in cloudy weather. The expense of the measurement is greatly reduced, since fewer men are required, the work of preparing the ground and the work of measuring are much lessened, and the rapidity of measuring is increased manyfold. The diminished cost makes it practicable to measure much longer bases, thus decreasing the number of stations required in the expansion. This method allows, also, the measurement of base lines at shorter intervals in the triangulation.

The tape in use has a length of 300 feet. It should be carefully compared, at an observed temperature, with a standard, both before and after its use in base measurement. Preferably, the site for the base line should be selected along a railway tangent, as such location is approximately level, and the railway ties afford an excellent support for the tape. If such a location is not available, one should be selected that will fill the requirements previously mentioned, should be cleared of brush and undergrowth, and if necessary its sharp inequalities should be leveled. The tape should be supported at intervals of not more than 25 feet by a series of low stools whose legs are prest into the ground, or by a series of laths driven into the ground, each bearing a horizontal support made of a wire nail, while similar stools or laths should sustain each end of the tape.

The measurement of a base with the steel tape is a simple matter. Provision must, however, be made, first, for the proper alignment of the base; second, for the proper tension of the tape; and, third, for the measurement of temperature.

The alignment also is simple, and should be marked upon the ground in advance of the work of measurement. In cases where a railway tangent furnishes the site for the base line, no alignment is needed beyond the provision for keeping the tape always at a uniform distance from one of the rails.

For insuring a uniform tension of the tape, an ordinary spring balance should be attached to the forward end of the tape, where a tension of 20 pounds is applied. In order to apply this uniformly and to insure against slip of the tape, an apparatus has been devised by Mr. H. L. Baldwin, jr., of the U. S. Geological Survey. It is




DEVICE FOR STRETCHING TAPE IN BASE MEASUREMENT.

Designed and used by H. L. Baldwin, Jr.

shown in Pl. II. For its use, it is necessary to obtain strips of board about 5 feet long and 4 inches wide, in number equal to the number of lengths of tape of which the base line consists. Numbered strips of zinc, each about 8 inches long and 1 inch wide, are tacked to the boards, and the boards are nailed to the railway ties if measurement is to be made along a railway tangent. These boards are designed to support the devices for maintaining the tension, and the contacts are marked upon the strips of zinc. Mr. Baldwin's apparatus consists essentially of a wheel worked by a lever and held by ratchets in any desired position. This wheel is attached to the spring balance in such way that by turning it the strain is put upon the spring balance, which is held at the desired tension by the ratchets. A small mechanism at the rear end of the tape holds the zero of the tape at the opposite mark. The great length of the tape, 300 feet, allows considerable friction or drag when the supports are frequent, and in order to insure a reasonably uniform distribution of the strain upon the tape it should be raised and allowed to fall with the strain on.

The measurements should be made at night or during cloudy days, in order that the temperature of the air, which is that indicated by the thermometers, and that of the tape shall be as nearly as possible the same. The temperature must be carefully observed by at least two thermometers at each tape length, in order that the best possible data for temperature correction may be obtained.

The base should be measured at least twice, and the two results compared by sections of 1,200 feet, or four tape lengths. The ends of the base should, if possible, be permanently marked by means of stone monuments set into the ground so that their surfaces are but a few inches above ground level, and the exact position of the ends should be indicated by a cross cut in a copper bolt embedded in the head of the stone, in order that the base may be preserved for future references.

A line of levels must be run over the site or over the stools which support the tape, for the purpose of obtaining its profile and thereby the means for deducing its horizontal length.

### PERSONNEL OF PARTY.

The personnel required in the measurement of a base line should be, in an ordinary case, as follows:

The chief of the party, who exercises a general supervision over the work, marks the extremities of the tape, and provides the necessary precautions against errors in the measurement, as hereafter stated.

The rear tapeman, who adjusts the rear end of the tape to the contact marks and who carries and reads one of the thermometers.

The head tapeman, who adjusts the forward end of the tape, exerts the requisite tension upon it, and carries and reads a second thermometer.

A recorder.

## REDUCTION OF BASE-LINE MEASUREMENT.

### REDUCTION TO STANDARD.

The first correction to be applied is that of reduction to a standard. The correction for this should be obtained by comparison with the standard of the Bureau of Standards. The correction for the entire line is in proportion to the correction as obtained by comparison with the standard. If the tape be longer than the standard the correction will be positive; if shorter, negative.

### CORRECTION FOR INCLINATION.

Data are obtained by running a line of levels over the base line. This line of levels gives the rise or fall, in feet and decimals of a foot, between the points of change in inclination. From this and the measured distance the angle of inclination is computed from the formula, $\sin\theta = \frac{h}{R}$; R being the distance and $h$ the difference in height, both given in feet. The correction in feet to the distance is then computed by the equation,

correction $= \frac{\sin^2 1'}{2} \theta^2 R$ or $0.00000004231\ \theta^2 R$, $\theta$ being express in minutes. (See Lee's Tables, p. 83.)

### CORRECTION FOR TEMPERATURE.

Steel expands for each degree of temperature .0000063596 of its length. This fraction multiplied by the average number of degrees of temperature at the time the base line was measured above or below 62 degrees, which is taken as the normal temperature, gives the proportion in which the base line is to be diminished or extended on account of this factor. Care must be taken to obtain correctly this average temperature. It must be the mean of all the thermometric readings, taken at uniform intervals of distance during the measurement. If the temperature be above the normal the correction is positive, and vice versa.

### REDUCTION TO SEA LEVEL.

The base line is measured on an arc of a circle parallel to the sea surface and raised above it, at an elevation which is known at least approximately. This arc with radii drawn from its extremities to the center of the earth forms approximately a triangle similar to that formed by the radii of the earth with the sea surface. The length at sea level is derived with a sufficient approximation to correctness by the proportion,

$$R : h :: K : \text{correction},$$

R being the radius of the earth, $h$ the mean height of the base line above sea level, and K its measured length. (See Report United States Coast and Geodetic Survey, 1882, Appendix 9, p. 196.)

### EXAMPLE OF REDUCTION OF BASE-LINE MEASUREMENT.

The following is an example taken from the records of measurement of a base near Spearville, Kans., together with the reduction of this base for inclination, temperature, and elevation above sea level:

*Record of measurement and reduction of Spearville base, Kansas.*

[Section 1. Stations 0-10. October 16, 1889. Light rain falling.]

| No. of tape. | Time, a. m. | Tension. | Thermometers. A. | Thermometers. B. | Temperature correction. | Total length of section. |
|---|---|---|---|---|---|---|
| | *h. m.* | *Lbs.* | ° | ° | | |
| 1......... | 10 13 | 19.75 | 50.5 | 50.0 | | |
| 2......... | 20 | 20.00 | 50.5 | 50.0 | Mean temp.=50.51° | |
| 3......... | 26 | 20.00 | 50.5 | 50.0 | | |
| 4......... | 31 | 20.25 | 50.5 | 50.0 | | |
| 5......... | 37 | 20.00 | 50.7 | 50.5 | 62°−50.51°=11.49° | |
| 6......... | 42 | 20.125 | 51.5 | 50.6 | | *Feet.* |
| 7......... | 47 | 20.25 | 51.0 | 50.8 | −11.49°×3000. | 1 tape length ............ =300.0617 |
| 8......... | 51 | 20.00 | 50.8 | 50.2 | ×.000006 | |
| 9......... | 55 | 20.125 | 50.8 | 50.0 | | 10×300.0617 .............. =3,000.617 |
| 10......... | 58 | 20.00 | 50.7 | 50.5 | =−.207 foot. | Temperature correction. −.207 |
| | | | | | | Result first measurement =3,000.410 |

[Second measurement, October 17, 1889.]

| No. of tape. | Time, p. m. | Tension. | Thermometers. A. | Thermometers. B. | Temperature correction. | Total length of section. |
|---|---|---|---|---|---|---|
| | *h. m.* | *Lbs.* | ° | ° | | |
| 1......... | 12 13 | 20.00 | 52.3 | 52.4 | | |
| 2......... | 21 | 20.25 | 53.3 | 52.9 | Mean=53.96° | Tape set back from sta. 0 .85 inch. |
| 3......... | 25 | 20.00 | 53.8 | 54.0 | | =.071 foot. |
| 4......... | 29 | 19.75 | 55.0 | 54.8 | | |
| 5......... | 33 | 20.00 | 55.0 | 53.2 | 62° −53.96°=8.04° | |
| 6......... | 36 | 20.00 | 53.8 | 54.0 | | |
| 7......... | 38 | 20.00 | 54.0 | 54.0 | −8.04°×3000. | *Feet.* |
| 8......... | 41 | 20.12 | 54.5 | 54.0 | ×.000006 | 10×300.0617 ................ =3,000.617 |
| 9......... | 45 | 19.75 | 55.1 | 54.4 | | Set back .................... −.071 |
| 10......... | 50 | 20.13 | 54.5 | 54.1 | =−.145 foot. | Temperature correction... −.145 |
| | | | | | | Result second measurement=3,000.401 |

*Correction for inclination, Spearville base, Kansas.*

$$\text{Correction}=\frac{\sin^2 1'}{2}\theta^2 \times \text{Distance.}$$

| Approximate distance. | Difference of elevation. | Angle θ | Log θ | 2 log θ | Log $\frac{\sin^2 1'}{2}$ | Log dist. | Log correction. | Correction. |
|---|---|---|---|---|---|---|---|---|
| *Feet.* | *Feet.* | ′ ″ | | | | | | |
| 200 | 0.8 | 13 34 | 1.1326 | 2.2652 | 2.6264 | 2.3010 | 7.1926 | .0015 |
| 4,200 | 4.2 | 2 22 | 0.3674 | 0.7348 | Constant | 3.6232 | 6.9844 | .0010 |
| 4,000 | 12.0 | 10 08 | 1.0052 | 2.0104 | | 3.6021 | 8.2389 | .0173 |
| 1,000 | 1.0 | 3 23 | 0.5250 | 1.0501 | | 3.0000 | 6.6765 | .0005 |
| 2,000 | 3.0 | 5 04 | 0.7024 | 1.4049 | | 3.3010 | 7.3323 | .0021 |
| 4,200 | 22.0 | 12 23 | 1.0917 | 2.1834 | | 3.6232 | 8.4330 | .0271 |
| 2,800 | 7.0 | 8 27 | 0.9263 | 1.8527 | | 3.4472 | 7.9263 | .0084 |
| 1,000 | 0.0 | 0 00 | 0.0000 | 0.0000 | | 3.0000 | 0.0000 | .0000 |
| 1,000 | 1.0 | 3 23 | 0.5250 | 1.0500 | | 3.0000 | 6.6764 | .0005 |
| 4,200 | 20.0 | 11 16 | 1.0504 | 2.1008 | | 3.6232 | 8.3504 | .0224 |
| 3,800 | 6.0 | 5 20 | 0.7267 | 1.4535 | | 3.5798 | 7.6597 | .0046 |
| 2,000 | 4.0 | 6 45 | 0.8293 | 1.6586 | | 3.3010 | 7.5860 | .0038 |
| 5,400 | 31.4 | 19 39 | 1.2934 | 2.5867 | | 3.7324 | 8.9455 | .0882 |
| 2,000 | 2.6 | 4 24 | 0.6437 | 1.2874 | | 3.3010 | 7.2148 | .0016 |
| 135 | 0.05 | 1 18 | 0.1072 | 0.2144 | | 2.1303 | 4.9712 | .0000 |
| | | | | | | | | .1790 |

*Reduction to sea level.*

Correction ........ $=\frac{K\,h}{R}$
Log K (meters) ........ =4.05956
Log $h$ (meters) ........ =2.87599
Colog R ........ =3.19660

Log 1.356 meters ........ =0.13215
Log meters to feet ........ =0.51599

Log 4.448 feet ........ =0.64814

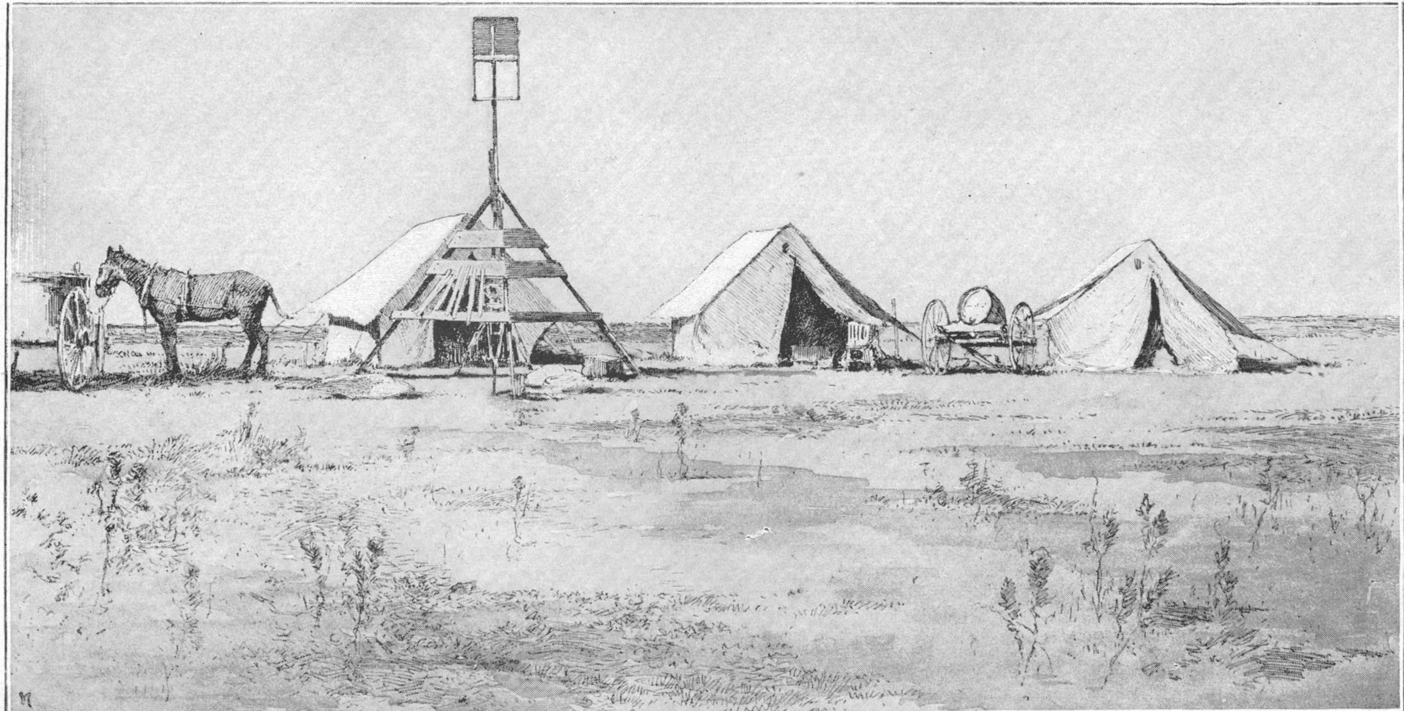
*Spearville base: Summary by sections.*

[Corrected for temperature.]

| Stations. | First measure. | Second measure. | Difference. |
|---|---|---|---|
| | ′ | ′ | *First—Second.* ′ |
| 1 to 10 | 3,000.410 | 3,000.401 | +.009 |
| 10 20 | .418 | .393 | +.025 |
| 20 30 | .431 | .431 | +.000 |
| 30 40 | .426 | .446 | −.020 |
| 40 50 | .437 | .478 | −.041 |
| 50 60 | .417 | .455 | −.038 |
| 60 70 | .369 | .392 | −.023 |
| 70 80 | .366 | .356 | +.010 |
| 80 90 | .955 | .938 | +.017 |
| 90 100 | .676 | .667 | +.009 |
| 100 110 | 3,000.899 | 3,000.898 | +.001 |
| 110 119 | 2,700.581 | 2,700.571 | +.010 |
| 119 126 | 2,100.244 | 2,100.234 | +.010 |
| | 37,806.629 | 37,806.660 | −.031′=.372″ |

Mean of 2 measurements .......... =[a]37,806.645
Reduction from S. W. base to △ .. −168.235
Reduction from N. E. base to △ .. − 2.864
Correction for inclination ......... − 0.179
Reduction to sea level ............ − 4.448
Corrected length .................. = 37,630.919

[a] Corrected for temperature.



FRAMED SIGNAL FOR USE WHERE OBSERVING TOWER IS NOT NECESSARY.

## PRIMARY TRIANGULATION.

The base line having been measured, the next step is the expansion. This work, as well as the body of the triangulation, consists in the selection of stations, the erection of signals, and the measurement of angles. Each triangle from the base line outward will, when the angular measurement is completed, have one side and the three angles known, from which the other two sides can be computed by means of a simple trigonometric formula.

The expansion differs from the body of the triangulation only in the fact that the average length of the sides of the triangles is less. As the expansion progresses away from the base line the sides of successive triangles become gradually longer, until the average length of side of the triangulation is reached. Since the sides are increasing in length, and hence since any inaccuracy in the measurement of the base is multiplied, this work must be planned and executed with greater care than the body of the triangulation requires.

A base line measured as above prescribed requires little expansion, since from the extremities of a 5-mile base one can observe directly on points 8 to 10 miles away, a distance as great as the average side of a triangle. Ordinarily, from the ends of the base, the surveyor can observe directly upon stations in his scheme of triangulation.

In the Western mountain region of the United States, where the sides of triangles may be 20 to 50 miles in length, an expansion is required.

### SELECTION OF STATIONS.

In the selection of triangulation stations two sets of requirements must be observed.

(1) Such stations must be so selected as to afford what is known as strong figures, in order to reduce to a minimum the errors which will creep into an extended system. In order to insure intervisibility, they should, if possible, be located upon hill or mountain summits; the most commanding in the neighborhood. No triangle on which dependence is placed for the location of a station should have at that station an angle of less than 30 degrees or more than 150 degrees.

The stations should, if practicable, be grouped into simple figures, as quadrilaterals or pentagons, with an interior station, etc. Where an area is being covered with triangulation such groupings naturally occur, but in certain cases the triangulation takes the form of narrow belts of figures, and then the belt may consist of simple triangles or quadrilaterals, as more complex figures are rarely desirable.

(2) Since the sole object of triangulation is the control of the topographic map, the location of stations should be adjusted to the needs of the topographers as far as is consistent with accuracy. This requirement affects most seriously the distance between stations.

Every plane-table sheet should contain at least three primary stations, and a fourth is desirable. Thus, for controlling the sheets on the scale 1:62500, the stations should not be more than 10 or 12 miles apart, and should be located with direct reference to the control of certain sheets. Again, since the primary stations must be occupied by topographers for intersecting on numerous points, they should be selected with reference to this requirement. They should command an extended view, especially of points suitable for cutting in, such as hill and mountain summits, houses, churches, etc.

Wherever possible, the instrument should be accurately centered under the signal. Whenever it is necessary to set up off center the direction and distance to the signal should be carefully measured and recorded.

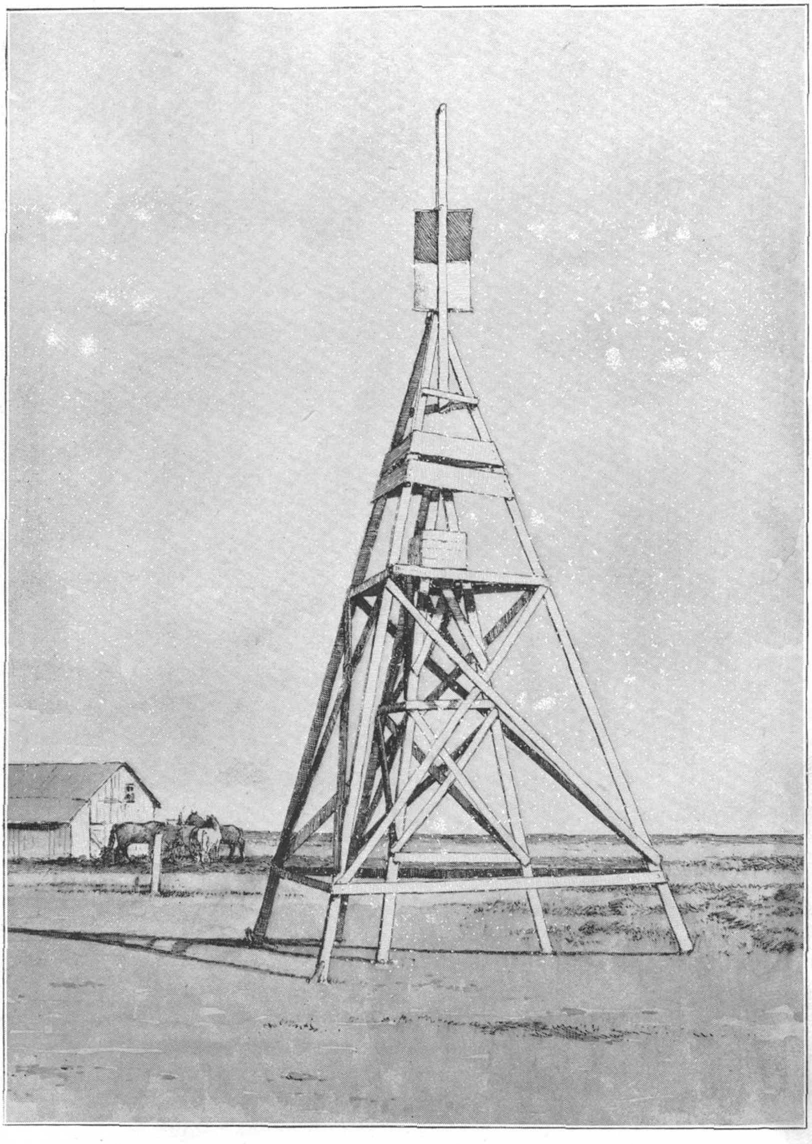
### SIGNALS.

While signals should be as simple and inexpensive as possible, their form and material must depend upon the requirements and the materials at hand. In a mountainous country where the summits are treeless, simple cairns of stone 7 to 10 feet in height are employed. Where the summits are wooded it is frequently convenient to clear them, leaving a single tree to serve as a signal. In such cases it is advisable to trim the tree of branches, with the exception of a tuft at the top. Where the station is clear but with green timber easily accessible, it is advisable to make a tripod of small trees, each with a tuft at its top. In undulating and hilly country it is often necessary to erect scaffolds. These should be built of sawed lumber and framed in simple fashion. If the lines are short a pole with a flag may be set in the top. If the lines are long the tower itself may serve as a signal, in which case its upper part should be clothed in black and white cotton.

Pl. III shows a form of framed signals adapted to the treeless plains of Kansas and the rolling open hills of New England and elsewhere where observing towers are not necessary.

It is frequently necessary to raise the instrument to a considerable elevation in order to overlook surrounding obstacles. In such cases the structures for supporting the instrument should be combined with the signals, and hence they may properly be described and figured here. These observing towers should be in two parts—an interior structure, solidly built of sawed lumber if available, for the immediate support of the instrument, and a framework surrounding it and supporting a platform just below the instrument stand for the observer. The two should be separate, in order that the jarring incident to moving about on the platform may not be communicated to the instrument. Such a type of observing tower is shown in Pl. IV.

In the Sierra Nevada of California, among the sugar-pine forests, a



SIGNAL COMBINED WITH TOWER FOR OBSERVER AND INSTRUMENT.

support for the instrument is frequently obtained by sawing off the top of a high tree and setting the instrument upon the stump, 50 or 75 feet above the ground, the tree being guyed by wire cables to prevent swaying. Neighboring trees, sawed off, support the platform for the observer. Similar devices are resorted to in the forests of West Virginia, Kentucky, and Tennessee. In secondary triangulation in these regions the instrument's support in many cases is provided as above described, while the observer's platform is attached to the same tree instead of having an independent support. This is objectionable, but is often the best plan available.

In other cases it is more economical to support the instrument upon the ground, and to have openings made thru the forest upon the station hill, in the directions of the sight lines, or even to have the whole summit cleared.

### HELIOTROPES.

It is often necessary to use more elaborate forms of signals, especially when the point observed upon is below the horizon line, so that the background, instead of being the sky, consists of forests or brown plains. In such cases resort is had to heliotropes. These are nothing more than devices for reflecting the sunlight to the observer at the instrument. The simplest form is a circular mirror with a screw hinged at the back, giving a universal motion. This is screwed into a stake or tripod over the center of the station to be observed upon, and a ray of sunlight is thrown thru a small hole in a board nailed to a stake 10 or 15 feet away and in the direction of the observer at the distant station. This form has the advantage of simplicity, as the simplest backwoodsman can manage it; and the triangulator can firmly fix all range stakes upon one visit to the station, and be sure of seeing the flash as he observes from each of the surrounding stations in turn.

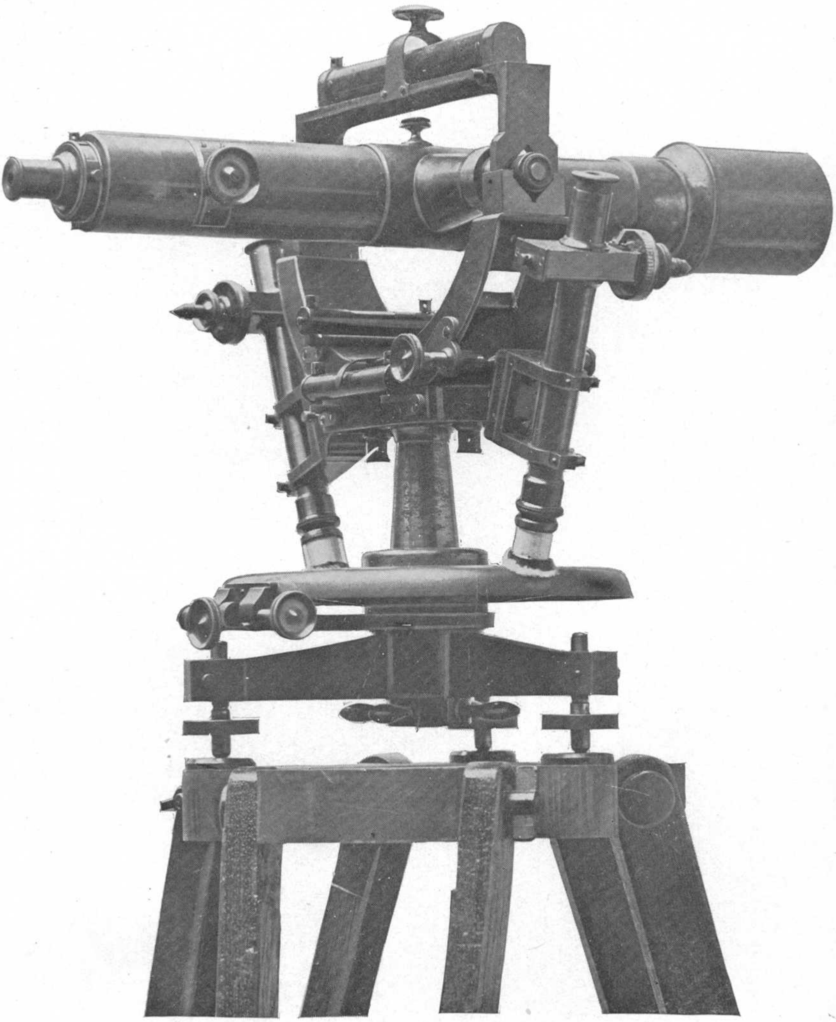
Two other forms are in use—the Coast Survey type and the Steinheil. The former consists of a telescope which is provided with a screw for fastening it into any convenient support or upon the theodolite. Upon the telescope are a mirror and two rings, the axis of the rings as well as the center of support of the mirror being parallel to the line of sight of the telescope. The telescope being directed upon the observing station, the mirror is so turned as to reflect the sunlight thru the rings and necessarily to the observing station. In many cases the use of a second mirror is necessary, owing to the relative position of the two stations and the sun, and such a mirror forms a part of the outfit. This form is little used, on account of its liability to get out of adjustment. The Steinheil heliotrope is a compact little instrument which can be easily carried in a case. It consists of a small sextant mirror, the two surfaces of which are as nearly absolutely parallel as possible. This mirror has a small hole in the center of the reflecting surface. Below this hole is a small lens in the shaft carrying the mirror, and below the lens is some white reflecting material, as plaster of Paris. The mirror is so mounted that it has four motions, two about its horizontal axis and two about its vertical axis, each of which can be separately bound or controlled by clamps or friction movements. To use the Steinheil, it is screwed into some wooden upright, as the side of a tree in such a position that the main axis carrying the lens and plaster of Paris reflector shall be parallel with the sun's rays. The observer, standing behind the mirror, receives from the rear surface of the glass a reflection of the sun, producing an imaginary sun. The mirror should not be moved until this imaginary sun, moving with it, appears to rest on the object to which it is desired to cast the flash, as the hill on which the triangulator is standing. As the surfaces of the mirror are parallel, the true reflected rays of the sun will also be cast on the object sighted to.

This instrument is in great favor, especially with the Western parties, where portability is of moment, first, because it is light and convenient to carry and use, and, second, because there are no movable parts to get out of adjustment by jarring. The latter is a serious defect in the Coast Survey instrument, since it requires frequent testing to make sure that neither of the two rings has been moved, as such movement would cause the reflections to be cast out of parallelism with the line of sight of the telescope.

The use of heliotropes presupposes the employment of men to operate them, thus increasing materially the expense of the work. Misunderstandings continually arise between the heliotropers and the observer, causing vexatious delays, and therefore their employment should be avoided whenever possible.

## THEODOLITES FOR TRIANGULATION.

Instruments differing widely in power and degree of accuracy have been in use by the United States Geological Survey for the measurement of angles in the primary triangulation. Formerly theodolites having circles 6, 7, 8, 10, and 11 inches in diameter and reading by vernier to 10 seconds were employed, and the results were reduced and adjusted by least squares. Subsequently it appeared desirable to employ instruments of a higher class and thus obtain more accurate results, which would render unnecessary this tedious adjustment. Pursuant to this decision the use of these vernier theodolites has been largely discontinued, and theodolites having 8-inch circles, reading by micrometer microscopes, have been substituted almost universally in the primary work. (See Pl. V.)



EIGHT-INCH THEODOLITE.

The circle, as above stated, has a diameter of 8 inches, and is subdivided to 10 minutes. The object glass is 2 inches in diameter and its focal distance is $16\frac{1}{2}$ inches. The telescope with the eyepiece commonly used has a power of about 30 diameters.

The circle is read by means of two microscopes placed opposite each other. Within the field of the microscope is a comb stretching over the space of 20 minutes. This comb has ten teeth, divided into two parts by a depression, each corresponding to 2 minutes. Parts of a minute down to 2 seconds are read by means of a micrometer screw moving a pair of fine threads in the field of the microscope.

### INSTRUCTIONS FOR MEASUREMENT OF HORIZONTAL ANGLES.

The following general precautions should be observed in the measurement of all horizontal angles in the primary triangulation:

The instrument should have a stable support, which may be a stone pier, a wooden post, or a good tripod. If a portable tripod is used its legs should be set firmly in the ground.

The instrument should be protected from the direct rays of the sun by means of an umbrella or a piece of canvas like a tent fly. It should also be shielded from winds that might jar or twist either it or its support.

The foot screws of the instrument, after it is leveled for work, should be tightly clamped. Looseness of the foot screws and tripod is a common source of error, especially with small instruments.

The alidade, or the part of the instrument carrying the telescope and verniers or microscopes, should move freely on the vertical axis. Clamps should likewise move freely when loosened. Whenever either of these moves tightly the instrument needs cleaning, oiling, or adjusting.

The observer should always have a definite preliminary knowledge of the objects or signals observed. The lack of it may lead to serious error and entail cost much in excess of that involved in getting such knowledge.

Great care should be taken to insure correctness in the degrees and minutes of an observed angle. The removal of an ambiguity in them is sometimes a troublesome or expensive task.

The errors to which measured angles are subject may be divided into two classes—those dependent on the instrument used, or instrumental errors; and those arising from all other sources, which, for the sake of distinction, may be called extrainstrumental errors.

The best instruments are more or less defective, and all adjustments on which precision depends are liable to derangement; hence the general practise of arranging observations in such a manner that the errors due to instrumental defects will be eliminated in the end results.

The principal errors of this kind and the methods of avoiding their effects are enumerated below.

Measurements made with a graduated circle are subject to certain systematic errors commonly called periodic. Certain of these errors are always eliminated in the mean (or sum) of the readings of the equidistant verniers or microscopes, and both of the latter should be read with equal care in precise work. Certain other errors of this class are not eliminated in the mean of the microscope readings, and only these need consideration. Their effect on the mean of all the measures of an angle may be rendered insignificant by making the number of individual measures with the circles in each of $n$ equidistant positions separated by an interval equal to $\frac{360°}{mn}$, where $m$ is the number of equidistant verniers or microscopes. Thus, if $m=2$, the circle should be shifted after each measure by an amount equal to $\frac{180°}{n}$, which, for example, is 45° if $n=4$ and 30° if $n=6$. The degree of approximation of this elimination increases rapidly with $n$. The effect of errors of this class is always nil on an angle equal to the angular distance between consecutive microscopes or a multiple thereof. Other things being equal, therefore, we should expect the measures of such special angles to show less range than the measures of other angles.

Besides the instrumental errors of the periodic class, there are also accidental errors of graduation. These are in general small, however, in the best modern circles and their effect is sufficiently eliminated by shifting the circle in the manner explained in the preceding paragraph.

The effect of an error of collimation on the circle reading for any direction varies as the secant of the altitude of the object observed. The effect on an angle between two objects varies as the difference between the secants of their altitudes. This effect is eliminated either by reversing the telescope in its wyes, or by transmitting it without changing the pivots in the wyes, the same number of measures being obtained in each of the two positions of the telescope. The latter method is the better one, especially in determining azimuth, since it eliminates at the same time errors due to inequality of pivots and to inequality in height of the wyes.

The effect of the error of inclination on the circle reading for any direction varies as the tangent of the altitude of the object observed. If the inclination is small, as it may always be by proper adjustment, its effect will be negligible in most cases. But if the objects differ much in altitude, as in azimuth work, the inclination of the axis must be carefully measured with the striding level, so that the proper correction can be applied. The following formula includes the corrections to the circle reading on any object for collimation and inclination of telescope axis:

$$c \sec h + b \tan h;$$

$c$ = collimation in seconds of arc,

$b$ = inclination of axis in seconds of arc,

$h$ = altitude of object observed.

Parallax of wires occurs when they are not in the common focal plane of the eyepiece and objective. It is detected by moving the eye to and fro sidewise while looking at the wires and the image of the object observed. If the wires appear to move in the least, an adjustment is necessary. The eyepiece should always be first adjusted to give distinct vision of the cross wires. This adjustment is entirely independent of all others and requires only that enough light to illuminate the wires enter the telescope or microscope tube. This adjustment is dependent on the eye and is in general different for different persons; hence maladjustment of the eyepiece can not be corrected by moving the cross wires with reference to the objective. Having adjusted the eyepiece, the image of the object observed may be brought into the plane of the cross wires by means of the rack-and-pinion movement of the telescope. A few trials will make the parallax disappear.

When circles are read by micrometer microscopes it is customary to have them so adjusted that an even number of revolutions of the screw will carry the wires over the image of a graduation space. If the adjustment is not perfect an error of run will be introduced. This may in all cases be made small or negligible since, by means of the independent movements of the whole microscope and the objective with respect to the circle, the image may be given any required size. In making this adjustment some standard space, or space whose error is known, should be used. At least once at each station where angles are read observations should be made for run of micrometers.

Tangent and micrometer screws should move freely, but never loosely. In making a pointing with the telescope the tangent screw should always move against or push the opposing spring. Likewise, bisections with the micrometer wires should be made always by making the screw pull the micrometer frame against the opposing spring or springs.

Extrainstrumental errors may be divided into four classes—errors of observation, errors from twist of tripod or other support, errors from centering, and errors from unsteadiness of the atmosphere.

Barring blunders or mistakes, the errors of observation are in general relatively small or unimportant. With practised observers in angular measurements, such errors are the least formidable of all the unavoidable errors, and their elimination in the end results is usually well-nigh perfect. The recognition of this fact is very important, for observers are prone to attribute unexpected discrepancies to bad observation rather than to the much more probable cause. After learning how to make good observations the observer should place the utmost confidence in them, and never yield to the temptation of changing them because they disagree with some preceding observations. Such discrepancies are in general an indication of good rather than poor work.

Stations or tripods which have been unequally heated by the sun or other source of heat usually twist more or less in azimuth. The rate of this twist is often as great as a second of arc per minute of time, and it is generally nearly uniform for intervals of ten to twenty minutes. The effect of twist is to make measured angles too great or too small according as they are observed by turning the microscopes in the direction of increasing graduation or in the opposite direction. This effect is well eliminated, in general, in the mean of two measures, one made by turning the microscopes in the direction of increasing graduation and followed immediately by turning the microscopes in the opposite direction. Such means are called combined measures or combined results, and all results used should be of this kind. As the uniformity in rate of twist can not be depended on for any considerable interval, the more rapidly the observations on an angle can be made the better will be the elimination of the twist. The observer should not wait more than two or three minutes after pointing on one signal before pointing on the next. If for any reason it should be necessary to wait longer, it will be best to make a new reading on the first signal.

The precision of centering an instrument or signal over the reference or geodetic point increases in importance inversely as the length of the triangulation lines. Thus, if it is desired to exclude errors from this source as small as a second, one must know the position of the instrument within one-third of an inch for lines a mile long or within 6 inches for lines 20 miles long. The following easily remembered relations will serve as a guide to the required precision in any case:

1 second is equivalent to 0.3 inch at the distance of 1 mile.

1 second is equivalent to 3.0 inches at the distance of 10 miles.

1 second is equivalent to 6.0 inches at the distance of 20 miles.

1 minute is equivalent to 1.5 feet at the distance of 1 mile.

The notes should always state explicitly where the instrument and signals are and give their coordinates (preferably polar coordinates) if they are not centered.

Objects seen thru the atmosphere almost always appear unsteady, and sometimes this unsteadiness is so great as to render the identity of the object doubtful. It is usually greatest during the middle of the day, and generally subsides or ceases an hour or two before sundown. There is also frequently a short interval of quietude about sunrise, and on cloudy days many consecutive hours of steady atmosphere may occur. For the best work observations should be made only when the air causes small or imperceptible displacements of signals. In applying this rule, however, the observer must use his discretion. Errors of pointing increase rapidly with increase of unsteadiness, but it will frequently happen that time may be saved by counterbalancing errors from this source by making a greater number of observations. Thus, if signals are fairly steady it may be economical to make double the number of observations rather than wait for better conditions.

The best results in a triangulation are to be obtained by measuring the angles separately and independently. Thus, if the signals in sight around the horizon are in the order A, B, C, etc., the angles A to B B to C, etc., are by this method observed separately, and whenever there is sufficient time at the observer's disposal this method should be followed.

Besides measuring single angles, it is desirable to measure independently combined angles—i. e., angles which consist of the sum of two or more single angles. Thus, supposing O to be the observing station and A, B, and C stations sighted on, the observer should measure not only the angles AOB and BOC, but the combined angle AOC. This is necessary not only because this angle may be used directly in the triangulation, but because it will be needed in forming conditions for adjusting the angles about the observing station, or the station adjustment, as it is called.

In order to secure the elimination of the errors mentioned above, the following procedure must be strictly adhered to:

Point on A and read both microscopes.

Point on B and read both microscopes.

Transit telescope and turn microscopes 180°.

Point on B and read both microscopes.

Point on A and read both microscopes.

Shift circle by $\frac{180^\circ}{n}$ and proceed as before until $n$ such sets of measures have been obtained.

Then measure the angles B to C, C to D, etc., including the angle necessary to close the horizon, in the same manner.

A form for record and computation of the results is given on page —.

When repeating instruments are used, the same procedure will be followed, except that there should be five pointings instead of one on each of A and B, the circle being read for the first pointing on A and the fifth on B, and again for the sixth pointing on B and the tenth on A.

The importance of having the measures of a set follow in quick succession must be constantly borne in mind. Under ordinarily favorable conditions an observer can make a pointing and read the microscopes once a minute, and a set of five repetitions should be made in five minutes or less.

When several stations or signals are visible and a nonrepeating instrument is used, time can be saved without material loss of precision in the angles, by observing on all the signals successively according to the following procedure, the signals being supposed in the order A, B, C, etc., as above:

Point on A and read microscope.
Point on B and read microscope.
Point on C and read microscope.
Point on A and read microscope.
Transit telescope and turn microscopes 180°.
Point on A and read microscope.
Point on B and read microscope.
Point on C and read microscope.
Point on A and read microscope.

Shift circle by $\frac{180°}{n}$ and proceed as before until $n$ such sets have been obtained.

The angles A to B, B to C, etc., read in this way may be computed as in the first method, always combining the measure A to B with the immediately succeeding measure B to A to eliminate twist. There is a theoretic objection to this process of deriving angles, founded on the fact that they are not independent, but in secondary work this objection may be ignored as of little weight.

For the 11-inch theodolite and for the 8-inch instruments made by Fauth & Co., all of which read by micrometer microscopes, four sets of measures on as many different parts of the circle will be required; and for the repeating theodolite six sets of measures will be required, all measures being made according to the procedures given above.

Under ordinary circumstances and with due care in centering, angles measured as specified above should show an average error of closure of less than 5 seconds. Under specially unfavorable conditions the number of sets of measures should be increased, care being always taken to shift the circle so as to eliminate periodic errors.

The practise of starting the measurement of an angle or series of angles with the microscopes reading 0° and 180°, 90°, and 270°, etc., should be avoided; otherwise the errors of these particular divisions will affect many angles. In shifting the circle it is neither necessary nor desirable to have the new positions differ from the preceding one by exactly $\frac{180^\circ}{n}$. A difference of half a degree either way is unimportant as respects periodic errors, and it is advantageous to have the minutes and seconds differ for the different settings.

Field notes should be clear and full. The date, place, name, and number of instrument used, and the names of observer and recorder should be recorded at the beginning of each day's work at a station. The positions of the instrument and signals observed should be defined, either by a full statement or by a reference to such, in each day's notes. The time of observations should be noted at intervals, to show that the instrument does not stand too long between pointings.

A preliminary reduction of all observations should be made in the field, in order to detect and correct gross errors. A plat of the triangulation should be made as the work progresses.

When mistakes are made in the record, the defective figures should not be erased, but simply crost out, and an explanation furnished in the column of remarks. Great care should be taken not only to avoid "cooking" or "doctoring" notes, but to avoid suspicion thereof.

The following example of form of record is taken from the primary triangulation executed in western Kansas:

*Record of measurement of horizontal angle.*

Station: Township corner, Kansas, July 1, 1889. Fauth 8-inch theodolite No. 362, one division of micrometer head = 2 seconds.]

| Station. | Micr. A. | Micr. B. | Mean reading. | Angle. | Mean. |
|---|---|---|---|---|---|
| | Telescope direct. | | | | |
| | ° ′ *Div.* | ° ′ *Div.* | ° ′ ″ | ° ′ ″ | ″ |
| Walton | 93 12 11.3 | 273 12 09.9 | 93 12 21.2 | 36 29 03.9 | |
| Newt | 129 41 11.9 | 309 41 13.2 | 129 41 25.1 | | 05.9 |
| Newt | 129 41 15.6 | 309 41 12.1 | 129 41 27.7 | 08.0 | |
| Walton | 93 12 10.6 | 273 12 09.1 | 93 12 19.7 | | |
| | Telescope reversed. | | | | |
| Walton | 138 27 03.2 | 318 26 28.0 | 138 27 01.2 | | |
| Newt | 174 56 02.8 | 354 55 28.9 | 174 56 01.7 | 00.5 | |
| Newt | 174 56 06.2 | 354 55 29.5 | 174 56 05.7 | | 01.8 |
| Walton | 138 27 05.2 | 318 26 27.4 | 138 27 02.6 | 03.1 | |
| | Telescope reversed. | | | | |
| Walton | 183 07 03.0 | 3 06 27.2 | 183 07 00.2 | | |
| Newt | 219 36 05.0 | 39 35 29.8 | 219 36 04.8 | 04.6 | |
| Newt | 219 36 08.1 | 39 35 29.5 | 219 36 07.6 | | 03.9 |
| Walton | 183 07 06.4 | 3 06 28.1 | 183 07 04.5 | 03.1 | |
| | Telescope direct. | | | | |
| Walton | 228 24 28.1 | 48 24 22.6 | 228 24 50.7 | | |
| Newt | 264 53 27.4 | 84 53 26.1 | 264 53 53.5 | 02.8 | |
| Newt | 264 54 01.1 | 84 53 26.1 | 264 53 57.2 | | 04.3 |
| Walton | 228 24 29.3 | 48 24 22.1 | 228 24 51.4 | 05.8 | |

4 | 15″.9

Mean of 4 combined measures[a] ........ =36° 29′ 03″.98

[a] Instrument over center of station.

## ORGANIZATION OF PARTIES AND PROSECUTION OF WORK.

A party for carrying on primary triangulation usually comprizes only the chief and an assistant, with the addition of a driver and a cook in case the party is living in camp. Frequently, however, it is found economical to employ a man to superintend the construction of signals. The chief of party is expected to select the stations and direct what forms of signals shall be erected, and to measure angles. In a mountainous country the selection of stations is usually a simple matter. From the summit of a mountain the chief of party may be able to select stations for considerable distances ahead and to order the erection of signals, leaving to the man employed for that purpose the erection of them. On the other hand, in a densely wooded region such as the Cumberland Plateau, where the summits have approximately the same elevation, the selection of stations is extremely difficult, requiring great ability and experience and involving an immense amount of labor. In such a region the chief of party finds it necessary to travel great distances, visit many hills, and even to climb to the summits of the highest trees, in order to select intervisible stations.

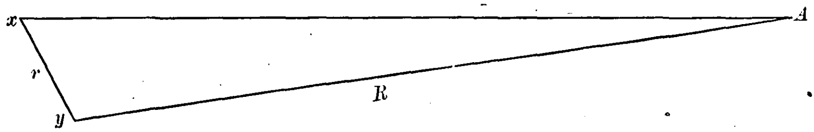
The selection of stations must be kept in advance of the reading of angles, but it is not advisable to keep it too far ahead, on account of the danger of the destruction of signals before angles have been read upon them. Therefore the chief of party finds it necessary to alternate between the two kinds of work, selecting and preparing three or four stations, then returning and measuring the angles.

When it is necessary to use heliotropes the party must be increased by one man for each such instrument employed. The proper management of such a party then calls for the exercise of much judgment on the part of the triangulator. If it is convenient for the chief of party to place each heliotroper before observing angles, and to show him where to direct his instrument, men of ordinary intelligence may be employed, and the work is one calling for time rather than for skill. Where, however, the party is moving frequently the observer and heliotropers occupying different stations nearly every day, as is possible in the dry atmosphere usually prevailing in the West, the chief of party has to arrange a schedule for each man, showing the order in which he is to occupy the stations and in what direction he is to flash from each. In this case the heliotroper must be a man having some topographic and technical skill, so that he may find his point, set up on center, and direct his flashes to the right place, besides exercising a goodly amount of judgment. A simple code of signals having been agreed upon, it becomes easy for the triangulator to let the heliotroper know that the work is completed, when he at once moves to the next designated station.

## REDUCTION OF PRIMARY TRIANGULATION.

### REDUCTION TO CENTER.

In case any station is occupied off center, the directions as read must first be reduced to center. In the diagram, let $x$ be the point



occupied, $y$ the station, $r$ the distance between them, $A$ the point to which the direction is laid and the angle at that point, and $R$ its distance, approximately known. Then, from the relations between the sides and the angles of the triangle,

$$R : r :: \sin x : \sin A,$$

$$\sin A = \frac{r \sin x}{R}, \text{ and } A \text{ (in seconds)} = \frac{r \sin x}{R \sin 1''}$$

correction in seconds of arc.

The following example taken from the triangulation in Kansas will illustrate the form of effecting this reduction. The references are to the diagram on page 59:

*Reduction to center of station at Walton △, Kansas.*

[See explanation: Appendix No. 9, page 167, U. S. Coast and Geodetic Survey report for 1882.]

Distance, inst. to center=0'.48 log = 9.6812
log feet to meters = 0.5160

Distance, inst. to center log meters = 9.1652 = log $r$.

| Direction. | x to n 7°. | x to o 73°. | x to p 105°. | x to q 185°. | x to r 273°. | x to s 306°. |
|---|---|---|---|---|---|---|
| Log sin angle | 9.0859 | 9.9806 | 9.9849 | 8.9403 | 9.9994 | 9.9080 |
| Colog distance | 5.9321 | 5.9182 | 6.4228 | 6.2434 | 6.0079 | 6.2514 |
| Log r | 9.1652 | 9.1652 | 9.1652 | 9.1652 | 9.1652 | 9.1652 |
| Colog sin 1″ | 5.3144 | 5.3144 | 5.3144 | 5.3144 | 5.3144 | 5.3144 |
| | 9.4976 | 0.3784 | 0.8873 | 9.6633 | 0.4869 | 0.6390 |
| Correction to direction | 0″.31 | 2″.39 | 7″.71 | 0″.46 | 3″.06 | 4″.36 |

Correction to angle
$a = n$ to $o$ $-0.31'' +2.39'' = +2.08''$
$b = o$ to $p$ $-2.39 +7.71 = +5.32$
$g = n$ to $p$ $-0.31 +7.71 = +7.40$
$c = p$ to $q$ $-7.71 -0.46 = -8.17$
$d = q$ to $r$ $+0.46 -3.06 = -2.60$
$e = r$ to $s$ $+3.06 -4.36 = -1.30$
$h = q$ to $s$ $+0.46 -4.36 = -3.90$
$f = s$ to $n$ $+4.36 +0.31 = +4.67$

### SPHERICAL EXCESS.

The angles are measured on a spherical surface, and the sum of the three measured angles of each triangle should equal 180° plus the spherical excess. The latter, however, needs to be computed and subtracted from the sum of the angles only for the purpose of testing the accuracy of closure of the triangle, as in the reduction the angles are treated as plane angles. When the area of the triangle is large the spherical excess in seconds ($E$) should be computed by the equation,

$$E = \frac{S}{r^2 \sin 1''},$$

where S is the area of the triangle in square miles, and $r$ the radius of curvature of the earth in miles. When the triangle (being within the United States) has an area less than 500 square miles, $r$ may be assumed as constant, and the spherical excess may be obtained by dividing the area in square miles by 75.5.

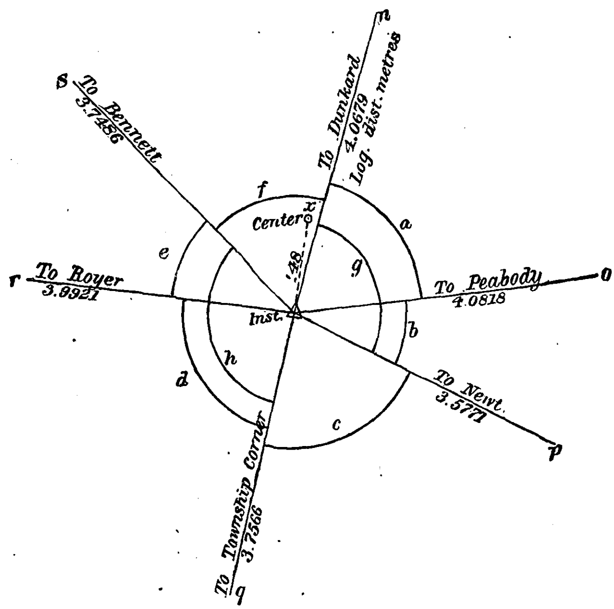
### STATION ADJUSTMENT.

The next step is the adjustment of the angles about the observing station, or the station adjustment, as it is called. Referring to the diagram, which represents the angles read at Walton station, in Kansas, it is seen that eight angles were measured, as follows:

*Record of angles measured at Walton, Kansas.*

| | Obs. angle. | | | Station adjustment. | Correction to center. | Angles locally adjusted and reduced to center. | | |
|---|---|---|---|---|---|---|---|---|
| | ° | ′ | ″ | ″ | ″ | ° | ′ | ″ |
| *a* Dunkard—Peabody | 65 | 45 | 28.37 | +.51 | +2.08 | 65 | 45 | 30.96 |
| *b* Peabody—Newt | 31 | 47 | 58.50 | +.52 | +5.32 | 31 | 48 | 04.34 |
| Sum | 97 | 33 | 26.87 | | | 97 | 33 | 35.30 |
| *g* Dunkard—Newt (meas.) | 97 | 33 | 28.39 | −.49 | +7.40 | 97 | 33 | 35.30 |
| Difference | | | −1.52 | | | | | 00.00 |
| *d* Township corner—Royer | 87 | 44 | 57.41 | −.56 | −2.60 | 87 | 44 | 54.25 |
| *e* Royer—Bennett | 34 | 00 | 03.35 | −.56 | −1.30 | 34 | 00 | 01.49 |
| Sum | 121 | 44 | 60.76 | | | 121 | 44 | 55.74 |
| *h* Township corner—Bennett | 121 | 44 | 59.05 | +.59 | −3.90 | 121 | 44 | 55.74 |
| Difference | | | +1.71 | | | | | 00.00 |
| *f* Bennett—Dunkard | 61 | 09 | 26.17 | +.02 | +4.67 | 61 | 09 | 30.86 |
| *g* Dunkard—Newt | 97 | 33 | 28.39 | −.49 | +7.40 | 97 | 33 | 35.30 |
| *c* Newt—Township corner | 79 | 32 | 06.25 | +.02 | −8.17 | 79 | 31 | 58.10 |
| *h* Township corner—Bennett | 121 | 44 | 59.05 | +.59 | −3.90 | 121 | 44 | 55.74 |
| Sum | 359 | 59 | 59.86 | | | 360 | 00 | 00.00 |
| | | | −0.14 | | | | | 00.00 |

Of these $a+b$ should $=g$, $d+e$ should $=h$, and $g+c+h+f$ should $=360°$. Thus are formed in this case three conditions affecting eight unknown quantities. The method by which are found the corrections which fulfil these conditions is that known as the method of least squares, for a full discussion of which, showing its application to triangulation, see "The Adjustment of Observations," by T. W. Wright and J. F. Hayford, pp. 180–260, (New York, D. Van Nostrand, 1906). It is unnecessary to explain the theory of this method, but it is desirable to show how it is applied in the class of cases under consideration,




which can best be done by tracing a case thru. There are here three equations of conditions, as follows:

$$(1)\quad a+b-g-1''.52 \quad =0;$$
$$(2)\quad d+e-h+1''.71 \quad =0;$$
$$(3)\quad f+g+c+h-0''.14=0;$$

in which the letters represent, not angles, as in the diagram, but unknown corrections to the angles. The coefficient of each of these corrections is unity. Arrange them in tabular form, the letters at the top referring to the equations, thus forming what is called a table of correlates. Now multiply each coefficient by itself and every other in the same horizontal line and sum them. Three normal equations result as follows:

| | $w$ | $y$ | $z$ |
|---|---|---|---|
| $a$ | 1 | | |
| $b$ | 1 | | |
| $c$ | | | 1 |
| $d$ | | 1 | |
| $e$ | | 1 | |
| $f$ | | | 1 |
| $g$ | −1 | | 1 |
| $h$ | | −1 | 1 |

| | | | | | |
|---|---|---|---|---|---|
| 1 | $+3.00w$ | | $-1.00z$ | $-1''.52$ | $=0$ |
| 2 | | $+3.00y$ | $-1.00z$ | $+1''.71$ | $=0$ |
| 3 | $-1.00w$ | $-1.00y$ | $+4.00z$ | $-0''.14$ | $=0$ |

These three equations involving three unknown quantities are then solved by elimination, with results as follows:

$$w = +\overset{\prime\prime}{.515}.$$
$$y = -.562.$$
$$z = +.023.$$

These values can now be substituted in the table of correlates, columns 1, 2, 3; the algebraic sum of lines *a*, *b*, *c*, *d*, etc., giving corrections to the angles *a*, *b*, *c*, *d*, etc.

| | 1 | 2 | 3 | Corrections to angles. |
|---|---|---|---|---|
| | | | | ″ |
| *a* | +.515 | | | +.515 |
| *b* | +.515 | | | +.515 |
| *c* | | | +.023 | +.023 |
| *d* | | −.562 | | −.562 |
| *e* | | −.562 | | −.562 |
| *f* | | | +.023 | +.023 |
| *g* | −.515 | | +.023 | −.492 |
| *h* | | +.562 | +.023 | +.585 |

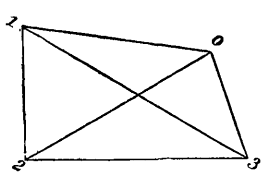
## FIGURE ADJUSTMENT.

The measurement of the angles having been executed by instruments and methods much better than the needs of the map require, it is not ordinarily necessary to make any figure adjustment further than an equal distribution of the error of each triangle among the three angles. Still, as the necessity for a more elaborate adjustment may arise, a description of the method of applying the least squares to adjustment of geometric figures in triangulation is here given, with a simple example of its application.

Each geometric figure in a system of triangulation is composed of a number of triangles. The measured angles of each triangle should equal 180° plus the spherical excess. Each triangle, therefore, furnishes an equation of condition, which is known as an angle equation. The number of angle equations in any figure is equal to the number of closed triangles into which it can be resolved. But since certain of these are a consequence of the others, the number of angle conditions which it is desirable to introduce is less than the number of triangles.

The number of angle equations in any figure is equal to the number of closed lines in the figure plus one, minus the number of stations. Thus, in a closed quadrilateral, the number of angle equations is $6+1-4=3$.

There is another class of conditions, known as side equations, which can be best explained by reference to a figure. In the diagram, suppose the figure 0, 1, 2, 3 to represent the projection of a pyramid, of which 1, 2, 3 is the base and 0 the apex. A geometric condition of such figure is that the sums of the logarithmic sines of the angles about the base, taken in one direction, must equal the similar sums taken in the other direction—i. e., the product of the sines must be equal. In



the present case, log sin 0, 1, 2 + log sin 0, 2, 3 + log sin 1, 3, 0 should equal log sin 1, 2, 0 + log sin 2, 3, 0 + log sin 0, 1, 3.

The number of side equations which can be formed in any figure is equal to the number of lines in the figure, plus 3, minus twice the number of stations in it, or $l+3-2n$. In a quadrilateral, $6+3-8=1$.

The numerical term in each angle equation is the difference between the sum of the observed angles on the one hand and 180° plus the spherical excess on the other. This is positive when the sum of the observed angles is the greater and vice versa. The coefficients of the unknown corrections are in each case unity, unless weights are assigned.

The numerical term in each side equation is the difference between the sums of the logarithmic sines, taken in the two directions. The coefficients of the unknown corrections are the differences for one second in the logarithmic sines of the angles.

The method of making up and solving these equations and applying the corrections to the angles can best be shown by means of an example. That here given is the simplest case involving both angle and side equations, namely, the case of a quadrilateral. The method of forming correlatives and normal equations, and their solution, is similar to that employed in station adjustment, and therefore the details are omitted.

In the equations of conditions and correlatives the angles are designated by directions to which the corrections are finally applied. Thus the angle of 3, 0, 2 is designated $-3/0+2/0$, the sign $-$ being given to the left-hand and the sign $+$ to the right-hand direction.

EXAMPLE OF FIGURE ADJUSTMENT BY LEAST SQUARES.

| | | Observed angles. | | |
|---|---|---|---|---|
| | | ° | ′ | ″ |
| (*a*) | 3·0·1 | 120 | 39 | 14.781 |
| | 0·1·3 | 21 | 26 | 17 806 |
| | 1·3·0 | 37 | 54 | 37.180 |
| | | 180 | 00 | 09.767 |
| | Spherical excess | | = | − 0.148 |
| | Closure error | | | + 9.619 |
| (*b*) | 0·1·2 | 81 | 52 | 51.222 |
| | 1·2·0 | 62 | 22 | 38.500 |
| | 2·0·1 | 35 | 44 | 45.861 |
| | | 180 | 00 | 15.583 |
| | | | | − 0.189 |
| | Closure error | | | +15.394 |
| | 1·2·3 | 91 | 28 | 38.000 |
| | 2·3·1 | 28 | 95 | 10.360 |
| | 3·1·2 | 60 | 26 | 33.416 |
| | | 180 | 00 | 21.776 |
| | | | | − 0.234 |
| | Closure error | | | +21.542 |
| (*c*) | 2·3·0 | 65 | 59 | 47.540 |
| | 3·0·2 | 84 | 54 | 28.920 |
| | 0·2·3 | 29 | 05 | 59.500 |
| | | 180 | 00 | 15.960 |
| | | | | − 0.193 |
| | Closure error | | | +15.767 |

*Side equation.*

[Taking 0 as the pole.]

| | Angle. | Log sines of spherical angle | Tabular difference for 1″. | Corrections to log sines. | Corrected log sines of spherical angles. | Spherical excess. | Log sines of plane angles. |
|---|---|---|---|---|---|---|---|
| (d). | 0. 1. 2......... | 9. 9956249. 7 | +3. 0 | − 25. 0 | 9. 9956224. 7 | −. 063 | 9. 9956224 |
| | 0. 2. 3......... | 9. 6869340. 0 | 37. 9 | −127. 9 | 9. 6869212. 1 | −. 065 | 9. 6869210 |
| | 1. 3. 0......... | 9. 7884705. 9 | 27. 0 | − 1. 2 | 9. 7884704. 7 | −. 050 | 9. 7884703 |
| | Sum......= | 29. 4710295. 6 | | | 29. 4710141. 5 | | 29. 4710137 |
| | 1. 2. 0......... | 9. 9474437. 5 | 11. 0 | − 59. 4 | 9. 9474378. 1 | −. 063 | 9. 9474378 |
| | 2. 3. 0......... | 9. 9607184. 9 | 9. 4 | − 77. 7 | 9. 9607107. 2 | −. 064 | 9. 9607107 |
| | 0. 1. 3......... | 9. 5628859. 2 | 53. 7 | −203. 0 | 9. 5628656. 2 | −. 049 | 9. 5628653 |
| | Sum......= | 29. 4710481. 6 | | | 29. 4710141. 5 | | 29. 4710137 |
| | From above... | 29. 4710295. 6 | | | | | |
| | Difference... | 00. 0000186. 0 | | | 000. 0 | | 0000. |

(d)..............................$0=+186.0\ -3.0\left(\frac{0}{1}\right)+03.0\left(\frac{2}{1}\right)-37.9\left(\frac{0}{2}\right)+37.9\left(\frac{3}{2}\right)-27.0\left(\frac{1}{3}\right)+27.0\left(\frac{0}{3}\right)$
$-\left[-11.0\left(\frac{1}{2}\right)+11.0\left(\frac{0}{2}\right)-9.4\left(\frac{2}{3}\right)+9.4\left(\frac{0}{3}\right)-53.7\left(\frac{0}{1}\right)+53.7\left(\frac{3}{1}\right).\right]$

## *Equations of condition.*

(a)..............................$0=+9''.619-\frac{3}{0}+\frac{1}{0}-\frac{0}{1}+\frac{3}{1}-\frac{1}{3}+\frac{0}{3}$
(b)..............................$0=+15\ .394-\frac{0}{1}+\frac{2}{1}-\frac{1}{2}+\frac{0}{2}-\frac{2}{0}+\frac{1}{0}$
(c)..............................$0=+15\ .767-\frac{2}{3}+\frac{0}{3}-\frac{3}{0}+\frac{2}{0}-\frac{0}{2}+\frac{3}{2}$

Collecting terms in (*d*) and dividing thru by 100 so as to avoid dealing with large numbers.

(d)..............................$0=+1.86+.507\left(\frac{0}{1}\right)+.030\ \frac{2}{1}\ -.489\left(\frac{0}{2}\right)+.379\left(\frac{3}{2}\right)-.270\left(\frac{1}{3}\right)$
$+.176\left(\frac{0}{3}\right)+.110\left(\frac{1}{2}\right)+.094\left(\frac{2}{3}\right)-.537\left(\frac{3}{1}\right).$

*Table of correlatives.*

| Direction. | *a.* | *b.* | *c.* | *d.* |
|---|---|---|---|---|
| 0-1 | −1 | −1 | ........ | +. 507 |
| 0-2 | ........ | +1 | −1 | −. 489 |
| 0-3 | +1 | ........ | +1 | +. 176 |
| 1-0 | +1 | +1 | ........ | ........ |
| 1-2 | ........ | −1 | ........ | +. 110 |
| 1-3 | −1 | ........ | ........ | −. 270 |
| 2-0 | ........ | −1 | +1 | ........ |
| 2-1 | ........ | +1 | ........ | +. 030 |
| 2-3 | ........ | ........ | −1 | +. 094 |
| 3-0 | −1 | ........ | −1 | ........ |
| 3-1 | +1 | ........ | ........ | −. 537 |
| 3-2 | ........ | ........ | +1 | +. 379 |

Forming the normal equations in the usual manner, we have:

| | | | | | |
|---|---|---|---|---|---|
| | ″ | | | | |
| (a)............... | 0=+ 9. 619 | +6. 000 | +2. 000 | +2. 000 | −0. 598 |
| (b)............... | 0=+15. 394 | +2. 000 | +6. 000 | −2. 000 | −1. 076 |
| (c)............... | 0=+15. 767 | +2. 000 | −2. 000 | +6. 000 | +0. 950 |
| (d)............... | 0=− 1. 860 | −0. 598 | −1. 076 | +0. 950 | +1. 054 |

Solving, we find the following values:

$$
\begin{aligned}
& \qquad\ {}'' \\
a &= +\ 1.900 \\
b &= -\ 4.386 \\
c &= -\ 5.208 \\
d &= +\ 3.059
\end{aligned}
$$

Substituting the values of *a*, *b*, *c*, *d*, in the table of correlatives.

| Direction. | *a*. | *b*. | *c*. | *d*. | Correction to each direction. |
|---|---|---|---|---|---|
| | | | | | ″ |
| $\frac{0}{1}$ | −1.900 | +4.386 | ............ | +1.551 | +4.037 |
| $\frac{0}{2}$ | ............ | −4.386 | +5.208 | −1.496 | −0.674 |
| $\frac{0}{3}$ | +1.900 | ............ | −5.208 | +0.538 | −2.770 |
| $\frac{1}{0}$ | +1.900 | −4.386 | ............ | ............ | −2.486 |
| $\frac{1}{2}$ | ............ | +4.386 | ............ | +0.336 | +4.722 |
| $\frac{1}{3}$ | −1.900 | ............ | ............ | −0.826 | −2.726 |
| $\frac{2}{0}$ | ............ | +4.386 | −5.208 | ............ | −0.822 |
| $\frac{2}{1}$ | ............ | −4.386 | ............ | +0.092 | −4.294 |
| $\frac{2}{3}$ | ............ | ............ | +5.208 | +0.288 | +5.496 |
| $\frac{3}{0}$ | −1.900 | ............ | +5.208 | ............ | +3.308 |
| $\frac{3}{1}$ | +1.900 | ............ | ............ | −1.643 | +0.257 |
| $\frac{3}{2}$ | ............ | ............ | −5.208 | +1.159 | −4.049 |

| | Observed angles. | Corrections. | Corrected spherical angles. | Sph. excess. | Plane angles. |
|---|---|---|---|---|---|
| | ° ′ ″ | ″ ″ | ° ′ ″ | ″ | ° ′ ″ |
| 3.0.1 | 120 39 14.781 | −3.308−2.486 | 120 39 08.986 | −.049 | 120 39 08.94 |
| 0.1.3 | 21 26 17.806 | −4.037+0.257 | 21 26 14.026 | −.049 | 21 26 13.98 |
| 1.3.0 | 37 54 37.180 | +2.726−2.770 | 37 54 37.136 | −.050 | 37 54 37.08 |
| | | | 180 00 00.148 | −.148 | 180 00 00.00 |
| 0.1.2 | 81 52 51.222 | −4.037−4.294 | 81 52 42.891 | −.063 | 81 52 42.83 |
| 1.2.0 | 62 22 38.500 | −4.722−0.674 | 62 22 33.104 | −.063 | 62 22 33.04 |
| 2.0.1 | 35 44 45.861 | +0.822−2.486 | 35 44 44.194 | −.063 | 35 44 44.13 |
| | | | 180 00 00.189 | −.189 | 180 00 00.00 |
| 1.2.3 | 91 28 38.000 | −4.722−4.049 | 91 28 29.229 | −.078 | 91 28 29.15 |
| 2.3.1 | 28 05 10.360 | −5.496−2.726 | 28 05 02.138 | −.078 | 28 05 02.06 |
| 3.1.2 | 60 26 33.416 | −0.257−4.294 | 60 26 28.865 | −.078 | 60 26 28.79 |
| | | | 180 00 00.232 | −.234 | 180 00 00.00 |
| 2.3.0 | 65 59 47.540 | −5.496−2.770 | 65 59 39.274 | −.064 | 65 59 39.21 |
| 3.0.2 | 84 54 28.920 | −3.308−0.822 | 84 54 24.794 | −.064 | 84 54 24.73 |
| 0.2.3 | 29 05 59.500 | +0.674−4.049 | 29 05 56.125 | −.065 | 29 05 56.06 |
| | | | 181 00 00.193 | −.193 | 180 00 00.00 |

## COMPUTATION OF DISTANCES.

In each triangle, starting with the base line, there is known at least one side and the three angles. The remaining sides are computed by the well-known proportion of sides to sines of opposite angles, or, exprest mathematically, $a=\frac{b \sin A}{\sin B}$. In this computation distances should be used in meters, and seven-place logarithms should be employed.

The following is an example of the correction of the angles and the computation of the sides of triangles, taken from the work in Kansas:

*Correction of angles and computation of sides of triangles.*

| Station. | Angles locally adjusted and reduced to center. | ⅓ error. | Plane angles. | Log sines. |
|---|---|---|---|---|
| | ° ′ ″ | ″ | ° ′ ″ | |
| Township corner | 36 29 04.0 | +.5 | 36 29 04.5 | 0.2257704 |
| Newt | 63 58 56.2 | +.6 | 63 58 56.8 | 9.9535952 |
| Walton | 79 31 58.1 | +.6 | 79 31 58.7 | 9.9927124 |
| | 179 59 58.3<br>Error=−1.7 | | | |

| | |
|---|---|
| Log dist. Newt-Walton | 3.5771611 |
| Log sin Newt | 9.9535952 |
| a. c. log sin Township corner | 0.2257704 |
| Log dist. Township corner-Walton | 3.7565267 |
| Log dist. Newt-Walton | 3.5771611 |
| Log sin Walton | 9.9927124 |
| a. c. log sin Township corner | 0.2257704 |
| Log dist. Township corner-Newt | 3.7956439 |

## COMPUTATION OF GEODETIC COORDINATES.

The next step is the computation of the latitude and longitude of the stations and the azimuth or direction of the lines connecting them. Initially, the latitude and longitude of some point are determined by astronomic observations, and this point is connected with the triangulation. The azimuth, or angle with a south line, of a line connecting this point with some station in the triangulation is also determined by astronomic observations. These, with the observed angles and the computed distances between the stations, form the data from which the latitudes and longitudes of the stations and the azimuths of the lines connecting them are computed. The difference in latitude between two adjoining stations is obtained from the following equation, based upon the Clarke spheroid:

$$-dL = K \cos a' \, B + K^2 \sin^2 a' \, C + (\delta L)^2 D - hK^2 \sin^2 a' \, E,$$

in which

$dL$ is the difference in latitude;

K, the distance between the stations in meters;

$a'$, the fore azimuth of the line connecting them, measured around clockwise from the south thru the west;

$h$, the first term;

$\delta L$, the approximate difference in latitude, being the sum of the first two terms;

B, C, D, and E, constants derived from the dimensions and figure of the earth (these are given for various latitudes in tables in Appendix 9 of United States Coast and Geodetic Survey Report for 1894).

The difference in longitude is obtained by means of the formula,

$$dM = \frac{K \sin a' \; A'}{\cos L'},$$

in which

$dM$ is the difference in longitude;
L′, the newly determined latitude;
A′, a constant, from tables; and the others as above.

The azimuths at the ends of a line differ from each other because of the convergence of the meridians. That first determined is known as the fore azimuth, the other as the back azimuth. All azimuths are measured from the south point around clockwise.

The back azimuth is computed from the formula,

$$- da = dM \frac{\sin \frac{(L+L')}{2}}{\cos \frac{1}{2}\, dL},$$

in which

M is the longitude of the first station;
L, the latitude, and
L′, the latitude of the second station.

The constants used are those of the Clarke spheroid of 1866.

These formulas are derived and explained in Appendix No. 9, Report United States Coast and Geodetic Survey for 1894.

The following are examples of the use of the formulas, taken from the triangulation in New Mexico:

*Examples of computation of geodetic coordinates.*

| | | | ° | ′ | ″ |
|---|---|---|---|---|---|
| (1) | Azimuth $a$: | Nell—Chusca. | 159 | 29 | 08.728 |
| | Spherical angle: | | 120 | 54 | 13.980 |
| | Azimuth $a'$: | Nell—Zuni. | 38 | 34 | 54.748 |
| | $\delta\, a + 180°$ | | 179 | 50 | 02 124 |
| | Azimuth $(a)$: | Zuni—Nell. | 218 | 24 | 56.872 |

GEODETIC COORDINATES.

| LATITUDE. | ° | ′ | ″ | | LONGITUDE. | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|---|
| L: | 35 | 25 | 13.443 | Nell. Geo. Pos. No. 5. | M: | 108 | 37 | 24.925 |
| $d$ L | | −17 | 47.546 | | $d$ M | + | 17 | 15.360 |
| L′ | 35 | 07 | 25.927 | Zuni. Geo. Pos. No. 6. | M′ | 108 | 54 | 40.285 |

Computation for latitude:

| | |
|---|---|
| log K | 4.6236305 |
| " B | 8.5111933 |
| " cos $a'$ | 9.8930500 |
| log (I) | 3.0278738 |
| log K² | 9.24726 |
| " C | 1.25696 |
| " sin² $a'$ | 9.58986 |
| log (II) | 0.09408 |
| log D | 2.3679 |
| " [I+II]² | 6.0568 |
| log (III) | 8.4247 |
| log E | 6.0124 |
| " K² sin² $a'$ | 8.8371 |
| " (I) | 3.0279 |
| log (IV) | 7.8774 |

| | ″ |
|---|---|
| (I) | 1066.286+ |
| (II) | 1.242+ |
| (III) | .026+ |
| (IV) | .008− |
| −$d$L | 1067.546+ |

| | |
|---|---|
| [I+II] | 1067.528 |
| log " | 3.0283792 |
| " [I+II]² | 6.0567584 |

Computation for longitude:

| | |
|---|---|
| log K | 4.6236305 |
| " sin $a'$ | 9.7949286 |
| " A′ | 8.5092394 |
| " sec L′ | 0.0872944 |
| Corr. for diff. arc & sine= | −15 |
| log (V) | 3.0150914 |
| $d$ M | 1035″.360 |

Computation of azimuth:

| | | |
|---|---|---|
| log (V) | | 3.015091 |
| " sin | $\left(\frac{L+L'}{2}\right)$ | 9.761522 |
| " sec | $\left(\frac{dL}{2}\right)$ | 0.000001 |
| log (VI) | | 2.776614 |
| $da$ | | − 597″.876 |
| | | −9′ 57″.876 |

Azimuth check.

Check:
Spher. angle
at ———

Computation of Azimuth $a$, in Book —, page —.
Spherical angle and distance=K, in Book —, page —, Triangle No. —.
——— Station; Computed by ———.

(2)

| | | ° | ′ | ″ |
|---|---|---|---|---|
| Azimuth $a$: | Chusca—Nell. | 339 | 21 | 40.150 |
| Spherical angle: | | 25 | 11 | 38.601 |
| Azimuth $a'$: | Chusca—Zuni. | 4 | 33 | 18.751 |
| $d\,a$+180° | | 179 | 57 | 25.650 |
| Azimuth ($a$): | Zuni—Chusca. | 184 | 30 | 44.401 |

GEODETIC COORDINATES.

| LATITUDE. | ° | ′ | ″ | | LONGITUDE. | ° | ′ | ″ |
|---|---|---|---|---|---|---|---|---|
| L: | 35 | 53 | 06.746 | Chusca. Geo. Pos. No. 4. | M: | 108 | 50 | 14.518 |
| $d$ L | − | 45 | 40.818 | | $d$M | + | 4 | 25.768 |
| L′ | 35 | 07 | 25.928 | Zuni. Geo. Pos. No. 6. | M′ | 108 | 54 | 40.286 |

Computation for latitude:

| | |
|---|---|
| log K | 4.9280539 |
| " B | 8.5111594 |
| " cos $a'$ | 9.9986260 |
| log (I) | 3.4378393 |
| log K² | 9.85610 |
| " C | 1.26435 |
| " sin² $a'$ | 7.79982 |
| log (II) | 8.92027 |
| log D | 2.3703 |
| " [I+II]² | 6.8757 |
| log (III) | 9.2460 |
| log E | 6.0214 |
| " K² sin² $a'$ | 7.6559 |
| " (I) | 3.4378 |
| log (IV) | 7.1151 |

| | ″ | | | |
|---|---|---|---|---|
| (I) | 2740.560+ | | | |
| (II) | .083+ | | | |
| (III) | .176+ | [I+II] | | 2740.643 |
| (IV) | .001− | log | " | 3.4378525 |
| −$d$ L | +2740.818 | " | [I+II]² | 6.875705 |

Computation for longitude:

| | |
|---|---|
| log K | 4.9280539 |
| " sin $a'$ | 8.8999280 |
| " A′ | 8.5092394 |
| " sec L′ | 0.0872944 |
| Corr. for diff. arc & sine | −129 |
| log (V) | 2.4245028 |
| $d$M | +265″.768 |

Computation of azimuth:

| | |
|---|---|
| log (V) | 2.424503 |
| " sin $\left(\frac{L+L'}{2}\right)$ | 9.764002 |
| " sec. $\left(\frac{d\,L}{2}\right)$ | 0.000009 |
| log (VI) | 2.188514 |
| $d\,a$ | − 154″.350 |
| | −2′ 34″.350 |

Azimuth check:

| | ° | ′ | ″ |
|---|---|---|---|
| | 218 | 24 | 56.872 |
| | 184 | 30 | 44.401 |
| Check: | 33 | 54 | 12.471 |
| Spher. angle at Zuni | 33 | 54 | 12.469 |

Computation of Azimuth $a$, in Book 67, page 4.
Spherical angle and distance= K, in Book 64, page 12, Triangle No. 3.
——— Station; Computed by H. M. W.

When the lines are not more than 20 miles in length the equation for latitude may be simplified without appreciable error by dropping the last two terms.

## TRAVERSE LINES FOR PRIMARY CONTROL.

In a level country, especially if covered with forests, it is very expensive to carry on triangulation, and in some cases practically impossible to do so. Under such circumstances traverse lines afford the only means of obtaining adequate control for maps.

A traverse line consists of a series of direction and distance measurements. Each course, as the direction and the accompanying distance are called, depends upon the one immediately preceding it, and a continuous chain is thus formed. Traverse lines are largely used in topographic work proper for making minor locations. The primary traverse differs from these only in the fact that it is much more elaborately executed.

The initial point of a primary traverse must be located either by triangulation or by astronomic determinations. The end of the line should, if possible, be a point similarly well located. The line should, if practicable, follow a railroad, in order to obtain the easiest possible grades, and thus avoid errors incident to slope.

The instruments used should be a 20- or 30-inch transit, a 300-foot steel tape, a spring balance, two flag poles, a thermometer, and a watch.

The party should consist of one chief to act as transitman, one recorder, two tapemen, and a flagman. At each station the transitman should proceed as follows:

The telescope should be set on the rear flag, both verniers read, the telescope transited, pointed on the front flag, and both verniers read. The circle should be shifted and the same angle remeasured with the telescope reversed.

Along a railroad the operation of measuring should be conducted as follows:

The front tapeman should attach to the front end of the tape a 20-pound tension with the spring balance. He should make a check mark on the rail or drive a tack or nail in the tie. The distance which he records should be checked by the transitman and by at least one other member of the party. The rails should be counted as a check upon the distance.

Along highways or across open country the tape should be kept level. The plumb bob should be used on steep slopes to bring the tape vertically over the point. Tape lengths should be marked on the measuring board with the marking needle. Where slopes are too steep for leveling with the 300-foot tape a shorter tape should be used. The temperature of the tape should be taken every hour during progress of the work.

Observations for azimuth should be made at the close of each day's work, if possible, and azimuth stations should be not more than 10 miles apart except on long tangents.

Where the line traversed is very crooked the instrument should be fitted for observation with a solar attachment, and this should be used at least twice each day, weather permitting, in addition to the observations on Polaris.

Careful notes should be taken, with a description of the starting point of the line, at the beginning and ending of each day's work. The location of each railroad station should be noted, also each milepost, switch, wagon road, stream, land or county line crost, and connection made with corners of the public-land surveys. Permanent marks should be placed at each end of the line and also at prominent junction joints, from which other primary-control lines may be started.

Lines of traverse exceeding 100 miles in length should be reduced by computation. The distances should be corrected for error of tape and for temperature and slope, and should be reduced to sea level, in the same manner as described in treating of the reduction of base lines, in case these corrections are of sufficient amount to affect the length appreciably upon the map.

The courses should be corrected for convergence of meridians. Then, commencing at the initial point, the latitude and departure of each station, one from another, should be computed in feet. The sum of the latitudes converted into seconds of latitude gives the difference in latitude, and the sum of the departures converted into seconds of longitude gives the difference in longitude.

Short lines of traverse may be platted with minute-reading protractors, but in this platting the utmost care should be exercised.

## PRECISE AND PRIMARY ELEVATIONS.

For the control of elevations level lines of three orders of accuracy are employed:

(1) Precise levels, which are run from the seacoast or from previously established precise benches to distant fields of work. These are of the highest order of accuracy obtainable.

(2) Primary levels, which are run across quadrangles or around their boundaries for the immediate purpose of furnishing primary points for heights. Under the law at least two primary bench marks should be established in each land-office township, and an equal number in regions where no land surveys have been made.

(3) Flying levels, which are run over traverse lines for the purpose of locating contours.

The instruments used in precise and primary levels are in all respects similar to those employed by the Coast and Geodetic Survey, which are figured and described in the United States Coast and Geodetic Survey Reports for 1899 and 1903.

In connection with these descriptions are general instructions for the use of these instruments in running precise lines, which differ only in minor details from the instructions followed by the United States Geological Survey.

In regions where secondary triangulation is practicable heights may be measured with the plane table directly from datum points established by the primary leveling and be carried thruout the work by means of this instrument.

When the control of the map is effected by means of primary traversing, such traverse should be accompanied by a level line.

# CHAPTER V.

## SECONDARY TRIANGULATION AND TRAVERSE AND STADIA MEASUREMENTS.

### SECONDARY TRIANGULATION.

The work of making secondary locations by intersection is done mainly by plane table. The use of the theodolite for this purpose is restricted to those cases where but little of this kind of location can be effected, and where, therefore, it seems scarcely worth while to prepare plane-table sheets.

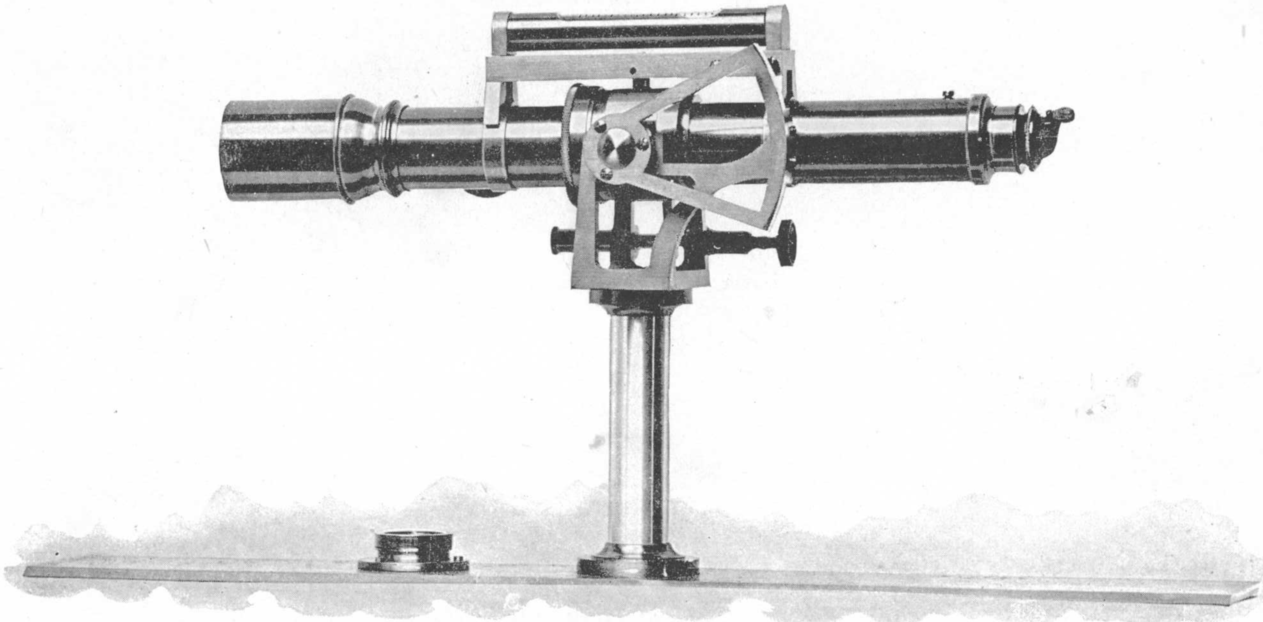
By means of the primary triangulation, three or four points are usually located upon each plane-table sheet. Within this primary triangulation, and depending upon it, are then located a large number of points, either by intersection, by traverse, or by both methods, forming a geometric framework upon which the sketching of the map depends.

Location by intersection should be carried as far as practicable—that is, all points capable of being located in this manner should be so placed, in order to afford the most ample control possible for the traverse lines, by which the intervening areas are to be filled in, it being understood that location by intersection is more accurate and more rapid than location by traverse, and consequently in every way more economical.

### THE PLANE TABLE.

Much misapprehension exists regarding the character and application of the plane table. This apparently arises from the fact that it is little known. It is applicable to all kinds of country, to all methods of work, and to all scales. It is a simple, direct, and economical instrument, rendering possible the making of a map directly from the country as copy, and doing away with elaborate notes, sketches, photographs, etc., which not only are more expensive, but produce inferior results. (See Pl. VII.)

The plane table consists of a board upon which is fastened a sheet of drawing paper. This board is mounted upon a tripod, which, in the more elaborate forms of the instrument, possesses great stiffness and stability. It should be capable of being leveled, of being turned in azimuth, and of being clamped in any position. On the paper is produced directly in miniature a representation of the country.



TELESCOPIC ALIDADE.

When set up at different places within the area in process of being mapped, any edge of the board must always extend in the same direction—that is, a certain edge of the board must always be set at the same angle with the north and south line. This is called orienting the board.

Directions are not read off in degrees and minutes, but platted directly on the paper. The instrument used for this purpose is known as the alidade, and consists of a ruler with a beveled edge, to which are attached for rough work two raised sights, and for the higher class of work a telescope, whose axis is fixt parallel to the edge of the ruler. This telescope carries also a delicate level and a vertical arc for the measurement of angles in the vertical plane, from which relative heights are obtained. The method of using this instrument is extremely simple in principle and not difficult in practise. (See Pl. VI.)

The making of locations from intersections obtained by means of the plane table requires that the instrument have the utmost stability consistent with lightness and portability. It demands an alidade equipped with a telescope of considerable power and good definition. In short, it is necessary that the plane table be in every respect of the best modern type in order that the highest degree of accuracy may be attained. Various forms of plane-table movement have been in use, including the heavy and cumbersome but stable movement employed by the United States Coast and Geodetic Survey, and the light but unstable movement used by the same organization in its less important work. At present a table is in general use which was invented by Mr. W. D. Johnson, of the United States Geological Survey, which combines, in a remarkable degree, the elements of stability, lightness, and facility of operation. (See Pl. VII and fig. 2.) The movement is essentially an adaptation of the ball-and-socket principle, so made as to furnish the largest practicable amount of bearing surface. It consists of two cups, one set inside the other, the inner surface of one and the outer surface of the other being ground so as to fit exactly. The inner cup is in two parts, or rather consists of two rings, one outside the other, one controlling the movement in level and the other that in azimuth. From each of these rings a screw projects beneath the movement, and upon each of these screws is a nut by which it is clamped. There is no tangent screw for either the leveling or the azimuth motion, as none is required. The movement is sustained by a light hard-wood tripod with split legs. The boards used generally accommodate a full-sized atlas sheet, but they necessarily differ in size, owing to the different scales of field work adopted. The largest board used for this movement holds an atlas sheet on a scale of 1:45000, and is 24 by 36 inches in size.

The kind of paper for plane-table sheets is of great importance, especially in intersection work, as paper which expands and contracts differently in different directions under varying conditions of moisture will easily produce errors of magnitude in the work. It matters little if the paper contracts and expands, provided it does so uniformly in all directions, but all paper is made with more or less fiber, and accordingly expands and contracts more in one direction than in another. To counteract this, two thicknesses of paper are used—

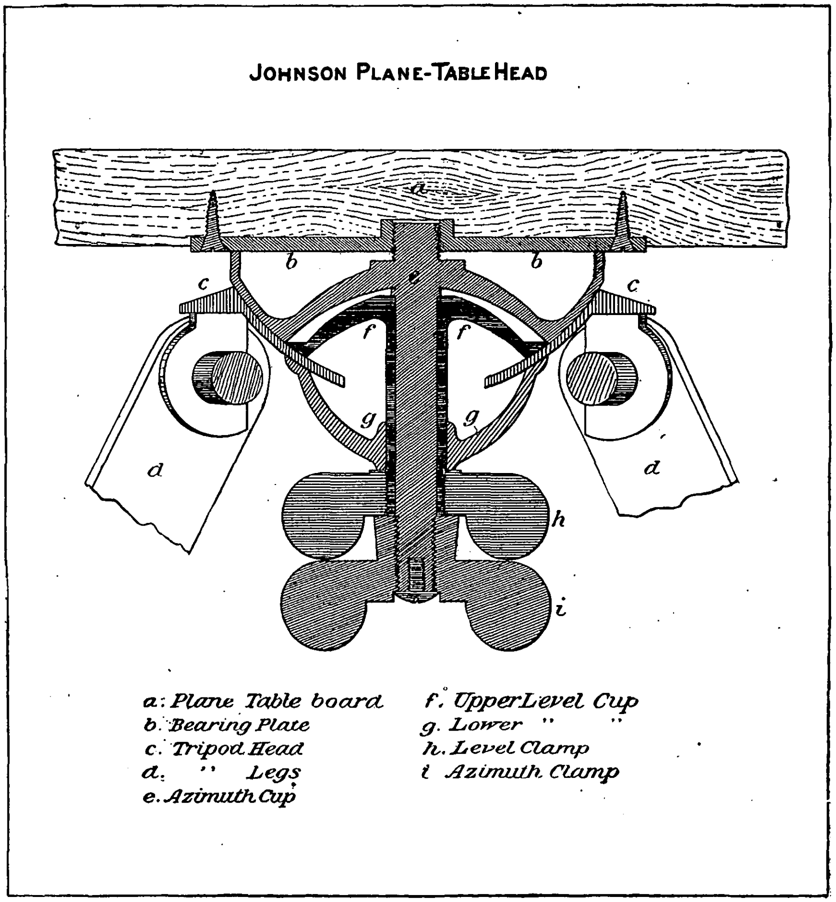
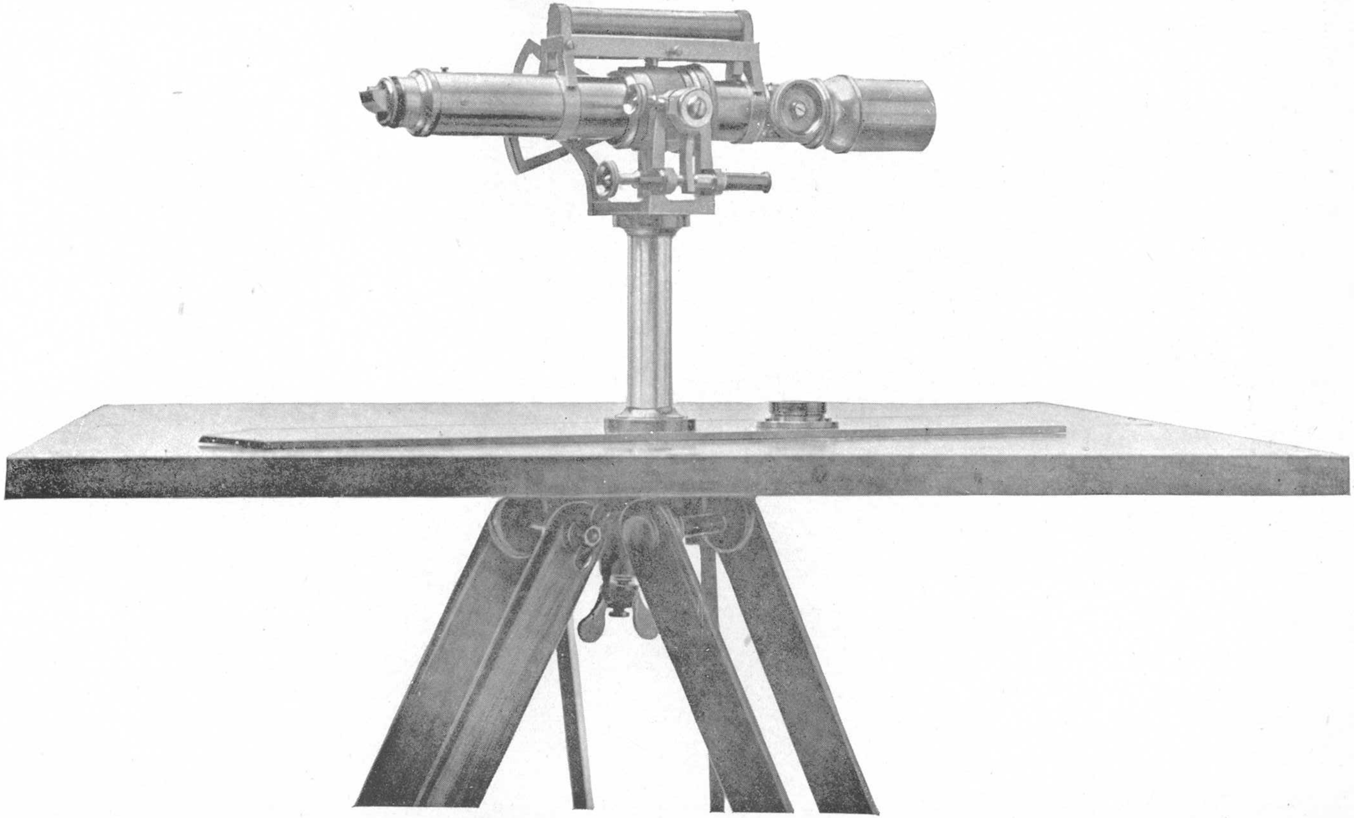



FIG. 2.—Johnson plane-table movement (section).

preferably that known as Paragon paper—mounted with the grain of one sheet at right angles to that of the other, and with cloth between the layers. In sheets so prepared it has been found that there is practically no distortion, even under the most severe tests.

The board is generally made of seasoned white pine, from one-half to five-eighths of an inch thick, with cleats across the ends, fastened in such way as to allow the body of the board to contract and expand freely, thereby preventing warping. Into the corners of this board



JOHNSON PLANE TABLE AND TELESCOPIC ALIDADE.

and on the edges at points halfway between the corners are set female screws for holding the paper to the board. At corresponding points in the plane-table sheet are punched holes half an inch in diameter, thru which pass screws with broad heads fitting into the female screws in the board. The holes in the paper, being larger than the screws, allow the paper to expand or contract freely when the screws are loose; when tightened, the broad heads of the screws bind the paper firmly in place.

## THE ALIDADE.

The alidade used with the plane table (see Pls. VI, VII) consists of a ruler of brass or steel 18 inches to 2 feet in length, sometimes graduated upon a chamfered edge to suit the scale of work, and carrying upon a column a telescope having a focal distance of 12 to 15 inches and a power of about 15 diameters. It has a vertical arc of 60 degrees, reading by vernier to single minutes, and a delicate level upon the telescope, the latter turning in a sleeve for the adjustment of vertical collimation. The arc is graduated in one direction only, reading from 0 to 60 degrees, and the reading is approximately 30 degrees when the telescope is horizontal. This is to obviate any possible error in the sign of the correction for level.

Upon the plane-table sheet is constructed a projection on the scale of the field work, and upon that are platted such of the primary points as fall on the sheet, each plane-table sheet being made to correspond to an atlas sheet. These primary points are first occupied by the plane tabler.

The instrument is set over one of these stations, leveled, and clamped in level. The ruler edge of the alidade is then laid upon the line connecting this station with a neighboring one on the sheet, and the table turned in azimuth until the other station is upon the vertical wire in the telescope. The instrument is then oriented and, after clamping in azimuth, is ready for work. Keeping the ruler upon the occupied station on the sheet, the telescope is then turned upon other objects which it is desirable to locate, and lines are drawn in turn toward them. The instrument is then taken up and moved to a second station, where it is again set up, leveled, and oriented, as before. A sight is then taken, and a line drawn in the direction of each point sighted from the first station. The intersection of each pair of sight lines is the true position of the corresponding point on the map. In this way station after station is occupied by the plane table, and numerous points are located by intersection. If possible, each point thus located should be intersected from at least three stations, in order to verify its location.

Any point located on the map may be used afterwards as a station. For the location of points which have not been fixt many different methods have been proposed, all of which are graphic solutions of the three-point problem. Probably the simplest and easiest of application is the following, which requires that three located points be visible, and that their distances be approximately known:

Set up, level, and clamp the instrument in level and azimuth, after orienting it as nearly as possible. Set the alidade successively on the three points and draw lines from their platted positions. Then the desired location will be found at a distance from each of these lines, measured at right angles to it, proportional to the distance on the ground between the point occupied and the known point. Thus the point of intersection of these lines at right angles to the lines of sight is the desired position. Were the distances from the known points to the occupied point accurately known, its position on the paper would be obtained at the first trial. Practically, however, two or three approximations are commonly found necessary.

Another method often used is as follows:

Fasten upon the plane-table board, which necessarily has not yet been oriented, a piece of tracing linen. Assume a point on this linen to represent the station, take sights upon and draw lines to all located points within the range of vision, and then, loosening the linen from the board, move it about over the map until these sight lines fall upon the proper points on the map. Then prick thru the position of the station from the linen to the map underneath. This location should then be tested by sighting from the point thus found to the various objects, to see if the sight lines fall upon the points as marked on the map. This method has the advantage of permitting the use of all located points that may be in sight, but it is not regarded as so accurate as the method first described.

In case one line of sight upon the required station has been obtained, that sight line may be utilized in making the location as follows, by resection: Having leveled the table, place the alidade upon this sight line already drawn, with the telescope pointing toward the object from which the sight was taken. Then turn the table in azimuth until the telescope falls upon this point, and clamp it. The table is now oriented, but the position of the present station is unknown further than that it is on this line. Then select some station whose direction makes a wide angle with this line, and move the alidade until the cross wire falls upon this selected station, while the ruler at the same time is upon the representation of the station on the map. The ruler will then cross the sight line at the point desired. By way of check, repeat the process with another station or located point. For this purpose a point in a suitable direction is valuable in proportion to its proximity.

Using the instrument as described above, the topographer locates all possible points. Then visiting in turn such of them as he finds necessary, perhaps a dozen or twenty, he locates by intersection points all over the sheet in as great number and as well distributed as possible, and with special reference to the needs of the traverse men, who will come after him and whose work will be located by means of his determinations. All this work must be done with the utmost nicety and precision. The setting of the alidade upon the station must bisect the needle hole by which it is marked and the lines of direction must be drawn with a sharp-pointed pencil.

The necessity for precision will be recognized when it is understood that any error introduced in the early part of the plane-table triangulation will be not only perpetuated but increased many times over as the work progresses, and as soon as an error becomes appreciable it produces difficulties and uncertainties in making locations, which may lead to embarrassing delays and ultimately require that all the work be repeated.

## MEASUREMENT OF ALTITUDES.

While making horizontal locations of points with the plane table, their heights also, relative to that of the point occupied, must be measured. This is done by means of the vertical arc of the alidade and the level on the telescope. Pointing upon the object whose relative height is to be measured, the telescope must first be brought to a horizontal position and the index error read. The telescope is then raised or deprest to the point and the reading obtained. This reading of the index error must be done for each point, as the table can not be leveled with sufficient accuracy and can not be expected to maintain its level so as to dispense with it. Knowing the horizontal distance to the point and the angle of elevation and depression, the difference in height is obtained by the solution of a right-angled triangle, thus:

$$h = d \tan a,$$

$h$ being the difference in height, $d$ the distance, and $a$ the angle of elevation or depression. This distance is then to be corrected for curvature of the earth and for refraction by the atmosphere. The correction for the former is obtained with sufficient accuracy by the following empirical rule: The curvature in feet equals two-thirds the square of the distance in miles. It is always positive in sign, whatever may be the sign of the difference in height.

Refraction is an uncertain and variable quantity. It is usually greatest at morning and night and least at midday. It is greater the nearer the line of sight is to the ground. Often in desert regions it is excessive. It is usually assumed as one-seventh the curvature and is negative.

Tables for the solution of vertical-angle work are used. These give differences in height for all angles and distances which should be employed, with corrections for curvature and refraction.

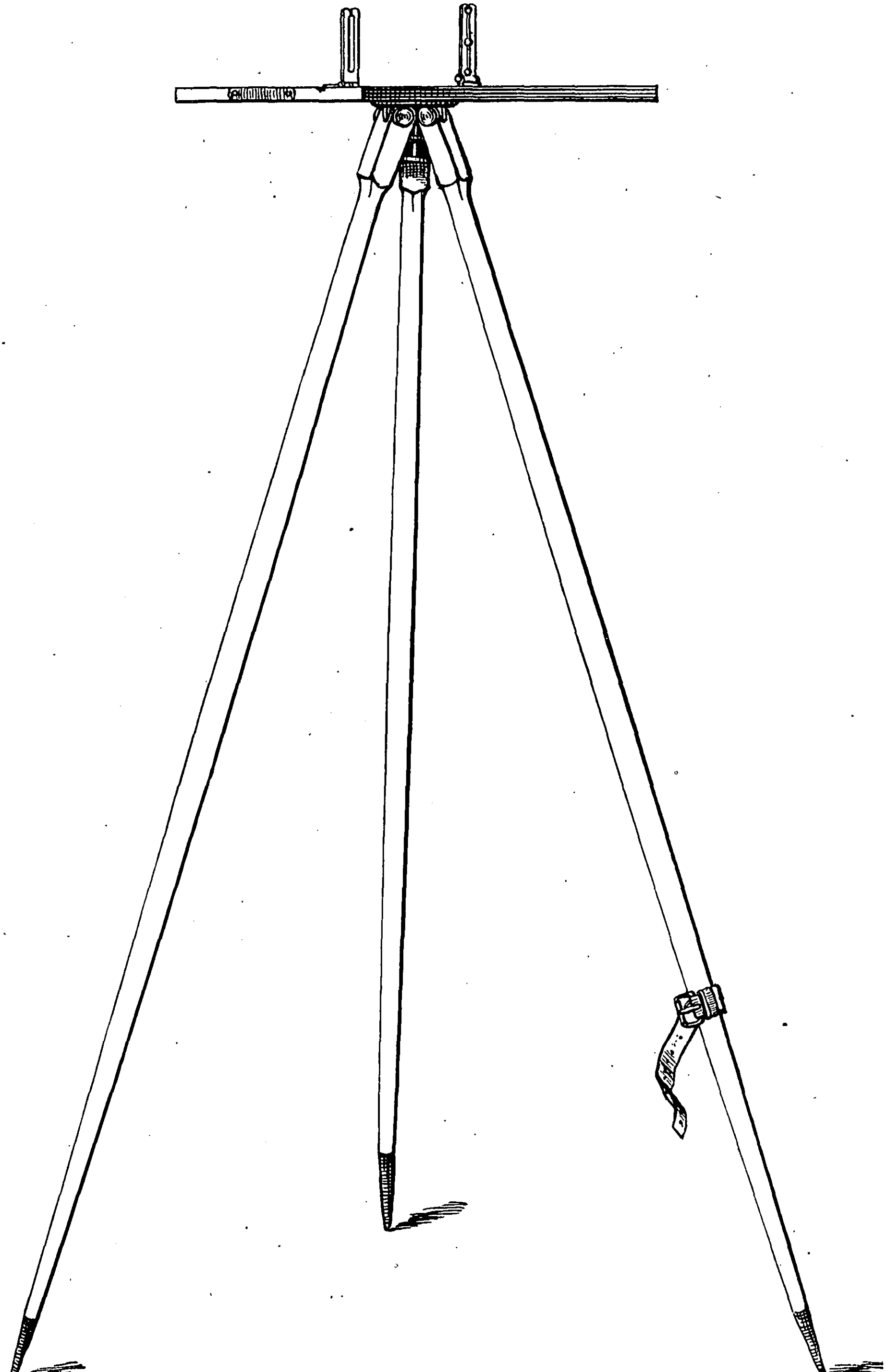
Differences of height should not be measured at greater distances than 10 miles if it can be avoided. An error of 1′ in the measurement of the angle is at this distance about 15 feet, while the uncertainty of refraction in such a length of line is necessarily great.

## TRAVERSE WORK.

As stated under the head of traverses for primary control (p. 67), a traverse line consists of a series of direction and distance measurements depending upon one another. These lines should be connected wherever possible with triangulation points in order to check up accumulated errors. If it were practicable or economical to carry on all the work of location by intersection, this would be the most accurate and on most accounts the best way to effect it, but it is only in limited localities, such as high-mountain regions where bold topographic forms predominate and where there is little or no culture, that the method of intersection is practicable for locating all necessary points. It is probable that in nine-tenths of the area of the United States it will be found necessary to locate the details of topography, culture, and drainage by means of traverse lines. In different parts of the country the relative extent to which the two methods can be applied depends upon various circumstances, principally the amount of relief of the surface and the prevalence of forests. Thus on the Atlantic plain, which is densely covered with forest and is practically level, it is necessary to use the traverse method exclusively, including even the primary control. Passing from this as an extreme case, thru rolling and hilly country, to the high, sharp mountains of the West, the triangulation method becomes more and more applicable, while the traverse method finally becomes used but little except in the details of roads and other cultural features.

For executing traverse work various instruments have been in use for measuring directions and distances. For direction there have been used theodolites of various forms and prismatic compasses, and for distances the stadia and the wheel.

All secondary traverse work should be done with plane tables upon which the directions and distances are platted directly. The Johnson plane table, with telescopic alidade, may be used for this work. When so used, a compass should be set on the edge of the board or of the alidade, for orienting, as orientation by backsighting is not sufficiently accurate.



TRAVERSE PLANE TABLE AND RULER ALIDADE.

### TRAVERSE PLANE TABLE.

Another plane table convenient for this purpose is of the simplest possible form, consisting of a board about 15 inches square, into one edge of which is set a narrow box containing a compass needle 3 inches in length. (See Pl. VIII.) This table is supported by a tripod of light construction, without leveling apparatus, the leveling of the instrument being effected with sufficient accuracy by the tripod legs. A single screw fastens the board to the tripod head, and the adjustment in azimuth is made by simply turning the board with the hand. It is held in place by friction. The table is adjusted in azimuth, or oriented, by means of the compass needle; that is, it is turned until the needle rests opposite the zero marks in the compass box, and is thus always made approximately parallel to itself, provided the magnetic declination remains constant.

The alidade consists of a brass ruler, 6 to 12 inches long, with folding sights. The edge of the ruler is graduated to facilitate platting of distances. Ordinary drawing paper backed with cloth is used for plane-table sheets, and is attached to the board by thumb tacks.

Distances may be measured by stadia, by tape, or by counting the revolutions of a carriage wheel. When traversing is done along roads, as is commonly the case, the distance is frequently measured by counting the revolutions of a wheel, usually one of the front wheels of a buggy or buckboard. For counting the revolutions, various automatic devices have been in use.

A traverseman is generally assigned a tract of country within which he is instructed to run traverses of all public roads and of such private roads as appear to be necessary in order to control the entire tract. If practicable, he is furnished with the positions of the points located within his tract, properly platted on his plane-table sheet, or, if these can not be furnished, with such descriptions of them as are necessary to enable him to recognize them and close his lines upon them or connect with them by triangulation. He is furnished with a horse and buggy or buckboard, traverse plane table, and aneroid. He has no rodman, but is expected to sight natural objects. Setting up his instrument at his initial station, he levels it roughly by means of the tripod legs, orients it by turning the table until the compass needle is on the zero marks in the compass box, and then, marking a point on the paper to represent his initial station and placing his alidade upon it, he points it to an object selected as his second station and draws a line in that direction. Driving along the road, he passes the point sighted at, noting the distance to it by a count of the revolutions of the wheel, and the height as recorded by the aneroid, and passes on, selecting some point from which he can see the point sighted at. There he stops, sets up his table as before, orients it, and sights upon the same signal which he sighted from his initial station. He plots the distance to the signal along the sight line from his initial station; then from the location of the signal as thus established he plots his second station by the distance measurement and the reverse of the observed direction. In this way the work progresses, a hundred stations or more being occupied in the course of the day. In this work one should aim to make as few stations and to take as long sights as possible consistent with accuracy. Bends of the road between stations can be sketched with all needful accuracy.

During the progress of the work all points off the line which are capable of being located by intersection must be located by sights taken from stations, and special care must be taken to connect them with the points located by the secondary triangulation, in order to afford as many checks as possible to the accuracy of the traverse line.

Traverse lines should close with but trifling error—1 per cent being as great an error as should be permitted—and all errors of closure should be shown. No line should be arbitrarily closed on the traverse sheet.

The traverseman should sketch or locate all country houses, and should note all road intersections and all railroad crossings, specifying by simple conventions whether over, under, or grade crossing. He should similarly describe all stream crossings, distinguishing fords, ferries, and bridges.

## MEASUREMENTS OF ALTITUDES IN CONNECTION WITH TRAVERSE WORK.

### VERTICAL ANGLES.

Height measurements in connection with traverse lines are effected in one of two ways—either by vertical angles with the telescopic alidade or by the use of the aneroid.

In regions where little or no secondary triangulation can be done, it becomes necessary to accompany certain of the traverse lines by profiles determined by vertical angles. Such profiles should be surveyed at intervals of 4 or 5 miles where the contour interval is 20 feet, and at intervals of 8 or 10 miles where it is 50 feet.

The plan of the traverse is run precisely as above sketched, except that often a rodman is employed. In running the profile, which is done coincidently with the plan, the points sighted for elevation may be the same as those used for the plan. If a rodman is employed, the target on the rod should be set at the height of the instrument, to simplify record and computation.

It must not be understood, however, that it is at all necessary that the survey of the profile should establish the height of all the points located by the traverse. The profile should give the elevations of all valleys and summits and of all road crossings. The line should be carried forward and these points measured by as few and as long lines of sight as possible. Often the roof of a house will furnish a datum point for use for a mile or two. Indeed, in an open, settled country the line can frequently be carried forward continuously by using housetops as targets.

The reduction of the profile must keep pace with the field work, so that on arriving at a check point the amount of the error may be shown at once. If this is not more than one-fourth or one-fifth of the contour interval, it is not considered as of material account. If, however, it reaches half a contour interval, the work should be examined, and if the error be not discovered the line should be resurveyed.

The heights as determined should be written in ink on the plane-table sheet in their proper places.

### THE ANEROID.

In some traverse work heights are measured with aneroids. The aneroid consists of a vacuum box of thin corrugated metal, which is comprest by an increase and expanded by a decrease in the pressure

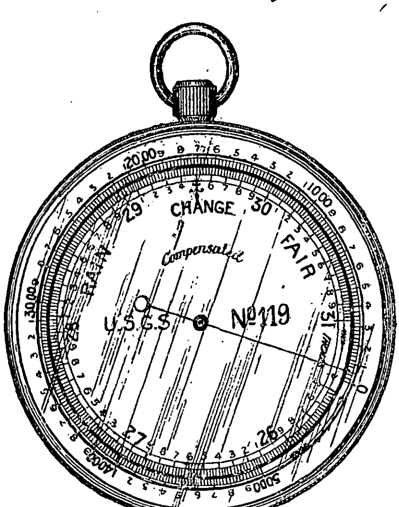



FIG. 3.—Aneroid.

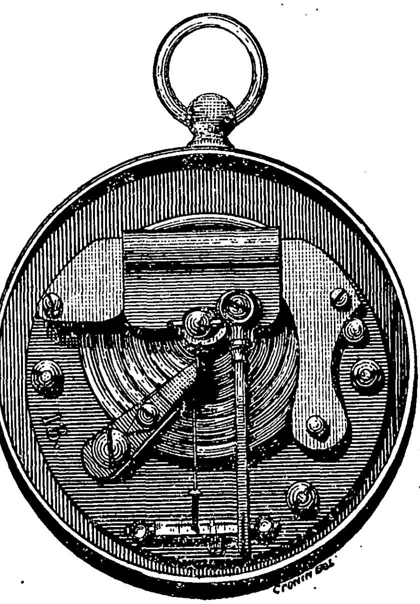


FIG. 4.—Works of the aneroid.

of the atmosphere. A train of mechanism magnifies this trifling movement enormously and moves an index upon a graduated dial. This dial is graduated to feet of elevation and also to inches of barometric pressure. (See figs. 3 and 4.) Several sizes of aneroids are made; that having a diameter of $2\frac{1}{2}$ inches is on the whole found the most satisfactory.

Owing mainly to its extreme delicacy the aneroid is a very uncertain instrument. It should be used differentially only, and for small differences in height and small intervals of time. Its indications should be checked by reference to known elevations whenever opportunity is afforded during the day, and at the beginning and end of each day's work.

On commencing work the movable scale on the aneroid should be set at the known height of the starting point and a note made of its reading on the inch scale. Elevations should then be read directly from the scale of feet. The heights of all points along the line of traverse that will be required in making the contour sketch should be read and written upon the traverse. Every depression and elevation, road crossing, etc., should thus be measured. There is, however, no necessity for reading the aneroid at every station in the traverse; that would merely encumber the work with a mass of useless data.

Upon reaching a check point, comparison should be made with the indications of the aneroid. If the difference is considerable—more than a contour interval—the error should be distributed backward along the line in proportion to distance. If it is small, it may be neglected.

In all this work notebooks are not required except as a convenient form of carrying paper on which to make the trifling computations required. The plane-table sheets comprize all the records necessary. The work, as it progresses, criticizes itself by its closures in position and elevation, and wherever necessary is revised immediately.

## ORGANIZATION OF PARTIES AND DISTRIBUTION OF WORK.

Secondary triangulation, traversing, measuring of heights, and sketching are commonly carried on by one party, consisting of the chief, who directs all operations and does most of the sketching; an assistant who carries on the secondary triangulation, selected as possessing special fitness for that work; and one, two, or more assistants who are engaged in traversing, the number depending upon the rapidity with which the country can be sketched relative to the rate at which the traversing progresses. If possible, the different items of work of such a party should follow one another in a certain order. The secondary triangulation should be done first, in order that the traversemen may be furnished with positions and heights for locating and checking their traverse lines. The traversing should follow, in order that all the control may be furnished to the chief of party for his use in sketching. This order requires that the members of the party be scattered over a considerable area, and if they are living in camp it necessitates their remaining away from it a considerable part of the time, or else that a large amount of traveling be done in order to reach camp at night. Where they are not living in camp, the most economical disposition is to scatter them at various places within their fields of work. In any case constant communication must be had between the chief of party and his assistants, in order that they may work in accord.

## STADIA MEASUREMENTS.

Under certain circumstances it is found advisable to use the stadia method for measuring distances in place of the wheel. This is the case where lines are to be run without reference to roads, and consequently where the wheel can not be employed with advantage. It has been used, too, in southern Louisiana, where peculiar methods of work imposed by the nature of the topography have made its employment economical. Any instrument carrying a telescope may be used for the stadia or telemeter method of measuring distances. To the reticule of the telescope are added two or more fixt horizontal wires placed at certain distances apart. A rod or board subdivided to suit the interval between the wires and painted in glaring colors forms part of the outfit. When this rod is set up at a distance from the telescope, that distance is ascertained from the number of subdivisions of the rod which are included between the wires of the telescope, the value of each division of the rod being known. All alidades are equipped with stadia wires. These wires are three in number, the intervals between them being equal. The rods are 14 feet in length and hinged so as to close to 7 feet. The intervals on the rods are 1 foot each. For convenience, the wires in the telescope should be so spaced that when the rod is at a distance of 100 feet the space between the two extreme wires will subtend 1 foot on the rod. At a distance of 1,400 feet, therefore, this space will subtend the entire length of the rod, while at a distance of 2,800 feet two adjacent wires in the telescope will subtend the entire length of the rod. Distances less than 100 feet are estimated by means of the fractional part of a foot on the rod which is included between the wires. The distances are read off on the rod by the surveyor at the instrument.

In measuring distance on slopes correction must be made to reduce the inclination measured to horizontal distance. Where the slope is slight, it is not regarded as necessary to make this reduction, especially where there are frequent points for checking and correcting the line.

The rod may be used also for measurement of the profile of the traverse line. For this purpose a point should be marked upon it at the same height as the telescope of the instrument and vertical angles taken to this point.

The foregoing matter applies most directly to maps on scales ranging from 1 to 8 miles to an inch and covering large areas of country. It applies also, tho less fully and directly, to maps on larger scales and those made for specific purposes. Among the latter class may be mentioned maps of mining districts, of watersheds, for drainage purposes or water supply, of railroad and canal lines, of parks, town sites, etc. In making such maps, whose scales are exprest in hundreds of feet to an inch instead of miles to an inch, we deal with inches instead of feet in both horizontal and vertical measurements, and consequently greater accuracy must be obtained as exprest in units of the ground surveyed, but in units of the map the standard of accuracy is unchanged. An error of one-fiftieth of an inch on the map is as large as should be permitted. The enlarged scale and reduced area under survey permits the use of certain instruments which are not economical for smaller scales, and also permits more extended use of certain others. The use of the stadia instrument in connection with the plane table here becomes economical, and it should be used extensively. After executing a primary triangulation the work should be relegated to the plane table, with which such secondary triangulation may be effected as seems expedient. Then the stadia comes into play, and, using one, two, or more rodmen, as may be economical, the topographer will locate points by direction and distance measurements from his station, reaching out perhaps one-fourth of a mile in all directions and measuring heights by vertical angles on the rod at the same time that he locates the horizontal positions of the points. In this way he will locate points which control the contour or points on the contours directly, as seems expedient. If the country is nearly level, it may be best to do the latter, but if there is much relief he will find it more expedient to locate only controlling points and to sketch the contours with reference to them.

From one station he will pass to another, and so cover the country mainly by stadia locations.

This is by far the most accurate, thoro, and rapid method of making detailed maps of small areas.

In making a location map of a railroad or a canal line, the method is necessarily varied to some extent. The line itself is measured with accuracy with a tape, and should be platted on the plane table as measured, and will thus serve as a continuous base line. The topography of either side should be taken by stadia, two rodmen being employed, one on either side of the line.

# CHAPTER VI.

## SKETCHING, OFFICE WORK, AND PROJECTIONS.

### SKETCHING.

Since sketching is by far the most important part of the work of map making, it should be done by the most competent man for this work in the party—as a rule, by its chief. Besides the fact that he is presumably the best sketcher in the party, there is another reason for requiring that he should execute the sketching—he is held responsible for the work, not only for the quality of the sketching but also for the accuracy and sufficiency of the control, and in the sketching of the map he has the best possible opportunity for examining into the condition of the control and of remedying any weaknesses.

Upon completing the secondary triangulation, the traverse work, and the measurement of heights—in short, the control—within an area, which may be large or small according to convenience, but preferably should comprize a quarter sheet, he should cause all this control to be assembled on one sheet. The traverse lines with all points located from them should be adjusted to the secondary locations, and all measurements of height should be platted upon this skeleton, thus presenting in complete form all the control within the area. With this sheet upon a sketching board the chief of party should go over the ground, sketching the drainage, culture, and forms of relief, and generalizing the features to suit the scale of his map. The relief should be sketched in actual continuous contours directly from the country as copy, so that upon leaving the sketching stations the only work remaining to complete the map will be inking and lettering. In heavy country, however, where the contours follow one another closely, it will often be sufficient to put in at the stations only a part of the contours—every fifth one, for instance—in order to economize time in the field. Stations for sketching may be selected with the utmost freedom. An exact location is unnecessary. Any point on or off the road which affords an outlook will serve. As a rule, frequent stations should be made, and one should not attempt to sketch at great distance unless the conditions are favorable, as in a country of large, bold features. It may be necessary to travel over all the roads that have been traversed and to climb many hills in order to sketch the entire area satisfactorily. On the other hand, in a different region the entire area may be sketched by a limited amount of travel or from a few elevated points. In a low country of small features much travel will be required, as these details must be sketched from near points. In a bold country of high relief, which may be sketched entirely from a few points, care must be exercised in the selection of sketching stations. From a great altitude the lower details will be dwarfed and will measurably disappear, while from low points the relations, forms, and masses of the greater elevations can not be properly seen. In such a country stations at different elevations must be selected in order to see all parts of the country to the best advantage. The extreme summits will prove of little service as sketching stations.

Sketching is artistic work. The power of seeing topographic forms in their proper shapes and proportions and of faithfully transferring these impressions to paper is one of the most difficult acquirements to obtain. The difficulty is increased by the necessity of expressing form by means of continuous contour lines at fixt intervals. This work involves knowledge of the elements of structural geology and good judgment in applying them.

Whatever its scale may be, every map is a representation reduced from nature, and consequently there is more or less generalization. It is therefore impossible to make any map an accurate, faithful picture of the country it represents. Moreover, the smaller the scale the higher must be the degree of generalization, and the further must the map necessarily depart from the original.

Now, it is in this matter of generalization that the judgment of the topographer is most severely tested. He must be able to take a broad as well as a detailed view of the country; he must understand the meaning of its broad features, and then must be able to interpret details in the light of those features. Only such a man is competent to make just generalizations and to decide what details should be omitted and what preserved, and, where details are omitted, what to put in their places in order to bring out the dominant features.

It is not possible to define the degree of detail which maps should represent. The limit commonly given—that imposed by the scale of the map—is not always the best. In representing country which has little plan or system, such as moraines or sand dunes, it is well to work in as much detail as the scale will bear; but where the country shows system in its structure, to which the minor detail is subordinate, the omission of some of this detail will give greater prominence to the larger features. The amount of detail thus omitted must necessarily be left to the judgment of the topographer, but no more should be omitted than is necessary to give full expression to the general features of the country.

## SCALE OF FIELD WORK.

The scale on which the field work is executed is generally larger than that on which the maps are to be published, because the drawing in the field is necessarily coarser than is possible in the office. The field scale is commonly greater than the publication scale; that is, plane-table sheets which are to be published on the scale of 1:62500 are made in the field on the scale of 1:48000 and those to be published on the scale of 1:125000 are made in the field on the scale of 1:96000.

## OFFICE WORK.

The office work of the topographers consists in the reduction and transfer of the work from plane-table sheets to the original maps. The reduction is usually effected by photography, this method having been found the most accurate and economical. It is often the practise simply to complete the plane-table sheet by inking and lettering it, making it serve as the original sheet. On many accounts this practise is advisable.

The original sheets are to serve as the original record of the work and as manuscript for publication. To answer these purposes they are made complete in all respects. Every original sheet should contain within itself all matter which is to be engraved or placed on record.

The method by which the map is to be published should have much influence upon the preparation of the original map. In case it is to be photolithographed or photo-engraved, it must be drawn by a skilled draftsman on a scale much larger than that of publication. If it is to be engraved, however, it is unnecessary to waste the time of a skilled draftsman upon it, as the engraver requires merely clearness and legibility.

## PROJECTIONS.

The projection almost universally used is the polyconic.

The construction of a projection for a limited area is a simple matter, but requires care and accuracy and the use of the best drafting instruments.

Tables for this projection are published by the United States Coast and Geodetic Survey, which are applicable to all scales of map, the measurements given being the absolute lengths on the earth's surface, which must be reduced to the scale of the map. Tables are published in Bulletin No. 234 of the United States Geological Survey, which are directly applicable to certain scales without reduction.

First draw a line down the middle of the sheet. Lay off on this line the length of the several projection spaces in latitude by taking from the projection table for the scale the length of, say, 5 minutes of latitude and lay it off repeatedly, thus establishing the points of intersection of parallels at each 5 minutes with the middle meridian. Thru these points draw lines across the sheet at right angles to the middle meridian, using beam compasses for this purpose. Lay off on these lines the dm's corresponding to the latitude for 2′ 30″ and 7′ 30″ from the middle meridian on each side, and at these points erect short perpendiculars. On these lay off the dp's corresponding to the dm's, and thru the points thus obtained draw and ink the projection lines.

For other areas the process is similar, but when a large area, such as that of the United States, is to be projected the mechanical difficulties greatly increase.

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